Large-scale structure

Marko Simonović
University of Florence

ISAPP 2024: Particle Candidates for Dark Matter

Lecture 3

One loop-matter power spectrum

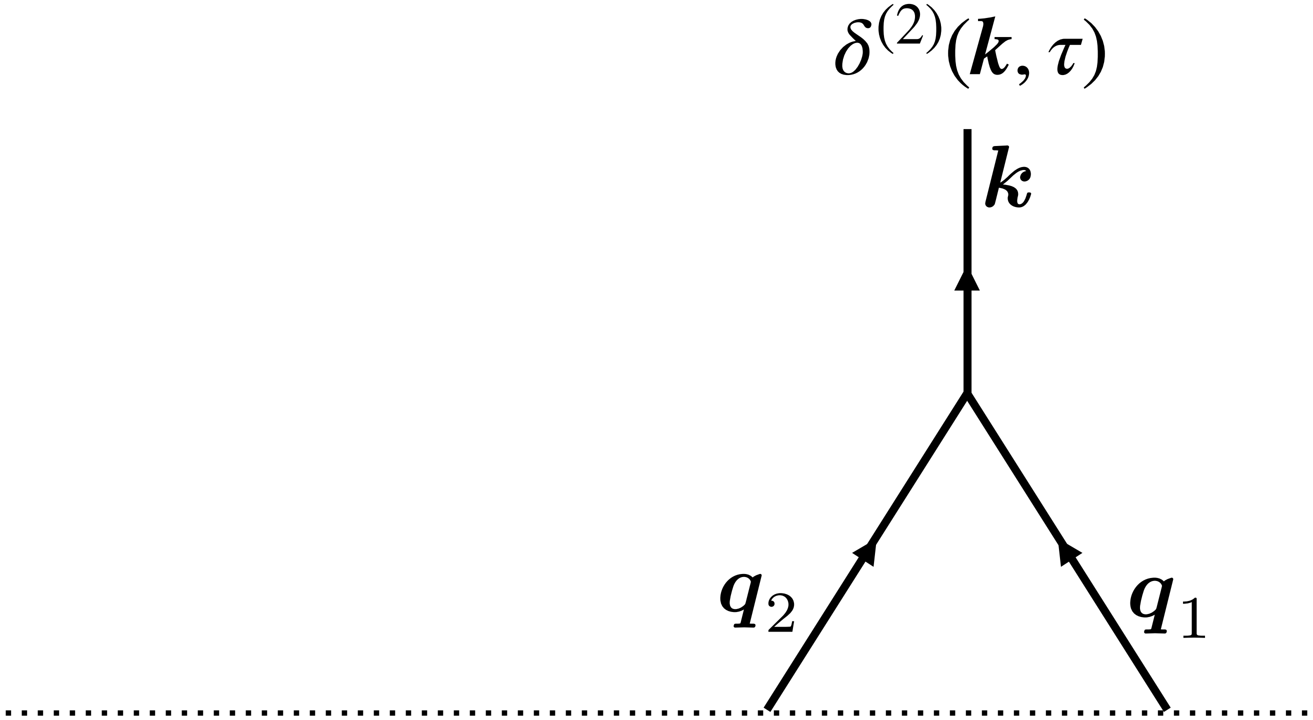
Problems with SPT and EFTofLSS

Infrared resummation

Nonlinear solutions

$$\delta^{(2)}(\boldsymbol{k},\tau) = \int_{\boldsymbol{q}_1}\int_{\boldsymbol{q}_2} (2\pi)^3\delta^D(\boldsymbol{k}-\boldsymbol{q}_1-\boldsymbol{q}_2)\, F_2(\boldsymbol{q}_1,\boldsymbol{q}_2)\, \delta^{(1)}(\boldsymbol{q}_1,\tau)\delta^{(1)}(\boldsymbol{q}_2,\tau)$$

$$F_2(\boldsymbol{q}_1,\boldsymbol{q}_2) = \frac{5}{7} + \frac{1}{2}\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{q_1 q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right) + \frac{2}{7}\frac{(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)^2}{q_1^2 q_2^2}$$

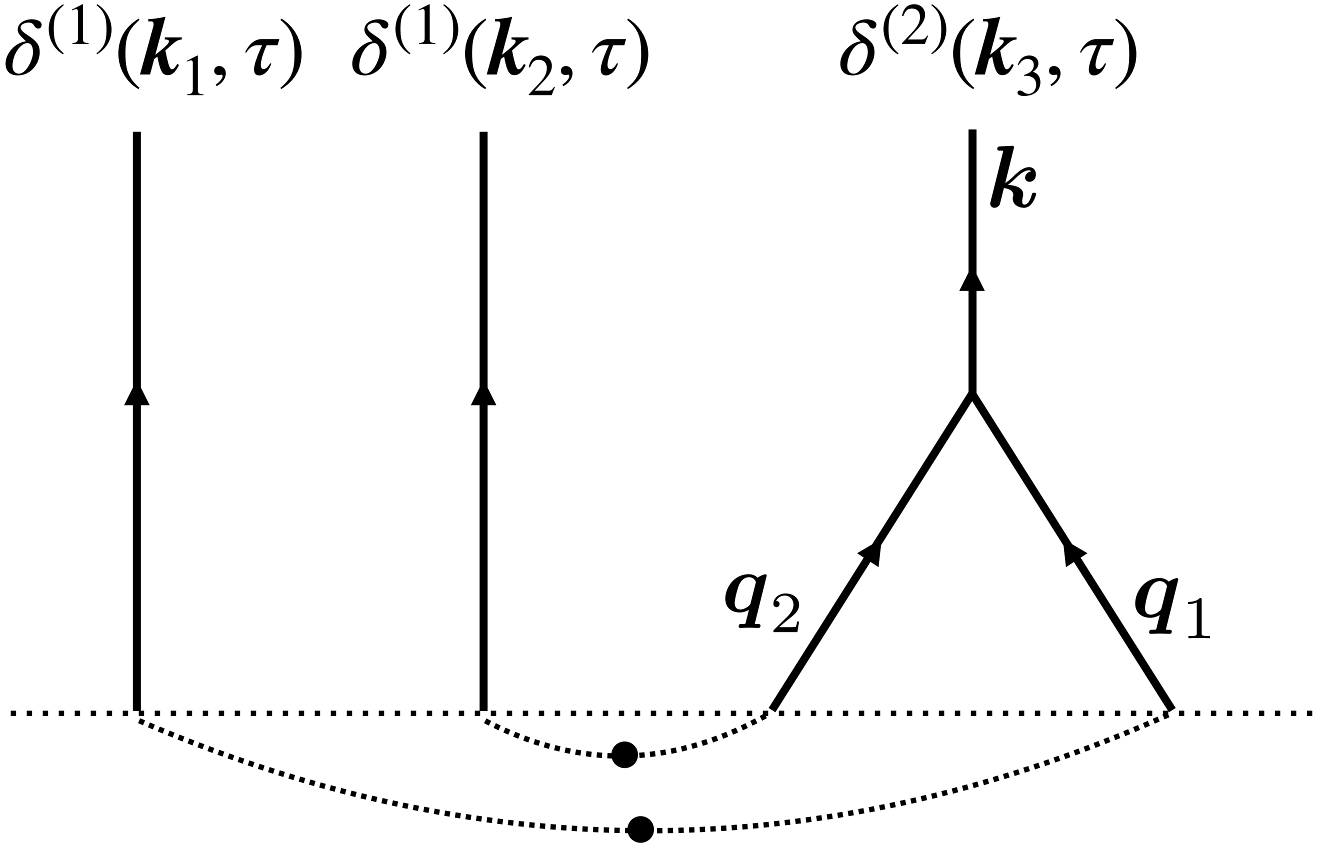


Bispectrum

$$\langle \delta_{\rm nl}(\boldsymbol{k}_1,\tau)\delta_{\rm nl}(\boldsymbol{k}_2,\tau)\delta_{\rm nl}(\boldsymbol{k}_3,\tau)\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_{\rm nl}(k_1,k_2,k_3,\tau)$$

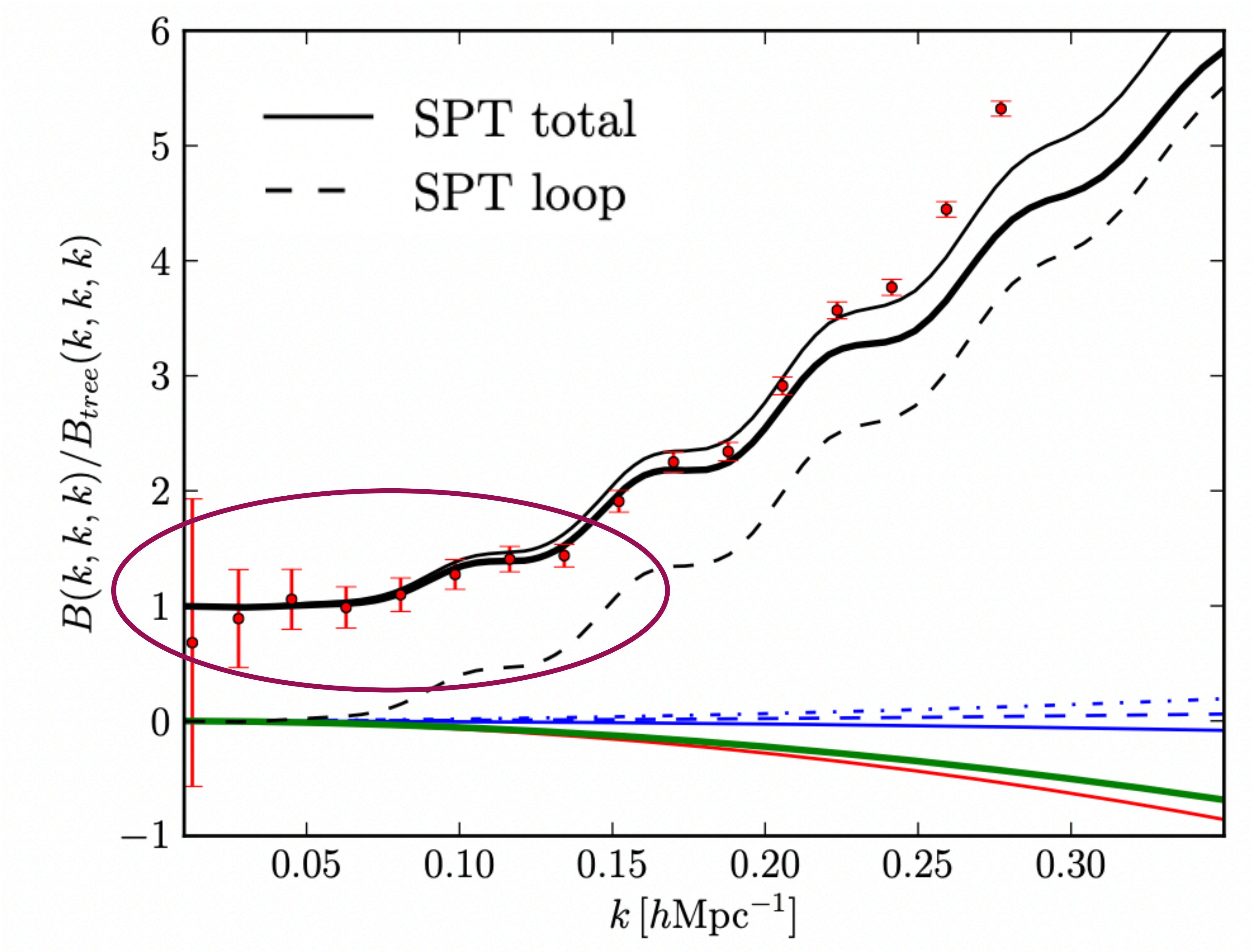
$$B_{\rm nl}(k_1,k_2,k_3,\tau) = B_{\rm tree}(k_1,k_2,k_3,\tau) + B_{1-{\rm loop}}(k_1,k_2,k_3,\tau) + \cdots$$

$$B_{\rm tree}(k_1,k_2,k_3,\tau) = 2F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)P_{\rm lin}(\boldsymbol{k}_1,\tau)P_{\rm lin}(\boldsymbol{k}_2,\tau) + 2\text{ perms}\,.$$



Wick contractions

Bispectrum

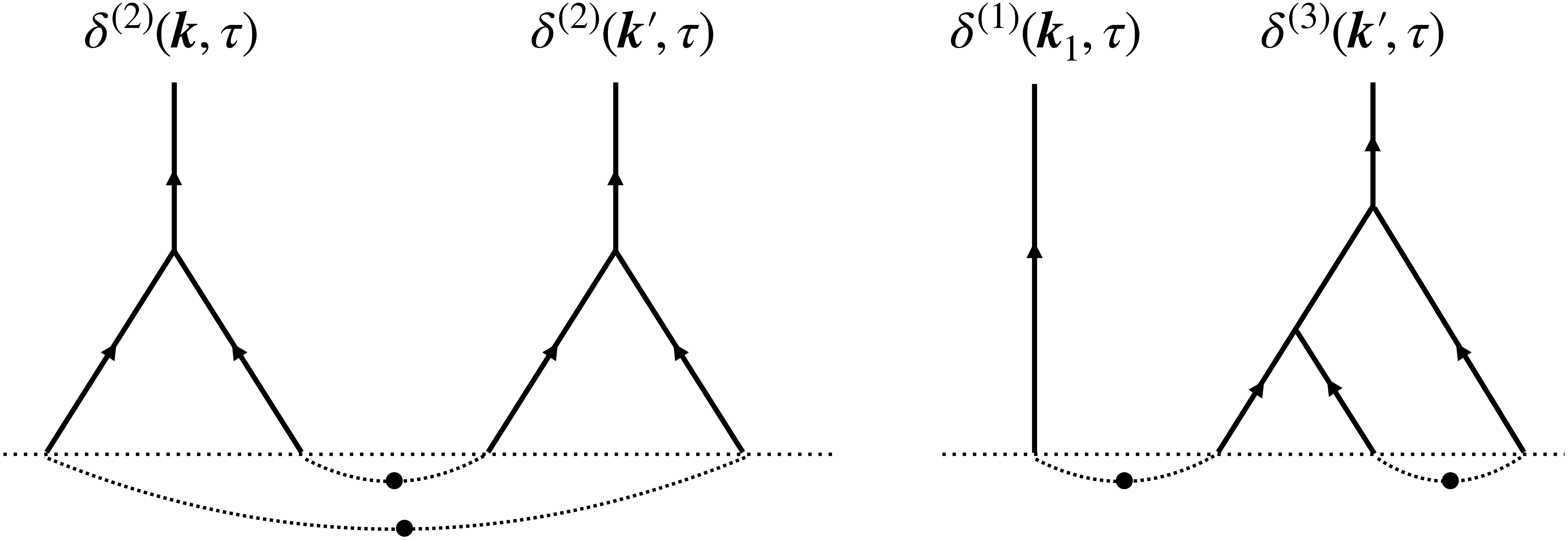

SPT total
SPT loop
$B(k,k,k)/B_{tree}(k,k,k)$
$k\,[h\mathrm{Mpc}^{-1}]$
6
5
4
3
2
1
0
−1
0.05
0.10
0.15
0.20
0.25
0.30

One-loop power spectrum

$$\langle \delta_{\rm nl}(\boldsymbol{k},\tau)\delta_{\rm nl}(\boldsymbol{k}',\tau)\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')P_{\rm nl}(k,\tau)$$

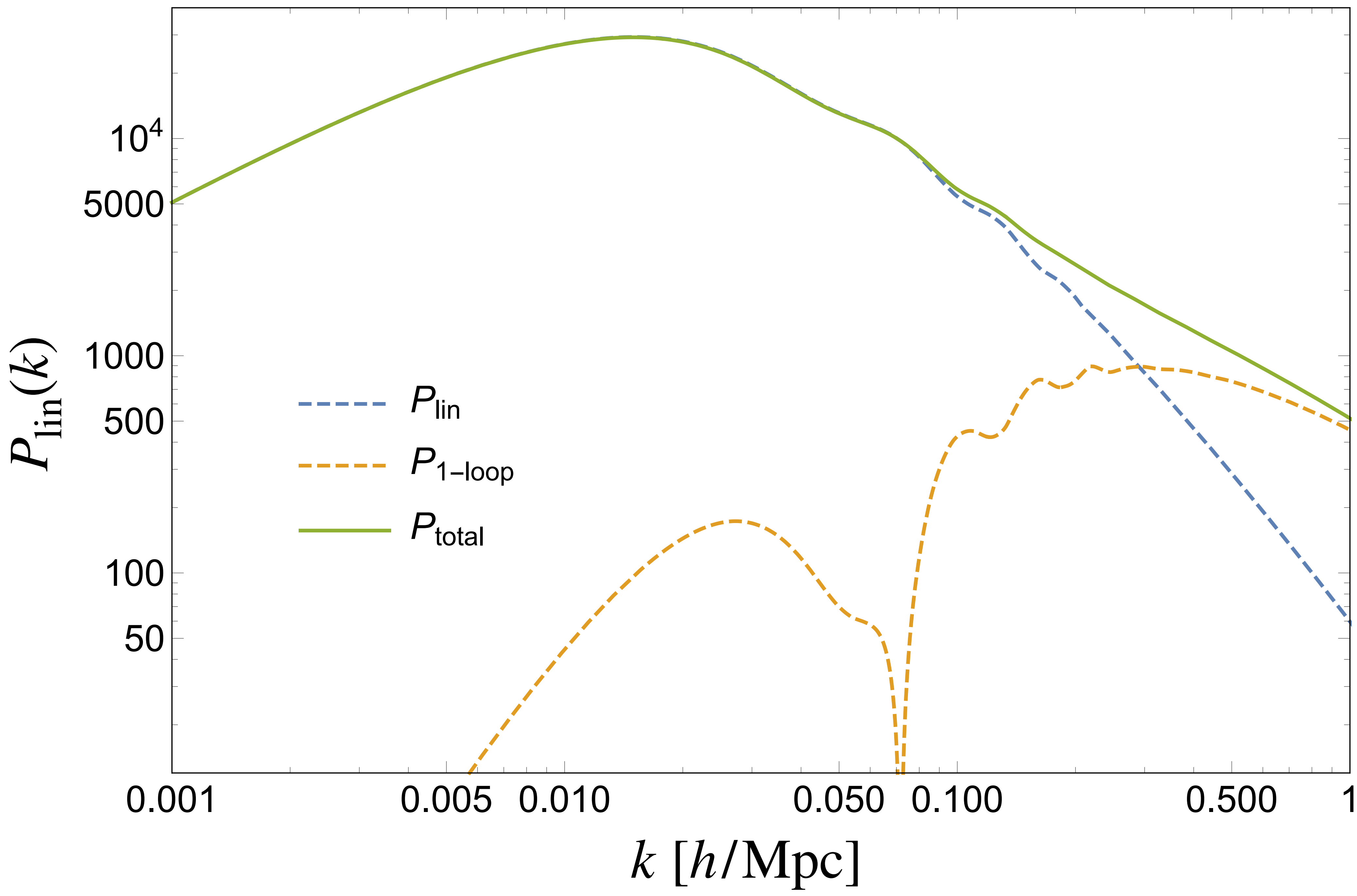
$$P_{\rm nl}(k,\tau) = P_{\rm lin}(k,\tau) + P_{1-{\rm loop}}(k,\tau) + \cdots$$

$$P_{1-{\rm loop}}(k,\tau) = P_{22}(k,\tau) + 2P_{13}(k,\tau)$$



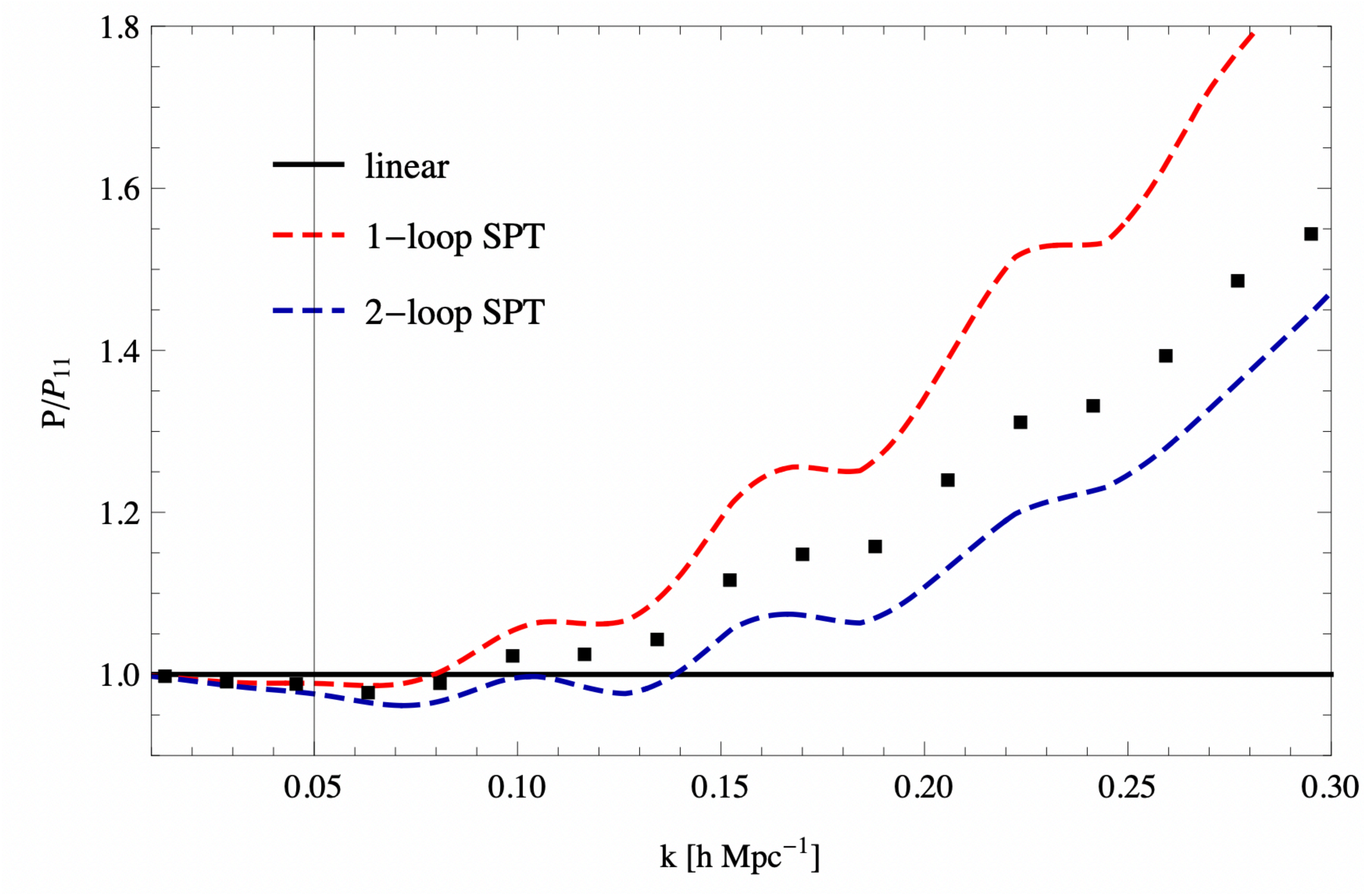
One-loop power spectrum

At redshift zero, $\Delta^2(k) \approx 1$ for $k \approx 0.3\ h/\mathrm{Mpc}$



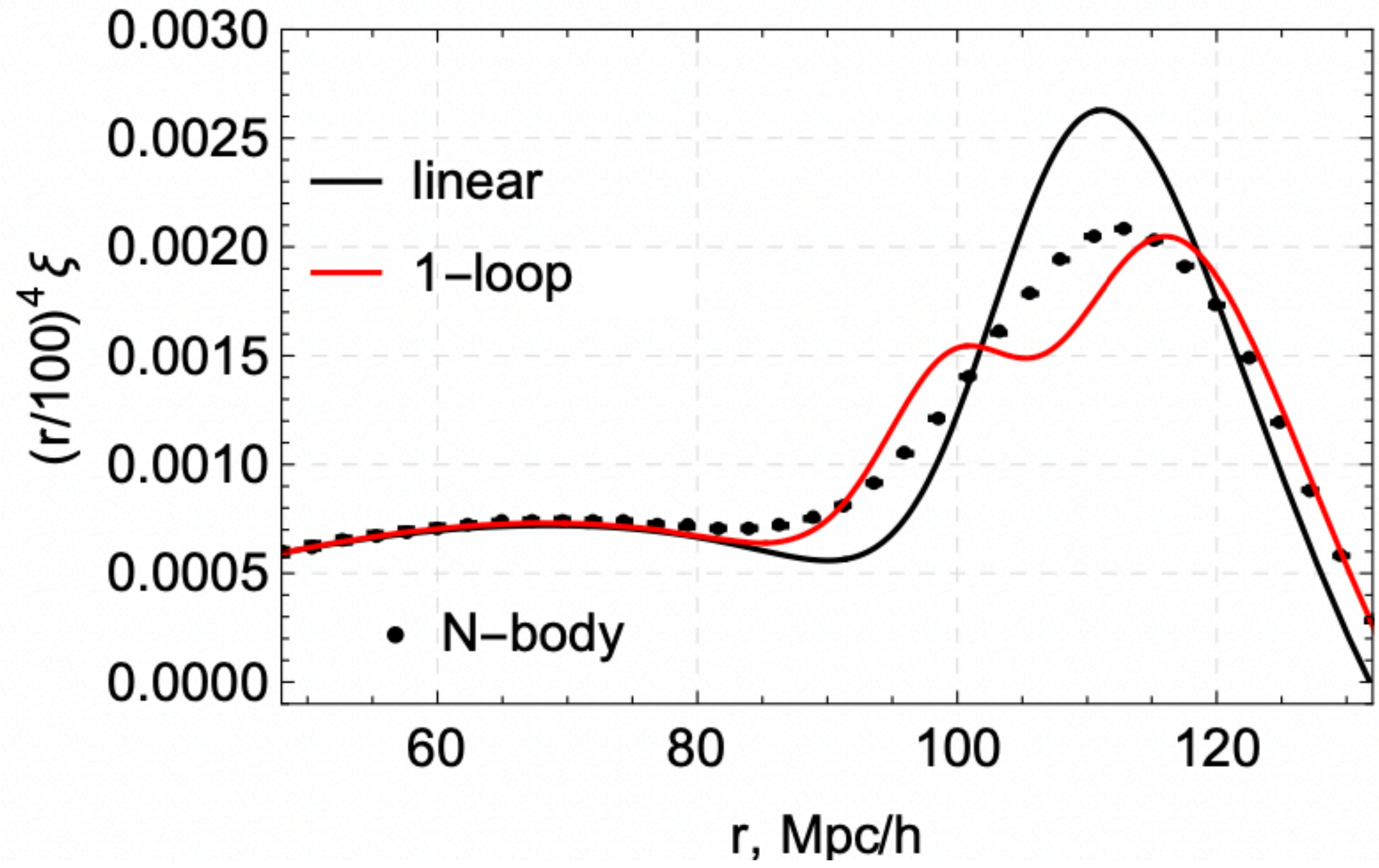
Problems of SPT

The broadband is wrong on all scales, PT does not converge…



Problems of SPT

The BAO peak is completely wrong…



Problems of SPT

Why is this happening?

Loops contain UV modes, but the theory is wrong there…

We have no free parameters to absorb this UV dependence!

Our equations of motion must be incomplete/inconsistent

Are we sure that $\Delta^2(k)$ is the only expansion parameter? What else?

Resolving these problems led to a lot of progress in the last ~15 years

Problems of SPT

Let us take a closer look at the equations of motion

$$\delta' + \nabla_i((1+\delta)v_i) = 0$$

$$v_i' + \mathscr{H} v_i + v_j \nabla_j v_i = -\nabla_i \Phi$$

$$\nabla^2 \Phi = \frac{3}{2} \mathscr{H}^2 \Omega_m(\tau)\, \delta$$

Assuming ideal fluid, these equations are correct for $f = \bar{f} + \delta f$

However, we want the EOM for the long-wavelength fields!

Problems of SPT

We want to split $f = f_l + f_s$ and average over f_s

$$f_l(\boldsymbol{x}) = \int d^3\boldsymbol{r}\, W_R(|\boldsymbol{x} - \boldsymbol{r}|)\, f(\boldsymbol{r}) \qquad W_R(x) \sim e^{-\frac{x^2}{2R^2}}$$

The average of product of fields is not the product of average fields

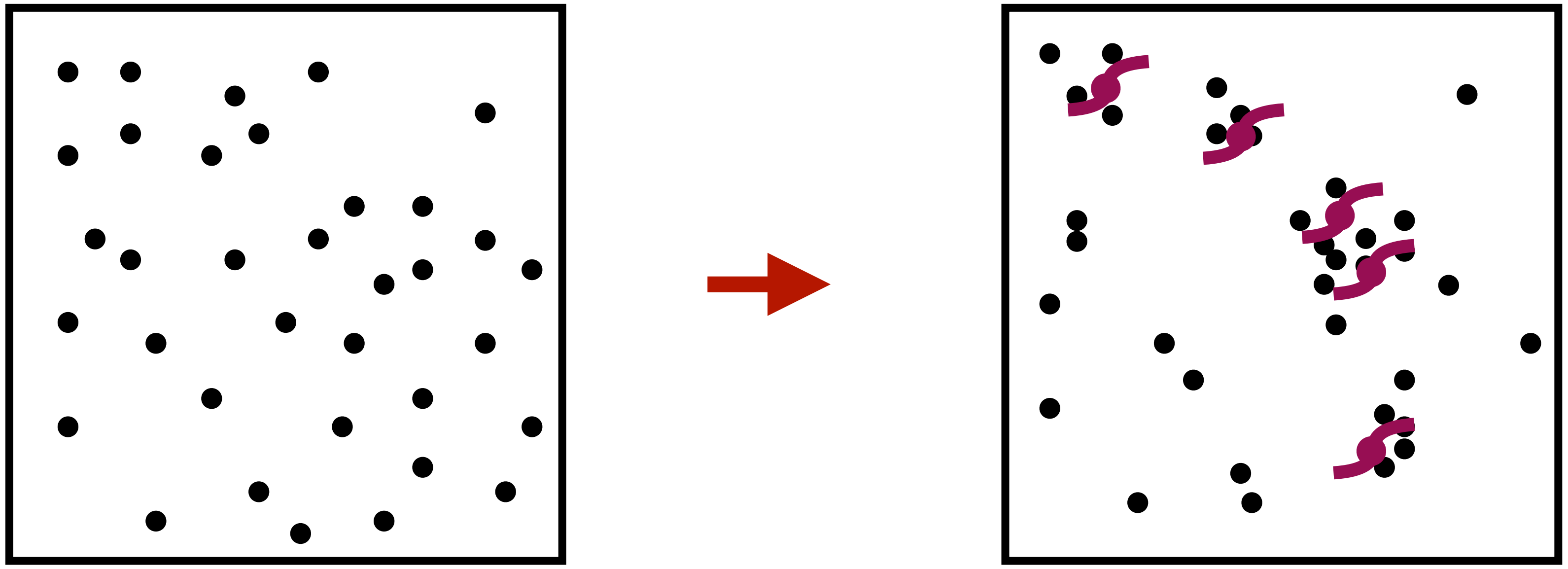
$$(fg)_l = f_l g_l + R^2 \nabla_i f_l \nabla_i g_l + (f_s g_s)_l + \cdots$$

↑

new terms with new free parameters!

Effective Field Theory of LSS

Just DM particles in an expanding universe



UV description: collisionless Boltzmann eq. $d/dt\, f(\boldsymbol{x}, \boldsymbol{p}, t) = 0$

gravity $\nabla^2 \Phi \propto \int d^3\boldsymbol{p}\, f(\boldsymbol{x}, \boldsymbol{p}, t)$

Mean free path effectively set by the age of the universe

Most of the motion are coherent bulk flows

Gravity helps by “gluing” DM particles which form DM halos

Effective Field Theory of LSS

This allows to truncate Boltzmann hierarchy

$$\delta_l' + \nabla_i((1+\delta)v_i)_l = 0$$

$$v_{i,l}' + \mathscr{H} v_{i,l} + v_{i,l}\nabla_j v_{i,l} = -\nabla_i\Phi_l \boxed{- c_s^2(\tau)\,\nabla_i\delta_l + \cdots} \longleftarrow$$

$$\nabla^2\Phi_l = \frac{3}{2}\mathscr{H}^2\Omega_m(\tau)\,\delta_l$$

new nonlinear terms with free coefficients

Baumann, Nicolis, Senatore, Zaldarriaga (2010)

Carrasco, Hertzberg, Senatore (2012)

Expansion parameters: δ, $\partial/k_{\rm NL}$

$$k_{\rm NL} \approx 1/R$$

Small-scale DM physics encoded in c_s^2

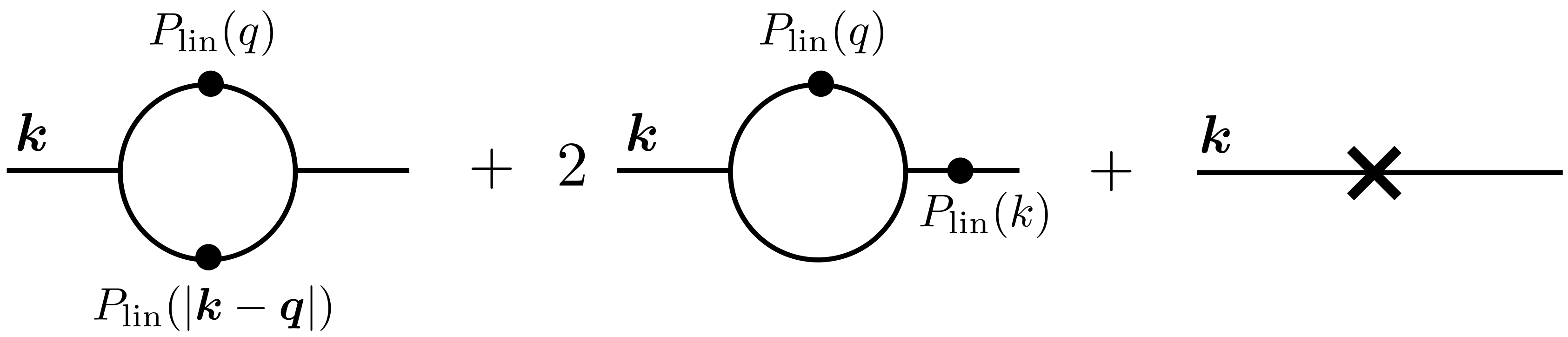
The same equations for any UV model (DM, fluid, axions…)

Effective Field Theory of LSS

Correlation functions in perturbation theory

Carrasco, Hertzberg, Senatore (2012)

$P_{1-\text{loop}}(k) =$



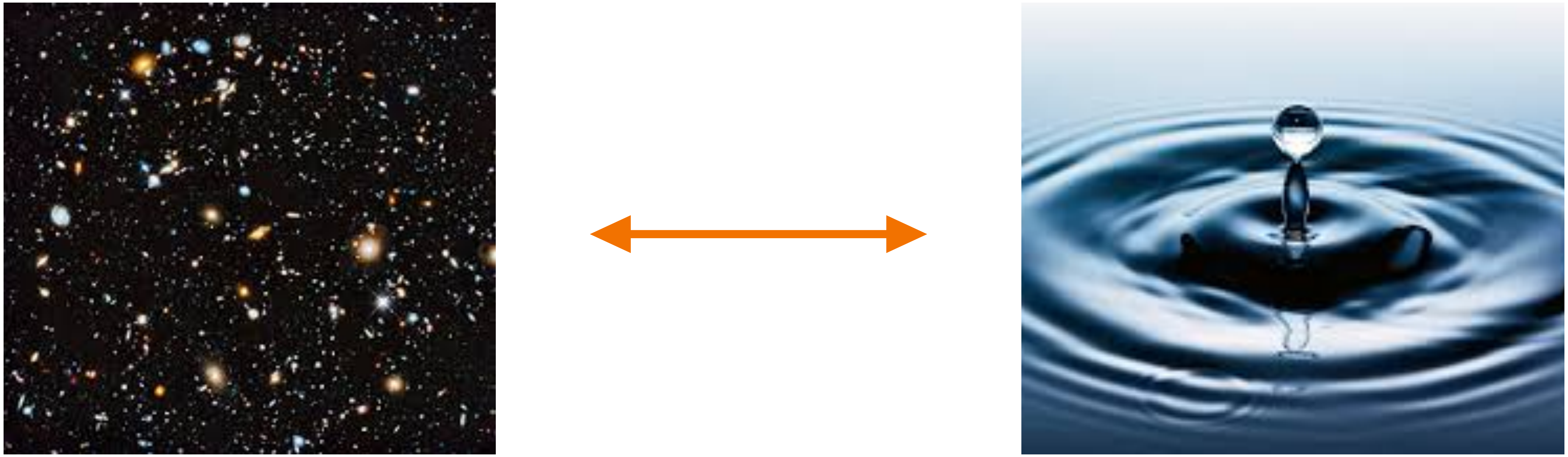
$$P_{13}^{\text{UV}}(k) = -\frac{61}{630\pi^2} P_{\text{lin}}(k) k^2 \int_0^\infty dq P_{\text{lin}}(q)$$

$$P_{1-\text{loop}}(k) = P_{22}(k) + P_{13}(k) + 2R^2 k^2 P_{\text{lin}}(k)$$

↑

time integral of c_s^2 and Green's functions

Effective Field Theory of LSS



Baumann, Nicolis, Senatore, Zaldarriaga (2010)
Carrasco, Hertzberg, Senatore (2012)
Senatore, Zaldarriaga (2014)
Senatore (2014)
Mirbabayi, Schmidt, Zaldarriaga (2014)
Baldauf, Mirbabay, MS, Zaldarriaga (2015)

Large distance dof: δ_g

EoM are fluid-like, including gravity

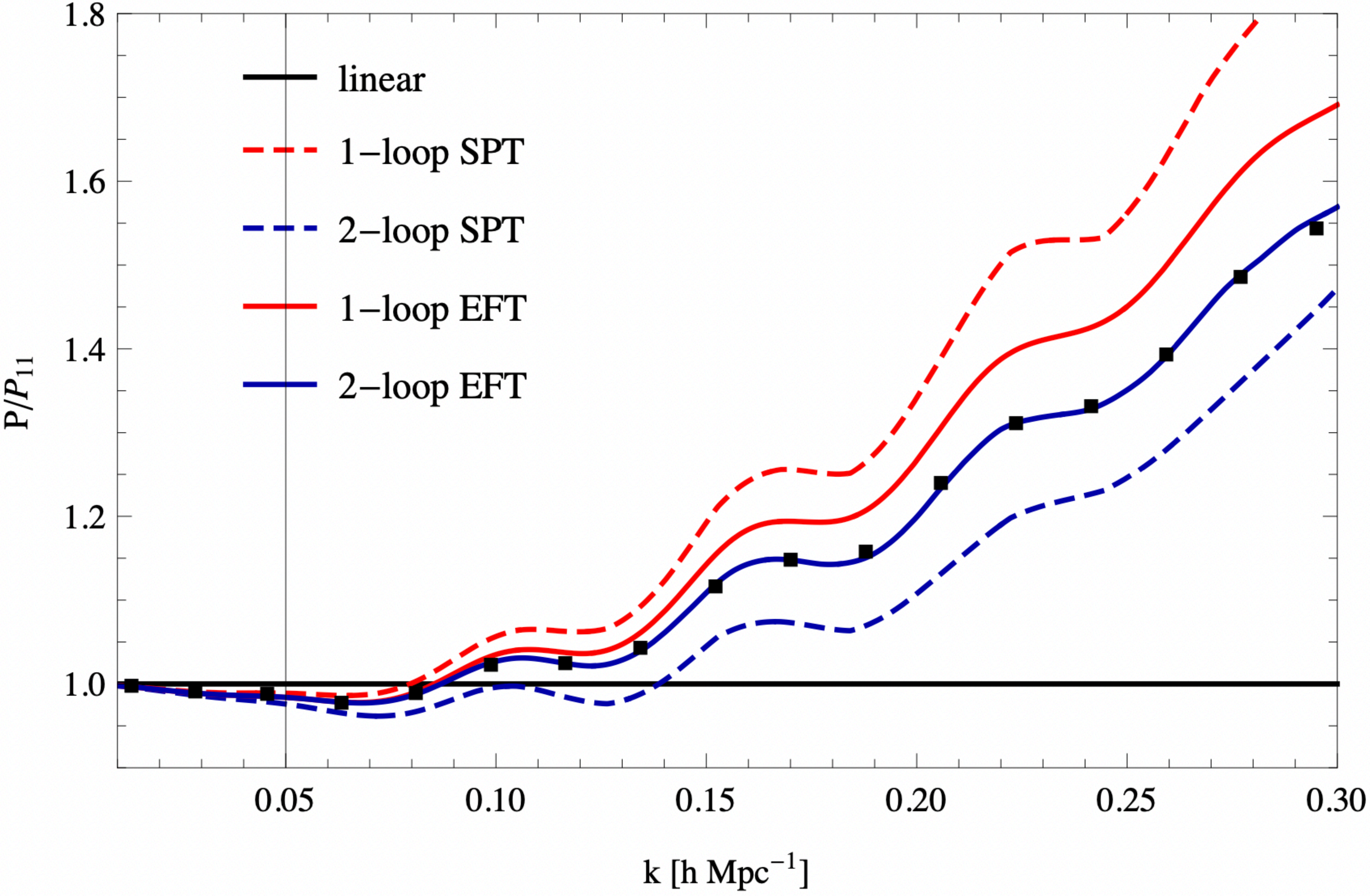
Symmetries, Equivalence Principle

Expansion parameters: δ_g, $\partial/k_{\rm NL}$

All “UV” dependence is in a handful of free parameters

On scales larger than $1/k_{\rm NL}$ this is the universal description of galaxy clustering

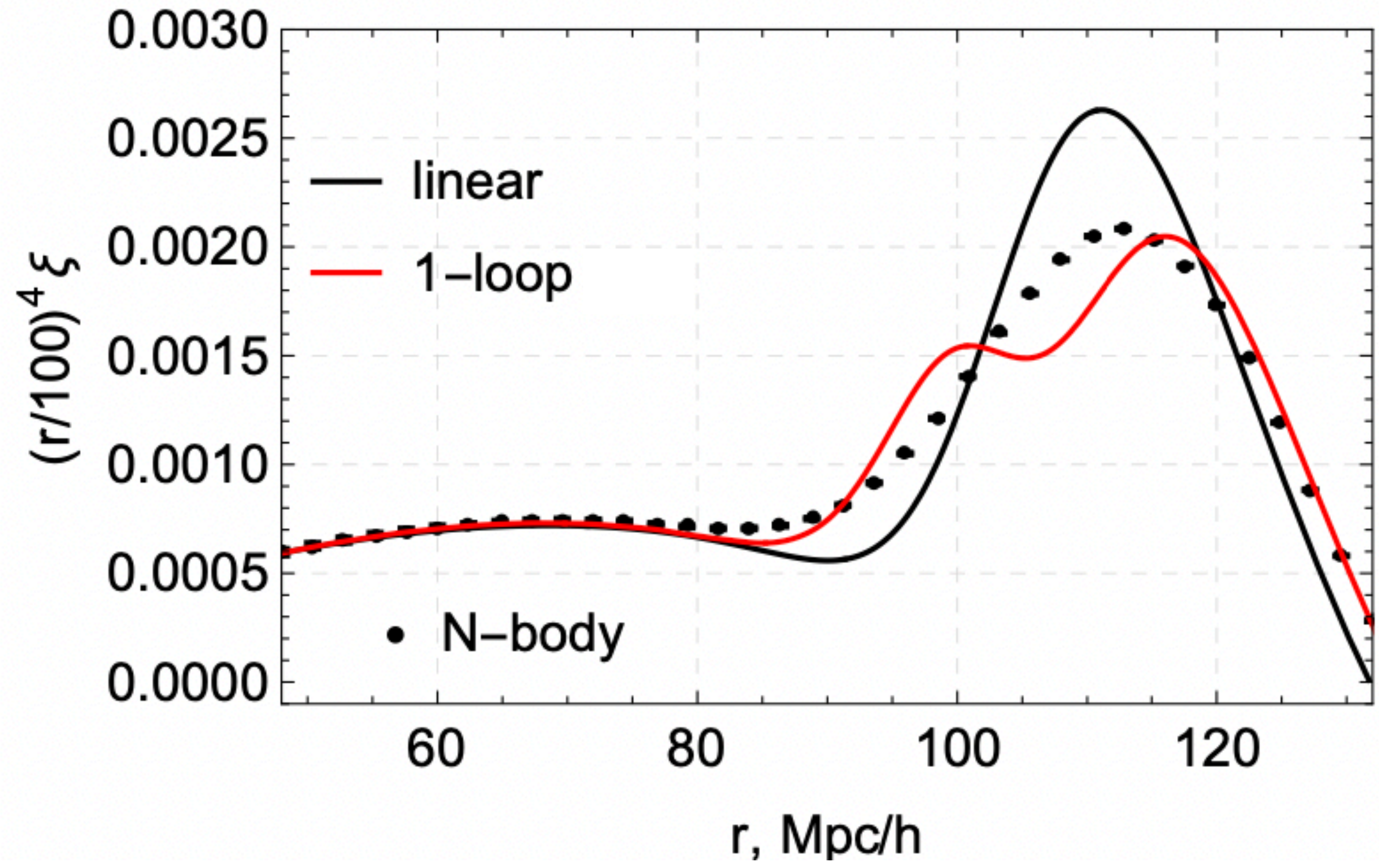
Effective Field Theory of LSS



Infrared resummation

Addition of counterterms does not solve the issue with the BAO peak

What do we do about it?



Infrared resummation

On large scales: $\delta' + \nabla_i v_i = 0$

How far DM particles move under the influence of gravity?

$$\psi_i = \int_0^\tau d\tau' v_i = -\frac{\nabla_i}{\nabla^2}\delta$$

Typical displacements are large!

$$\langle \psi_i^2 \rangle = \frac{1}{2\pi^2}\int dk P_{\text{lin}}(k,\tau)$$

very different from $\Delta^2(k)$!

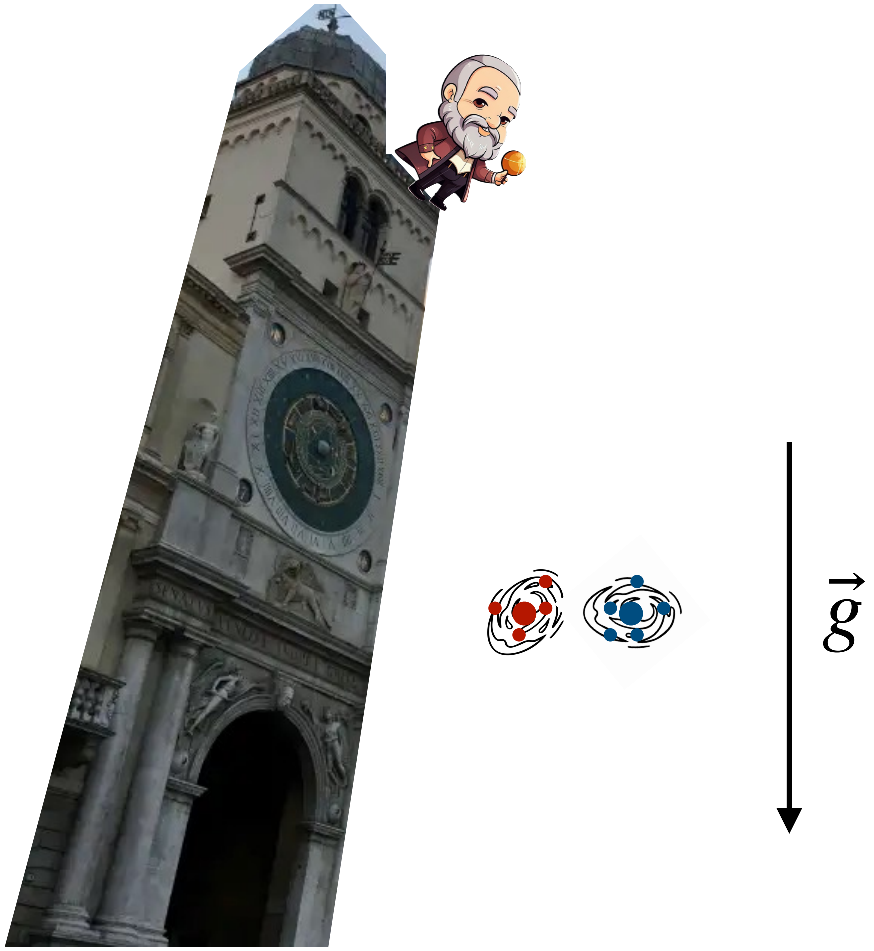
↑

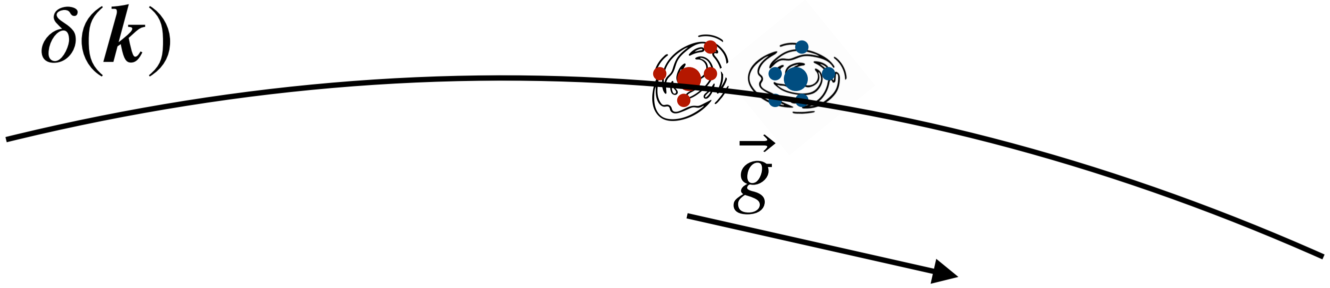
dominated by the “infrared” modes with $k \sim k_{\text{eq}}$.

Infrared resummation

What is the effect of these modes on smaller scales?

Just a universal displacement as dictated by the Equivalence Principle





One can re-sum all displacement contributions in PT

Infrared resummation

What is being resummed?

$$F_2(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{5}{7} + \frac{1}{2}\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1 q_2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\frac{(\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)^2}{q_1^2 q_2^2}$$

$$F_2(\boldsymbol{q}_1, \boldsymbol{q}_2)\Big|_{q_1 \ll q_2} = \frac{1}{2}\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1^2} + \mathcal{O}(1)$$

$$F_n(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_n)\Big|_{q_1 \ll q_i} = \frac{1}{n!}\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1^2} \cdots \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_n}{q_n^2} + \mathcal{O}(1)$$

$\delta(x_i) \to \delta(x_i + \psi_i)$ in Fourier space becomes: $\delta(\boldsymbol{k}) \to \delta(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{\psi}}$

Infrared resummation

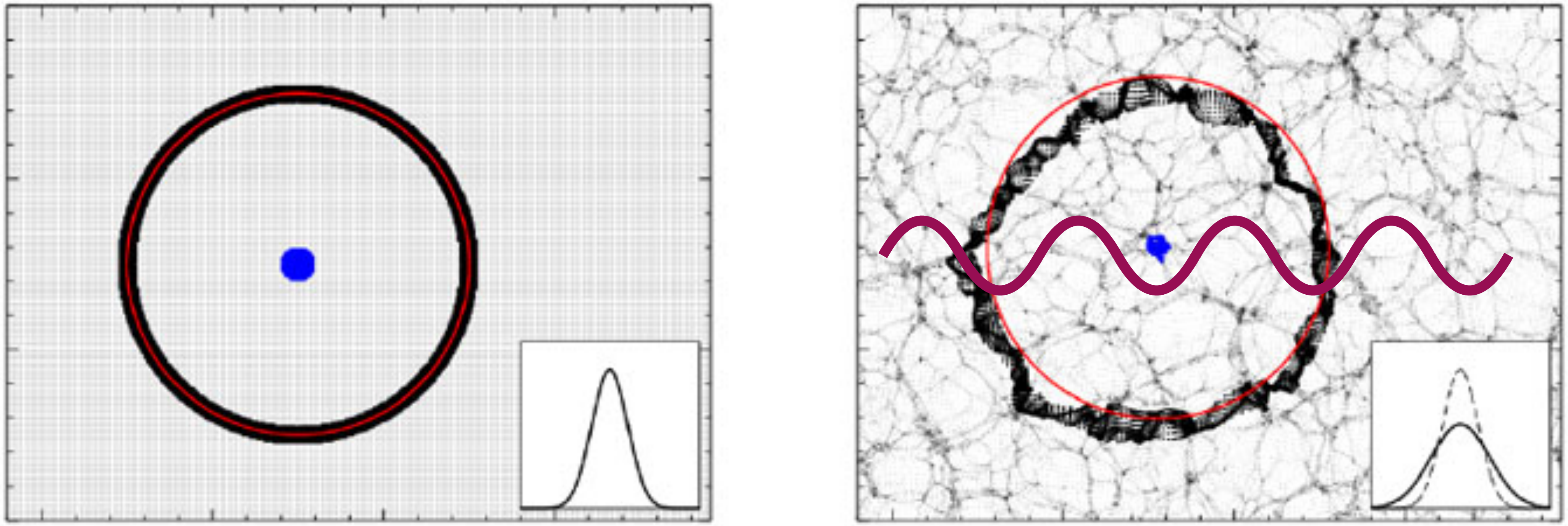
Displacements do not affect the smooth part of correlation functions

Displacements are observable only in the presence of features!

$$\Sigma_\Lambda^2 \approx \frac{1}{6\pi^2}\int_0^\Lambda \mathrm{d}q P_{\mathrm{lin}}(q)[1-j_0(q\ell_{\mathrm{BAO}})+2j_2(q\ell_{\mathrm{BAO}})]$$

new parameter

$$2\pi/\ell_{\mathrm{BAO}} < q \ll 2\pi/\sigma$$



Infrared resummation

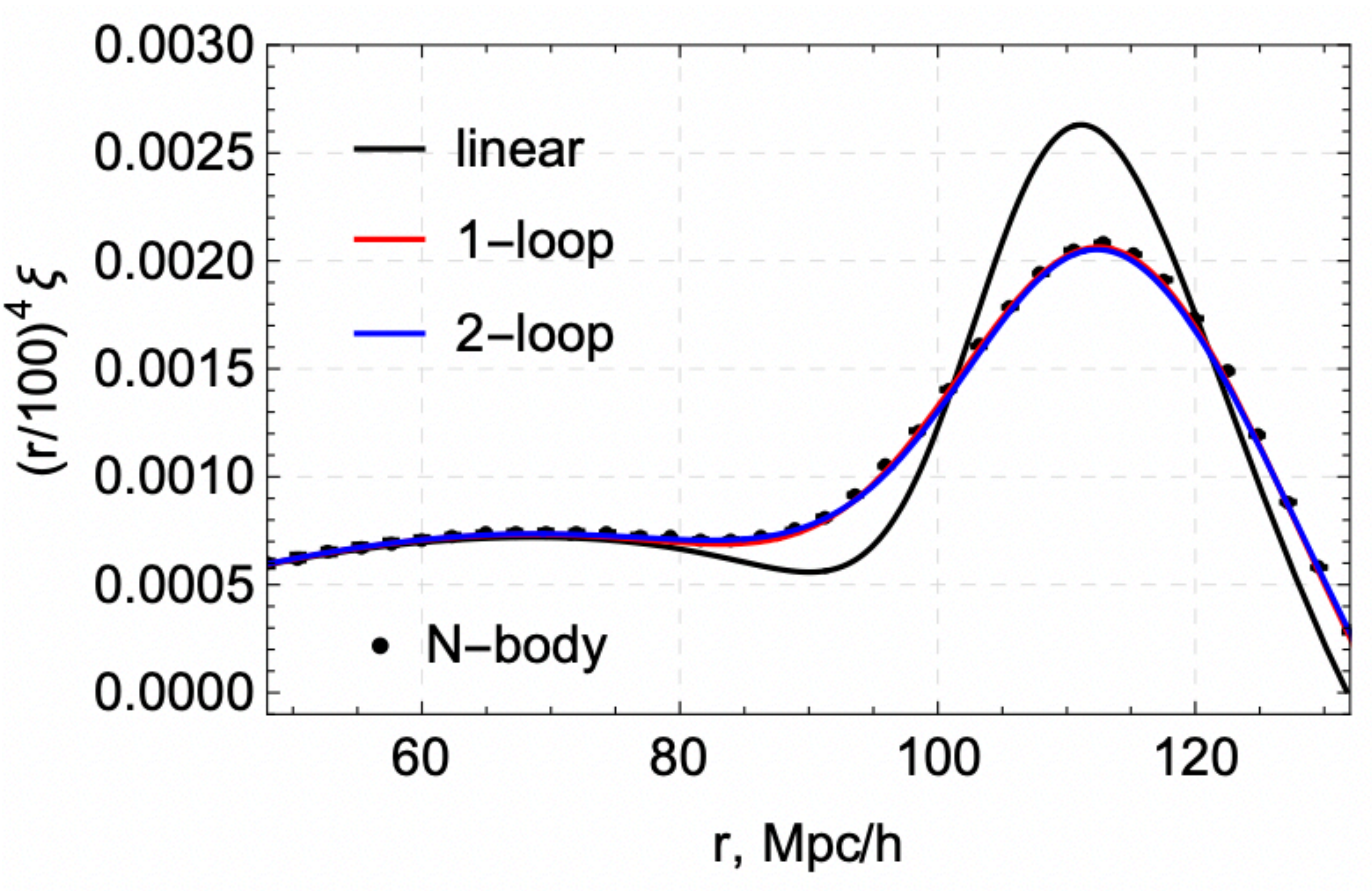
Senatore, Zaldarriaga (2014)
Baldauf, Mirbabayi, MS, Zaldarriaga (2015)
Vlah, Seljak, Chu, Feng (2015)
Blas, Garny, Ivanov, Sibiryakov (2016)
Senatore, Trevisan (2017)

The IR resummed 1-loop power spectrum:

$$\tilde{P}(k) = P^{nw}_{\rm lin}(k) + P^{nw}_{1-\rm loop}(k)$$
$$+e^{-\Sigma^2_{\epsilon k}k^2}(1+\Sigma^2_{\epsilon k}k^2)P^{w}_{\rm lin}(k) + e^{-\Sigma^2_{\epsilon k}k^2}P^{w}_{1-\rm loop}(k)$$

PT in tidal fields, nonperturbative in displacements

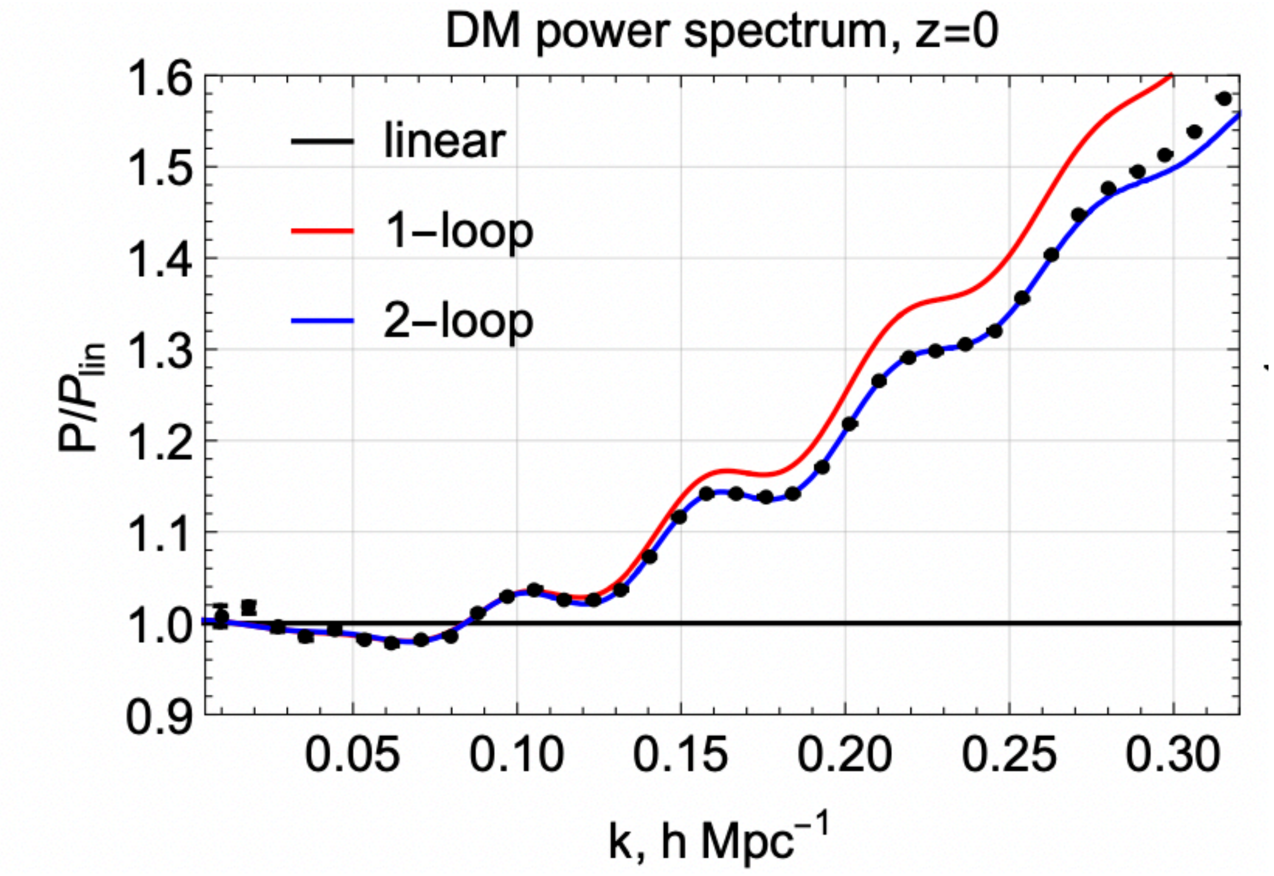
Infrared resummation

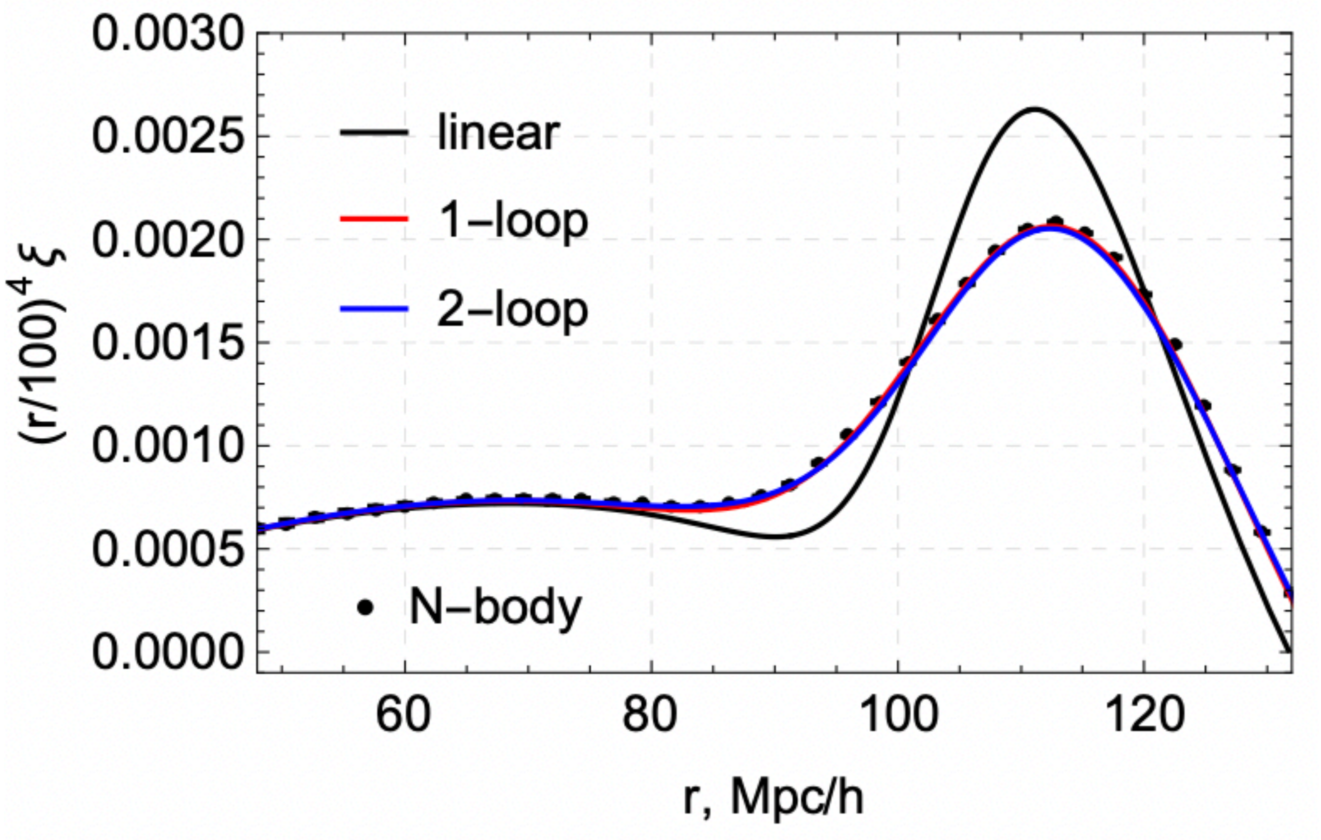


IR resummed 2-loop power spectrum

1% precision up to $k \approx 0.25\ h/\mathrm{Mpc}$

Perfect description of the BAO peak





The same principles hold for galaxies