

Probing anomalous transport of (integrable) spin chains

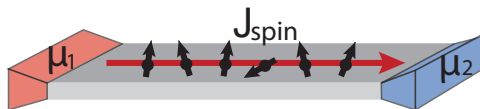
Zala Lenarčič

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October 2023



Probing transport properties via tensor network calculations



Boundary driven open systems, Znidarič, PRL 20211

$$\hat{\mathcal{L}}\rho = -i[H, \rho] + \hat{\mathcal{D}}\rho, \quad \hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\}$$

with

$$\begin{aligned} L_1 &= \sqrt{1 + \mu} S_1^- & L_3 &= \sqrt{1 - \mu} S_L^- \\ L_2 &= \sqrt{1 - \mu} S_1^+ & L_4 &= \sqrt{1 + \mu} S_L^+ \end{aligned}$$

Probe:

- Nature of diffusion in integrable and nearly integrable systems
- Stability of superdiffusion in nearly integrable systems
- Subdiffusion in tilted (Stark) interacting chains

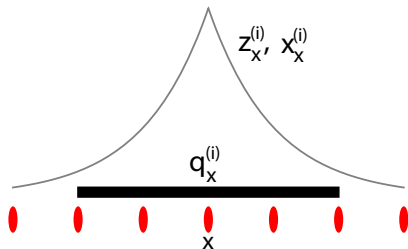
Integrable systems

- **Macroscopic number of conservations laws** $[H, C_i] = 0$
- Paradigmatic example: Heisenberg model

$$H_{XXZ} = \sum_i J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z)$$

Three types of conservation laws J. Stat. Mech. (2016) 064008

$$C_i = \begin{cases} Q_i = \sum_x q_x^{(i)}, & \text{local} \\ X_i = \sum_x x_x^{(i)}, & \text{quasi-local, even under spin reversal symm} \\ Z_i = \sum_x z_x^{(i)}, & \text{quasi-local, odd under spin reversal symm} \end{cases}$$



Non-trivial transport properties

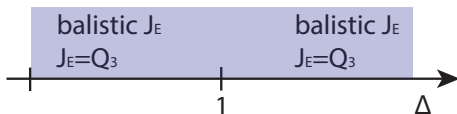
$$H_{XXZ+B} = \sum_i J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) + BS_i^z = \sum_i h_{i,i+1}$$

- Ballistic transport of energy current, PRB 55, 11029 (1997)

$$J_E = \sum_i (\vec{S}_{i-1} \times \vec{S}_i) \cdot \vec{S}_{i+1} + BJ_S, \quad \frac{dh_{i,i+1}}{dt} = i[H, h_{i,i+1}] = -(j_{i+1}^E - j_i^E)$$

$$J_S = \sum_i S_i^x S_{i+1}^y - S_i^y S_{i+1}^x, \quad \frac{dS_i^z}{dt} = i[H, S_i^z] = -(j_{i+1}^S - j_{i-1,i}^S)$$

- Important $J_E = Q_3$, implying ballistic energy transport



Non-trivial transport properties

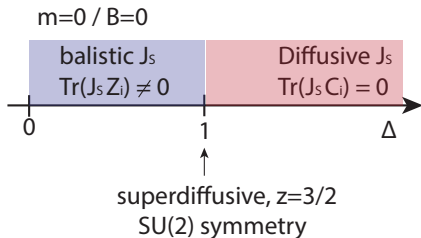
$$H_{X X Z + B} = \sum_i J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) = \sum_i h_{i,i+1}$$

- Ballistic transport of energy current, PRB 55, 11029 (1997)

$$J_E = \sum_i (\vec{S}_{i-1} \times \vec{S}_i) \cdot \vec{S}_{i+1}, \quad \frac{dh_{i,i+1}}{dt} = i[H, h_{i,i+1}] = -(j_{i+1}^E - j_i^E)$$

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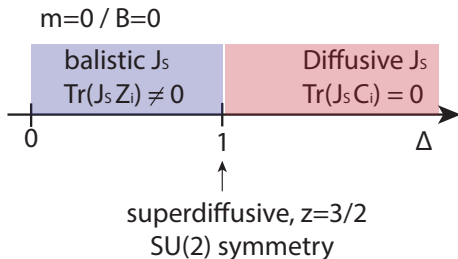
- Very rich spin transport RMP 93, 25003 (2021)



What happens to spin transport under perturbations?

$$H = H_{\text{XXZ}} + H'$$

- Different transport regimes
- Different symmetry classes of perturbations
- Finite / zero magnetization



Approaches

- **Full ED** or Microcanonical Lanczos method (MCLM)

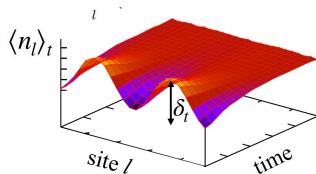
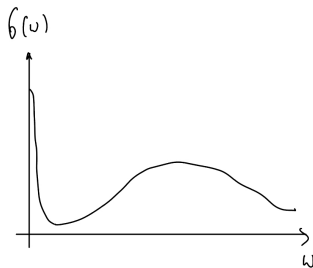
$$\sigma(\omega) = \frac{\beta}{L} \int_0^\infty dt e^{i\omega t} \langle J_S(t) J_S(0) \rangle_\beta = 2\pi D \delta(\omega) + \sigma_{reg}(\omega)$$

- Spin stiffness for ballistic transport

$$D^* = \frac{D}{\chi_0}, \quad \chi_0 = \frac{1}{\pi} \int d\omega \sigma(\omega)$$

- Diffusion constant

$$D = \frac{\sigma_{reg}(\omega = 0)}{\chi_0}$$



Approaches



- **Boundary driven open systems**

$$\hat{\mathcal{L}}\rho = -i[H, \rho] + \hat{\mathcal{D}}\rho, \quad \hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\}$$

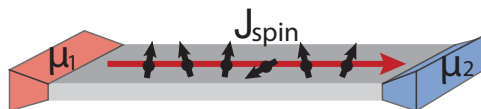
with

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Current scaling, Znidaric, PRL 2011

$$\text{tr}[J_s \rho_{\text{NESS}}] \sim L^{-\gamma} \text{ with } \begin{cases} \gamma = 0, & \text{ballistic,} \\ 0 < \gamma < 1, & \text{super-diffusive,} \\ \gamma = 1, & \text{diffusive,} \\ \gamma > 1, & \text{sub-diffusive,} \end{cases}$$

Approaches



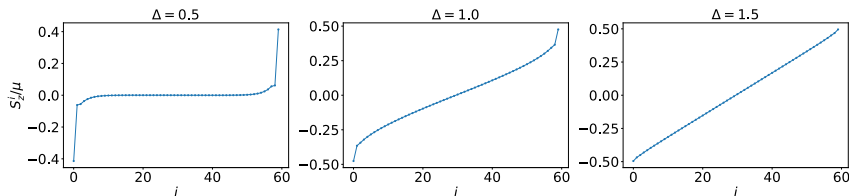
- **Boundary driven open systems**

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with

$$L_1 = \sqrt{1 + \mu} S_1^- \quad L_3 = \sqrt{1 - \mu} S_L^-$$
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Spin profiles for unperturbed Heisenberg model



Approaches

- **Generalized hydrodynamics:** elastic scattering of quasi-particles

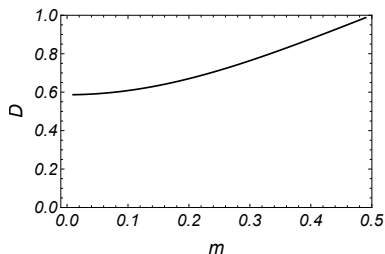
Types of conservation laws \leftrightarrow types of quasi-particles

PRL 115, 157201 (2015), J. Stat. Mech. (2021) 084001

$$\left\{ \begin{array}{l} Q_i = \sum_x q_x^{(i)}, \quad (s = 1) \text{ magnons } \uparrow\uparrow\downarrow\uparrow\uparrow \\ X_i = \sum_x x_x^{(i)}, \quad (s > 1) \text{ bound states of magnons } \uparrow\uparrow\downarrow\uparrow\uparrow, \\ Z_i = \sum_x z_x^{(i)}, \quad \text{doublet with eff. mag. moment PRL 119, 020602 (2017)} \end{array} \right.$$

Drude weight

$$D = \frac{\beta}{2} \sum_{s=1}^{s_{\max}} \int d\theta \rho_s^{\text{tot}}(\theta) n_s (1 - n_s) (v_s^{\text{eff}}(\theta) m_s^{\text{dr}})^2.$$

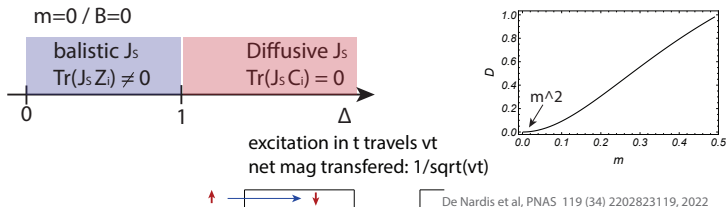


What happens to spin transport under perturbations?

$$H = H_{XXZ} + H'$$

- **Diffusive regime $\Delta > 1$**

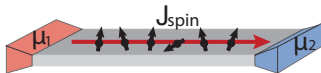
- Prelovšek, Nandy, Lenarčič, Mierzejewski, and Herbrich, PRB 106, 245104 (2022)



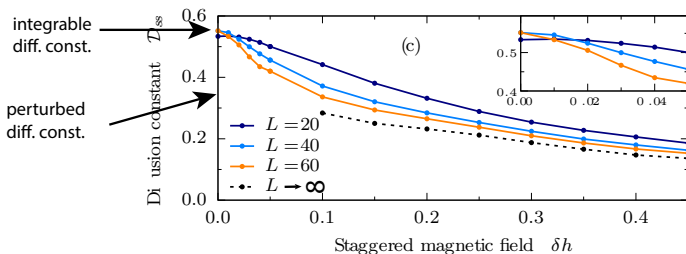
Results: perturbed diffusive spin transport $\Delta > 1$

$$H = J \sum_i \left[\frac{1}{2} (S_{i+1}^+ S_i^- + \text{H.c.}) + \Delta S_{i+1}^z S_i^z \right] + \delta h J H' \quad H' = \sum_i (-1)^i S_i^z$$

Main result from open systems analysis, $\mathcal{D} = -\frac{\text{tr}[J_s \rho_{NESS}]}{\nabla S^z}$

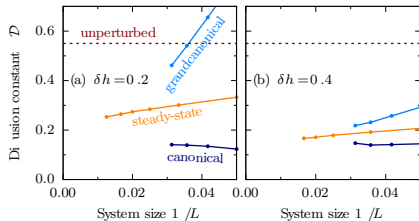
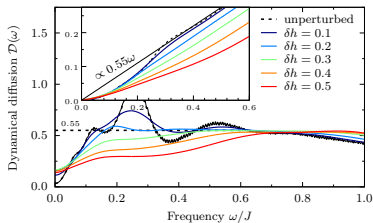


- jump in diffusion constant PRB 106, 245104 (2022)
- see also De Nardis et al, PNAS 119 (34) 2022



Results: perturbed diffusive spin transport $\Delta > 1$

Comparison to ED results

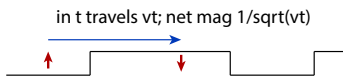


What happens to spin transport under perturbations?

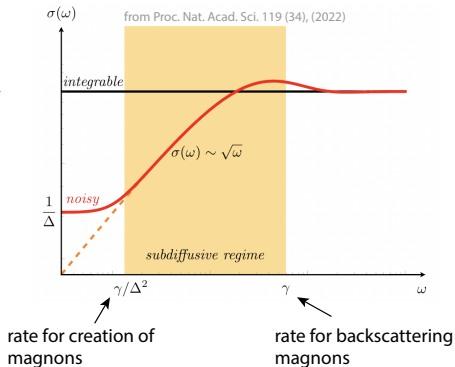
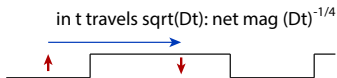
De Nardis et al, Proc. Nat. Acad. Sci. 119 (34), (2022)

- for $\Delta = \infty$: subdiffusive transport
- finite $\Delta > 1$: intermediate subdiffusive transport

integrable case: diffusion



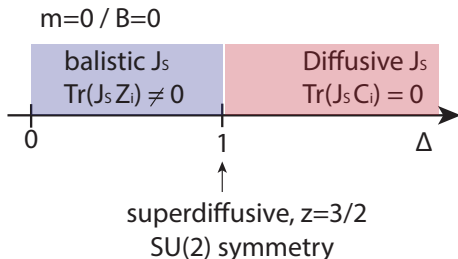
perturbed case: subdiffusion



Superdiffusion

$$H = H_{\text{XXZ}} + H'$$

- **Different symmetry classes of perturbations**
 - Nandy et al, Rev. B 108, L081115 (2023)
 - See also De Nardis et al, PRL 127, 057201 (2021)
- **Finite / zero magnetization**
 - Nandy et al, Rev. B 108, L081115 (2023)

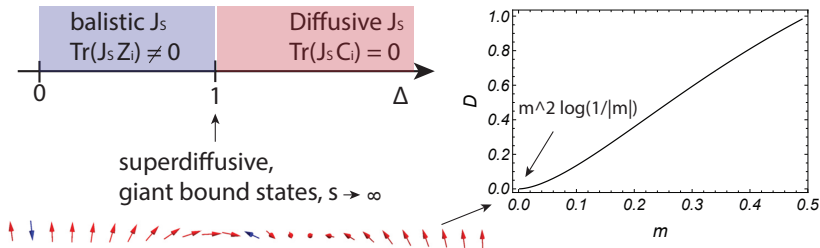


Superdiffusion

- **Generalized hydrodynamics:** elastic scattering of quasi-particles J.
Stat. Mech. (2021) 084001
Drude weight

$$D = \frac{\beta}{2} \sum_{s=1}^{s_{\max} \rightarrow \infty} \int d\theta \rho_s^{\text{tot}}(\theta) n_s (1 - n_s) (v_s^{\text{eff}}(\theta) m_s^{\text{dr}})^2.$$

- Non-analytical dependence on Drude weight at $m \rightarrow \infty$
- Contribution from $s_{\max} \rightarrow \infty$, i.e., giant magnons.



Results: role of symmetry of perturbations

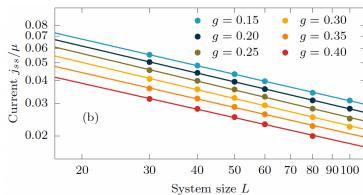
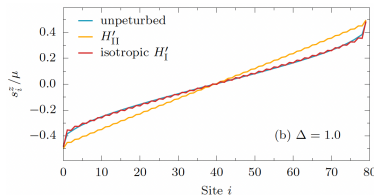
$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + g J H',$$

$$H'_{\text{is}} = \sum_i (-1)^i \vec{S}_i \cdot \vec{S}_{i+1},$$

$$H'_{\text{an}} = (1/2) \sum_i (-1)^i (S_{i+1}^+ S_i^- + \text{H.c.})$$

Symmetry of perturbation very important!

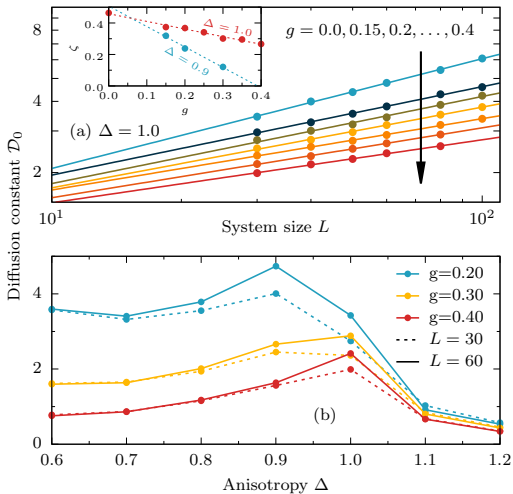
- symm. breaking: diffusive transport
- symm. preserving: superdiffusive transport, $j_s \sim L^{-1/2}$
- Relevant for cold atom experiment, *Science*, 376(6594), 716-720 (2022)



Results: role of symmetry of perturbations

Superdiffusive scaling of diffusion constant $\mathcal{D} = -\frac{\text{tr}[J_s \rho_{NESS}]}{\nabla_s z} \sim L^\zeta$

→ superdiffusion robust at significant perturbations, PRB 108, L081115 (2023)



Results: role of symmetry of perturbations

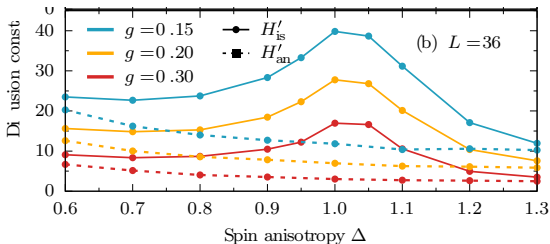
$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + gJH',$$

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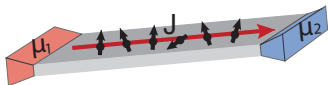
Main ED result: symmetry of perturbation very important!

→ diffusion constant at finite magnetization

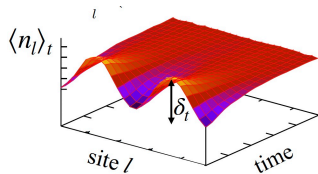


Transport in tilted chains

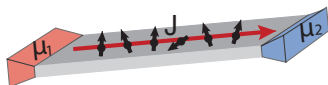
$$H = \sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l \right) + V \tilde{n}_l \tilde{n}_{l+1} + F \left(l - \frac{L}{2} \right) \tilde{n}_l + H', \quad \tilde{n}_l = n_l - 1/2$$



$$H_F(t) = \sum_l \left(e^{-iFt} c_l^\dagger c_{l+1} + e^{iFt} c_{l+1}^\dagger c_l + V n_l n_{l+1} \right) + H'.$$

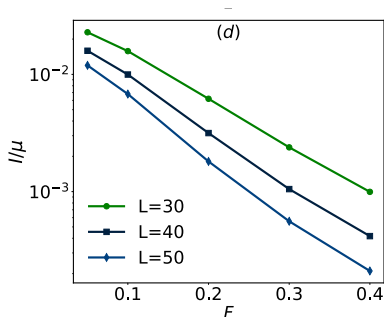


Crossover from diffusive to subdiffusive transport

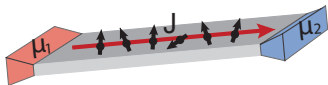


$$H = \sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l \right) + V \tilde{n}_l \tilde{n}_{l+1} + F \left(l - \frac{L}{2} \right) \tilde{n}_l + H', \quad \tilde{n}_l = n_l - 1/2$$

- Current exponentially suppressed with $F \leftrightarrow$ no Stark localization, PRL 122, 040606 (2019)

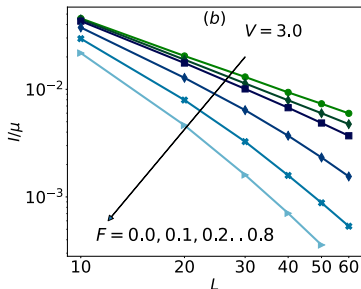
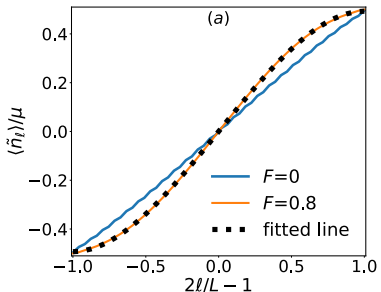


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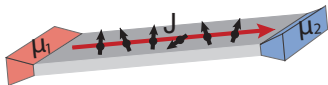


$$H = \sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l \right) + V \tilde{n}_l \tilde{n}_{l+1} + F \left(l - \frac{L}{2} \right) \tilde{n}_l + H', \quad \tilde{n}_l = n_l - 1/2$$

- Profile: diffusive \rightarrow subdiffusion
- Current: F and L dependent scaling $I \sim L^{1-z}$
- Define L dependent dynamical exponent z

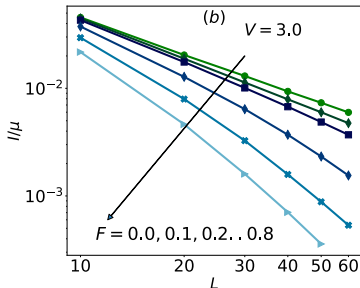
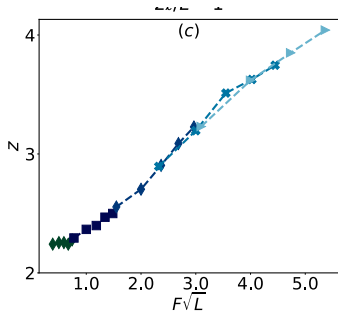


Crossover from diffusive to subdiffusive transport



$$H = \sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l \right) + V \tilde{n}_l \tilde{n}_{l+1} + F \left(l - \frac{L}{2} \right) \tilde{n}_l + H', \quad \tilde{n}_l = n_l - 1/2$$

- Universal scaling of $z(F\sqrt{L})$
- $z = 4$: fractonic hydrodynamics due to conserved M at large F ?
- Cold atom experiment, Phys. Rev. X 10, 011042 (2020).



$z(F\sqrt{L})$ dependence and bounds on dynamics of M

$$\frac{d\langle M \rangle_t}{dt} = \frac{1}{F} \frac{d\langle H - H_0 \rangle_t}{dt} = -\frac{1}{F} \frac{d\langle H_0 \rangle_t}{dt}$$

- variation of dipol moment: $\langle M \rangle_t = \langle \psi(0) | e^{iHt} M e^{-iHt} | \psi(0) \rangle$

$$|\langle M \rangle_t - \langle M \rangle_{t'}| < \delta_M = \frac{\alpha L}{F}$$

- width of the spectrum of M , $M|\psi_n\rangle = d_n|\psi_n\rangle$

$$\sigma_M^2 = \frac{1}{Z} \text{Tr}(M^2) = \sum_{l=-\frac{L}{2}+1}^{\frac{L}{2}} \frac{l^2}{4} \simeq \frac{L^3}{48}.$$

- When fractonic dynamics sets in

$$\frac{\delta_M}{\sigma_M} = \frac{4\alpha\sqrt{3}}{F\sqrt{L}}.$$

Proof of M conservation in $T = \infty$ state

$$\lim_{L \rightarrow \infty} \frac{\langle M(t)M \rangle_{T=\infty}}{\langle MM \rangle_{T=\infty}} = 1, \quad M(t) = e^{iHt} M e^{-iHt}$$

Proof of M conservation in $T = \infty$ state

$$\lim_{L \rightarrow \infty} \frac{\langle M(t)M \rangle_{T=\infty}}{\langle MM \rangle_{T=\infty}} = 1, \quad M(t) = e^{iHt} M e^{-iHt}$$

Proof:

$$\|H\|^2 = \|H_0\|^2 + F^2 \|M\|^2, \quad \frac{\|H_0\|}{\|M\|} \propto \frac{L^{1/2}}{L^{3/2}}$$

$$\|M\|^2 = \|M^{\parallel}\|^2 + \|M^{\perp}\|^2$$

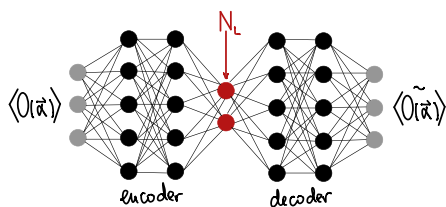
$$M^{\parallel} = \frac{\langle MH \rangle}{\langle HH \rangle} H, \quad M^{\perp} = M - M^{\parallel}$$

$$\begin{aligned} \langle [M^{\parallel} + M^{\perp}(t)]M \rangle &\geq \|M^{\parallel}\|^2 - |\langle M^{\perp}(t)M \rangle| \\ &\geq \|M^{\parallel}\|^2 - \|M^{\perp}\| \|M\|, \end{aligned}$$

$$1 \geq \frac{\langle M(t)M \rangle}{\|M\|^2} \geq 1 - \frac{\|M^{\perp}\|}{\|M\|} - \frac{\|M^{\perp}\|^2}{\|M\|^2}$$

ML assisted reconstruction of H from measurements

ML assisted reconstruction of H from measurements



Input x : expectation values $\text{tr}[O(\alpha)\rho]$ of local operators

$$O(\alpha) = \sigma_1^{\alpha_1} \dots \sigma_{|S|}^{\alpha_{|S|}}, \quad \alpha = (\alpha_1, \dots, \alpha_{|S|}) \in \{0, x, y, z\}^{|S|}$$

- data element: $\langle O(\alpha) \rangle$ of N_O operators

Bottleneck: N_L neurons

- Dimensional reduction $N_O \rightarrow N_L$
- Latent (compressed) representation

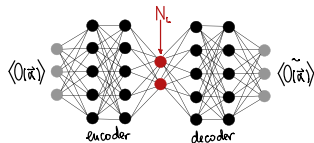
Loss function

$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} (f_\theta(x) - x)^2$$

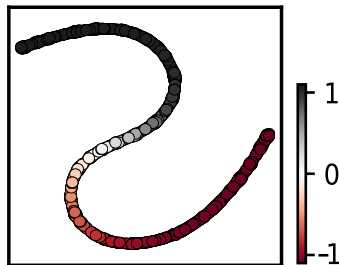
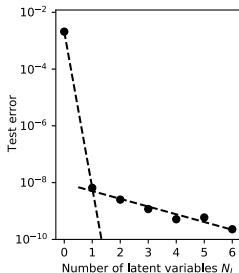
Latent representation of thermal states

Input: different thermal state of H

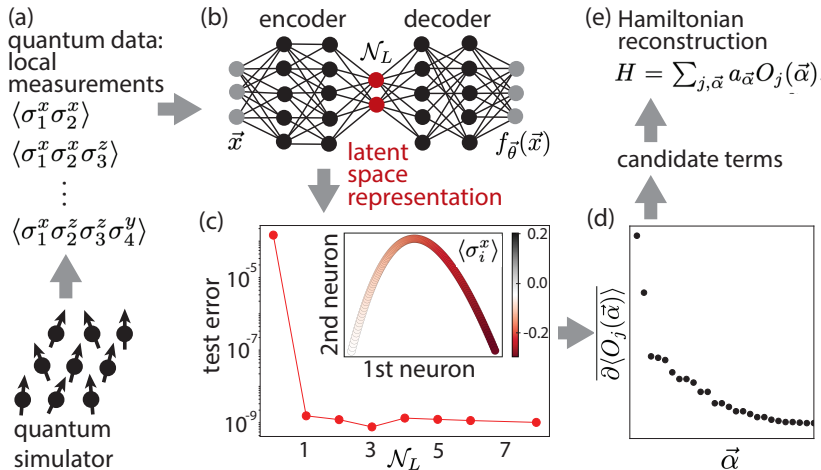
$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \langle O(\alpha) \rangle_\beta = \langle O(\alpha) \frac{1}{Z} e^{-\beta H} \rangle,$$



- Test error: $\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} (f_\theta(x) - x)^2$
- Latent representation: 1D manifold
- Data elements ordered by energy $\langle H \rangle / N$, PRB 106, L041110 (2022)

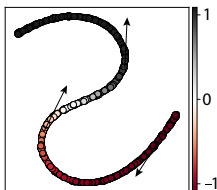


H reconstruction procedure

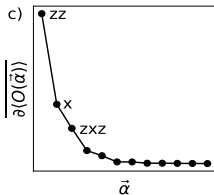


H reconstruction procedure

1. 1D latent representation



2. Find H candidate terms as operators $\langle O(\alpha) \rangle$ with largest gradient along the latent representation:



3. Root finding to fix prefactors a_α ,

$$H = \sum_{\alpha} a_{\alpha} O(\alpha),$$

$$\text{tr} \left[O(\alpha') \frac{e^{-\beta \sum_{\alpha} a_{\alpha} O(\alpha)}}{Z} \right] - \langle O(\alpha') \rangle = 0$$

(4.) If $\frac{\text{Var}(a_{\alpha})}{\mathbb{E}(a_{\alpha})} \sim O(1)$, drop those terms

(5.) Redo the root finding without the fake terms

(6.) Overall prefactor not set; potentially from dynamics.

Test Hamiltonian reconstruction

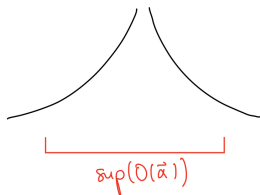
Strictly local Hamiltonian, e.g. $H = \sum_i J \sigma_i^z \sigma_{i+1}^z + h \sigma_i^x$

- Correct reconstruction, if $\max(\mathcal{O}(\alpha)) \geq \sup(H)$

$$\frac{\sup(H)}{\sup(\mathcal{O}(\vec{\alpha}))}$$

Long-range interactions, e.g., $H = \sum_i \sum_d \frac{1}{d^\gamma} (a \sigma_i^x \sigma_{i+d}^x + b \sigma_i^y \sigma_{i+d}^y)$

- small error in reconstruction due to finite support of $\langle \mathcal{O}(\alpha) \rangle$
- Qualitatively ok

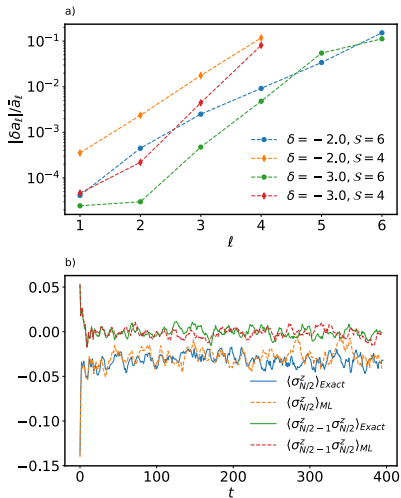

$$\sup(\mathcal{O}(\vec{\alpha}))$$

Test Hamiltonian reconstruction

Reconstructing

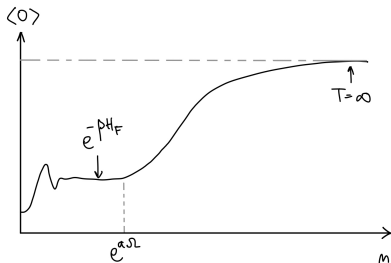
$$H_L = \sum_i \left(h_z \sigma_i^z + \sum_{r=1}^{\mathcal{R}-1} (J_1 \sigma_i^x \sigma_{i+r}^x + J_2 \sigma_i^y \sigma_{i+r}^y) \right).$$

Error estimate



Application to Floquet H learning

- Floquet engineering via periodic driving: $H(t + T) = H(t)$
- Floquet Hamiltonian, $U_{t_0+T, t_0} = U = e^{-iTH_F}$
Eckardt, Rev. Mod. Phys. 89, 011004 (2017)
 - H_F from high frequency $\Omega = 1/T \gg 1$ expansion (Magnus...)
 - Valid on prethermal plateau, up to timescales $e^{\alpha\Omega}$



What is the effective Hamiltonian beyond the Floquet prethermal regime?

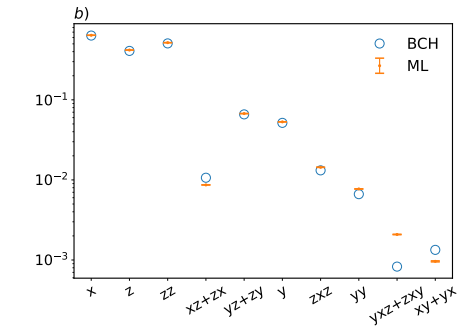
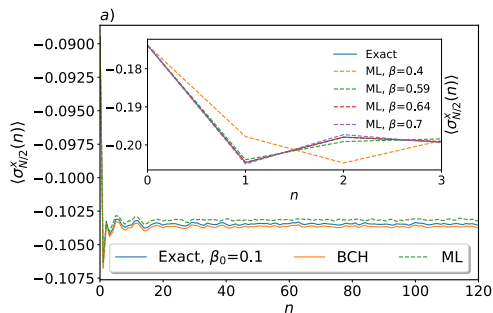
Application to Floquet H learning

Floquet protocol

$$U = e^{-iH_1 T/2} e^{-iH_2 T/2}$$

$$H_1 = \sum_{j=1}^N J \sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x + h_z \sigma_j^z$$

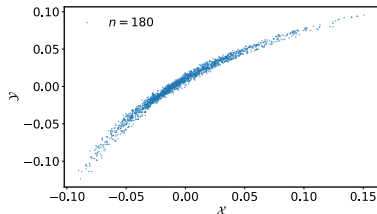
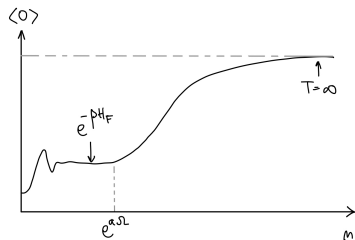
$$H_2 = \gamma \sum_{j=1}^N \sigma_j^x$$



Floquet H learning in the heating regime

Model:
$$U = \exp(-iH_1 T/4) \exp(-iVT/2) \exp(-iH_1 T/4),$$
$$H_1 = \sum_i J \sigma_i^z \sigma_{i+1}^z + h_z \sigma_i^z + h_x \sigma_i^x, \quad V = \epsilon \sum_j \sigma_j^x$$

Heating regime for larger ϵ :



A single latent variable sufficient for the whole time span

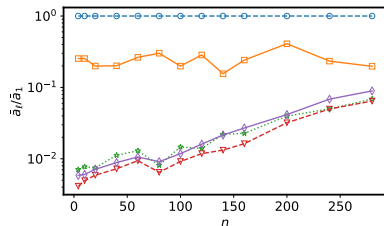
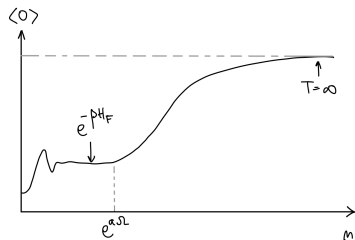
- thermal states throughout the heating regime PRB 103, 144307 (2021)

Does H_F become less local in the heating regime?

Floquet H learning in the heating regime

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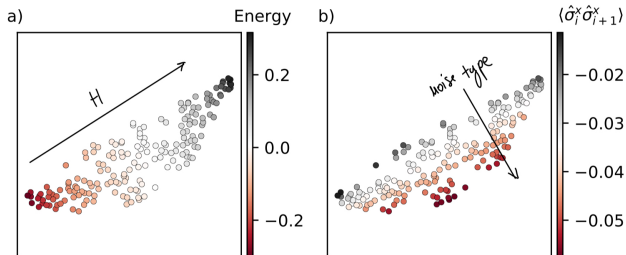
Open setup: noise-type reconstruction

$$H = \sum_i J_z S_i^z S_{i+1}^z + h_x S_i^x + h_z S_i^z$$

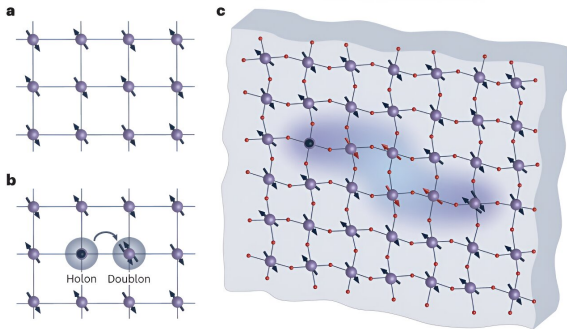
with Lindblad operators that favour AFM $\sigma_i^x \sigma_{i+1}^x$ correlations

$$\hat{\mathcal{L}}\rho = -i[H, \rho] + \epsilon \hat{\mathcal{D}}\rho = 0, \quad \hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\}$$

$$\begin{aligned} L_i^{(1a)} &= S_i^{+,x} P_{i+1}^{\downarrow,x}, & L_i^{(1b)} &= P_i^{\downarrow,x} S_{i+1}^{+,x}, \\ L_i^{(2a)} &= S_i^{-,x} P_{i+1}^{\uparrow,x}, & L_i^{(2b)} &= P_i^{\uparrow,x} S_{i+1}^{-,x}, \\ L_i^{(3)} &= S_i^z \end{aligned} \tag{1}$$



Hubbard excitons in Hubbard systems



Credit: Nature Physics (2023). DOI: 10.1038/s41567-023-02187-0

Collaboration with CalTech experimental group, Nat. Phys. (2023).

See poster by Madhumita Sarkar

Collaborators

P. Prelovšek, S. Nandy, Z. Lenarčič, M. Mierzejewski, and J. Herbrych, Phys. Rev. B 106, 245104 (2022)

S. Nandy, Z. Lenarčič, E. Ilievski, M. Mierzejewski, J. Herbrych, P. Prelovšek, Phys. Rev. B 108, L081115 (2023)

S. Nandy, J. Herbrych, Z. Lenarčič, A. Głódkowski, P. Prelovšek, M. Mierzejewski, arXiv:2310.01862 (2023)

S Nandy, M Schmitt, M Bukov, Z Lenarčič, arXiv:2308.08608 (2023)



Dr. Sourav Nandy

ERC and QuantERA PhD and postdoc positions

ERC DrumS: Weakly driven quantum symmetries

Tensor Networks in Simulation of Quantum matter (T-NiSQ)

Bañuls, Cirac, Bloch (Munich), Ringbauer, Blatt (Innsbruck), Montangero (Padova), Ortega (Bilbao).

Quantum simulation with engineered dissipation (QuSiED)

Chang (Barcelona), Marino (Mainz), Nägerl (Innsbruck), Hemmerich (Hamburg), Zarand (Budapest).



Probing transport properties via tensor network calculations

- Diffusion in nearly integrable systems
 - Jump in diffusion constant
 - **PRB 106, 245104 (2022)**
- Superdiffusion in nearly int. systems
 - Superdiffusion stable for symmetry preserving perturbations
 - **PRB 108, L081115 (2023)**
- Subdiffusion in tilted (Stark) chains
 - Fractonic hydro
 - Universal $z(F\sqrt{L})$ dependence and TD subdiffusion
 - **arXiv:2310.01862 (2023)**
- ML assisted Hamiltonian reconstruction,
 - **arXiv:2308.08608 (2023)**

