Slow Stable Hybrid Stars

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- Introduction
- Hybrid Stars
- Equation of State
- SSHS

Introduction to Neutron Star

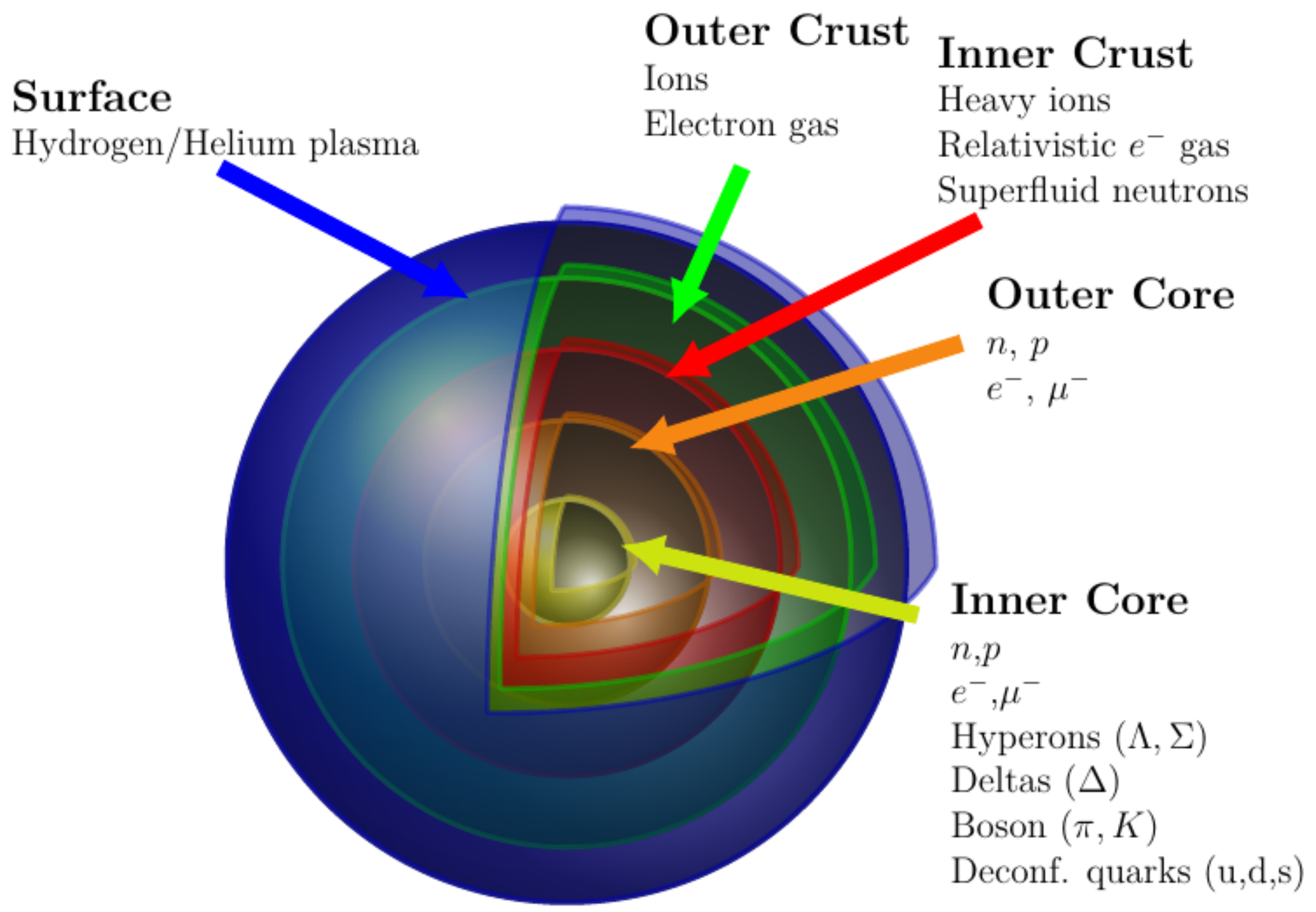
- Formed in: Type II, Ib or Ic SN (remnant of $M \approx 8 - 30 M_\odot$)

- **Mass:** $M \approx 2M_\odot$ **(or even more)**

- **Radius:** $R \approx 10 - 12 km$

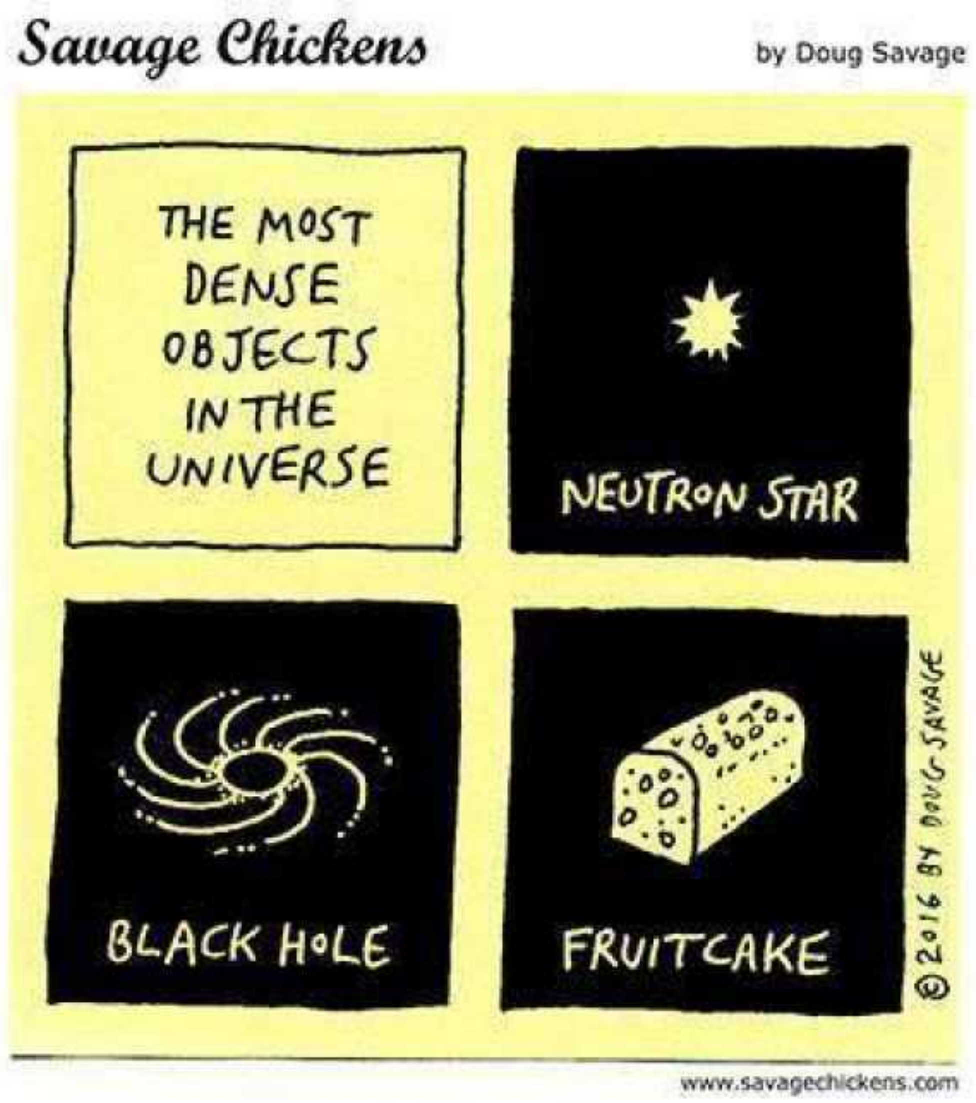
- **Density:** $\rho \approx 10^{14} - 10^{15} g/cm^3$

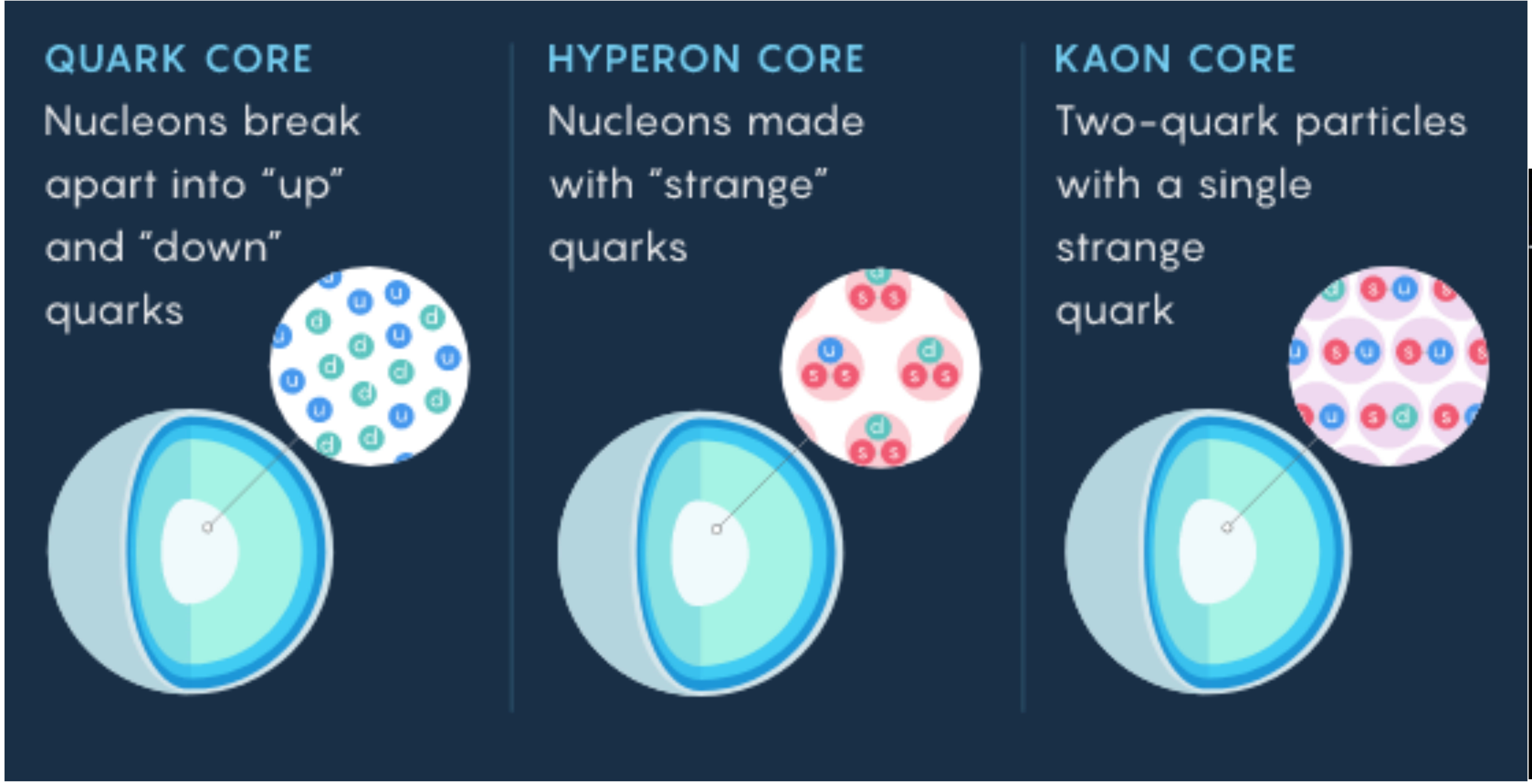
- **Magnetic Field:** $B \approx 10^{15} - 10^{18} G$

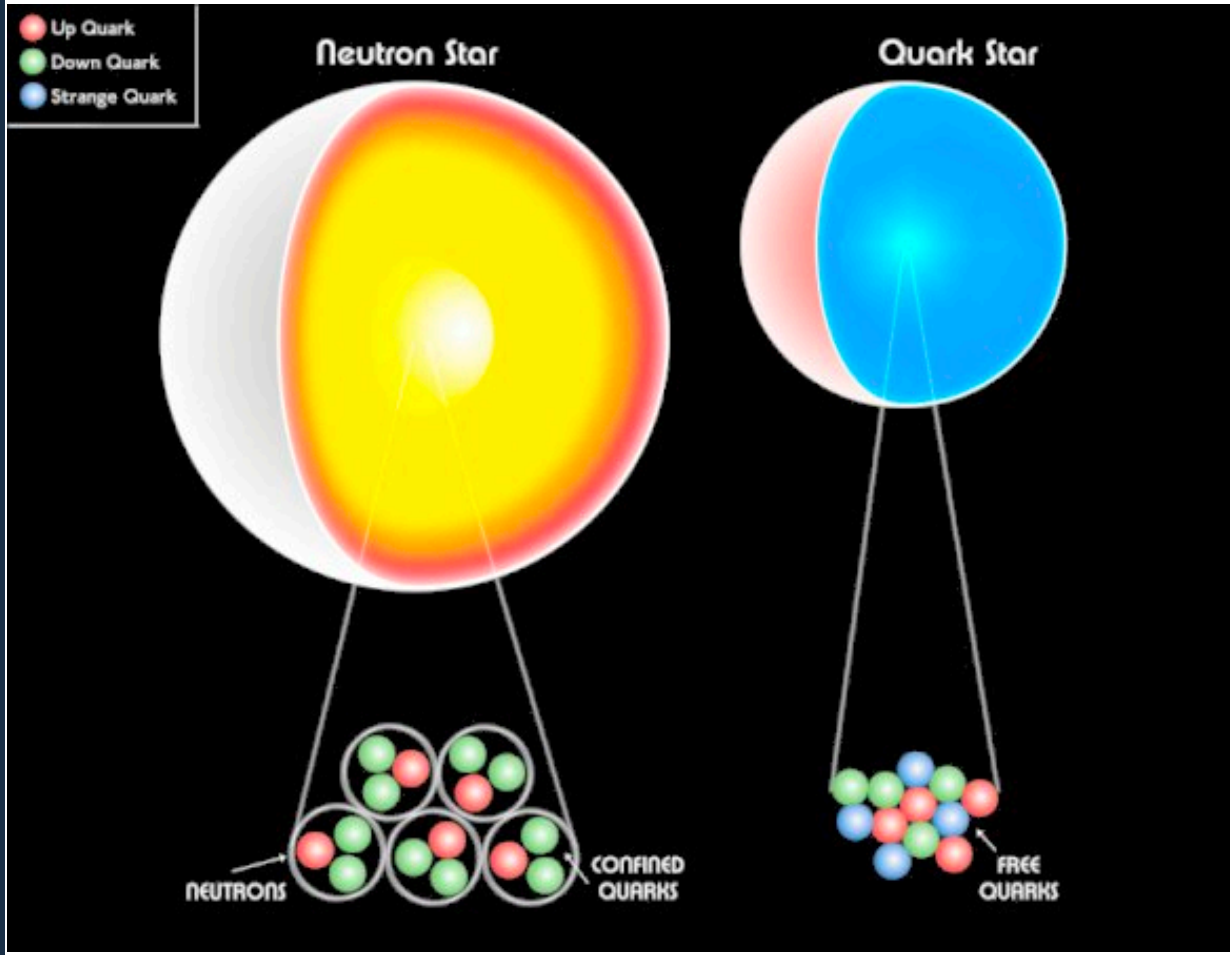


- NS has 5 major regions:

- **The Atmosphere & The Envelope**: shaping the emergent photon spectrum
- **The Crust:** extending about 1-2 km, primarily contains nuclei.
- **The Inner & Outer Core:** contains 99% of stars mass, radius = 12 kms, density = 10 ρ_0 and exotic phases.



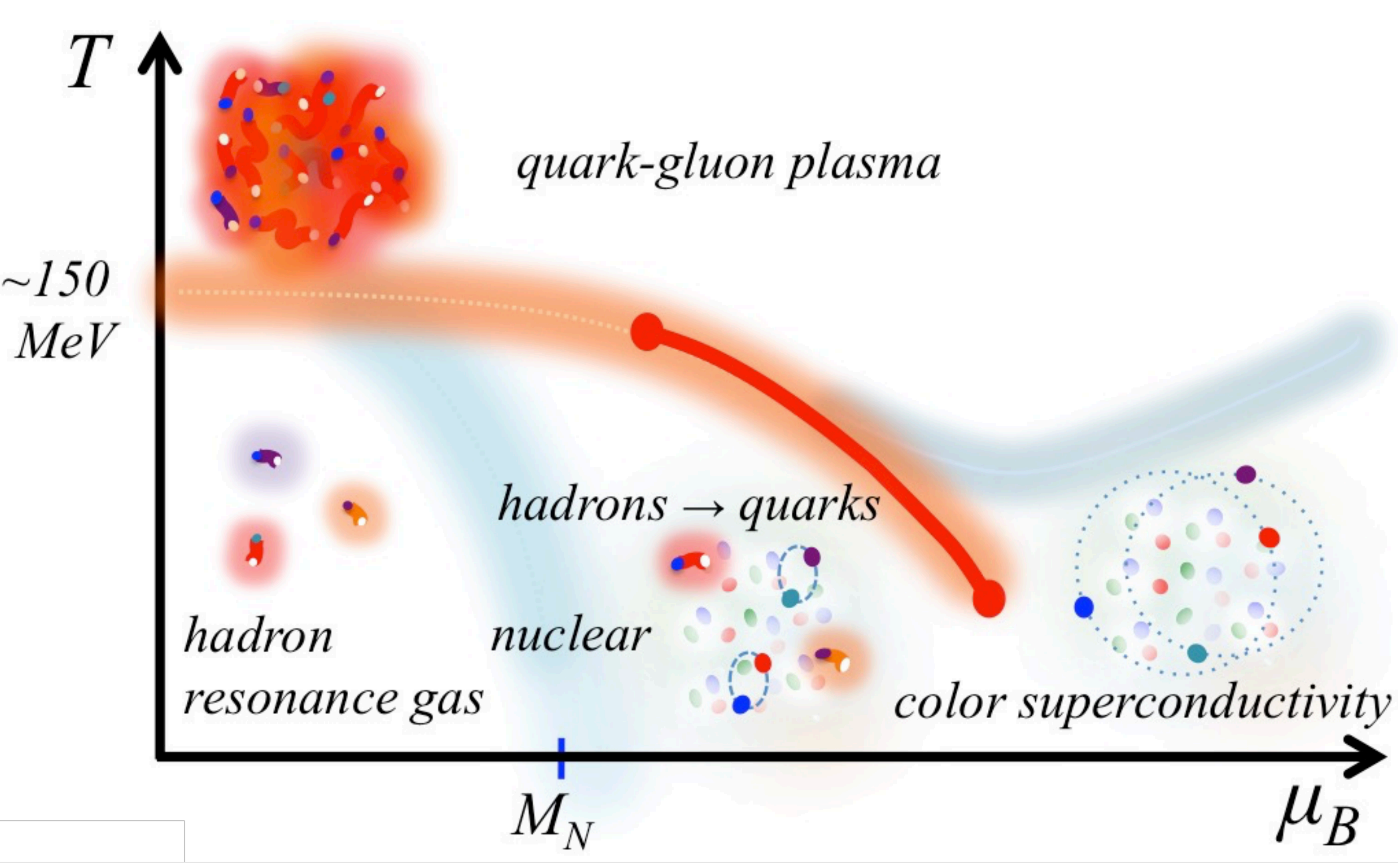




Strange Stars: Stars made up of pure Quark matter. **APJ** 310:261-272,1986

Hybrid Stars: Stars with Hadronic layers followed by a mixed Hadron-Quark phase. **Compact stars**. N. K. Glendenning

Twin Stars: Special Hybrid stars **Kämpfer 1981**



Hadronic Models

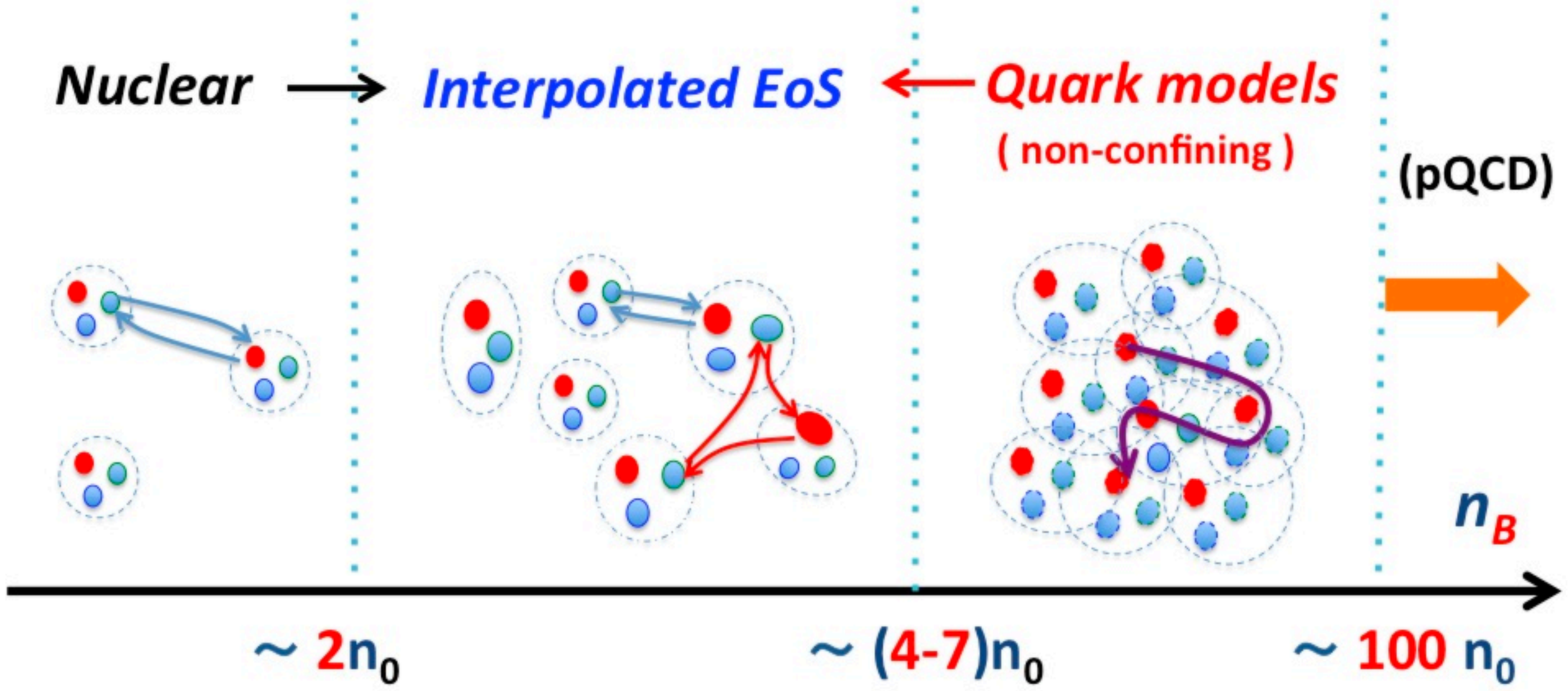
Relativistic Mean-Field (RMF)
Density-Dependent Relativistic Mean-Field (DDRMF)
Chiral Mean-Field (CMF)…

J. Boguta and A. R. Bodmer, Nucl. Phys. A 292, 413 (1977)., R. Brockmann and H. Toki, Phys. Rev. Lett. 68, 3408 (1992).

Quark Models

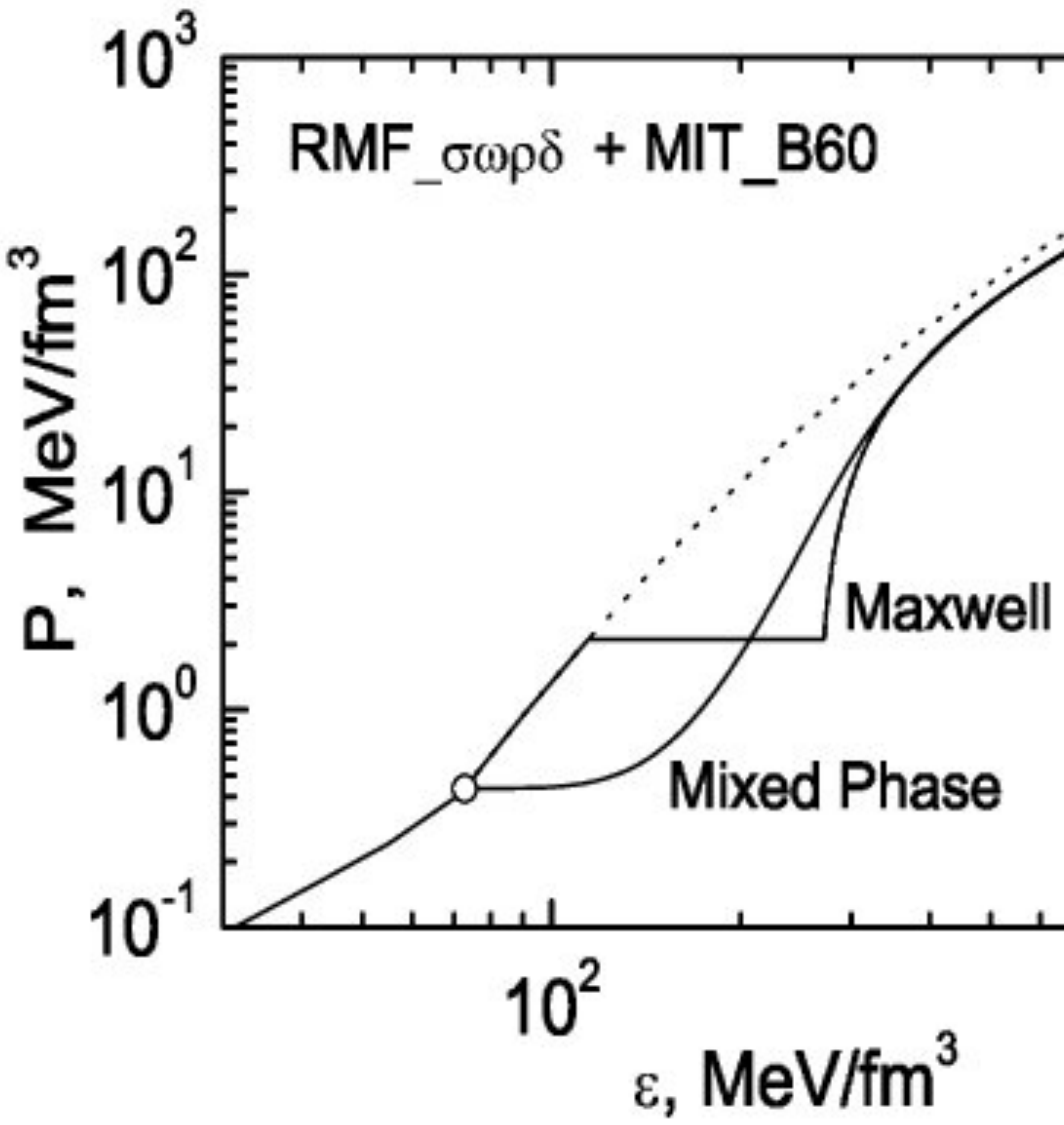
MIT Bag Model
NJL Model
DDQM…

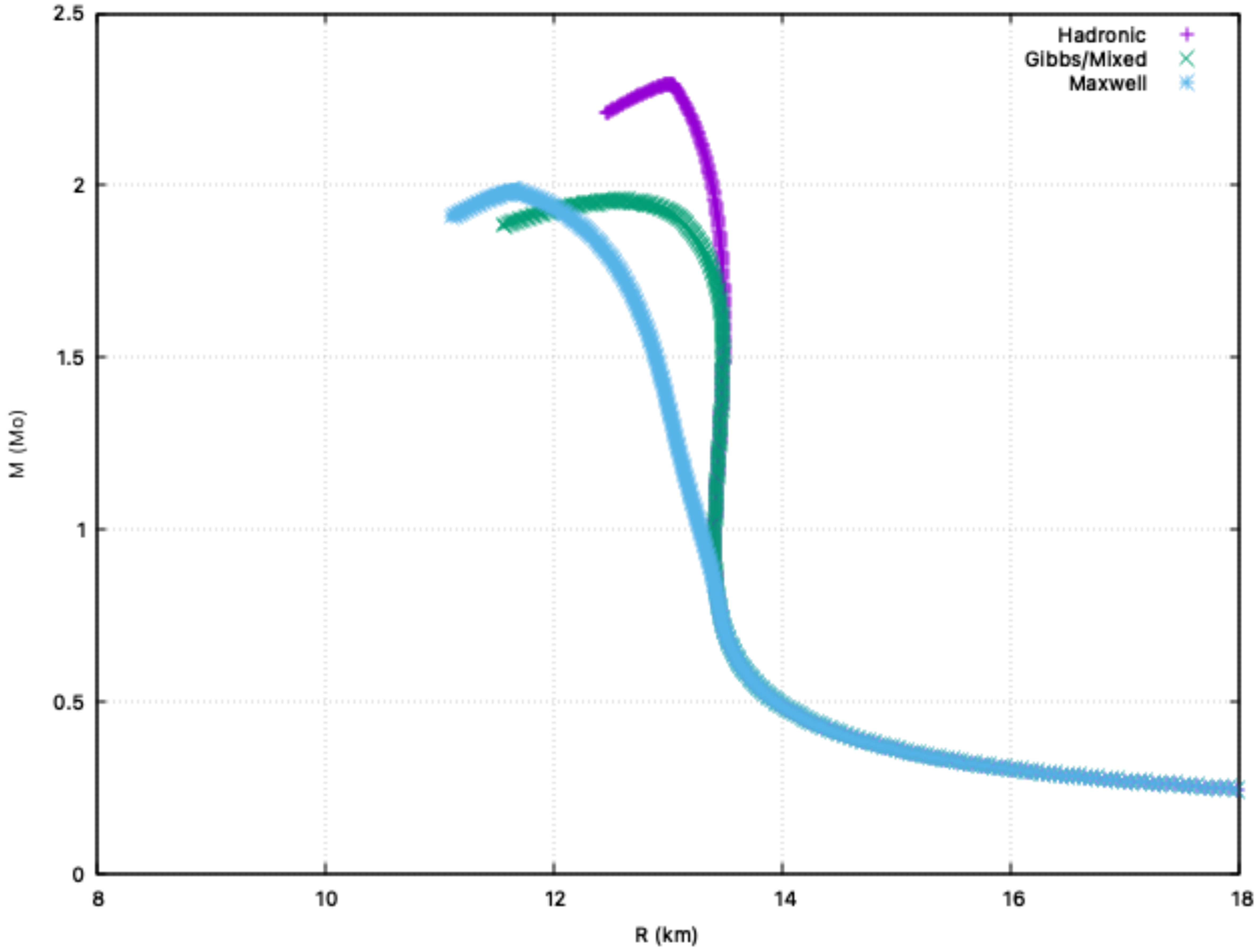
Rosenhauer et al., 1992; Aziz et al., 2019), Orsaria et al., 2013



Hybrid Stars

- Stars with external layers made of hadronic matter and a core of pure Quark matter.
- The region between them is a Hadron-Quark mixed Phase.





Ishfaq Rather et.al, *J.Phys.G* 48 (2021)

Equation of State

Hadronic Model

$$\mathscr{L}_{\rm RMF} = \sum_{b\in H} \bar{\psi}_b \Big[i\gamma^\mu \partial_\mu - \gamma^0 (g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b} I_{3b}\rho_{03}) - (m_b - g_{\sigma b}\sigma_0) \Big] \psi_b$$

Spin-1/2 baryon octet

$$-\frac{i}{2} \sum_{b\epsilon\Delta} \bar{\psi}_{b\mu} \Big[\varepsilon^{\mu\nu\rho\lambda} \gamma_5 \gamma_\nu \partial_\rho - \gamma^0 \left(g_{\omega b}\omega_0 + g_{\rho b} I_{3b}\rho_{03} \right) - (m_b - g_{\sigma b}\sigma_0)\, \varsigma^{\mu\lambda} \Big] \psi_{b\nu}$$

Spin-3/2 baryon decuplet

Rarita-Schwinger-type int. Lag.

M. G. de Paoli, et al., J. Phys. G 40, 055007 (2013).

$$+ \sum_{\lambda} \bar{\psi}_\lambda \left(i\gamma^\mu \partial_\mu - m_\lambda \right) \psi_\lambda - \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2$$

Lepton admix + pure mesonic terms

DD couplings

$$g_{ib}(n_B) = g_{ib}(n_0) \frac{a_i + b_i(\eta + d_i)^2}{a_i + c_i(\eta + d_i)^2}$$

$$g_{\rho b}(n_B) = g_{ib}(n_0) \exp\left[-a_\rho (\eta - 1) \right]$$

$$n_b = \frac{\lambda_b}{2\pi^2} \int_0^{k_{F_b}} dk\, k^2 = \frac{\lambda_b}{6\pi^2} k_{F_b}^{\,3}$$

Baryon density

$$n_b^s = \frac{\lambda_b}{2\pi^2} \int_0^{k_{F_b}} dk \frac{k^2 m_b^*}{\sqrt{k^2 + m_b^{*2}}}$$

Scalar density

Equation of State

$$\varepsilon_B = \sum_b \frac{\gamma_b}{2\pi^2}\int_0^{k_{F_b}} dk k^2\sqrt{k^2+m_b^{*2}} + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_{F_\lambda}} dk k^2\sqrt{k^2+m_\lambda^2} + \frac{m_\sigma^2}{2}\sigma_0^2 + \frac{m_\omega^2}{2}\omega_0^2 + \frac{m_\phi^2}{2}\phi_0^2 + \frac{m_\rho^2}{2}\rho_{03}^2$$

$$P = \sum_i \mu_i n_i - \epsilon + n_B \Sigma^r$$

$$\Sigma^r = \sum_b \left[\frac{\partial g_{\omega b}}{\partial n_b}\omega_0 n_b + \frac{\partial g_{\rho b}}{\partial n_b}\rho_{03} I_{3b} n_b + \frac{\partial g_{\phi b}}{\partial n_b}\phi_0 n_b - \frac{\partial g_{\sigma b}}{\partial n_b}\sigma_0 n_b^s\right]$$

Rearrangement term

$$\mu_b^* = \mu_b - g_{\omega b}\omega_0 - g_{\rho b} I_{3b}\rho_{03} - g_{\phi b}\phi_0 - \Sigma^r$$

Effective Chemical Potential

DD-ME2 parameter

G. A. Lalazissis, *et al.*, Phys. Rev. C 71, 024312 (2005).

i	m_i(MeV)	a_i	b_i	c_i	d_i	$g_{iN}(n_0)$
σ	550.1238	1.3881	1.0943	1.7057	0.4421	10.5396
ω	783	1.3892	0.9240	1.4620	0.4775	13.0189
ρ	763	0.5647	—	—	—	7.3672

Quantity	Constraints [44, 49]	This model
n_0 (fm^{-3})	0.148–0.170	0.152
$-B/A$ (MeV)	15.8–16.5	16.4
K_0 (MeV)	220–260	252
S_0 (MeV)	31.2–35.0	32.3
L_0 (MeV)	38–67	51

M. Dutra, Physical Review C 90, 055203 (2014), M. Oertel , Rev. Mod. Phys. 89, 015007 (2017)

Density-Dependent Quark mass Model

- noninteracting gas of quasiparticles with density-dependent masses.
- Overcomes the consistency between zero pressure and energy minimum
- To include quark interactions in a simple way.

G. N. Fowler, S. Raha, and R. M. Weiner, Z. Phys. C 9, 271 (1981),
S. Chakrabarty et al., Phys. Lett. B 229, 112 (1989),
S. Chakrabarty, Phys. Rev. D 43, 627 (1991)
O. G. Benvenuto and G. Lugones, Phys. Rev. D. 51, 1989 (1995)

$$m_i = m_{i0} + \frac{D}{n_B^{1/3}} + C n_B^{1/3} = m_{i0} + m_I$$

B C Backes et al . J. Phys. G: Nucl. Part. Phys. 48 (2021) 055104

Dictates linear confinement (D)

Leading order perturbative interactions (C)

Current quark mass (m_{i0})

DD term (m_I)

$$\mathscr{E} = \Omega_0(\{\mu_i^*\}, m_i) + \sum_i \mu_i^* n_i = \Omega_0(\{\mu_i^*\}, m_i) - \sum_i \mu_i^* \frac{\partial \Omega_0}{\partial \mu_i^*}$$

$$P = -\Omega_0 + \sum_{i,j} \frac{\partial \Omega_0}{\partial m_j} n_i \frac{\partial m_j}{\partial n_i}$$

$$\Omega_0 = -\sum_i \frac{g_i}{24\pi^2}\left[\mu_i^* \nu_i\left(\nu_i^2 - \frac{3}{2} m_i^2\right) + \frac{3}{2} m_i^4 ln\left(\frac{\mu_i^* + \nu_i}{m_i}\right)\right],$$

Radial Eqs. with PT

Phase Conversion

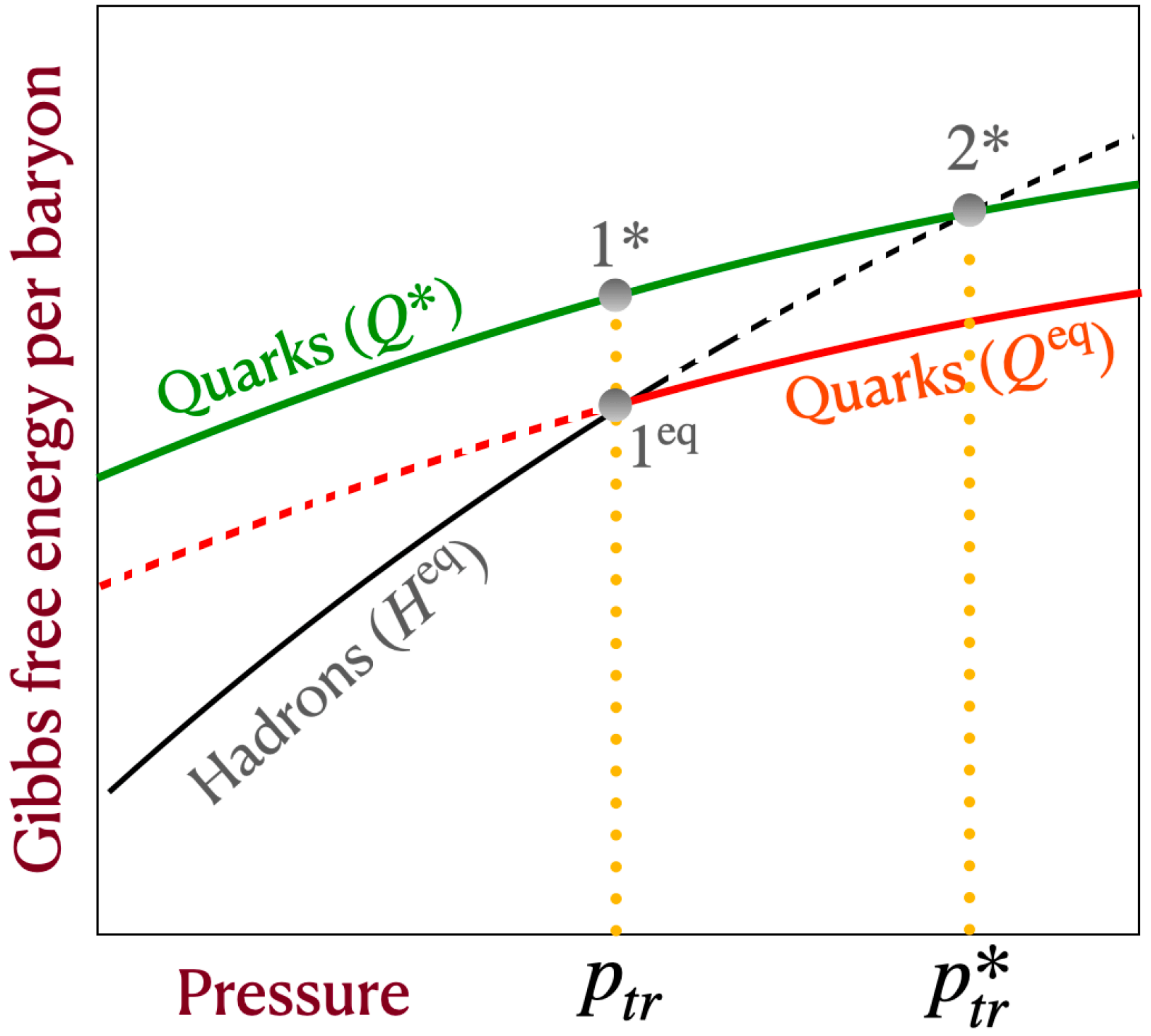
1) Conversion timescale (τ_{conv}) >> Oscillation period (τ_{osc}) (fluid elements keep their nature) → **slow phase transition**

2) $\tau_{conv} << \tau_{osc}$ (fluid elements are easily converted) → **Rapid phase transition**

p_{tr} & p_{tr}^*

Sufficiently different → **Rapid phase transition**

Close enough → **slow phase transition**



Rapid phase transition

$$\frac{\partial M}{\partial \mathcal{E}_c} > 0 \implies \omega_0^2 \geq 0$$ (stable star)

$$\frac{\partial M}{\partial \mathcal{E}_c} < 0 \implies \omega_0^2 < 0$$ (unstable star)

slow phase transition

$$\frac{\partial M}{\partial \mathcal{E}_c} > 0 \implies \omega_0^2 \geq 0$$ (stable star)

$$\frac{\partial M}{\partial \mathcal{E}_c} < 0 \implies \omega_0^2 > 0$$ (stable star)

G. Lugones et al. *Universe* **2021**, *7*(12), 493

Slow-stable hybrid stars

Germán Lugones *et al* JCAP, 03 (2023) 028

J. P. Pereira, *ApJ.* 860 (2018) 12

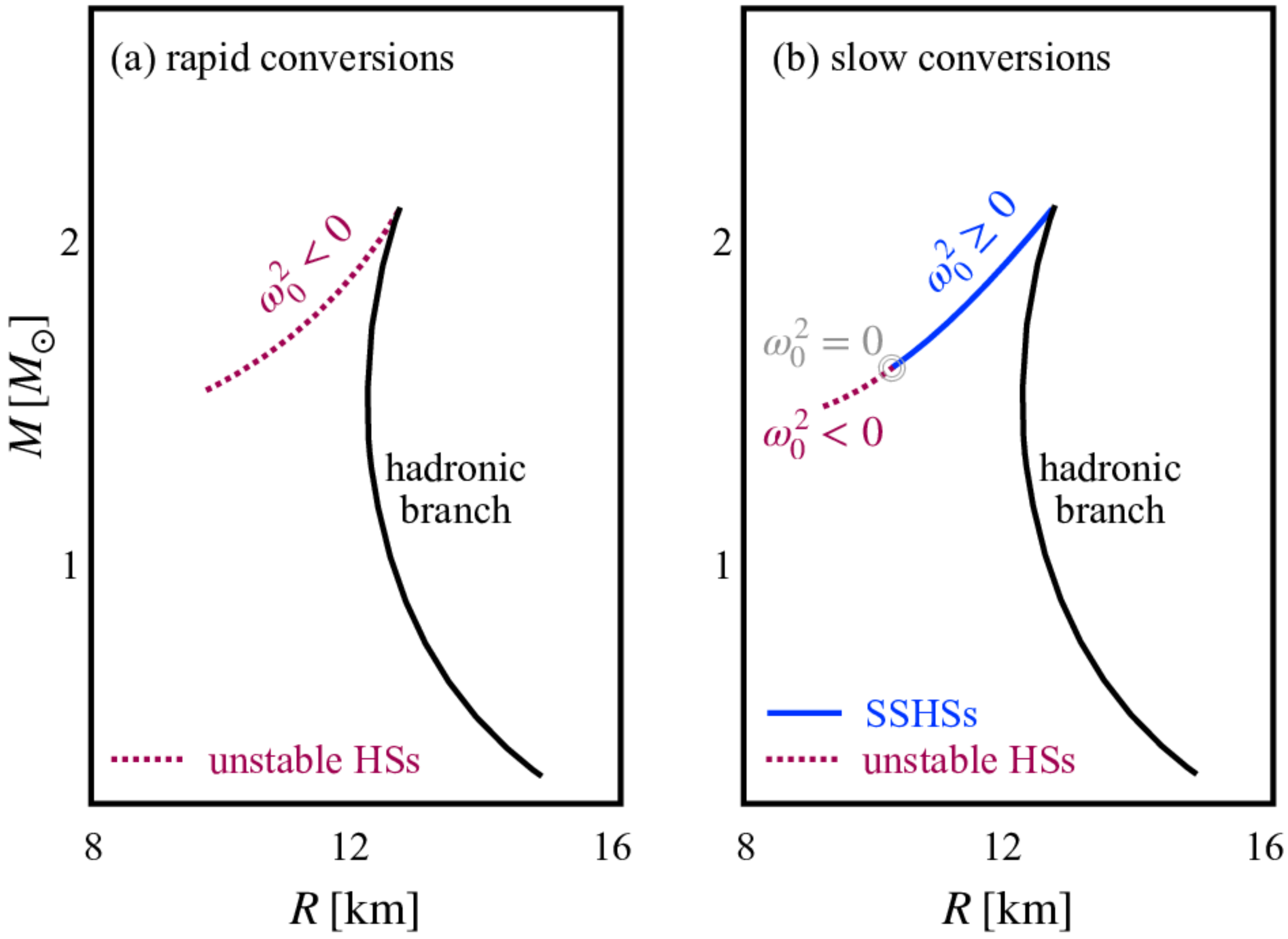
Junction conditions:

Rapid: $$[\xi]_-^+ = \Delta p \left[\frac{1}{p_0'}\right]_-^+ \qquad [\Delta p]_-^+ = 0$$

Slow: $$[\xi]_-^+ = 0 \qquad [\Delta p]_-^+ = 0$$

radially unstable configurations are radially stable under small perturbations

hadron-quark transition at a high density



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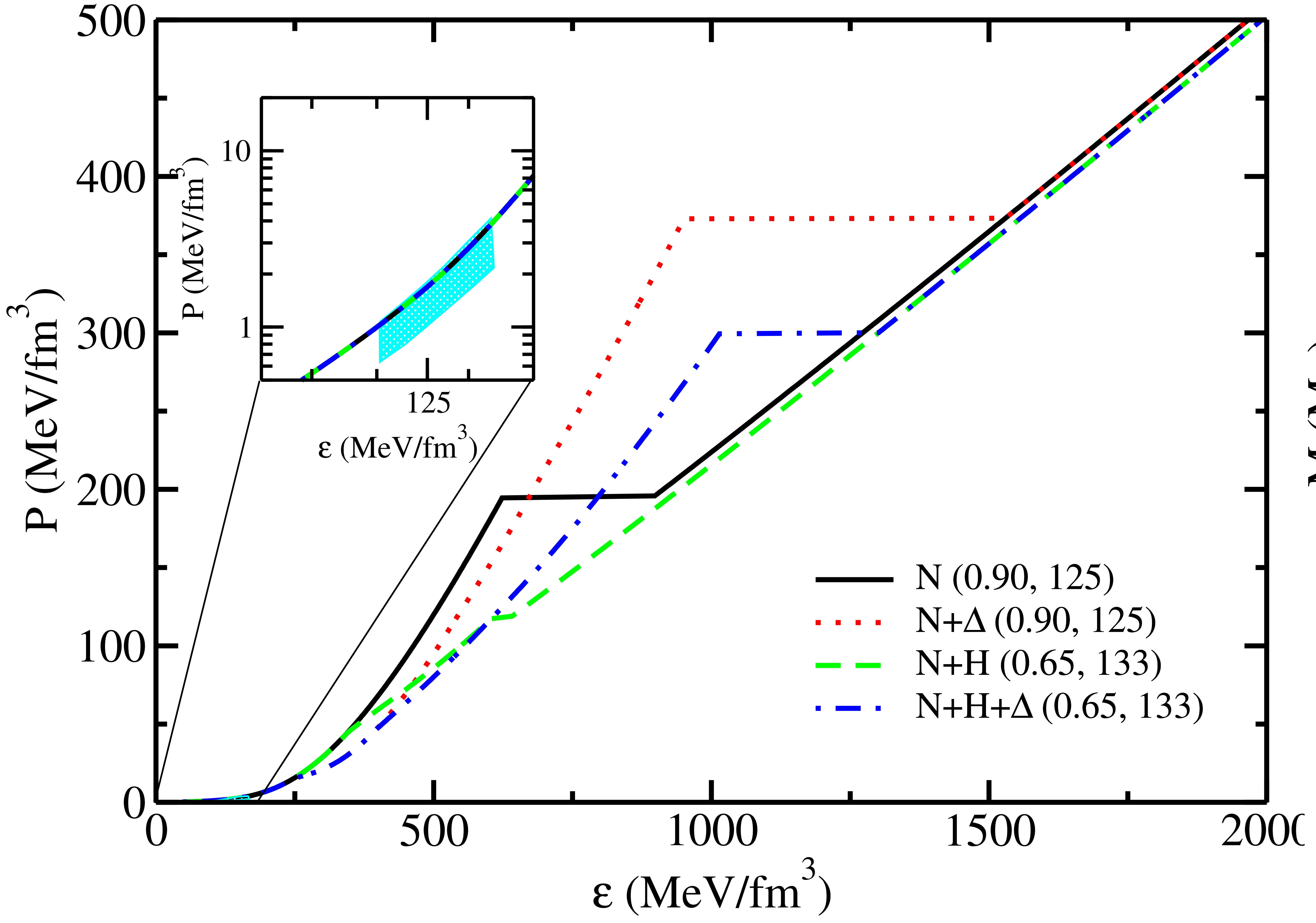
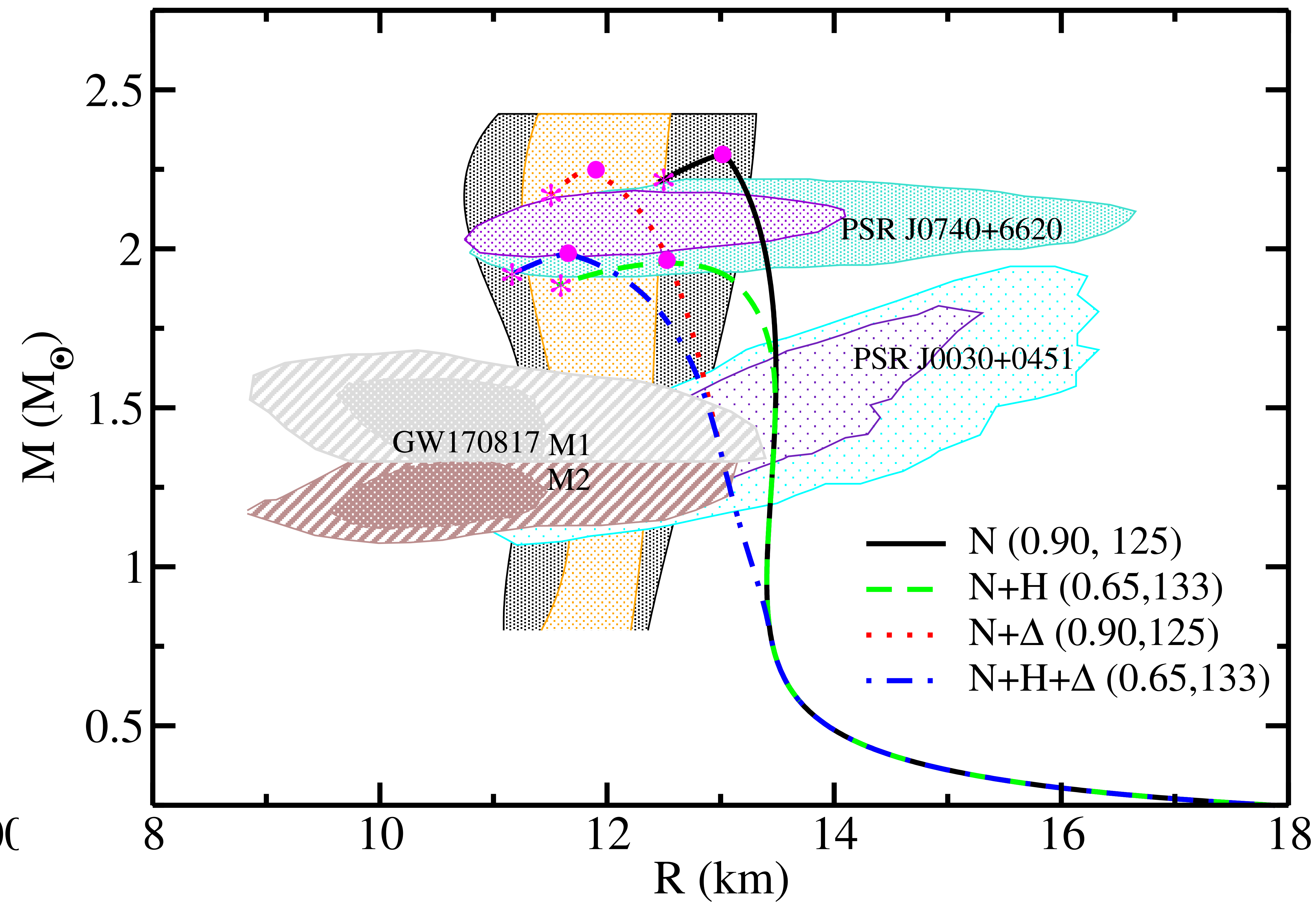
Results





Length of the SSHS branch

- ∝ 1/(Energy density jump between two phases).
- ∝ Stiffness of the quark EoS.


Slow conversion
Rapid conversion
ν_0 (kHz)
M (M_O)
N (0.90, 125)
N+ Δ (0.90, 125)
N + H (0.65, 133)
N + H + Δ (0.65, 133)
N+Δ (0.90, 125)
N+H (0.65, 133)
N+H+Δ (0.65, 133)
Stable
Non-vanishing
f-mode frequency
Zero f-mode
frequency at M_max

Summary

- Observation of a new class of phase in Neutron Stars: **Slow stable hybrid stars**.
- Oscillation functions of Δ-inclusive nuclear (N +Δ) and hyperonic matter (N + H + Δ).
- Radial oscillations with Δ-baryons and Phase transition to the Quark matter.

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