

Axion Theory and Pheno

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From axions and ALPs to Dark Matter

J. Jaeckel*

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Durham, ^TTohoku U., ^{YY}MPI Muenchen+CERN, ^{ff}Fermilab



The strong CP Problem and the Axion solution

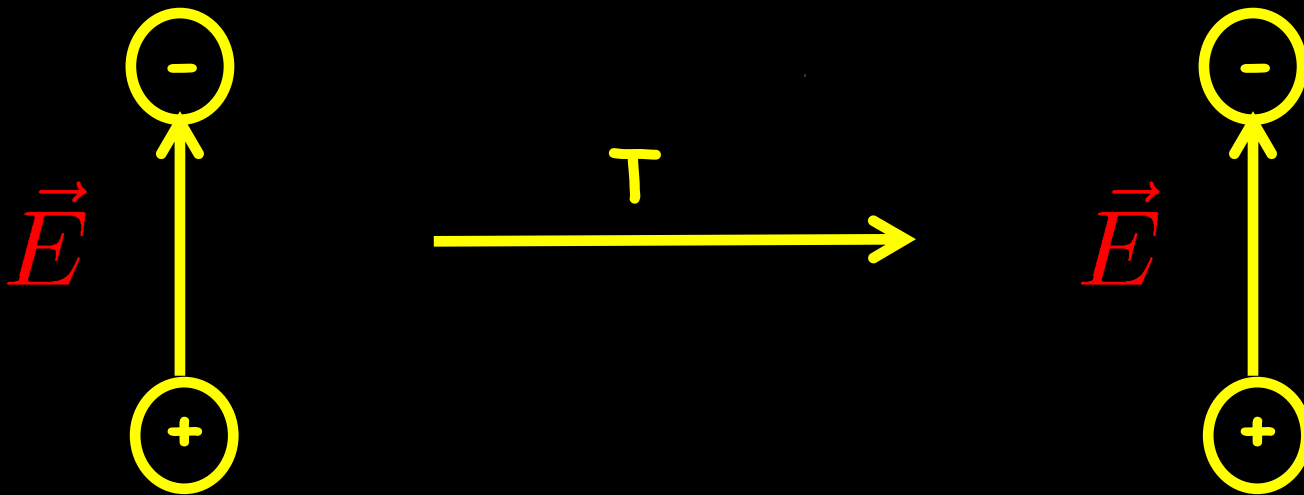
The strong CP Problem

A dirty little secret...

$$S = \int d^4x \left[-\frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{\theta}{4} G^{\mu\nu} \tilde{G}_{\mu\nu} + i\bar{\psi} D_\mu \gamma^\mu \psi + \bar{\psi} M \psi \right]$$

” $\sim \theta \vec{E} \cdot \vec{B}$ ”

- The θ -term violates time reversal (T=CP)!

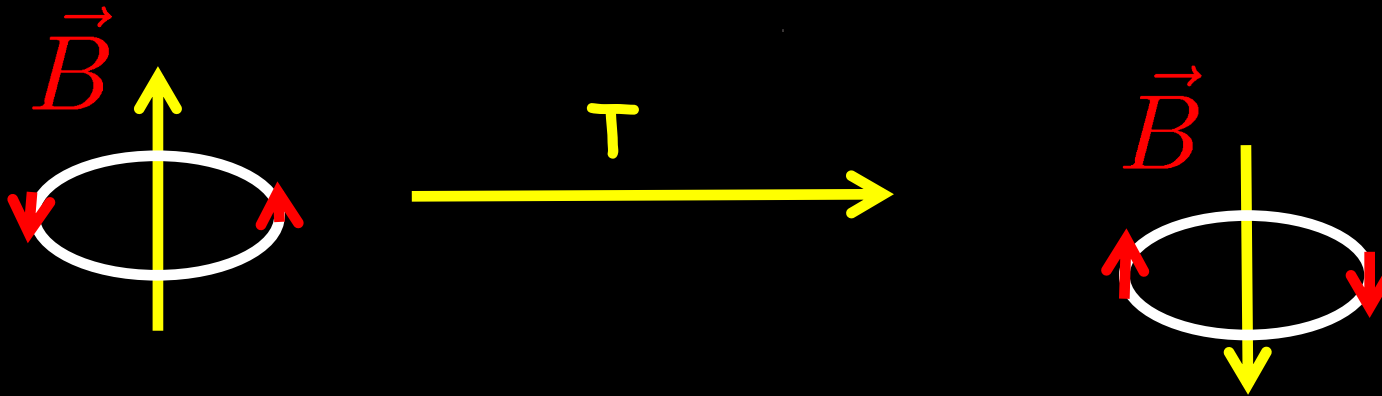


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” $\sim \theta \vec{E} \cdot \vec{B}$ ”

- The θ -term violates time reversal (T=CP)!

$$\begin{array}{ccc} & \text{T=CP} & \\ \vec{E} & \longrightarrow & \vec{E} \\ \vec{B} & \longrightarrow & -\vec{B} \\ \vec{E} \cdot \vec{B} & \longrightarrow & -\vec{E} \cdot \vec{B} \end{array}$$

A dirty little secret...

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” $\sim \theta \vec{E} \cdot \vec{B}$ ”

- The θ -term also violates parity P!

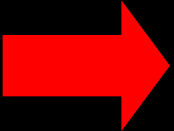
$$\begin{array}{ccc} & \text{T=CP} & \\ \vec{E} & \longrightarrow & -\vec{E} \\ \vec{B} & \longrightarrow & \vec{B} \quad \text{Cross product!} \\ \vec{E} \cdot \vec{B} & \longrightarrow & -\vec{E} \cdot \vec{B} \end{array}$$

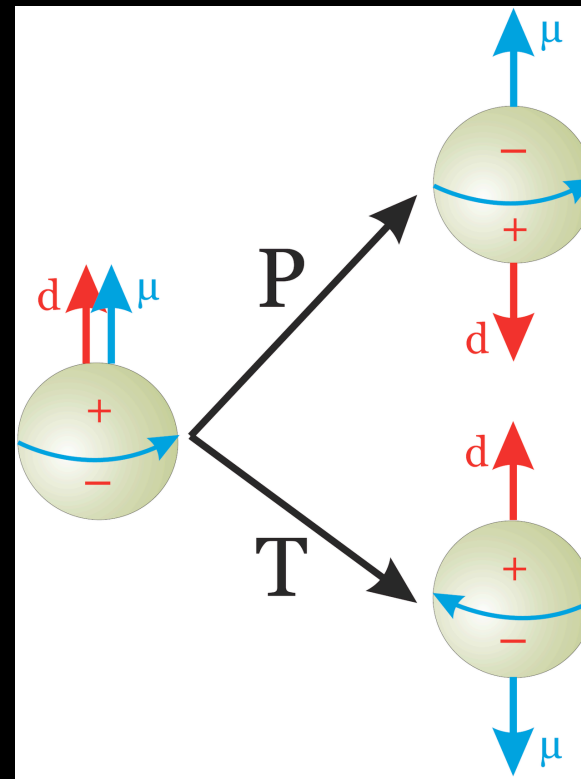
A dirty little secret...

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” $\sim \theta \vec{E} \cdot \vec{B}$ ”

- The θ -term violates time reversal ($T=CP$)!
- Connected to strong interactions!

 **Electric dipole moment of the neutron!**



Wait a minute...

- Neutron not elementary
- Does the argument still work?
- States with Spin S and Vector V

$$|\mathbf{S}, \mathbf{V}\rangle \quad |\mathbf{S}, -\mathbf{V}\rangle$$

- QM with Parity: Eigenstates

$$|\text{even}\rangle = \frac{1}{\sqrt{2}}(|\mathbf{S}, \mathbf{V}\rangle + |\mathbf{S}, -\mathbf{V}\rangle) \quad |\text{odd}\rangle = \frac{1}{\sqrt{2}}(|\mathbf{S}, \mathbf{V}\rangle - |\mathbf{S}, -\mathbf{V}\rangle)$$

Interact with electric field...

Dipole moment \sim vector

$$\mathbf{d} = cq\mathbf{V}$$

Hamiltonian

$$H = \begin{pmatrix} H_{ee} & cq\mathbf{V} \cdot \mathbf{E} \\ cq\mathbf{V} \cdot \mathbf{E} & H_{oo} \end{pmatrix}$$

Case 1

$$H_{ee} \approx H_{oo} = H$$

\rightarrow $E_{\pm} = H \pm cq\mathbf{V} \cdot \mathbf{E}$ \leftarrow Looks like an EDM!

Interact with electric field...

Dipole moment ~ vector

$$\mathbf{d} = cq\mathbf{V}$$

Hamiltonian

$$H = \begin{pmatrix} H_{ee} & cq\mathbf{V} \cdot \mathbf{E} \\ cq\mathbf{V} \cdot \mathbf{E} & H_{oo} \end{pmatrix}$$

Case 2 (applicable to neutron)

$$H_{ee} - H_{oo} = \Delta H \gg cq\mathbf{E} \cdot \mathbf{V}$$

→
$$E_1 = H_{ee} + \frac{(cq\mathbf{E} \cdot \mathbf{V})^2}{\Delta H}$$

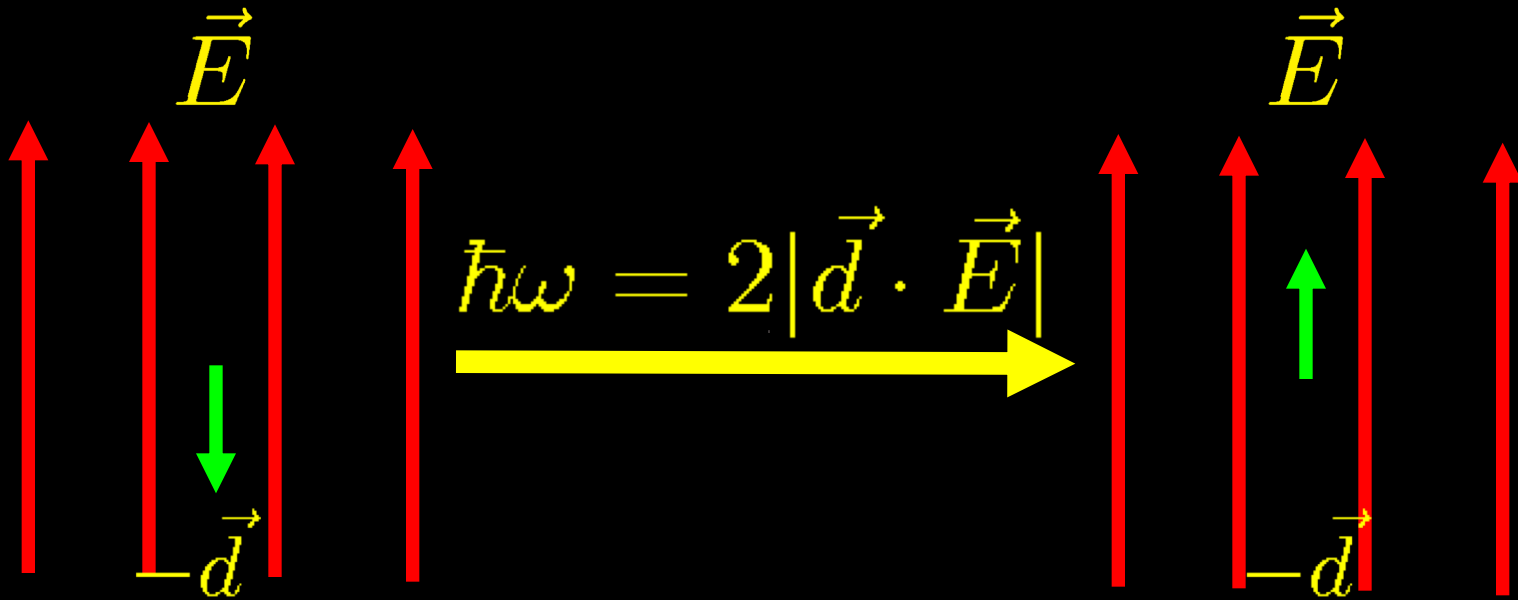
$$E_1 = H_{oo} - \frac{(cq\mathbf{E} \cdot \mathbf{V})^2}{\Delta H}$$

No linear term
No EDM

Tiny energy change

Measure neutron electric dipole moment

- θ would cause neutron EDM \longrightarrow Experiment



\longrightarrow Measure transition frequency.

No neutron electric dipole moment...

$$\begin{aligned} |\vec{d}| &\lesssim 10^{-26} e \text{ cm} \\ &= 10^{-13} e \text{ fm} \end{aligned}$$

Measurement of the Permanent Electric Dipole Moment of the Neutron

C. Abel (Sussex U.), S. Afach (PSI, Villigen and Zurich, ETH), N.J. Ayres (Sussex U. and Zurich, ETH), C.A. Baker (Rutherford), G. Ban (Caen U.) et al. (Jan 31, 2020)

Published in: *Phys.Rev.Lett.* 124 (2020) 8, 081803 • e-Print: [2001.11966](https://arxiv.org/abs/2001.11966) [hep-ex]

What do we expect?

- Two mass scales in the game:

$$m_q \sim 1 - 10 \text{ MeV}$$

$$\Lambda_{\text{QCD}} \sim 300 \text{ MeV}$$

$$d_n \sim e \times \text{length} \times \theta \sim e \times \frac{m_q}{\Lambda_{\text{QCD}}^2} \times \theta$$

$$\sim (1 - 10) \times 10^{-16} \text{ e cm } \theta$$

Wait a minute... it's a total derivative

- In Feynman rules it looks like this

$$vertex \sim \int d^4x \partial_\mu [\phi_1(x) \phi_2(x) \cdots \phi_n(x)]$$

- And in momentum space

$$vertex \sim (p_1 + \dots p_n) \delta(p_1 \dots p_n) = 0$$

➔ ????

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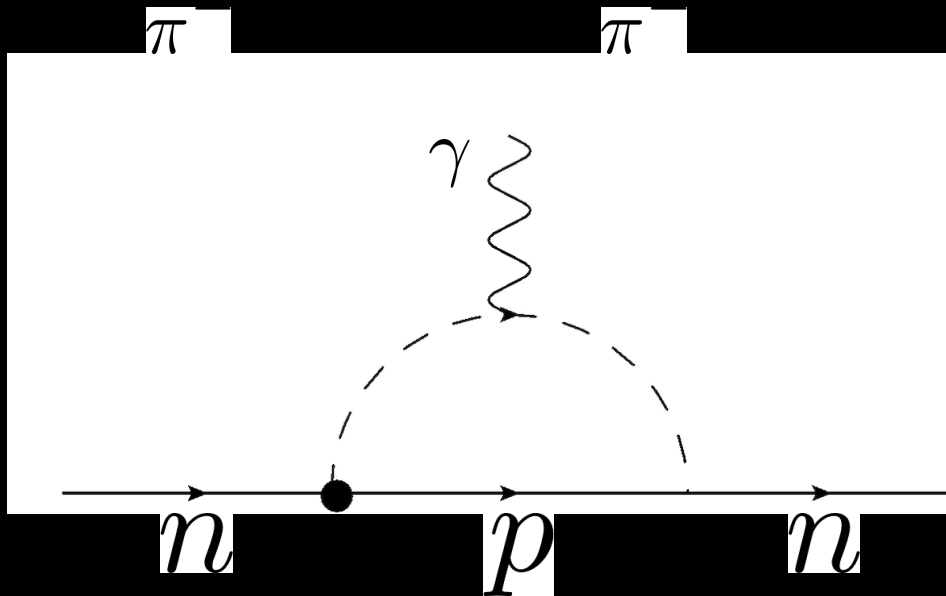
- And in momentum space

$$vertex \sim (p_1 + \dots p_n) \delta(p_1 \dots p_n) = 0$$

→ It must be something non-perturbative!

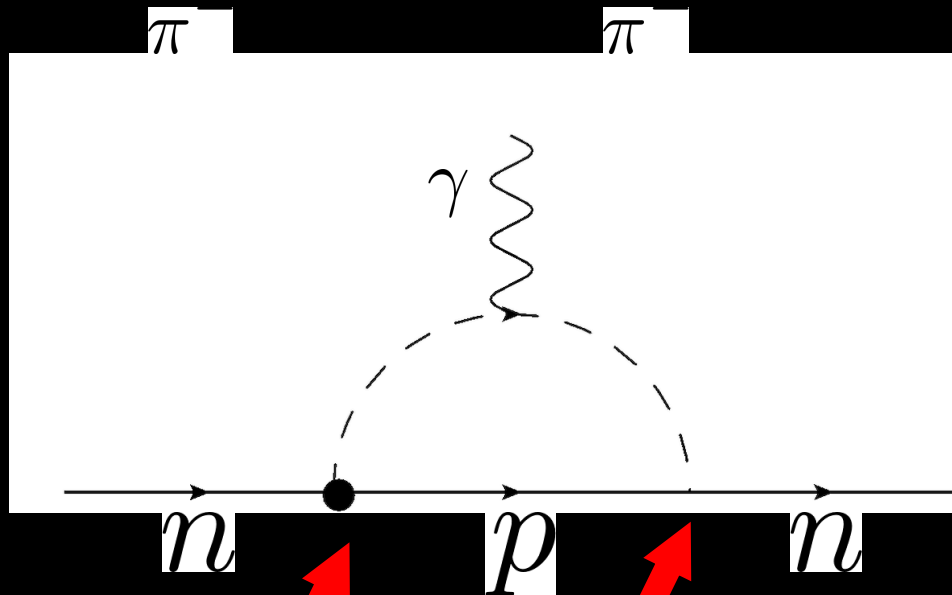
Electric dipole moment "calculation"

Take one step back and do it
in the effective theory.



Electric dipole moment "calculation"

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in the effective theory.



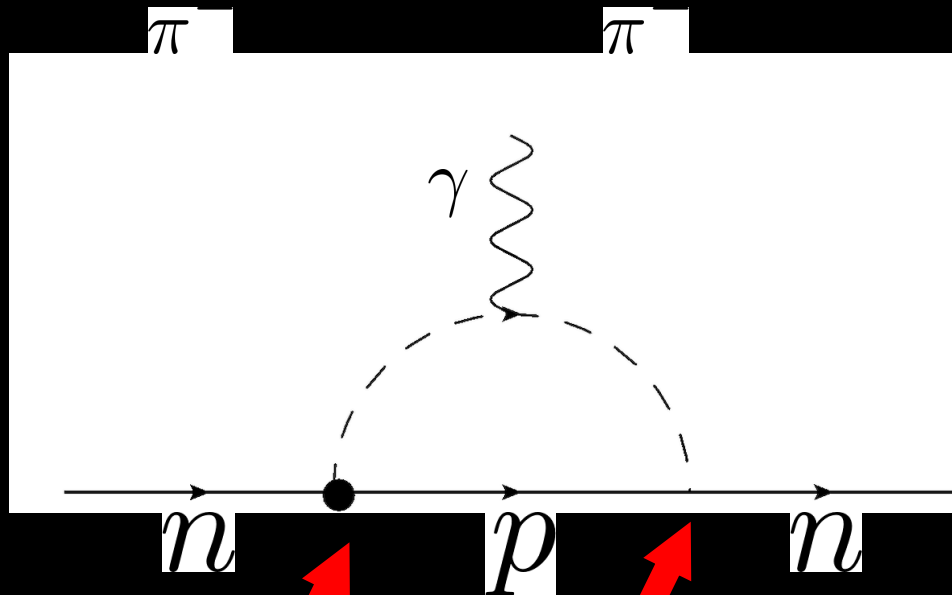
scalar

pseudoscalar

Otherwise only
magnetic dipole
CP violation needed

Electric dipole moment "calculation"

Take one step back and do it
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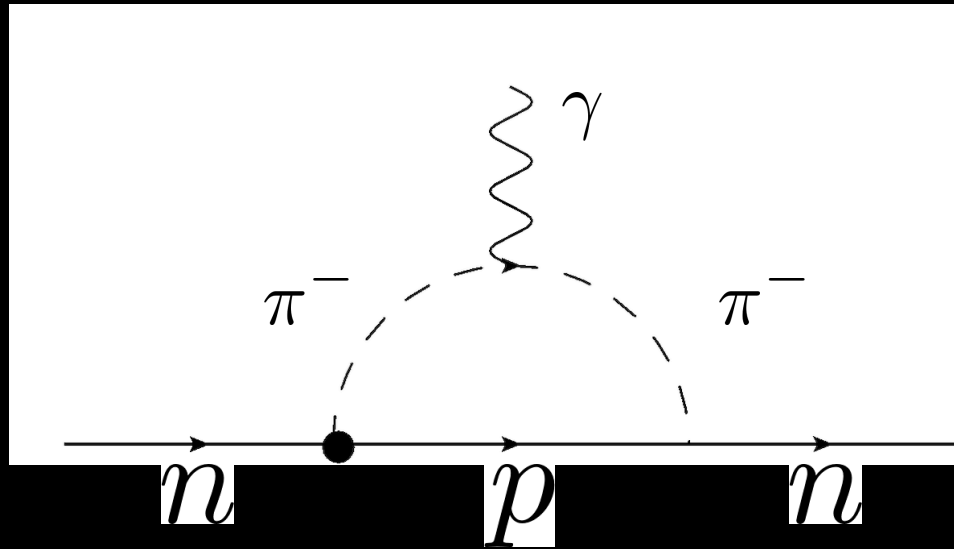


scalar
CP viol.
here

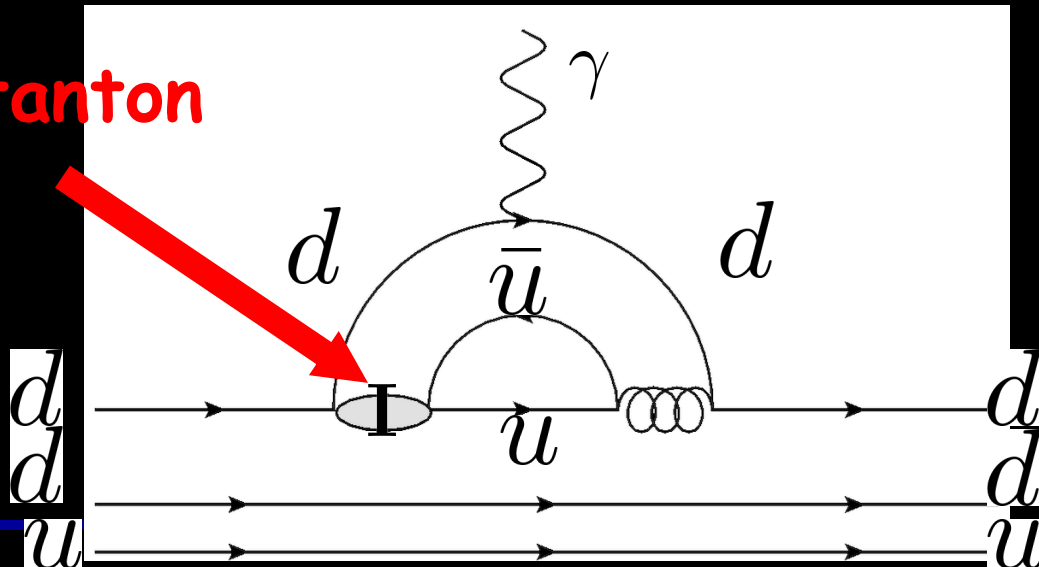
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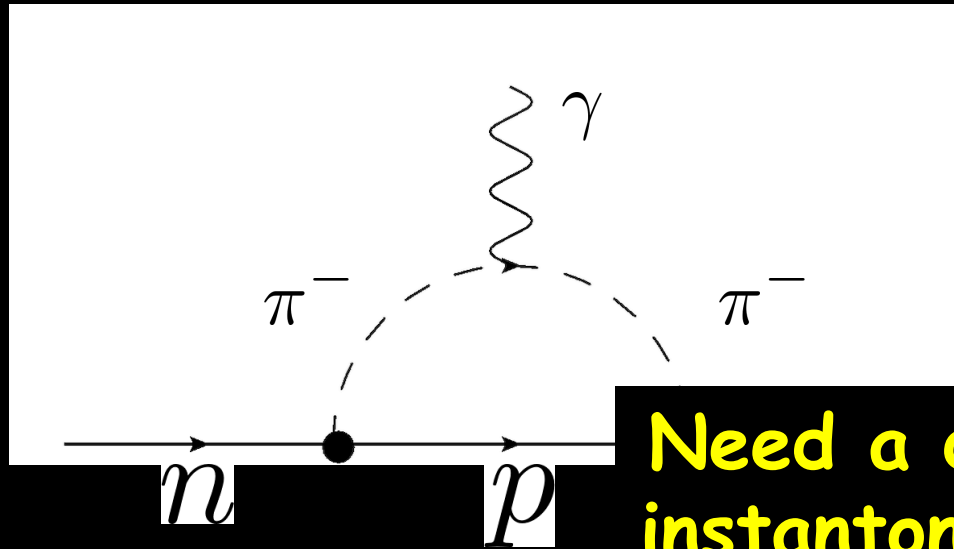
Electric dipole moment "calculation"



Instanton

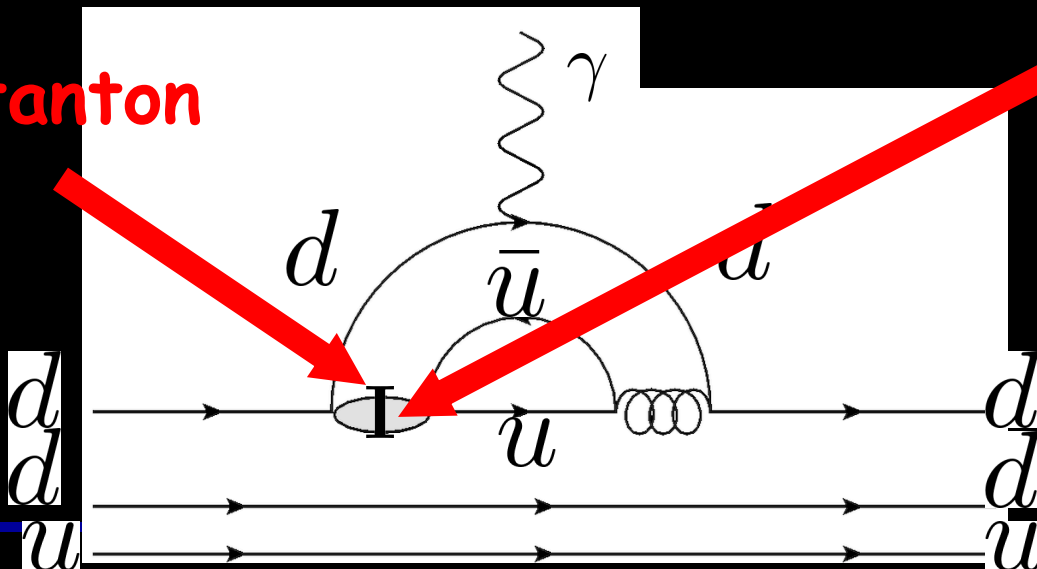


Electric dipole moment "calculation"



Need a difference between instantons and antiinstantons
→ CP viol

Instanton



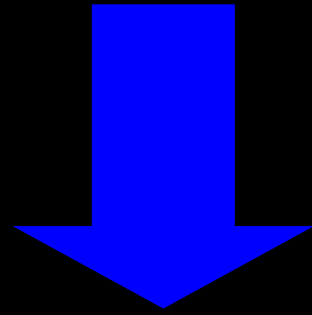
Implications

- Detailed calculation gives

$$|\vec{d}| \sim 1 - 10 \times 10^{-16} e cm \theta$$

 $|\theta| \lesssim 10^{-10}$

 **Extremely unnatural!**



Strong CP Problem

The problem is even worse...

- What we measure is actually

$$\theta_{\text{eff}} = \theta + \text{Arg Det}(M_u M_d)$$


Up and down type quark mass matrices

Changing quark mass phases

- **A fermion mass term**

$$\mathcal{L} \supset m \bar{\psi}_L \psi_R + h.c.$$

- **A chiral rotation**

$$\psi' = \exp\left(-i\frac{\beta}{2}\gamma_5\right) \psi \quad \rightarrow \quad \begin{aligned} \bar{\psi}_L &\rightarrow \exp(-i\beta/2)\bar{\psi}_L \\ \psi_R &\rightarrow \exp(-i\beta/2)\psi_R \end{aligned}$$

$$\rightarrow m \rightarrow \exp(-i\beta)m$$

\rightarrow Rotate all phases away???

Chiral rotations anomalous

- Chiral symmetry is symmetry of classical Lagrangian
 - **BUT: Not** of Quantum Theory
-

- Adler-Bell-Jackiw anomaly

$$\partial_\mu j^\mu = \frac{g^2}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

- Chiral rotations not a good symmetry: it is anomalous

$$d\mu' = \mathcal{D}\psi' \mathcal{D}\bar{\psi}' = d\mu \exp\left(-\frac{i}{4} \int_x \frac{\beta}{2} \frac{e^2}{8\pi^2} \text{Tr} F^{\mu\nu} \tilde{F}_{\mu\nu}\right)$$

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$$\rightarrow \theta \rightarrow \theta + \beta$$

Chiral rotations anomalous

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$$\psi' = \exp \left(-i \frac{\beta}{2} \gamma_5 \right) \psi$$

$$\rightarrow \theta \rightarrow \theta + \beta$$

$$\rightarrow \text{Arg}(m) + \theta \quad \text{is invariant}$$

The problem is even worse...

- What we measure is actually

$$\theta_{\text{eff}} = \theta + \text{Arg Det}(M_u M_d)$$


Up and down type quark mass matrices

They actually have $O(1)$ complex phases;
The CKM phase is $O(1)$!!

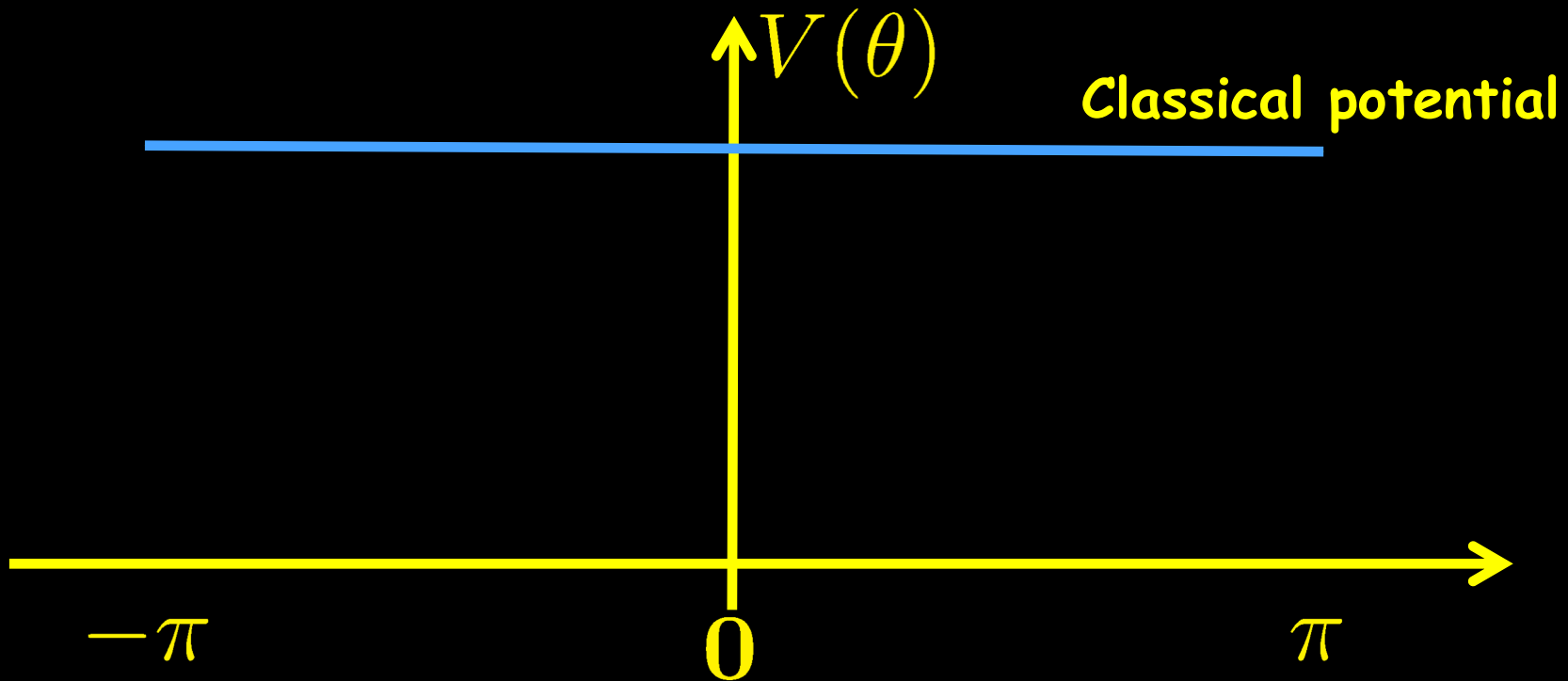
Why should this specific
combination of phases vanish???

The axion solution
to the strong CP problem

In pictures...

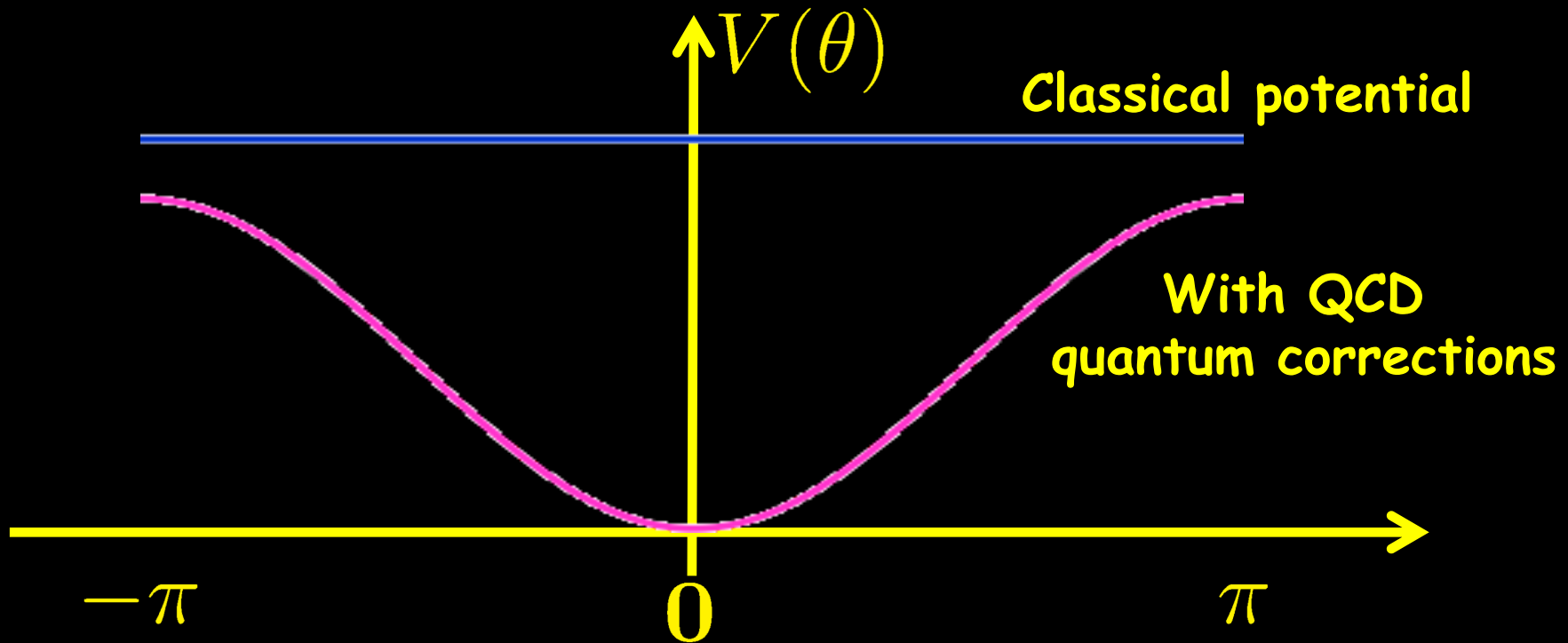
The axion solution to the strong CP problem

- Make θ dynamical \rightarrow it can change its value



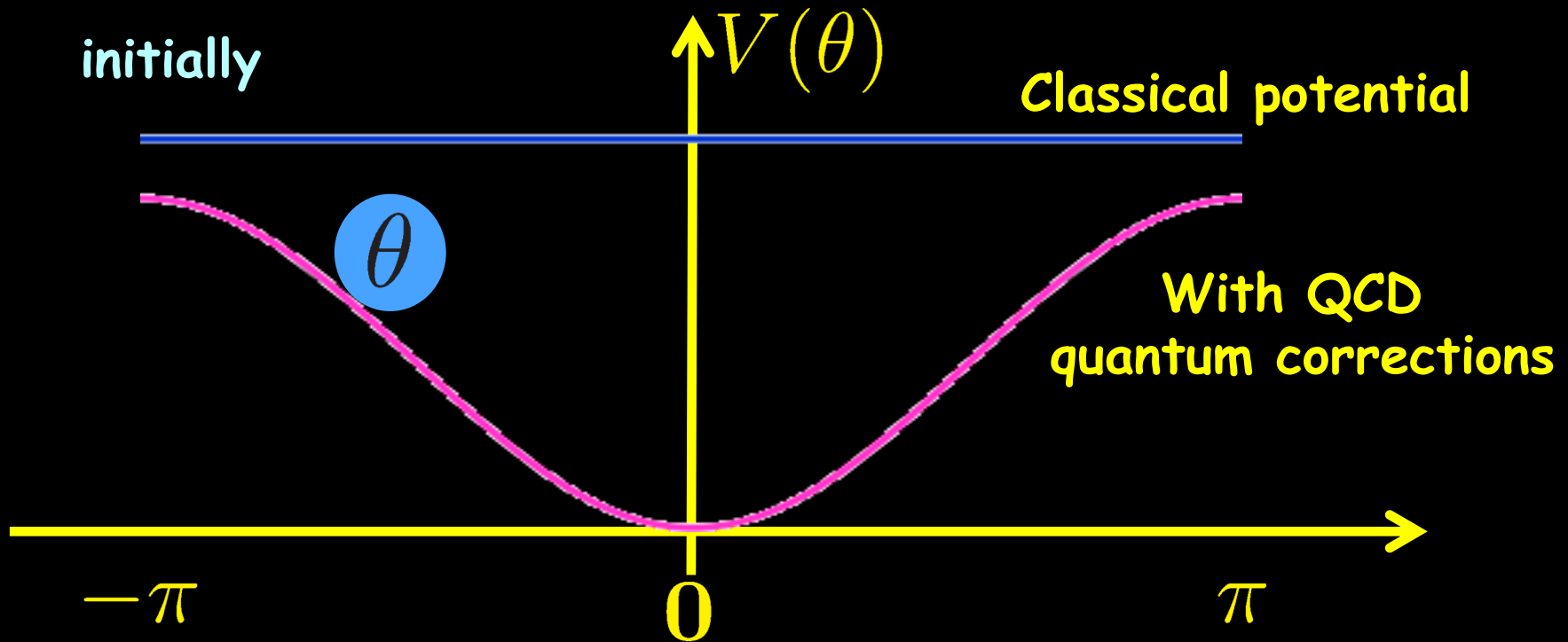
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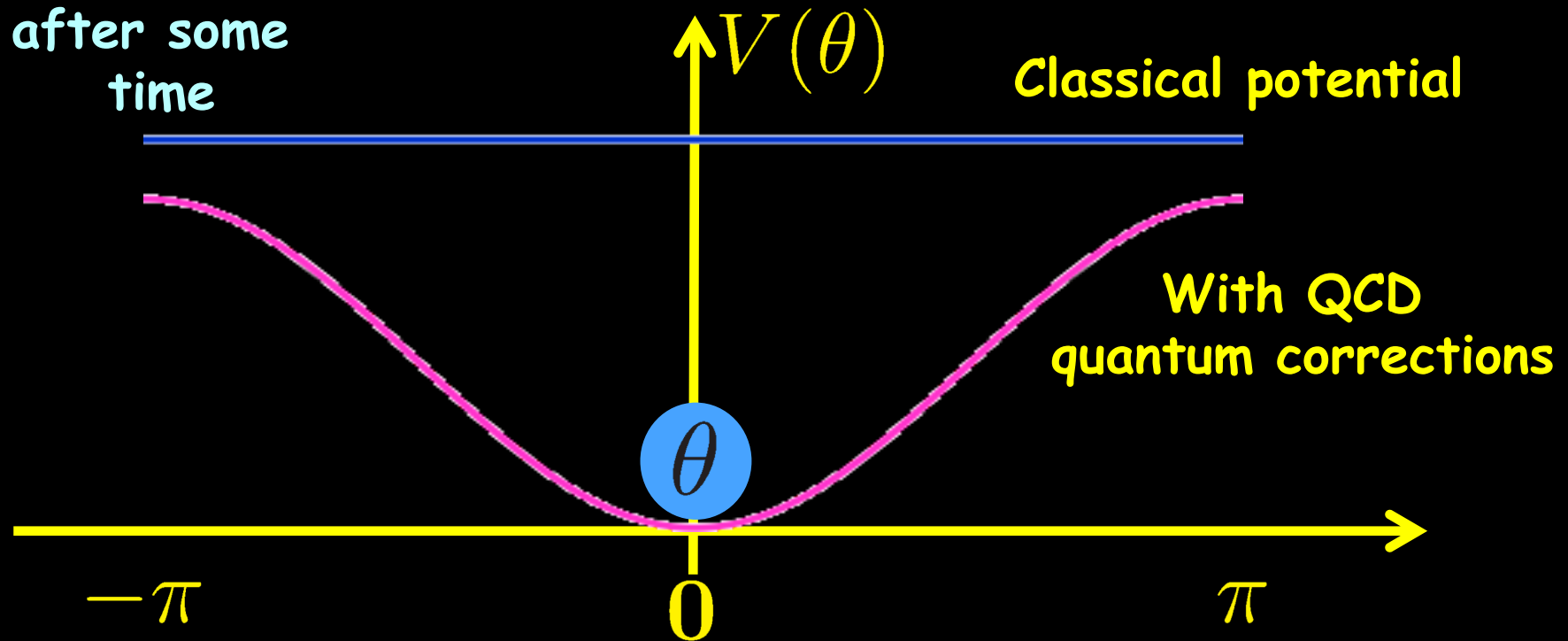
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The axion solution to the strong CP problem

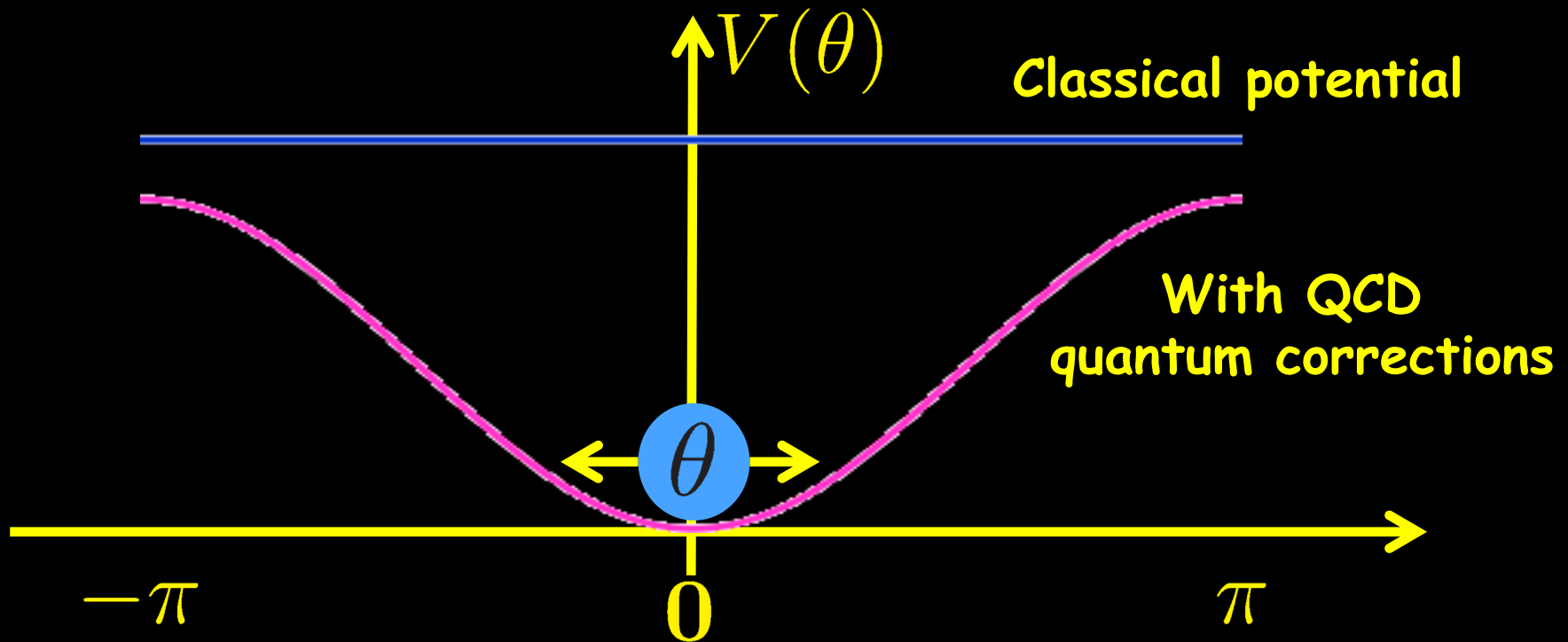
- Make θ dynamical \rightarrow it can change its value



\rightarrow QCD likes to be CP conserving (if we allow it)

The axion solution to the strong CP problem

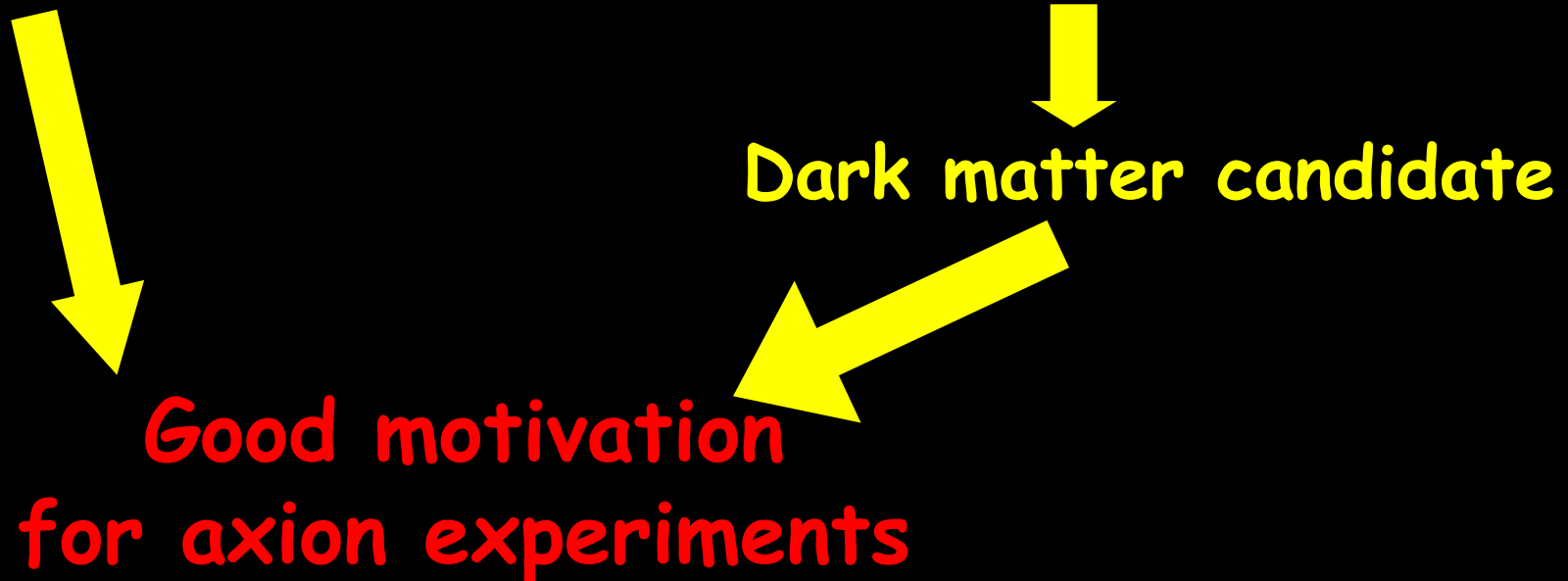
- Make θ dynamical \rightarrow it can change its value



\rightarrow Can still move

\rightarrow new particle = axion

- Classical flatness from symmetry
- Quantum corrections are small
- New **light** particle: **The Axion**
(it's a **Weakly Interacting Sub-eV Particle**)



In Equations...

A Dynamical θ

- **Idea:**
 - Make θ a dynamical degree of freedom
 - Let θ have no tree level potential
 - Let θ have only derivative couplings
- **Then:**

$$\begin{aligned}\exp\left(-\int_x V(\theta)\right) &= \left| \int \mathcal{D}A_\mu \exp(-S_{eff}[\phi, A^\mu]) \exp\left(-i\theta \frac{g^2}{32\pi^2} \int_x G^{\mu\nu} \tilde{G}_{\mu\nu}\right) \right| \\ &\leq \int \mathcal{D}A_\mu \left| \exp(-S_{eff}[\phi, A^\mu]) \exp\left(-i\theta \frac{g^2}{32\pi^2} \int_x G^{\mu\nu} \tilde{G}_{\mu\nu}\right) \right| \\ &\leq \int \mathcal{D}A_\mu \exp(-S_{eff}[\phi, A^\mu]) \\ &\leq \exp\left(-\int_x V[0]\right)\end{aligned}$$

A Dynamical θ

- Idea:
 - Make θ a dynamical degree of freedom a .
 - Let θ have no tree level potential
 - Let θ have only derivative couplings
- Canonically normalize $\theta = a/f_a$

➔ $V[a/f_a = \theta = 0] \leq V[\theta] \quad \forall \theta$

➔ $\theta = a/f_a$ will evolve to $a = \theta = 0$

➔ CP is conserved

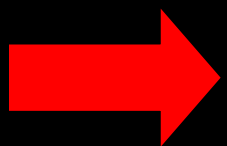
What is a?

- Properties:

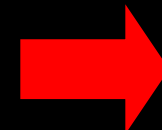
- Let a be a dynamical degree of freedom.
- Let a have no tree level potential
- Let a have only derivative couplings

- $a/f_a \in [0, 2\pi]$ since

$$\frac{g^2}{32\pi^2} \int d^4x G_{\mu\nu} \tilde{G}^{\mu\nu} = n \in \mathbb{Z}$$



a is Goldstone boson
of a $U(1)$ symmetry



Axion!

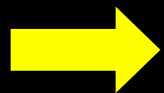
Peccei-Quinn Symmetry

- Toy model:

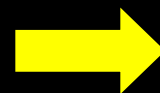
$$\mathcal{L} = -\frac{1}{4}F^2 + i\bar{\psi}D_\mu\gamma^\mu\psi - |\partial_\mu\phi|^2 - \mu^2|\phi|^2 - \lambda|\phi|^4 \\ + \bar{\psi} \left(Y\phi\frac{1+\gamma_5}{2} + Y^*\phi^*\frac{1-\gamma_5}{2} \right) \psi$$

- **U(1):** $\phi \rightarrow \exp(i\beta)\phi$
 $\psi \rightarrow \exp\left(-i\frac{\beta}{2}\gamma_5\right)\psi$

- If $\mu^2 < 0$ we have SSB



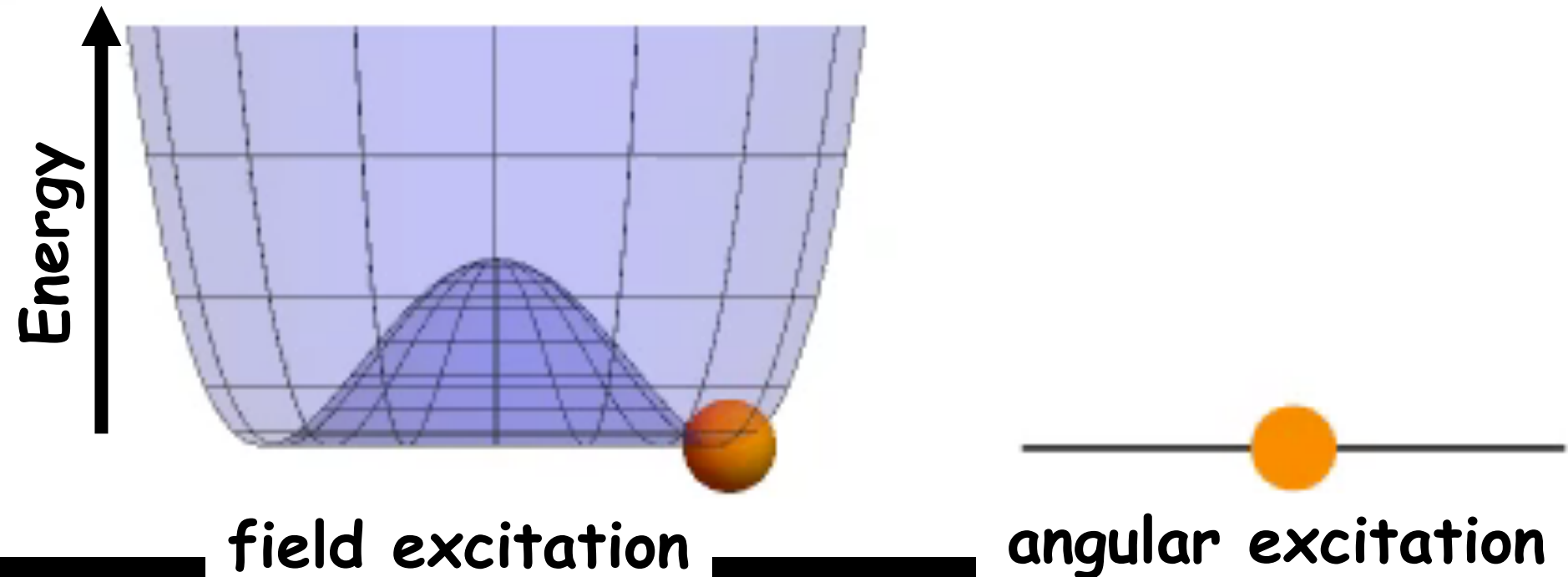
Phase is Goldstone



Use it as Axion

What is a Goldstone Boson?

- Let us start with a $U(1)$ /rotation symmetric potential



field excitation

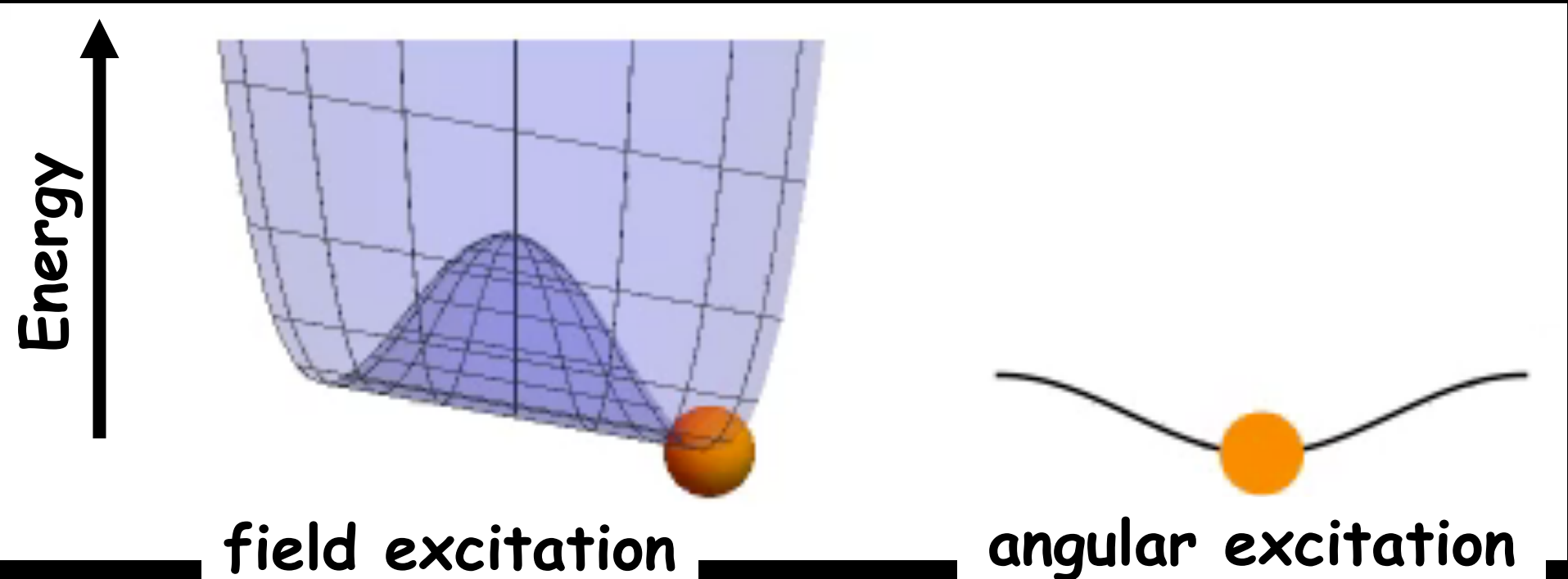
angular excitation

→ If you move along the minimum,
it costs no energy to move around

→ Particle is massless

What is a **pseudo-Goldstone Boson**?

- Add a **small breaking** of $U(1)$ /rotation symmetry

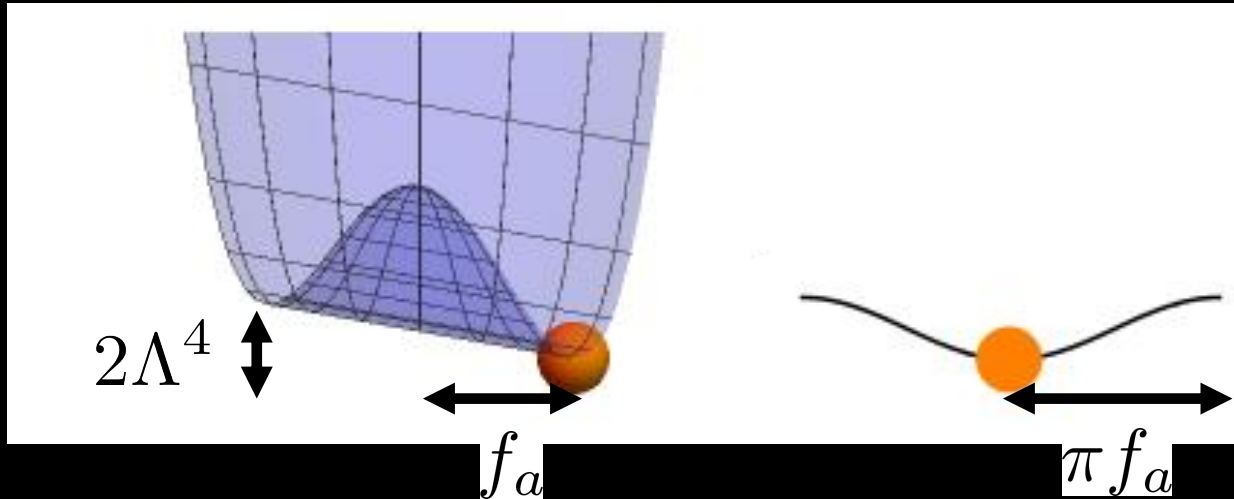


→ If you move along the minimum,
it costs a little bit of energy

→ Particle has a small mass

What is a pseudo-Goldstone Boson?

- Add a small breaking of U(1)/rotation symmetry



$$V(a) = \Lambda^4 \left[1 - \cos\left(\frac{a}{f_a}\right) \right]$$

very small

$$\text{mass}^2 = m_X^2 = V''(0) = \frac{\Lambda^4}{f_a^2}$$

small (pointing to Λ^4)
large (pointing to f_a^2)

Goldstone bosons...

Scalar part of Lagrangian

$$\mathcal{L} = -|\partial_\mu\phi|^2 - \mu^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4$$

U(1) symmetry

$$\phi \rightarrow \exp(i\alpha)\phi$$

Spontaneously broken

$$\mu^2 < 0$$

Vacuum expectation value

$$\langle|\phi|\rangle = \sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{1}{\sqrt{2}}f_a$$

Goldstone bosons...

We can always write

$$\phi = |\phi| \exp(-i\alpha(x))$$




$$\partial_\mu \phi^* \partial^\mu \phi \rightarrow (\partial_\mu |\phi|)^2 + |\phi|^2 (\partial_\mu \alpha(x))^2$$

Kinetic term



$$\alpha(x) \rightarrow \frac{a(x)}{f_a}$$

Properly normalize



$$\mathcal{L} \supset (\partial_\mu |\phi|)^2 + (\partial_\mu a)^2$$

Goldstone bosons...

- Look at potential

$$V(\phi) = V(|\phi|) = \text{independent of } a$$

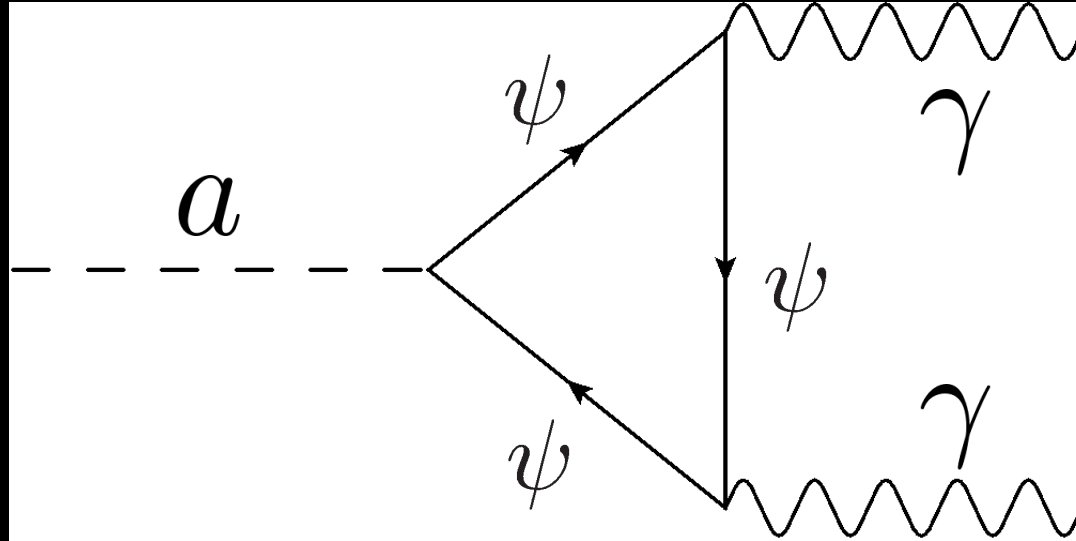
- ϕ massive
- a massless



Only interesting degree of freedom at low energies

The Coupling to $F\tilde{F}$ ($G\tilde{G}$ analog)

- A diagram



- And a dimensional argument:

$$g \sim \frac{1}{\text{mass}} \sim \frac{1}{f_a}$$

The Coupling to $F \tilde{F}$

- Adler-Bell-Jackiw anomaly

$$\partial_\mu j^\mu = \frac{g^2}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

- Chiral rotations not a good symmetry: it is anomalous

$$d\mu' = \mathcal{D}\psi' \mathcal{D}\bar{\psi}' = d\mu \exp\left(-\frac{i}{4} \int_x \frac{\beta}{2} \frac{e^2}{8\pi^2} \text{Tr} F^{\mu\nu} \tilde{F}_{\mu\nu}\right)$$

$$\psi' = \exp\left(-i\frac{\beta}{2}\gamma_5\right)\psi$$

The Coupling to $F \tilde{F}$

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$$\psi' = \exp\left(-i\frac{\beta}{2}\gamma_5\right) \psi = \frac{a}{f_a}$$


The Coupling to $F \tilde{F}$

- Adler-Bell-Jackiw anomaly

$$\partial_\mu j^\mu = \frac{g^2}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

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$$\mathcal{L} \supset -\frac{1}{4} \frac{\alpha}{4\pi f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

The mass of the Axion

- $U(1)_{PQ}$ is not exact. It's anomalous!

➡ Goldstone ➡ Pseudogoldstone

- Dimensional considerations

- SSB scale

$$\sim f_a$$

- Quark masses

$$\sim m_q$$

- QCD scale

$$\sim \Lambda_{\text{QCD}} \sim f_\pi$$

➡ PseudoGoldstone mass

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

*Axion mass from
Topological Suszeptibility*

The ``topological'' axion mass

$$\begin{aligned}\frac{d^2}{d\theta^2} \exp\left(-\int_x V(\theta)\right) &= \frac{d^2}{d\theta^2} \exp(-V(\theta)\mathcal{V}) \\ &= (-V''(\theta)\mathcal{V} + (V'(\theta)\mathcal{V})^2) \exp(-V(\theta)\mathcal{V})\end{aligned}$$

→ Evaluate at $a = \theta = 0$ (minimum)
normalize $V(0)=0$

$$\begin{aligned}&= -V''(\theta)\mathcal{V} \\ &= -m_a^2 f_a^2 \mathcal{V}\end{aligned}$$

The axion mass

$$\begin{aligned}\exp\left(-\int_x V(\theta)\right) &= \int \mathcal{D}A_\mu \exp(-S_{eff}[\phi, A^\mu]) \exp\left(-i\theta \frac{g^2}{32\pi^2} \int_x G^{\mu\nu} \tilde{G}_{\mu\nu}\right) \\ &= \int \mathcal{D}A_\mu \exp(-S_{eff}[\phi, A^\mu]) \exp(-i\theta Q)\end{aligned}$$

$$\begin{aligned}\frac{d^2}{d\theta^2} \exp\left(-\int_x V(\theta)\right) &= \int \mathcal{D}A_\mu (-iQ)^2 \exp(-S_{eff}[\phi, A^\mu]) \exp(-i\theta Q) \\ &= -\langle Q^2 \rangle \\ &= -\mathcal{V} \chi_{top}\end{aligned}$$

$$\Rightarrow m_a^2 f_a^2 = \chi_{top}$$

*Axion mass from
Chiral Perturbation Theory*

Problem...

- QCD is hard to solve
 - Topological susceptibility not immediately accessible
 - Try to express things with **measured** low energy quantities
 - Chiral Perturbation Theory
-

QCD+ axion Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\partial_\mu a\partial^\mu a + \frac{g^2}{32\pi^2} \left[\frac{a}{f_a} + \theta \right] G^{\mu\nu} \tilde{G}_{\mu\nu} \\ & + \frac{k_u}{f_a} (\partial_\mu a) \bar{u} \gamma^\mu \gamma^5 u + \frac{k_d}{f_a} (\partial_\mu a) \bar{d} \gamma^\mu \gamma^5 d \\ & + i\bar{u} \not{D} u + i\bar{d} \not{D} d - m_u \bar{u} u - m_d \bar{d} d,\end{aligned}$$

Space-time dependent chiral rotations

$$\psi \rightarrow \exp\left(\frac{i\alpha(x)}{2}\gamma^5\right)\psi$$

$$i\bar{\psi}\not{D}\psi \rightarrow i\bar{\psi}\not{D}\psi - \frac{\partial_\mu\alpha(x)}{2}(\bar{\psi}\gamma^\mu\gamma^5\psi)$$

Space-time dependent chiral rotations

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$$\sim \frac{a}{f_a}\partial_\mu\bar{\psi}\gamma^\mu\gamma^5\psi$$

Typical derivative
Goldstone interaction!

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$$\sim \frac{a}{f_a}\partial_\mu\bar{\psi}\gamma^\mu\gamma^5\psi$$

Typical derivative
Goldstone interaction!

Mass term

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}\exp\left(2i\gamma^5\alpha(x)\right)\psi$$

Space-time dependent chiral rotations

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$$\sim \frac{a}{f_a}\partial_\mu\bar{\psi}\gamma^\mu\gamma^5\psi$$

Typical derivative
Goldstone interaction!

Mass term

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}\exp(2i\gamma^5\alpha(x))\psi \sim m\bar{\psi}\psi + \frac{2mi}{f_a}a\bar{\psi}\gamma^5\psi$$

Yukwa type interaction!

Space-time dependent chiral rotations

$$\psi \rightarrow \exp\left(\frac{i\alpha(x)}{2}\gamma^5\right)\psi$$

$$i\bar{\psi}\not{D}\psi \rightarrow i\bar{\psi}\not{D}\psi - \frac{\partial_\mu\alpha(x)}{2}(\bar{\psi}\gamma^\mu\gamma^5\psi)$$

$$\sim \frac{a}{f_a}\partial_\mu\bar{\psi}\gamma^\mu\gamma^5\psi$$

Transformation
exchanges!



Mass term

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}\exp(2i\gamma^5\alpha(x))\psi \sim m\bar{\psi}\psi + \frac{2mi}{f_a}a\bar{\psi}\gamma^5\psi$$

Next step

QCD+ axion Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g^2}{32\pi^2} \left[\frac{a}{f_a} + \theta \right] G^{\mu\nu} \tilde{G}_{\mu\nu} \\ & + \frac{k_u}{f_a} (\partial_\mu a) \bar{u} \gamma^\mu \gamma^5 u + \frac{k_d}{f_a} (\partial_\mu a) \bar{d} \gamma^\mu \gamma^5 d \\ & + i\bar{u} \not{D} u + i\bar{d} \not{D} d - m_u \bar{u} u - m_d \bar{d} d,\end{aligned}$$

Do chiral rotations

$$u \rightarrow \exp(i\gamma^5 \alpha_u(x)) u$$

$$d \rightarrow \exp(i\gamma^5 \alpha_d(x)) d$$



$$\theta + \frac{a}{f_a} \rightarrow \theta + 2\alpha_u + 2\alpha_d + \frac{a}{f_a} = 0$$

$$2\alpha_u + 2\alpha_d = - \left[\frac{a}{f_a} + \theta \right]$$

Eliminates gluon coupling

What happens to the quarks?

Write:

$$\alpha_u = -\frac{c_u}{2} \left[\frac{a}{f_a} + \theta \right], \quad \alpha_d = -\frac{c_d}{2} \left[\frac{a}{f_a} + \theta \right], \quad \text{with } c_u + c_d = 1$$

$$\rightarrow +\frac{k_u}{f_a} (\partial_\mu a) \bar{u} \gamma^\mu \gamma^5 u + \frac{k_d}{f_a} (\partial_\mu a) \bar{d} \gamma^\mu \gamma^5 d \longrightarrow +\frac{k'_u}{f_a} (\partial_\mu a) \bar{u} \gamma^\mu \gamma^5 u + \frac{k'_d}{f_a} (\partial_\mu a) \bar{d} \gamma^\mu \gamma^5 d$$

$$k_u \rightarrow k'_u = k_u - \frac{c_u}{2}, \quad k_d \rightarrow k'_d = k_d - \frac{c_d}{2}$$

$$\begin{aligned} & -m_u \bar{u} u - m_d \bar{d} d \\ & \rightarrow -m_u \bar{u} \exp \left[-i c_u \left(\theta + \frac{a}{f_a} \right) \gamma^5 \right] u - m_d \bar{d} \exp \left[-i c_d \left(\theta + \frac{a}{f_a} \right) \gamma^5 \right] d. \end{aligned}$$

More Goldstone bosons...

- For $m_{u,d}$ small QCD features an approximate global chiral $SU(2)_L \times SU(2)_R$ symmetry
- Symmetry is spontaneously broken by QCD interactions $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
 $\langle \bar{u}u \rangle \sim \langle \bar{d}d \rangle \sim v$
- \rightarrow 3 Goldstone bosons: π^0, π^+, π^-

More Goldstone bosons...

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 $\langle \bar{u}u \rangle \sim \langle \bar{d}d \rangle \sim v$
- \rightarrow 3 Goldstone bosons: π^0, π^+, π^-
- Replacement rules

$$\bar{u}u \rightarrow v \cos\left(\frac{\pi^0}{f_\pi}\right),$$

$$\bar{u}\gamma^5 u \rightarrow iv \sin\left(\frac{\pi^0}{f_\pi}\right),$$

$$i\bar{u}\gamma^\mu\gamma^5 u \rightarrow \frac{1}{2}f_\pi\partial^\mu\pi^0,$$

$$\bar{d}d \rightarrow v \cos\left(\frac{\pi^0}{f_\pi}\right),$$

$$\bar{d}\gamma^5 d \rightarrow -iv \sin\left(\frac{\pi^0}{f_\pi}\right)$$

$$i\bar{d}\gamma^\mu\gamma^5 d \rightarrow -\frac{1}{2}f_\pi\partial^\mu\pi^0$$

Insert into QCD+axion

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial^\mu\pi^0\partial_\mu\pi^0 - \frac{1}{2}\partial^\mu a'\partial_\mu a' - \left(\frac{k'_u - k'_d}{2f_a}\right) f_\pi\partial^\mu a'\partial_\mu\pi^0 \\ - m_u v \cos\left(\frac{\pi^0}{f_\pi} - \frac{c_u a'}{f_a}\right) - m_d v \cos\left(\frac{\pi^0}{f_\pi} + \frac{c_d a'}{f_a}\right),$$

$$a' = a - \langle a \rangle = a + f_a \theta.$$

Pions we can measure!
This is the idea

Insert into QCD+axion

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial^\mu\pi^0\partial_\mu\pi^0 - \frac{1}{2}\partial^\mu a'\partial_\mu a' - \left(\frac{k'_u - k'_d}{2f_a}\right) f_\pi\partial^\mu a'\partial_\mu\pi^0 \\ - m_u v \cos\left(\frac{\pi^0}{f_\pi} - \frac{c_u a'}{f_a}\right) - m_d v \cos\left(\frac{\pi^0}{f_\pi} + \frac{c_d a'}{f_a}\right),$$

$$a' = a - \langle a \rangle = a + f_a \theta.$$

Eliminate via

$$c_u = \frac{1}{2} + k_u - k_d, \quad c_d = \frac{1}{2} + k_d - k_u$$

$$k'_u - k'_d = 0$$

Expand to second order...

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial^\mu\pi^0\partial_\mu\pi^0 - \frac{1}{2}\partial^\mu a\partial_\mu a - \frac{1}{2}(\pi^0, a') M^2 \begin{pmatrix} \pi^0 \\ a' \end{pmatrix} + \dots$$

$$M^2 = \begin{pmatrix} (m_u + m_d)\frac{v}{f_\pi^2} & (-m_u c_u + m_d c_d)\frac{v}{f_\pi f_a} \\ (-m_u c_u + m_d c_d)\frac{v}{f_\pi f_a} & (m_u c_u^2 + m_d c_d^2)\frac{v}{f_a^2} \end{pmatrix}$$

Expand to second order...

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial^\mu\pi^0\partial_\mu\pi^0 - \frac{1}{2}\partial^\mu a\partial_\mu a - \frac{1}{2}(\pi^0, a') M^2 \begin{pmatrix} \pi^0 \\ a' \end{pmatrix} + \dots$$

$$M^2 = \begin{pmatrix} (m_u + m_d)\frac{v}{f_\pi^2} & (-m_u c_u + m_d c_d)\frac{v}{f_\pi f_a} \\ (-m_u c_u + m_d c_d)\frac{v}{f_\pi f_a} & (m_u c_u^2 + m_d c_d^2)\frac{v}{f_a^2} \end{pmatrix}$$

Diagonalize mass matrix

$$m_\pi^2 = \frac{(m_u + m_d)v}{f_\pi^2}$$



Known

$$m_a^2 = \frac{v}{f_a^2} \frac{m_d m_u}{m_d + m_u} = \frac{f_\pi^2}{f_a^2} \frac{m_d m_u}{(m_d + m_u)^2} m_\pi^2$$



Only f_a unknown

Learn something about QCD...

- Topological susceptibility argument

$$m_a^2 = \frac{\chi_{top}}{f_a^2}$$

- Chiral perturbation theory

$$m_a^2 = \frac{v}{f_a^2} \frac{m_d m_u}{m_d + m_u} = \frac{f_\pi^2}{f_a^2} \frac{m_d m_u}{(m_d + m_u)^2} m_\pi^2$$

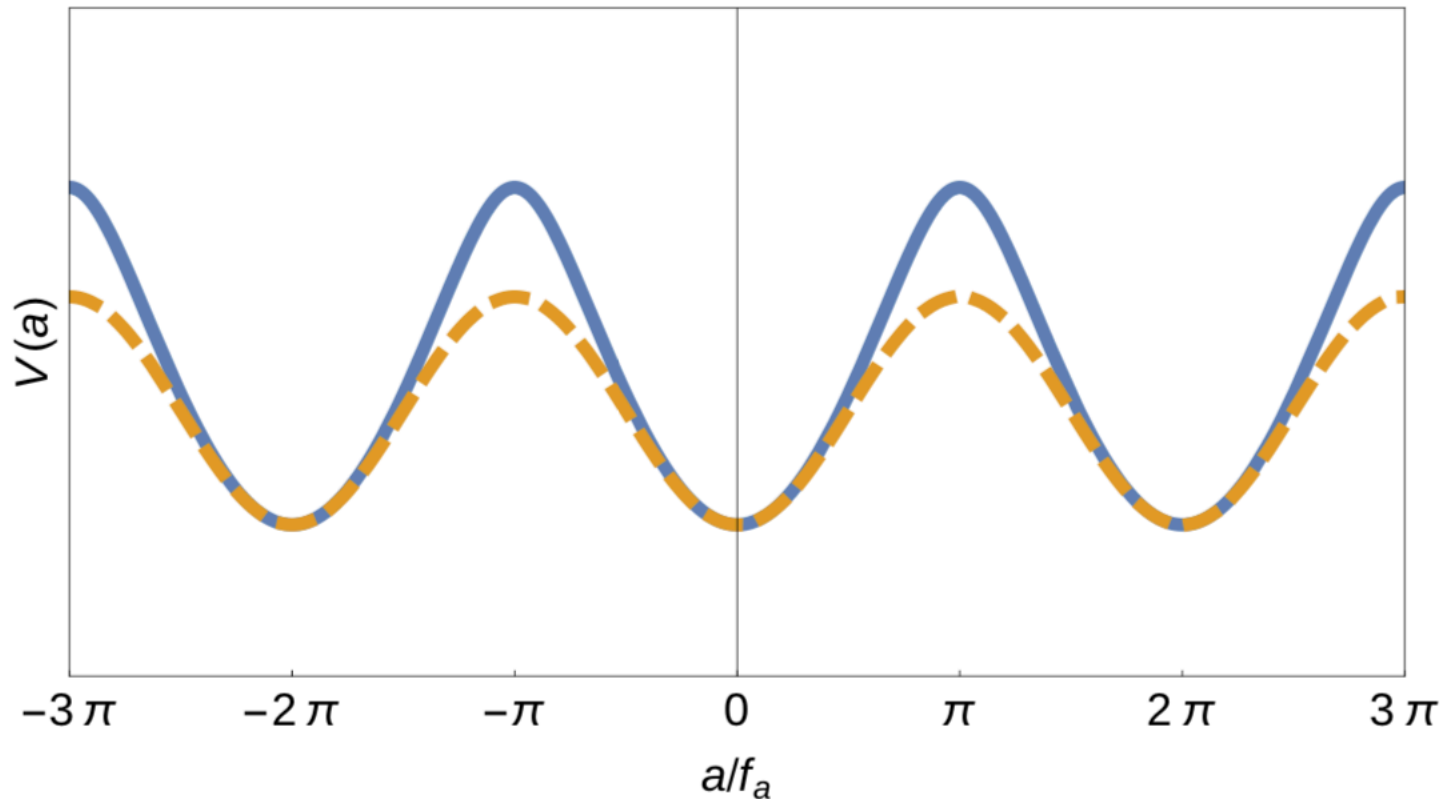
$$\rightarrow \chi_{top} = f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

One can actually calculate the potential

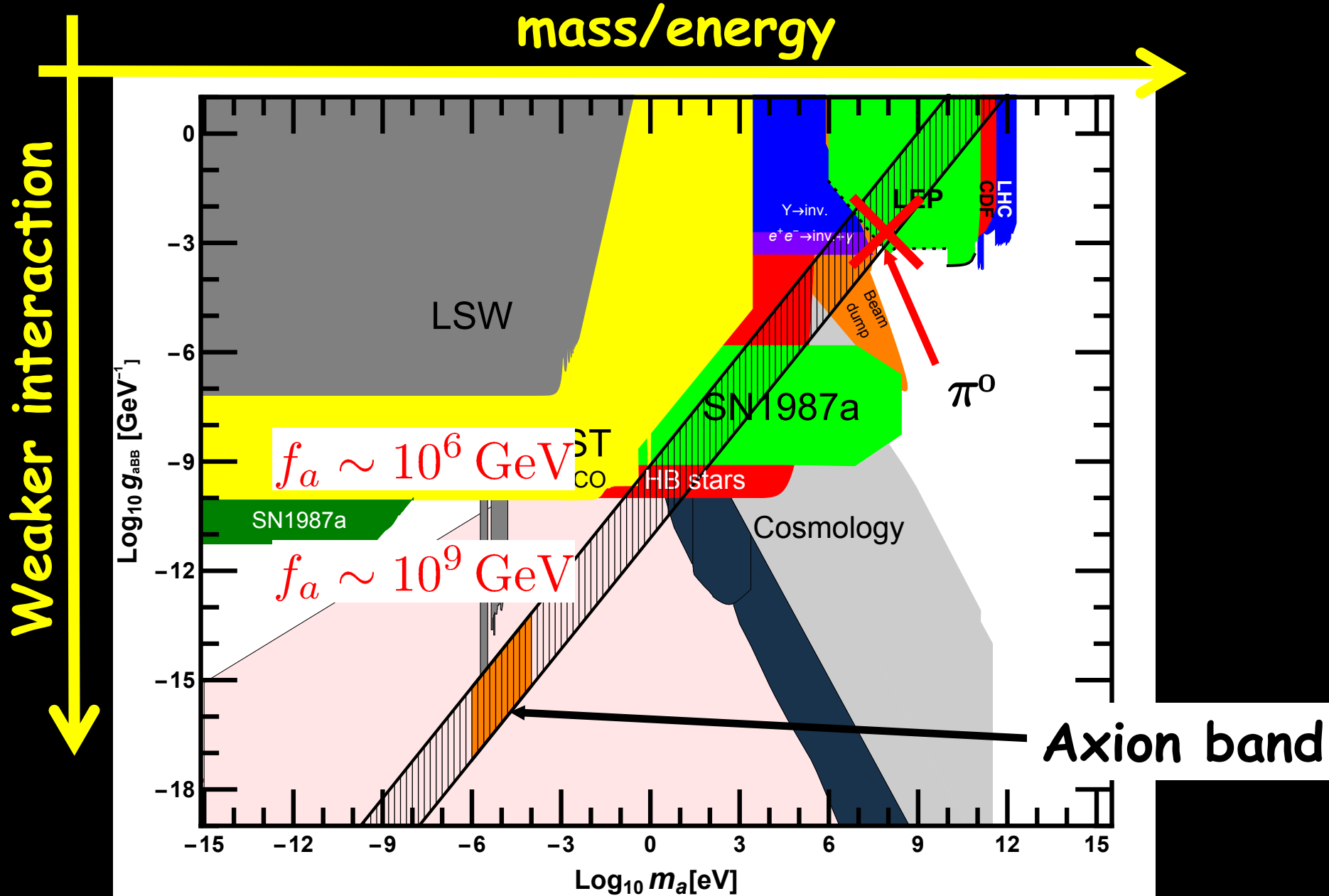
$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left(\frac{a}{2f_a} \right)}$$

The QCD axion, precisely

Giovanni Grilli di Cortona^a, Edward Hardy^b,
Javier Pardo Vega^{a,b} and Giovanni Villadoro^b



Axions and axion-like Particles



Dirty Little Secrets

Axions and the electroweak hierarchy

- Lagrangian may/will contain interactions

$$\mathcal{L} \supset \lambda |\phi_{PQ}|^2 |H|^2$$

- After PQ breaking contribution to Higgs mass

$$\Delta m_H^2 \sim \lambda f_a^2 \gtrsim \lambda (10^9 \text{ GeV})^2 \sim 10^{14} v_{\text{EW}}^2$$

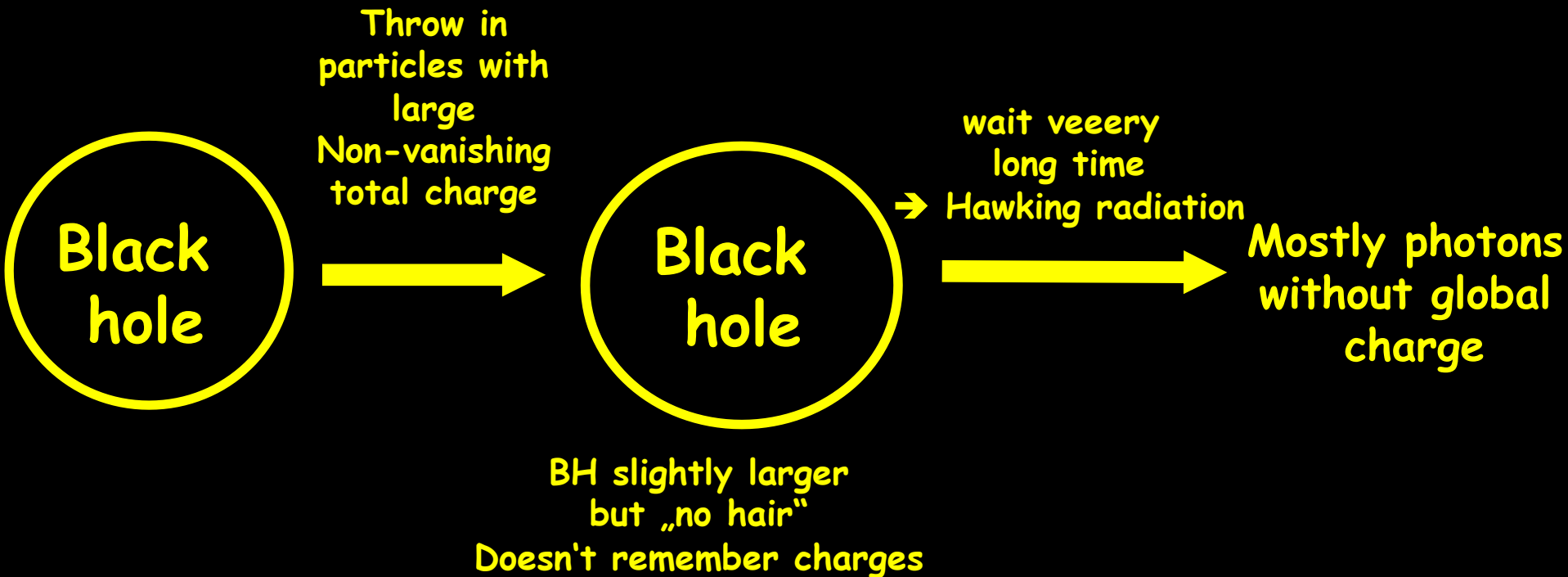
→ we just found an explicit large contribution to electroweak hierarchy problem

Wiggling out?

- We could have small $\lambda \ll 1$
→ Not really plausible
- Hope: Not really a new problem. Could be solved by the same mechanism that solved electroweak hierarchy...

Gravity breaks all global symmetry

- Imagine a particle charged under global symmetry (no gauge field attached)



Gravity breaks global symmetries

- Initial state is charges
- Final state has no/different charge
- Noether current conservation violated
- Global symmetry is explicitly broken


Gravity breaking Lagrangian

$$\begin{aligned}\mathcal{L}_{PQ-viol} &\supset -\frac{1}{2M_P^{k-4}} \left(c\phi^k + c^*(\phi^*)^k \right) \\ &= -|c|f_a^4 \left(\frac{f_a}{M_P} \right)^{k-4} \cos \left(k\frac{a}{f_a} + \text{Arg}(c) \right)\end{aligned}$$

Minimum typically not at $a=0$!!!

Strong CP problem re-appears

$$V(a) = m_a^2 f_a^2 \left(1 - \cos \left(\frac{a}{f_a} \right) - |c| \frac{f_a^2}{m_a^2} \left(\frac{f_a}{M_P} \right)^{k-4} \cos \left(k \frac{a}{f_a} + \text{Arg}(c) \right) \right)$$


$$\begin{aligned} \theta_{\min} = \frac{a_{\min}}{f_a} &\approx -|c| k \frac{f_a^2}{m_a^2} \left(\frac{f_a}{M_P} \right)^{k-4} \sin(\text{Arg}(c)) \\ &\sim 10^{-6} \left(\frac{f_a}{10^{10} \text{ GeV}} \right)^{10} \quad c \sim 1, \quad k = 10 \end{aligned}$$

**Very large power suppression needed
to fulfill EDM constraint!!!**

But c may be $\ll 1$

- The gravitational symmetry breaking seems non-perturbative

- \rightarrow Expect

$$c \sim \exp(-S)$$

Euclidean action of object
responsible for generation of symmetry breaking

But c may be $\ll 1$

- The gravitational symmetry breaking seems non-perturbative

- → Expect

$$c \sim \exp(-S)$$

Euclidean action of object responsible for generation of symmetry breaking

- Case of wormholes

$$S = \frac{\pi\sqrt{6}}{8} \frac{M_P}{f_a} \sim 0.96 \frac{M_P}{f_a}$$

Wormholes and masses for Goldstone bosons

Rodrigo Alonso (CERN), Alfredo Urbano (CERN) (Jun 22, 2017)

Published in: *JHEP* 02 (2019) 136 • e-Print: [1706.07415](#) [hep-ph]

Euclidean wormholes, baby universes, and their impact on particle physics and cosmology

Arthur Hebecker (Heidelberg U.), Thomas Mikhail (Heidelberg U.), Pablo Soler (Heidelberg U.) (Jul 2, 2018)

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But c may be $\ll 1$

- The gravitational symmetry breaking seems non-perturbative

- → Expect

$$c \sim \exp(-S)$$

Euclidean action of object responsible for generation of symmetry breaking

- Case of wormholes

$$S \sim \frac{M_P}{f_a}$$

→ Breaking might be very small!

Wormholes and masses for Goldstone bosons

Rodrigo Alonso (CERN), Alfredo Urbano (CERN) (Jun 22, 2017)

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Summary I

Summary I

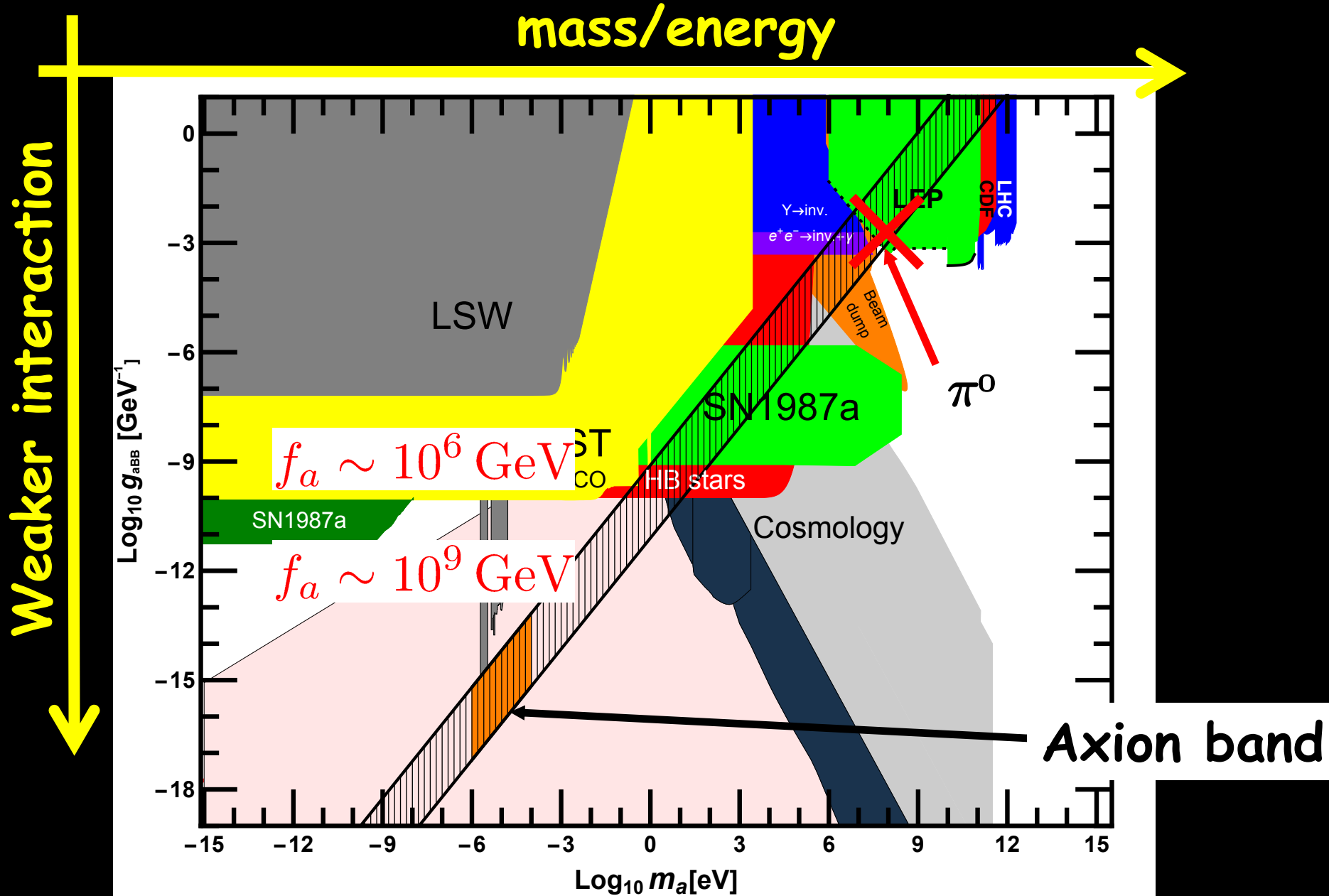
- Strong CP problem is a naturalness problem
- Requires cancellation of two very different contributions
- Dynamical solution: Axions

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

$$\text{Interaction} \sim -\frac{1}{4} \frac{\alpha}{4\pi f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

→ Testable?!

Axions and axion-like Particles



The ALPs

→ more general:
axion-like particles

- Crucial features of axions:
 - Low Mass
 - Tiny coupling
 - Both linked to large scale of SSB: f
 - Finite field range $2\pi f$
 - Typical of Goldstone-Bosons!
- ALPs: (Pseudo-)Goldstones of spontaneously broken global symmetries
-

A simple model (same as before)

- **Scalar + Fermions**

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}D^{\mu}\gamma_{\mu}\psi - |\partial_{\mu}\phi|^2 - \mu^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4 + \bar{\psi}\left(Y\phi\frac{1+\gamma^5}{2} + Y^*\phi^*\frac{1-\gamma^5}{2}\right)\psi.$$

- **U(1) symmetries**

- **Global**

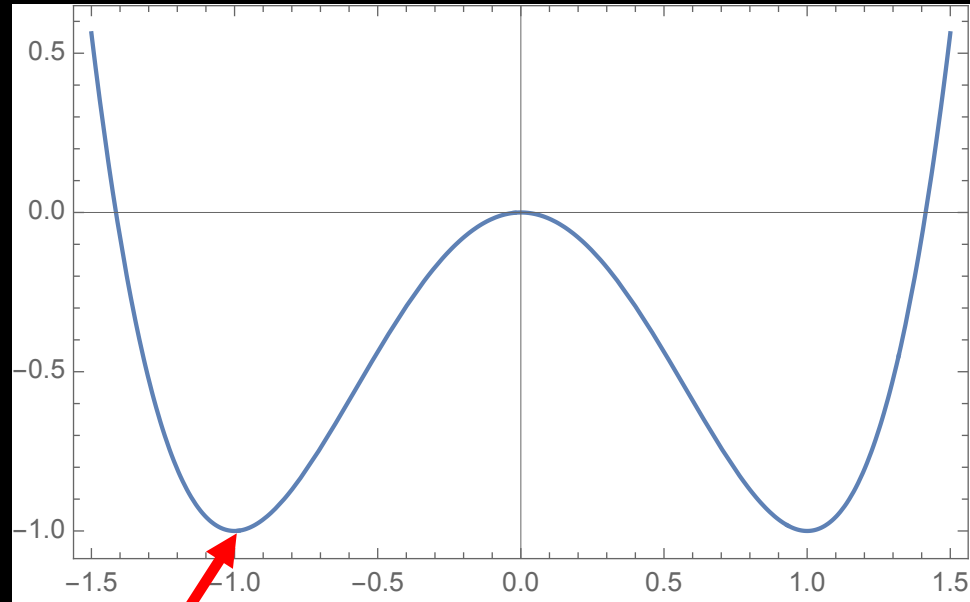
$$\phi \rightarrow \exp(i\alpha)\phi, \quad \psi \rightarrow \exp\left(-\frac{i}{2}\alpha\gamma^5\right)\psi.$$

- **Gauge**

$$\psi \rightarrow \exp(i\beta\phi) \quad \phi \rightarrow \phi$$

Spontaneous Symmetry Breaking

$$\mu^2 < 0$$



$$\langle |\phi| \rangle = \sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{1}{\sqrt{2}} f_a$$

Describing Goldstone Bosons

$$\phi = |\phi| \exp(-i\alpha(x))$$


Use modulus and phase
to describe field

$$m_{|\phi|} \sim \frac{f_a}{\sqrt{\lambda}}$$

Very heavy!

Describing Goldstone Bosons

$$\phi = |\phi| \exp(-i\alpha(x))$$

Use modulus and phase
to describe field

$$m_{|\phi|} \sim \frac{f_a}{\sqrt{\lambda}}$$

Very heavy!

But: $V(\phi) = V(|\phi|)$ Independent of α



$$m_\alpha = 0$$

Normalize field

- Still need to look at kinetic term

$$\partial_\mu \phi^* \partial^\mu \phi \rightarrow (\partial_\mu |\phi|)^2 + |\phi|^2 (\partial_\mu \alpha(x))^2 = (\partial_\mu |\phi|)^2 + \frac{1}{2} f_a^2 (\partial_\mu \alpha(x))^2$$

Not normalized



$$\alpha(x) \rightarrow \frac{a(x)}{f_a}$$



$$\mathcal{L}_{kin,\alpha} = \frac{1}{2} (\partial_\mu a)^2$$

Properly normalized

Goldstone interactions are suppressed

- Already known: anomalous interactions

$$\mathcal{D}\psi'\mathcal{D}\bar{\psi}' = \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left(-i \int \frac{\alpha}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}\right) = \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left(-i \int \frac{a}{4\pi f_a} F^{\mu\nu} \tilde{F}_{\mu\nu}\right)$$

- Symmetry preserving ones must be derivative interactions

$$\frac{\partial_\mu \alpha(x)}{2} (\bar{\psi} \gamma^\mu \gamma^5 \psi) = \frac{\partial_\mu a}{2 f_a} (\bar{\psi} \gamma^\mu \gamma^5 \psi)$$

$$\sim \frac{2m_\psi}{f_a} \bar{\psi} \gamma^5 \psi$$

Slightly cheating?

- In models like

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}D^{\mu}\gamma_{\mu}\psi - |\partial_{\mu}\phi|^2 - \mu^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4 + \bar{\psi}\left(Y\phi\frac{1+\gamma^5}{2} + Y^*\phi^*\frac{1-\gamma^5}{2}\right)\psi.$$

$$\rightarrow m_{\psi} \sim Y f_a$$

$$\rightarrow \frac{2m_{\psi}}{f_a} \sim Y$$

But: The ψ are the heavy particles that we are anyway not interested in

Light particles indeed have suppressed couplings via derivative argument!!!

Realization (DFSZ type)

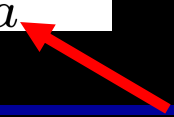
- Both PQ field ϕ and Higgs H carry PQ charge

$$\phi \rightarrow \exp(i\alpha), \quad H \rightarrow \exp(iq_H \alpha) H$$

$$f_a \sim \sqrt{\langle \phi \rangle^2 + \langle H \rangle^2} \sim \langle \phi \rangle$$

Axion still $\alpha \sim \frac{a}{f_a}$

- Couplings to SM fermions via Higgs

$$\sim Y H \bar{L} e \sim m_e \alpha \bar{e} e \sim \frac{m_e}{f_a} \bar{e} e$$


Pseudo-Goldstones

- A slightly broken symmetry

$$\mathcal{L} = |\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 + \frac{1}{2\sqrt{2}} b (\phi + \phi^*)$$

$$\begin{aligned} V(\phi) &= V_{||}(|\phi|) + b f_a \cos(\alpha) \\ &= V_{||}(|\phi|) + b f_a \cos\left(\frac{a}{f_a}\right) \end{aligned}$$

$$m_\alpha = \sqrt{\frac{b}{f_a}}$$

→ Suppressed by
- small b
- large f

Slightly cheating

- More generally

$$\mathcal{L} = |\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 + \frac{1}{2\sqrt{2^n}} \Lambda^{4-n} (\phi^n + \phi^{*,n})$$

- $\rightarrow \frac{m_a^2}{f_a^2} \sim \left(\frac{\Lambda}{f_a}\right)^{4-n}$

- Need $\Lambda \ll f_a$

$n=1$

$$m_a^2 \sim \frac{\Lambda^3}{f_a} \ll \Lambda^2$$

$n=3$

$$m_a^2 \sim \Lambda f_a \gg \Lambda^2$$

Anomalous/Non-perturbative breaking

to the rescue:

Luckily many models (e.g. QCD axion) do not take Λ as an input parameter

→ $\Lambda \sim M_0 \exp\left(-\frac{8\pi^2}{g^2}\right)$

→ $g \sim 0.1 - 1$ Can generate tiny values of Λ

Typical for: Anomalous breaking of a symmetry!

String Axions/ALPs

String theory: Moduli and Axions

- String theory needs Extra Dimensions

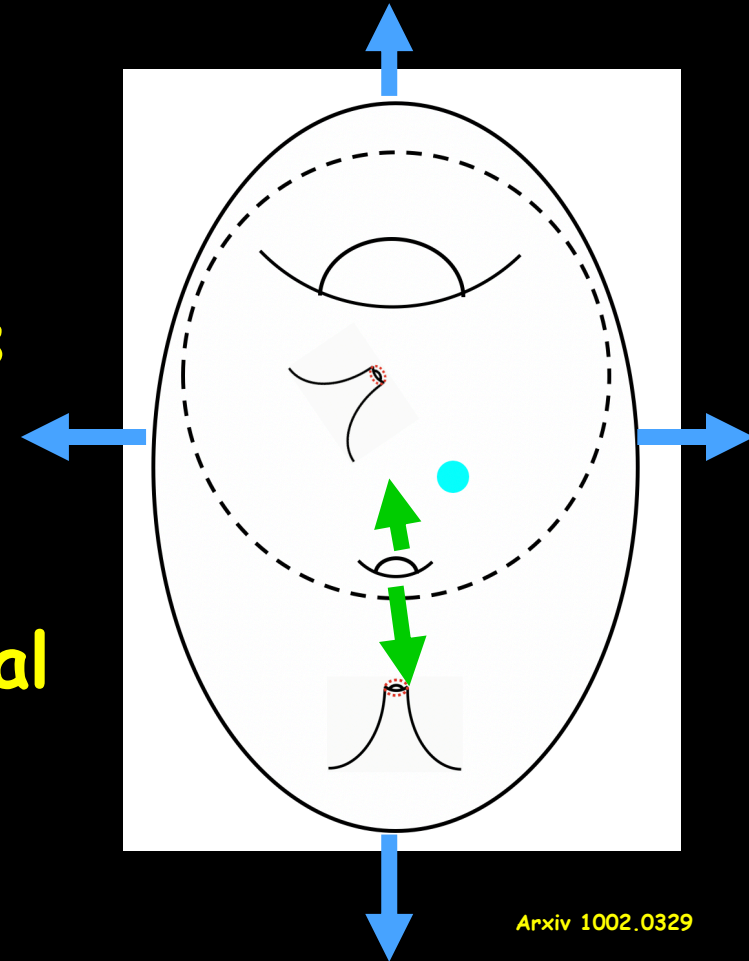


Must compactify

- Shape and size deformations correspond to fields:
Moduli (WISPs) and Axions
Connected to the fundamental scale, here string scale



Axion/ALP candidates



- Gauge field terms

$$\mathcal{L} = \frac{1}{g^2} F^2 + i\theta F \tilde{F}$$

- + Supersymmetry/supergravity

$$\mathcal{L} = \text{Re}[f(\Phi)] F^2 + \text{Im}[f(\Phi)] F \tilde{F}$$



Scalar ALP/moduli coupling + pseudoscalar ALP coupling

Axions/ALPs and Moduli

- Gauge couplings always field dependent
(no free coupling constants)
 - Axions/ALPs + Moduli always present in String theory
-

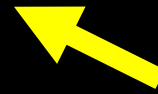
String Axions/ALPs

General Features

Need for large volume

- Generically

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}}$$



Volume in string units

→ If we want sub-Planckian axion we need large (even LARGE) volume

Small mass from shift symmetry

- Perturbatively axions feature shift symmetry

$$a \longrightarrow a + c$$

- This is essentially as for Goldstones
- Broken by non-perturbative effects

$$V(a) = M^4 \exp(-S) \exp\left(ik \frac{a}{f_a}\right) + h.c. = 2M^4 \exp(-S) \cos\left(k \frac{a}{f_a}\right)$$



small

String Axions
and the
Dark Radiation Hydra

Axionic/ALPy Dark Radiation

- Many (string) models feature a long-lived modulus Φ
- This reheats the Universe $\Phi \rightarrow SM$
- Significant branching ratio into axions/ALPs $\Phi \rightarrow a + a$
- These a are effective degrees of freedom visible in BBN and CMB
- often dangerous „Dark Radiation Problem“

M. Cicoli, J. P. Conlon, and F. Quevedo, “Dark radiation in LARGE volume models,” *Phys. Rev. D* **87** no. 4, (2013) 043520, arXiv:1208.3562 [hep-ph].

A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski, “Dark Radiation predictions from general Large Volume Scenarios,” *JHEP* **09** (2014) 140, arXiv:1403.6810 [hep-ph].

T. Higaki and F. Takahashi, “Dark Radiation and Dark Matter in Large Volume Compactifications,” *JHEP* **11** (2012) 125, arXiv:1208.3563 [hep-ph].

S. Angus, “Dark Radiation in Anisotropic LARGE Volume Compactifications,” *JHEP* **10** (2014) 184, arXiv:1403.6473 [hep-ph].

+ . . . + <https://arxiv.org/pdf/2203.08833.pdf>

The dark radiation problem concrete

- String models usually have **too much axionic dark radiation**
- Reason: Long-lived volume modulus ϕ_b dominates the Universe before reheating it

$$BR_{\phi_b \rightarrow aa} \sim \frac{\Gamma_{\phi_b \rightarrow aa}}{\Gamma_{\phi_b \rightarrow SM} + \Gamma_{\phi_b \rightarrow aa}} \sim \frac{1}{1 + 2z^2} \sim \mathcal{O}(1)$$



$$\Delta N_{\text{eff}} \sim 6.1 \left(\frac{11}{g_*^4 g_{*,S}^{-3}} \right)^{1/3} BR(\phi \rightarrow aa)$$

But: **CMB + Co. say:**

$$\Delta N_{\text{eff}} \lesssim 0.2 - 0.4$$



Solution: Decay to Higgses

- SUSY breaking generates coupling to Higgses

.... an actually not so long calculation...

$$m_H^2 \sim m_{3/2}^2 \left[c_0 + c_{\text{loop}} \ln \left(\frac{m_{\text{KK}}}{m_{3/2}} \right) \right]$$

$$m_H^2 \sim \left(\frac{W_0}{\mathcal{V}} \right)^2 \left[c_0 + c_{\text{loop}} \ln \left(\frac{\mathcal{V}^{1/2}}{W_0} \right) \right] M_P^2$$

$$\mathcal{V} \sim \tau_b^{3/2} \sim \exp \left(\sqrt{\frac{3}{2}} \phi_b \right)$$

$$\mathcal{L} \supset \sim \left(m_{3/2}^2 \frac{c_{\text{loop}}}{2} \sqrt{\frac{3}{2}} \right) h^2 \frac{\delta \phi_b}{M_P} \sim m_{3/2}^2 c_{\text{loop}} h^2 \frac{\delta \phi_b}{M_P}$$

$$\Gamma_{\phi_b \rightarrow hh} \sim \frac{m_{3/2}^4 c_{\text{loop}}^2}{m_{\tau_b} M_P^2} \sim (c_{\text{loop}} \mathcal{V})^2 \frac{m_{\tau_b}^3}{M_P^2} \gg \Gamma_{\phi_b \rightarrow a_b a_b}$$

Solution: Decay to Higgses

- SUSY breaking generates coupling to Higgses

.... an actually not so long calculation...

$$\Gamma_{\phi_b \rightarrow hh} \sim \frac{m_{3/2}^4 c_{\text{loop}}^2}{m_{\tau_b} M_P^2} \sim (c_{\text{loop}} \mathcal{V})^2 \frac{m_{\tau_b}^3}{M_P^2} \gg \Gamma_{\phi_b \rightarrow a_b a_b}$$



$$BR_{\phi_b \rightarrow aa} \ll 1$$

→ Problem solved!

Or is it?

- Well, it's a Hydra ;-)
-

Inflaton may be longest-lived modulus

- Inflaton decay now slower than volume modulus

$$\Gamma_{\text{inflaton}} \sim \mathcal{V}^{-4} \gtrsim \Gamma_{\phi_b} \sim c_{\text{loop}}^2 \mathcal{V}^{-2.5}$$

- May dominate Universe

$$BR(\text{inflaton} \rightarrow a + X) \sim \frac{1}{1+x}$$

Decay rates...

Decay rate	scaling	explicit
$\Gamma_{\phi_I \rightarrow \phi_b \phi_b}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	Γ_1
$\Gamma_{\phi_I \rightarrow \phi_b \phi_b}^{\text{pot}}$	$\sim (\ln \mathcal{V})^{5/2} \mathcal{V}^{-4} M_P$	$4\Gamma_1 / (\mathbf{a}_I \tau_I)^2$
$\Gamma_{\phi_I \rightarrow \phi_L \phi_L}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$4\Gamma_1$
$\Gamma_{\phi_I \rightarrow \phi_L \phi_L}^{\text{pot}}$	$\sim (\ln \mathcal{V})^{1/2} \mathcal{V}^{-4} M_P$	Γ_2
$\Gamma_{\phi_I \rightarrow a_b a_b}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	Γ_1
$\Gamma_{\phi_I \rightarrow a_L a_L}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$4\Gamma_1$
$\Gamma_{\phi_I \rightarrow AA}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$8N_g \Gamma_1$
$\Gamma_{a_I \rightarrow \phi_b a_b}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$2\Gamma_1$
$\Gamma_{a_I \rightarrow \phi_L a_L}^{\text{kin}}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$8\Gamma_1$
$\Gamma_{a_I \rightarrow AA}$	$\sim (\ln \mathcal{V})^{9/2} \mathcal{V}^{-4} M_P$	$8N_g \Gamma_1$


Inflaton may be longes-lived modulus

- Inflaton decay now slower than volume modulus

$$\Gamma_{\text{inflaton}} \sim \mathcal{V}^{-4} \gtrsim \Gamma_{\phi_b} \sim c_{\text{loop}}^2 \mathcal{V}^{-2.5}$$

- May dominate Universe

$$BR(\text{inflaton} \rightarrow a + X) \sim \frac{5}{8N_g} \sim 0.05$$


O(100)

Thanks to decays to **MANY SM gauge bosons!**

Dark Radiation
is useful

We expect some dark radiation

$$\Delta N_{\text{eff}} \sim 6.1 \left(\frac{11}{g_*^4 g_{*,S}^{-3}} \right)^{1/3} BR(\phi \rightarrow aa) \simeq 0.3 \left(\frac{11}{g_*^4 g_{*,S}^{-3}} \right)^{1/3} \simeq 0.14$$

$$g_* = g_{*,S} = 106.75$$

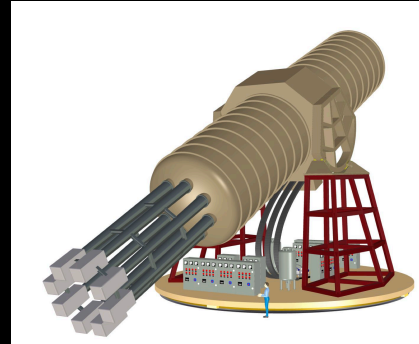
This dark radiation is made from axions.
A significant part is QCD axions



Detectable

Dark Radiation may be detectable + Useful

- For example in IAXO

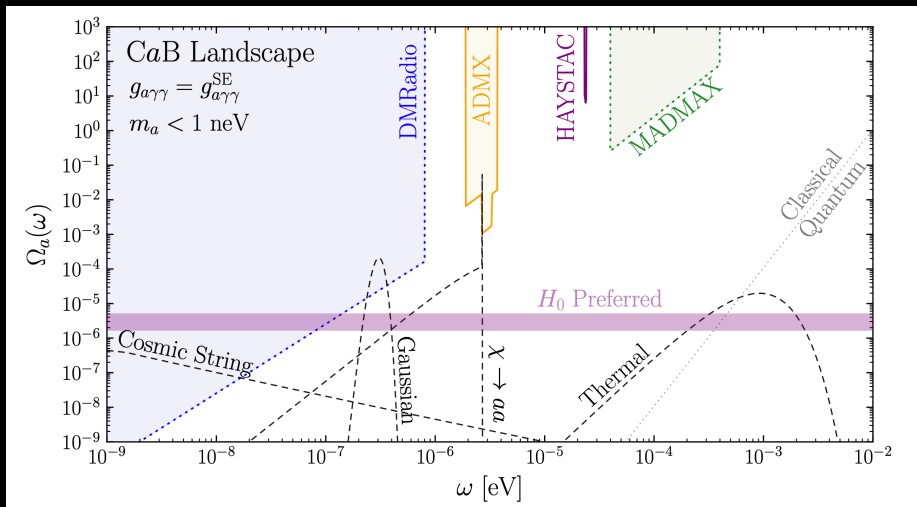


Physics potential of the International Axion Observatory (IAXO)

IAXO Collaboration • E. Armengaud (IRFU, Saclay) et al. (Apr 19, 2019)

Published in: JCAP 06 (2019) 047 • e-Print: 1904.09155 [hep-ph]

- But also other experiments



Cosmic axion background

Jeff A. Dror (UC, Santa Cruz and UC, Santa Cruz, Inst. Part. Phys. and UC, Berkeley and LBNL, Berkeley), Hitoshi Murayama (UC, Berkeley and LBNL, Berkeley and Tokyo U., IPMU), Nicholas L. Rodd (UC, Berkeley and LBNL, Berkeley)

- Might be interesting to think beyond scalar photon couplings!

New tool to probe Reheating

- This dark radiation may allow to get access to information about reheating

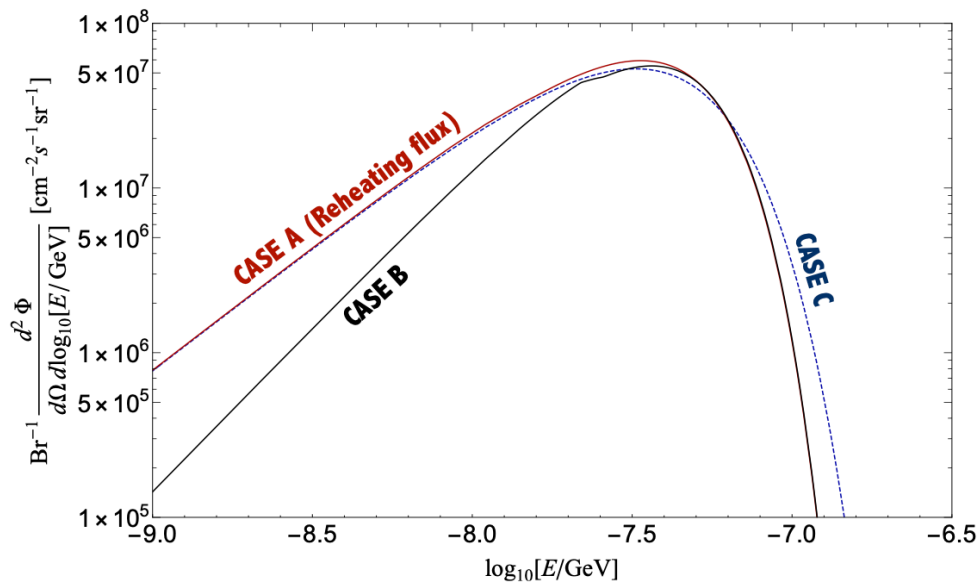
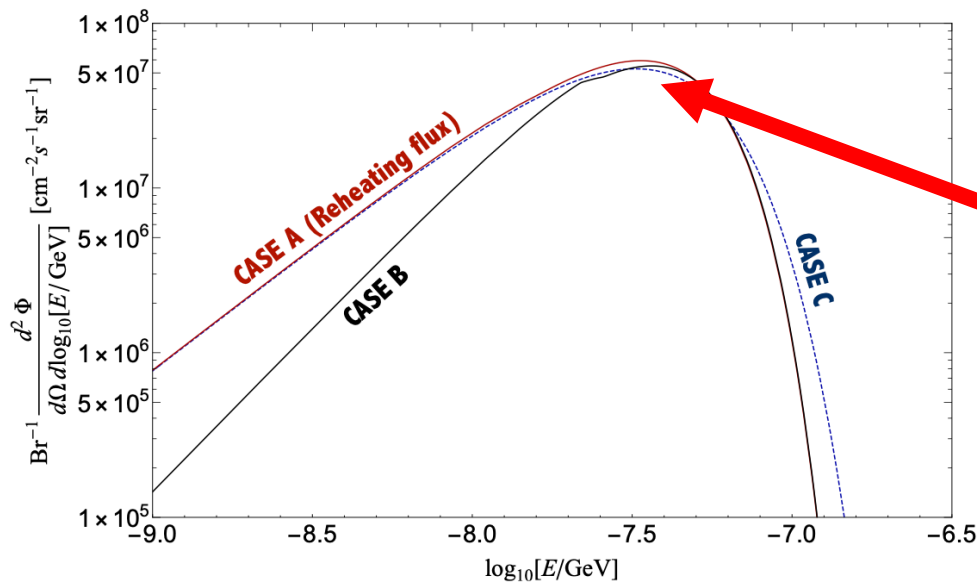


Figure. 1. The differential flux of the messenger particle, $d^2 \Phi / d \log_{10} E d \Omega$. CASE A (ϕ once dominated the Universe) and CASE B (ϕ never dominates the Universe and decay in the radiation dominant epoch) are shown in red and black lines, respectively. We also show the flux for CASE C where a subdominant ϕ decays in the matter dominant era as the blue dashed line.

New tool to probe Reheating

- This dark radiation may allow to get access to information about reheating



Measures

$$\frac{m_{\Phi}}{T_{\Phi}}$$

Figure. 1. The differential flux of the messenger particle, $d^2 \Phi / d \log_{10} E d \Omega$. CASE A (ϕ once dominated the Universe) and CASE B (ϕ never dominates the Universe and decay in the radiation dominant epoch) are shown in red and black lines, respectively. We also show the flux for CASE C where a subdominant ϕ decays in the matter dominant era as the blue dashed line.

Measure reheating temperature

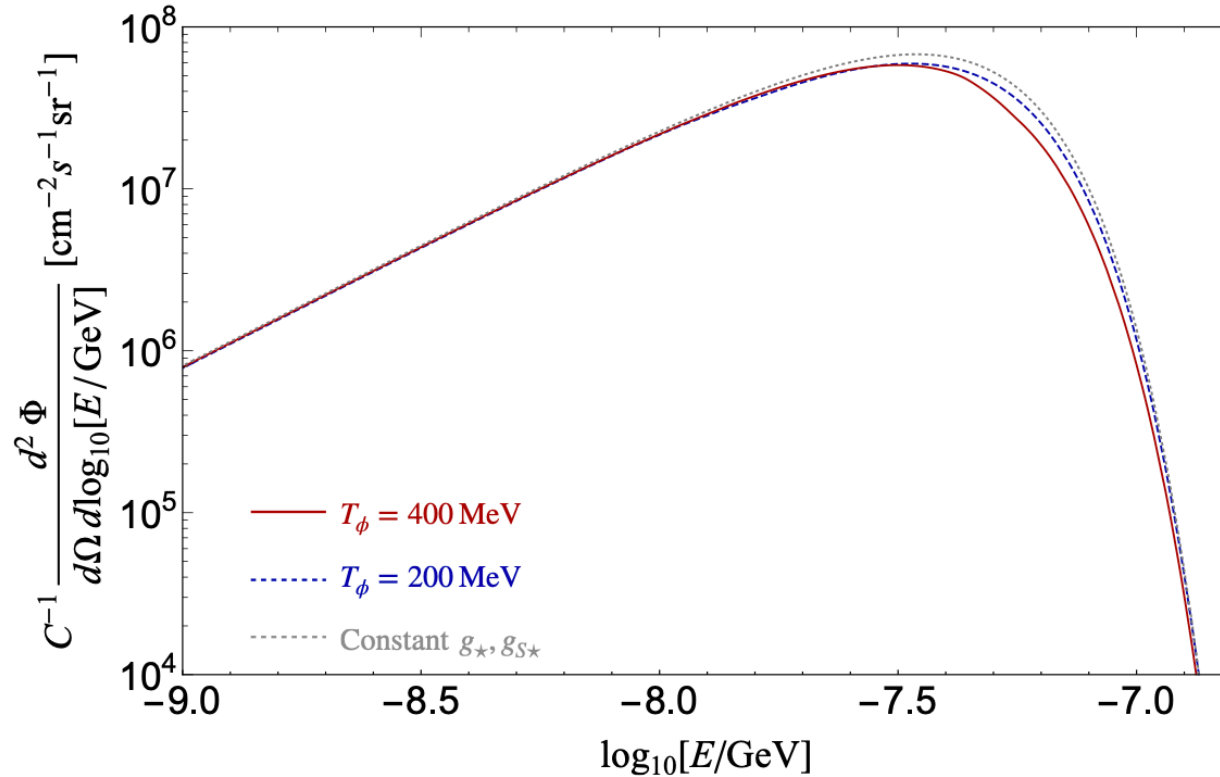


Figure. 2. The reheating flux dependence on the decoupling effect: $T_\phi = 400 \text{ MeV}$ (red-solid line) and $T_\phi = 200 \text{ MeV}$ (blue-dashed line, CASE A). We take $g_\star, g_{S\star}$ temperature in-

Summary II

Summary II

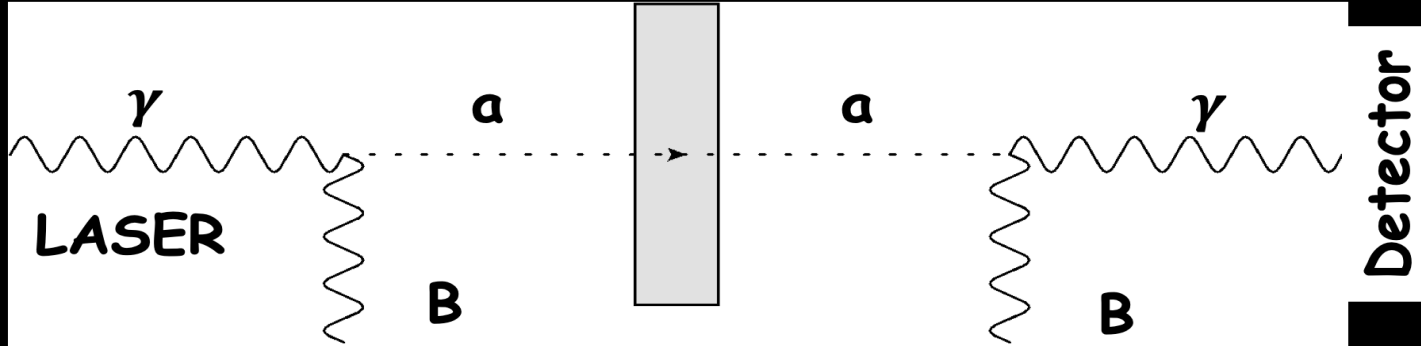
- Pseudo-Goldstone bosons are prototypes for ALPs
- Small mass natural (anomalous symmetry)
- Couplings of derivative type (suppressed by large scale f)
- Finite field range $[-\pi f, \pi f)$
- Generically appear in String theories
- String axions suggest pre-inflationary scenario and some dark radiation

How to find
axions and ALPs...

The Power of Low Energy Experiments

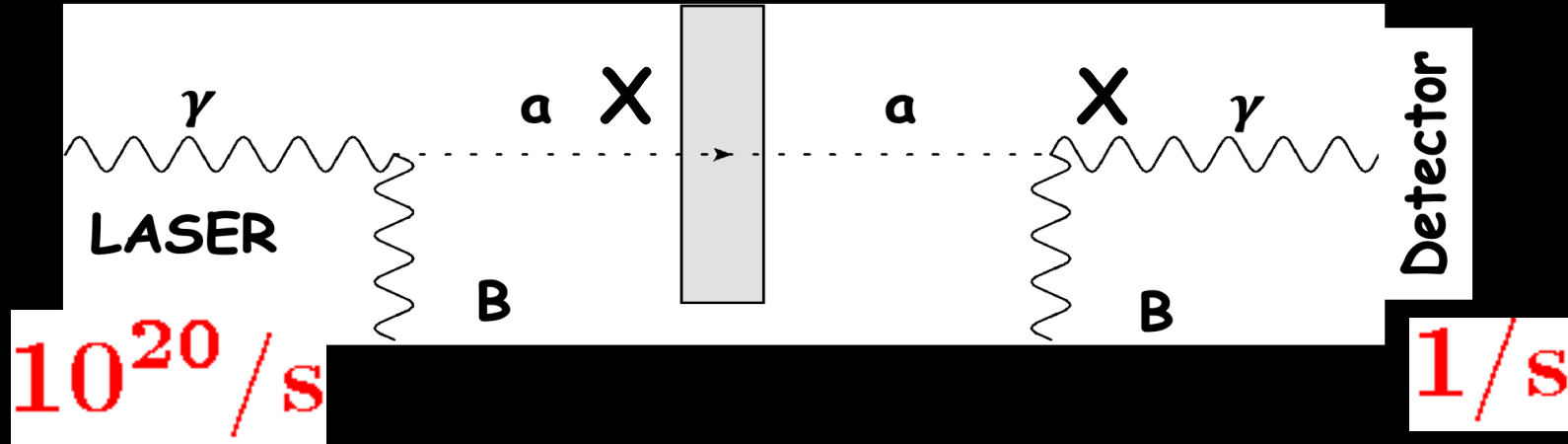
Light shining through walls

Light shining through walls



Light shining through walls

Light shining through walls



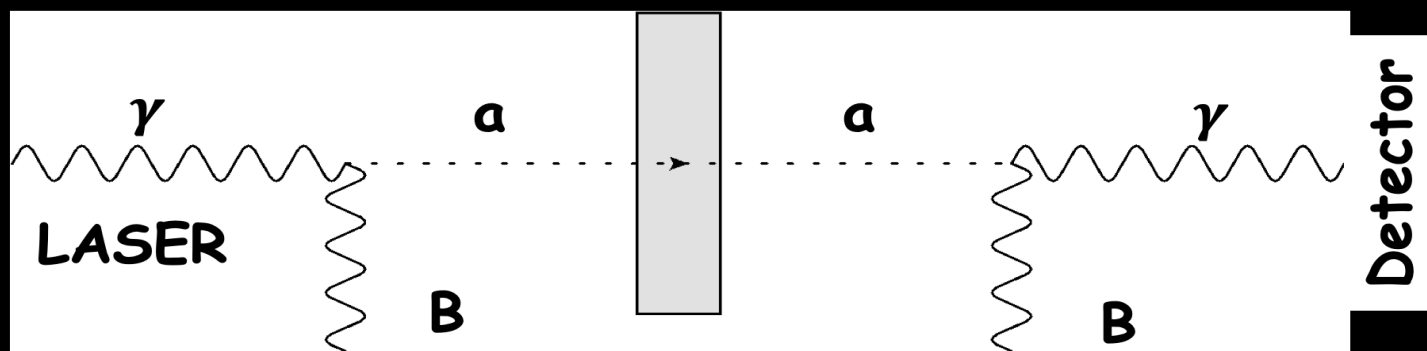
- **Test** $P_{\gamma \rightarrow X \rightarrow \gamma} \lesssim 10^{-20}$
- **Enormous precision!**
- **Study extremely weak couplings!**

Photons coming through the wall!

- It could be Axion(-like particle)s!

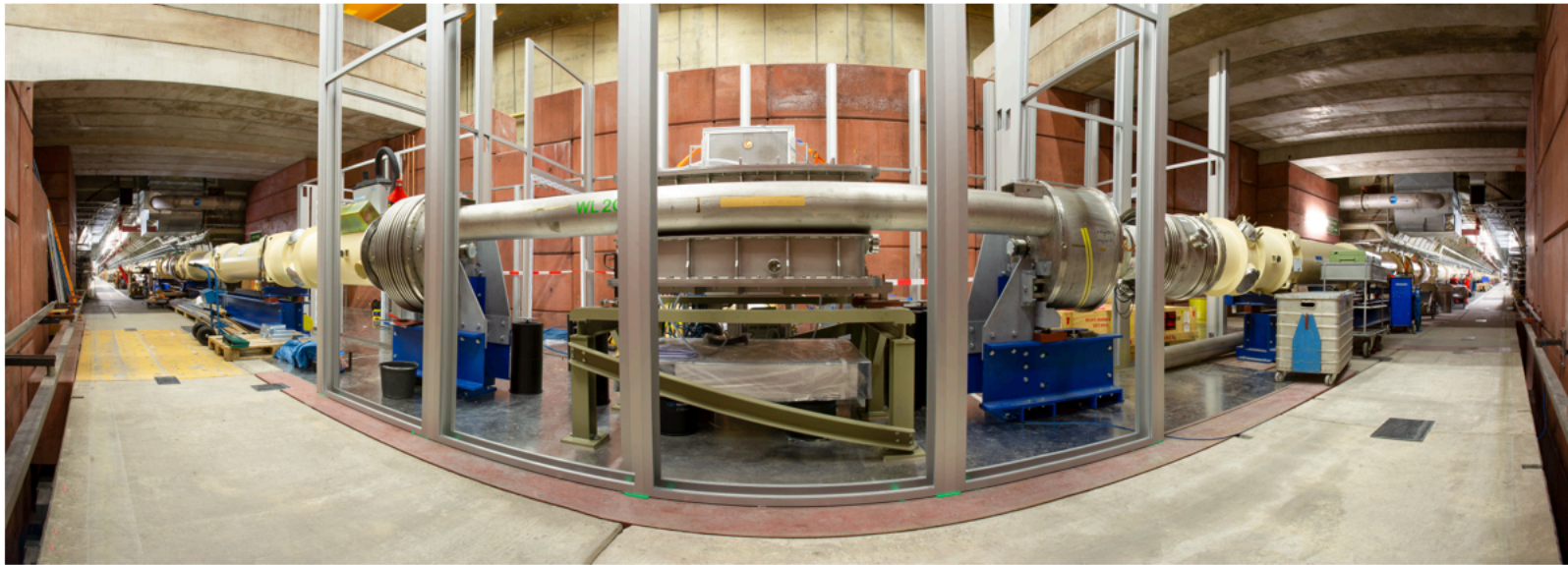
- Coupling to two photons: $\frac{1}{M} a \tilde{F} F \sim \frac{1}{M} a \vec{E} \cdot \vec{B}$

Light shining through walls

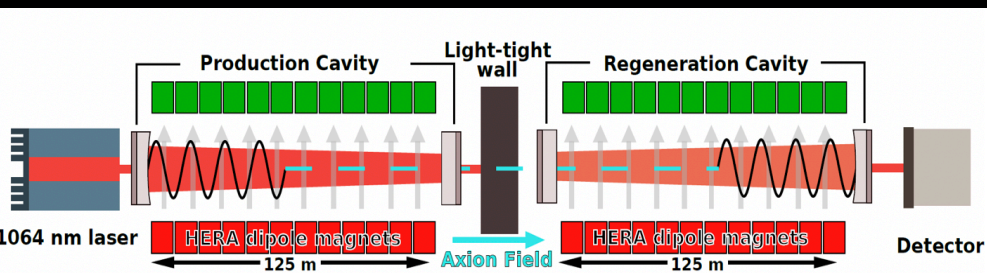


$$P_{\gamma \rightarrow a \rightarrow \gamma} \sim N_{\text{pass}} \left(\frac{BL}{M} \right)^4$$

ALPS II



ALPS II under construction as of October 2020 (Copyright DESY / M. Mayer)



DESY. KET Strategy 2022 | Axions | 19 Nov. 2022 | Axel Lindner

ALPS

2022-11-16 11:22:27

ACCESS
TUNNEL WEST / NORTH

ACCESS
ALPS EXPERIMENT

Legend

- Laser ON
- Laser OFF
- Pers. Interlock set

Optics Preparation

November 2022

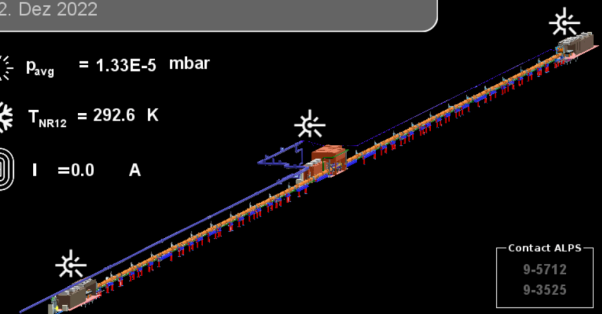
Magnet String Cool-down (Access restricted)

12. Dez 2022

$P_{avg} = 1.33E-5 \text{ mbar}$

$T_{NR12} = 292.6 \text{ K}$

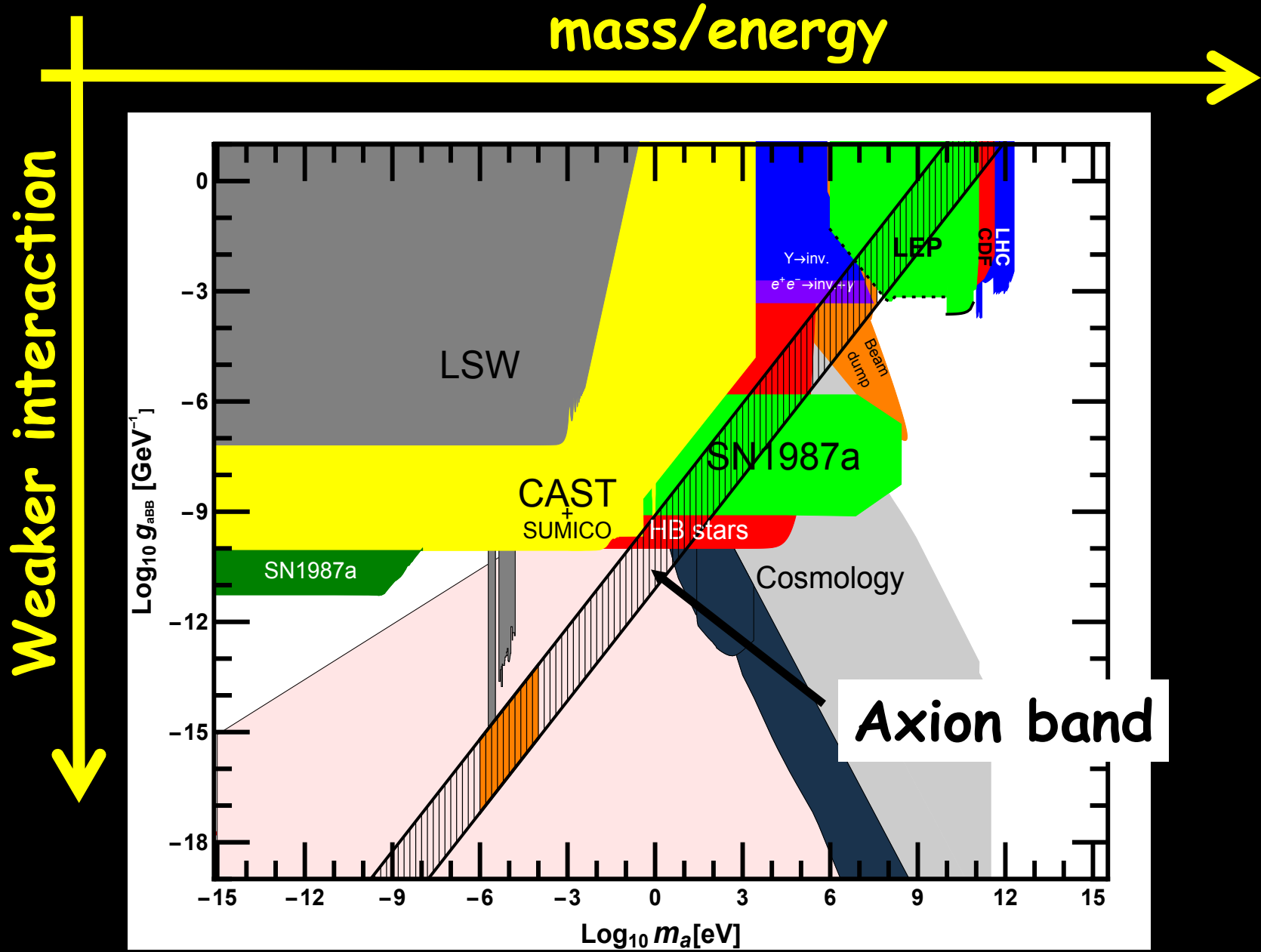
$I = 0.0 \text{ A}$



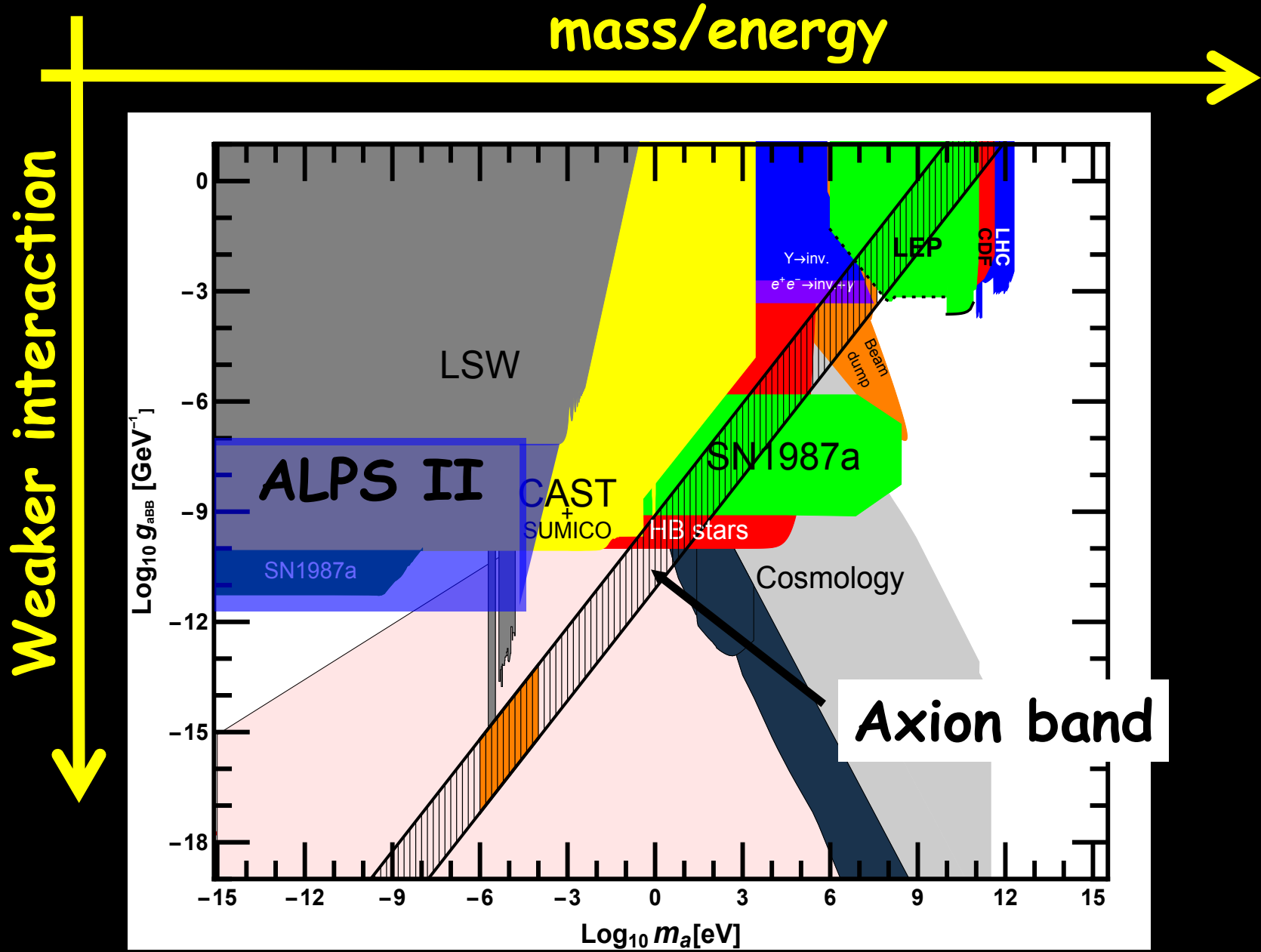
Contact ALPS

9-5712
9-3525

Small coupling, small mass



Small coupling, small mass

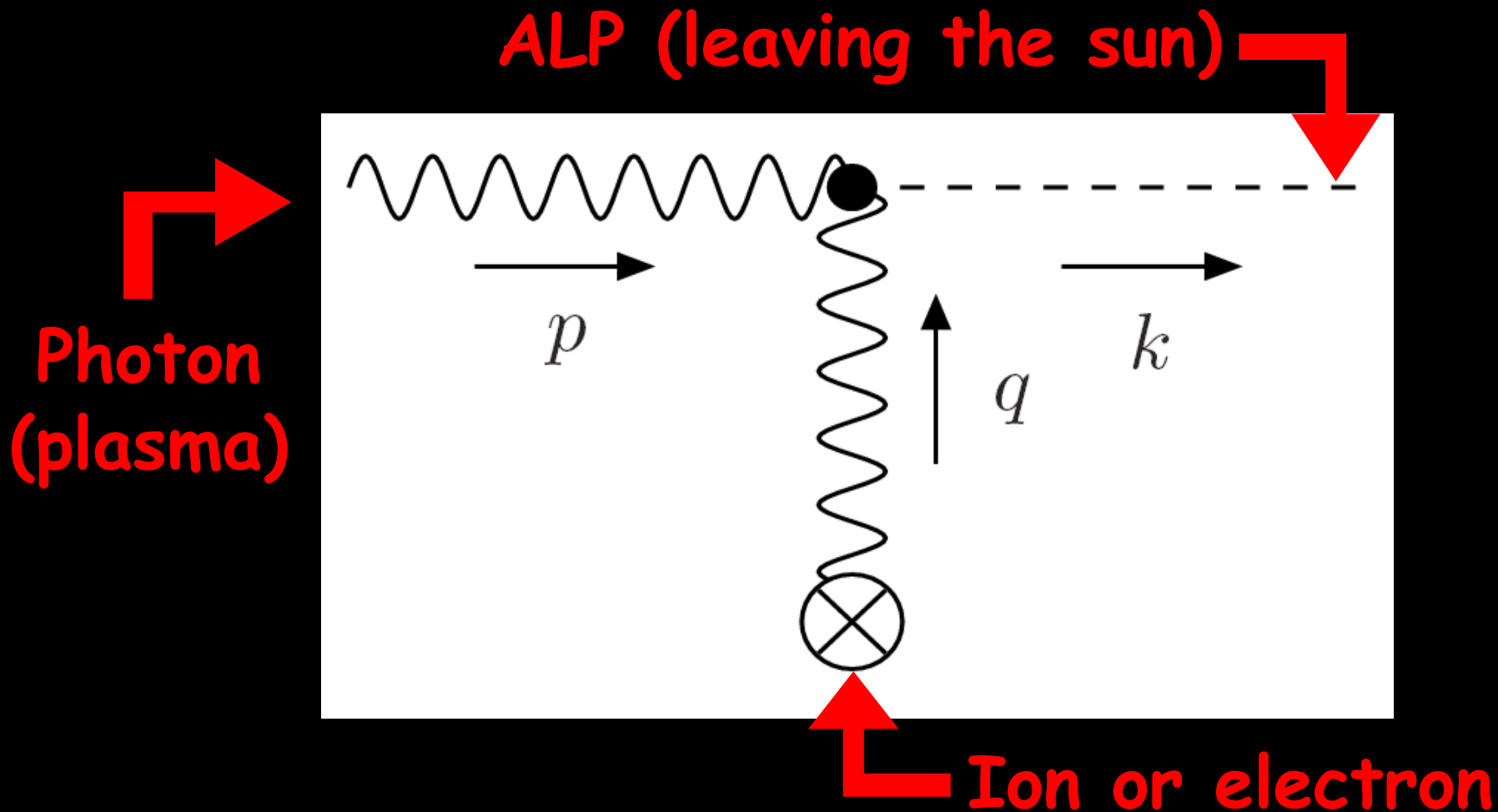


Interlude:

Think and make life hard
for experimentalists...

Energy loss in stars.

- Primakoff process (in the sun)



We would freeze...

- If the coupling g is too large the sun would have died long ago.
- Why?

Axions can leave the sun without further interaction (in contrast to photons)

- ➔ Large energy loss from axion emission
 - ➔ Sun burns fuel faster
 - ➔ Sun would have died long ago
-

A (Very) Moderate Bound

- Without ALPs sun has fuel for about 10^{10} years
- Energy loss via ALPs:

- Sun Lifetime with ALPs

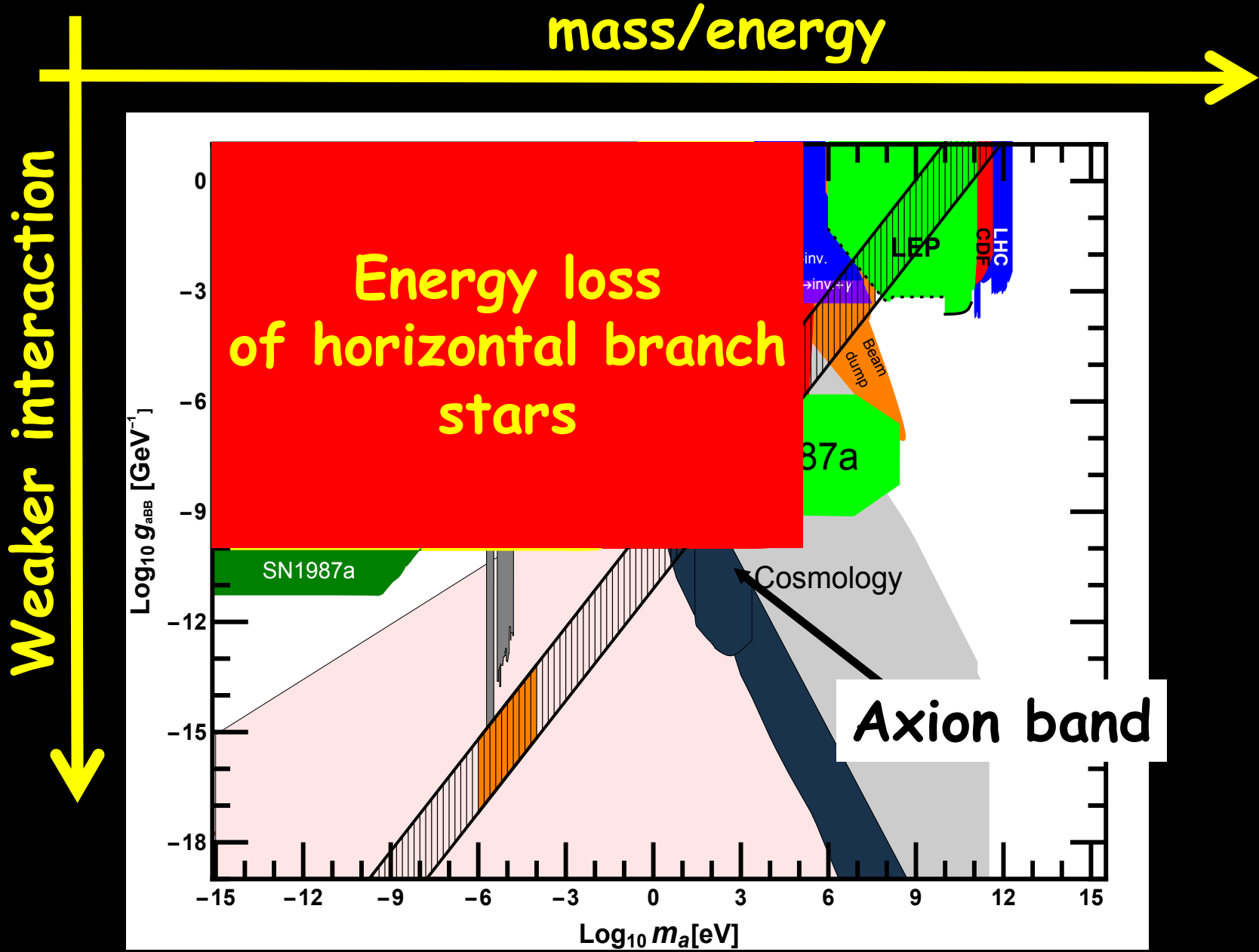
$$L_a \approx 1.7 \cdot 10^9 (g \cdot 10^4 \text{ GeV})^2 L_\gamma$$

- Pretty sure sun has been around for more than 10 years

$$t_{sun} \sim 10 \text{ years} \times (g \cdot 10^4 \text{ GeV})^{-2}$$

 $g \leq 10^{-4} \text{ GeV}^{-1}$

A Real killer bound



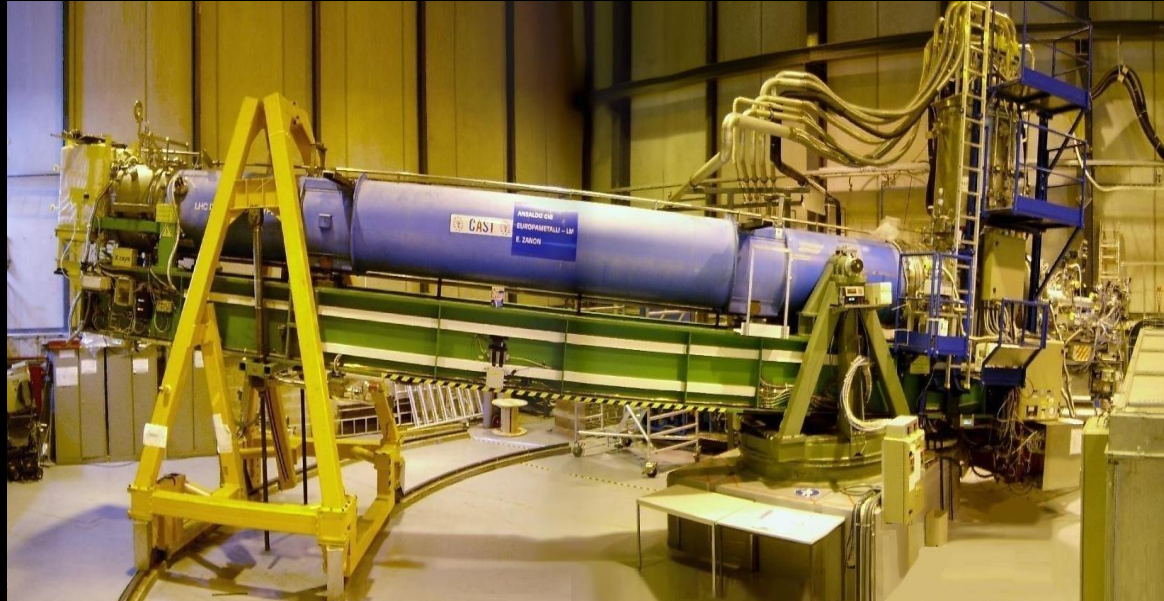
Back to experiments...

Helioscopes

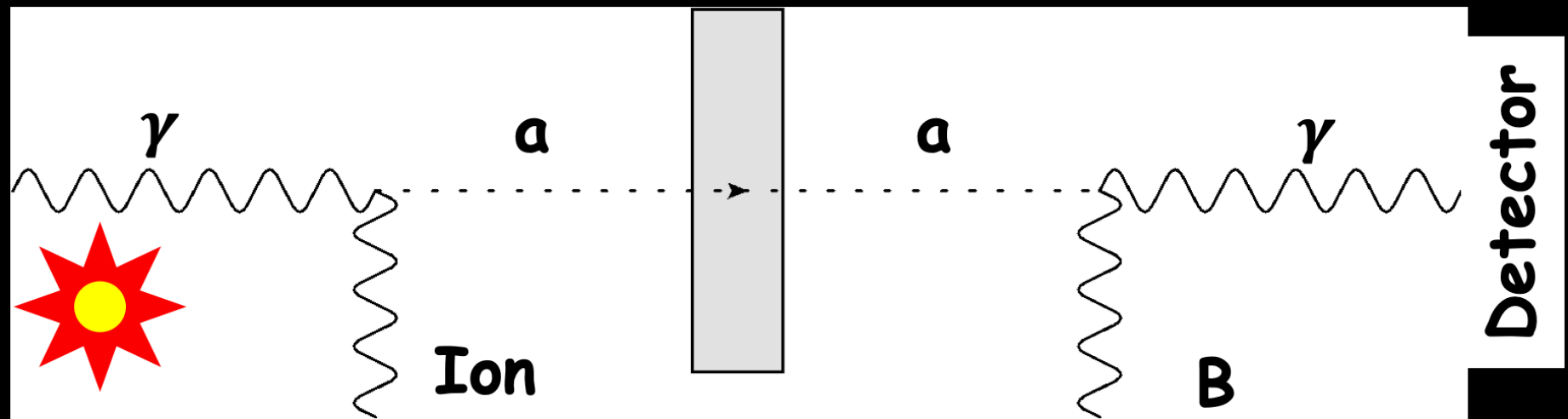
CAST@CERN

SUMICO@Tokyo

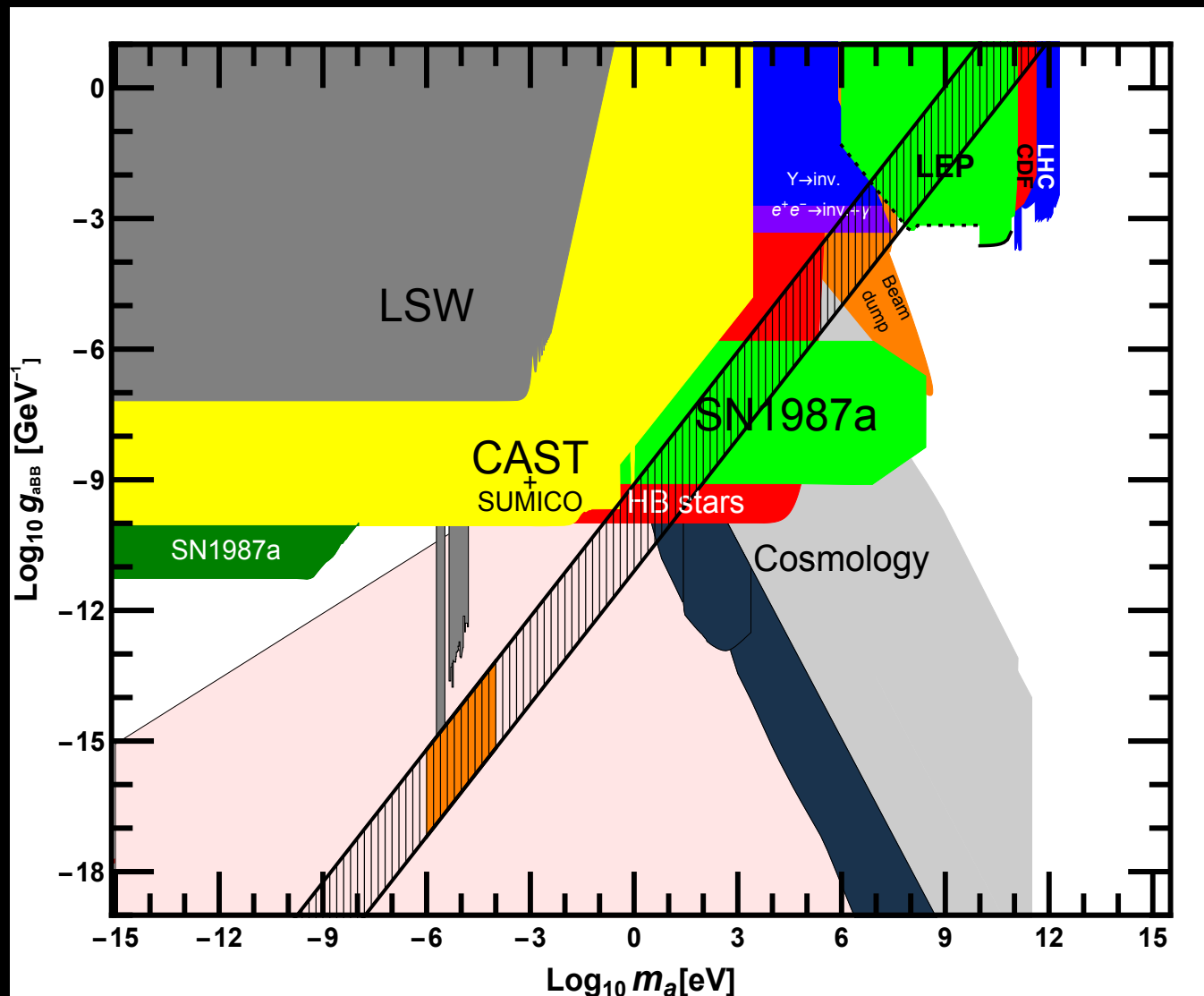
SHIPS@Hamburg



Light shining through walls



Sensitivity



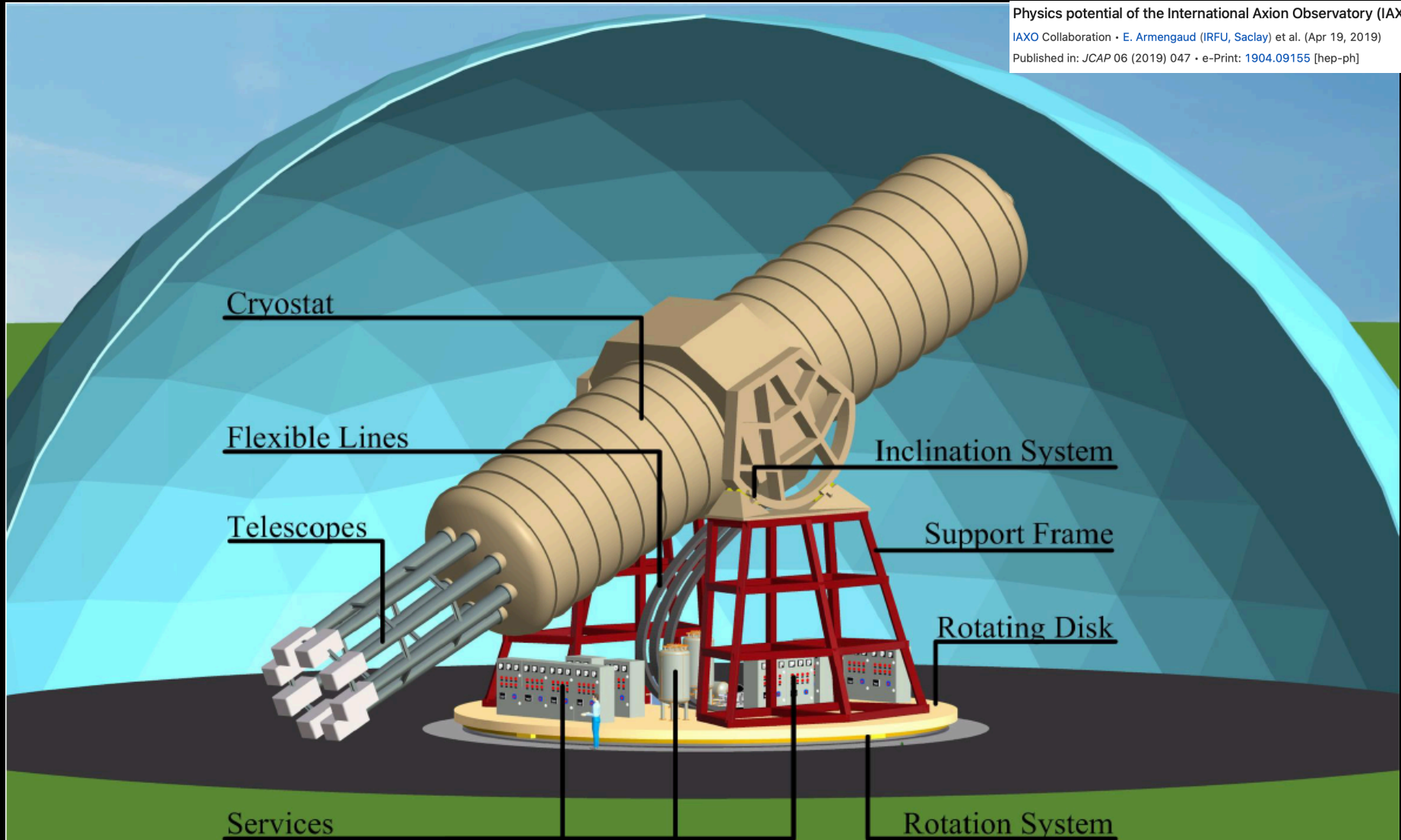
Going to the future: IAXO

The International Axion Observatory

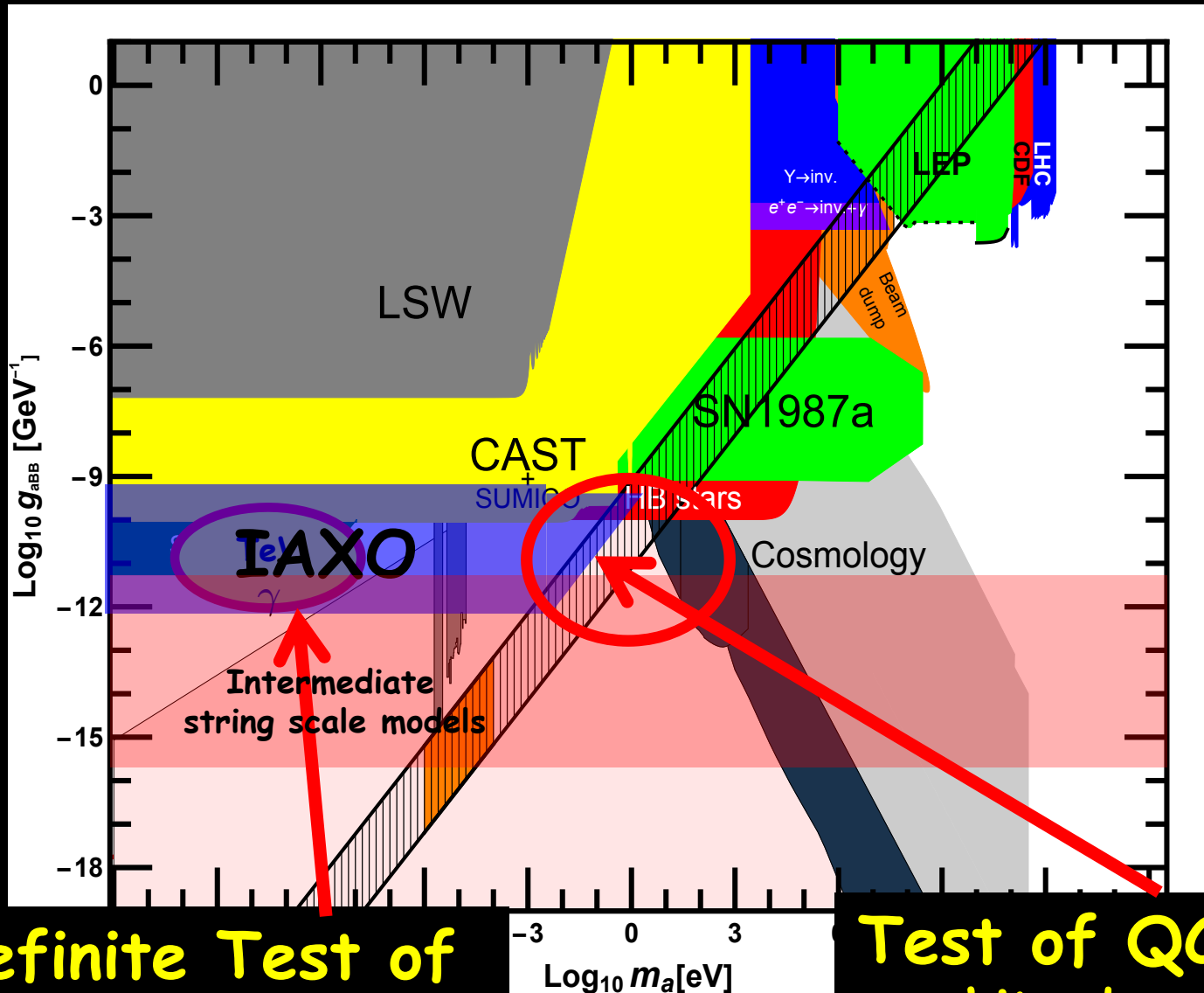
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Published in: JCAP 06 (2019) 047 • e-Print: 1904.09155 [hep-ph]



An interesting area...



Definite Test of
TeV transparency

Test of QCD axion
+ white dwarf anomaly

Can Dark Matter
be Axions/ALPs?

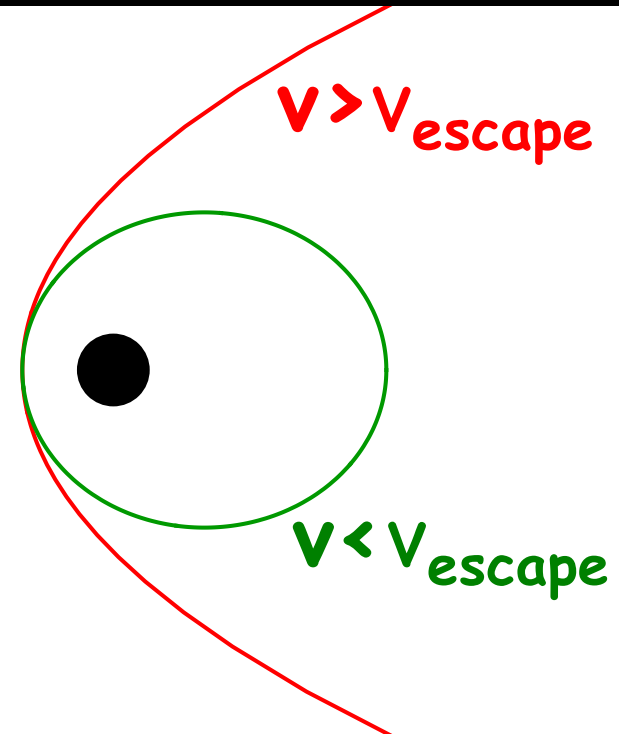
- Dark matter is dark, i.e.
it doesn't radiate!
(and also doesn't absorb)
- very, very weak interactions with light
and with ordinary matter
- Exactly the property of
Axions

A common prejudice

- Dark Matter has to be heavy: $m_{\text{DM}} \gtrsim \text{keV}$.
- Prejudice based on thermal production!
and/or fermionic DM!

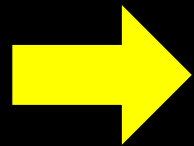
Both assumptions give
minimal velocity

→ galaxy,
i.e. structure,
formation inhibited!



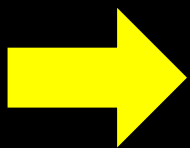
Weakly interacting sub-eV DM

- Has to be non-thermally (cold!!!) produced



See misalignment mechanism

- Bosonic!



Axion(-like particles)
Hidden Photons

Dark matter has to be heavy...

Dark matter has to be heavy $m_{\text{DM}} \gtrsim \text{keV}$?

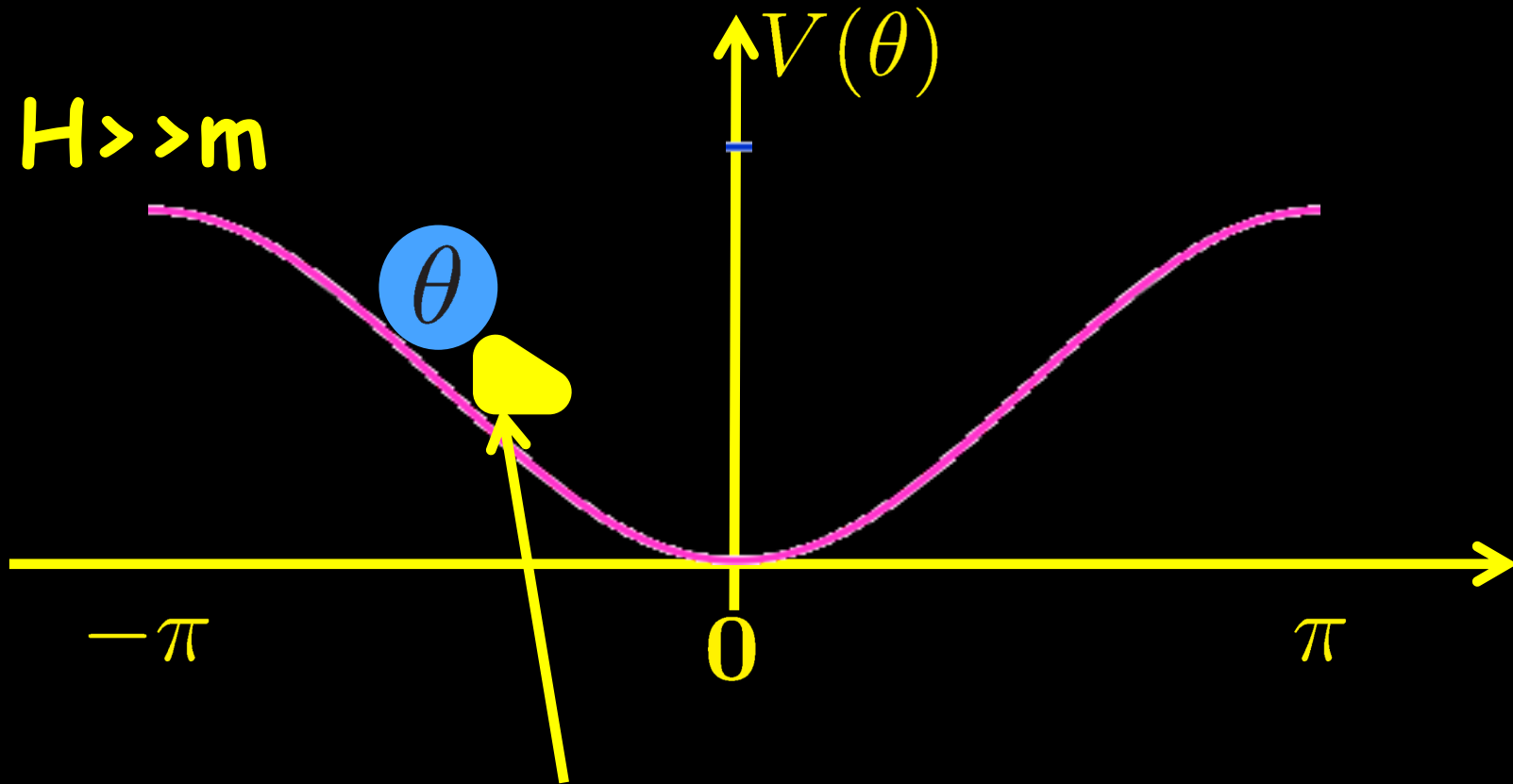
Dark matter has to be heavy...

Dark matter has heavy $m_{\text{DM}} \gtrsim \text{keV}$?

MYTH BUSTED

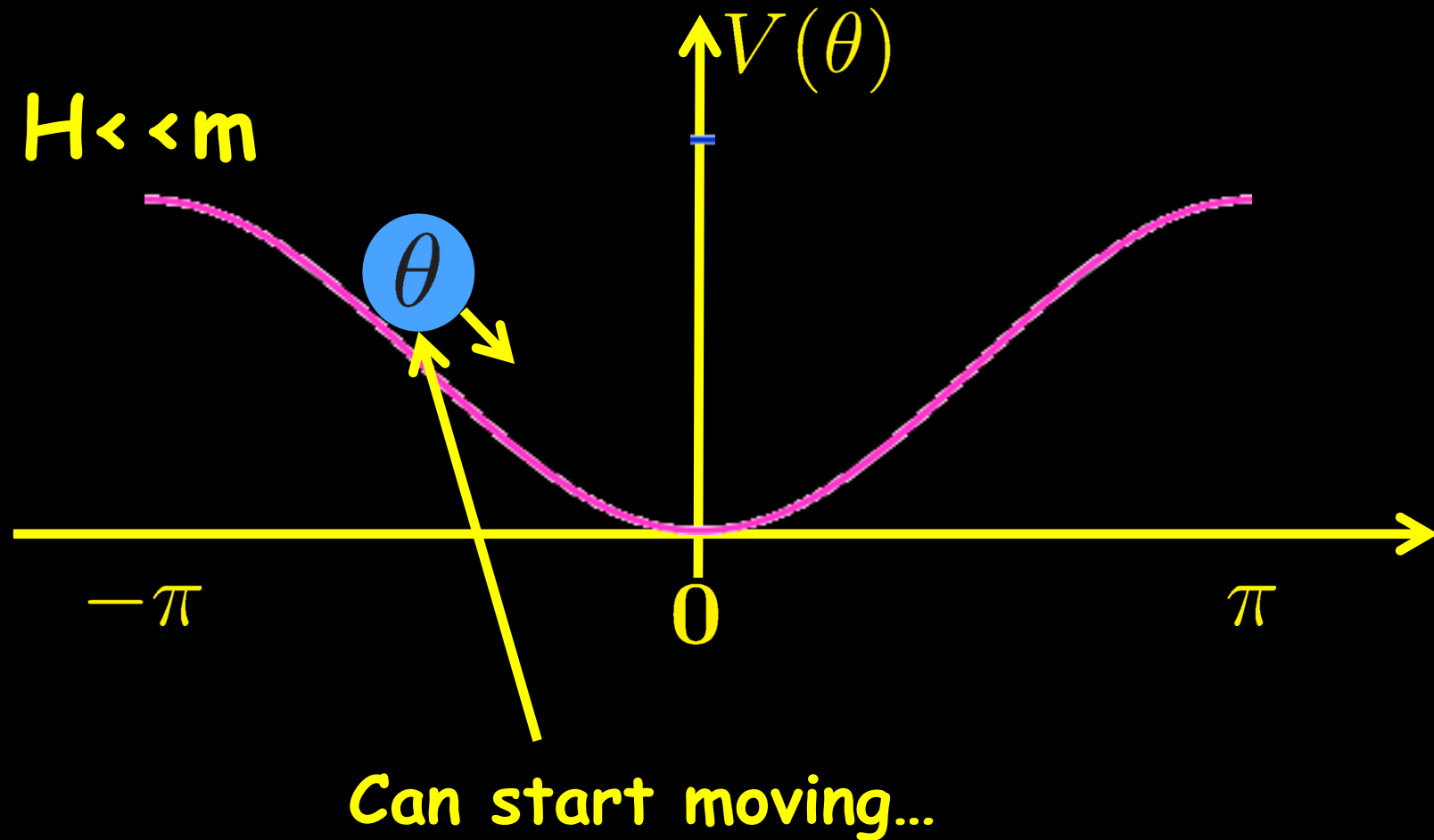
SuperCold Dark Matter

The axion has no clue where to start

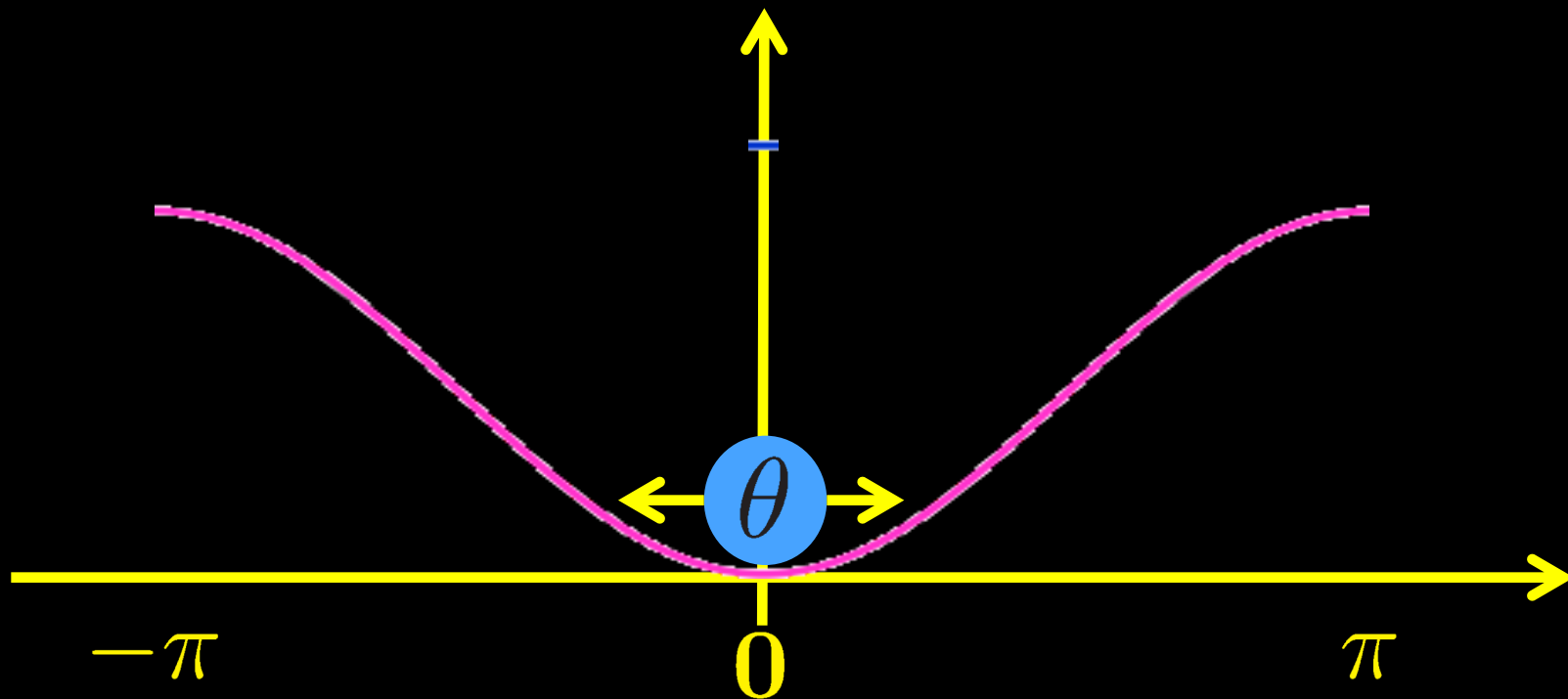


Field is stuck because of Hubble "breaking"

The axion has no clue where to start



The axion solution to the strong CP problem



- Oscillations contain energy
- behave like non-relativistic particles ($T=0$)

Axion Dark Matter

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

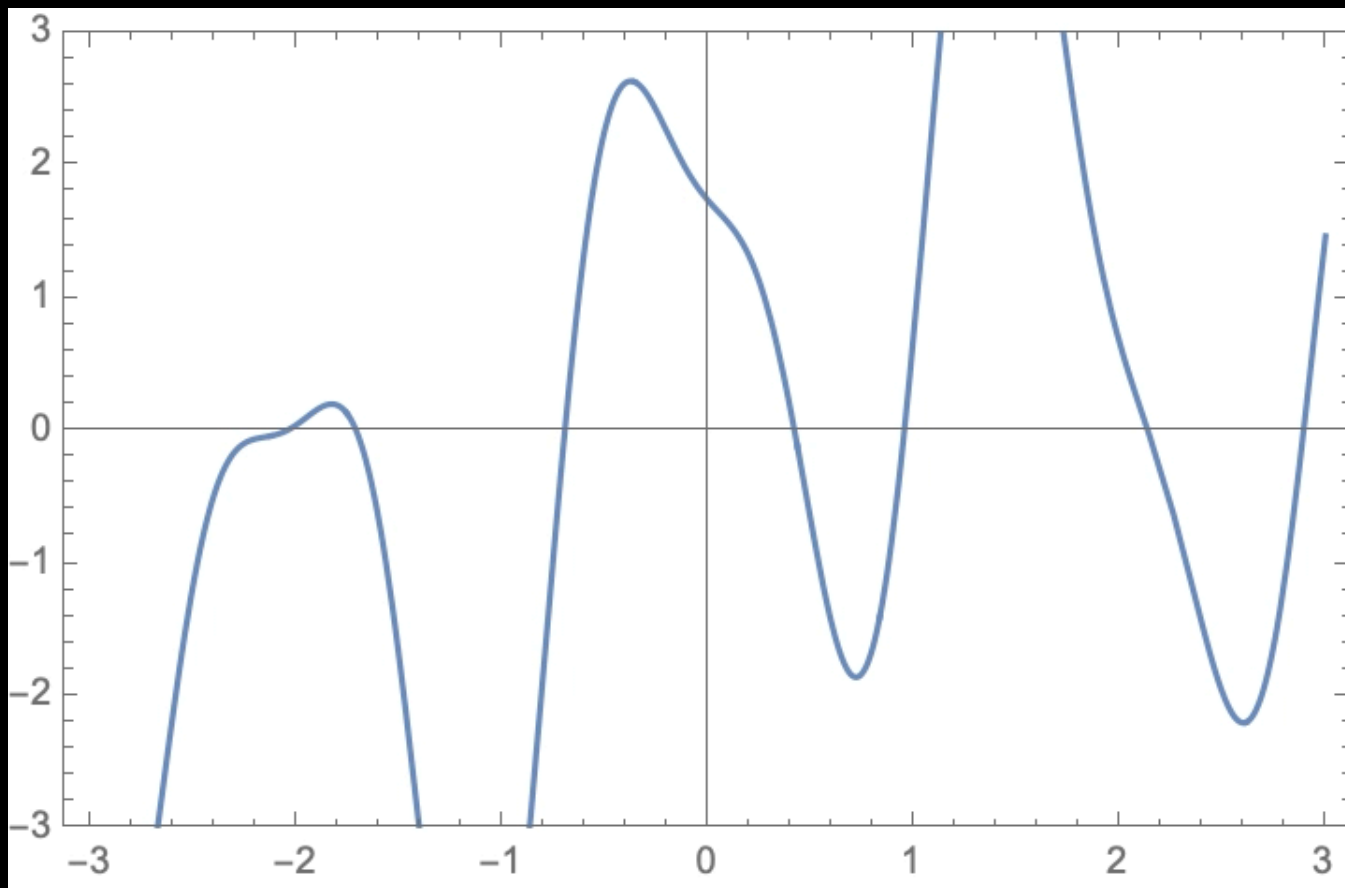
$$H = \frac{\dot{R}(t)}{R(t)}$$

- $H \gg m_a \rightarrow$ overdamped oscillator
- $H \ll m_a \rightarrow$ damped oscillator

$$\rho_a(t) = \frac{\rho_{ini}}{R^3(t)} \rightarrow \text{Dark Matter}$$

Why Cold? Inflation!

Field
value



space

$$velocity \sim \frac{p}{m} \sim \frac{\hbar}{m} \frac{d}{dx} \rightarrow 0$$

The Amount

- Determined by the initial density

$$\rho_1 \sim \frac{1}{2} m_1^2 \phi_1^2$$

- + when the oscillation/dilution starts

$$H(T_1) \sim m_1$$

→ Dilution

$$\frac{\rho_0}{\rho_1} \sim \frac{V_1}{V_0} \sim \frac{a_1^3}{a_0^3}$$

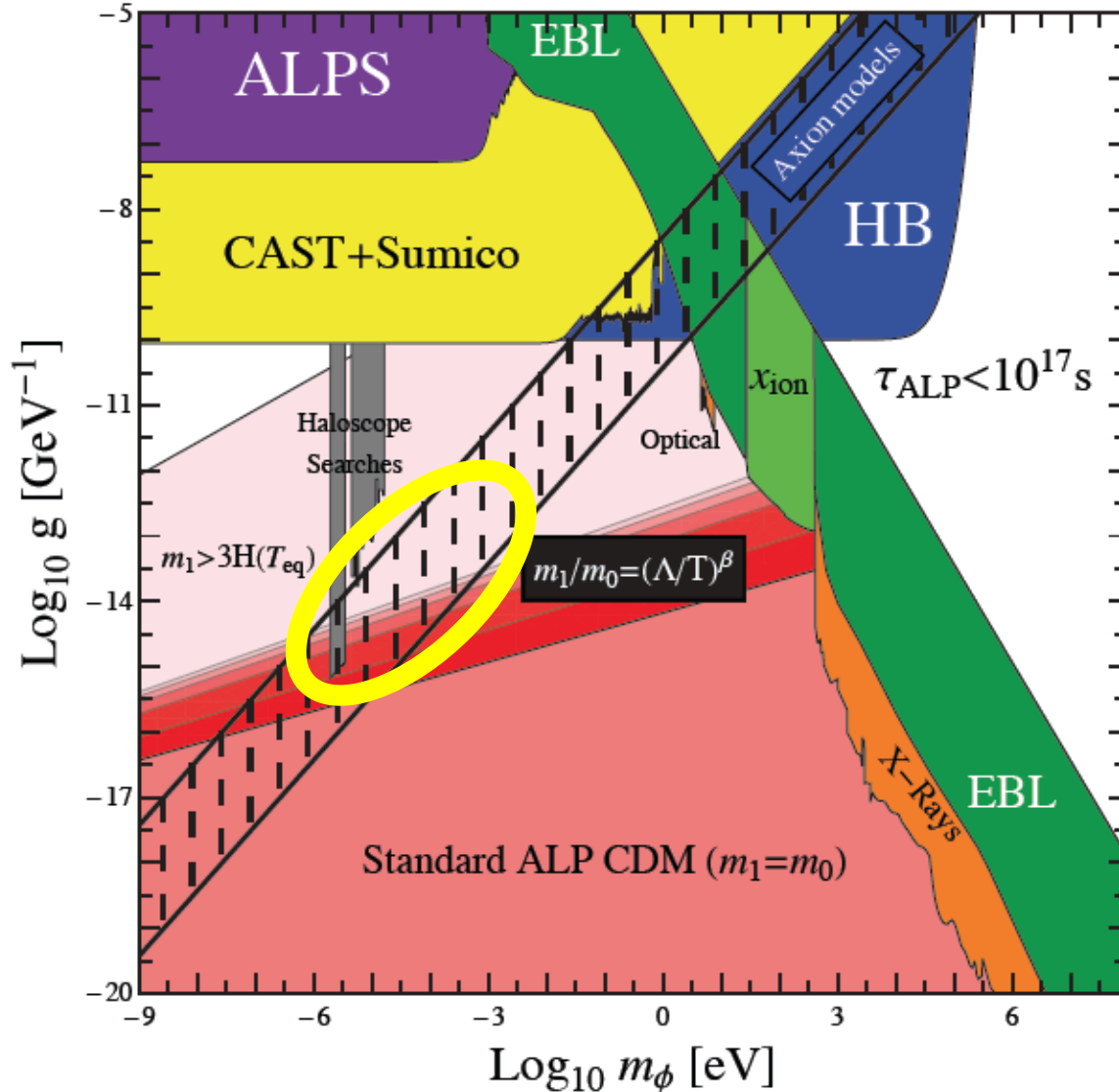
$$\rho_{\phi,0} \simeq 0.17 \frac{\text{keV}}{\text{cm}^3} \times \sqrt{\frac{m_0}{\text{eV}}} \sqrt{\frac{m_0}{m_1}} \left(\frac{\phi_1}{10^{11} \text{ GeV}} \right)^2 \mathcal{F}(T_1)$$

$$\mathcal{F}(T_1) = \frac{(g_*(T_1)/3.36)^{\frac{3}{4}}}{(g_{*S}(T_1)/3.91)}$$

Axion(-like particle) Dark Matter

$\sim 10^7 \text{ GeV}$

$\sim 10^{12} \text{ GeV}$



An underappreciated feature

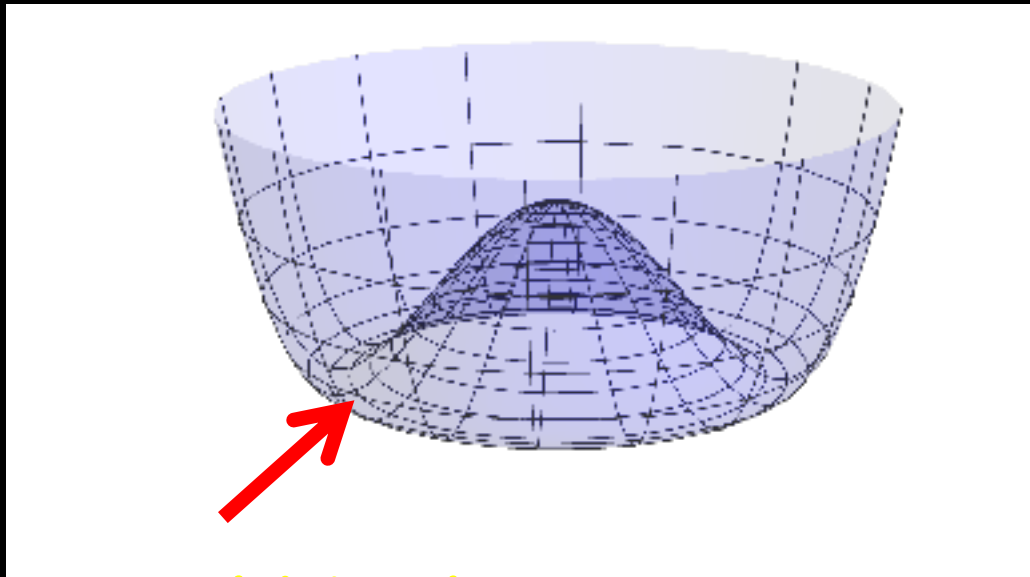
String axions have ``pre-inflationary'' cosmo!

Why?

- Actually do not result from the usual spontaneous symmetry breaking
- Exist during inflation
(otherwise don't understand string inflation)
- Axion string tension expected to be higher than string scale
- Too high temperature → Decompactification

Post-inflationary axions

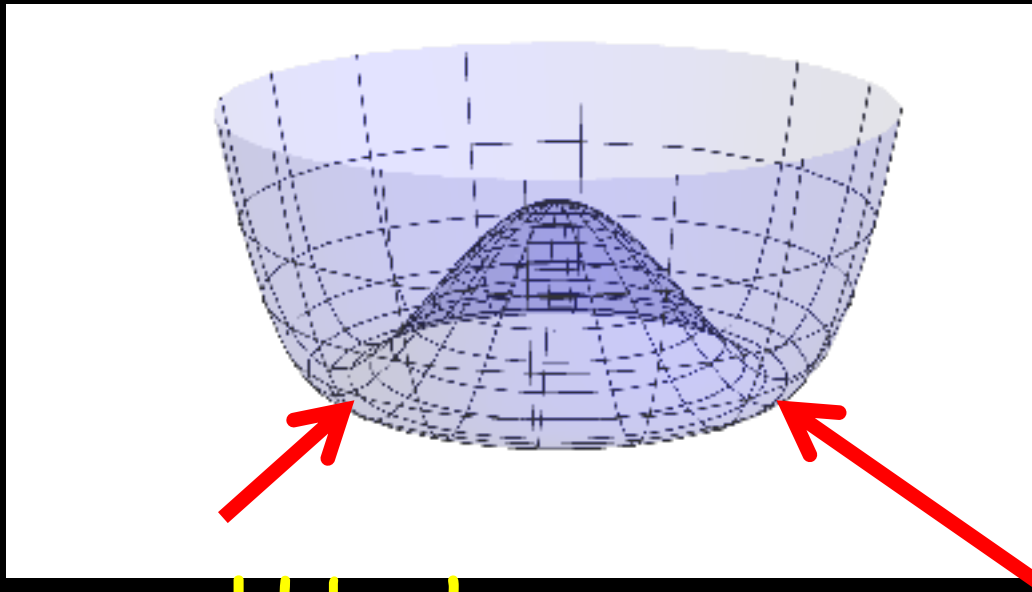
- (Last) Spontaneous symmetry breaking of Peccei-Quinn symmetry could also happen **after** inflation



Axion could be here

Post-inflationary axions

- (Last) Spontaneous symmetry breaking of Peccei-Quinn symmetry could also happen **after** inflation

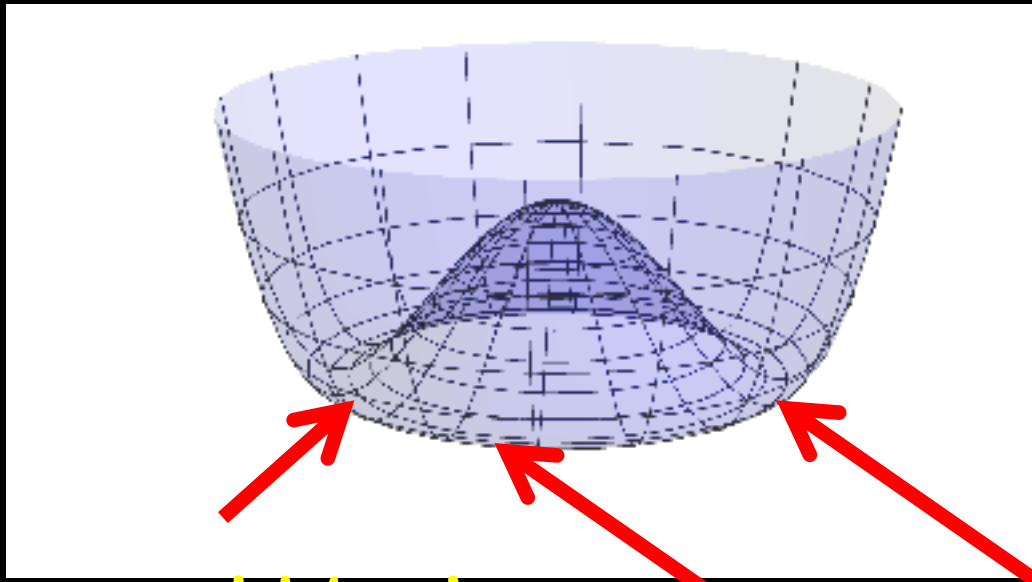


Axion could be here

Or here

Post-inflationary axions

- (Last) Spontaneous symmetry breaking of Peccei-Quinn symmetry could also happen **after** inflation

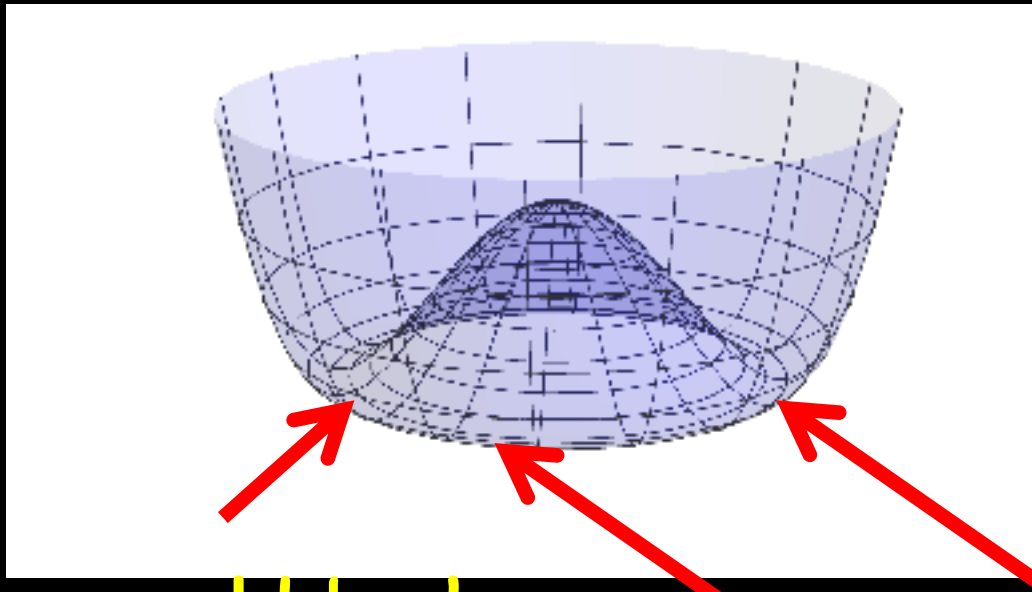


Axion could be here

Or here Or here

Post-inflationary axions

- (Last) Spontaneous symmetry breaking of Peccei-Quinn symmetry could also happen **after** inflation



Axion could be here

Or here Or here

Different Hubble patches \rightarrow different place

Features

- Average density predicted
- No initial condition problem

$$\rho_{average} \sim \langle \phi^2 \rangle \sim f_a^2 \langle \theta^2 \rangle \frac{2}{3} \pi^2 f_a^2$$

- But large fluctuations between different Hubble patches
-

Features

- Average density predicted
- No initial condition problem

$$\rho_{average} \sim \langle \phi^2 \rangle \sim f_a^2 \langle \theta^2 \rangle \frac{2}{3} \pi^2 f_a^2$$

- But large fluctuations between different Hubble patches

→ Problem? Isocurvature fluctuations?

Features

- Average density predicted
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$$\rho_{average} \sim \langle \phi^2 \rangle \sim f_a^2 \langle \theta^2 \rangle \frac{2}{3} \pi^2 f_a^2$$

- But large fluctuations between different Hubble patches

→ Problem? Isocurvature fluctuations?

→ NO. Patches today are still tiny; solar system sized → we do not know anything

Features

- Average density predicted
- No initial condition problem

$$\rho_{average} \sim \langle \phi^2 \rangle \sim f_a^2 \langle \theta^2 \rangle \frac{2}{3} \pi^2 f_a^2$$

- But large fluctuations between different Hubble patches
 - Some patches very overdense
 → collapse into “miniclusters”
-

Strings...

- $U(1)$ symmetry breaking leads to topological defects...

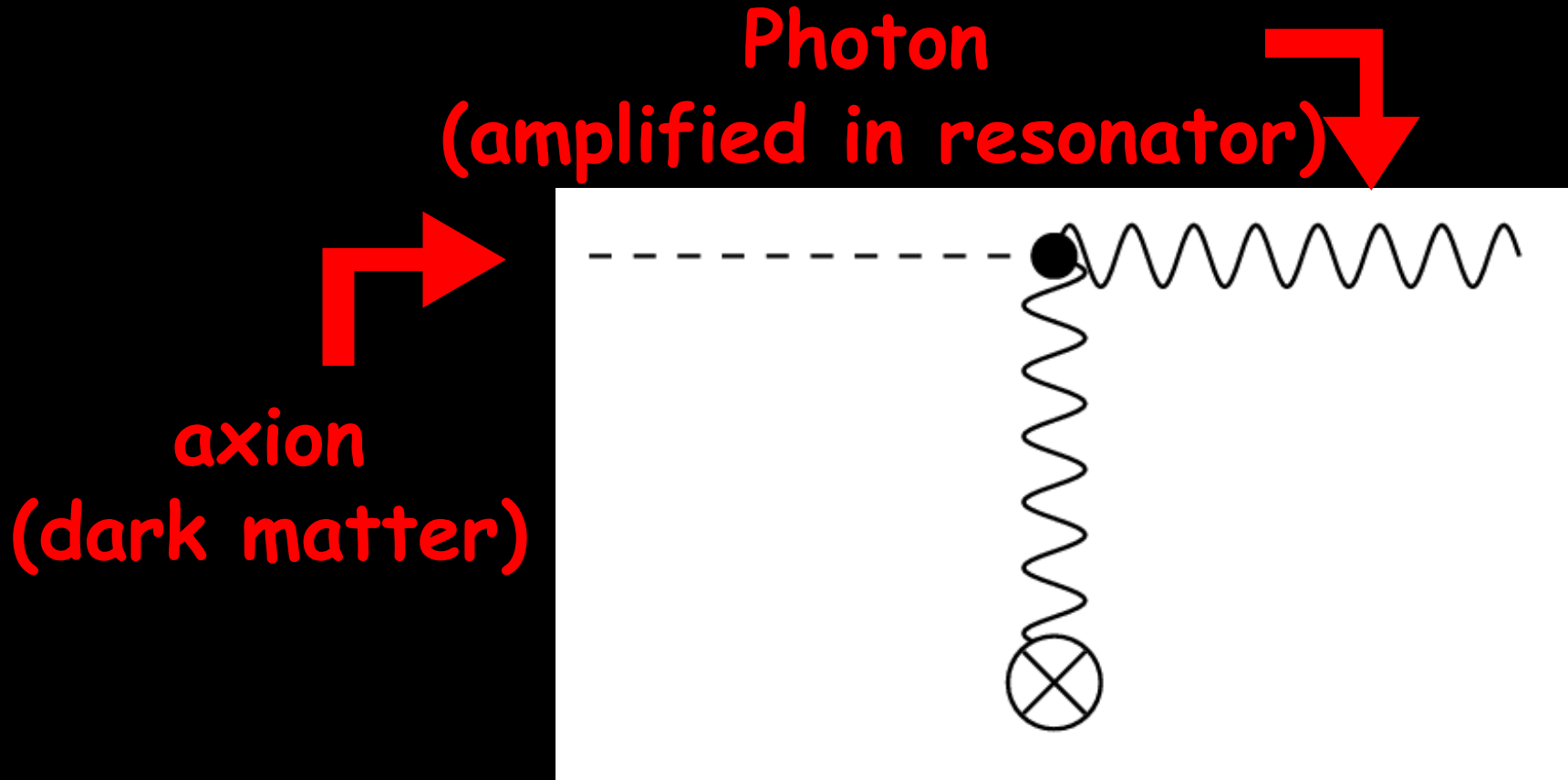
Strings...

- U(1) symmetry breaking leads to topological defects...
 - Strings could contribute significantly to energy density
 - But... Not yet able to fully calculate
-

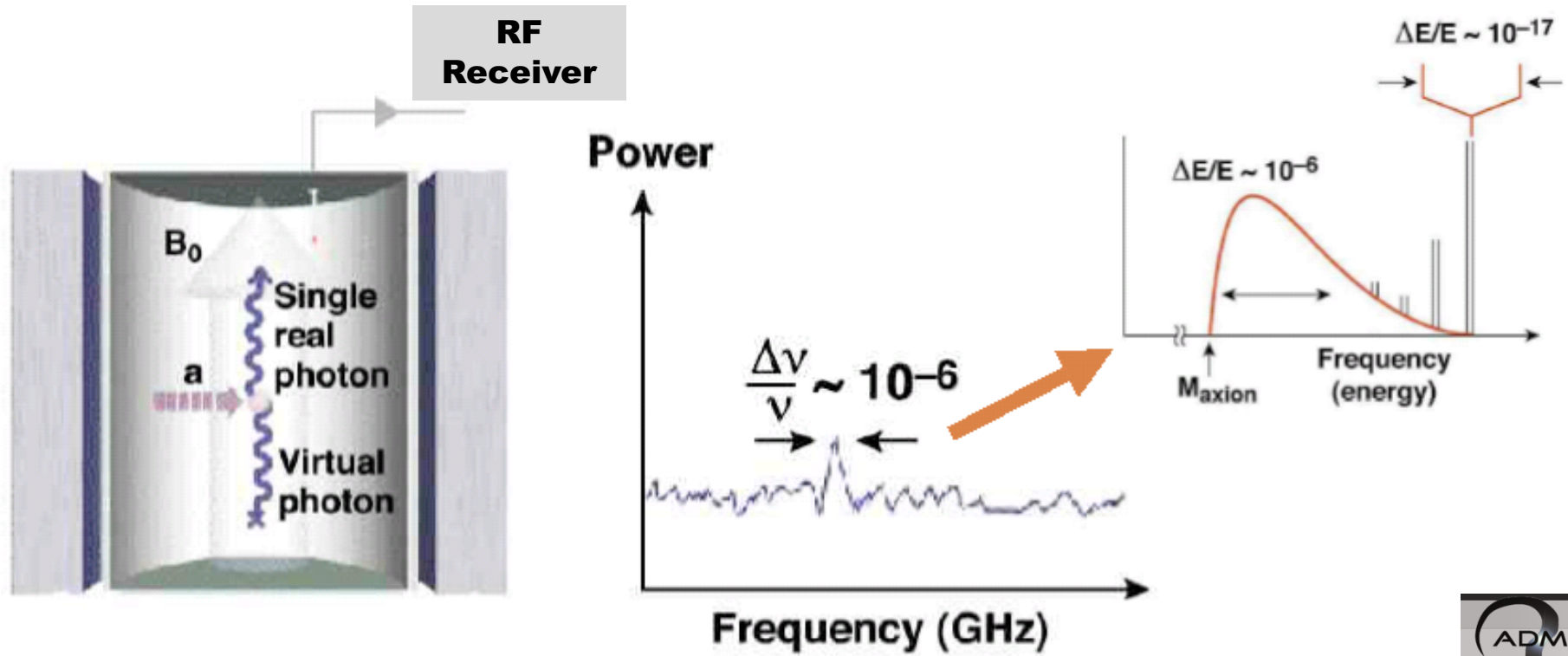
Detecting
Axion/WISPy
DM

Use a plentiful source of axions

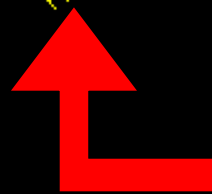
- Photon Regeneration



Signal: Total energy of axion

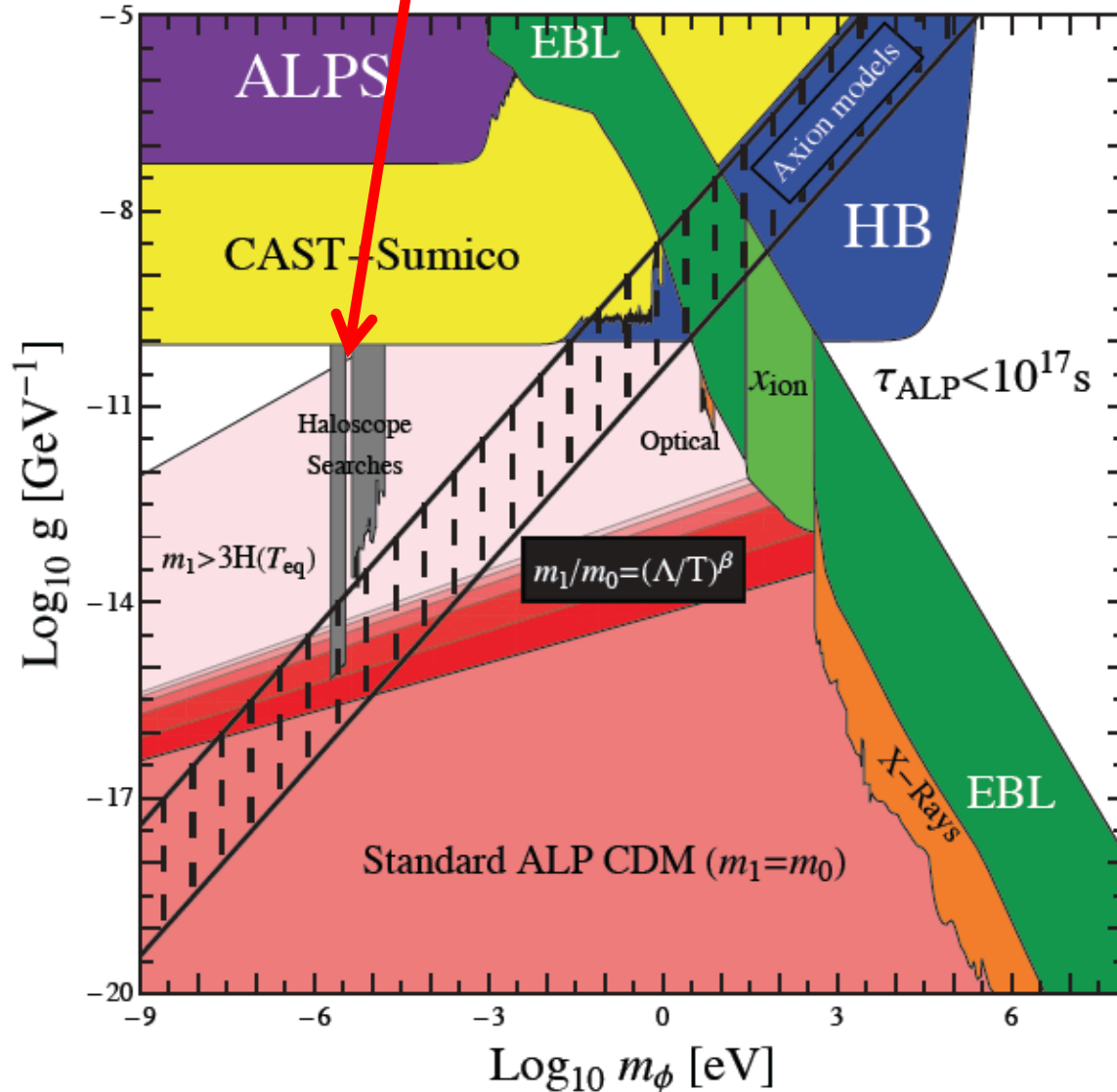


$$h\nu = m_a c^2 [1 + \mathcal{O}(\beta^2 \sim 10^{-6})]$$

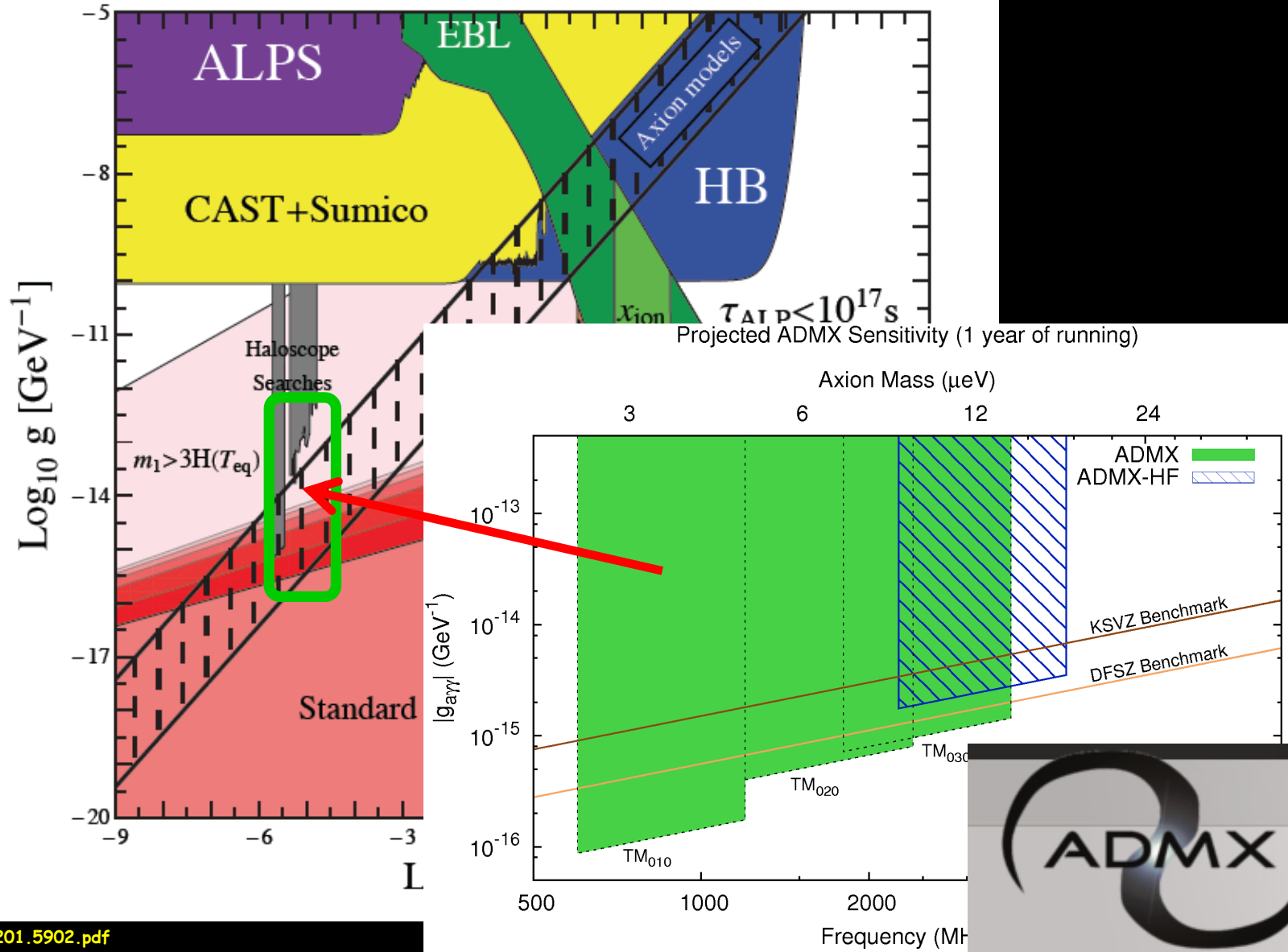


Virial velocity
in galaxy halo!

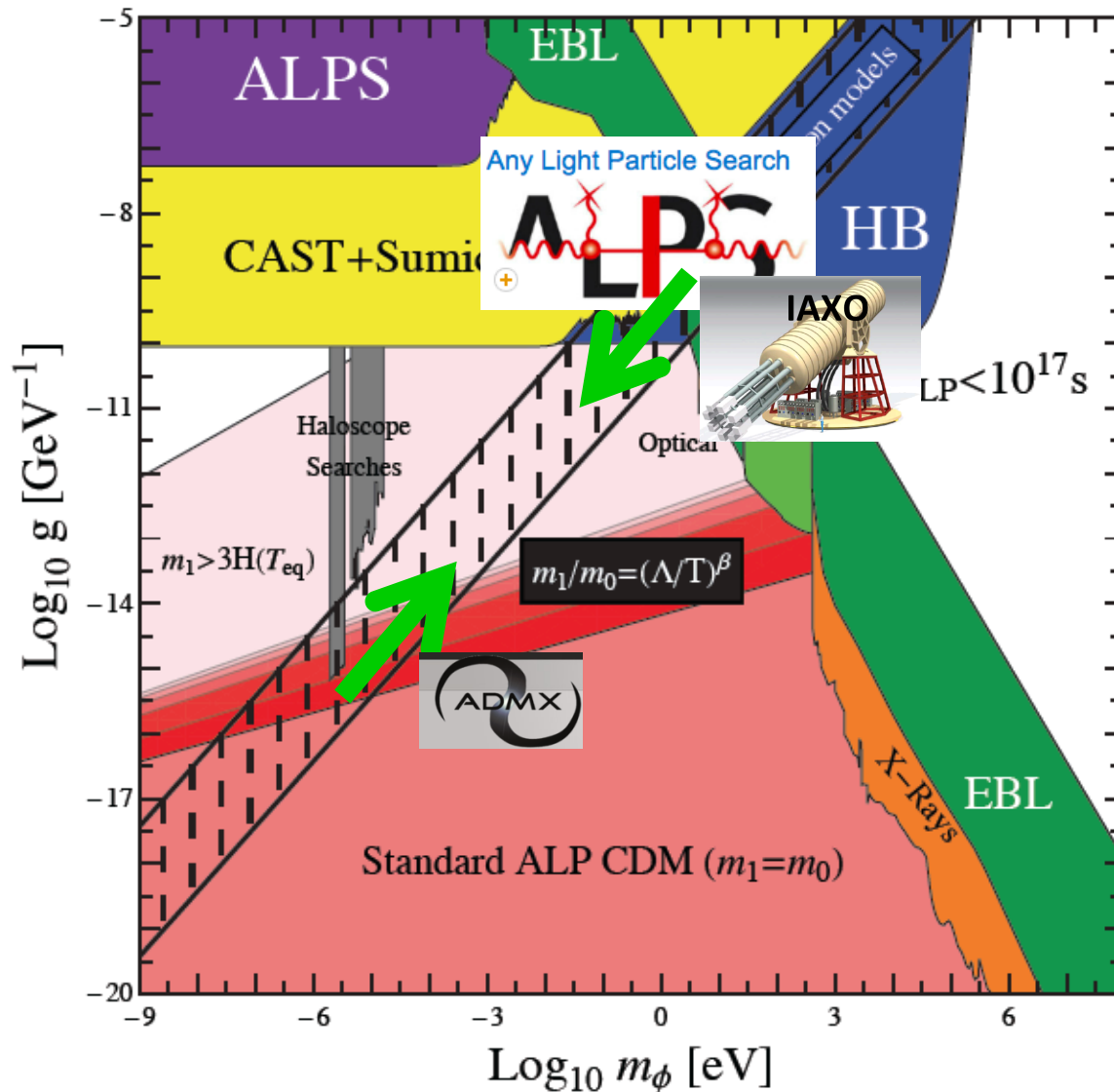
An extremely sensitive probe!!!



A discovery possible any minute!



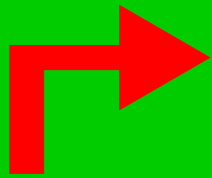
Encircling the axion...



Electricity from Dark Matter :-).

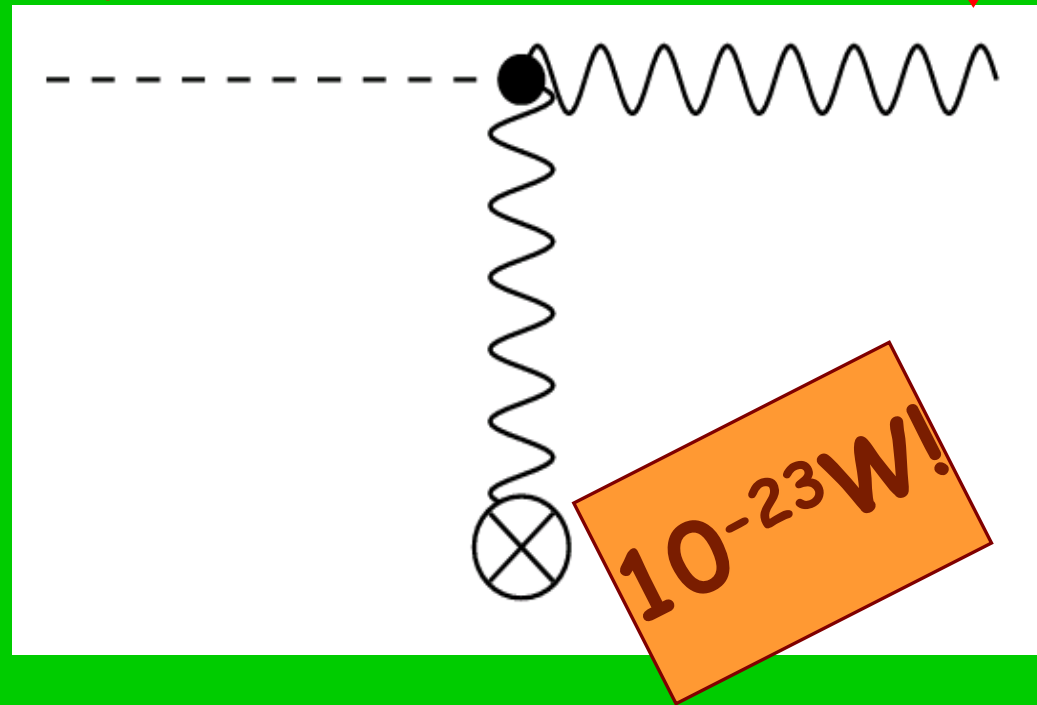
- Photon Regeneration

Photon
(amplified in resonator)



axion

(dark matter)



Really sustainable Energy

- Galaxy contains $(6-30) \times 10^{11}$ solar masses of DM

→ $(3-15) \times 10^{43}$ TWh

@100000 TWh per year (total world today)

→ 10^{38} years ☺

DM power

$$\rho * v \sim 300 \text{ MeV/cm}^3 * 300 \text{ km/s} \sim 10 \text{ W/m}^2$$

compared to 2 W/m^2 for wind

Summary III

Summary III

- Axions can be searched for in high sensitivity lab experiments
 - Axions can be dark matter
 - Dark matter axions even better testable
 - Dark Astronomy may follow soon
-

Conclusions

- The axion provides a solution to the tuning of the theta-angle
- This is a dynamical solution in the sense that theta-evolves to 0 during the Universe' evolution
- Predictive → Testable in the not too distant future!
- Makes a **super-cool** Dark Matter candidate
- Searches ongoing and could lead to "Dark Astronomy"