Top-quark loops for precision Higgs physics

UNIVERSITÀ DEGLI STUDI DI PADOVA

INFN PADOVA
Istituto Nazionale di Fisica Nucleare
Sezione di Padova

Marco Vitti - Padova University & INFN, Padova

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Outline

1. Precision Higgs Physics at the LHC
2. Example: $gg \to XY$ @ NLO QCD
3. pT expansion

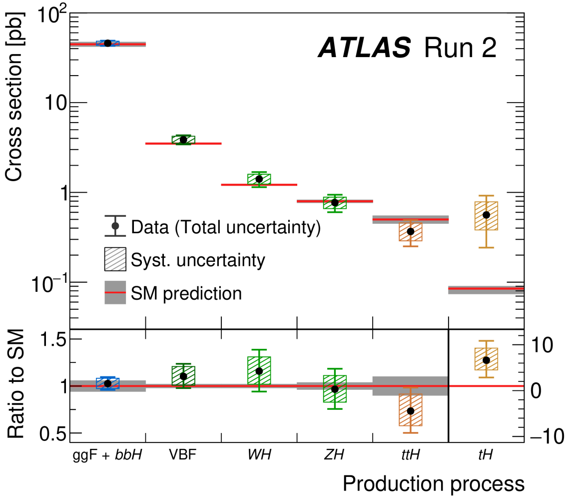
Work in collaboration with
L. Alasfar, L. Bellafronte, G. Degrassi, P.P. Giardino,
R. Gröber, X. Zhao

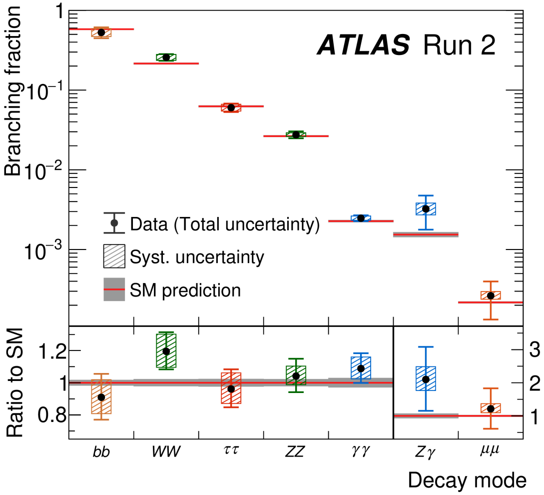
Higgs Physics at the LHC

Does the discovered Higgs boson behave as the SM predicts?

What we know after Run2 $(139\,\mathrm{fb}^{-1})$

- CP-even scalar
- Mass measured with **permille** precision
- Production and decay channels all compatible with SM predictions
- Experimental uncertainties in the 10-20% range

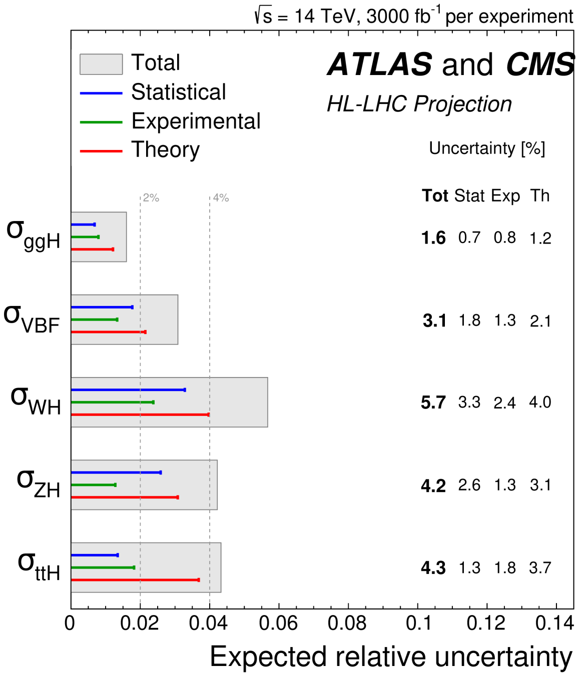


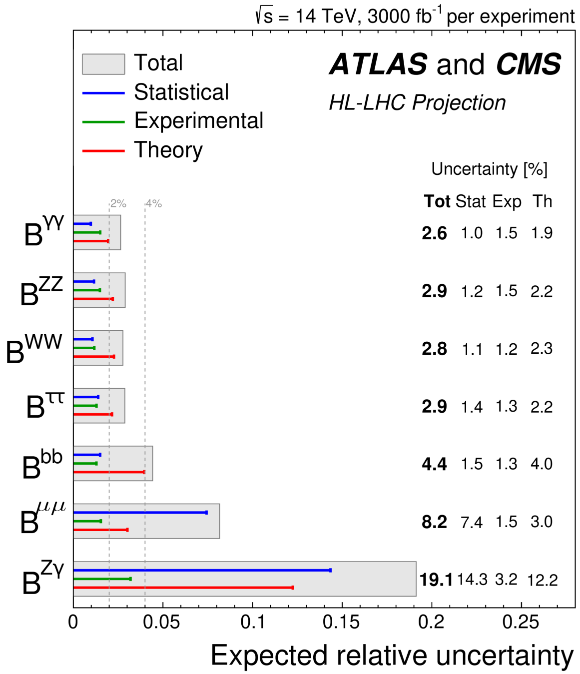


[ATLAS-2207.00092]

What next? Projections for High-Luminsoity LHC

- Systematic uncertainties will play important role





[Cepeda et al. - 1902.00134]

- Theory uncertainties need to be reduced $\Rightarrow$ Improve accuracy of SM predictions

GOAL : **percent precision**

Theory uncertainties

- Parametric uncertainties
- PDF determination
- Matching with parton showers
- Missing higher orders (MHO) in perturbative calculations [THIS TALK]
  Conventionally estimated by varying renormalization and factorization scales

Compute (multi-)loop Feynman diagrams ⇒ Reduce MHO uncertainties

$$\sigma = \sum_{ij} \int dx_1 dx_2 \; f_i(x_1, \mu_F) f_j(x_2, \mu_F) \; \hat{\sigma}_{ij}(x_1, x_2, Q, \mu_F, \mu_R) + \mathcal{O}\left(\Lambda_{QCD}/Q\right)$$

$$\hat{\sigma}_{ij}(\mu_F, \mu_R) = \alpha_S^k(\mu_R) \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)}(\mu_F, \mu_R) \alpha_S^m(\mu_R)$$

Where to look for improvements?

- Les Houches precision wishlist [Huss et al. - 2207.02122]

Table 1. Precision wish list: Higgs boson final states. $\mathrm{N}^x\mathrm{LO}_{\mathrm{QCD}}^{(\mathrm{VBF}*)}$ means a calculation using the structure function approximation. $V = W, Z$.

Process	Known	Desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO}^{(t)}_{\mathrm{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.) $\mathrm{NNLO}^{(b,c)}_{\mathrm{QCD}}$
$pp \to H + j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to H + 2j$	$\mathrm{NLO_{HTL}} \otimes \mathrm{LO_{QCD}}$ $\mathrm{N^3LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ (incl.) $\mathrm{NNLO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ $\mathrm{NLO}^{(\mathrm{VBF})}_{\mathrm{EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{N^3LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ $\mathrm{NNLO}^{(\mathrm{VBF})}_{\mathrm{QCD}}$
$pp \to H + 3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO}^{(\mathrm{VBF})}_{\mathrm{QCD}}$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to VH$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO}^{(t,b)}_{gg\to HZ}$	
$pp \to VH + j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Table 3. Precision wish list: vector boson final states. $V = W, Z$ and $V', V'' = W, Z, \gamma$. Full leptonic decays are understood if not stated otherwise.

Process	Known	Desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD}} + \mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $+ \mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V + j$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$

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$pp \to H+j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to H+2j$	$\mathrm{NLO_{HTL}} \otimes \mathrm{LO_{QCD}}$ $\mathrm{N^3LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ (incl.) $\mathrm{NNLO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ $\mathrm{NLO}^{(\mathrm{VBF})}_{\mathrm{EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{N^3LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ $\mathrm{NNLO}^{(\mathrm{VBF})}_{\mathrm{QCD}}$
$pp \to H+3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO}^{(\mathrm{VBF})}_{\mathrm{QCD}}$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
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$pp \to VH+j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$

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Process	Known	Desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD}} + \mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
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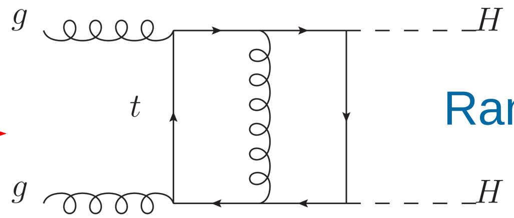
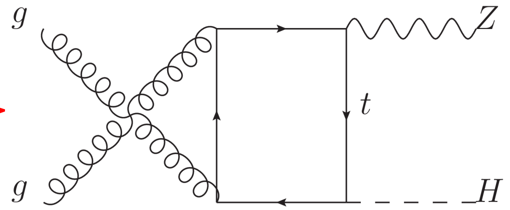
Where to look for improvements?

Process	Known	Desired
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$



Ramona's talk

Process	Known	Desired
$pp \to VH$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO}^{(t,b)}_{gg\to HZ}$	



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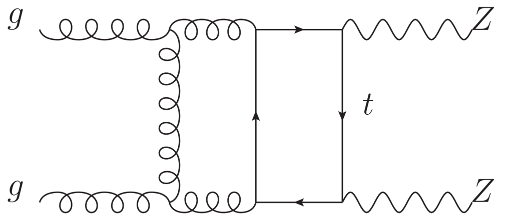
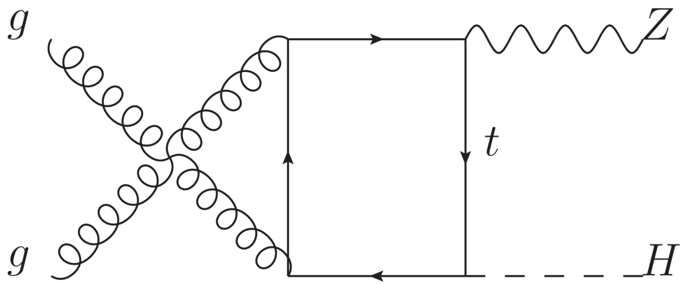


Gluon-initiated 2→2 processes
Two-loop diagrams with **massive** internal lines

Main problem in the NLO calculation
Multi-scale (m_Z, m_H, m_t, s, t) two-loop integrals
No full analytic results

$gg \rightarrow ZH$



Solutions

Numerical Evaluation [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]

- 🟢 Exact results
- 🔴 Demanding in terms of computing resources and time
- 🔴 Issues with flexibility

Analytic Approximations

🟢 Reduce the number of scales in the integrals by exploiting **hierarchies** of masses/kinematic invariants

🔴 Proliferation of integrals
🔴 Restricted to specific phase space regions

- Limit $m_t \to \infty$
  [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.50]
- Large mass expansion: add finite top-mass effects
  [Hasselhuhn, Luthe, Steinhauser - 1611.05881]
- High-energy expansion: $m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$
  [Davies, Mishima, Steinhauser - 2011.12314]
- Small-mass expansion: $m_Z, m_H \to 0$
  [Wang, Xu, Xu, Yang - 2107.08206]
- pT expansion: $m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}$
  [Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]
  [Bonciani, Degrassi, Giardino, Groeber - 1806.11564]

pT Expansion: calculation overview

Steps implemented in Mathematica code on a desktop machine

(1) Generation of Feynman diagrams contributing to the amplitude O(100 diags) (FeynArts [Hahn - 0012260])

(2) Lorentz decomposition of the amplitude: **projectors** and **scalar form factors** (FeynCalc [Mertig et al. ('91) ; Shtabovenko et al. - 1601.01167]): contractions, Dirac traces...

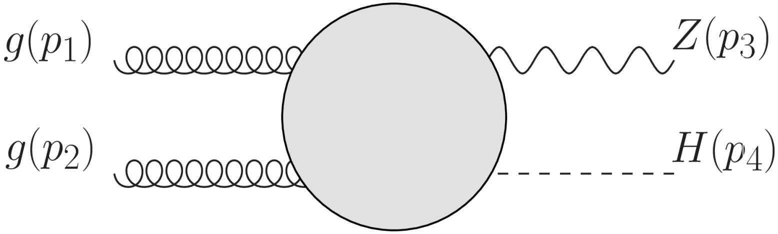
$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^{6} \mathcal{P}^{(i)}_{\mu\nu\rho} F^{(i)} \qquad F^{(i)} = \sum_{i=1}^{n} C^{(i)} \; I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

(3) Expansion of the form factors in the limit of small pT

(4) Decomposition of scalar integrals using integration-by-parts (IBP) identities (LiteRed [Lee - 1310.1145])

(5) Evaluation of master integrals [see talks by Henrik, Manoj, Giulio, Giacomo...]

pT Expansion



- We assume the limit of a **forward kinematics**

$$(p_1+p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1 \qquad \frac{p_T^2}{4m_t^2} \ll 1$$

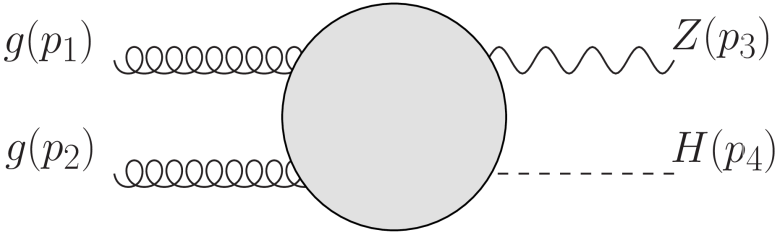
- After the expansion, scalar loop integrals depend on fewer scales

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2)$$

- The new scalar integrals are decomposed in MIs using IBP relations
- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **only one scale**

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2) \to \mathrm{MI}(\hat{s}/m_t^2)$$

pT Expansion



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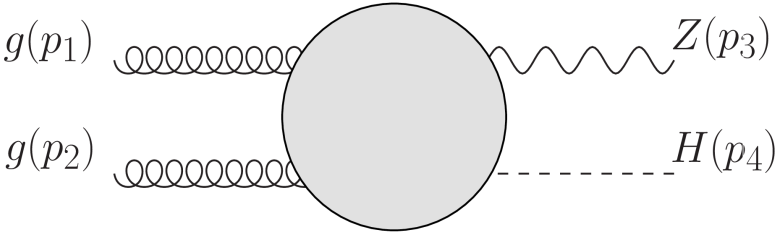
pT Expansion



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IBP Reduction

Major bottleneck in the computation

For first **three orders** in pT expansion (good for % accuracy)

- Search for the IBP reduction rules → O(week)
- Very large intermediate expressions → O(10 GB)
- Few MIs but huge coefficients → O(GB)
- Massaging and series expanding in ϵ O(week) (D= 4-2ϵ)

Still, size of the final results → O(100 kB)

Master Integrals

52 MIs already known in the literature

SAME MIs FOR $gg \to HH$, $gg \to ZH$, $gg \to ZZ$

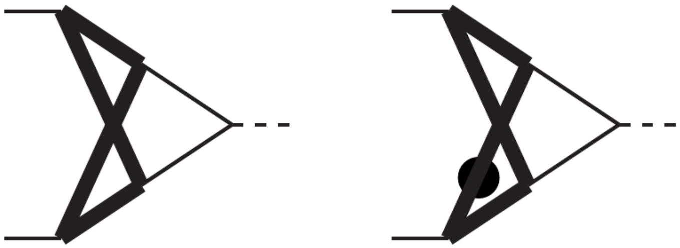
- 50 MIs expressed in terms of Generalized Polylogarithms (GPLs)

  [Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

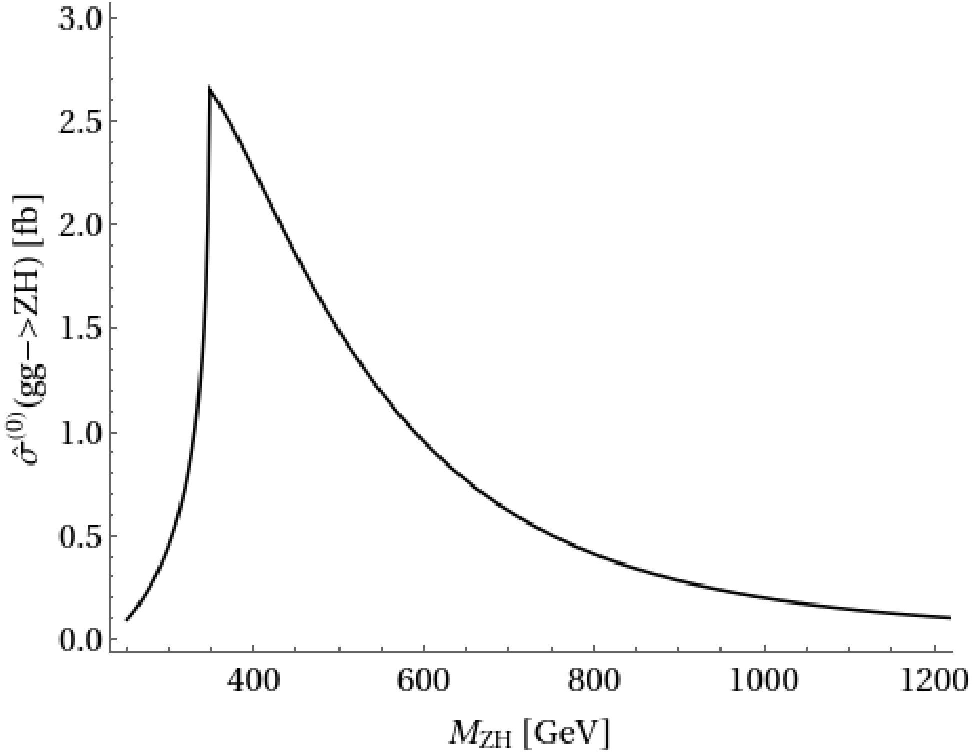
- Two elliptic integrals [von Manteuffel, Tancredi ('17)]

  Semi-analytical evaluation implemented in FORTRAN routine

  [Bonciani, Degrassi, Giardino, Gröber ('18)]



Comparing Validity Ranges



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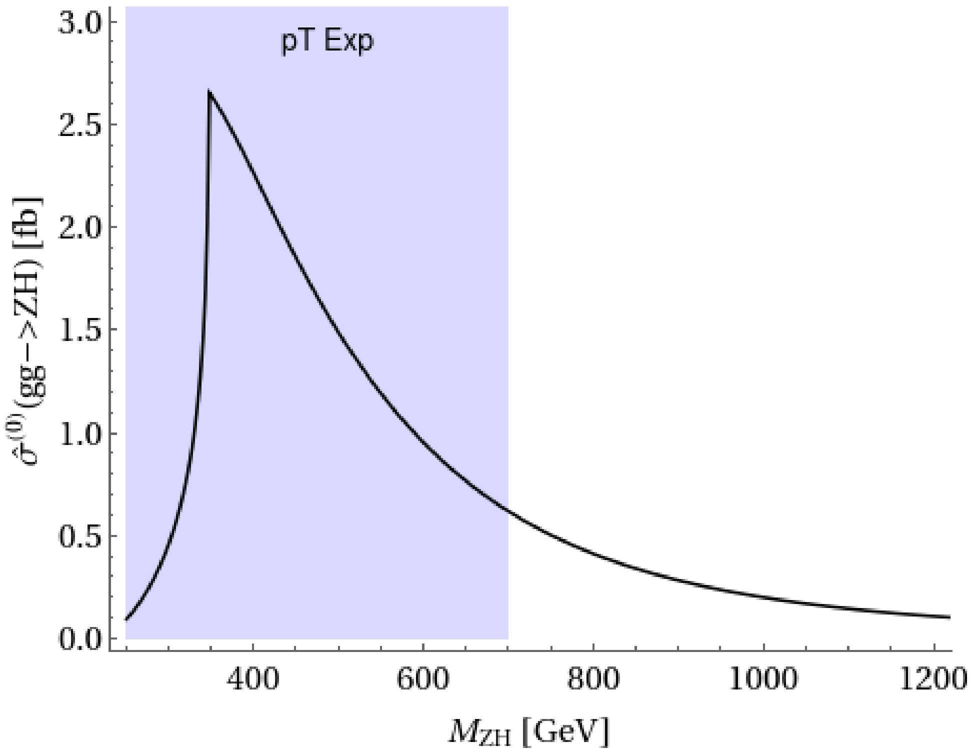


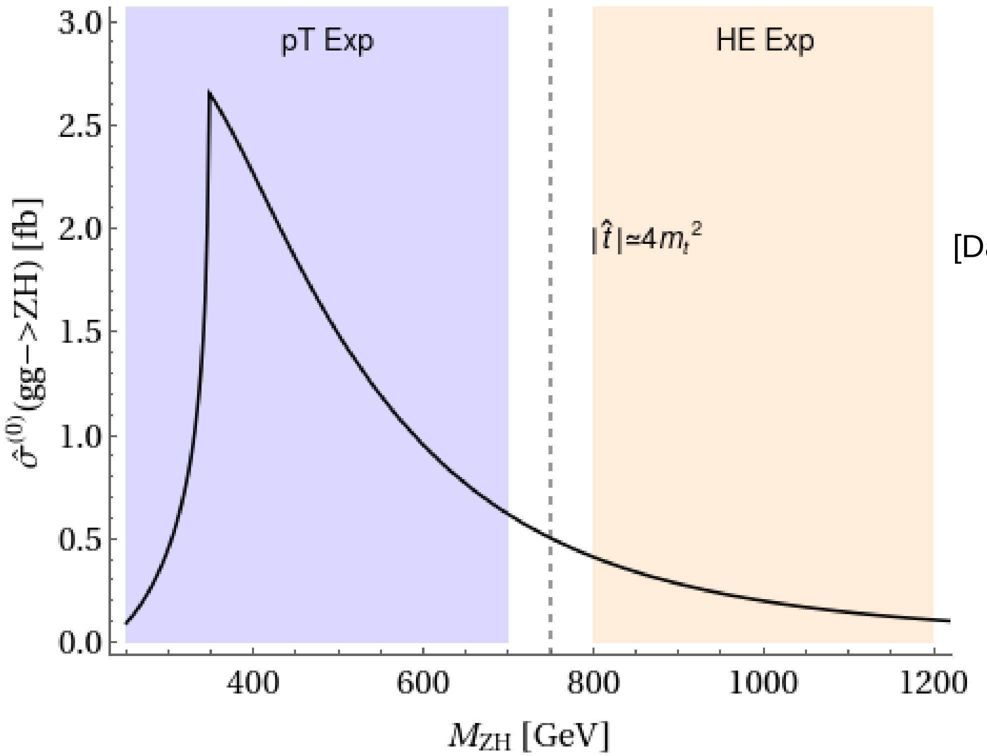
Comparing Validity Ranges

- p_T exp

  $p_T^2 \lesssim 4m_t^2$

  or

  $\hat{t} \lesssim 4m_t^2$



[Davies, Mishima, Steinhauser - 2011.12314]

- high-energy exp

  $\hat{t} \gtrsim 4m_t^2$

The two expansions can be combined!!

Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo

$gg \to ZH$ @NLO QCD - Top mass schemes

- Take deviations of $\overline{\rm MS}$ scheme wrt OS result as top mass scheme uncertainty

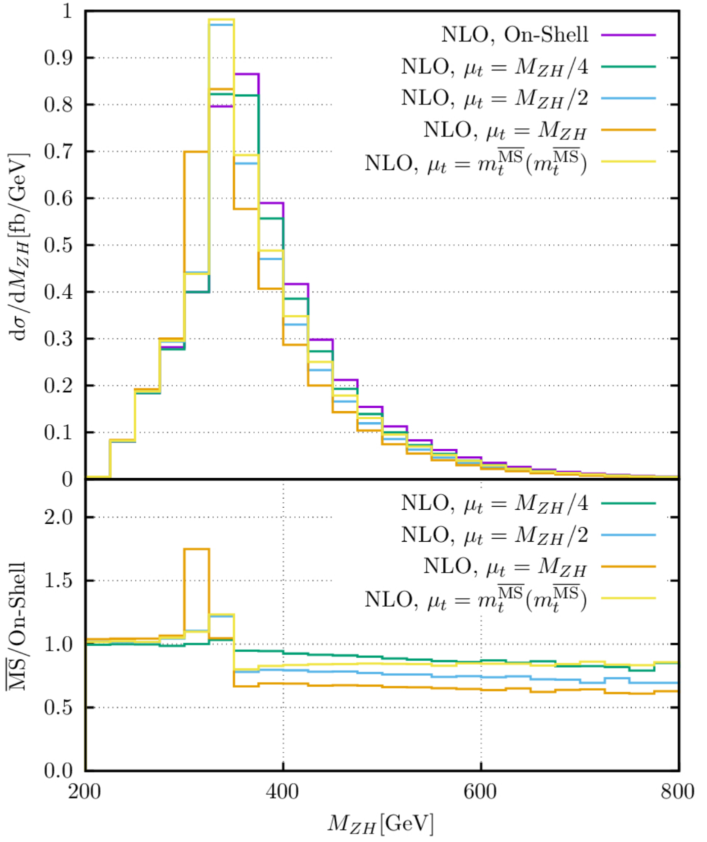
- Analytic results → change of top mass scheme is straightforward

$$F_i^{NLO,\overline{\rm MS}} = F_i^{NLO,{\rm OS}} - \frac{1}{4}\frac{\partial F_i^{LO}}{\partial m_t^2}\Delta_{m_t^2} \qquad \Delta_{m_t^2} = 2m_t^2 C_F\left[-4 + 3\log\left(\frac{m_t^2}{\mu^2}\right)\right]$$

- Same method already used for HH production
  [Baglio et al. - 1811.05692, 2003.03227]

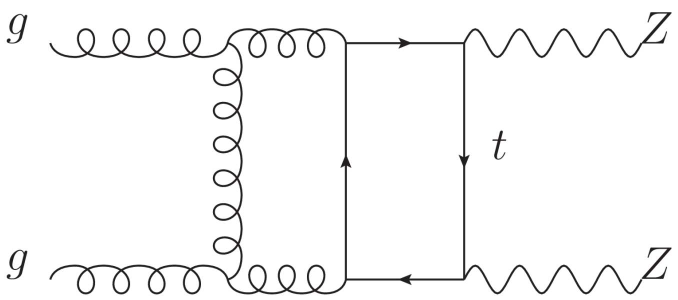
Bin Width [GeV]	LO	NLO
1	$64.01^{+15.6\%}_{-35.9\%}$	$118.6^{+17.2\%}_{-27.0\%}$
5	$64.01^{+15.3\%}_{-35.6\%}$	$118.6^{+14.7\%}_{-24.9\%}$
25	$64.01^{+14.0\%}_{-33.1\%}$	$118.6^{+10.9\%}_{-20.8\%}$
100	$64.01^{+2.0\%}_{-25.3\%}$	$118.6^{+0.6\%}_{-13.7\%}$
∞	$64.01^{+0\%}_{-23.1\%}$	$118.6^{+0\%}_{-12.9\%}$

Avoid overestimate of mt uncertainty



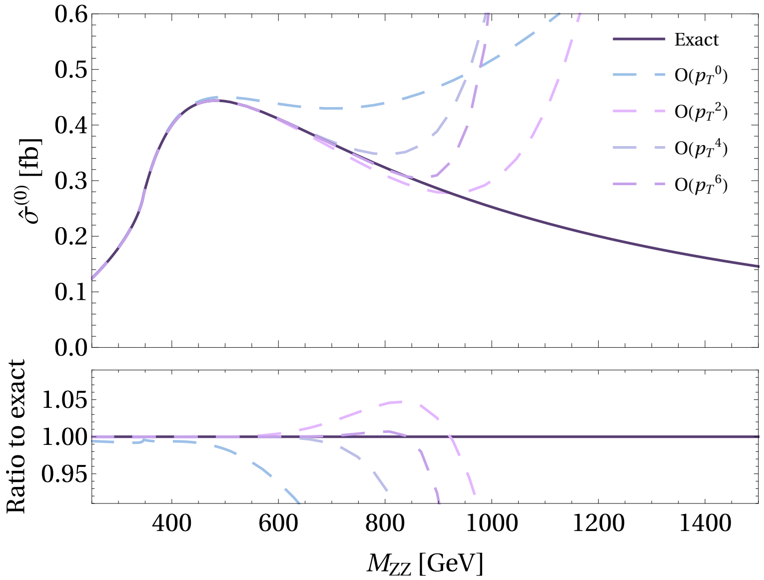
[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \rightarrow ZZ$



p_T expansion for $gg \to ZZ$

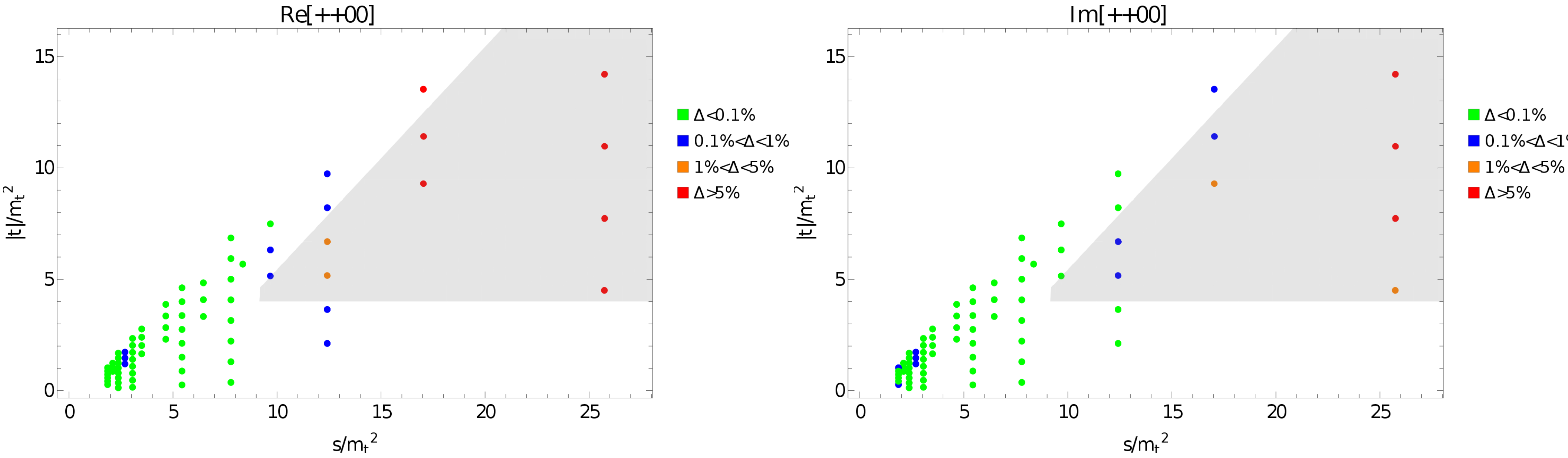
- More involved Lorentz structure → 16 form factors
- More involved intermediate expressions
- ~ 750.000 scalar integrals per form factor
- IBP leads to same 52 MIs as HH and ZH
- Permille accuracy at LO with three orders



Helicity amplitudes at NLO

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \boxed{\mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}} + \mathcal{O}(\alpha_s^3)$$

[Agarwal, Jones, von Manteuffel - 2011.15113]



[PRELIMINARY]

Conclusions & outlook

- Higgs precision measurements call for improved theoretical predictions
- $2 \to 2$ processes with **massive** loops are hard
- Analytic approximations are useful for flexibility and efficiency
- Found a way to combine pT and high-energy expansions
  see also [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

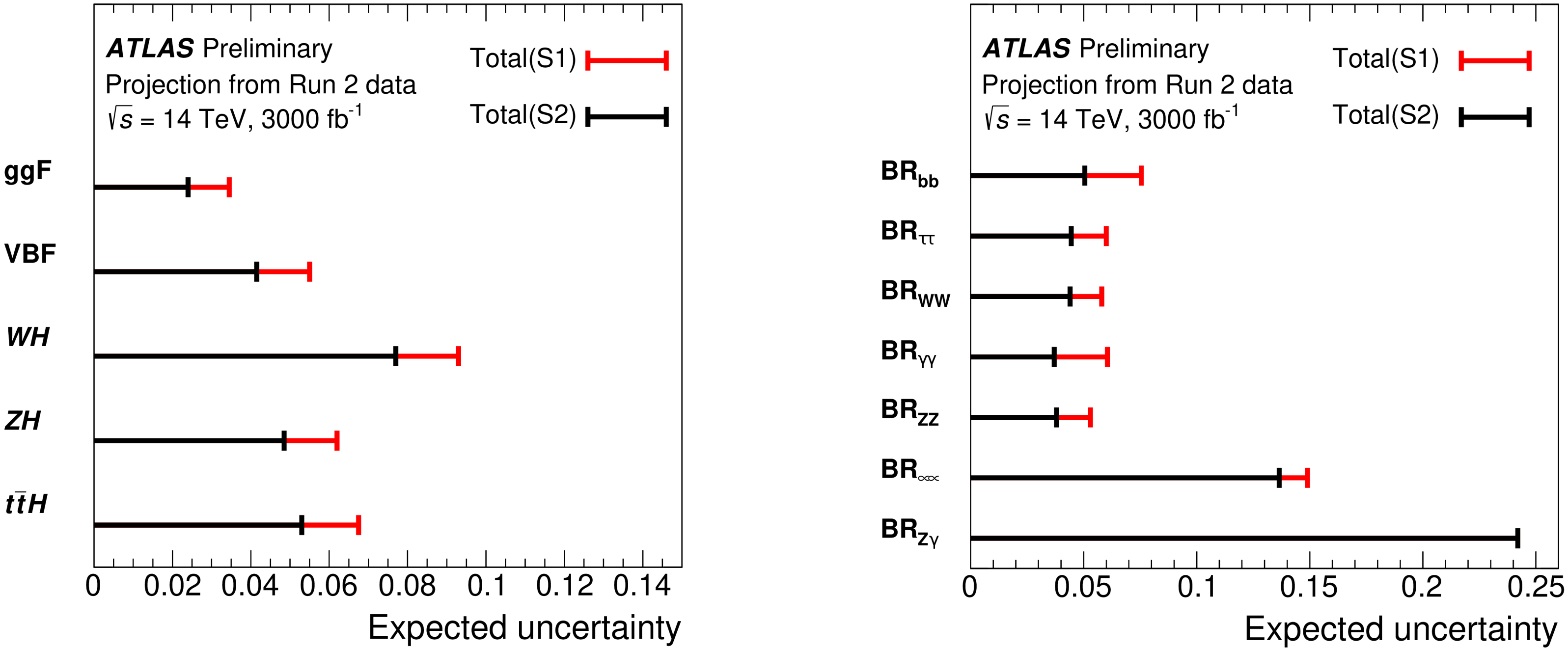
- Is NLO QCD sufficient for $gg \to ZH$? (is **3-loop** feasible?)
- $gg \to WW$? How to deal with both heavy and light quarks running in the loops
- EW corrections to $2 \to 2$ processes? Possibly different master integrals

We're gonna need New Ideas and Lots of RAM

Backup

What next? Projections for High-Luminsoity LHC

- Systematic uncertainties will begin to dominate



[ATLAS-PHYS-PUB-2018-0548-054]

- Scenario 1: systematics as in Run2 (conservative)
- Scenario 2: exp sys corrected; theo sys halved

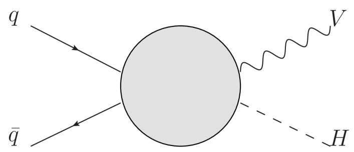
VH Production

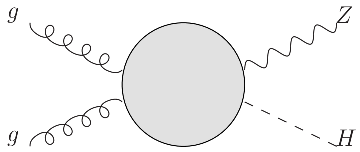
$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Two partonic channels in *pp->ZH*:

  $q\bar{q} \to ZH$ - dominant contribution

- $gg \to ZH$ - about 6% of $\sigma(pp \to ZH)$





- Theory prediction in MC codes:

  $q\bar{q} \to ZH$: NNLO accuracy [Han, Willenbrock- '91] [Brein, Djouadi, Harlander- 0307206]

  $gg \to ZH$: LO accuracy → Large scale uncertainties

Production mode	$\Delta_y^{<VH>}$
WH	$\pm 0.7\%$
$q\bar{q} \to ZH$	$\pm 0.6\%$
$gg \to ZH$	$\pm 25\%$

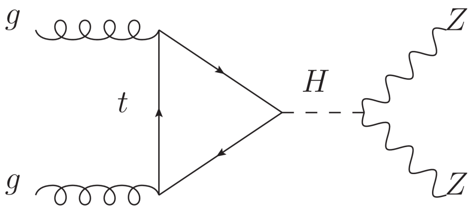
(No *gg*-channel for WH)

[CERN Yellow Report 4]

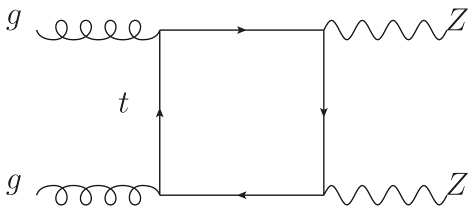
If we really want to improve the theory prediction **we need to go beyond LO in $gg \to ZH$**

ZZ Production

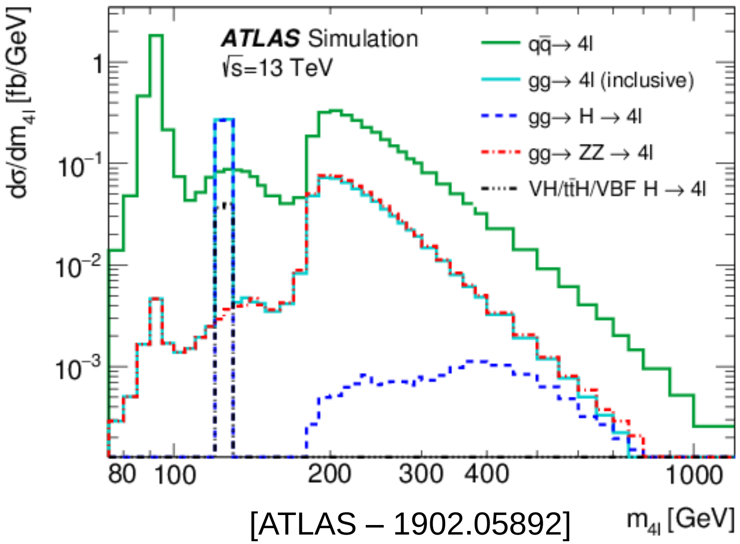
- $pp \to ZZ$ provides access to **single-Higgs** production via gluon fusion
- $q\bar{q} \to ZZ$ gives dominant contribution to hadronic cross section
- $gg \to ZZ$ is about 10% of $\sigma(pp \to ZZ)$



(Higgs-mediated)



(continuum)
Irreducible background



[ATLAS – 1902.05892]

- Knowledge of the background is important for **Higgs width** determination via off-shell measurements

$$gg \to ZH$$

Why $gg \to ZH$? VH Production at the LHC

$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]
(work in progress on $H \to c\bar{c}$ [ATLAS-2201.11428, CMS-2205.05550])

- O(pb) cross sections
- $W \to \ell\nu$
  $Z \to \ell\ell$ and $Z \to \nu\nu$
- VH theory uncertainties [Cepeda et al. - 1902.00134]

$pp \to W^+ H$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.831	$^{+0.74}_{-0.73}$	1.79
14	0.913	$^{+0.64}_{-0.76}$	1.78
27	1.995	$^{+0.43}_{-1.04}$	1.84

$pp \to ZH$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.880	$^{+3.50}_{-2.68}$	1.65
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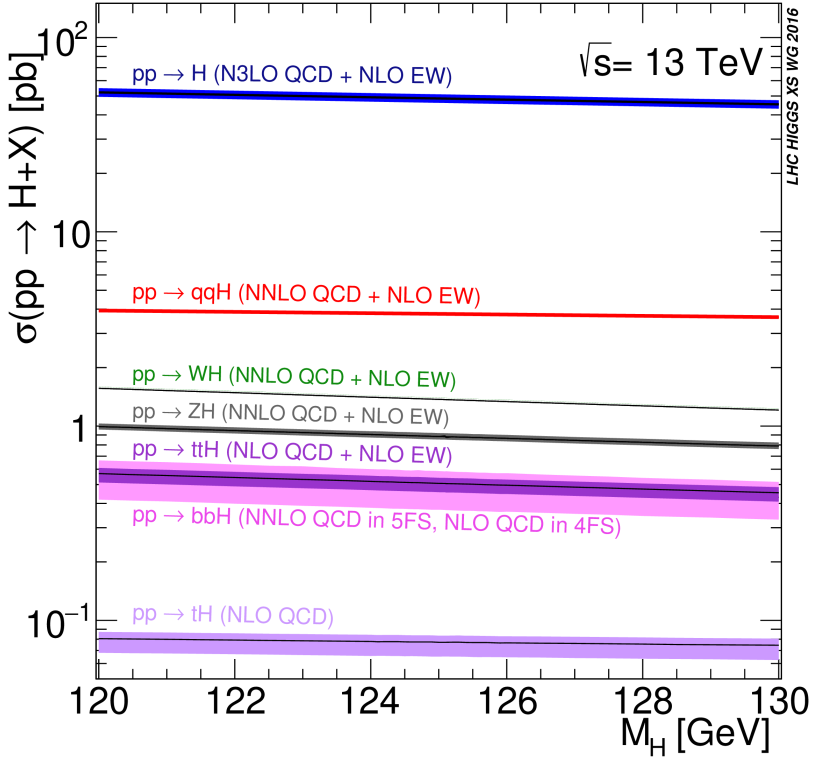

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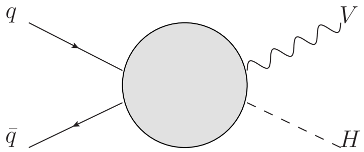
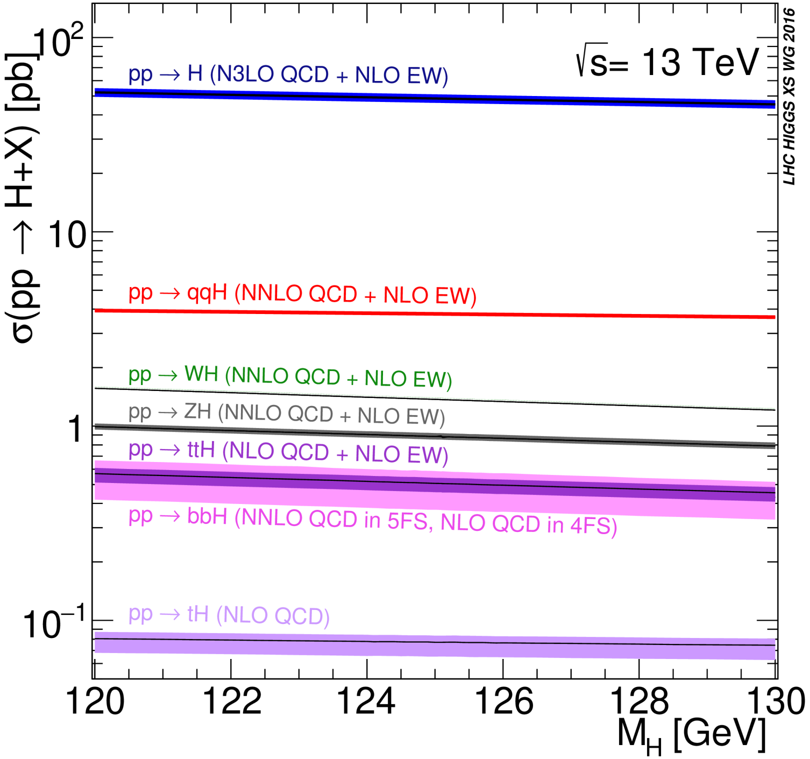


$pp \to ZH$



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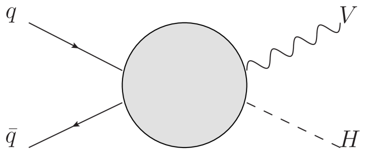
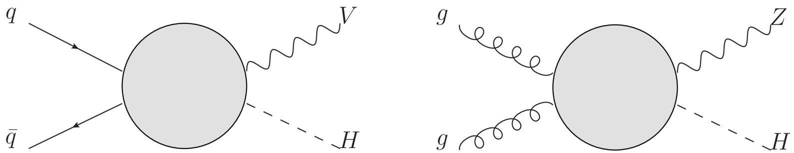
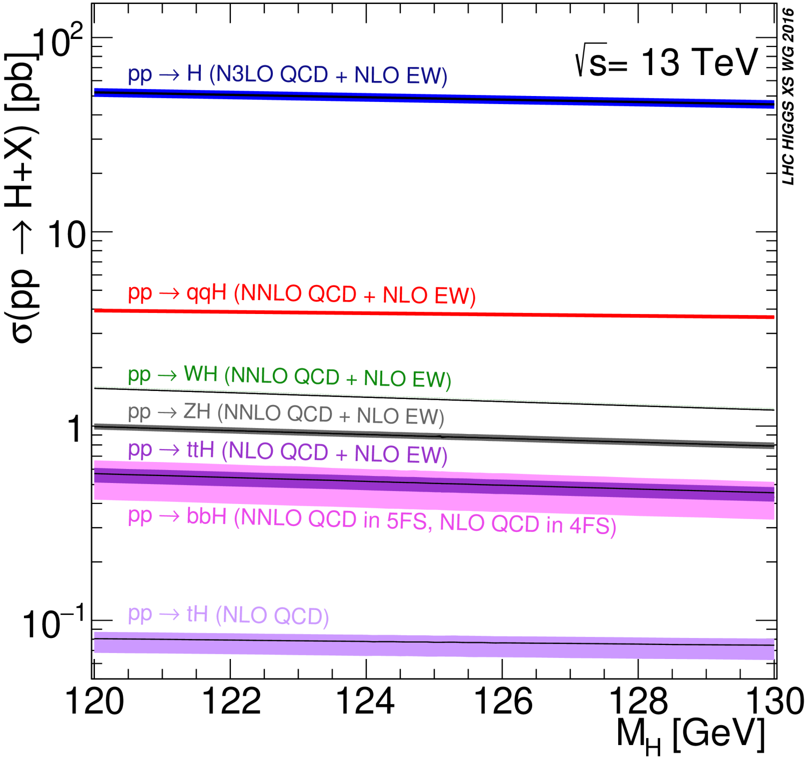


$pp \to ZH$

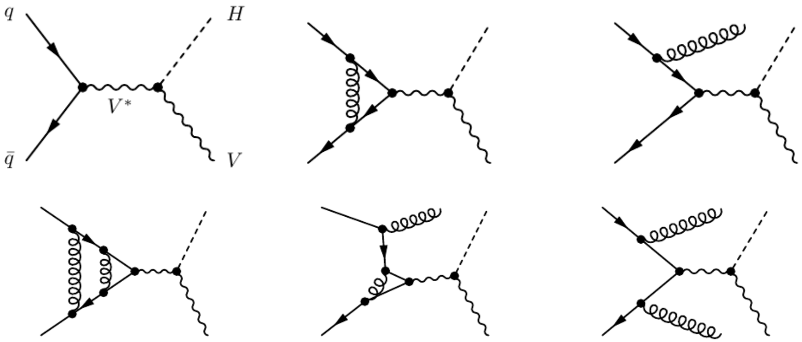




LO: quark-initiated tree-level contribution

QCD Effects: mainly due to Drell-Yan (DY) production followed by $Z^* \to ZH$ decay

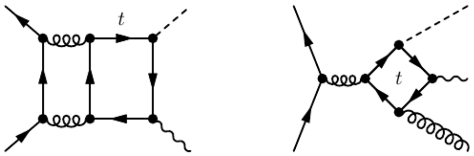
- **Drell-Yan:**



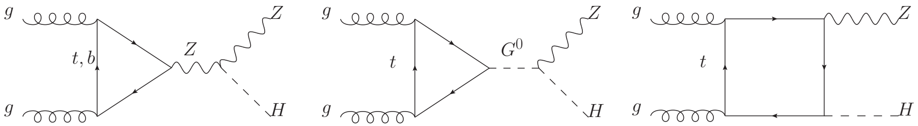
Known through NNLO (O(α_s^2))
(+30% wrt LO) [Han, Willenbrock ('91) ; Hamberg, van Neerven, Matsuura ('92) ; Brein, Djouadi, Harlander - 0307206]

- **Non Drell-Yan:**

Quark-initiated O(1%) wrt LO
[Brein, Harlander, Wiesemann, Zirke - 1111.0761]



Gluon-initiated



- **EW corrections:** known through NLO (-(5-10%) wrt LO) [Dittmaier et al. - 1211.5015]

Lorentz Structure and Projectors

- General expression for the $gg \to ZH$ amplitude

$$\mathcal{M} = i\sqrt{2} m_Z G_F \frac{\alpha_s(\mu_R)}{\pi} \delta^{ab} \mathcal{A}_{\mu\nu\rho}(p_1, p_2, p_3) \epsilon^{\mu}(p_1) \epsilon^{\nu}(p_2) \epsilon^{*\rho}(p_3)$$

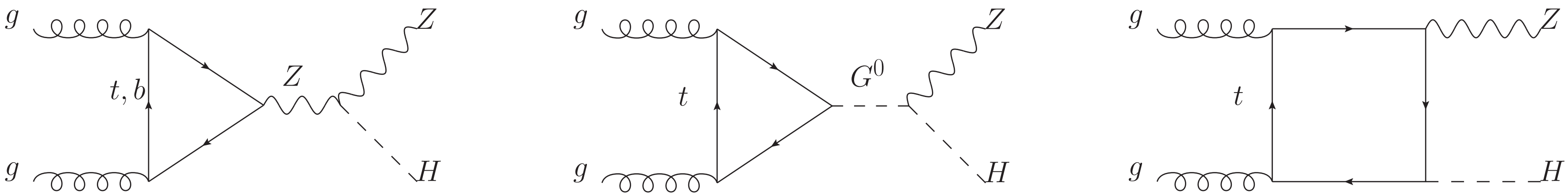
- Lorentz tensor $\mathcal{A}_{\mu\nu\rho}$ decomposed using six projector multiplying **scalar** form factors

$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^{6} \mathcal{P}^{(i)}_{\mu\nu\rho} F^{(i)}$$

- Form factors are linear combinations of scalar loop integrals

$$F^{(i)} = \sum_{i=1}^{n} C^{(i)} \; I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

$gg \rightarrow ZH$ @ LO



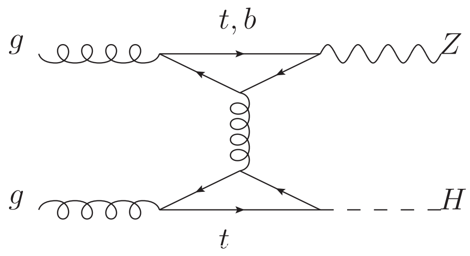
- Third generation gives dominant contribution [Kniehl ('90) - Dicus, Kao ('88)]
- $\mathcal{O}(\alpha_s^2)$ correction to $pp \rightarrow ZH$ cross section
- NNLO suppression wrt to $q\bar{q} \rightarrow ZH$ but gluon luminosity higher at LHC
- Contributes to about 6% of $\sigma(pp \rightarrow ZH)$ for $\sqrt{s} = 14$ TeV
- Only LO included in MC $\rightarrow$ scale variation leads to **25%** relative uncertainties
- NLO corrections expected to be large in *gg* processes (e.g. *H, HH*)

[Cepeda et al. - 1902.00134]

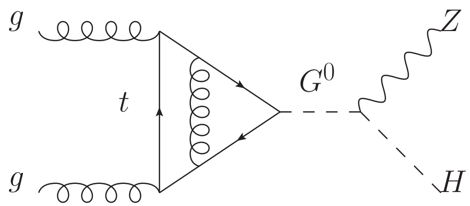
$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.123	$^{+24.9}_{-18.8}$	4.37
14	0.145	$^{+24.3}_{-19.6}$	7.47
27	0.526	$^{+25.3}_{-18.5}$	5.85

$gg \to ZH$ @ NLO QCD: Virtual Corrections

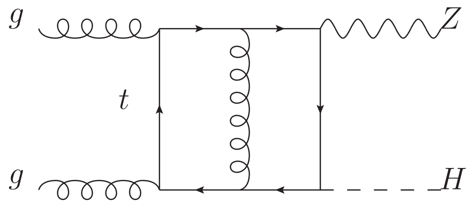
- Double-triangle $\rightarrow$ standard Passarino-Veltman technique



- 1PI triangle $\rightarrow$ known
  [Spira et al. - 9504378 ; Aglietti et al. - 0611266 ; Altenkamp et al. - 1211.5015]



- Two-loop boxes $\rightarrow$ very hard



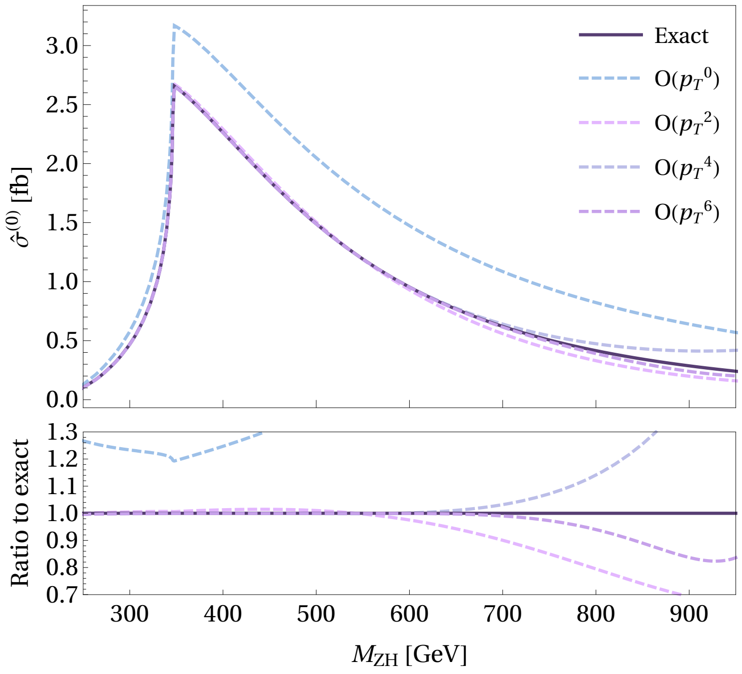
Main problem in the NLO calculation
Multi-scale (m_Z, m_H, m_t, s, t) two-loop integrals
No full analytic results

LO Validation

- Three orders sufficient for very good accuracy
- Reliable results for $M_{ZH} \lesssim 700 \text{ GeV}$
- For $M_{ZH} \gtrsim 700 \text{ GeV}$ the assumption

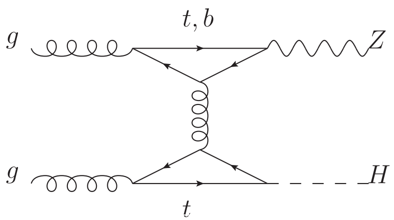
$$p_T^2 \ll 4m_t^2$$

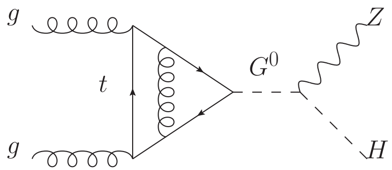
can be violated $\Rightarrow$ the p_T expansion **diverges** (but wait a few slides...)

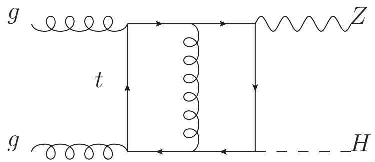


$gg \to ZH$ @ NLO in QCD: all ingredients

Virtual corrections ($2 \to 2$, two loops): merging pt+HE expansions





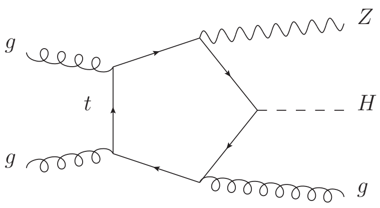


Real emission ($2 \to 3$, one loop): automated evaluation (RECOLA2, MadGraph5)

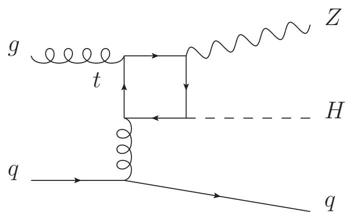
[Denner, Lang, Uccirati - 1711.07388]
[Alwall et al. - 1405.0301]

We included all diagrams that:

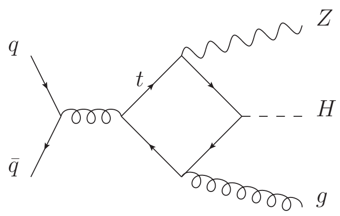
- give O(α_s^3) contribution to the cross section *pp->ZH*
- feature a closed fermion loop



$gg \to ZHg$



$qg \to ZHq$



$q\bar{q} \to ZHg$

Full NLO QCD Results

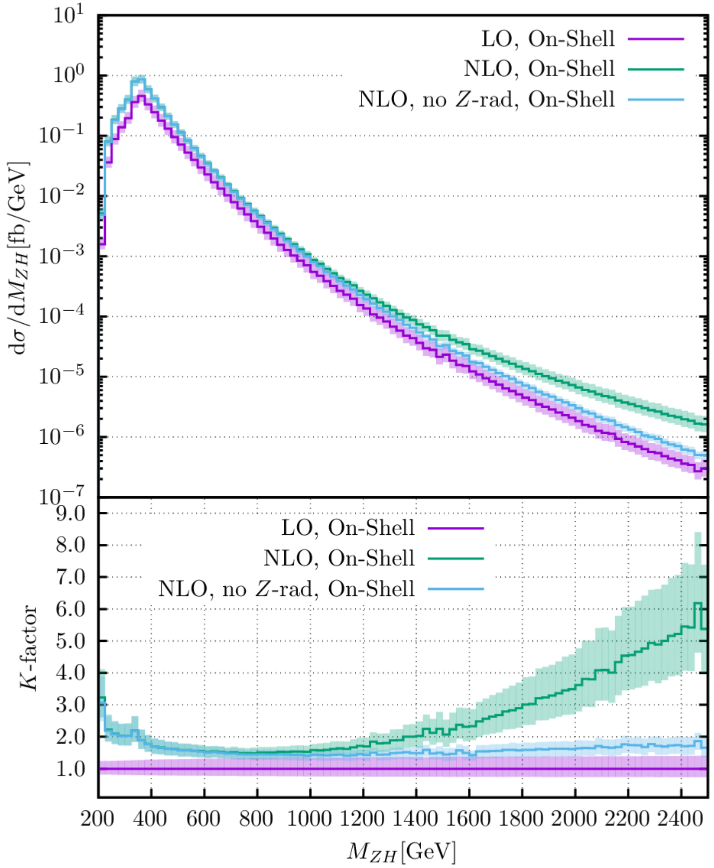
Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K=\sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- Top mass renormalized both in OS and $\overline{\text{MS}}$ scheme
- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 2/3 wrt LO
- Agreement with independent calculations
  [Wang et al. - 2107.08206] [Chen et al. - 2204.05225]

M_{ZH} distribution

- K-factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail (M_{ZH}>1 TeV)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Full NLO QCD Results

Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
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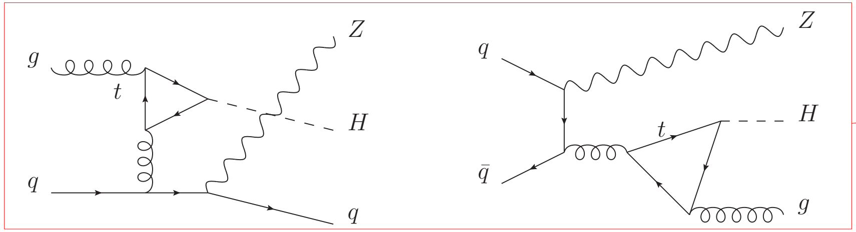
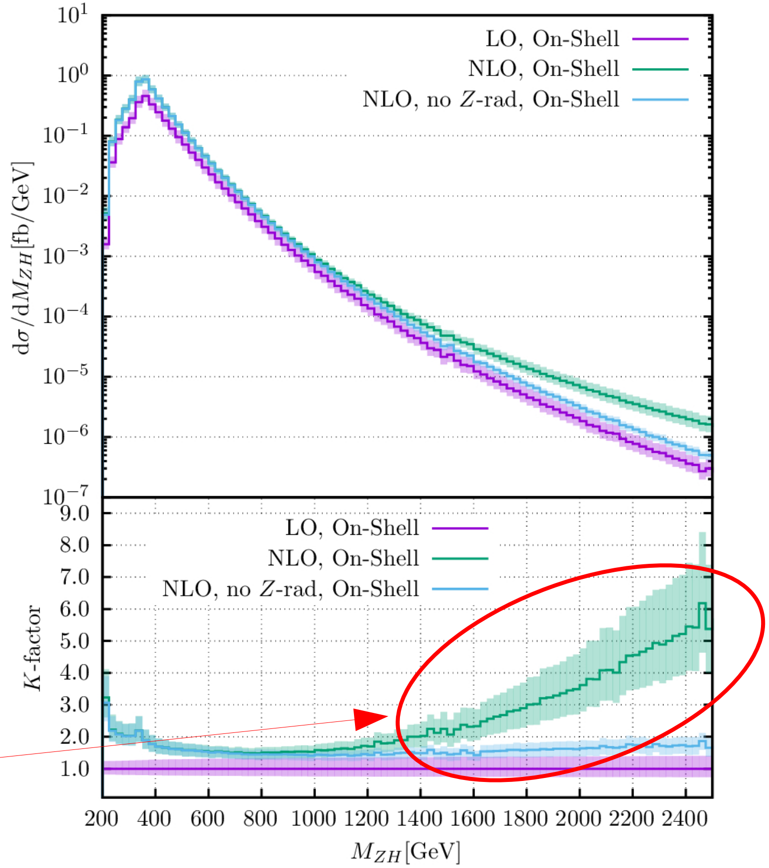
  [Wang et al. - 2107.08206] [Chen et al. - 2204.05225]

Z-radiated diagrams

- Large EW Logs?

$$\log^2\left(\frac{m_Z^2}{M_{ZH}^2}\right)$$

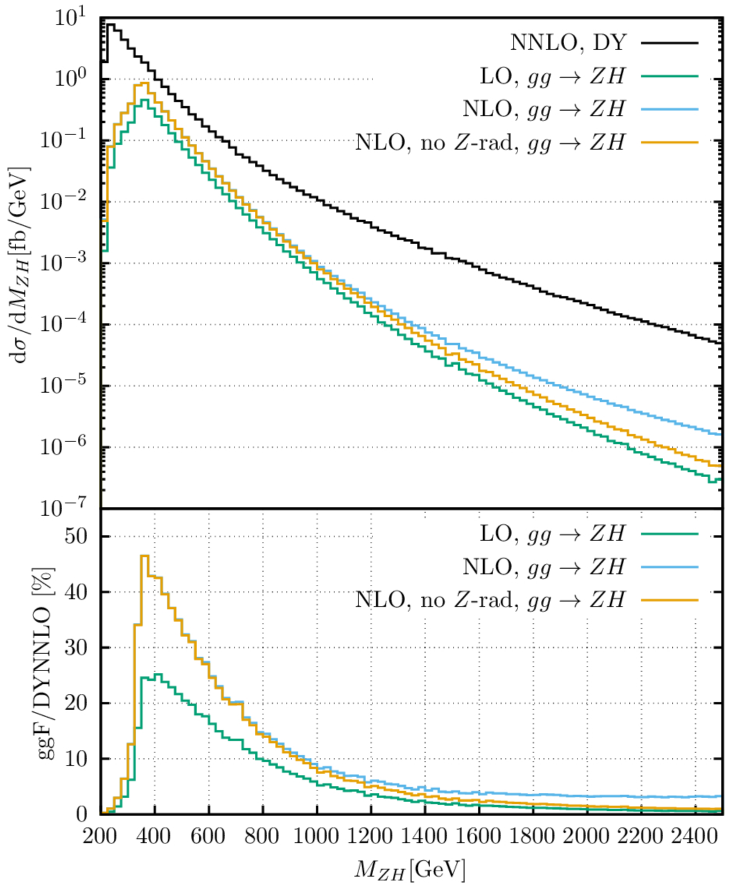




[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \to ZH$ @ NLO: comparing with Drell-Yan contribution

- ***gg → ZH*** is almost 50% of DY near $M_{ZH} \sim 2\, m_t$
- Because of Z -radiated diagrams the gg contribution falls off as rapidly as the DY one (ratio constant at ~ 2%)
- DY obtained using **vh@nnlo**
  [Harlander et al - 1802.04817]



[Degrassi, Gröber, MV, Zhao - 2205.02769]

The impact of $gg \to ZH$

Table 10: Cross-section for the process $pp \to ZH$. The predictions for the $gg \to ZH$ channel are computed at LO, rescaled by the NLO K-factor in the $m_t \to \infty$ limit, and supplemented by the $\mathrm{NLL_{soft}}$ resummation. The photon contribution is omitted. Results are given for a Higgs boson mass $m_H = 125.09$ GeV.

$\sqrt{s}$ [TeV]	$\sigma_{\mathrm{NNLO\ QCD \otimes NLO\ EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\mathrm{PDF}\oplus\alpha_s}$ [%]
13	0.880	$^{+3.50}_{-2.68}$	1.65
14	0.981	$^{+3.61}_{-2.94}$	1.90
27	2.463	$^{+5.42}_{-4.00}$	2.24

Table 11: Cross-section for the process $pp \to ZH$. The photon and $gg \to ZH$ contributions are omitted. Results are given for a Higgs boson mass $m_H = 125.09$ GeV.

$\sqrt{s}$ [TeV]	$\sigma_{\mathrm{NNLO\ QCD \otimes NLO\ EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\mathrm{PDF}\oplus\alpha_s}$ [%]
13	0.758	$^{+0.49}_{-0.61}$	1.78
14	0.836	$^{+0.51}_{-0.62}$	1.82
27	1.937	$^{+0.56}_{-0.74}$	2.37

Table 12: Cross-section for the process $gg \to ZH$. Predictions are computed at LO, rescaled by the NLO K-factor in the $m_t \to \infty$ limit, and supplemented by the $\mathrm{NLL_{soft}}$ resummation. Results are given for a Higgs boson mass $m_H = 125.09$ GeV.

$\sqrt{s}$ [TeV]	$\sigma_{\mathrm{NNLO\ QCD \otimes NLO\ EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\mathrm{PDF}\oplus\alpha_s}$ [%]
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γ^5 in Dimensional Regularization

- When $D \neq 4$ cannot have both $\{\gamma^\mu, \gamma^5\} = 0$ and cyclicity of Dirac trace

Larin scheme

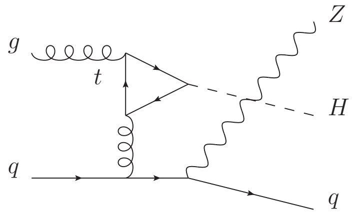
- γ^5 always on the right inside traces
- Substitute $\gamma^\mu \gamma^5 \to -i/6\, \varepsilon^{\mu\alpha\beta\rho} \gamma^\alpha \gamma^\beta \gamma^\rho$
- Contractions of $\varepsilon^{\mu\alpha\beta\rho}$ are taken in *D* dimensions
- Finite renormalization needed to restore Ward Identities for axial-vector current

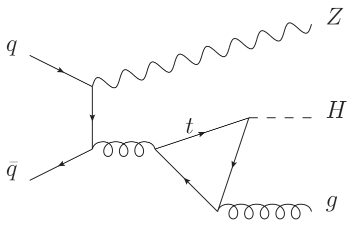
For $gg \to ZH$

$$\mathcal{A}_i^{\mathrm{NLO}} = \mathcal{A}_i^{\mathrm{NLO,\ ndr}} - \frac{\alpha_s}{\pi} C_F \mathcal{A}_i^{\mathrm{LO}}$$

Consistency check by comparing the LME results in **DimReg** and **Pauli-Villars**

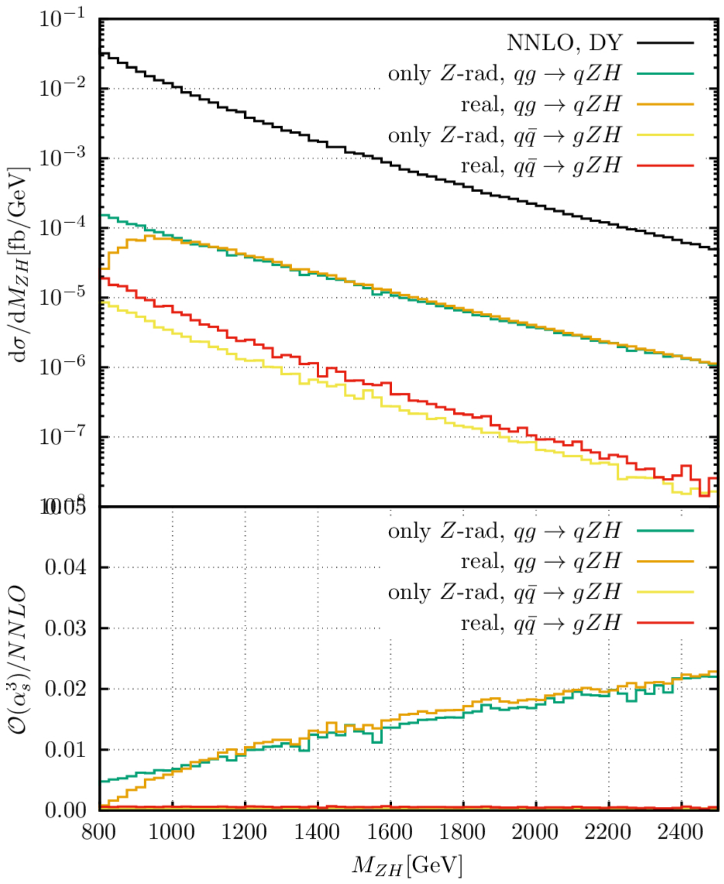
The effect of Z-radiated diagrams





In the high-energy tail (M_{ZH}> 1 TeV)

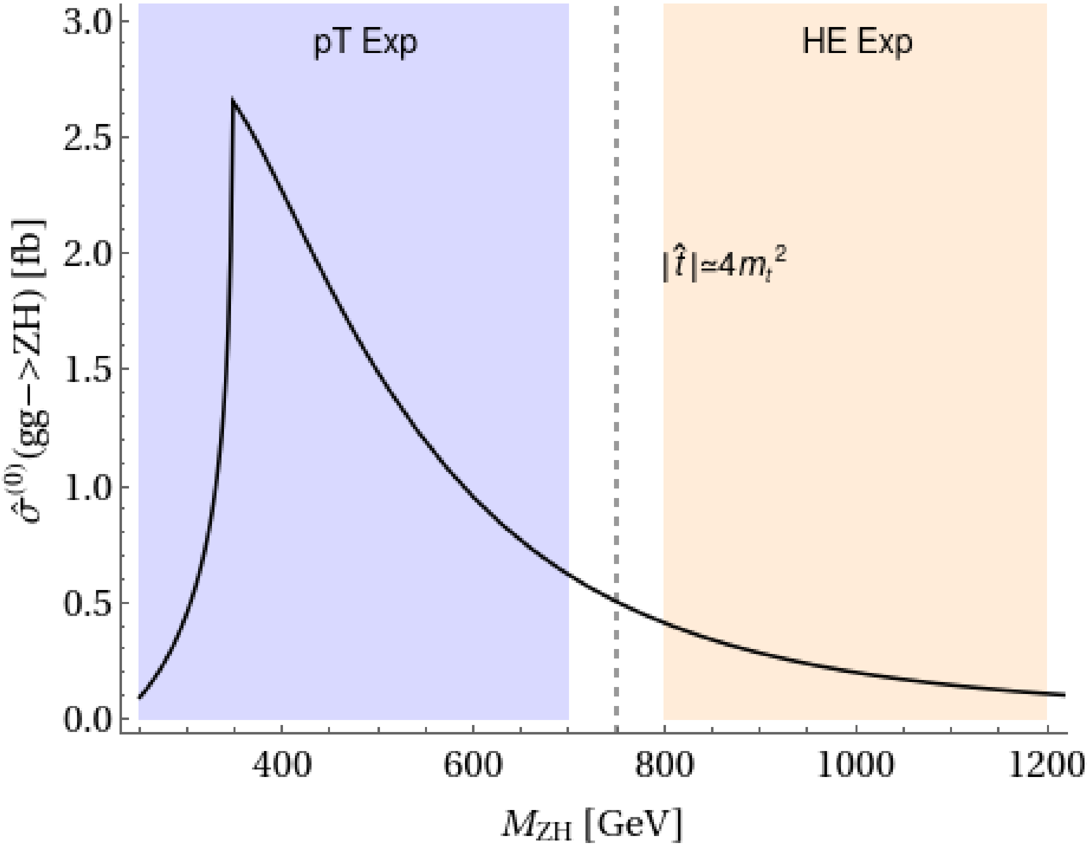
- **qg → ZHq channel**
  - Z-radiated diagrams dominate
  - Non-negligible contribution (up to 2% wrt DY)
- **$q\bar{q}$ → ZHg channel**
  - Z-radiated diagrams dominate
  - Negligible (PDF suppression)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Comparing validity ranges

In the low-energy region you test the SM



In the high-energy region you test BSM

Merging pT and HE expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \quad (q = m + n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

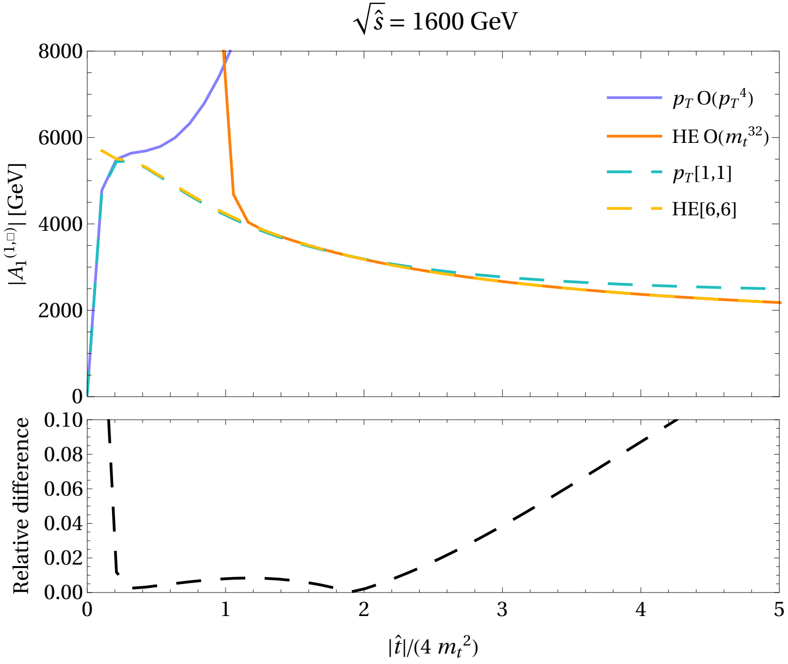
- For each FF we merged the following results

  - pT exp improved by [1/1] Padé

  - HE exp improved by [6/6] Padé

- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2 \rightarrow$ can switch without loss of accuracy (% level or below)

- Evaluation time for a phase-space point below 0.1 $s \Rightarrow$ suitable for Monte Carlo



Integration-by-Parts Reduction

Express a scalar integral as a function of denominator exponents

$$I(n_1, \ldots, n_N) = \int d^D k_1 \cdots d^D k_L \frac{1}{D_1^{n_1} \cdots D_N^{n_N}} \qquad (n_i \in \mathbb{Z})$$

Recurrence relations connecting scalar integrals with different n_i from differentiation

$$\int d^D k_1 \cdots d^D k_L \frac{\partial}{\partial k_i^\mu} \frac{q_j^\mu}{D_1^{n_1} \cdots D_N^{n_N}} = 0$$

The process can be **iterated** $\Rightarrow$ each scalar integral in the amplitude can be decomposed along a basis of master integrals

$$I(n_1, \ldots, n_N) = \sum_j C^{(j)} MI^{(j)}(\mathbf{z_1}, \ldots, \mathbf{z_N}) \qquad z_i \in \{0, 1, 2\}$$

- For $gg \to ZH$ @ NLO: from ~200.000 scalar integrals to 52 MIs
- First simplification with pT expansion $\Rightarrow$ simpler IBP $\Rightarrow$ simpler MIs

pT expansion: example

1) Consider a **one-loop** box integral

$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2}(q\cdot p_2)^{n_3}(q\cdot p_3)^{n_4}}{(q^2-m_t^2)[(q+p_2)^2-m_t^2][(q-p_1-p_3)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; explicit the transverse component wrt beam axis

$$\frac{(q\cdot p_3)^{n_4}}{[(q-p_1-p_3)^2-m_t^2]}$$

$$\begin{aligned} p_3^\mu &= \frac{u'}{s'}p_1^\mu + \frac{t'}{s'}p_2^\mu + r_\perp^\mu \\ &= -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + \frac{\Delta_m}{s'}p_1^\mu + r_\perp^\mu \end{aligned}$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

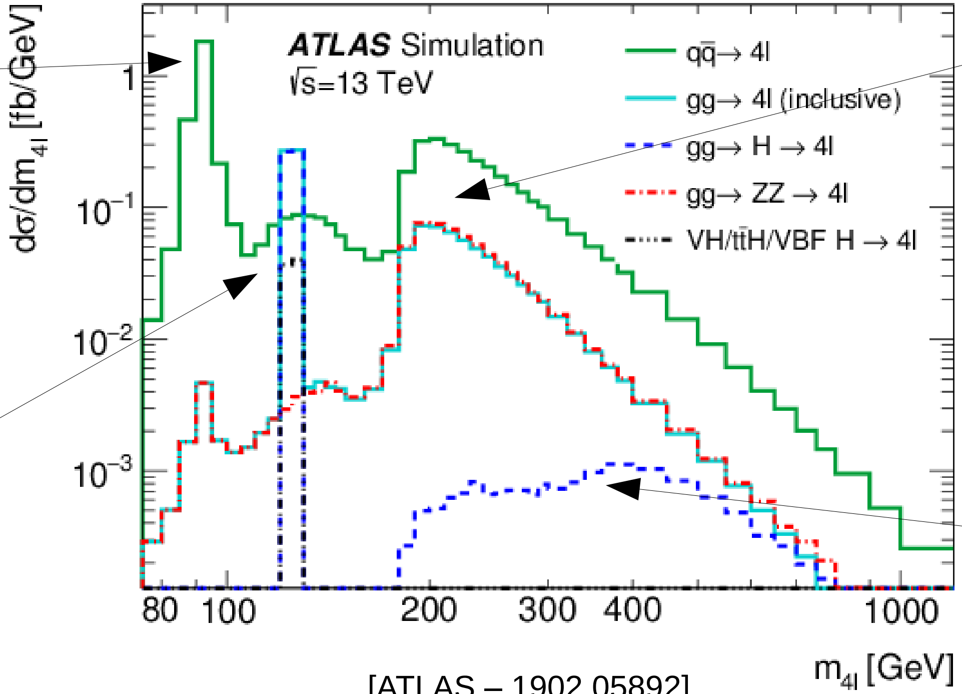
$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n'_2}(q\cdot p_2)^{n'_3}(q\cdot r_\perp)^{n'_4}}{(q^2-m_t^2)^{l_1}[(q+p_2)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

4) LiteRed searches for MIs with $n'_4=0$ $\rightarrow$ the MIs do not depend on $r_\perp$

$pp \rightarrow ZZ$

Single Z threshold

On-shell H production

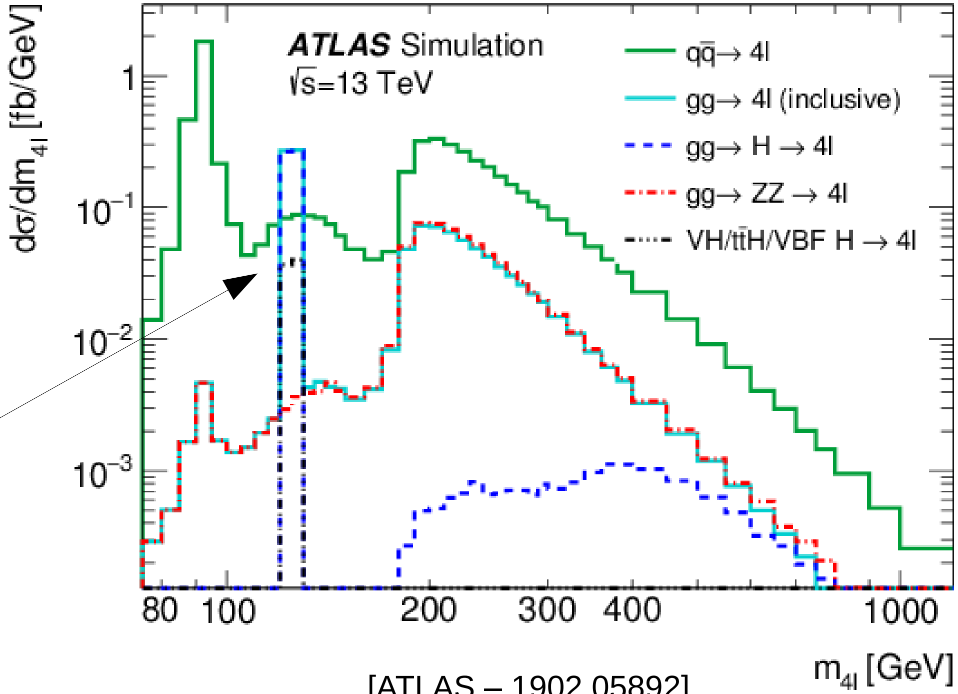


[ATLAS – 1902.05892]

ZZ on-shell threshold

- modifies the Higgs-peak region [Kauer, Passarino - 1206.4803]

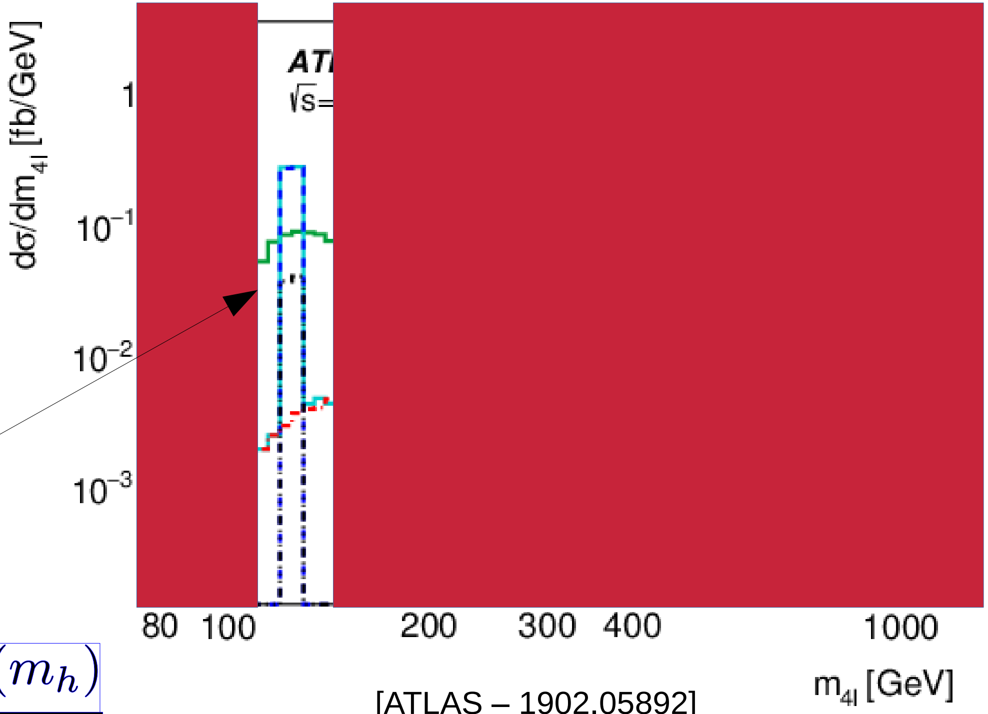
Top-pair threshold



On-shell H production

[ATLAS – 1902.05892]

Higgs width from off-shell measurements

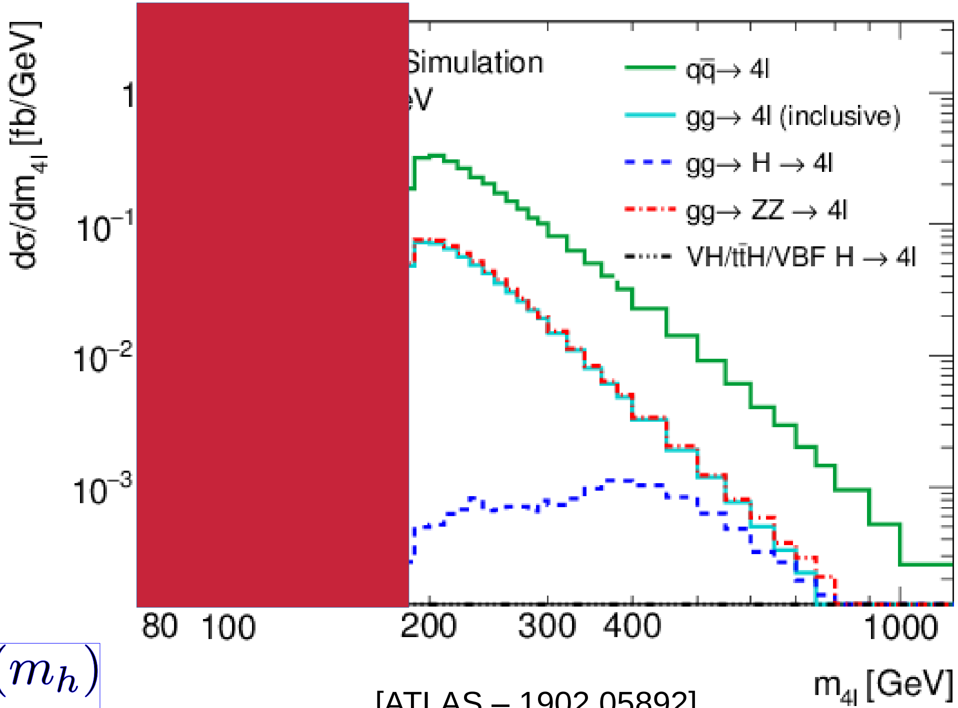


[ATLAS – 1902.05892]

On-shell H production

$$\mu_{\text{on}} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\text{SM}}}$$

Higgs width from off-shell measurements



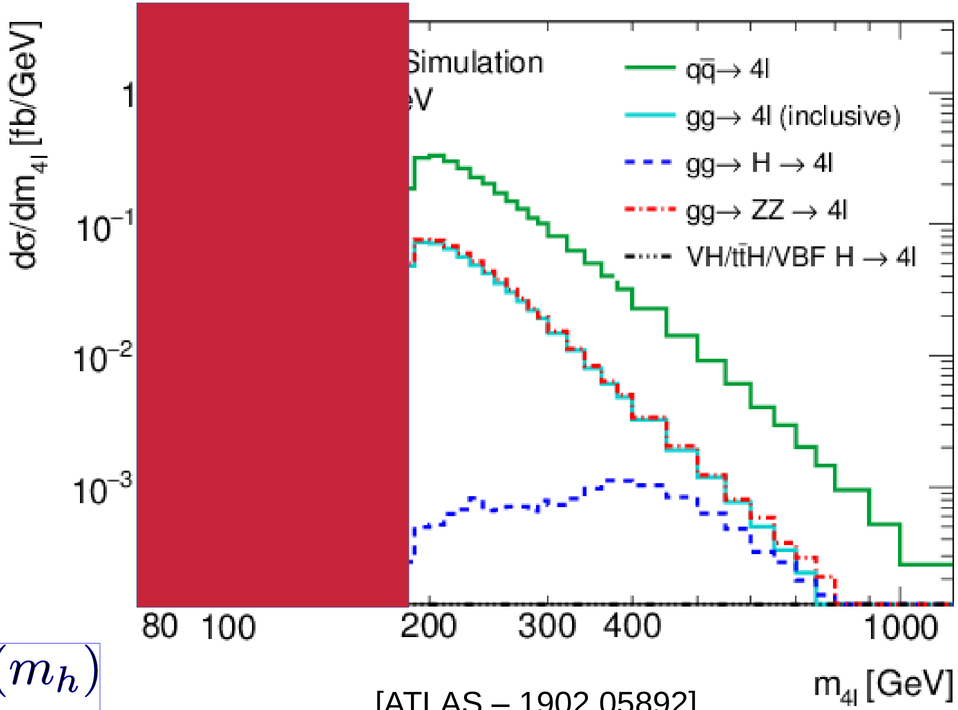
$$\mu_{\text{on}} = \frac{\kappa_{ggh}^2(m_h)\kappa_{hZZ}^2(m_h)}{\Gamma_h/\Gamma_h^{\text{SM}}}$$

[ATLAS – 1902.05892]

Off-shell region

$$\mu_{\text{off}} = \kappa_{ggh}^2(m_{ZZ})\kappa_{hZZ}^2(m_{ZZ})$$

Higgs width from off-shell measurements



[ATLAS – 1902.05892]

$$\mu_{\rm on} = \frac{\kappa_{ggh}^2(m_h)\kappa_{hZZ}^2(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

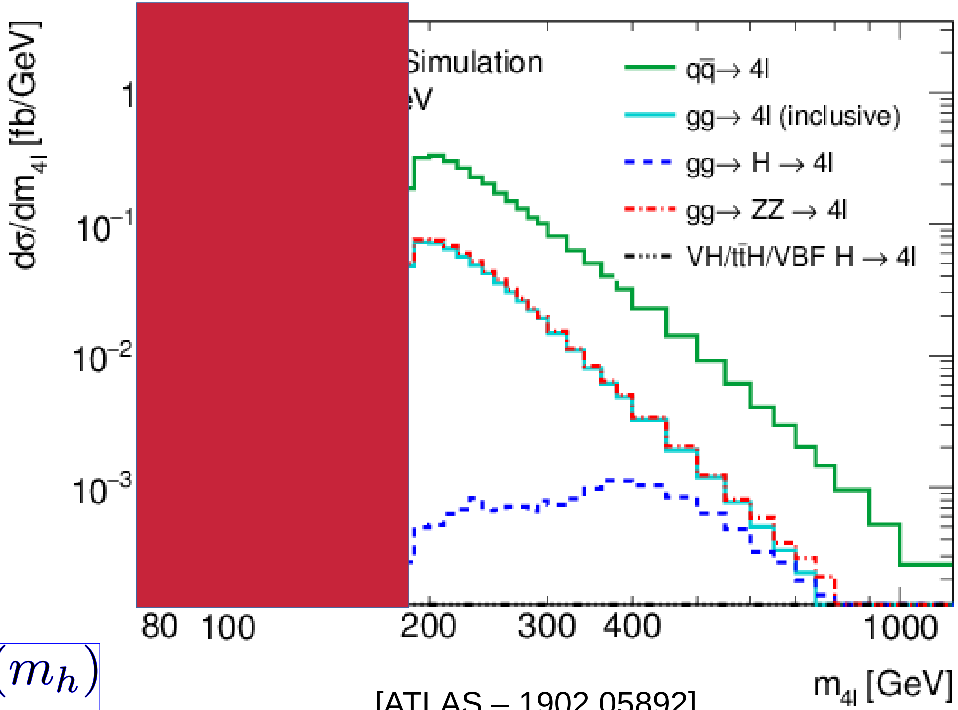
Off-shell region

$$\mu_{\rm off} = \kappa_{ggh}^2(m_{ZZ})\kappa_{hZZ}^2(m_{ZZ})$$

$$\frac{\mu_{\rm on}}{\mu_{\rm off}} \propto \frac{\kappa_{ggh}^2(m_h)\kappa_{hZZ}^2(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}} \frac{1}{\kappa_{ggh}^2(m_{ZZ})\kappa_{hZZ}^2(m_{ZZ})},$$

[Kauer, Passarino – 1206.4803]
[Caola, Melnikov – 1307.4935]
[Campbell. Ellis, Williams - 1311.3589]

Higgs width from off-shell measurements



[ATLAS – 1902.05892]

$$\mu_{\rm on} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

Off-shell region

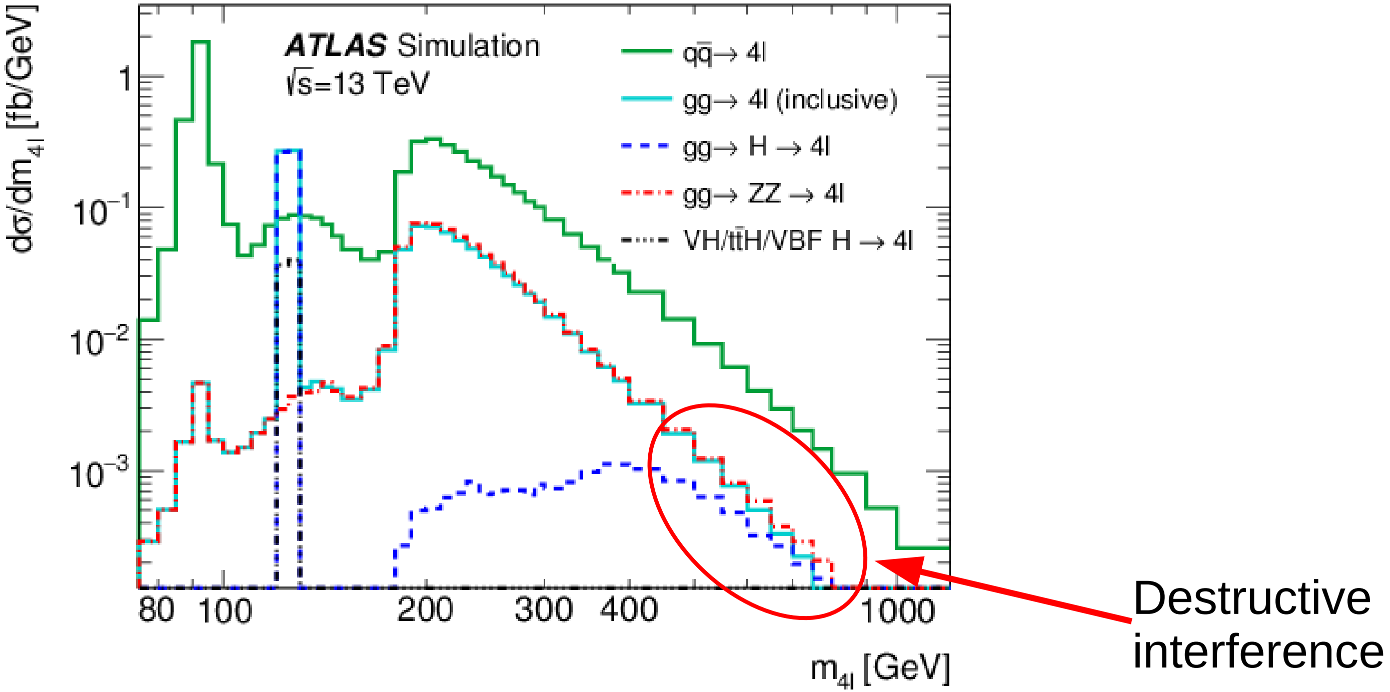
$$\mu_{\rm off} = \kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})$$

$$\Gamma_H = 3.2^{+2.4}_{-1.7}\,\mathrm{MeV}$$

[CMS - 2202.06923]

$$\Gamma_H = 4.5^{+3.3}_{-2.5}\,\mathrm{MeV}$$

[ATLAS - 2304.01532]



[ATLAS – 1902.05892]

Interference @ NLO: massless vs massive

Two-loop boxes are a problem (again)

- Light-quark (~massless) known fully analytically
  [Caola et al. - 1509.06734]

- Heavy quarks

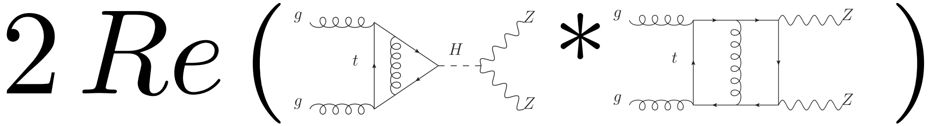
➔ Exact numerical results available
[Agarwal, Jones, von Manteuffel - 2011.15113 ; Brønnum-Hansen, Wang - 2101.12095]

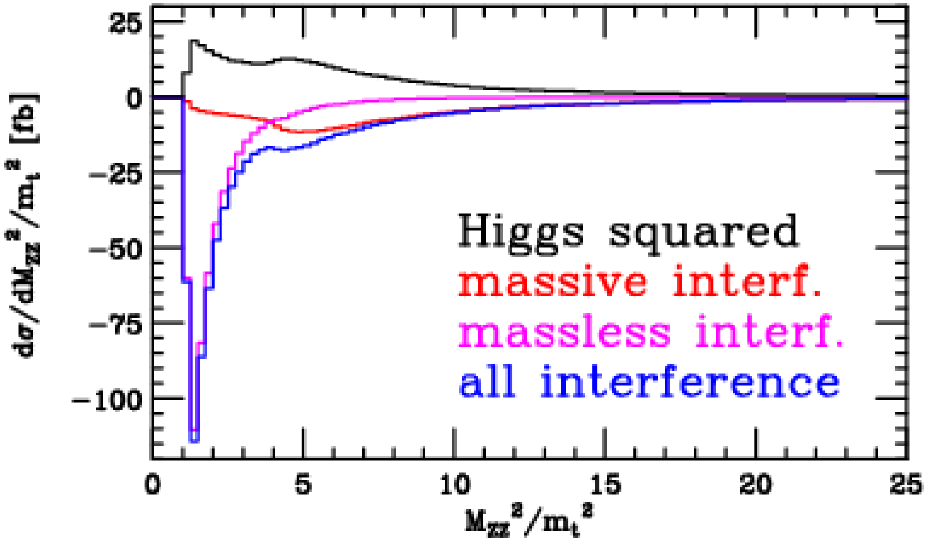
➔ Analytic approximations:

-LME [Melnikov, Dowling - 1503.01274 ; Gröber, Maier, Rauh - 1605.04610]

-High-energy exp [Davies et al. - 2002.05558]

$$m_Z^2 \ll m_t^2 \ll \hat{s}, \hat{t}$$



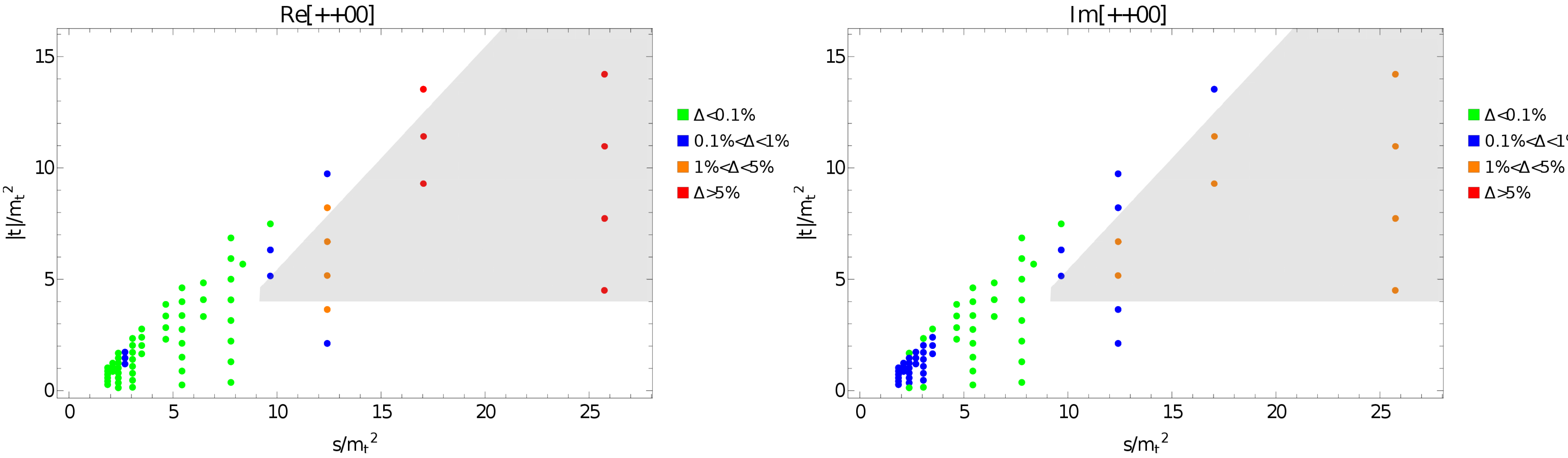


[Campbell et al. - 1605.01380]

Helicity amplitudes at NLO

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \boxed{\mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}} + \mathcal{O}(\alpha_s^3)$$

[Agarwal, Jones, von Manteuffel - 2011.15113]



[PRELIMINARY]