pfd-parallel and *NeatIBP*

two packages to shorten rational functions and IBP linear systems

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Sep. 25, 2023
IMathem/Amplitudes 2023

A tale of two packages

photo from USPS website

pfd-parallel

simplifies rational functions
in amplitudes computations,
by multivariate partial fraction

NeatIBP

generates small size
IBP systems based on
syzygy/module intersection

pfd-parallel

A package simplifies rational functions
in amplitudes computations
by multivariate partial fraction

Bendle, Boehm, Heymann, Ma, Rahn, Ristau, Wittmann, Wu, Xu and YZ

https://github.com/singular-gpispace/pfd-parallel/

Department of Mathematics, Univ. of Kaiserslautern
USTC

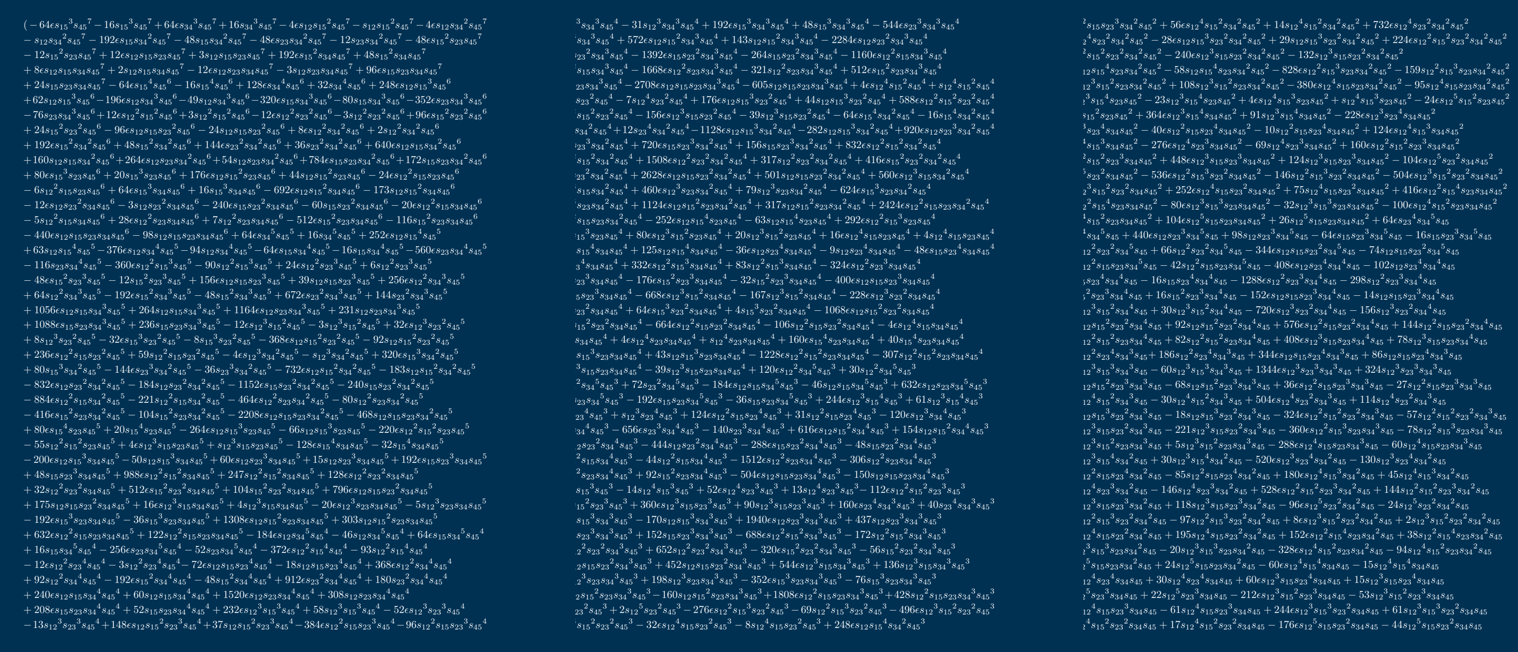
arXiv: 2104.06866
package finished on Oct. 2022
Accepted in Comp. Phys. Comm.

Why do we need multivariate partial fraction?

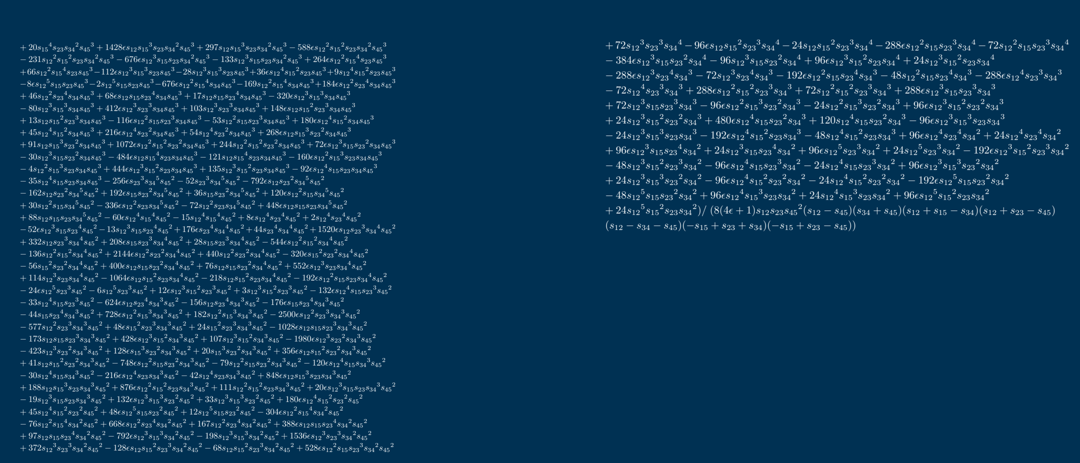
It is well known that sometimes in amplitudes computations, multivariate partial fraction can greatly simplify rational functions.

- simplify coefficients in Amplitudes
- simplify IBP reduction coefficients
- boost canonical DE construction

Meyer, CPC 222 (2018)
Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov, JHEP 05 (2019) 08
Agarwal, Buccioni, von Manteuffel, Tancred, PRL 127 (26) (2021) 26200
… …



An example from IBP reduction coefficient in
Boehm, Wittmann, Wu, Xu, YZ,
JHEP 12 (2020) 054



5 page long expression = 1/3 page expression

Choose a UT basis,
then IBP reduction coefficients
can be greatly simplified by partial fraction

pfd-parallel

Massively-Parallel
Partial Fraction Decomposition

a massively parallel framework for partial fraction decomposition of rational functions
based on the Singular/GPI-Space framework

Improved Leintars algorithm

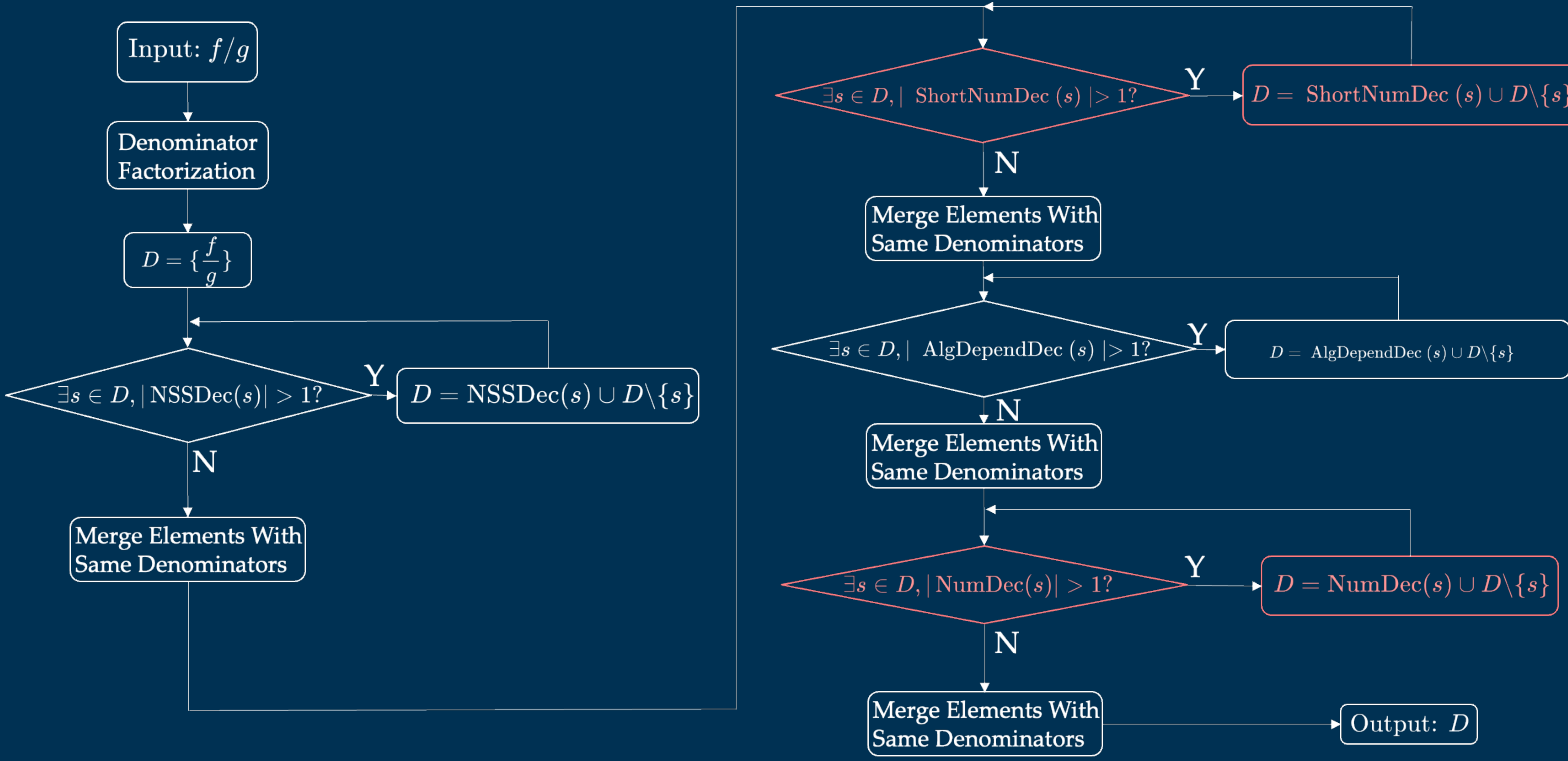
Boehm, Wittmann, Wu, Xu, and YZ, JHEP 12 (2020) 054

Heller and von Manteuffel: CPC. 271 (2022) 108174.

MultivariateApart algorithm

Please also try package 'MultivariateApart'

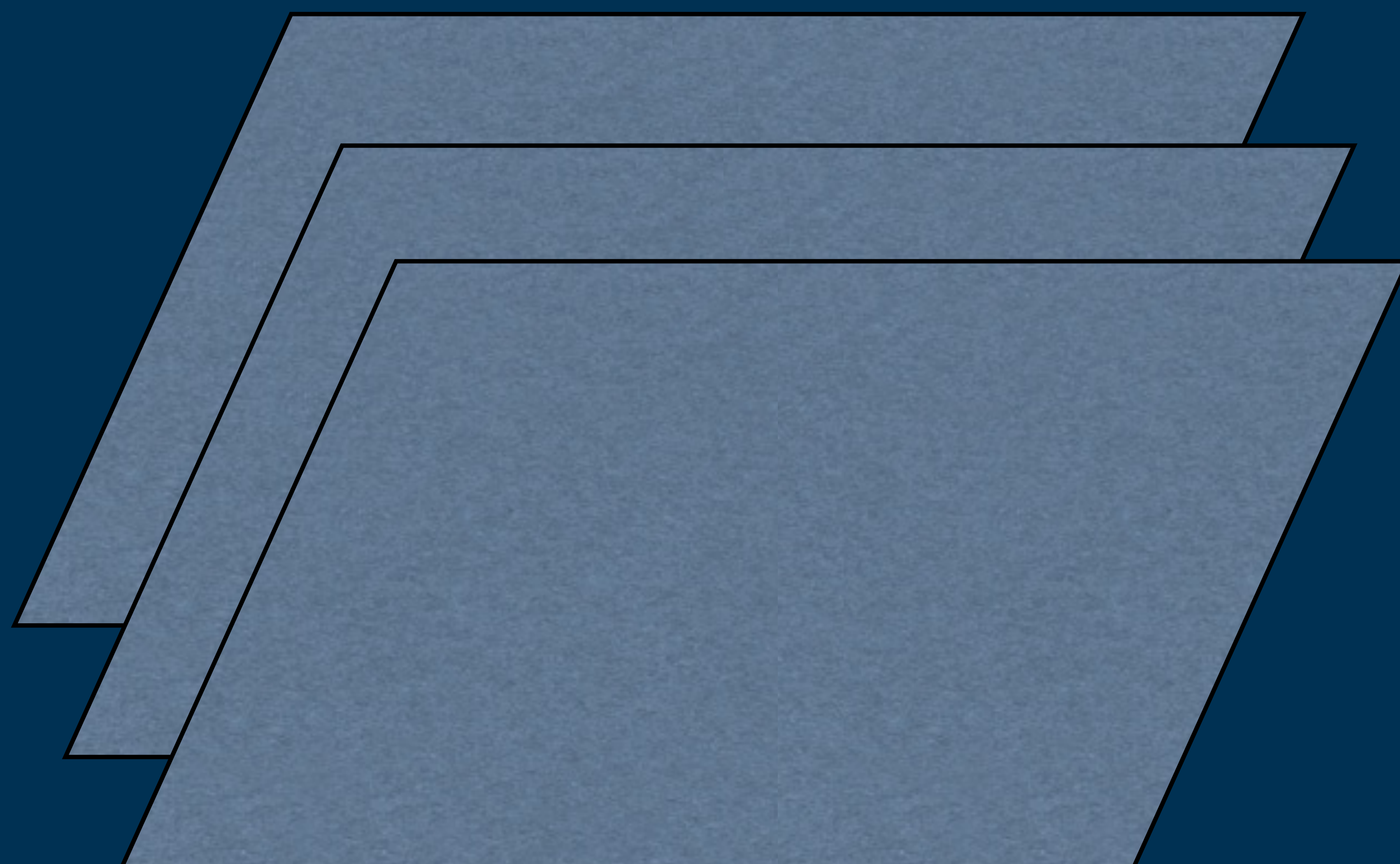
Improved Leintars' algorithm



Boehm, Wittmann, Wu, Xu, and YZ, JHEP 12 (2020) 054

pfd-parallel

Massively-Parallel
Partial Fraction Decomposition



Partial Fraction Decomposition can be a heavy computation,
so parallelization is important

We implement several layers of parallelization

parallelization between several rational functions
parallelization between terms in one rational function

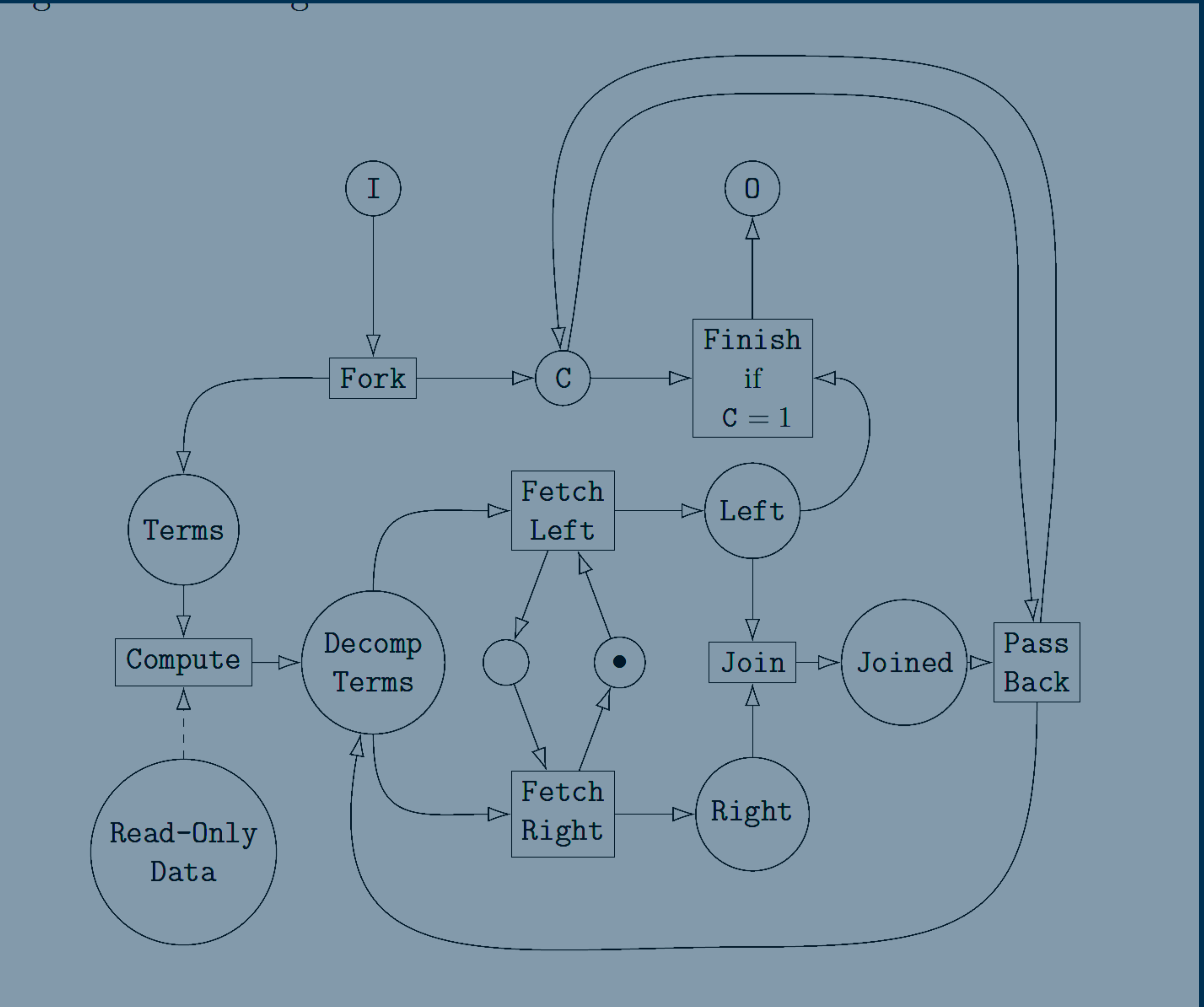
complicated workflow
so we implement the
workflow manager
GPI-space

pfd-parallel

Massively-Parallel
Partial Fraction Decomposition

GPI-Space, a workflow management system,
created by the Competence Center High Performance Computing of Fraunhofer ITWM.

Combine Singular with GPI-Space
to create a way of performing massively parallel computations in computer algebra.



For details,
refer to Janko Boehm's website

https://www.mathematik.uni-kl.de/~boehm/singulargpispace/

Petri-Net for the parallization over
the decomposition of terms

pfd-parallel Example

Simplifying coefficients in multi-loop scattering amplitudes

Two-loop leading color $pp \to W^{\pm} + \gamma + j$

from Badger, Hartanto, Krys, Zoia
JHEP 05 (2022) 035

6 kinematic variables

$$1096\ \text{MB} \xrightarrow{\textit{pfd-parallel}} 9\ \text{MB}$$

with MultivariateApart algorithm
implemented in GPI-space/Singular

Running on a node with 48 CPUs, ~ 5.5 hours

NeatIBP

A package generates small size IBP systems based on syzygy/module intersection

Wu, Boehm, Ma, Xu and YZ

https://github.com/yzhphy/NeatIBP

arXiv: 2305.08783

It is well know that IBP reduction could be the slowest step for an amplitude computation

$$\int \frac{d^D l_1}{i\pi^{D/2}} \cdots \int \frac{d^D l_L}{i\pi^{D/2}} \frac{\partial}{\partial l_i^\mu} \frac{v_i^\mu}{D_1^{\alpha_1} \dots D_k^{\alpha_k}} = 0$$

Chetyrkin, Tkachov (1981)

Standard Laporta algorithm may provide a system containing too many integrals other than target or master integrals.

Gaussian elimination with symbolic parameter is extremely difficult

Many way to improve the situation …

other than the standard Laporta algorithm

Non-commutative algebra to avoid Gaussian elimination by a reduction over the integral index operators

Smirnov, Smirnov , JHEP01(2006)001
…

Syzygy to decrease the number of integrals in an IBP system

Gluza, Kajda, Kosower, PRD. 83.045012
Schabinger, JHEP 01 (2012) 077
Larsen YZ, PRD 93 (2016) 4, 041701 …

Intersection theory to map the space of Feynman integrals to a twisted cohomology group

Mastrolia, Mizera JHEP 02 (2019) 139
Frellesvig, Gasparotto, Laporta, Mandal, Mastrolia, Mizera JHEP 05 (2019) 153
Frellesvig, Gasparotto, Mandal, Mastrolia, Mattiazzi, Mizera PRL. 123.201602
…

Auxiliary mass flow to find integrals relations based on the eta expansion

Liu, Ma Phys. Rev. D 99 (2019), no 7 071501
Guan, Liu, Ma, Chin. Phys. C 44(2020) 9, 093106
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Syzygy for IBP reduction

smart vector choice

$$\int \frac{d^D l_1}{i\pi^{D/2}} \cdots \int \frac{d^D l_L}{i\pi^{D/2}} \frac{\partial}{\partial l_i^\mu} \frac{v_i^\mu}{D_1^{\alpha_1} \ldots D_k^{\alpha_k}} = 0$$

Suppose that we want to forbid the increase of the propagator index α_i, we can require that,

$$\Big(\sum_{j=1}^{L} v_j^\mu \frac{\partial}{\partial l_j^\mu} D_i\Big) + g_i D_i = 0$$

Syzygy equation

Gluza, Kajda, Kosower, PhysRevD. 83.045012

where both v_j^μ and g_i contain polynomials in loop momenta.

ways to apply syzygy method

Syzygy from Linear Algebra, Schabinger, *JHEP 01 (2012) 077*

Module Intersection, Larsen, YZ, *Phys.Rev.D* 93 (2016) 4, 041701

Dual Conformal Symmetry, Bern, Enciso, Ita, Zeng, Phys. Rev. D 96, 096017 (2017)

… …

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… …

Module Intersection

polynomials

Require

1. no shifted exponent: $\sum_{j=1}^{k} a_j(z) \frac{\partial F}{\partial z_j} + \beta(z) F = 0$ These $\big(a_1(z), \ldots a_k(z)\big)$ form a module $M_1 \subset R^k$.

2. no propagator degree increase: $a_i(z) \in \langle z_i \rangle, \quad 1 \leq i \leq m$ These $\big(a_1(z), \ldots a_k(z)\big)$ form a module $M_2 \subset R^k$.

Both M_1 and M_2 are pretty simple ...

Larsen YZ
Phys.Rev.D 93 (2016) 4, 041701

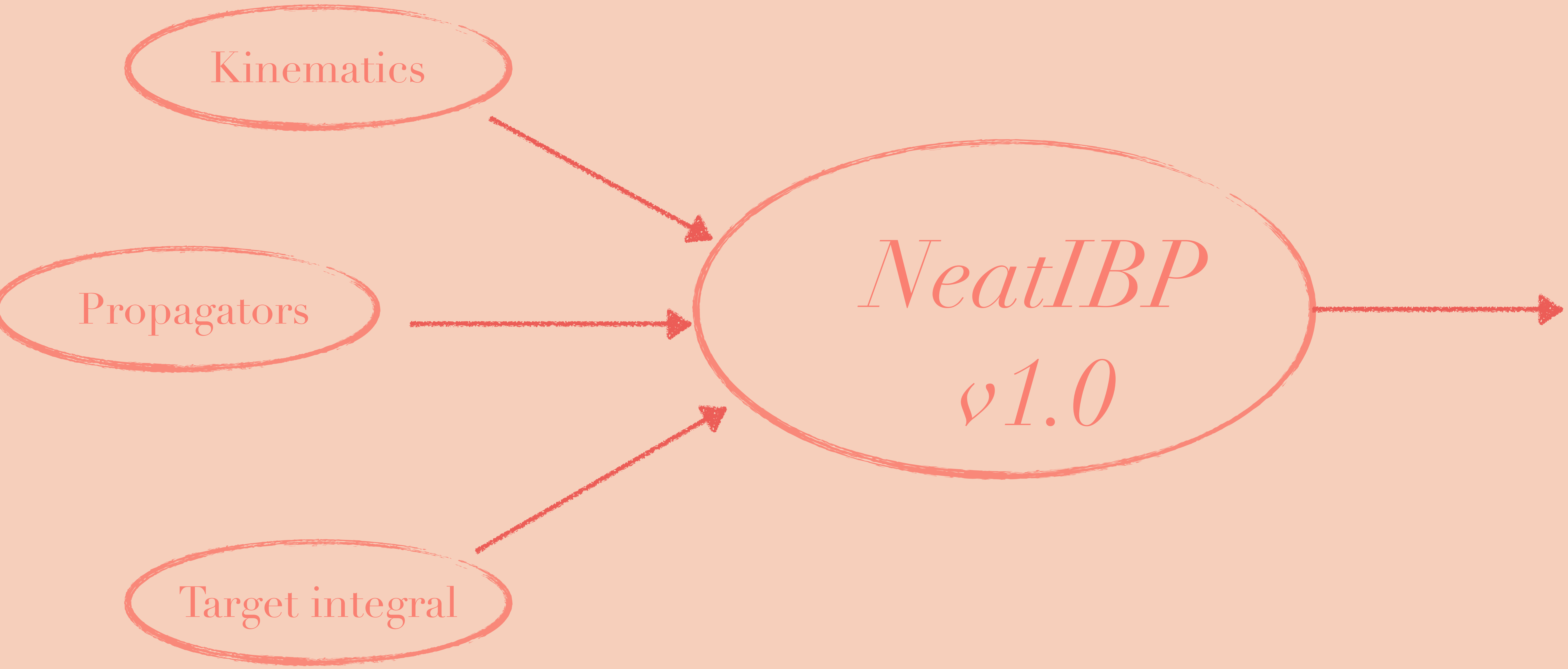
$$M_1 \cap M_2$$

Intersection of two modules over a polynomial ring

NeatIBP v1.0

an automatic package to generate small-size IBP based on syzygy/module intersection

Boehm, Ma, Wu, Xu and YZ



small size IBP relations with restricted propagator indices which is enough to reduce target to master integrals

https://github.com/yzhphy/NeatIBP

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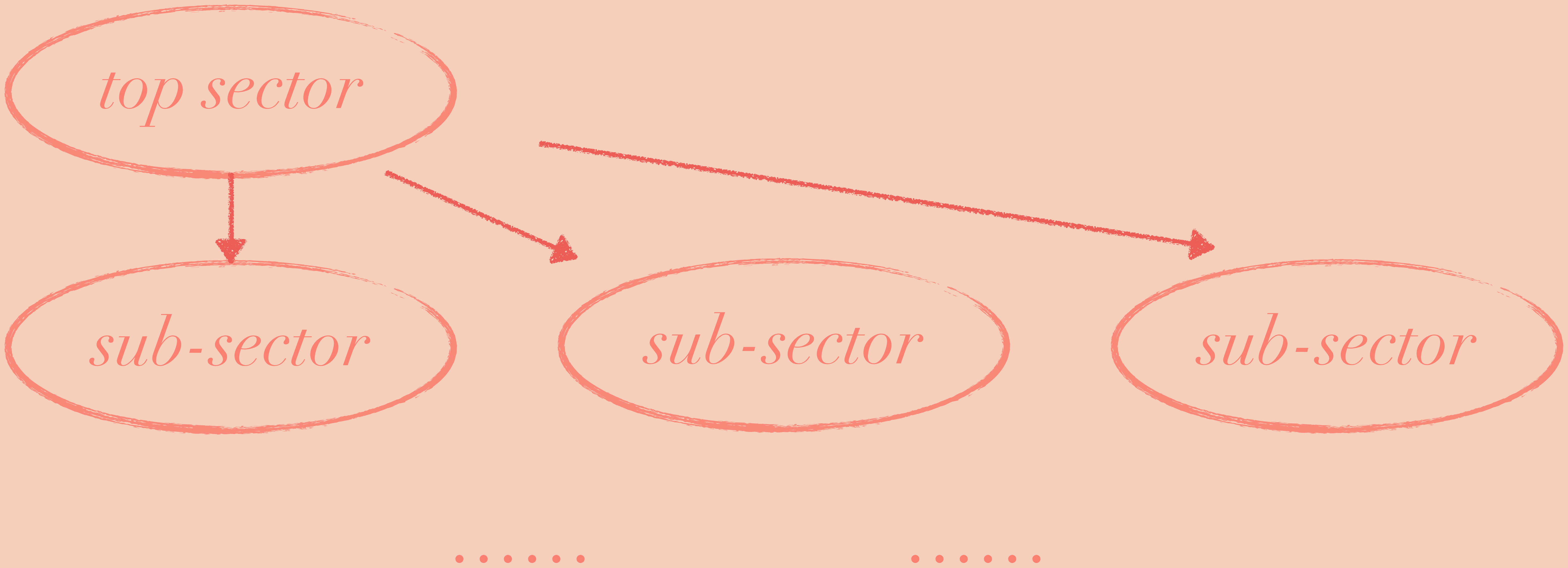


sector-by-sector module intersection

generate IBPs with propagator indices restricted

finite-field method to select useful and independent IBP relations

Determine the master integrals and accumulate IBP relations in the same time

NeatIBP v1.0

Implementation notes

Mathematica/Singular codes module intersection is done with the Schreyer's algorithm in Singular

Parallelization is over different sectors

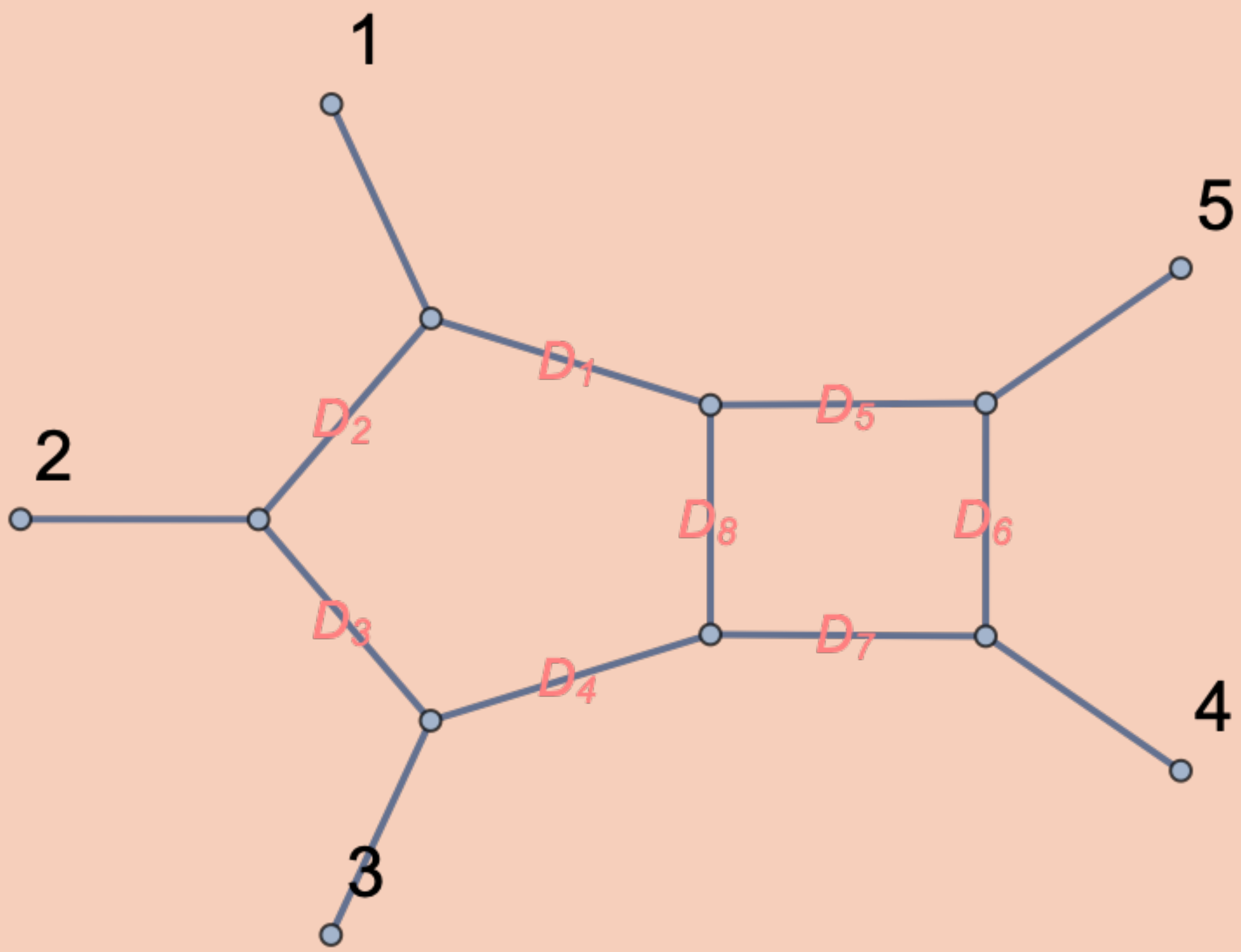
Finite Field Computations to select useful and independent IBPs powered by the package SpaSM https://github.com/cbouilla/spasm by Charles Bouillaguet

Unitarity cut applicable

Symmetry relation implemented

NeatIBP v1.0

Example 1 Massless pentagon-box



5 Mandelstam variables

(hardest ones with deg-5 numerator)

2483 target integrals without double propagators

on a workstation with 10 cores
NeatIBP running time ~27 minutes

14120 IBP relations without double propagators

short and sparse IBP system
with integer-valued Mandelstam variables
Finite-field reduction < 1 second

Comparison: Laporta algorithm uses >100 times more IBP relations for this task
FIRE6: 11207942 IBP relations

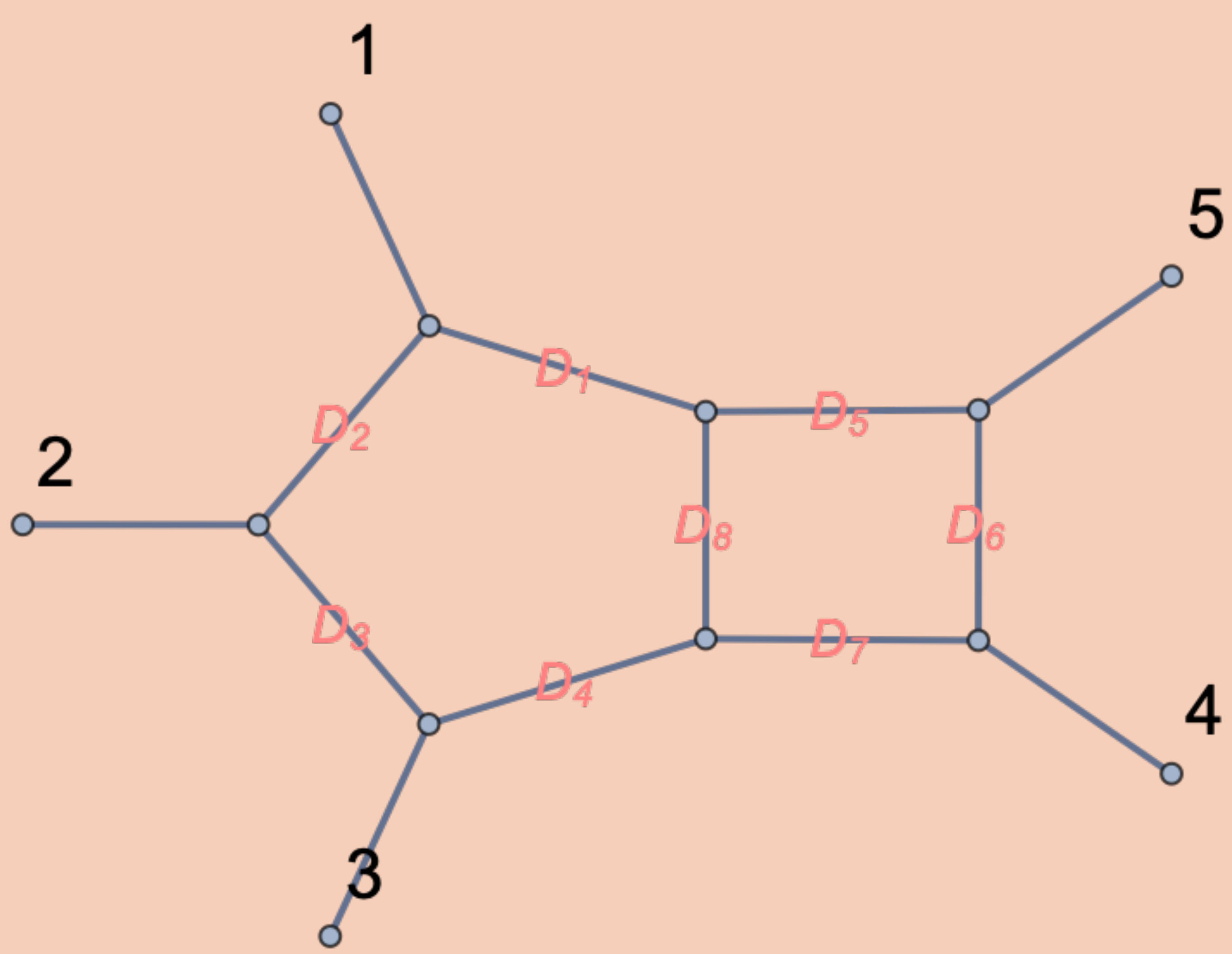
NeatIBP v1.0

Example 1' with double propagators

Slogan said: "syzygy IBP method does not apply for integrals with double propagators"

This is wrong.

syzygy IBP method does apply for integrals with double propagators



61 master integrals' derivatives in Mandelstam variables

880 target integrals; most of them with double propagators

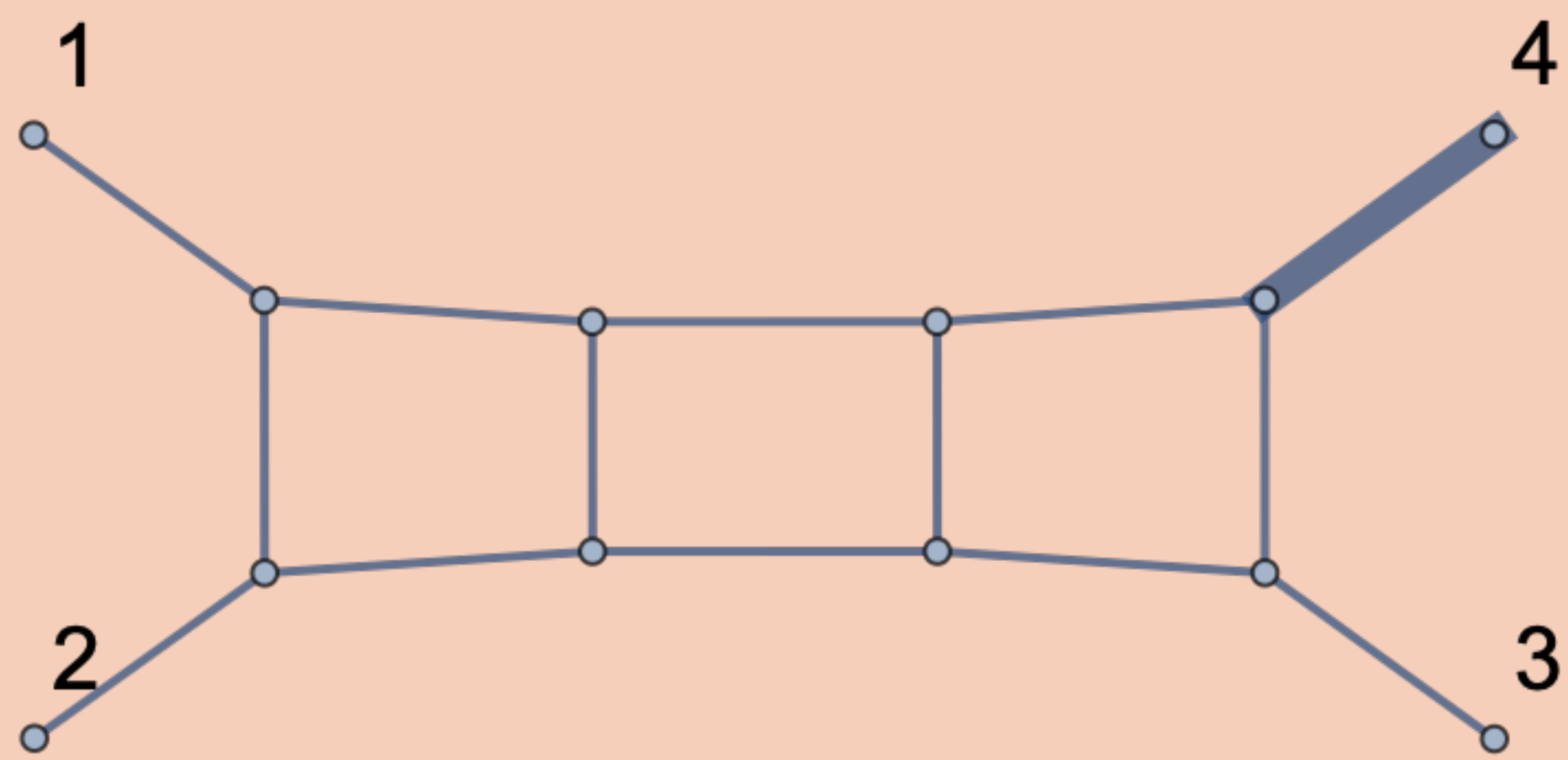
on a workstation with 10 cores
NeatIBP running time ~17 minutes

3313 IBP relations, enough

NeatIBP v1.0 also works for pentagon box with external massive legs.

NeatIBP v1.0

Example 2 Three loop



One massive external leg

21192 target integrals on various sectors,
most complicated
G[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, -6, 0, 0, 0] …

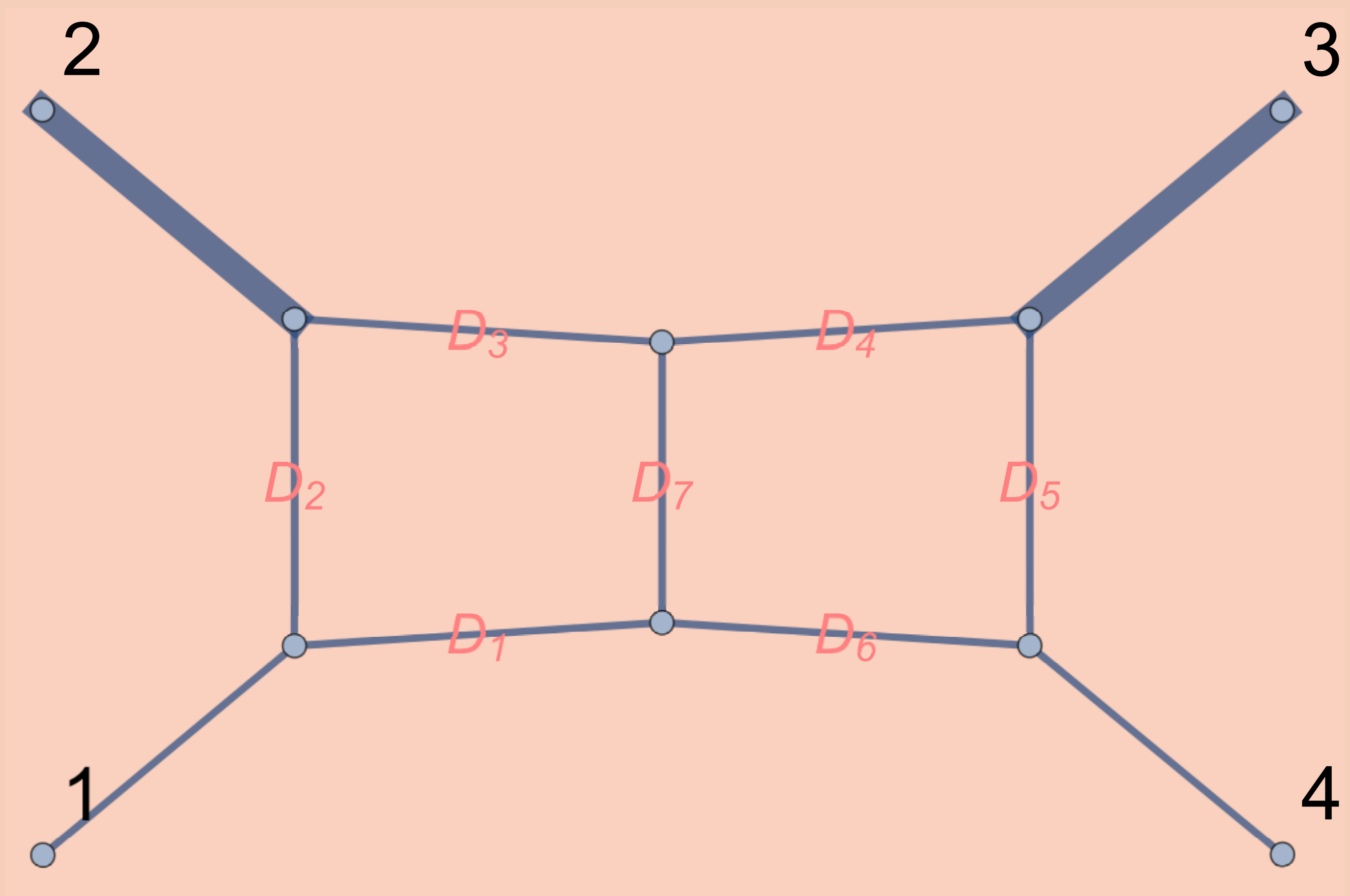
on a workstation with 50 cores
NeatIBP running time ~4.5 hour

192319 IBP generated

(83 master integrals detected)

NeatIBP v1.0 + KIRA/FireFly

What if we want the analytic result of IBP reduction? Try NeatIBP + Kira/FireFly



[Kira] Klappert, Lange, Maierhofer, and Usovitsch, Comp.Phys.Comm., 266:108024,
[FireFly] Jonas Klappert and Fabian Lange, Comp.Phys.Comm., 247:106951

An interface to Kira is prepared, and also interface to Blade, FiniteFlow on the way …

	Reduction Time	Reduction RAM usage
Kira/FireFly	$>$ 1 day	4 GB $\sim$ 5 GB
NeatIBP +Kira/FireFly	3 hours	600 MB

NeatIBP v1.0

Issues and outlook

- NeatIBP v1.0 uses Mathematica quite a lot. In future, we will focus on open-source Languages.
- NeatIBP v1.0 uses Singular's Schreyer algorithm. We have been testing linear algebra syzygy method for a long time, but not used in this version … *refer to works by Agarwal, von Manteuffel, Schabinger*
- Only a very small number of syzygies needed … NeatIBP 1.1. *Kosower's idea*
- We will apply the massive parallelization framework GPI-space for the future NeatIBP versions

Summary

photo by YZ
12.16.2015

pfd-parallel **and** *NeatIBP*

Simplify
Rational Functions
in parallel

Generate
Simple IBP relations

Based on algebraic geometry

aims at cutting-edge computation of Feynman integrals

Syzygy to IBP, with Baikov representation

$$\int \frac{d^D l_1}{i\pi^{D/2}} \cdots \int \frac{d^D l_L}{i\pi^{D/2}} \frac{1}{D_1^{\alpha_1} \ldots D_k^{\alpha_k}} \propto \int_\Omega dz_1 \ldots dz_k \frac{F^{\frac{D-L-E-1}{2}}}{z_1^{\alpha_1} \ldots z_k^{\alpha_k}}$$

$$0 = \int_\Omega dz_1 \ldots dz_k \sum_{i=1}^{k} \frac{\partial}{\partial z_i} \left(a_i(z) \frac{F^{\frac{D-L-E-1}{2}}}{z_1^{\alpha_1} \ldots z_k^{\alpha_k}} \right)$$

No boundary term

feel free to set some of z's to zero (unitary cut)

- much easier to compute the syzygies
- divide and conquer the problem by spanning cuts