Introduction to Cosmology (II)

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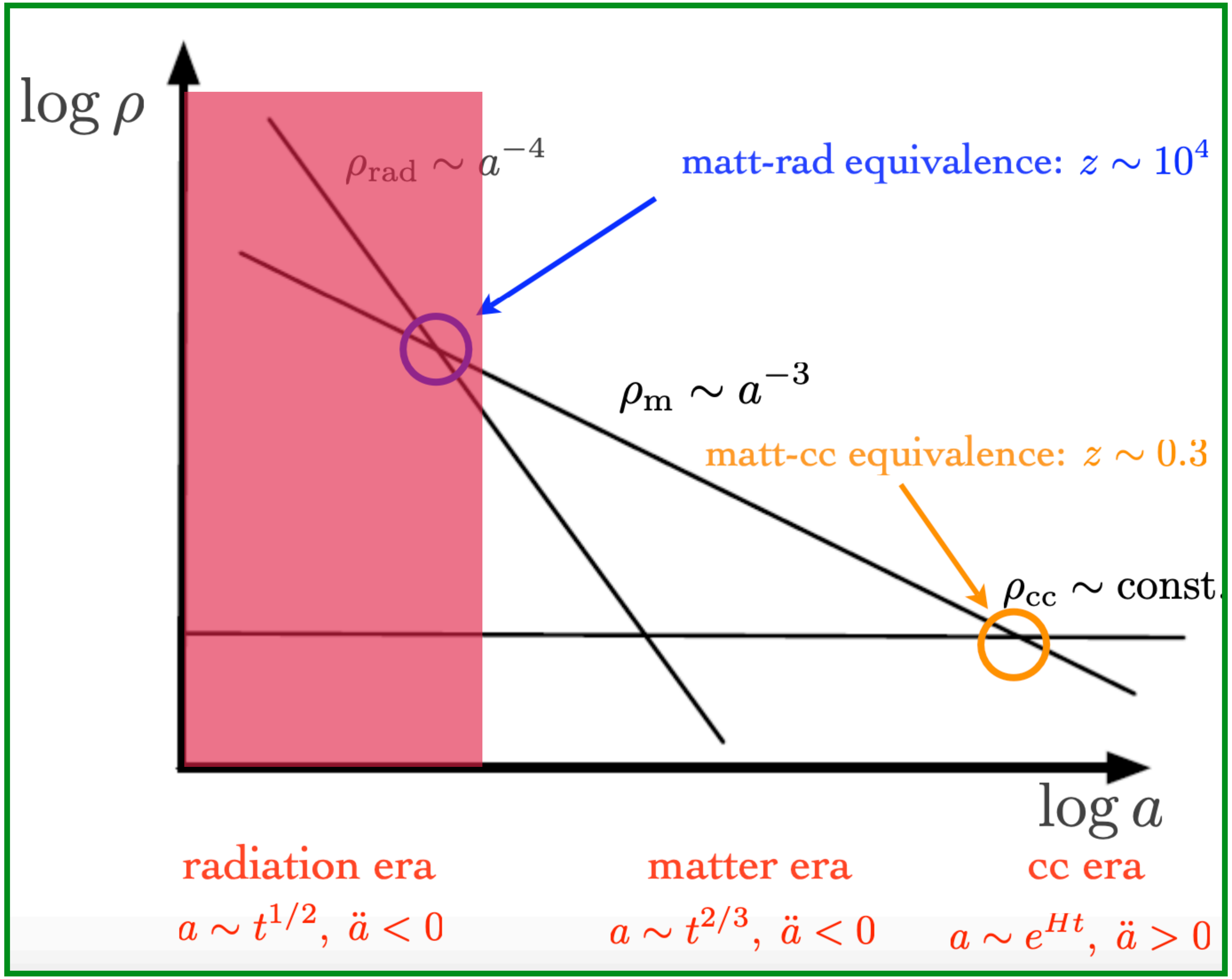
Outline

- FLRW Universe
- Thermal history of the Big Bang (=relics from the Early Universe)
- Inflation and cosmic perturbations


$\log\rho$
$\rho_{\rm rad} \sim a^{-4}$
matt-rad equivalence: $z \sim 10^4$
$\rho_{\rm m} \sim a^{-3}$
matt-cc equivalence: $z \sim 0.3$
$\rho_{\rm cc} \sim$ const.
$\log a$
radiation era
$a \sim t^{1/2},\ \ddot{a} < 0$
matter era
$a \sim t^{2/3},\ \ddot{a} < 0$
cc era
$a \sim e^{Ht},\ \ddot{a} > 0$

Equilibrium Thermodynamics

Even if the background is expanding, approximate equilibrium holds for particles with interactions much faster than the time scale of the expansion

interaction rate $\Gamma \gg H$ expansion rate

$$f(\vec{x},\, \vec{p},\, t,\, \mu) \xrightarrow[\text{kinetic equilibrium}]{\text{homogeneity, isotropy}} f_T(p,\, \mu) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

distribution function

$$E = \sqrt{p^2 + m^2}$$

$$p = |\vec{P}| \sim 1/a$$

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$$\rho = g \int \frac{d^3p}{(2\pi)^2} E\, f_T(p,\, \mu) =$$

$$\xrightarrow{T \gg m,\, \mu} \quad g\frac{\pi^2}{30}T^4 \times \begin{cases} 1 & \text{boson} \\ \frac{7}{8} & \text{fermion} \end{cases}$$

$$\xrightarrow{T \ll m} \quad m\, n = g\, m \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

energy density

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energy density

$$n = g \int \frac{d^3p}{(2\pi)^3}\, f_T(p, \mu) \underset{T \gg m, \mu}{\simeq} g\,\frac{2\zeta(3)}{\pi^2}\,T^3 \underset{\text{[fermion]}}{\left[\frac{3}{4}\right]}$$

number density

$$P = g \int \frac{d^3p}{(2\pi)^3}\, \frac{k^2}{3E}\, f_T(p, \mu) \underset{T \gg m, \mu}{\simeq} \frac{1}{3}\rho$$

pressure

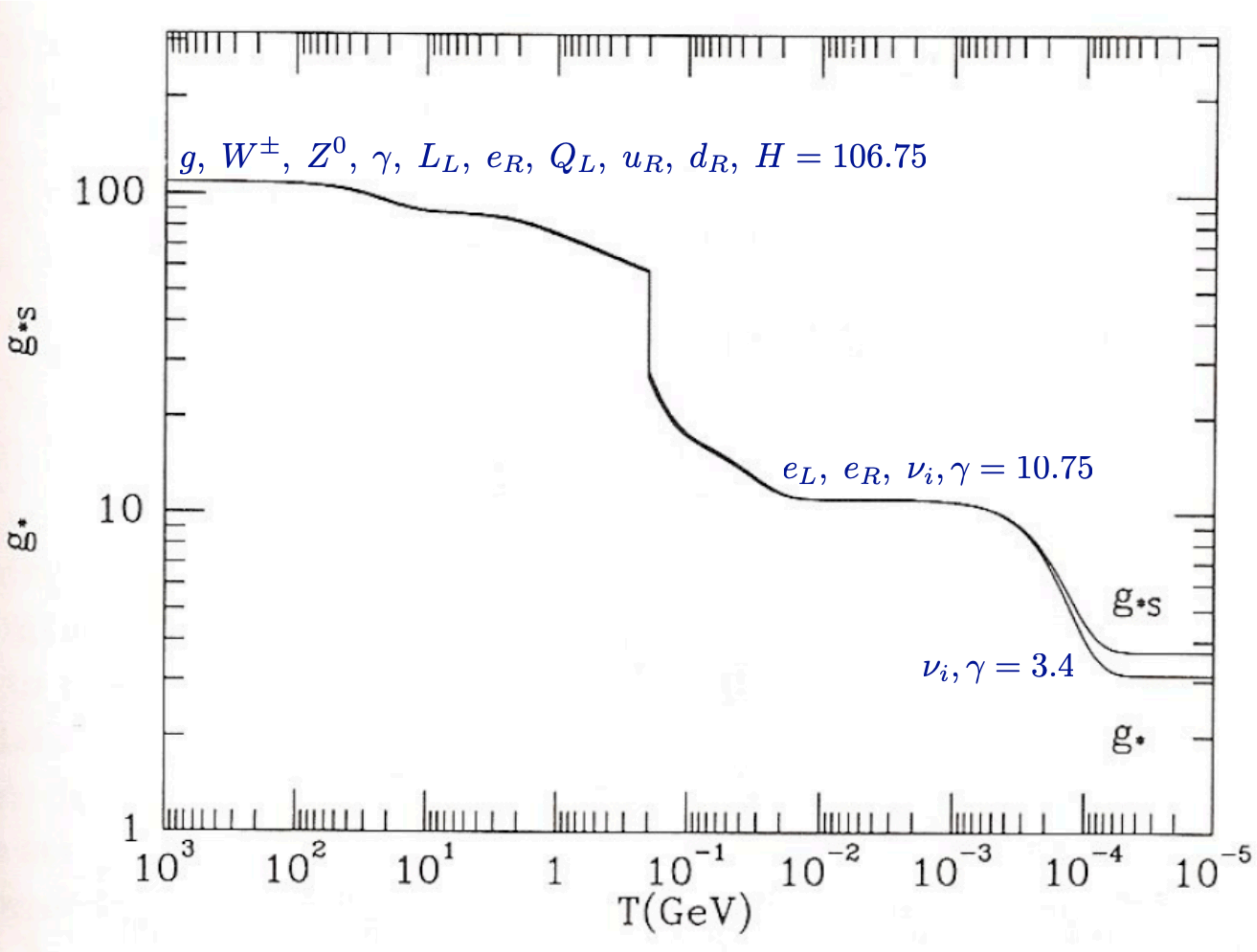
Relativistic degrees of freedom dominate

$$\rho_{\rm tot} = \sum_i \rho_i \simeq g_* \frac{\pi^2}{30} T^4$$

$$g_* = \sum_b g_b + \frac{7}{8} \sum_f g_f$$

sum over relativistic d.o.f's

Standard Model



Kolb & Turner

Entropy

Energy in a comoving volume is not conserved: $\rho_\gamma a^3 \sim 1/a$

Entropy in a comoving volume is conserved: $S = \dfrac{\rho + P}{T} a^3 \equiv \overset{\text{entropy density}}{\overset{\downarrow}{s}}\, a^3$

$$s = \frac{(\rho + p)}{T} = \frac{2\pi^2}{45} g_{*s} T^3 = 1.80\, g_{*s} n_\gamma \qquad g_{*s} = \sum_b g_b \left(\frac{T_b}{T}\right)^3 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^3$$

$s \sim a^{-3}$ comoving number density: $N = \dfrac{n}{s}$

$g_{*s}^{1/3}\, T\, a \sim \text{const}$ $T \sim 1/a$ in ‘normal’ times

Time-Scales

- expansion (rad. dom.): $H = \sqrt{\frac{8\pi G}{3}\rho} \simeq 0.33\, g_*^{1/2}\, \frac{T^2}{M_p}$ $\left[t/\mathrm{sec} \simeq (T/\mathrm{MeV})^{-2}\right]$

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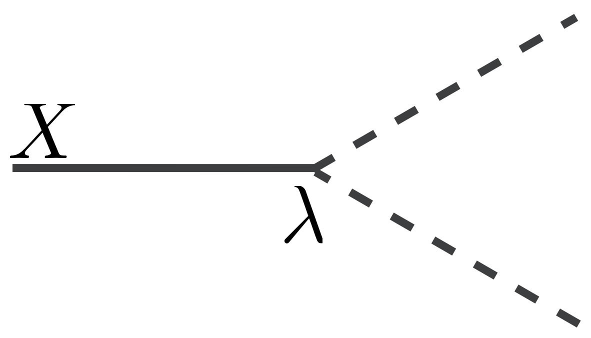
- interactions:



decay $\Gamma_D \sim \lambda^2 m_X \begin{cases} m_X/T & T \geq m_X \\ 1 & T \leq m_X \end{cases}$

inv. decay $\Gamma_{ID} \sim \Gamma_D \begin{cases} 1 & T \geq m_X \\ \left(\frac{m_X}{T}\right)^{3/2} e^{-\frac{m_X}{T}} & T \leq m_X \end{cases}$

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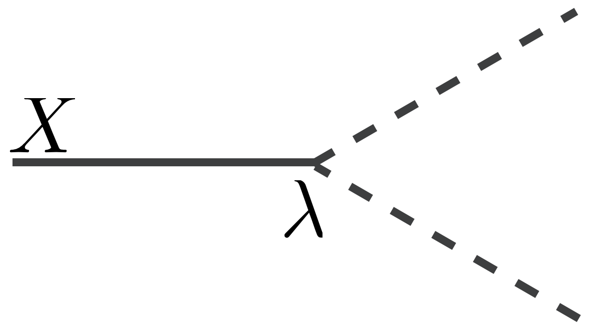

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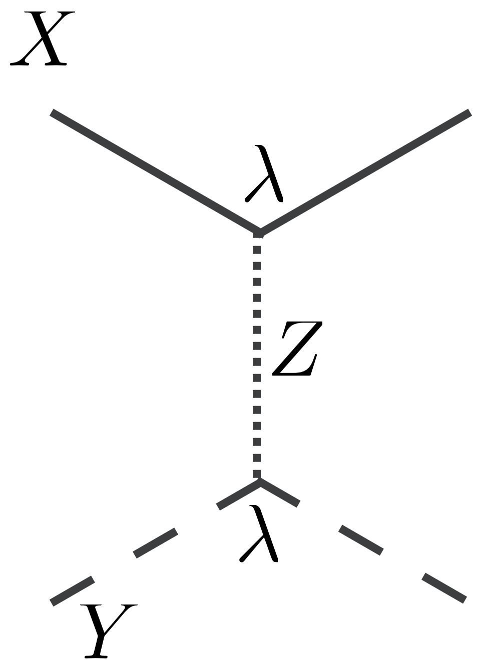


scattering (annihilation)

$$\Gamma_{S,X} \sim n_Y \lambda^4 \frac{T^2}{(T^2 + m_Z^2)^2} \sim \begin{cases} \lambda^4 T & T \gg m_{Y,Z} \\ \lambda^4 T^5/m_Z^4 & m_Y \ll T \ll m_Z \\ \propto e^{-m_Y/T} & T \ll m_Y \end{cases}$$

In or Out?

rule of thumb: in equilibrium if Γ ≫ H

- gauge interactions: $\Gamma \sim \alpha^2 T \gg H$ $\longrightarrow$ $T \ll \alpha^2 M_p/0.33/\, g_*^{1/2} \simeq O(10^{15}\,\mathrm{GeV})$

- weak interactions: $\Gamma \sim G_F^2 T^5 \gg H$ $\longrightarrow$ $T \gg 1\mathrm{MeV}$

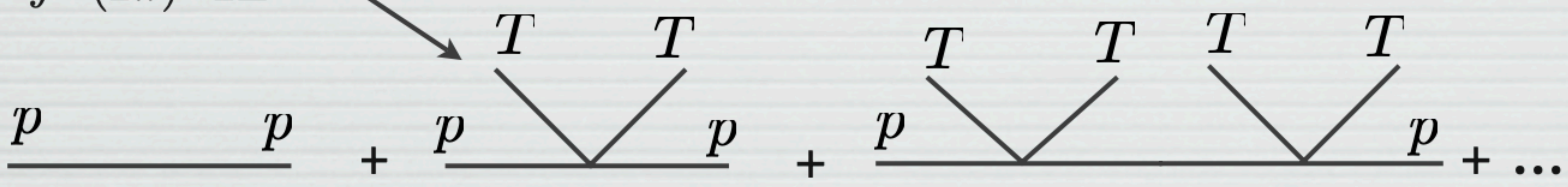
- baryon-photon coupling: $\Gamma \sim n_e\, \sigma_T \gg H$ $\longrightarrow$ $\overset{n_e \to 0}{O(1\mathrm{eV})} \ll T \ll O(10^{15}\mathrm{GeV})$

  free-electron density Thomson cross-section

- DM annihilations: $\Gamma \sim n_\chi \sigma_{\mathrm{ann}} \gg H$ $\longrightarrow$ $T \gg m_\chi$

Symmetry Breaking/Restoration

$$-i\lambda\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E}f(p)\propto -i\lambda T^2$$



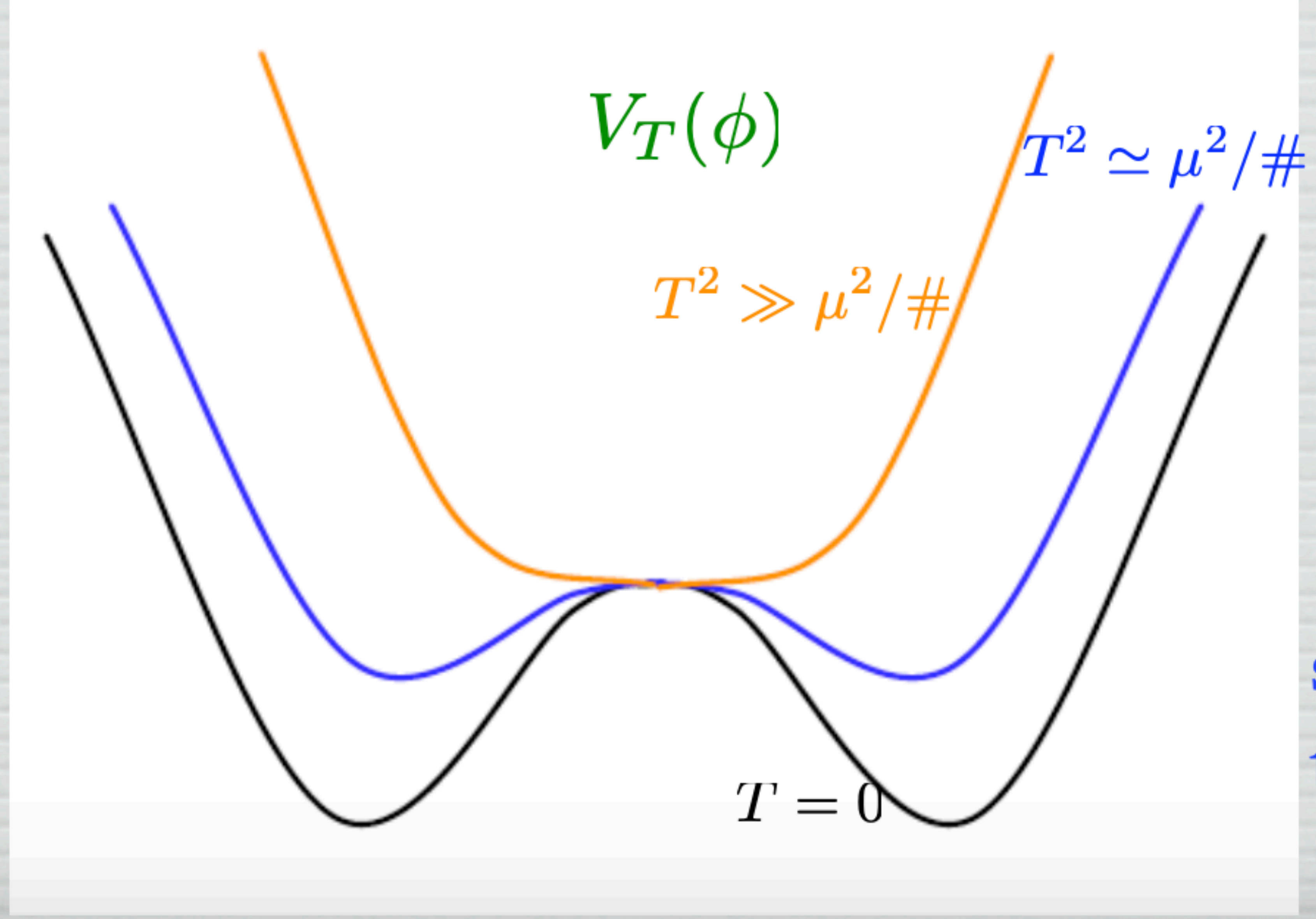
forward scattering

$$\frac{i}{p^2-m^2}+\left(\frac{i}{p^2-m^2}\right)^2(-i\lambda T^2)+\left(\frac{i}{p^2-m^2}\right)^3(-i\lambda T^2)^2+\ldots=\frac{i}{p^2-m^2-\lambda T^2}$$

thermal mass

$$V(\phi)=-\frac{1}{2}\mu^2\phi^2+\frac{1}{4!}\lambda\phi^4\xrightarrow{T\gg\mu}\frac{1}{2}(-\mu^2+\#T^2)\phi^2+\frac{1}{4!}\lambda\phi^4$$

positive contributions from all relativistic particles coupled to ϕ



symmetry breaking phase transitions in the early Universe!

Neutrino Decoupling

Neutrino coupled to the thermal bath via weak interactions as

$$n\,\nu \leftrightarrow p\,e^-$$

$$n \leftrightarrow p\,e^-\,\bar{\nu}$$

$$n\,e^+ \leftrightarrow p\,\bar{\nu}$$

$$\nu\,\bar{\nu} \leftrightarrow e^-\,e^+$$

$$\Gamma_W \sim G_F^2 T^5 < H \sim 0.33\,g_*^{1/2}\frac{T^2}{M_p}$$

$$\Downarrow$$

$$T < 0.8\mathrm{MeV}$$

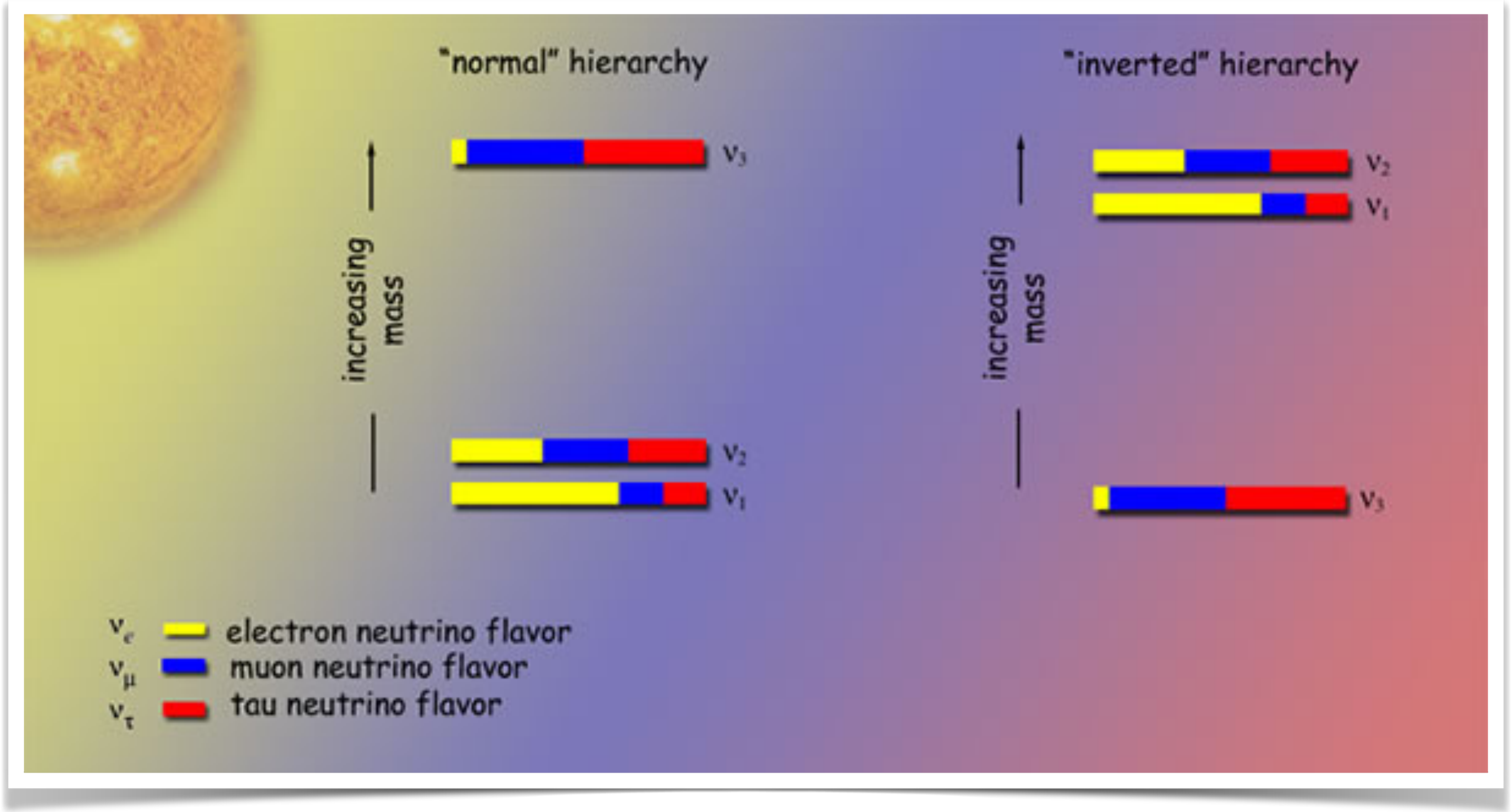
weak interaction decoupling

$^3H\ \beta$ decay $\quad m_\nu < 0.8\,\mathrm{eV}\ (90\%\ C.L.) \quad \left(m_\nu^2 = \sum_i |U_{ei}|^2\,m_i^2\right)$

oscillations

$|\Delta m_{12}^2| = (5.4 - 9.5) \times 10^{-5}\,\mathrm{eV}^2$ solar

$|\Delta m_{23}^2| = (1.2 - 4.8) \times 10^{-3}\,\mathrm{eV}^2$ atmosferic



ordering still unknown

Hot Relics

$m_\nu / T_W < 10^{-6}$ Neutrinos are relativistic at decoupling $\left.\frac{n_{\nu i}}{n_\gamma}\right|_{T_{\rm dec}} = \frac{2\times 3/4}{2} = \frac{3}{4}$

At $T \sim m_e < T_W$ electrons become non-relativistic: $e^+e^- \rightleftharpoons \gamma\gamma$ annihilate in photons (mostly)

Entropy conservation: $g_*^s T^3 a^3|_{\rm before} = g_*^s T^3 a^3|_{\rm after}$

$\Gamma_{\rm ann} \sim \alpha^2 m_e \gg H(T\sim m_e)$ annihilations are “instantaneous”: $a_{\rm before} = a_{\rm after}$

Neutrinos are already decoupled: $$\overset{\gamma}{\overbrace{\left(2+\underset{\substack{\uparrow\\ e^+,\, e^-}}{4}\,\frac{7}{8}\right)T^3_{\rm before} = 2T^3_{\rm after}}}$$

$$\boxed{\frac{T_\nu}{T_\gamma} = \frac{T_{\rm before}}{T_{\rm after}} = \left(\frac{4}{11}\right)^{1/3}} \qquad \boxed{\left.\frac{n_{\nu_i}}{n_\gamma}\right|_{\rm today} = \frac{3}{11}}$$

entropy of electrons transferred to photons
(% correction due to residual $e^+e^- \to \nu\bar{\nu}$)

Standard Model Neutrinos are NOT CDM

If all three neutrinos are non-relativistic today: $\rho_\nu = m_\nu \, n_\nu$ $\Omega_\nu \equiv \frac{\rho_\nu}{\rho_c} = \frac{m_\nu}{94 \, eV \; h^2}$ (3 families)

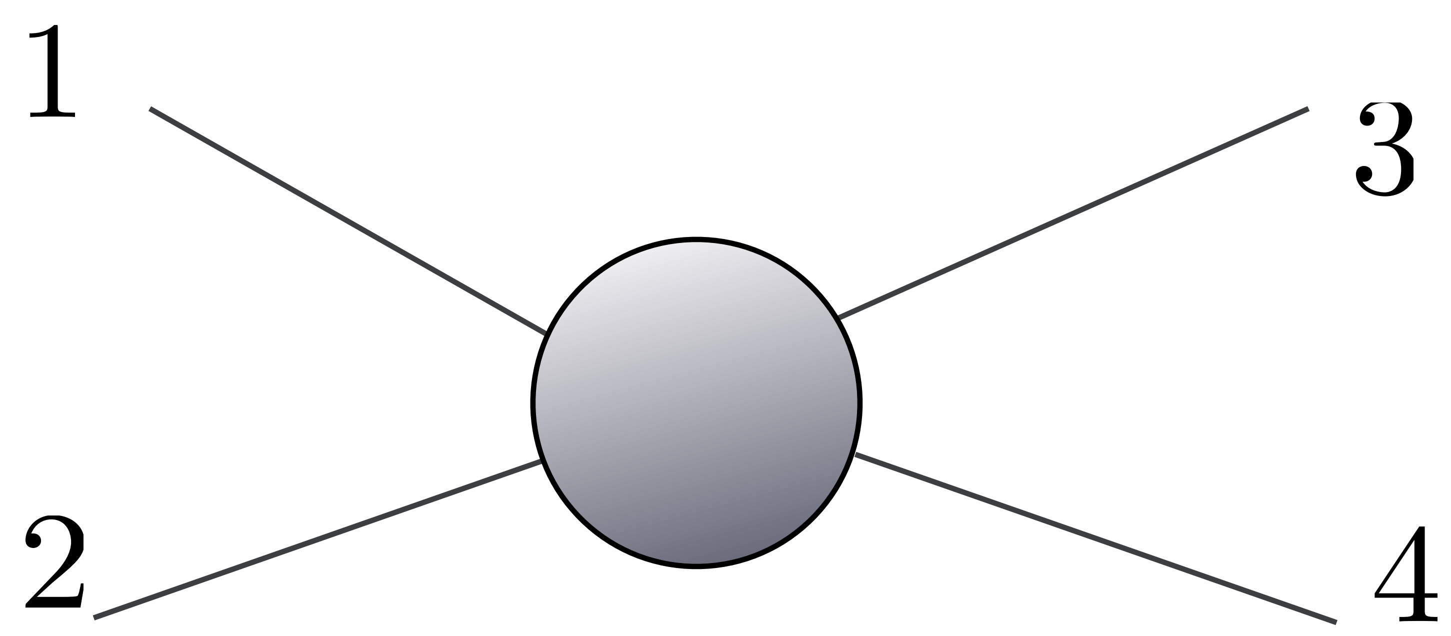
Neutrino free-streaming erases all structures up to mass scales: $M_{FS} = 3 \cdot 10^{18} \frac{M_\odot}{m_\nu^2(\mathrm{eV})} \gg M_{\mathrm{Gal}} = O(10^{12} \; M_\odot)$

Pauli exclusion principle forbids fermion dark matter of mass m<0.5 KeV to form dwarf spheroidal galaxies

(Tremaine-Gunn)

Cold relics: see Laura Covi's lectures

Boltzmann Equation



$$a^{-3}\frac{d(n_1a^3)}{dt}=\int\frac{d^3p_1}{(2\pi)^32E_1}\int\frac{d^3p_2}{(2\pi)^32E_2}\int\frac{d^3p_3}{(2\pi)^32E_3}\int\frac{d^3p_4}{(2\pi)^32E_4}$$

$$\times(2\pi)^4\delta^3(p_1+p_2-p_3-p_4)\delta(E_1+E_2-E_3-E_4)|\mathcal{M}|^2$$

$$\times\{f_3f_4[1\pm f_1][1\pm f_2]-f_1f_2[1\pm f_3][1\pm f_4]\}$$

number density

distribution functions

transition amplitude

$(\mathcal{M}_{12\to 34}=\mathcal{M}_{34\to 12})$ time-reversal! (CP)

Approximations

- kinetic equilibrium: $$f \to \frac{1}{e^{(E-\mu)/T} \pm 1}$$ enforced by fast scattering processes (momentum exchange)

- classical statistics: $$f \to e^{-(E-\mu)/T}$$ Boltzmann eq. typically necessary when T≲ E

➡ $$n_i = g_i\, e^{\mu_i/T} \int \frac{d^3p}{(2\pi)^3}\, e^{-E_i/T} = \; e^{\mu_i/T} n_i^{eq}$$

$$n_i^{eq} = \begin{cases} g_i (\frac{m_i T}{2\pi})^{3/2} e^{-m_i/T} & m_i \gg T \\ g_i \frac{T^3}{\pi^2} & m_i \ll T \end{cases}$$

$$f_3 f_4 [1 \pm f_1][1 \pm f_2] - f_1 f_2 [1 \pm f_3][1 \pm f_4] \to e^{-(E_1+E_2)/T} \left\{ \frac{n_3\, n_4}{n_3^{eq}\, n_4^{eq}} - \frac{n_1\, n_2}{n_1^{eq}\, n_2^{eq}} \right\}$$

$$= e^{-(E_1+E_2)/T} \left(e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T} \right)$$

Thermally averaged cross-section:

$$\langle \sigma\, v \rangle \equiv \frac{1}{n_1^{eq} n_2^{eq}} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} e^{-(E_1+E_2)/T}$$

$$\times (2\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)|\mathcal{M}|^2$$

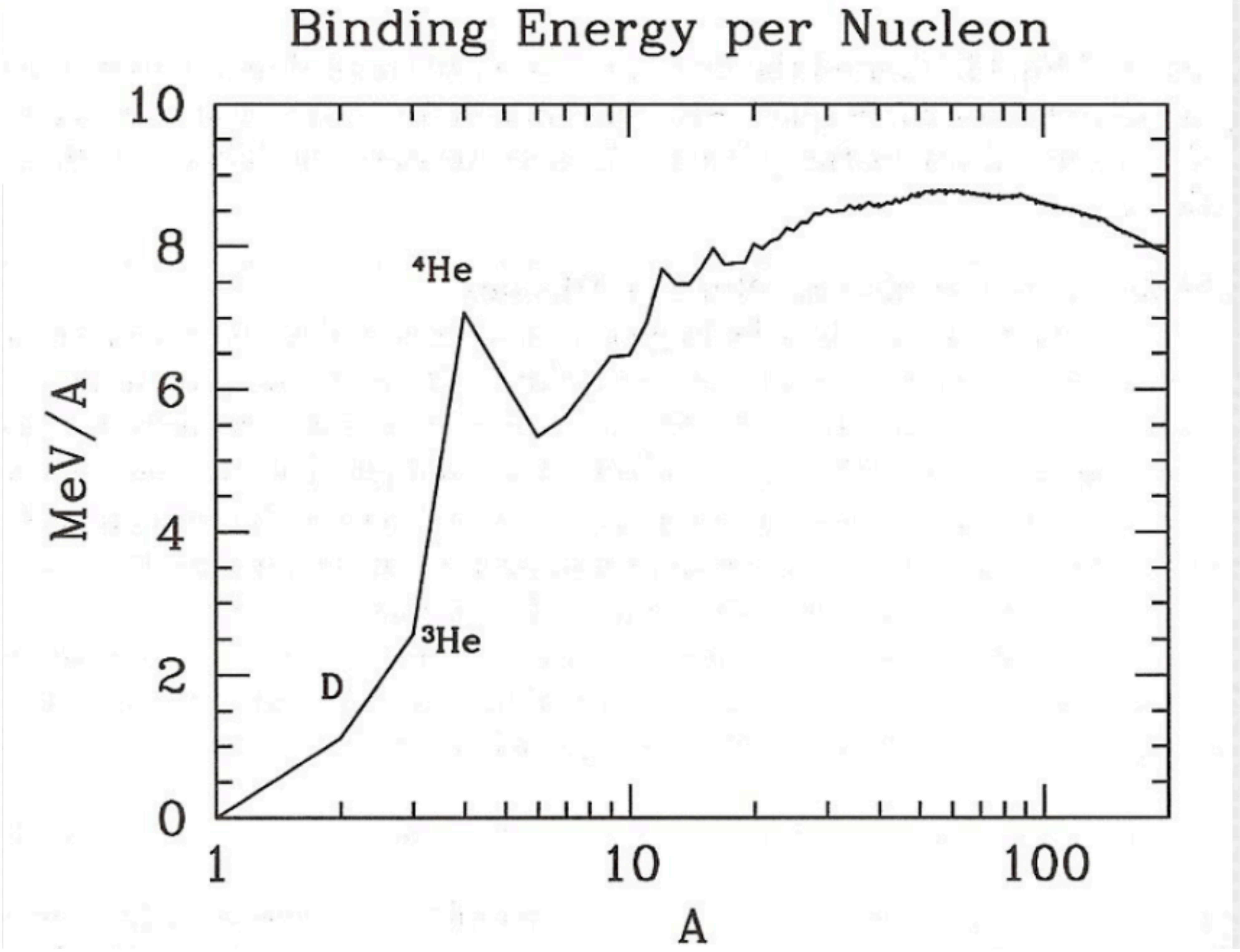
Boltzmann equation: $\dfrac{d\ln\left(n_1 a^3\right)}{d\ln a} = \dfrac{n_2\langle\sigma v\rangle}{H}\left[e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T}\right]$

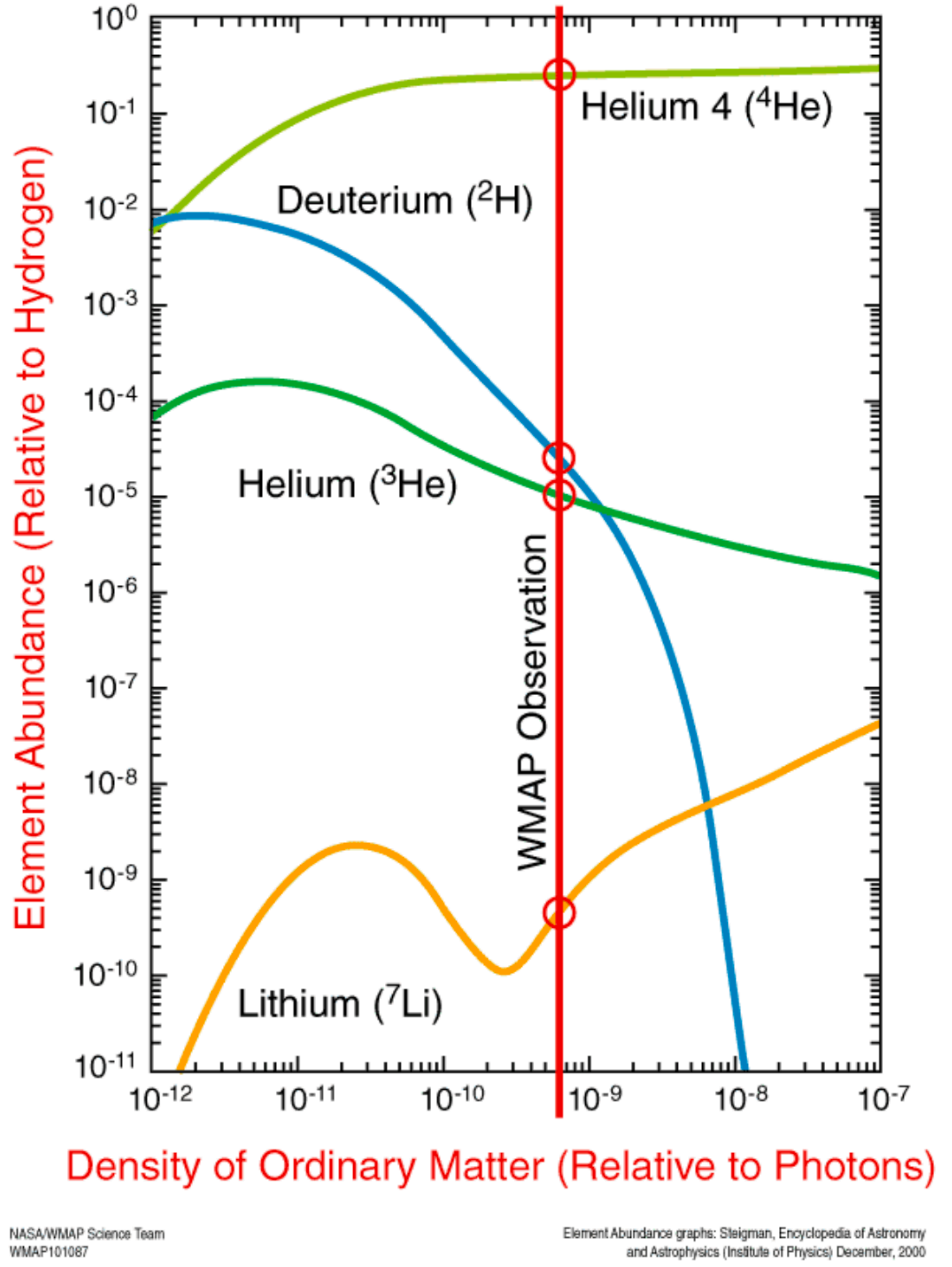
Equilibrium limit: $\Gamma(12 \to 34) = n_2\langle\sigma v\rangle \gg H$ $\Longrightarrow$ $\boxed{\mu_1 + \mu_2 = \mu_3 + \mu_4}$

Chemical equilibrium

Big Bang Nucleosynthesis

… or why there is something other than iron in the universe





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Element Abundance graphs: Steigman, Encyclopedia of Astronomy and Astrophysics (Institute of Physics) December, 2000

When did BBN start?

first nucleus. Deuterium: $n\,p \leftrightarrow D\,\gamma$ $\qquad B_D = m_n + m_p - m_D = 2.2\,\mathrm{MeV}$

Equilibrium: $\mu_n + \mu_p = \mu_D$ $\qquad \mu_\gamma = 0$ due to other processes, e.g. $e^+e^- \longleftrightarrow \gamma\gamma$

$$\Downarrow$$

$$\frac{n_D}{n_n\,n_p} = \frac{n_D^{eq}}{n_n^{eq}\,n_p^{eq}} \simeq \frac{3}{4}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_D/T}$$

$$n_p \simeq n_n \simeq n_b, \quad n_\gamma \sim T^3$$

$$\frac{n_D}{n_b} \sim \overbrace{\frac{n_b}{n_\gamma}}^{O(10^{-10})}\left(\frac{T}{m_p}\right)^{3/2} e^{B_D/T}$$

The large entropy density (n_γ/n_b) delays D production from T~B_D to T~0.1 MeV

Helium abundance

When nucleosynthesis starts, all neutrons go to Helium-4 (excellent approximation)

How many neutrons are around at T~0.1 MeV ?

$$n\,\nu \leftrightarrow p\,e^-$$
$$n \leftrightarrow p\,e^-\,\bar{\nu}$$
$$n\,e^+ \leftrightarrow p\,\bar{\nu}$$

$$\frac{n_n}{n_p} = e^{-\overset{1.293\text{ MeV}}{\Delta m}/T}\, e^{(\mu_n-\mu_p)/T}$$

$$\frac{\mu_n}{T} \simeq \frac{\mu_p}{T} \sim \frac{n_b}{n_\gamma} \sim 10^{-10}$$

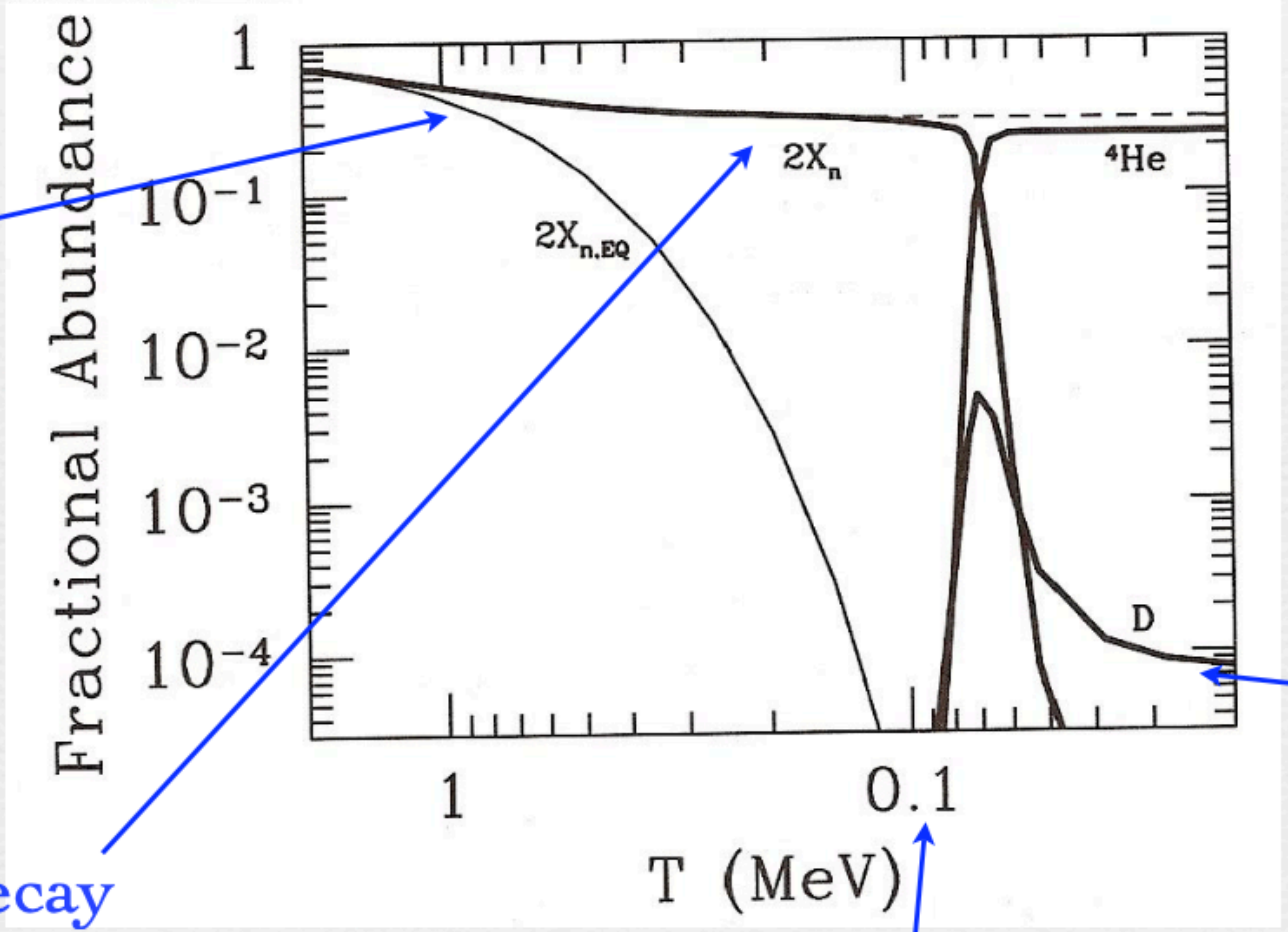
$$T_W \overset{\Gamma_W = H}{=} \left(0.33\,\frac{g_*^{1/2}}{G_F^2 M_p}\right)^{1/3} \sim 0.8\,\text{MeV}$$

$$\left.\frac{n_n}{n_p}\right|_{T=0.8\,\text{MeV}} \simeq \frac{1}{6} \quad \xrightarrow[\text{Boltzmann eq.}]{\text{n-decay}} \quad \left.\frac{n_n}{n_p}\right|_{T=0.1\,\text{MeV}} \simeq \frac{1}{8}$$

~ all neutrons finally go in ^{4}He

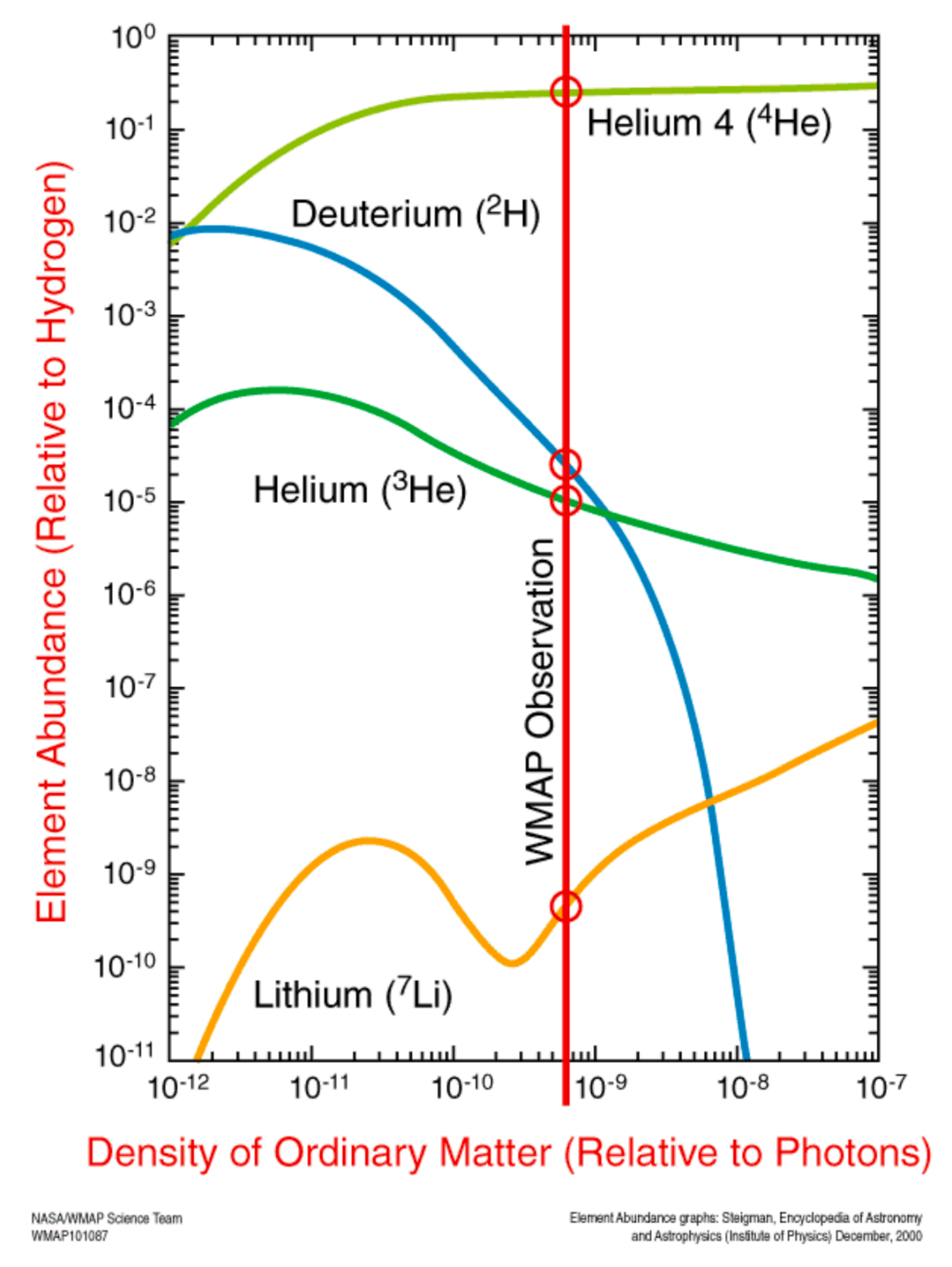
$$X_4 \equiv \frac{4\, n_{^4He}}{n_b} = \frac{2 n_n}{n_n + n_p} \simeq 0.22$$

decoupling



n decay

BBN starts



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Element Abundance graphs: Steigman, Encyclopedia of Astronomy and Astrophysics (Institute of Physics) December, 2000

residual D abundance

$$D\, p \to {}^3He\, \gamma$$

inefficient at low n_b/n_γ

$$T_W = \left(0.33\, \frac{g_*^{1/2}}{G_F^2 M_p}\right)^{1/3} \sim 0.8\,\mathrm{MeV}$$

$$g_* = g_*^{\mathrm{standard}} + \Delta g_*$$

$$T_W = T_W^{\mathrm{standard}} + \Delta T_W$$

$$\frac{n_n}{n_p} = \left.\frac{n_n}{n_p}\right|_{\mathrm{standard}} + \Delta \frac{n_n}{n_p}$$

$$X_4 = X_{4,\ \mathrm{st.}} + \Delta X_4$$

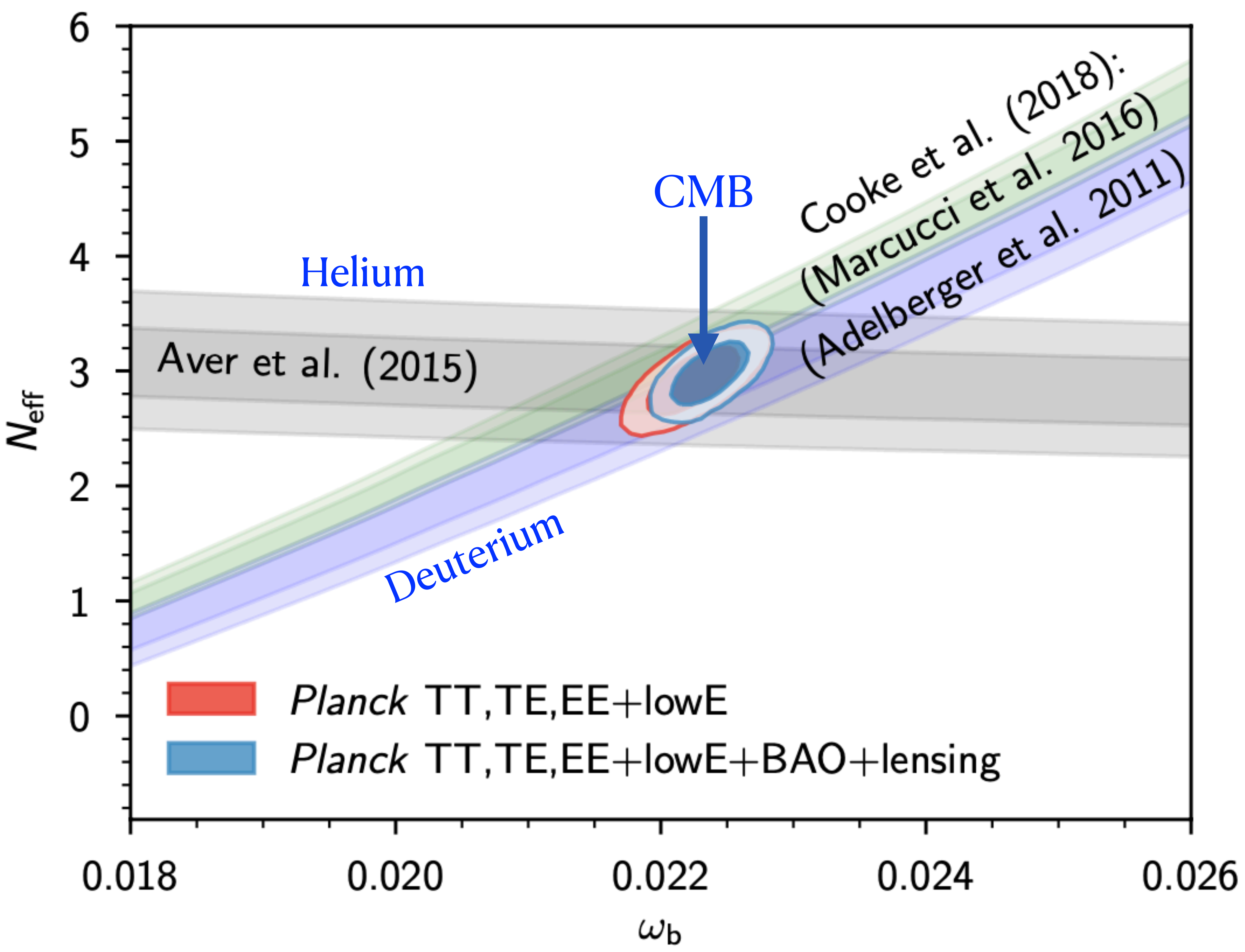
$$\Downarrow$$

$$\Delta N_\nu < 0.1$$

bound on extra relativistic species

BBN is a powerful probe of n_b/n_γ and of $H(T \sim 1\mathrm{MeV})$!

CMB+BBN: Baryons make up only ~ 4%



$$\omega_b = \Omega_b h^2$$

Recombination

first atom. Hydrogen: $e\, p \leftrightarrow H$ $\qquad$ $B = m_p + m_e - m_H = 13.6\,\mathrm{eV}$

charge neutrality: $n_e = n_p$

free-electron fraction: $X_e = \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H}$

Equilibrium: $\mu_e + \mu_p = \mu_H$ $\Rightarrow$ $\frac{n_e\, n_p}{n_H} = \frac{n_e^{eq}\, n_p^{eq}}{n_H^{eq}}$

$O(10^{-10})$

Saha equation: $\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \boxed{\frac{n_b}{n_\gamma}} \left(\frac{T}{m_e}\right)^{3/2} e^{B/T}$

The small n_b/n_γ delays recombination, defined as $X_e^{rec} = 0.1$ to $T_{\rm rec} = 0.3$ eV

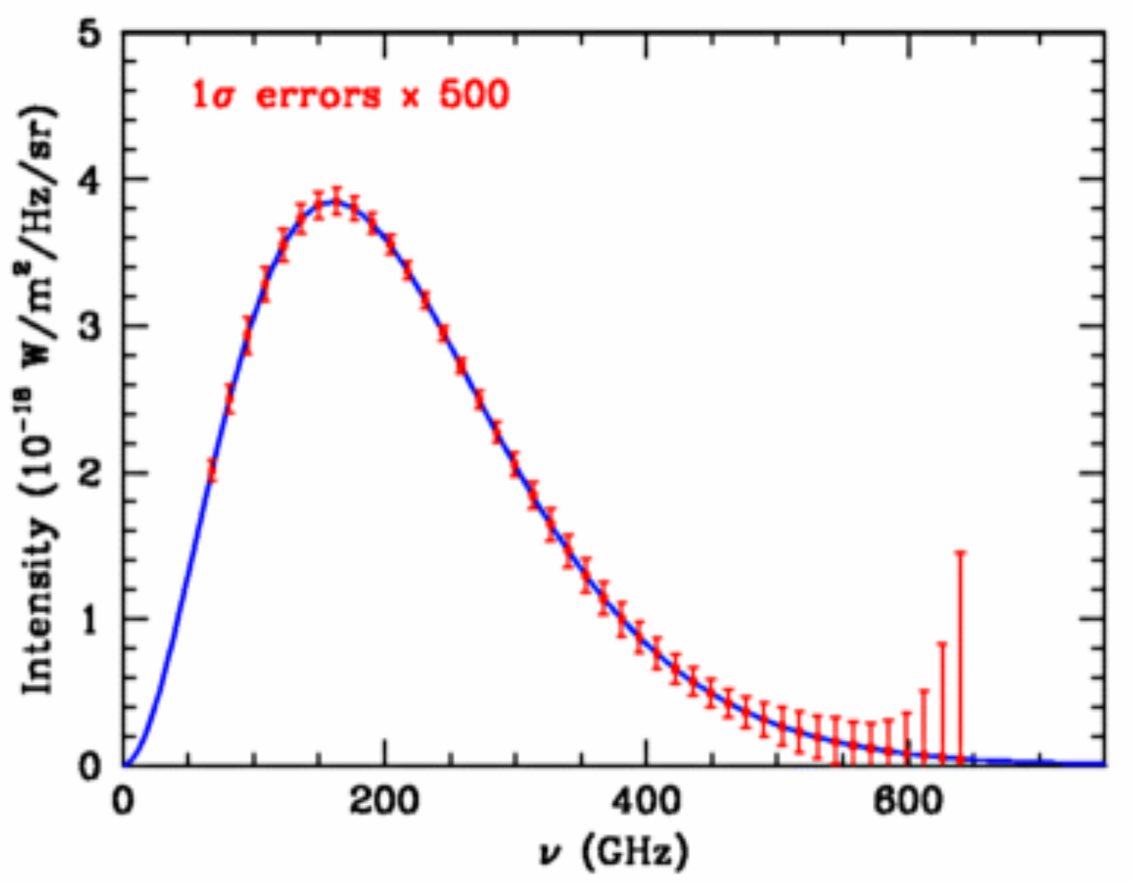
From recombination to decoupling

Compton scattering $\Gamma_T = n_e\,\sigma_T = X_e\,n_b\,\sigma_T$ $(\sigma_T = 0.665 \times 10^{-24}\mathrm{cm}^2)$

At recombination X_e drops, then $\Gamma_T < H$ and photons decouple from matter at $T_{\rm dec} \simeq 0.26\ \mathrm{eV}$

After decoupling, photon energies get redshifted

$$f_\gamma(E;\,t) = f_\gamma(E\,a(t)/a_{\rm dec};\,t_{\rm dec}) = \frac{1}{e^{Ea(t)/a_{\rm dec}T_{\rm dec}} - 1} = \frac{1}{e^{E/T(t)} - 1}$$



Blackbody spectrum at $T(t) = T_{\rm dec}\dfrac{a_{\rm dec}}{a(t)}$

Baryogenesis

Evidences of a baryon-asymmetric Universe

Direct searches:

Cosmic rays at E > O(100 MeV) probe galactic scales, $r_{\rm Gal} \sim 30\; kpc$

$\frac{n_{\bar{p}}}{n_p} \sim 3\; 10^{-4}$ compatible with secondary production in $p\,p \to p\,p\,p\,\bar{p}$

Indirect searches:

Look for: 1) γ's from $b\,\bar{b}$ annihilations;
2) CMB spectrum distortions due to Compton scattering.

no signal of galaxy-antigalaxy annihilation from Virgo cluster, or X-rays emitting clusters: $r_{\rm Clust} \sim 10\; Mpc$ (Steigman '76)

CMB+diffuse gamma ray background constrain matter-antimatter islands to be larger than ~ $O(10^3\;\; Mpc) \sim 1/H_0$ (Cohen, De Rujula, Glashow '98)

Do we really need to produce it?

thermal fluctuation ?

$O(10^{79})$ photons in a galactic volume

$n_b \sim n_\gamma$ at T > GeV, $\frac{n_b - n_{\bar{b}}}{s} \sim \frac{n_b^{1/2}}{g_* n_b} \sim 10^{-42}$

$$\text{WMAP+BBN:} B = \frac{n_b - n_{\bar{b}}}{s} = (8.6 \pm 0.4) \times 10^{-11}$$

initial condition ?

problem with inflation: $a_f \sim e^{60} a_i \rightarrow n_b^f \sim e^{-180} n_b^i$

$$\rho_b^i \sim e^{180} n_b^f m_p \ll V \sim g_* T_{RH}^4$$

$T_{RH} \gg e^{180} \frac{n_b}{s_f} m_p \sim 10^{65}\,\text{GeV}$

inflation is useless (monopoles, horizon...)
self-consistency problems $T_{RH} \gg M_p \sim 10^{19}\,\text{GeV}$

Sakharov's conditions

Necessary conditions for a theory of baryogenesis:

1) B-violating interactions;

2) C and CP-violating interactions;

C: $\Gamma[\, i \to f\,] = \Gamma[\,\bar{i} \to \bar{f}\,]$

CP ≡ T: $\Gamma[\, i \to f\,] = \Gamma[\, f \to i\,]$

no net result: $n_b - n_{\bar{b}} = 0$

3) departure from th. equilibrium;

th. equilibrium: $\begin{aligned} n_{b_i} &= n_{b_i}(E_i,\, \mu_i,\, T) \\ n_{\bar{b}_i} &= n_{\bar{b}_i}(E_i,\, -\mu_i,\, T) \end{aligned}$ + B-violation + charge neutrality

$$\longrightarrow \boxed{\mu_i = 0 \qquad\qquad n_{b_i} = n_{\bar{b}_i}}$$

Standard Model: B-violation

B (and L) are **accidental symmetries** of the SM: $\partial_\mu J^\mu_{B(L)} = 0$

they are broken at the quantum level by **triangle anomalies**:

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = -\frac{3}{32\pi^2} g^2 F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} \quad \text{(+ U(1)...)}$$

$$\Delta B = 3\,\Delta N_{cs} = \Delta L$$

$$\left(N_{cs}(t) = \frac{g^2}{96\pi^2} \int d^3x\, \epsilon_{abc} \epsilon^{ijk} A^a_i A^b_j A^c_k \right)$$

$$B = \int d^3x\, J^0_B$$

in vacuum to vacuum transitions ΔN_{cs} is integer.

effective interaction: $O_{B+L} = \Pi_{i=1\ldots3}(q_{L_i}\, q_{L_i}\, q_{L_i}\, l_{L_i})$

$$\boxed{\Delta(B+L) = 6\,, \quad \Delta(B-L) = 0}$$

Sphalerons

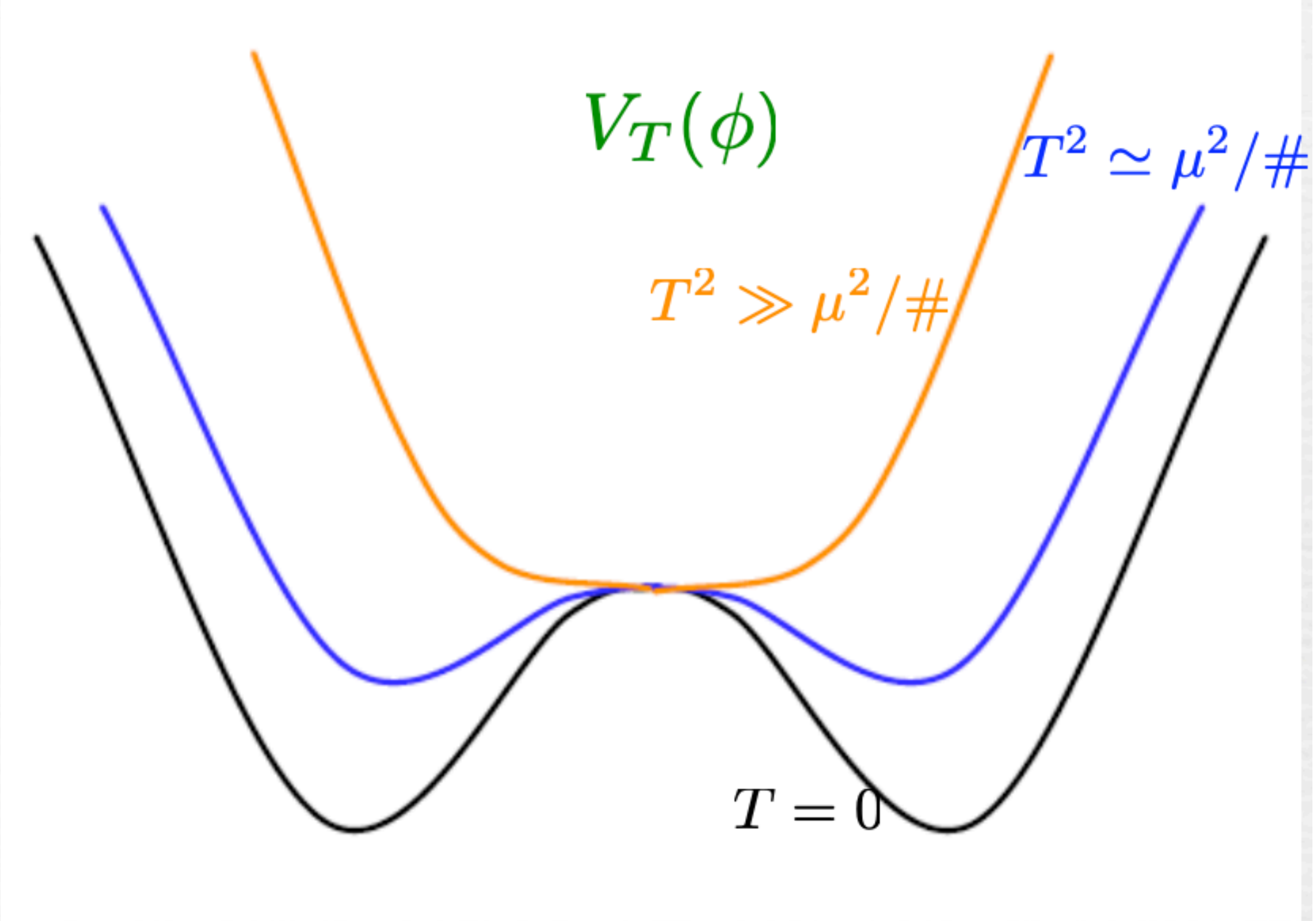
Higgs expectation value

different vacua are separated by an energy barrier $E_{sp} = \frac{4\pi v}{g} B(\lambda/g) = O(10\,\mathrm{TeV})$

$T = 0$: vacuum tunnelling $\Gamma \sim e^{4\pi/\alpha_w} = O(10^{-165})$

$T \neq 0$: thermal fluctuations $\Gamma \sim e^{-\frac{E_{sp}(T)}{T}}$

$E_{sp}(T) \propto v(T) \to 0$ at high T!



Sphaleron rate:

Broken phase
(Arnold, Mc Lerran)

$$\frac{\Gamma_{B+L}}{V} = \kappa \frac{M_W^7}{(\alpha T)^3} e^{-\beta E_{sph}(T)}$$

$\beta = 1/T,\ M_W = g^2 v_F(T)/2$

Symmetric phase
(Arnold, Yaffe)

$$\frac{\Gamma_{B+L}}{V} = (10.8 \pm 0.7) \left(\frac{gT}{m_D}\right)^2 \alpha^5 T^4 \left[\ln\left(\frac{m_D}{\gamma}\right) + 3.041 + \left(\frac{1}{\ln(1/g)}\right)\right]$$

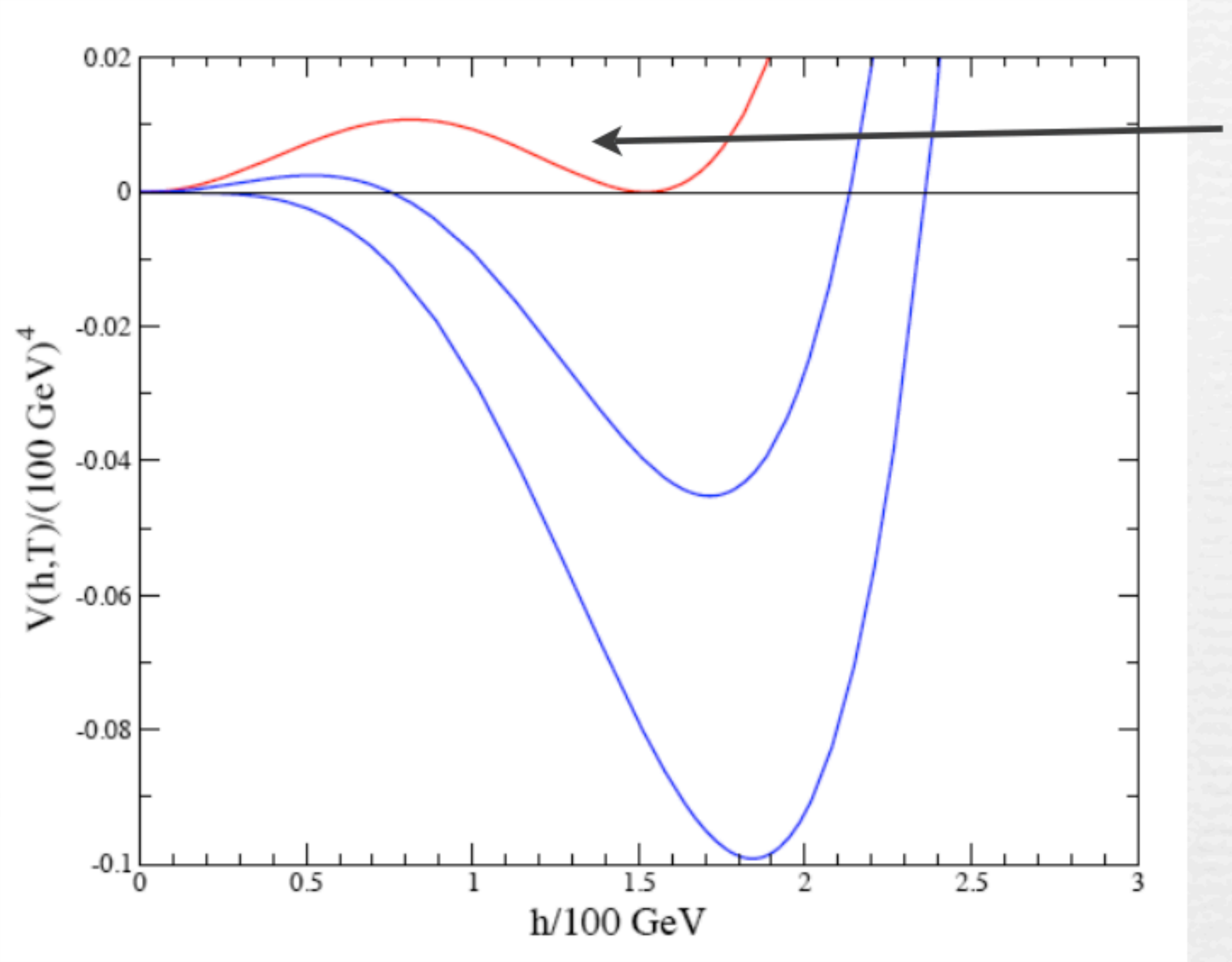
$m_D \sim gT$

$\gamma \sim g^2 T \ln(1/g)$

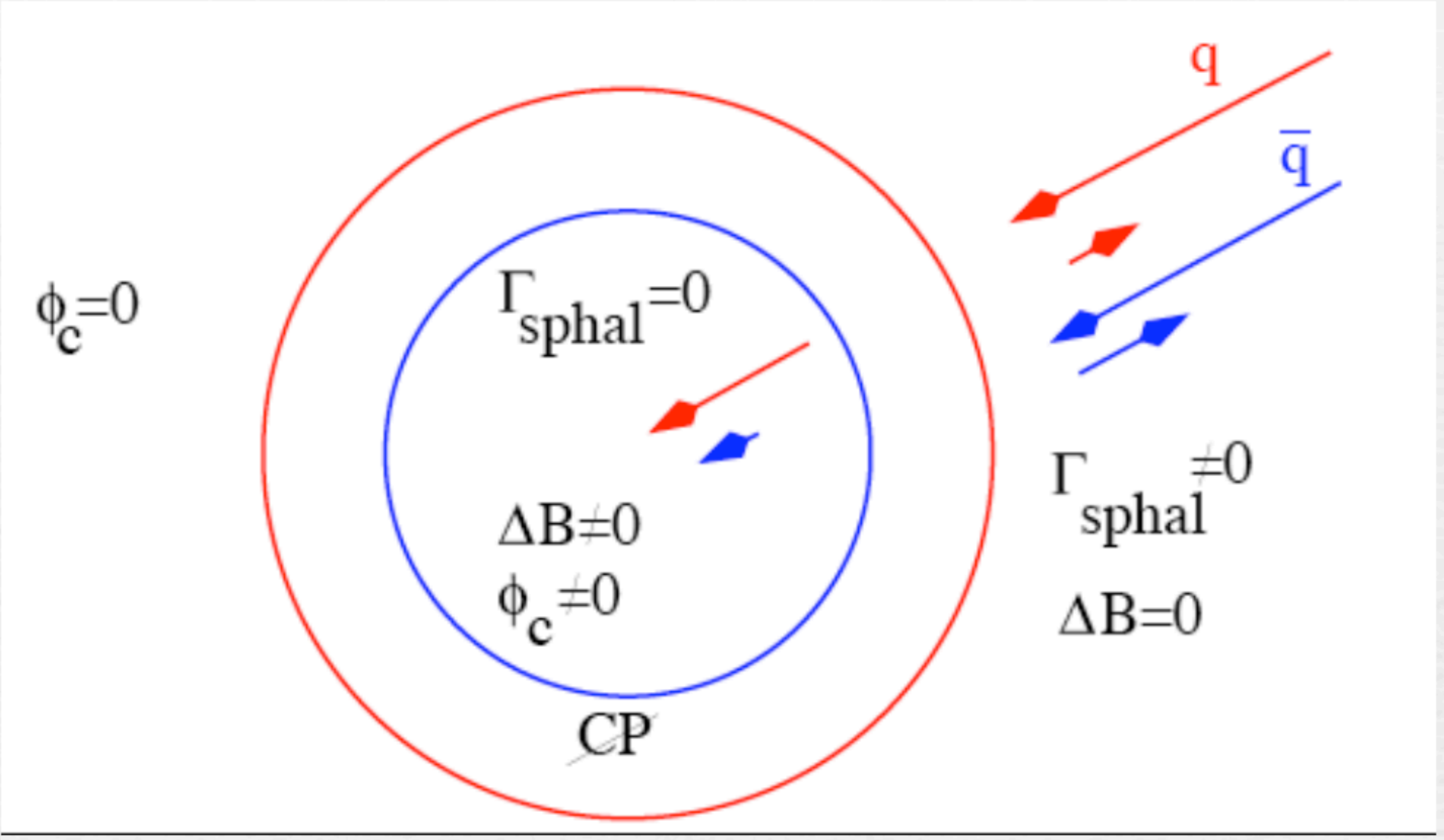
in equilibrium for $T_{\mathrm{EWPT}} \sim 100$ GeV $< T < O(10^{12})$ GeV

Standard Model: out-of-equilibrim

need to switch off the sphalerons in $\tau \ll H^{-1}$: first order phase transition



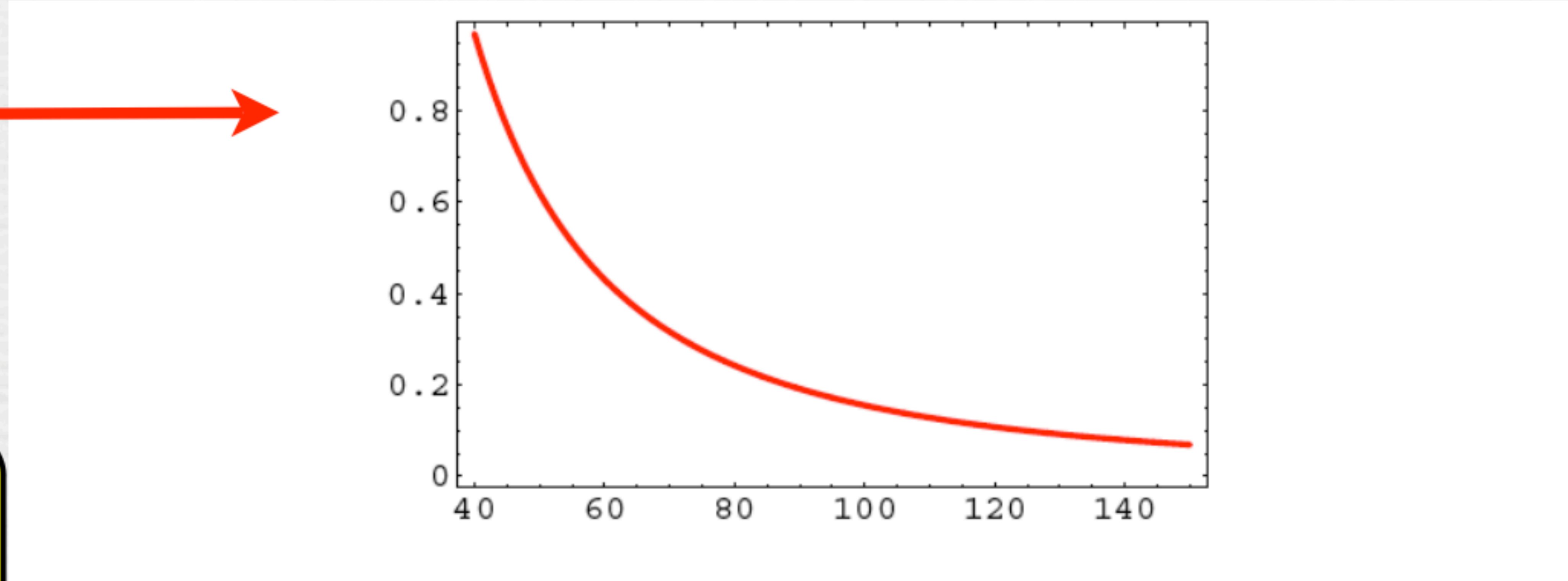
barrier due to \`cubic' term in $V_T(\phi)$



sphaleron erasure of the asymmetry avoided if $v(T_c)/T_c > 1$

moreover, CKM CP violation is not enough

Electroweak baryogenesis still a possibility in SM extensions (SUSY)



$v(T_c)/T_c$ as a function of m_H (in GeV) [one-loop]

$T_{\mathrm{BAU}} \gg T_{\mathrm{EW}}$: need B-L violation

$$n_i - \overline{n}_i = \frac{gT^3}{6}\begin{cases} \beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) \;, & \text{fermions} \\ 2\beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) \;, & \text{bosons} \;. \end{cases}$$

SM: $5N_f + 1$ chemical potentials.

each interaction in equilibrium gives a constraint.

$$\begin{aligned} B &= \sum_i (2\mu_{qi} + \mu_{ui} + \mu_{di}) \;, \\ L_i &= 2\mu_{li} + \mu_{ei} \;, \quad L = \sum_i L_i \;. \end{aligned}$$

assuming: equilibrium for sphalerons, Yukawa, gauge, QCD instantons + hypercharge neutrality one gets

$$B = c_s(B-L)\,, \qquad L = (c_s - 1)(B-L) \qquad c_s = \frac{8N_f + 4}{22N_f + 13}$$

... so, the problem is, where did B-L come from?

SM+right-handed neutrinos

L-violation!

lagrangian for the leptonic sector:

$$\begin{aligned}\mathcal{L} \;&=\; \bar{\ell}_{Li} i\partial\!\!\!/ \ell_{Li} + \bar{e}_{Ri} i\partial\!\!\!/ e_{Ri} + \bar{N}_{Ri} i\partial\!\!\!/ N_{Ri} \\ &\quad + f_{ij}\bar{e}_{Ri}\ell_{Lj}H^\dagger + \boxed{h_{ij}\bar{N}_{Ri}\ell_{Lj}H - \frac{1}{2}M_{ij}N_{Ri}N_{Rj}} + \text{h.c.}\end{aligned}$$

neutrino masses:

$$\begin{pmatrix} 0 & h\,v \\ h^T\,v & M \end{pmatrix}$$

$$\boxed{m_\nu \sim \frac{(hv)^2}{M}\,, \qquad M}$$

see-saw mechanism!

$$\left(\text{ex: } \frac{m_\nu}{5\cdot 10^{-2}\,\text{eV}} = \left(\frac{hv}{1\,\text{GeV}}\right)^2 \frac{2\cdot 10^{10}\,\text{GeV}}{M}\right)$$

CP-violation: f_{ij} , M_{ij} diagonal, 6 CP-violating phases in h_{ij}

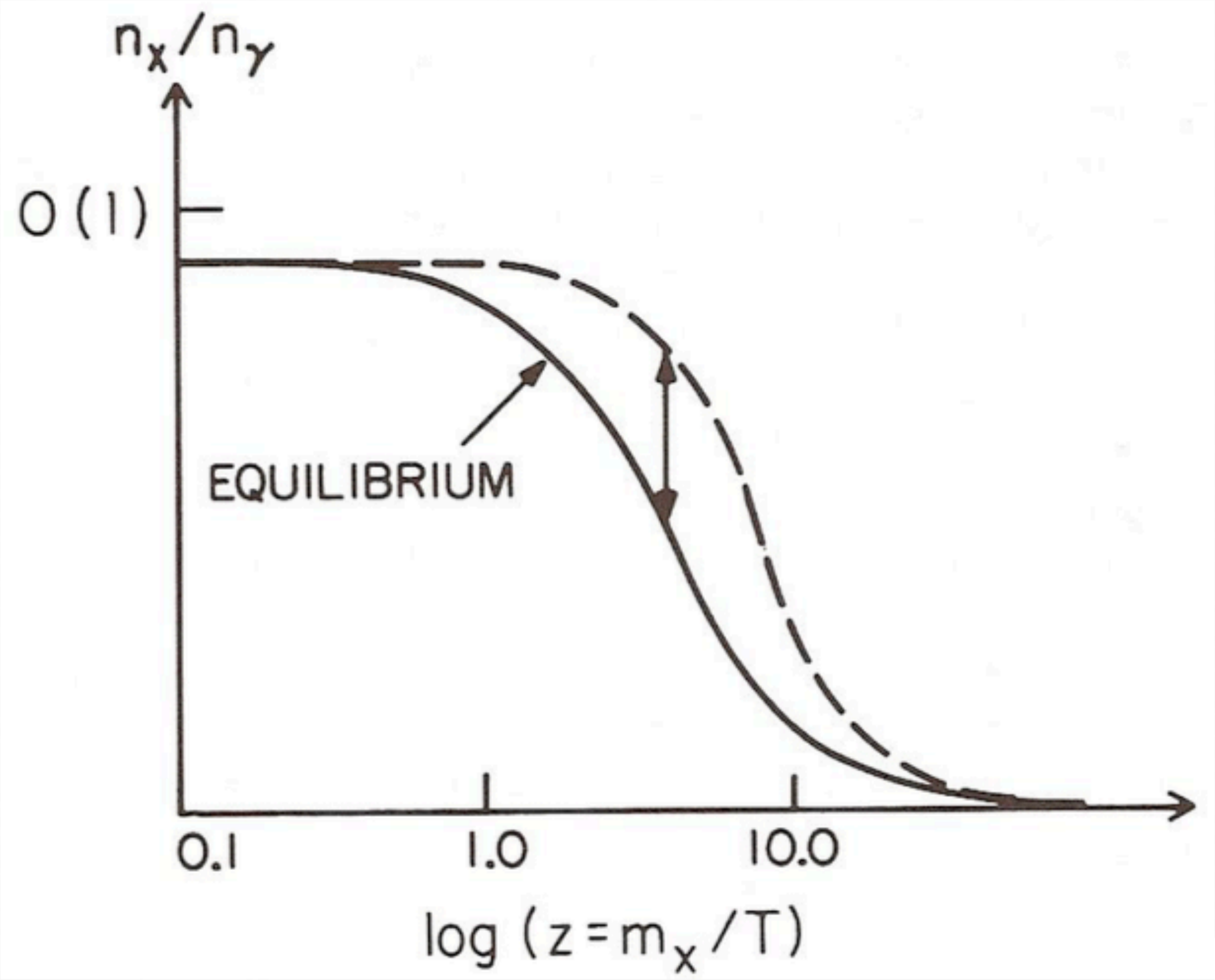
however, at low energies (<< M)

$$\mathcal{L}_{\text{eff}} = \bar{\ell}_{Li} i\partial\!\!\!/ \ell_{Li} + \bar{e}_{Ri} i\partial\!\!\!/ e_{Ri} + f_{ii}\bar{e}_{Ri}\ell_{Li}H^\dagger + \frac{1}{2}\sum_k h^T_{ik}h_{kj}\ell_{Li}\ell_{Lj}\frac{H^2}{M_k} + \text{h.c.}\,.$$

only 3 physical phases in $-\frac{1}{2}m_{\nu_{ij}}\ell_{Li}\ell_{Lj}\frac{H^2}{\langle H\rangle^2}$

no straight connection between high-energy and low-energy CP violation

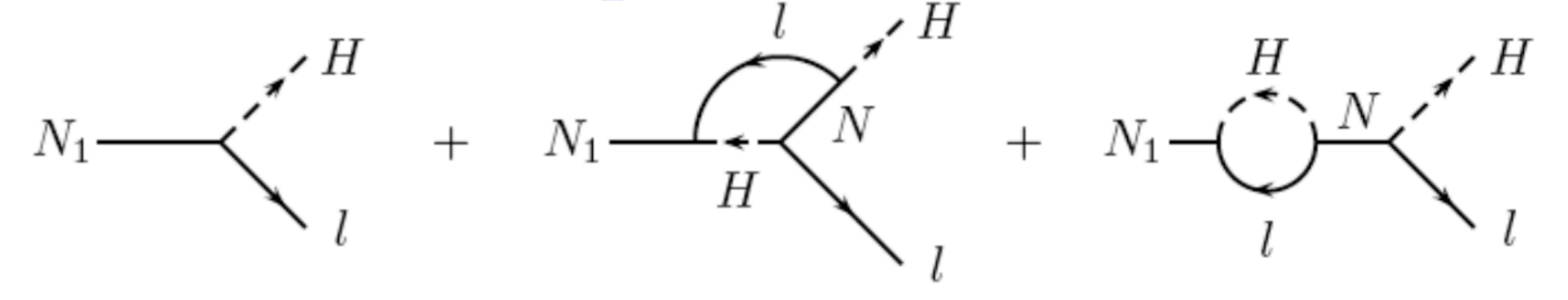
Out-of-equilibrium decay



$T_{\rm dec} < M$: inverse decays are out of eq.

$$\epsilon = \frac{\Gamma(N_1 \to H\, l) - \Gamma(\bar{N}_1 \to \bar{H}\, \bar{l})}{\Gamma(N_1 \to H\, l) + \Gamma(\bar{N}_1 \to \bar{H}\, \bar{l})}$$

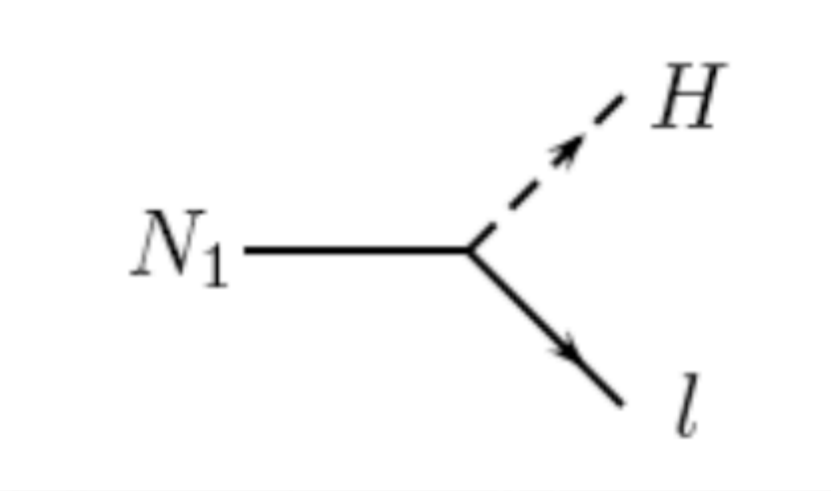
CP-violation is a quantum effect (interference)



$$L = \frac{n_l - n_{\bar{l}}}{s} \sim \frac{\epsilon\, n_N}{g_* n_N} = \frac{\epsilon}{g_*}$$ (fully efficient)

$$B - L = -L = -K\frac{\epsilon}{g_*}$$ (realistic) K=efficiency factor <1 (need Boltz. eq.)

Leptogenesis and light neutrinos



$$\Gamma_{\rm dec} \sim \frac{h^2}{8\pi} M \sim \frac{1}{8\pi} \frac{h^2 v^2}{M} \frac{M^2}{v^2}$$

$$H(T=M) \sim 1.66\, g_*^{1/2} \frac{M^2}{M_p}$$

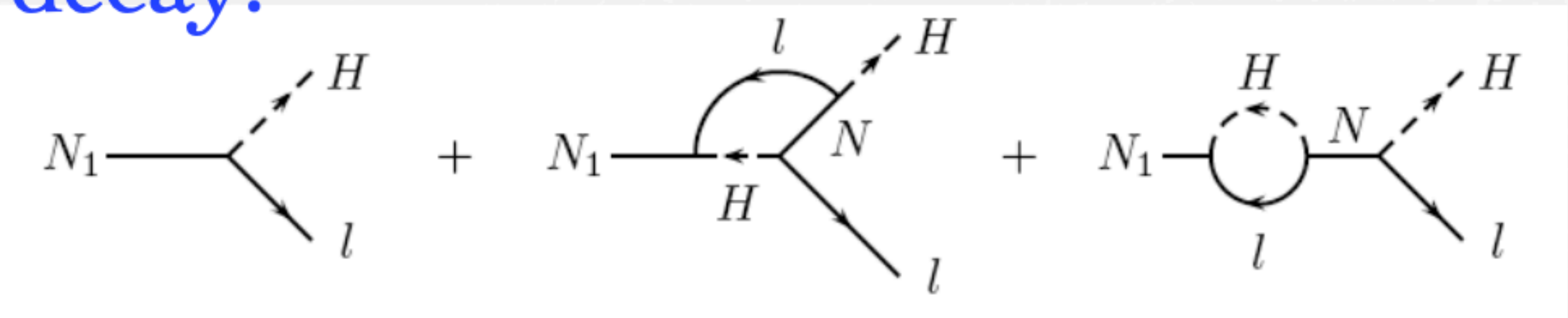
$$\boxed{\frac{\Gamma_{\rm dec}}{H(T=M)} \sim \frac{1}{1.66\, g_*^{1/2} 8\pi} \frac{M_p\, m_\nu}{v^2} \sim 1 \longrightarrow m_\nu = O(10^{-3}\,{\rm eV})}$$

Boltzmann equations

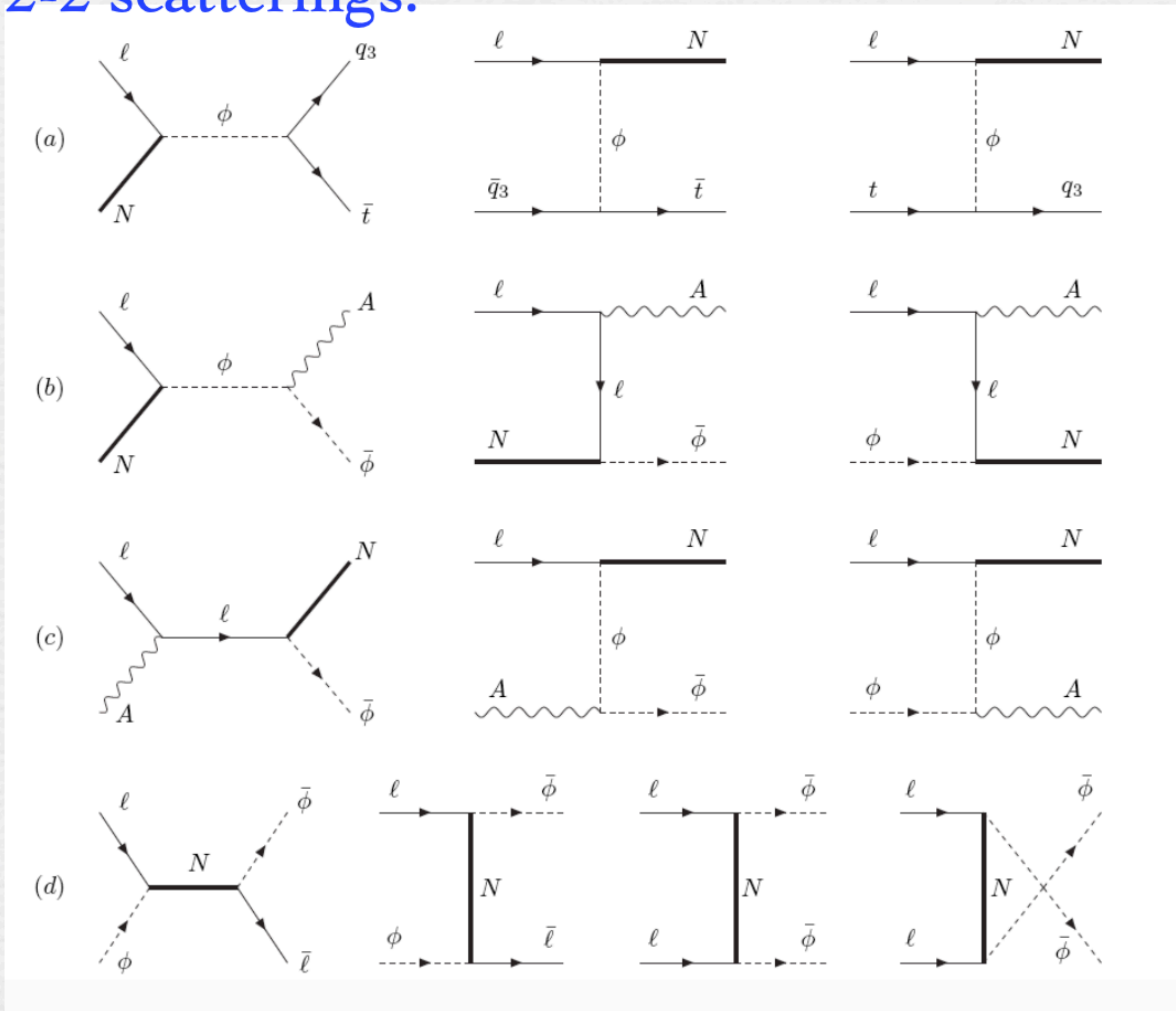
$z = M_{N_1}/T$

$$\boxed{\begin{aligned}\frac{dN_{N_1}}{dz} &= -(D+S)\,(N_{N_1}-N^{\rm eq}_{N_1})\,,\\ \frac{dN_{B-L}}{dz} &= -\varepsilon_1\,D\,(N_{N_1}-N^{\rm eq}_{N_1})-W\,N_{B-L}\end{aligned}}$$

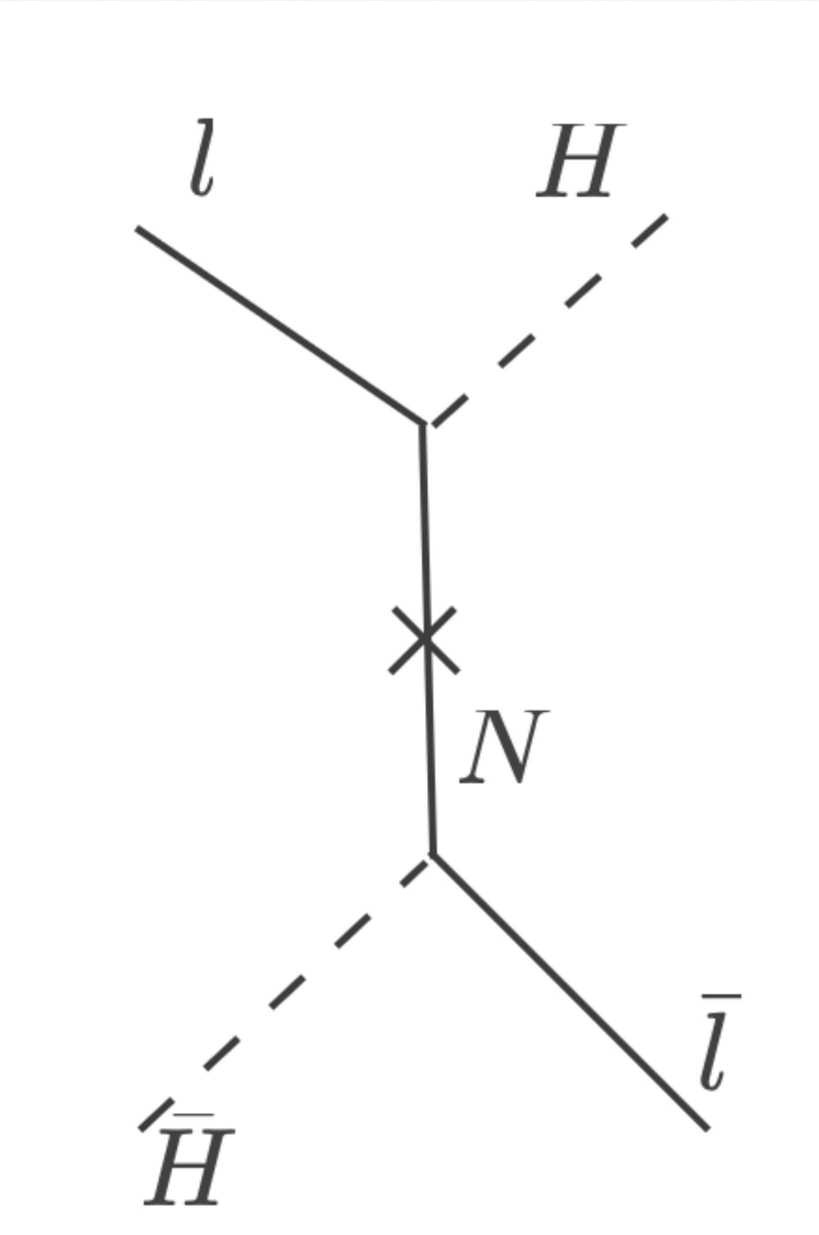
decay:



2-2 scatterings:

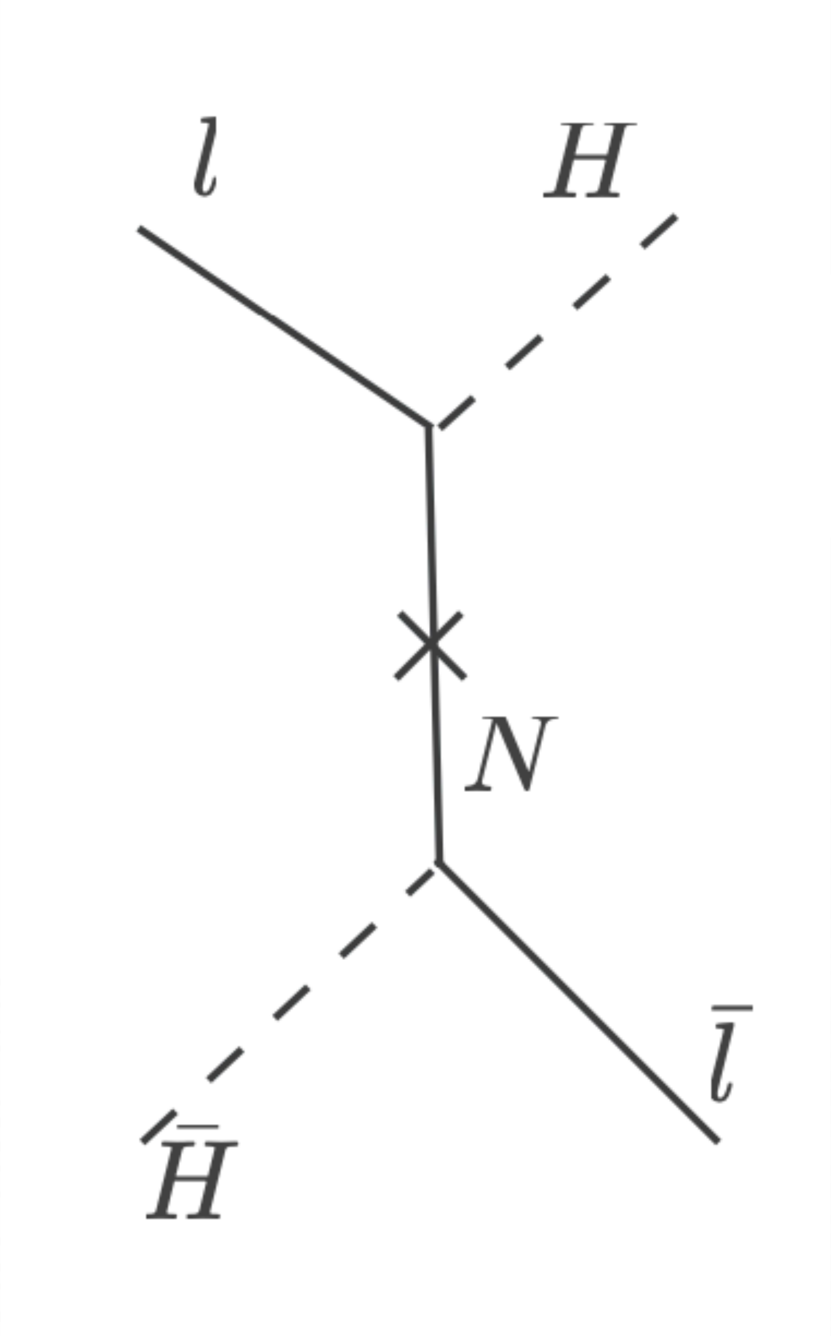


washout:



$$-\frac{1}{2}m_{\nu_{ij}}\ell_{Li}\ell_{Lj}\frac{H^2}{\langle H\rangle^2}$$

Washout and neutrino mass



$\Delta L = 2$ processes can erase the asymmetry

$$-\frac{1}{2} m_{\nu_{ij}} \ell_{Li} \ell_{Lj} \frac{H^2}{\langle H \rangle^2}$$

$$\Gamma_{\text{wo}} \sim \frac{m_\nu^2}{v^4} T^3 < H(T \simeq M)$$

upper bound on

$$m_\nu^2 = \left(\sum_{i=1}^{3} m_i^2 \right)^{1/2}$$

$$\boxed{10^{-3}\, eV < m_\nu < 0.2 \text{ eV}}$$

$(m_{\nu_e} < 2.5 \text{ eV from } \beta\text{-decay})$

independence from initial conditions

enough B typically requires $M > 4 \cdot 10^8$ GeV