Introduction to Cosmology (I)

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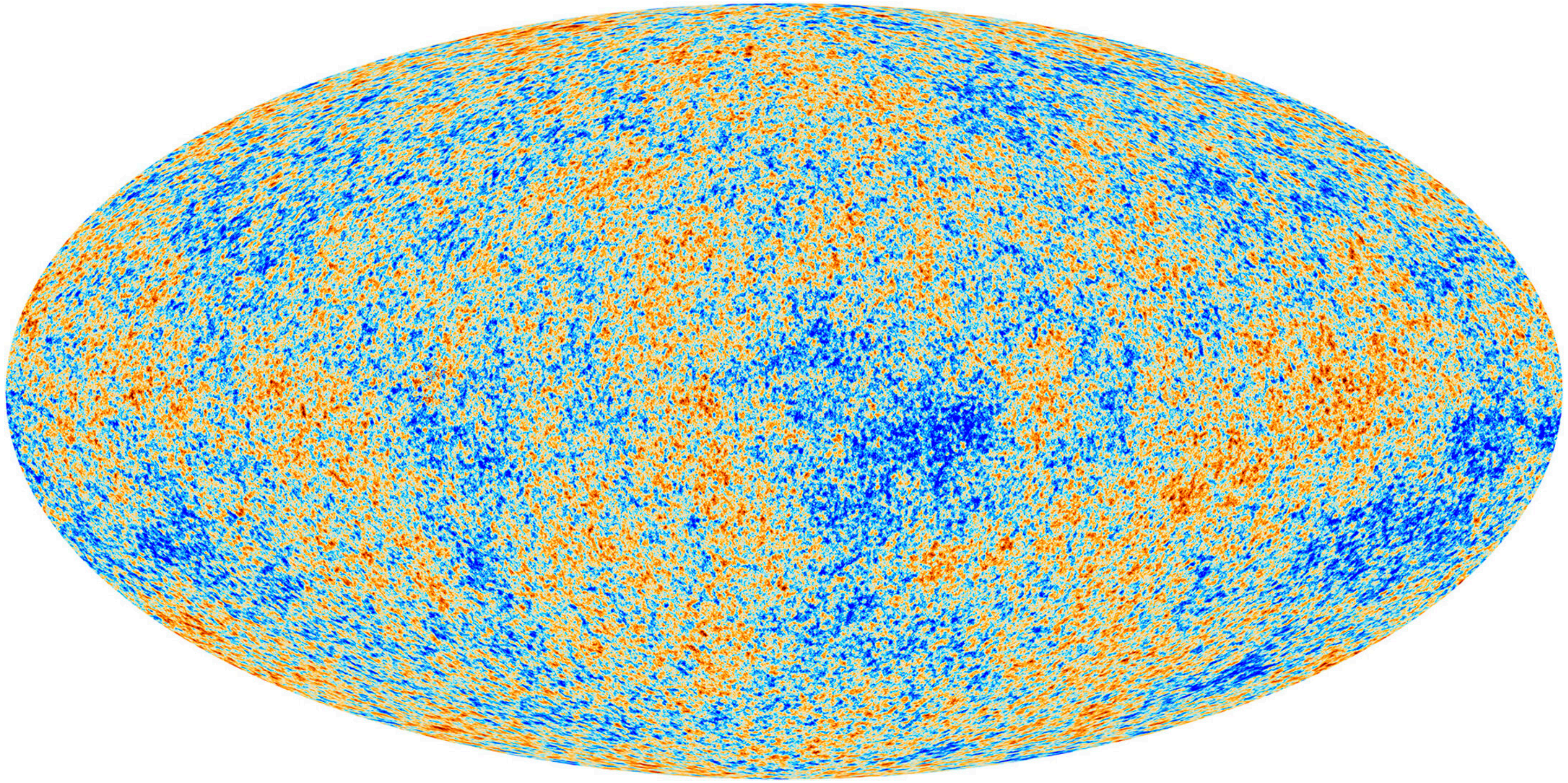
Outline

- FLRW Universe
- Thermal history of the Big Bang (= relics from the Early Universe)
- Inflation and cosmic perturbations

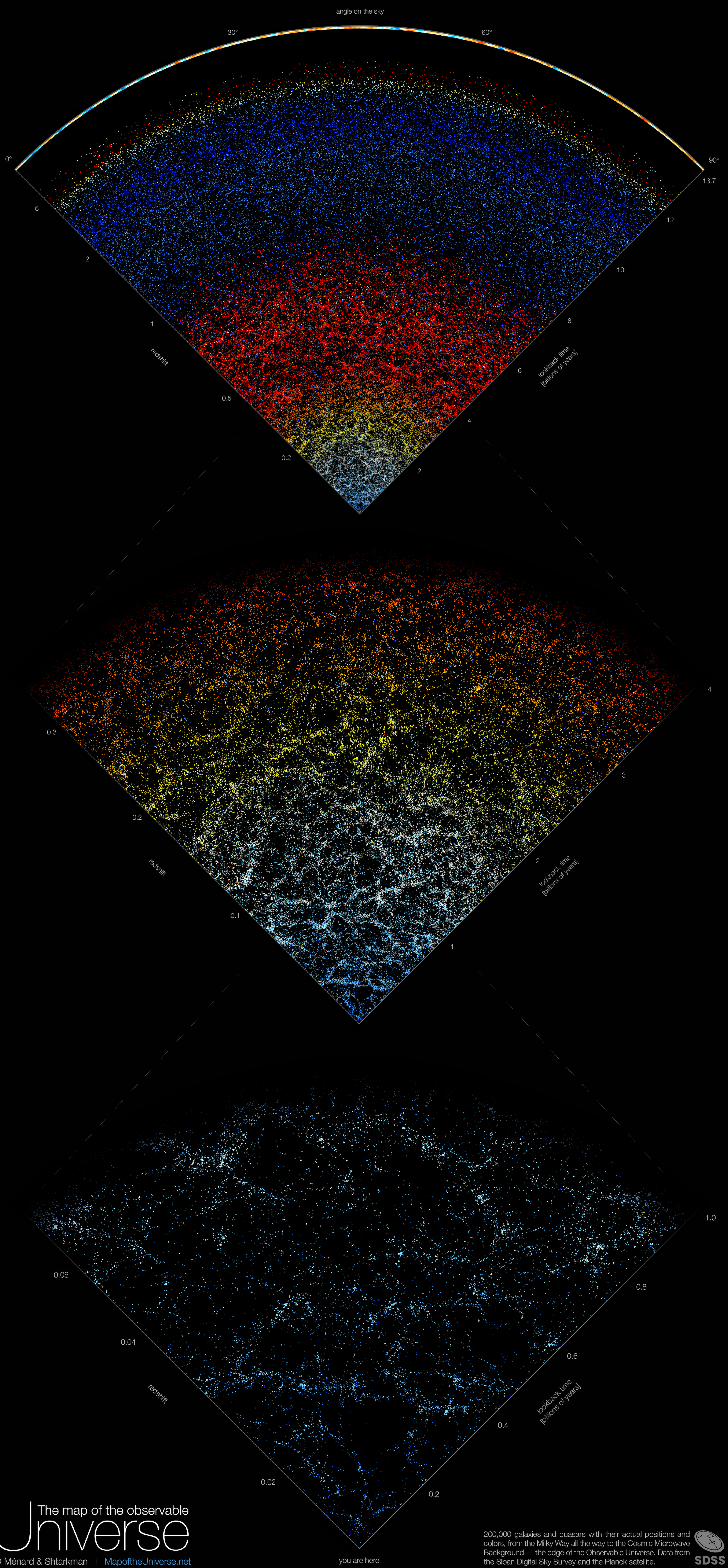
Some facts about our universe

It is isotropic and (most probably) homogeneous

CMB temperature map from PLANCK
~ 380,000 years after the Bang



$T \simeq 2.73\ K$ $\qquad \frac{\Delta T}{T} \simeq 10^{-5}$



The map of the observable Universe

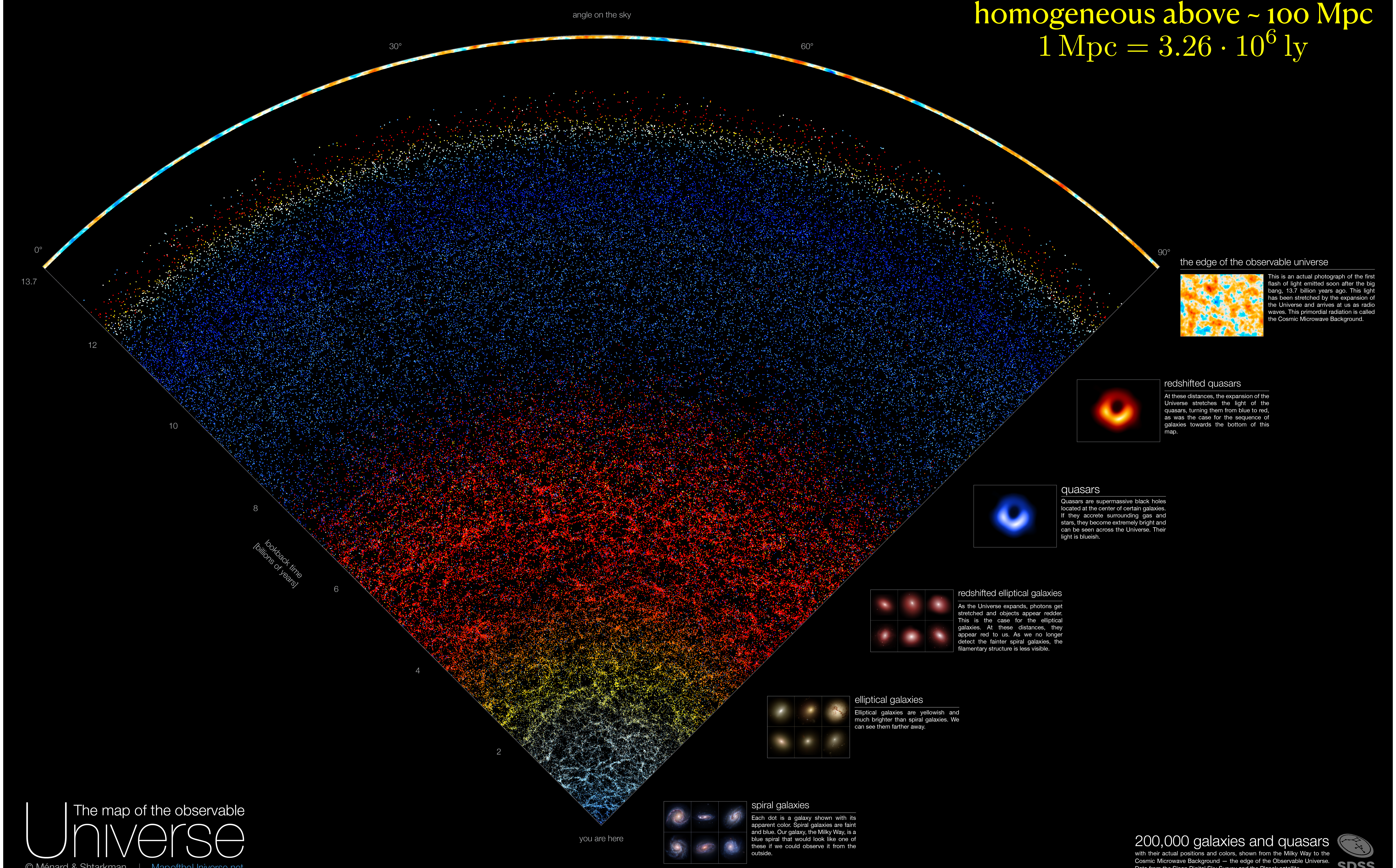
| MapoftheUniverse.net

200,000 galaxies and quasars with their actual positions and colors, from the Milky Way all the way to the Cosmic Microwave Background — the edge of the Observable Universe. Data from the Sloan Digital Sky Survey and the Planck satellite.

SDSS

homogeneous above ~ 100 Mpc

$1\,\mathrm{Mpc} = 3.26 \cdot 10^6\,\mathrm{ly}$



the edge of the observable universe

This is an actual photograph of the first flash of light emitted soon after the big bang, 13.7 billion years ago. This light has been stretched by the expansion of the Universe and arrives at us as radio waves. This primordial radiation is called the Cosmic Microwave Background.

redshifted quasars

At these distances, the expansion of the Universe stretches the light of the quasars, turning them from blue to red, as was the case for the sequence of galaxies towards the bottom of this map.

quasars

Quasars are supermassive black holes located at the center of certain galaxies. If they accrete surrounding gas and stars, they become extremely bright and can be seen across the Universe. Their light is blueish.

redshifted elliptical galaxies

As the Universe expands, photons get stretched and objects appear redder. This is the case for the elliptical galaxies. At these distances, they appear red to us. As we no longer detect the fainter spiral galaxies, the filamentary structure is less visible.

elliptical galaxies

Elliptical galaxies are yellowish and much brighter than spiral galaxies. We can see them farther away.

spiral galaxies

Each dot is a galaxy shown with its apparent color. Spiral galaxies are faint and blue. Our galaxy, the Milky Way, is a blue spiral that would look like one of these if we could observe it from the outside.

200,000 galaxies and quasars

with their actual positions and colors, shown from the Milky Way to the Cosmic Microwave Background — the edge of the Observable Universe. Data from the Sloan Digital Sky Survey and the Planck satellite.

SDSS

The map of the observable Universe

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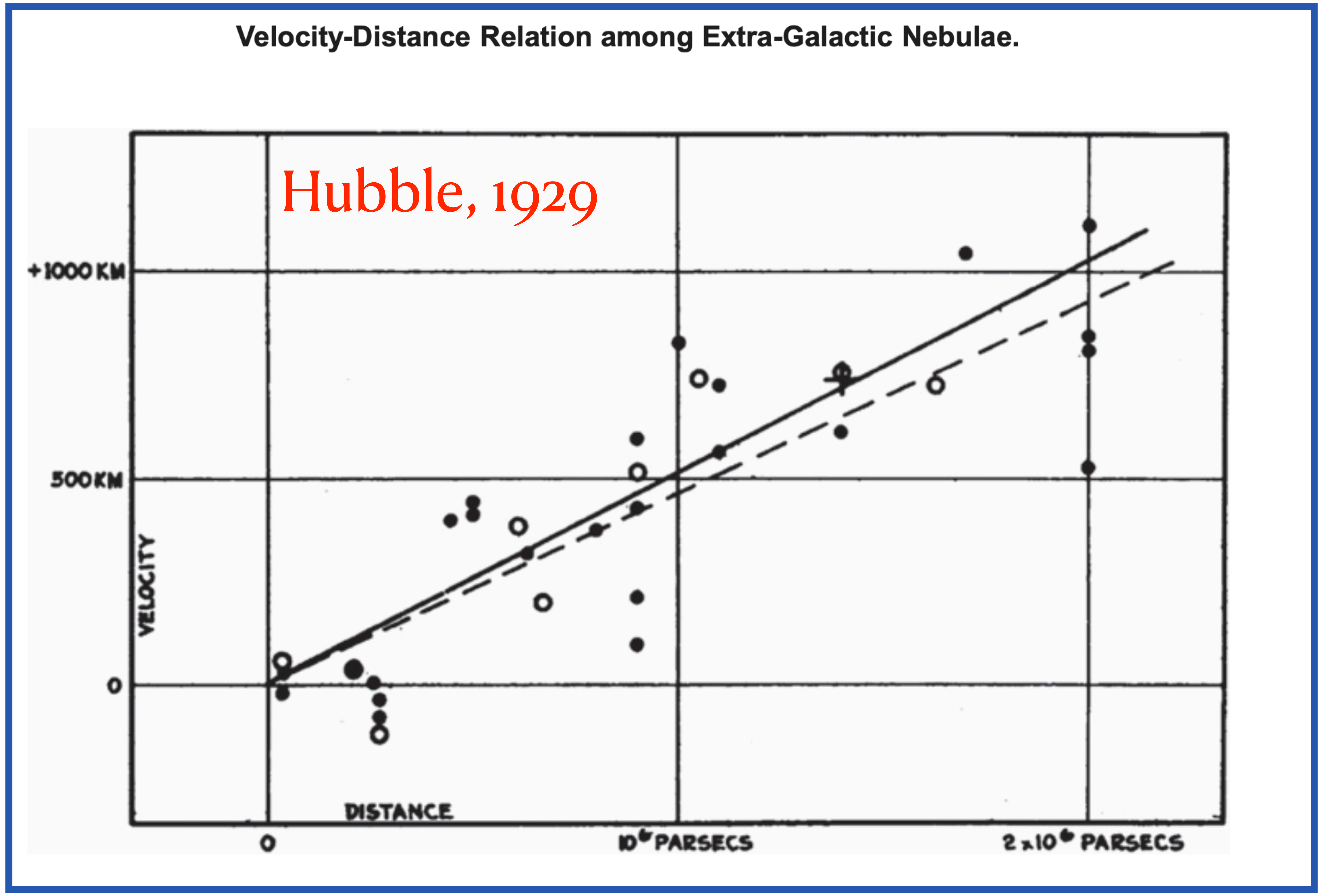
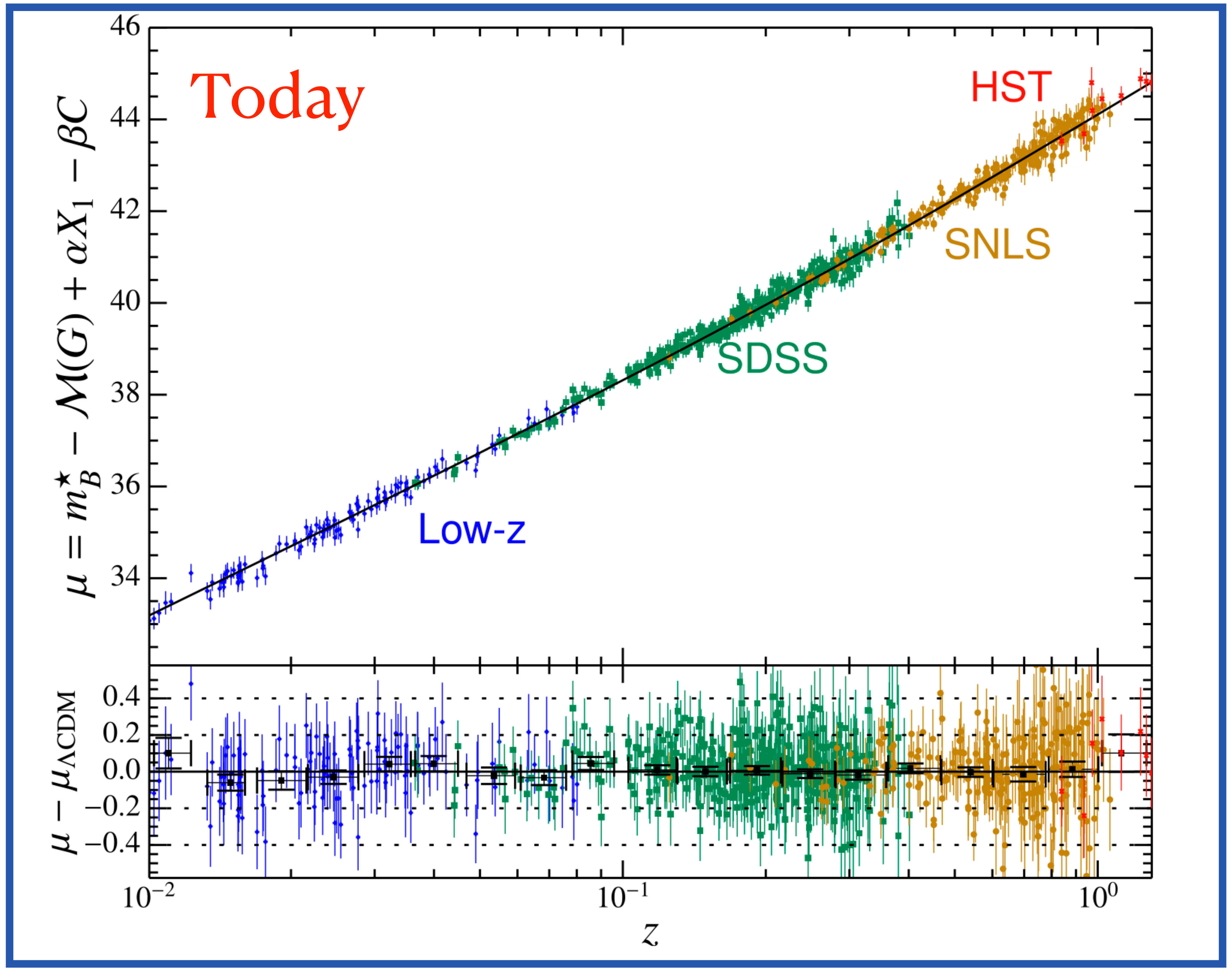
It expands

N.A. Bachall, PNAS 112 (2015), 3173

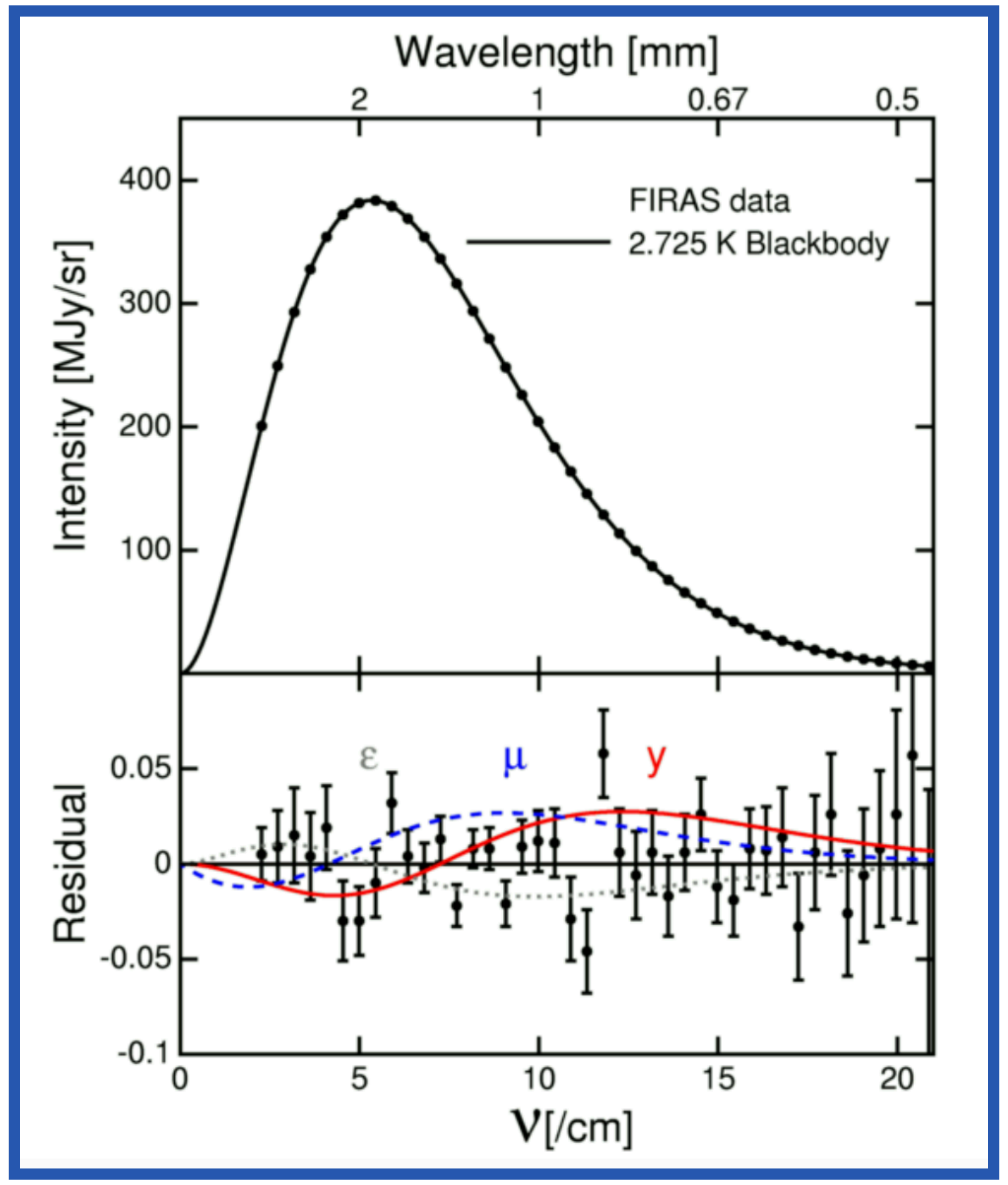




$$v = H_0\, d$$

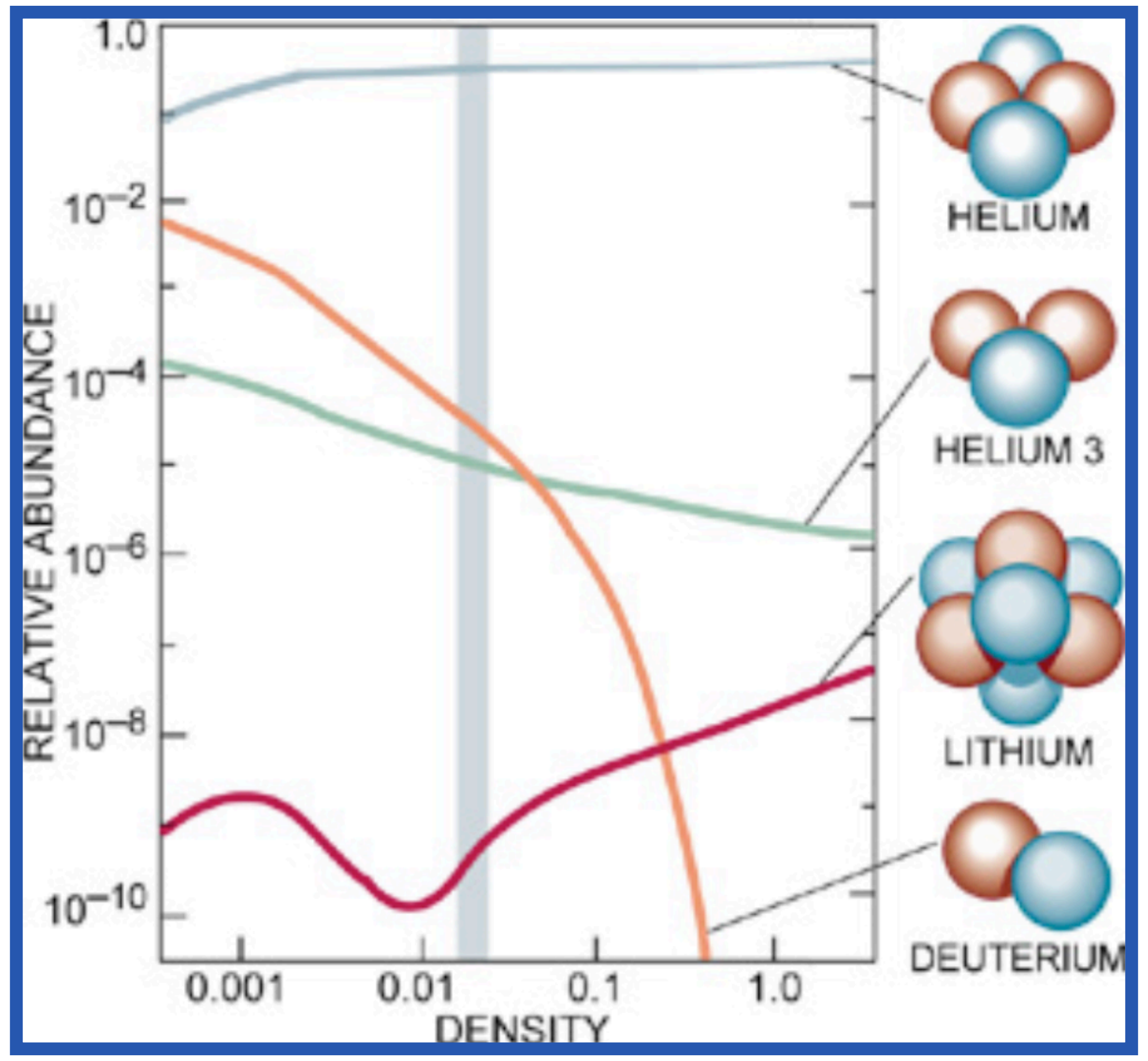
Hubble parameter: $H_0 = \mathrm{h}\,100\,\mathrm{Km/s/Mpc}, \qquad h \simeq 0.7$

It was denser and hotter in the past



COBE (Firas): CMB spectrum is the best black body ever measured!

380000 yrs after the Bang: T~ 3000 K



Primordial nucleosynthesis of light nuclei:
the universe was a nuclear furnace!

1 s after the Bang: T~ MeV (~10^{10} K)

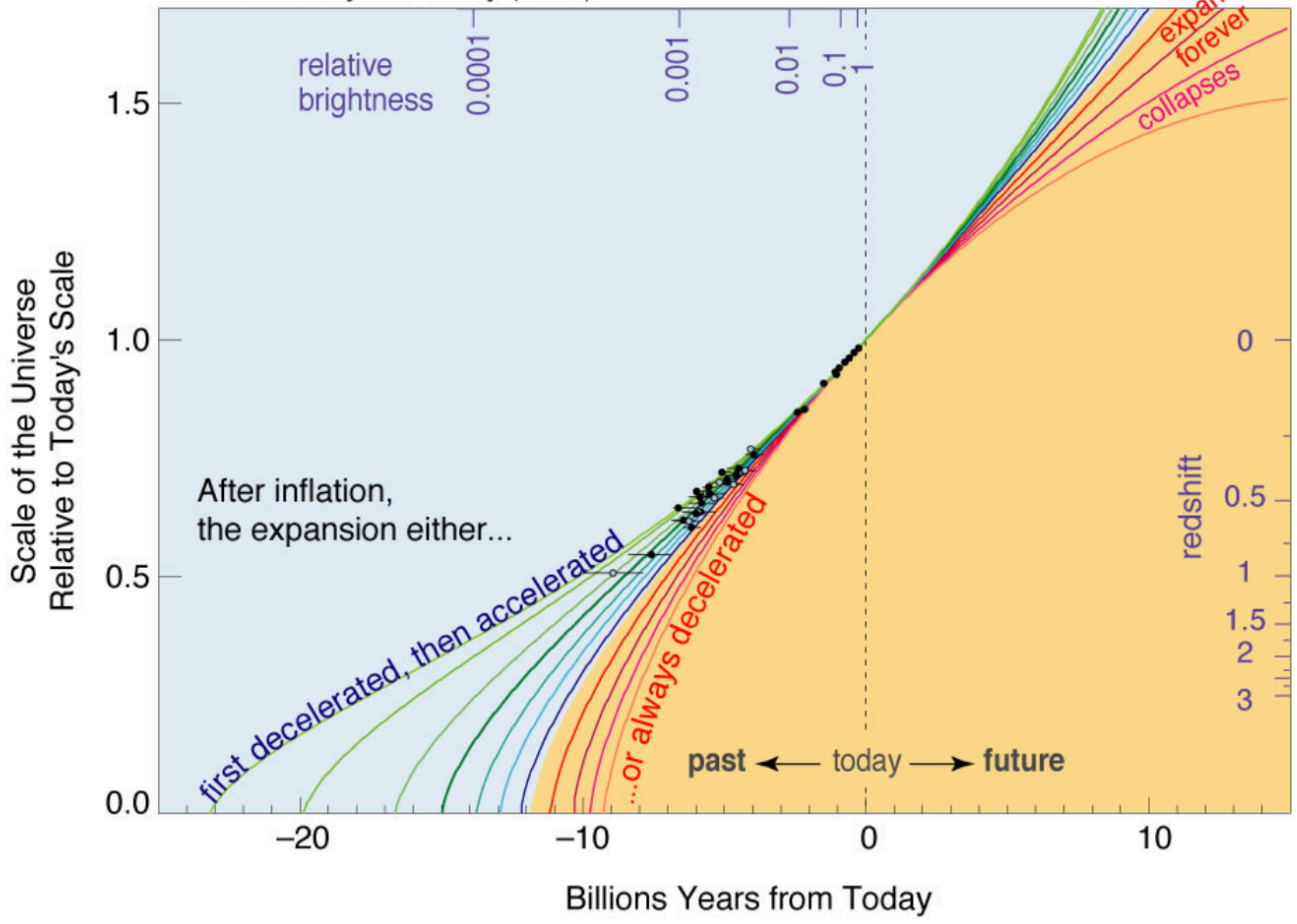
The Standard Universe

- It is homogeneous and isotropic on large scales;
- It expands;
- It was hot and close to thermal equilibrium in the past;
- Light nuclei (D, ^{3}He, ^{4}He, ^{7}Li) formed < few seconds after the Bang;
- Photons decoupled ~ 380000 yrs after the Bang

All this is described by Standard Cosmology + Standard Model of Particle Physics

Some more facts about our universe…

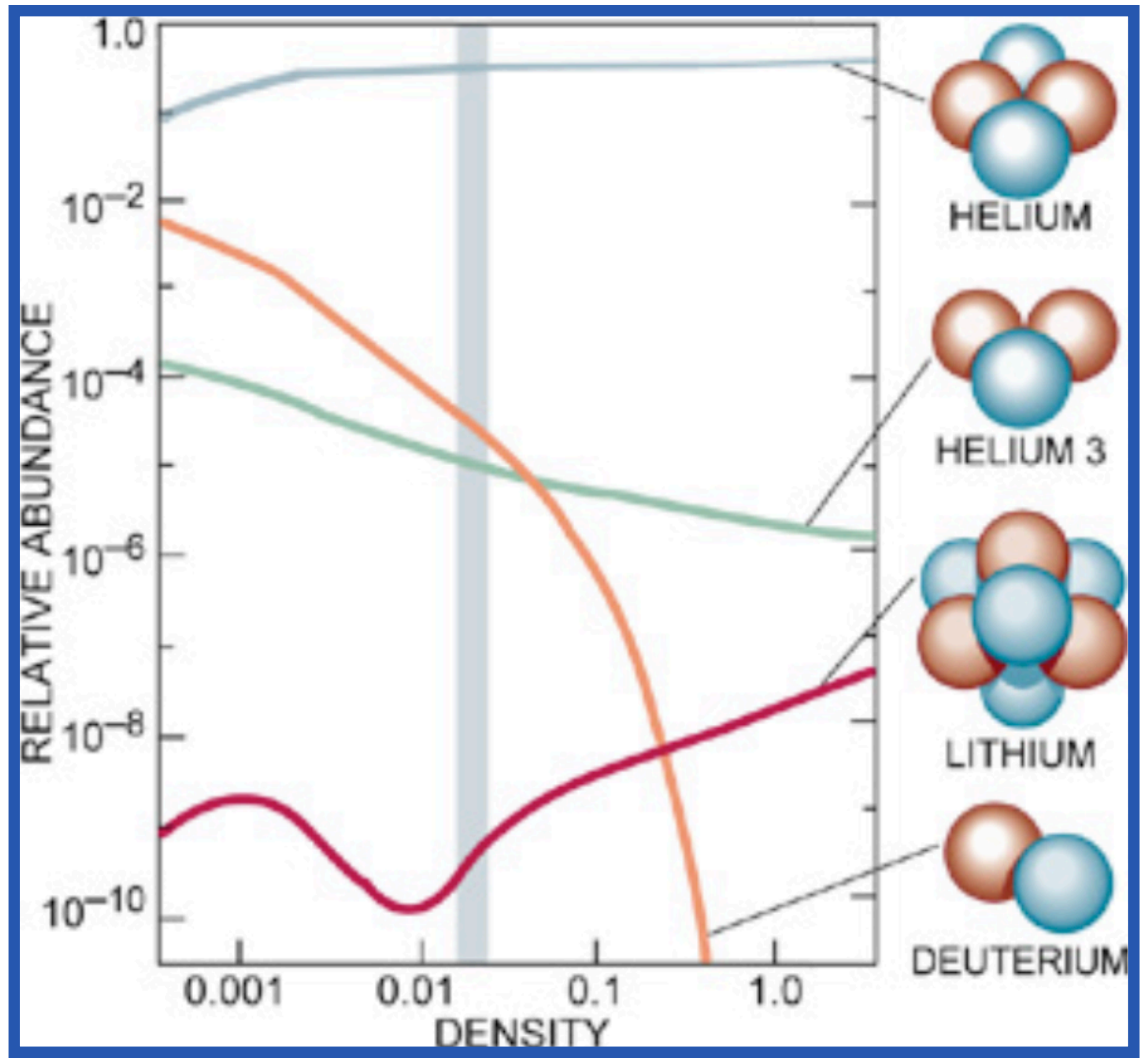
The expansion is accelerating



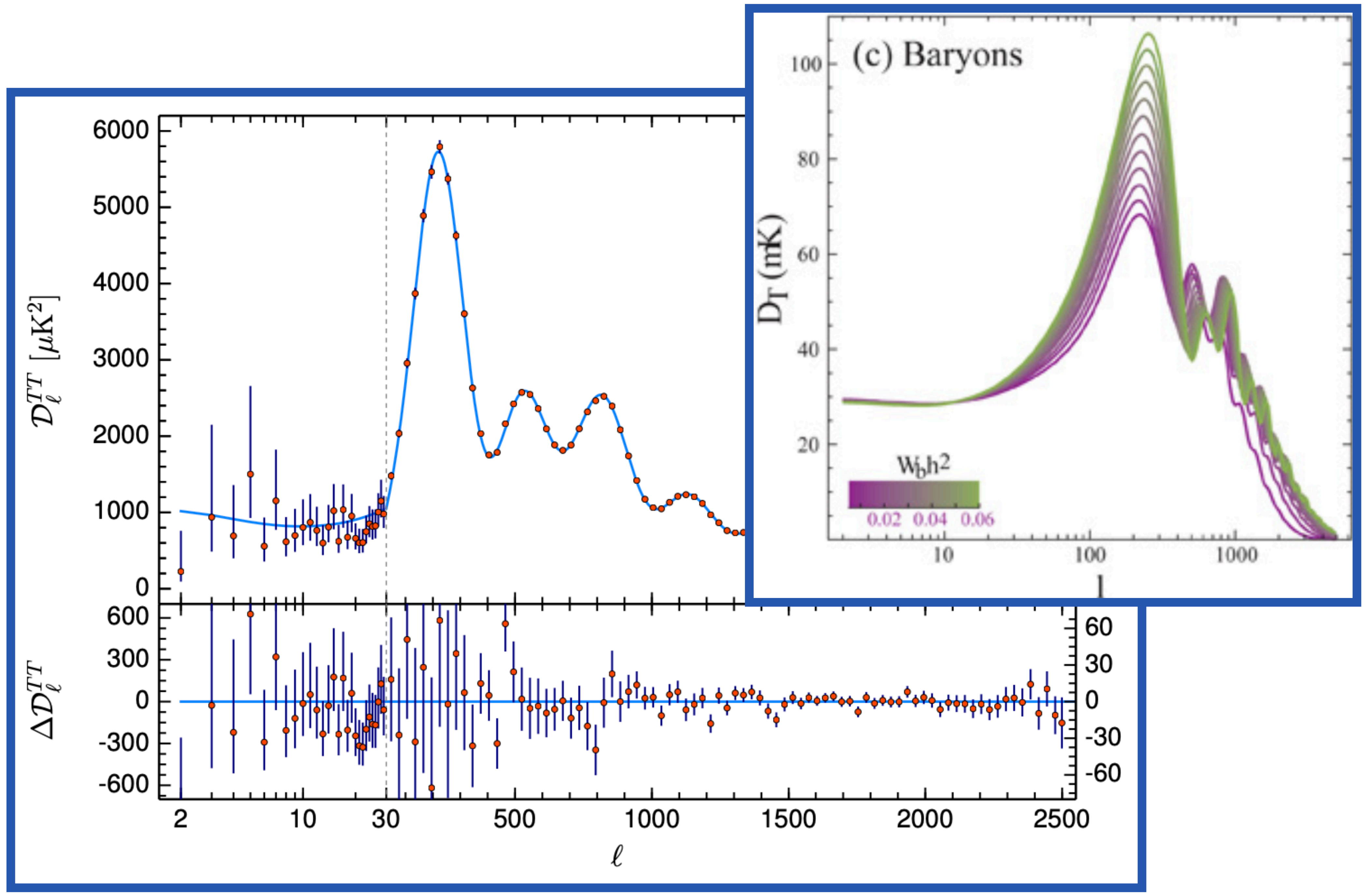
Perlmutter, Physics Today (2003)

In GR, it requires ~ 70% of the energy density of the universe to have negative pressure: Dark Energy!

Baryons make up only ~ 4%



t~1-100 sec
light nuclei abundances depend
on the primordial baryon density

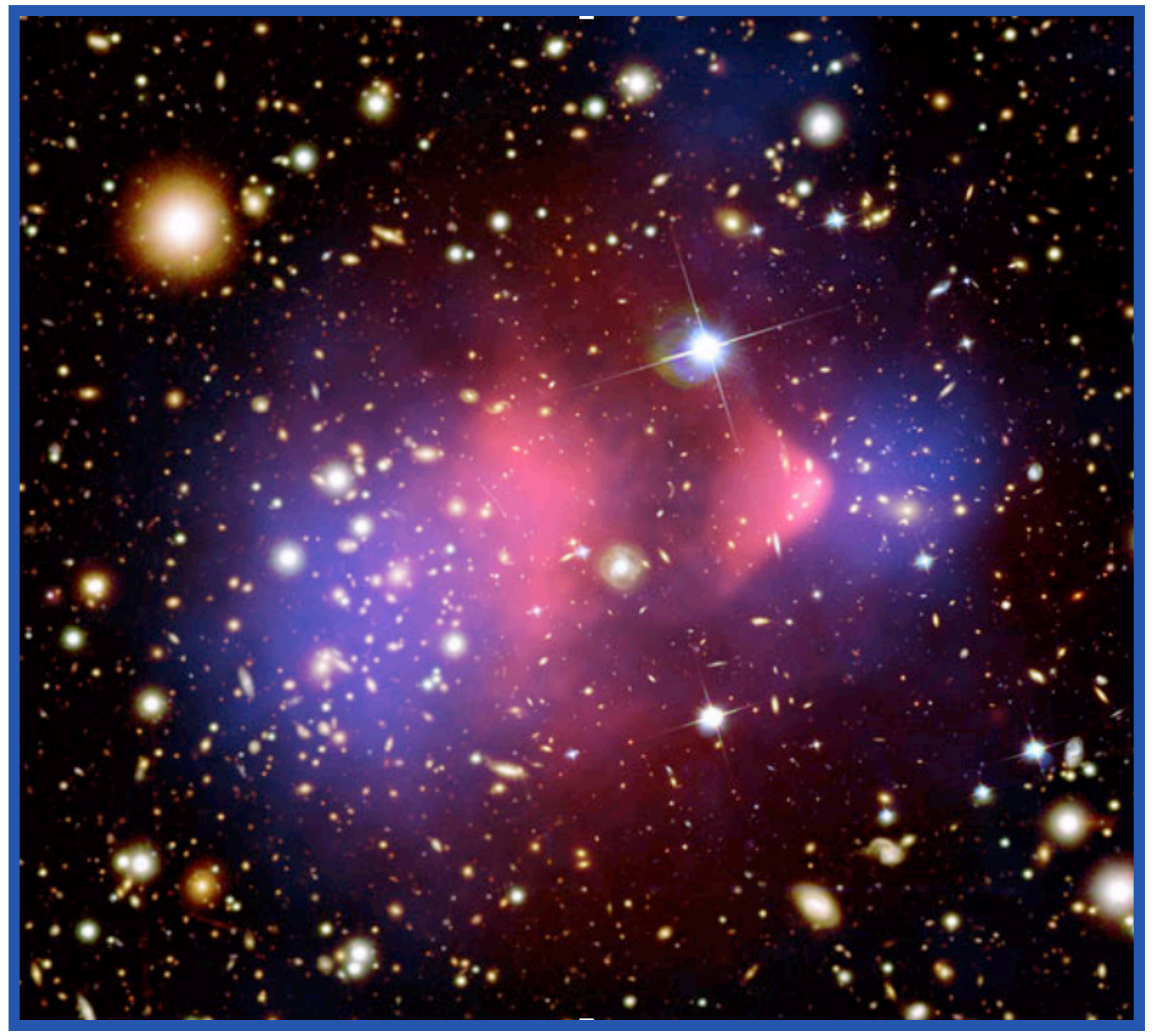


W. Hu, S. Dodelson

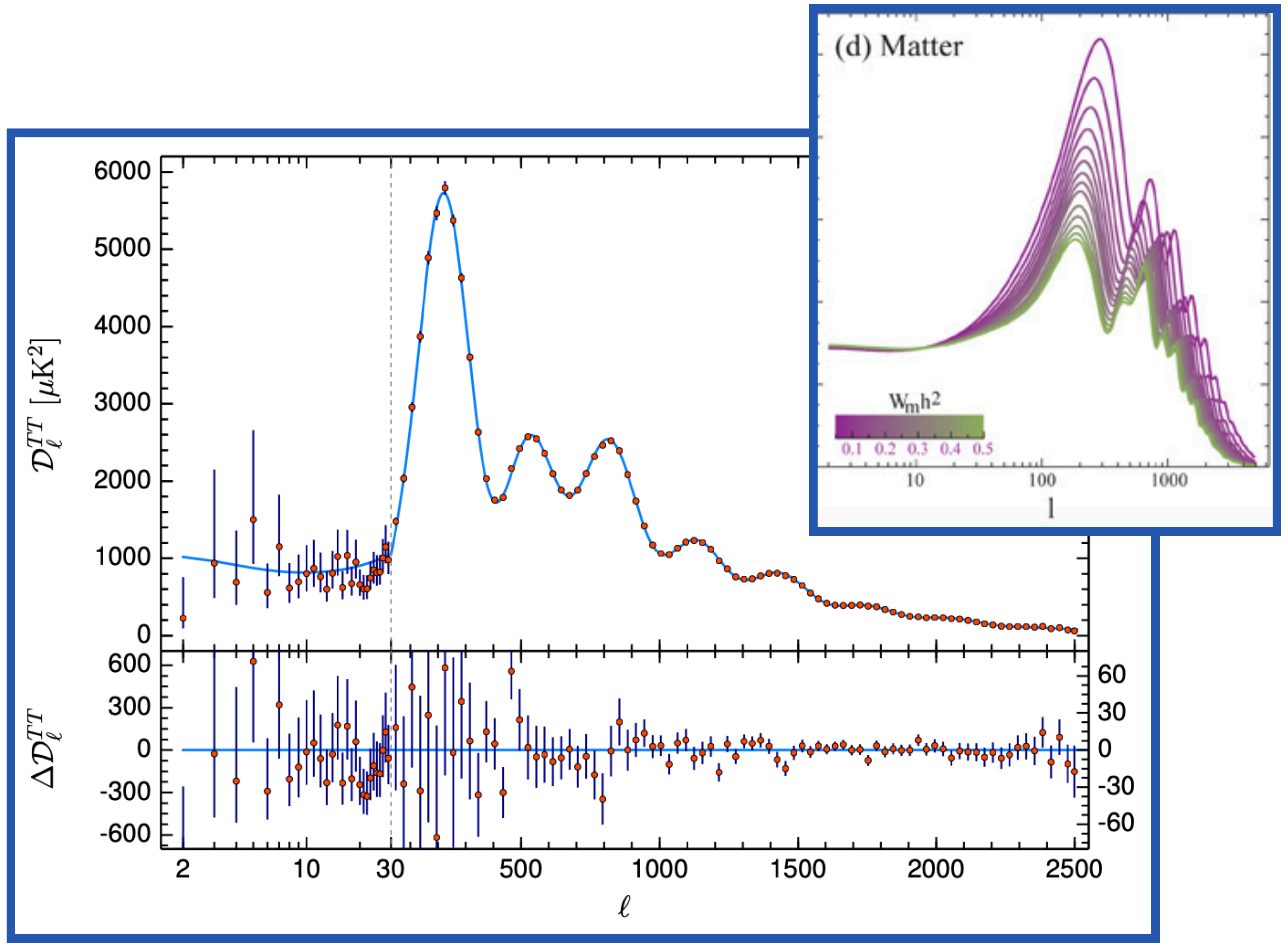
t~380,000 yrs
amplitude of acoustic oscillations
in the baryon-photon plasma
depends on baryon density

Moreover, there are no anti-baryons!

Most of the matter is dark



W. Hu, S. Dodelson



In GR, it requires ~ 25% of the energy density of the universe to be pressureless and non-baryonic: Dark Matter!

Puzzles from the Universe

- The expansion accelerates;
- Normal matter makes up less than 5% of the present energy content;
- Structure formation is driven by ~25% of an unknown pressureless component;
- Initial conditions on density and velocities is extremely unlikely;
- Antimatter is missing.

All this requires New Physics beyond the Standard Models!!

The homogeneous expanding universe

Most general metric: $ds^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu = g_{00}(x)\,dt^2 + 2g_{0i}(x)\,dt\,dx^i + g_{ij}(x)\,dx^i dx^j$ 10 comps

Synchronous frame: $x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x) \implies ds^2 = dt^2 + g_{ij}(x)\,dx^i dx^j$ 6 comps

Homogeneity and isotropy: the metric tensor is invariant under translations (3) and rotations (3)

$$ds^2 = dt^2 - a^2(t) f(R) \delta_{ij}\, dx^i dx^j$$

(cartesian coordinates)
$R^2 = \delta_{ij} x^i x^j$

scale factor

3-dim line element

$$f(R) = \frac{1}{\left(1 + \frac{KR^2}{4}\right)^2}$$

K=0, flat (euclidean)
K>0, closed (finite volume)
K<0, open

$$\int_0^\infty \frac{dR}{1 + \frac{KR^2}{4}} = \pi/\sqrt{K}$$

Friedmann-Lemaître-Robertson-Walker Metric

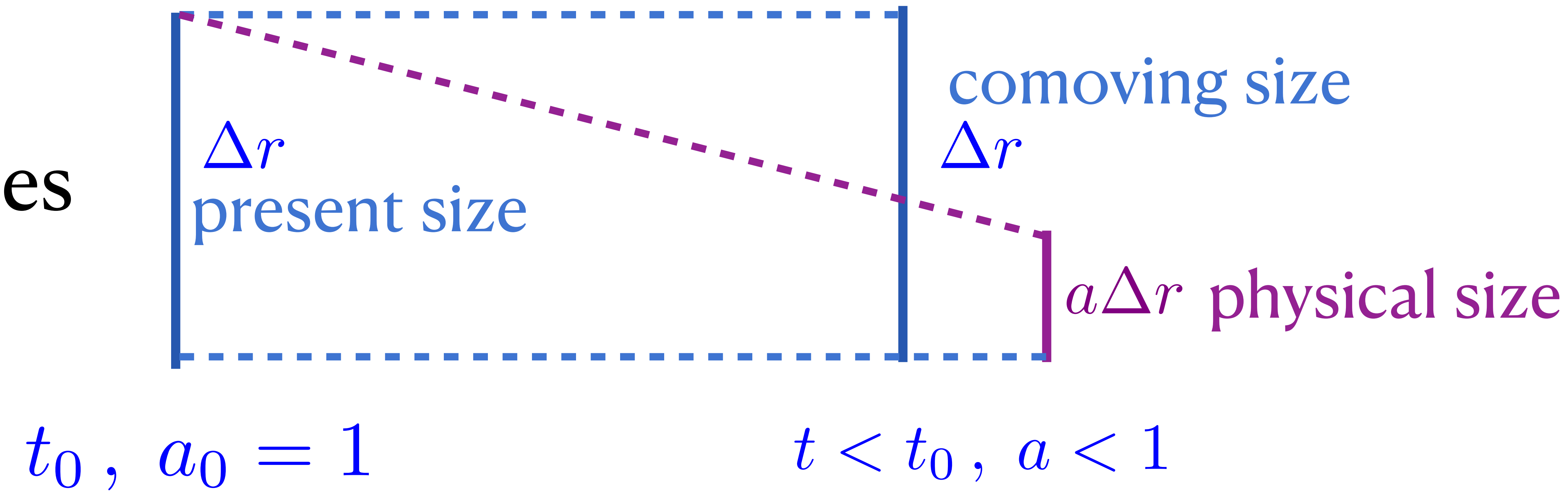
Polar coordinates: $r = R\left(1+\frac{KR^2}{4}\right)^{-1}$ $\quad dR\left(1+\frac{KR^2}{4}\right)^{-1} \to dr/\sqrt{1-Kr^2}$

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-Kr^2)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] = dt^2 - a(t)^2 d\vec{l}^2$$

scale factor

curvature constant: $[K]=[L]^{-2}$
K=0, flat (euclidean)
K>0, closed (finite volume)
K<0, open

$r,\ \theta,\ \phi$ "comoving" coordinates



FLRW space-time completely specified by a(t) and K

Kinematics on FLRW: photons

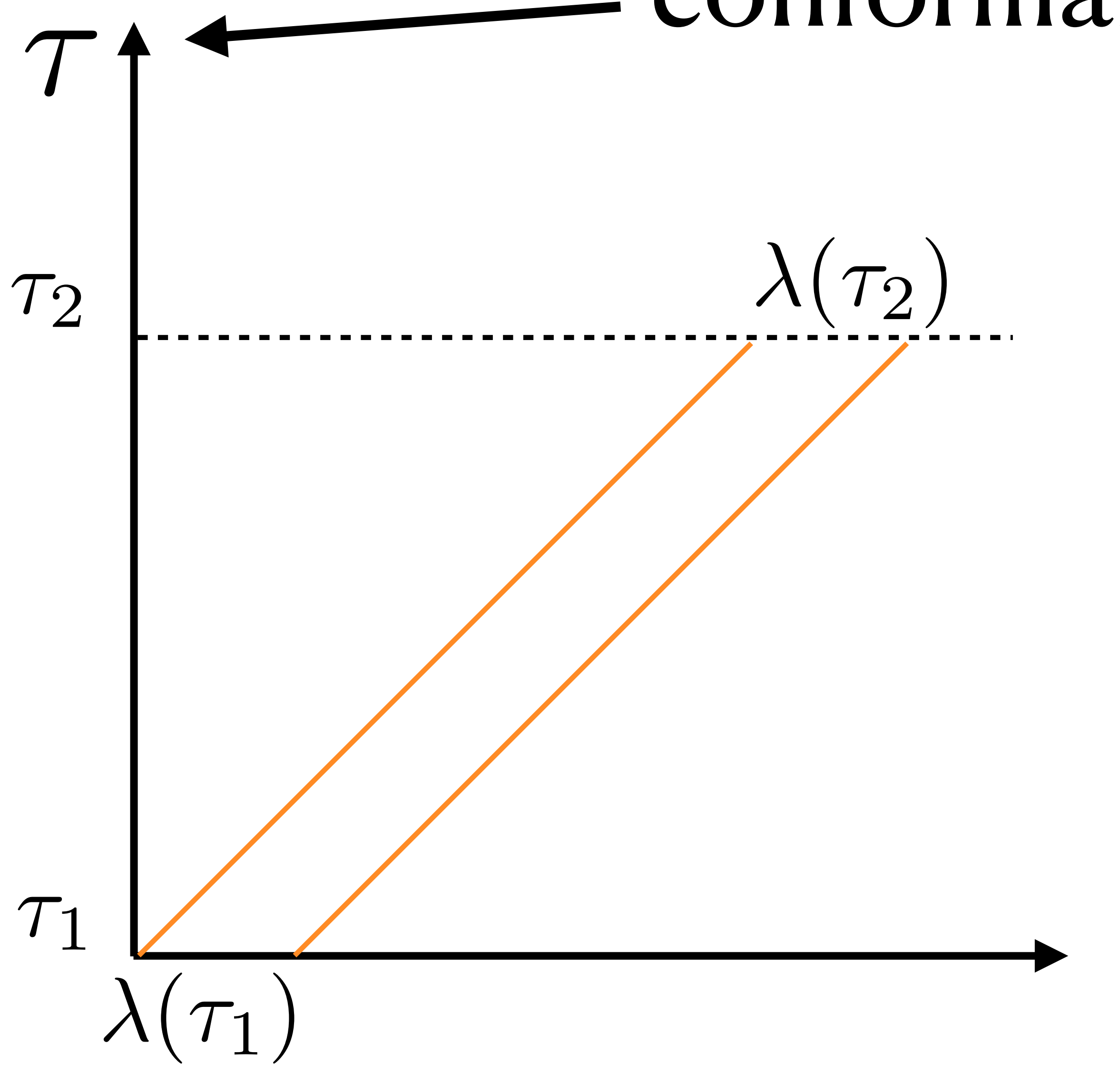
$$ds^2/a(t)^2 = dt^2/a(t)^2 - d\vec{l}^2 \equiv d\tau^2 - d\vec{l}^2$$

FLRW is “conformal” to Minkowski

conformal time

conformal diagrams:
photons (ds²=0) travel at 45 degrees

$$\lambda^{\rm ph}(\tau_2) = a(\tau_2)\lambda(\tau_2) = a(\tau_2)\lambda(\tau_1) = \frac{a(\tau_2)}{a(\tau_1)}\lambda^{\rm ph}(\tau_1)$$



comoving coordinates

cosmological redshift

$$\boxed{1 + z \equiv \frac{\lambda^{\rm ph}(t_0)}{\lambda^{\rm ph}(t)} = \frac{1}{a(t)}}$$

it is the “clock” of cosmological expansion

Kinematics on FLRW: massive particles

Action for a free particle: $S = -m \int ds = -m \int dt\, \sqrt{1 - a^2 \dot{x}^2} = \int dt\, L$

Conserved conjugate momentum: $P_i = \dfrac{dL}{d\dot{x}^i} \simeq m a^2\, \dot{x}^i$ (non relativistic limit)

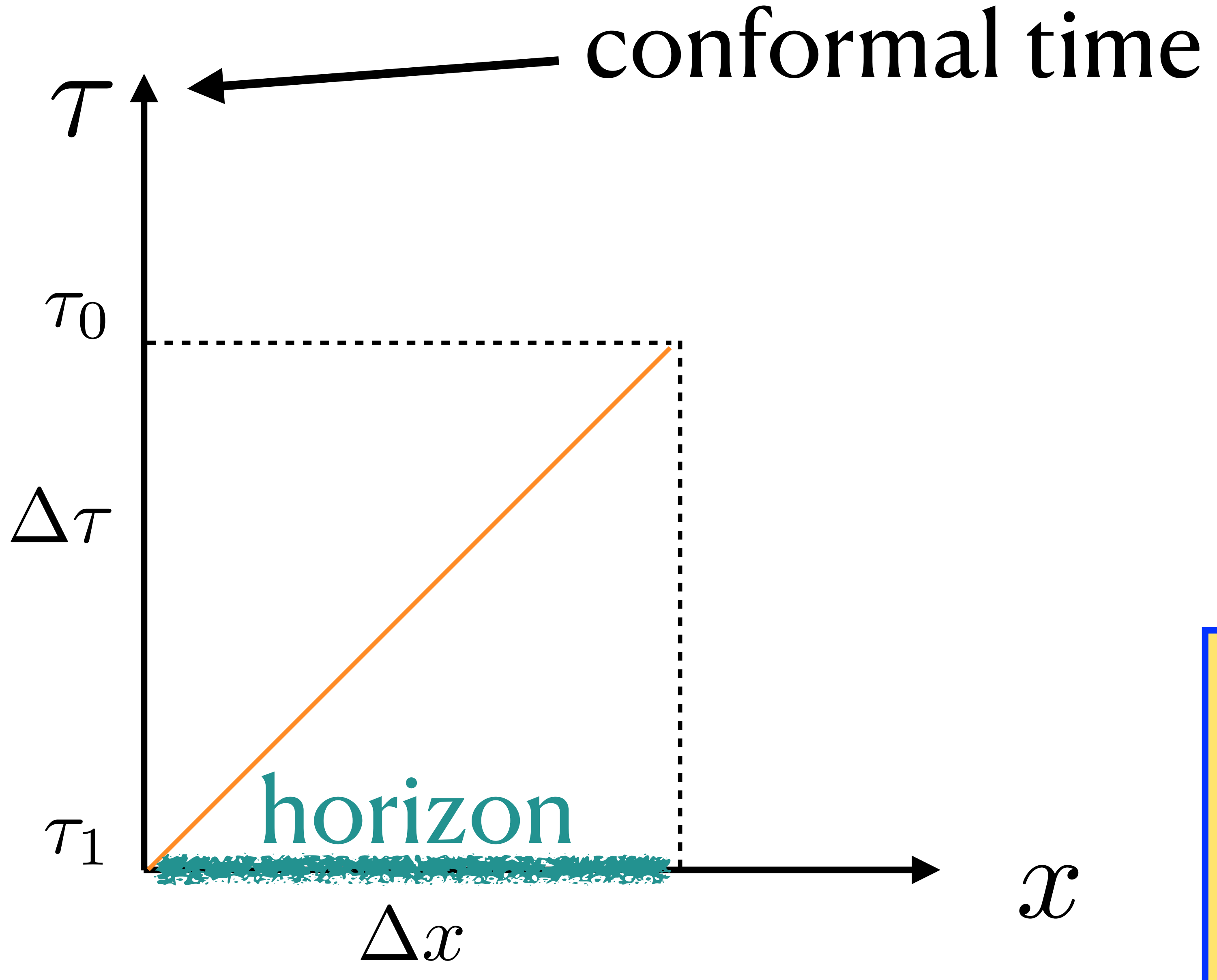
Peculiar momentum: $|\vec{P}|^2 = P_i P^i = g^{ij} P_i P_j = a^{-2} \delta_{ij} P_i P_j \sim a^{-2}$

Peculiar motions of free massive particles are damped by the expansion:

$$|\vec{P}| \sim a^{-1} \qquad E(t) = \sqrt{m^2 + (E_{\text{in}}^2 - m^2)\frac{a_{\text{in}}^2}{a(t)^2}}$$

Kinematics on FLRW: particle horizon

Comoving distance travelled by a photon from $t=0$ to $t=t_0$ (flat space, $K=0$)



$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(w+1)}}$$ power law scale factor

$$\Delta x = \tau_0 - \tau_1 = \int_{t_1}^{t_0} \frac{dt}{a(t)} = t_0 \frac{3(1+w)}{1+3w} \left(\frac{t}{t_0}\right)^{\frac{1+3w}{3(w+1)}} \Bigg|_{t_1}^{t_0}$$

	$w > -1/3$	$w < -1/3$
$t_1 \to 0$	$\Delta x \to t_0 \frac{3(1+w)}{1+3w}$	$\Delta x \to \infty$
	finite horizon $O(t_0)$	infinite horizon!

What about the future? Check it!

Kinematics on FLRW: comoving distance

Comoving distance: $ds = 0 \rightarrow \int_0^r \frac{dr'}{\sqrt{1 - Kr'^2}} = \Delta\tau = \int_0^t \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{H(z')}$

$H \equiv \frac{\dot a}{a}$

Hubble parameter

$$r(z) = \begin{cases} \dfrac{\sin\left(\sqrt{K}\Delta\tau(z)\right)}{\sqrt{K}} & K > 0 \quad \text{closed} \\ \Delta\tau(z) & K = 0 \quad \text{flat} \\ \dfrac{\sinh\left(\sqrt{|K|}\Delta\tau(z)\right)}{\sqrt{|K|}} & K < 0 \quad \text{open} \end{cases}$$

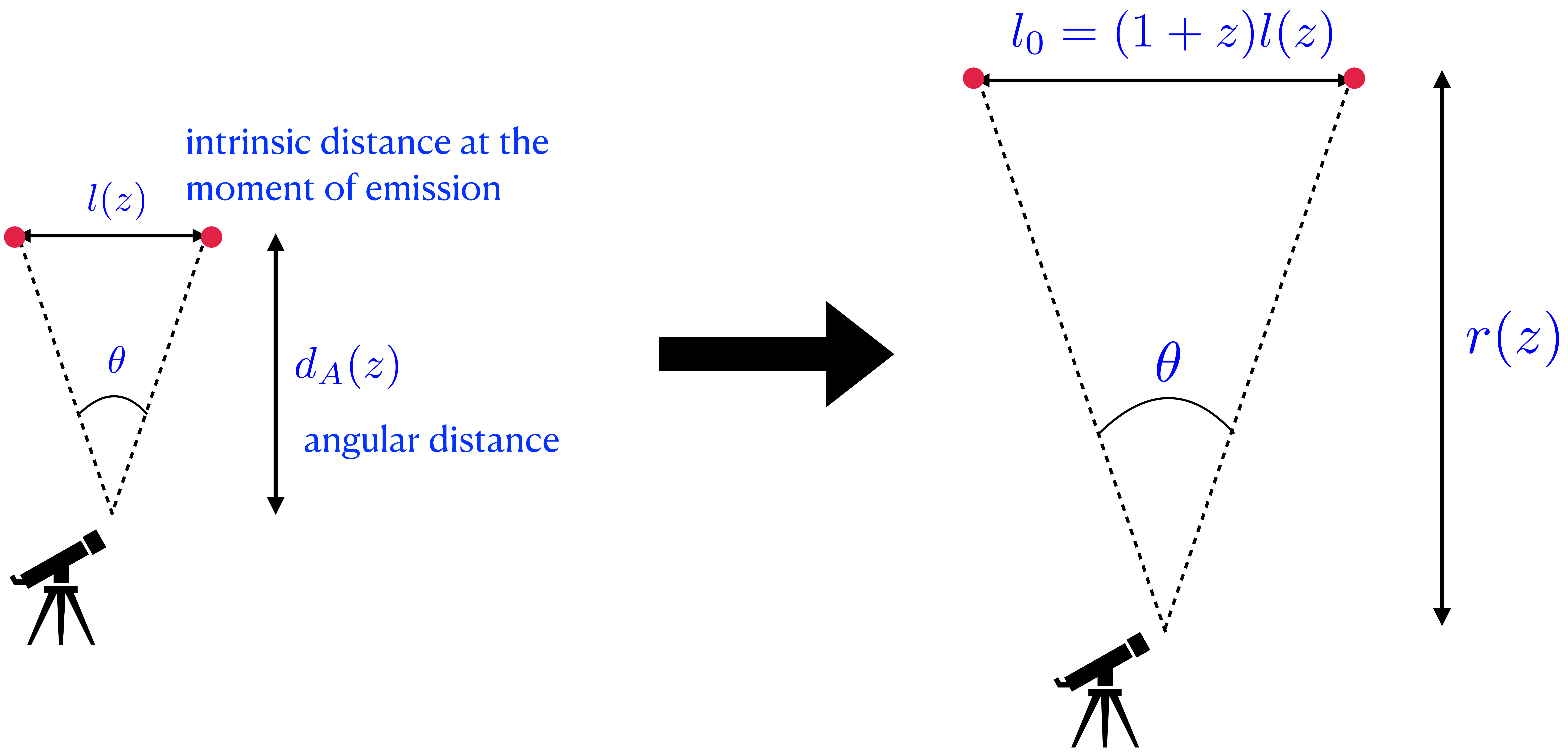
Cosmological information in K and in $\Delta\tau(z)$

For small z and K=0, $r(z) \simeq \frac{z}{H_0}\left(1 - \frac{1}{2}(1 + q_0)\, z + \cdots\right)$

Hubble's law

$q_0 \equiv -\frac{\ddot a_0}{H_0}$ deceleration parameter

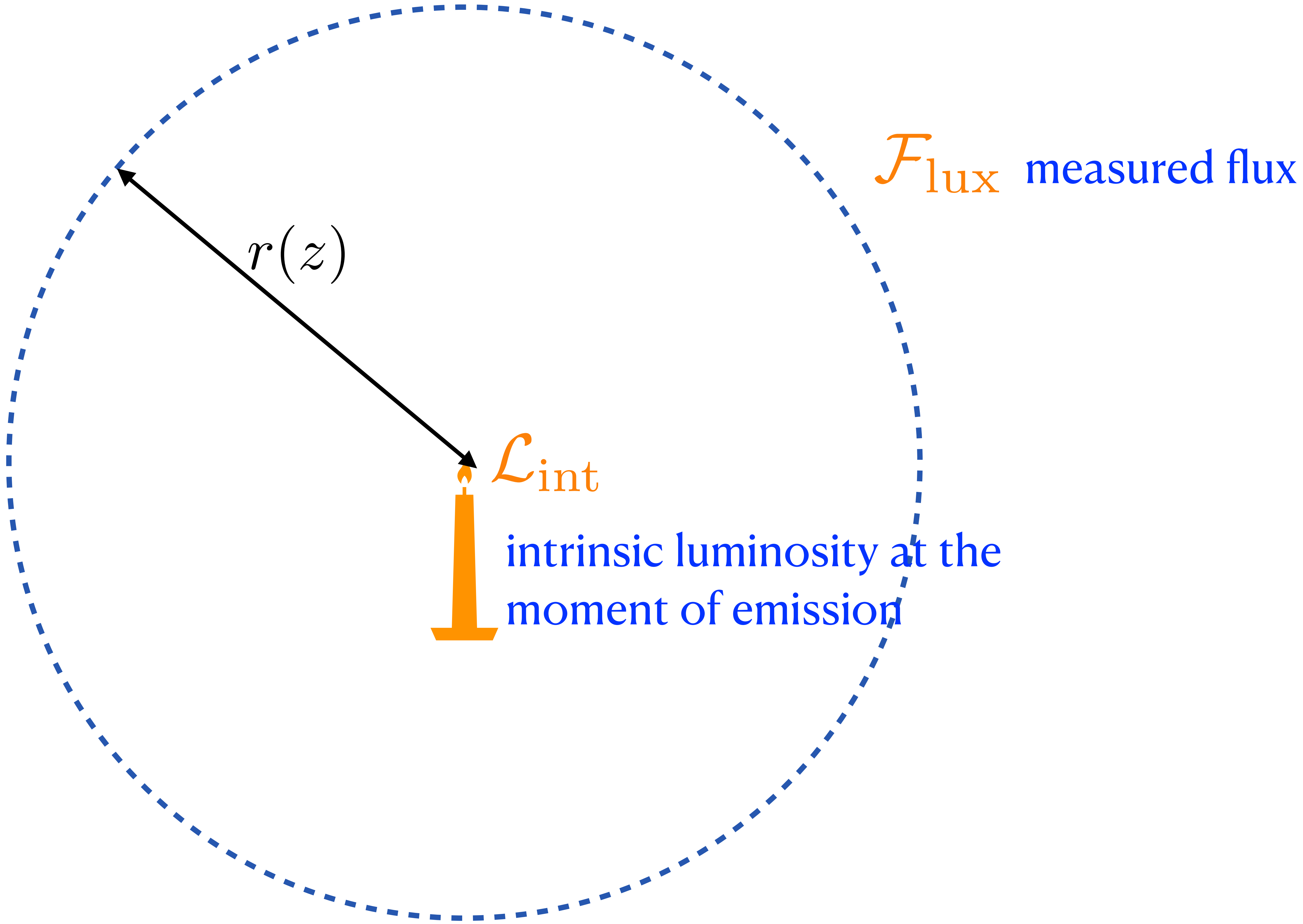
Kinematics on FLRW: angular distance



$$d_A(z) \equiv \frac{l(z)}{\theta} = \frac{l(z)}{l_0} r(z) = \frac{r(z)}{1+z}$$

Examples: CMB angular anisotropies, BAO’s

Kinematics on FLRW: luminosity distance



$\mathcal{F}_{\rm lux}$ measured flux

$r(z)$

$\mathcal{L}_{\rm int}$

intrinsic luminosity at the moment of emission

$$\mathcal{F}_{\rm lux} \equiv \frac{\mathcal{L}_{\rm int}}{4\pi d_L^2(z)} = \frac{\mathcal{L}_{\rm int}(1+z)^{-2}}{4\pi r^2(z)}$$

$$d_L(z) = (1+z)r(z) = (1+z)^2 d_A(z)$$

Example: type Ia supernovae

Energy-Momentum tensor in FLRW

Energy-momentum tensor: $T^{\mu}_{\nu} = -P g^{\mu}_{\nu} + (\rho + P) u^{\mu} u_{\nu} \xrightarrow{\text{FLRW frame}} \mathrm{diag}(\rho, -P, -P, -P)$

4-velocity
of the observer

$\rho(t)$ energy density
$P(t)$ pressure

$$d\left(\rho a^3\right) = -P d a^3$$

1st principle of thermodynamics
in an expanding universe

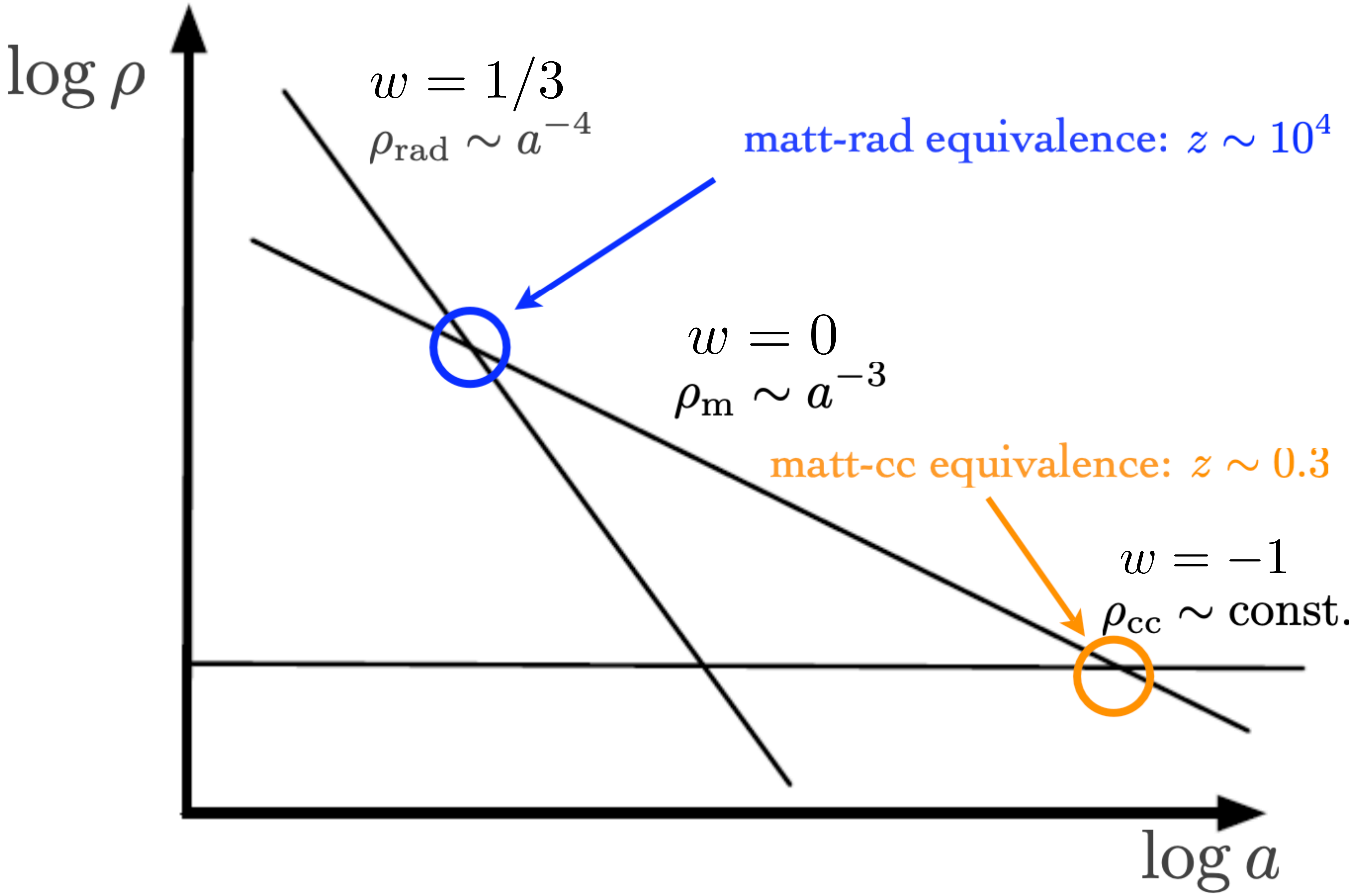
$$\Downarrow$$

$w(a) \equiv \dfrac{P(a)}{\rho(a)}$ equation of state $\Rightarrow$ $\rho(a) = \rho_0 e^{-3\int_1^a \frac{da'}{a'}(1+w(a'))} \xrightarrow{\text{w constant}} \rho_0\, a^{-3(1+w)}$

Cosmological epochs

ΛCDM Model: $\rho = \rho_{\rm m} + \rho_{\rm rad} + \rho_\Lambda = \rho^0_{\rm m} a^{-3} + \rho^0_{\rm rad} a^{-4} + \rho_\Lambda$

different components dominate at different epochs



Dynamics on FLRW: Friedmann equation

Einstein equations on FRLW: $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$

curvature term:
behaves as a component with w=-1/3

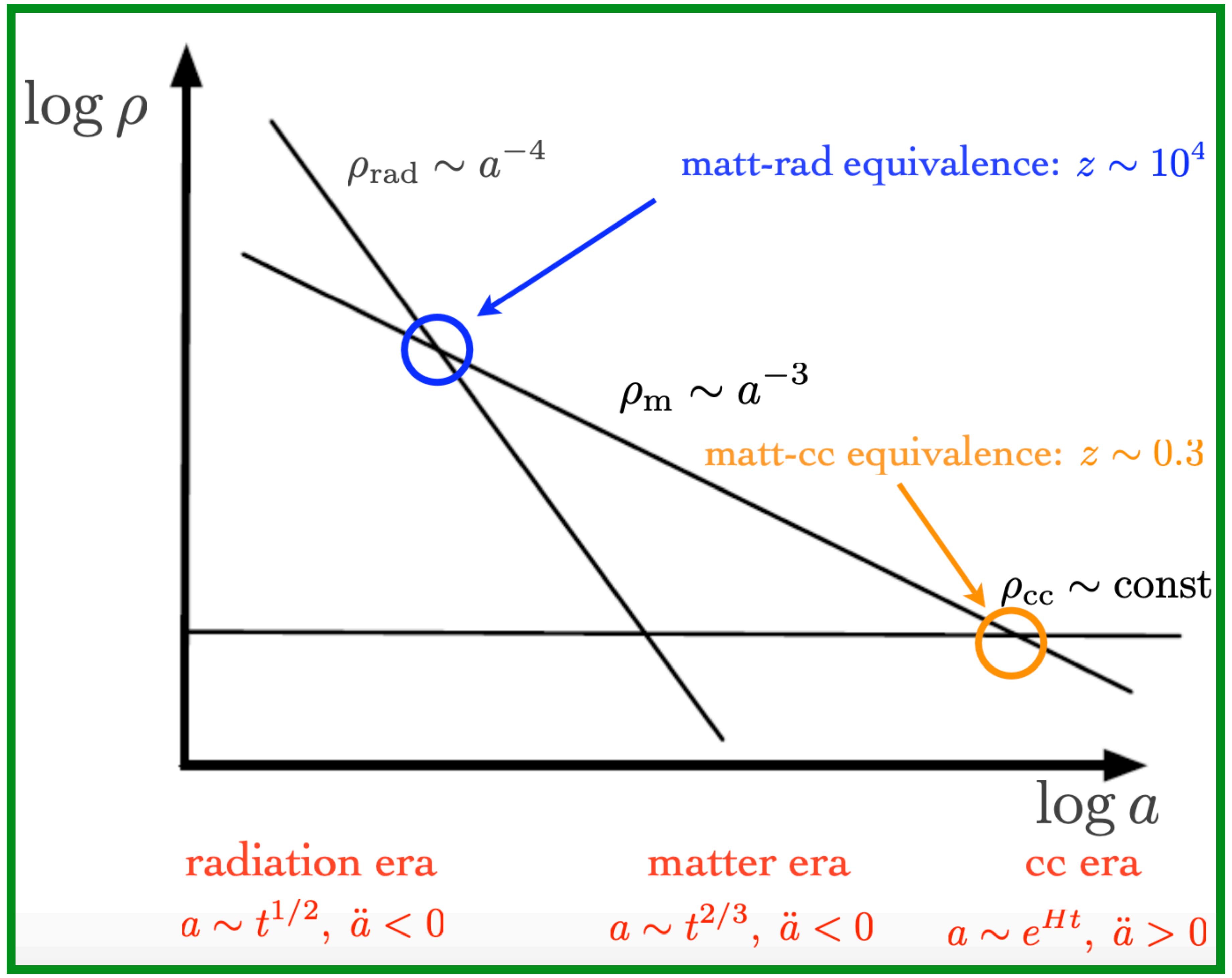
Continuity equation: $d\left(\rho a^3\right) = -P\,da^3$

Solution for constant $w = \frac{P}{\rho}$: $\boxed{\rho \sim a^{-3(w+1)}, \qquad a \sim t^{2/3(w+1)}}$ $a \sim e^{Ht}$ for $w = -1$

$$H^2 = H_0^2\left[\Omega_{\rm m} a^{-3} + \Omega_{\rm rad} a^{-4} + \Omega_{\rm K} a^{-2} + \Omega_\Lambda a^{-4}\right]$$

$$\Omega_i \equiv \frac{\rho_i^0}{\rho_c^0} \qquad \rho_c^0 \equiv \frac{3H_0^2}{8\pi G}$$

$\rho_c^0 = 1.88\, h^2 10^{-29}\, {\rm g}\cdot{\rm cm}^{-3} \simeq (10^{-3}{\rm eV})^4 h^2$ critical density today



The ΛCDM Model

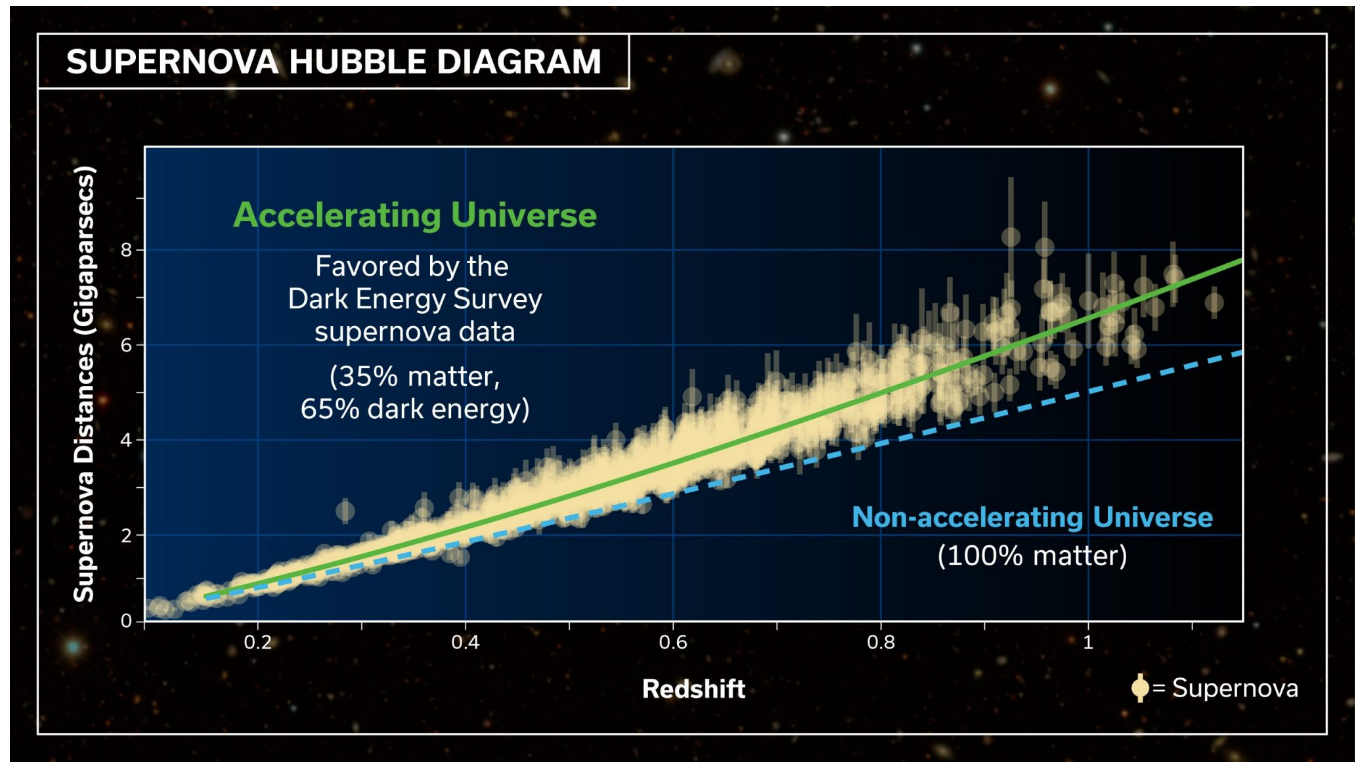


Image: DES collaboration

$d_L(z)$ from SNeIa

from Big Bang Nucleosynthesis and CMB

from CMB + LSS + ...

$\Omega_K = 0.0007 \pm 0.0019$ $d_A(z)$ from CMB and BAO's