Closing gaps in the GW spectrum:
Ideas to detect μHz and high f GWs

Diego Blas (UAB/IFAE)

Generalitat de Catalunya
Departament de Recerca i Universitats

Universitat Autònoma de Barcelona

IFAE

BIST Barcelona Institute of Science and Technology

Ayuda Beatriz Galindo Senior
BG20/00228

GOBIERNO DE ESPAÑA
MINISTERIO DE UNIVERSIDADES
MINISTERIO DE CIENCIA E INNOVACIÓN
AGENCIA ESTATAL DE INVESTIGACIÓN

Closing gaps in the GW spectrum:

Ideas to detect μHz gh f GWs

Diego Blas (UAB/IFAE)
w/ A. Jenkins (UCL)

based on 2107.04063 (PRL)/2107.04601 (PRD)

Generalitat de Catalunya
Departament de Recerca i Universitats

Universitat Autònoma de Barcelona

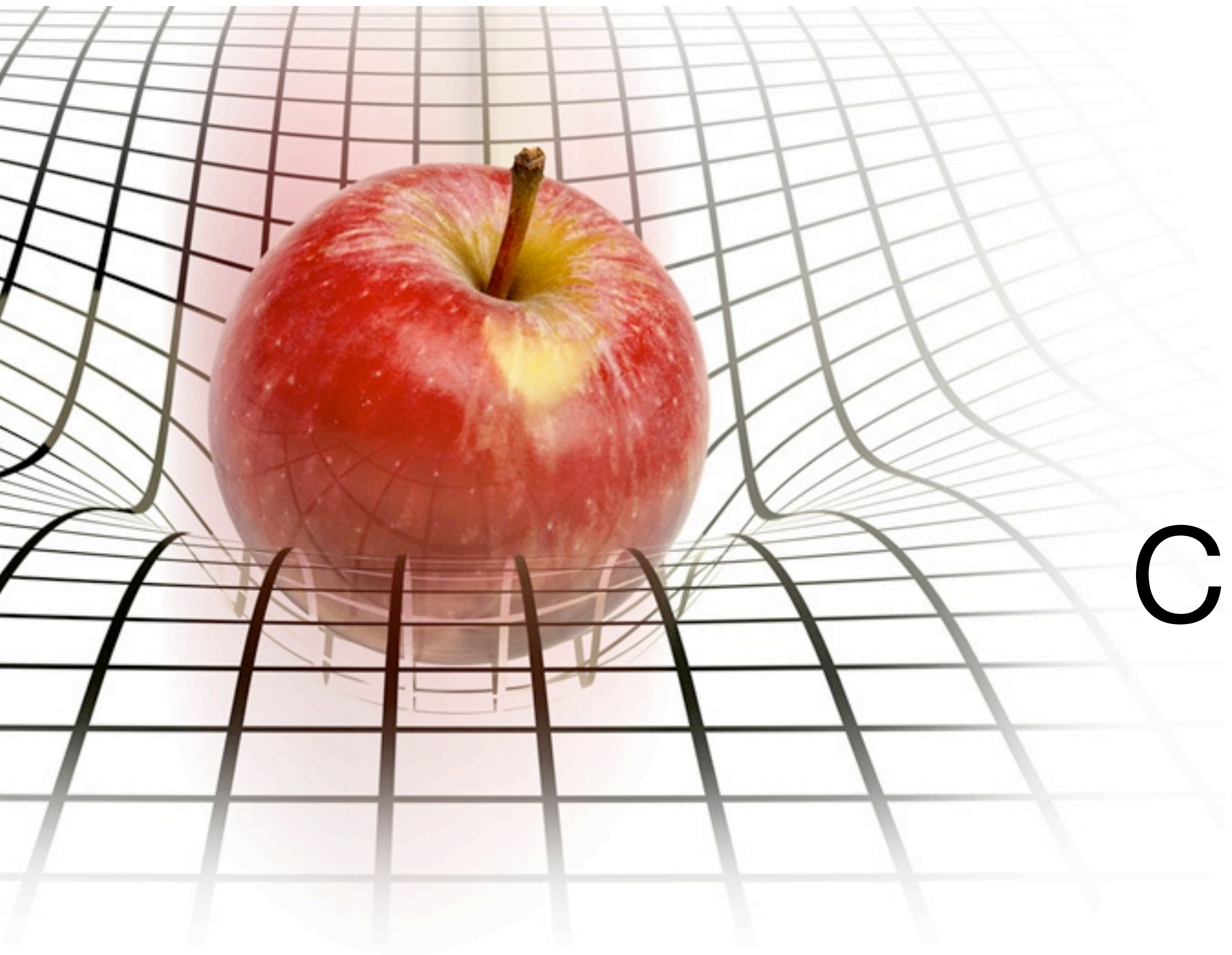
IFAE

BIST Barcelona Institute of Science and Technology

Ayuda Beatriz Galindo Senior
BG20/00228

GOBIERNO DE ESPAÑA
MINISTERIO DE UNIVERSIDADES
MINISTERIO DE CIENCIA E INNOVACIÓN
AGENCIA ESTATAL DE INVESTIGACIÓN

GWs (essentials)



Perturbations of space-time
travelling as waves of frequency f

Characterised by 2 polarizations $h_{+,\times}$ (dimensionless)

$$h_{+,\times} \overset{c=1}{\approx} h_0 \cos\left(2\pi f (t - z) + \phi\right)$$

GWs carry energy. They have **energy density**

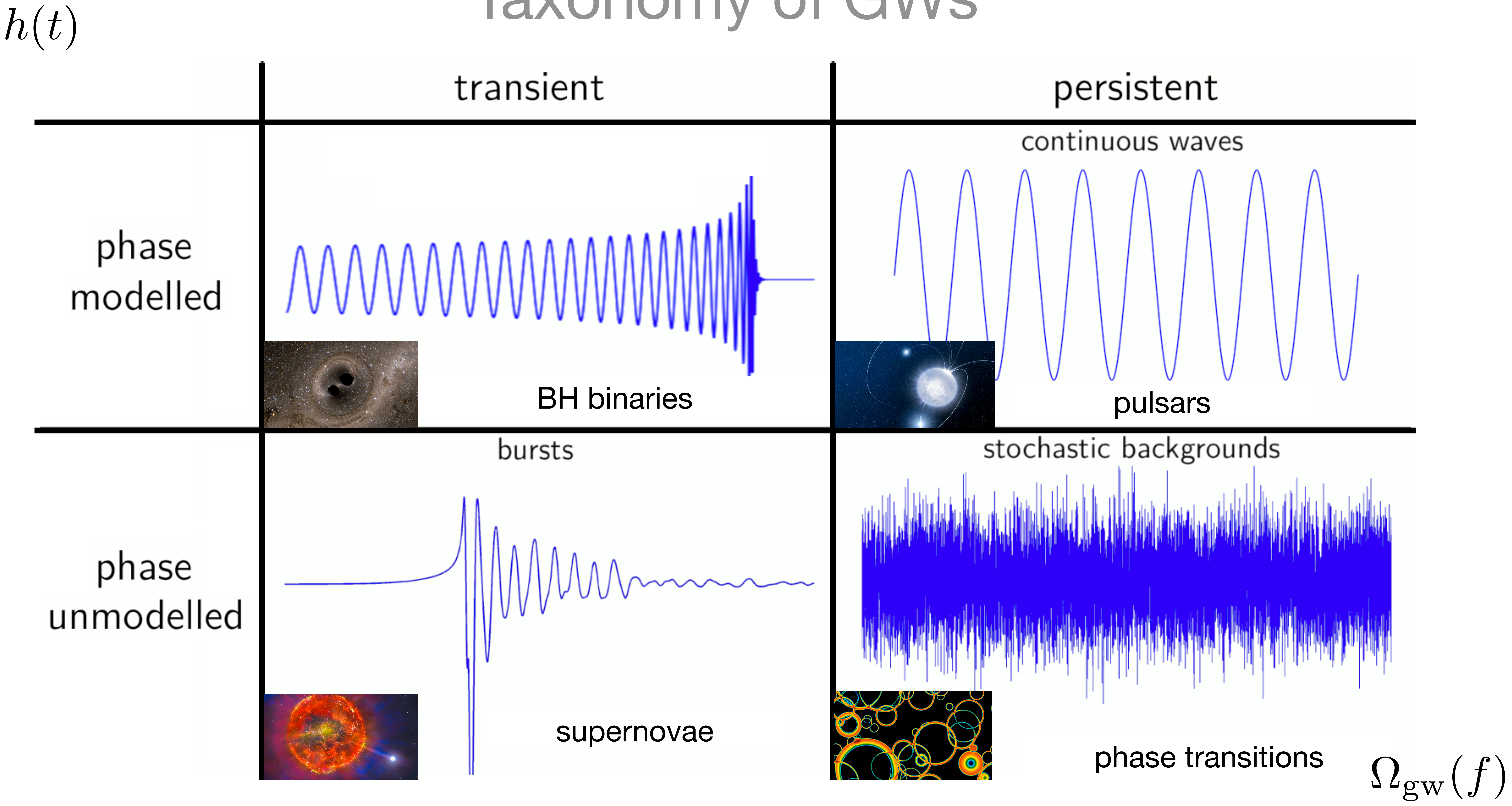
$$\rho_{\rm gw} = \frac{1}{16\pi G} \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c} \frac{{\rm d}\rho_{\rm gw}}{{\rm d}(\ln f)}$$

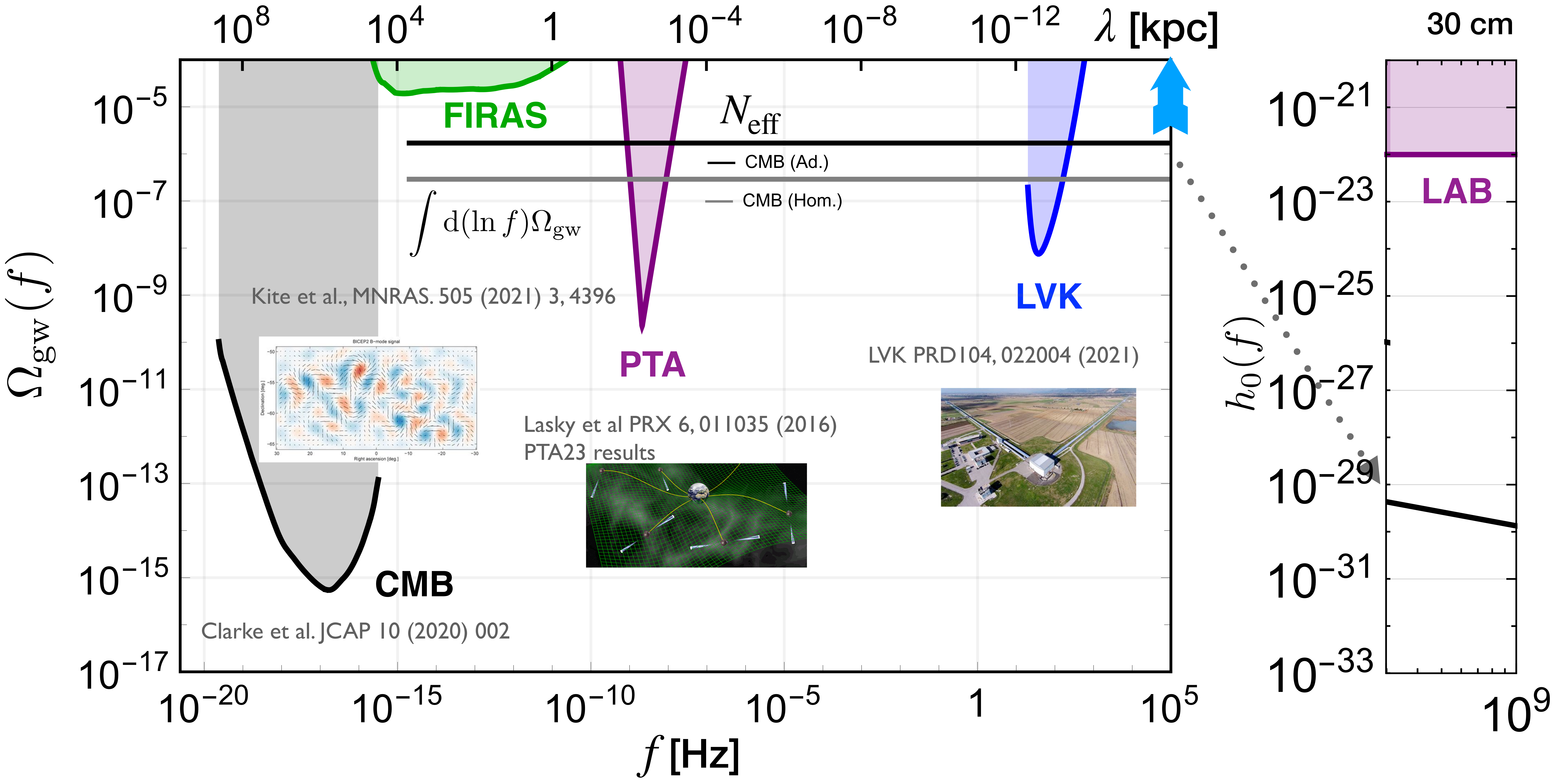
$h^2 \Omega_{\rm gw}(f)$ $\quad h \approx 0.67$

$$\rho_c = 1.2 \times 10^{11} M_\odot {\rm Mpc}^{-3}$$
$$\sim {\rm keV}/{\rm cm}^3$$

Taxonomy of GWs

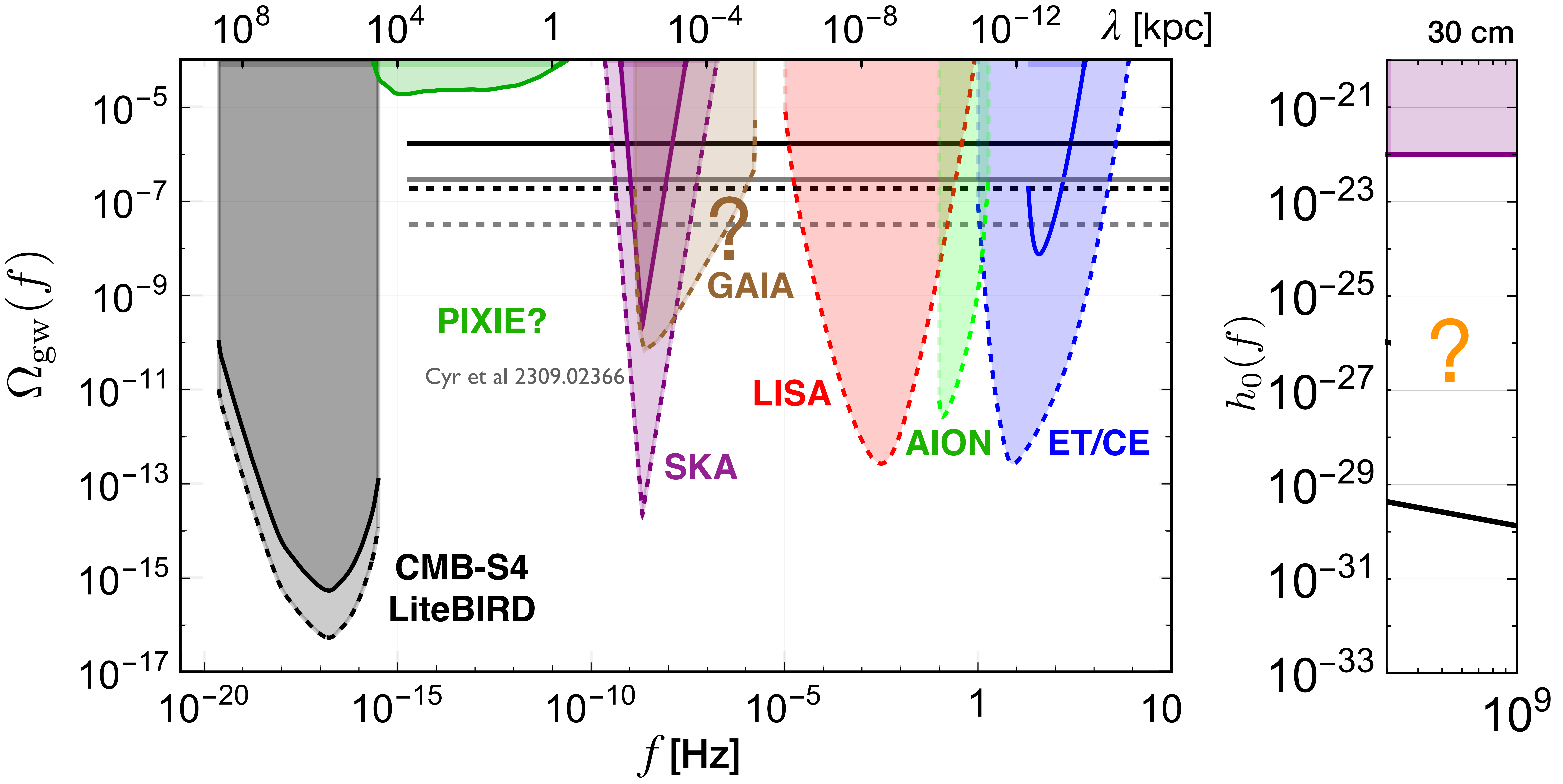


Soundscape of GWs today

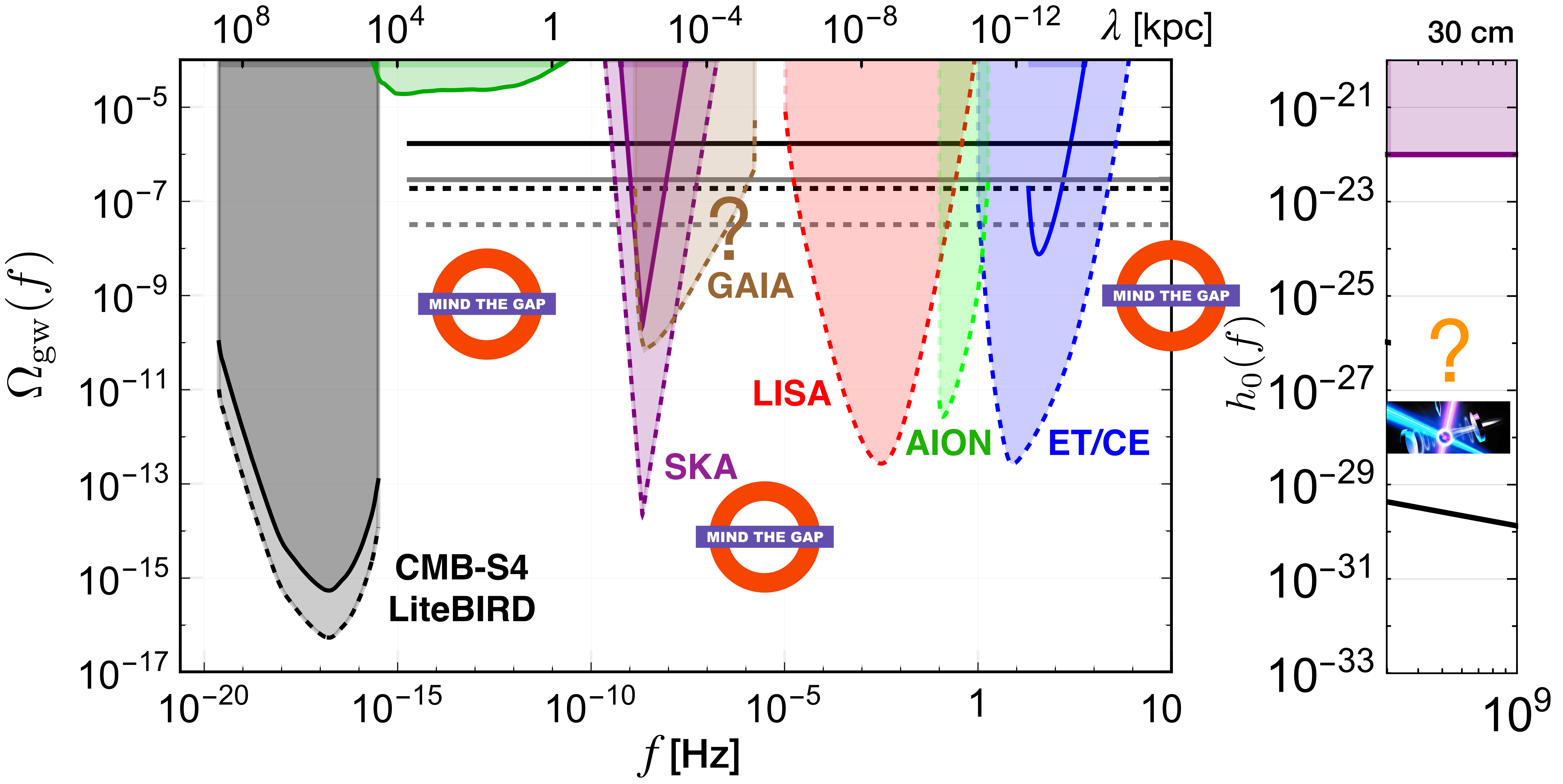


Soundscape of GWs today

Future soundscape (maybe 2040)?



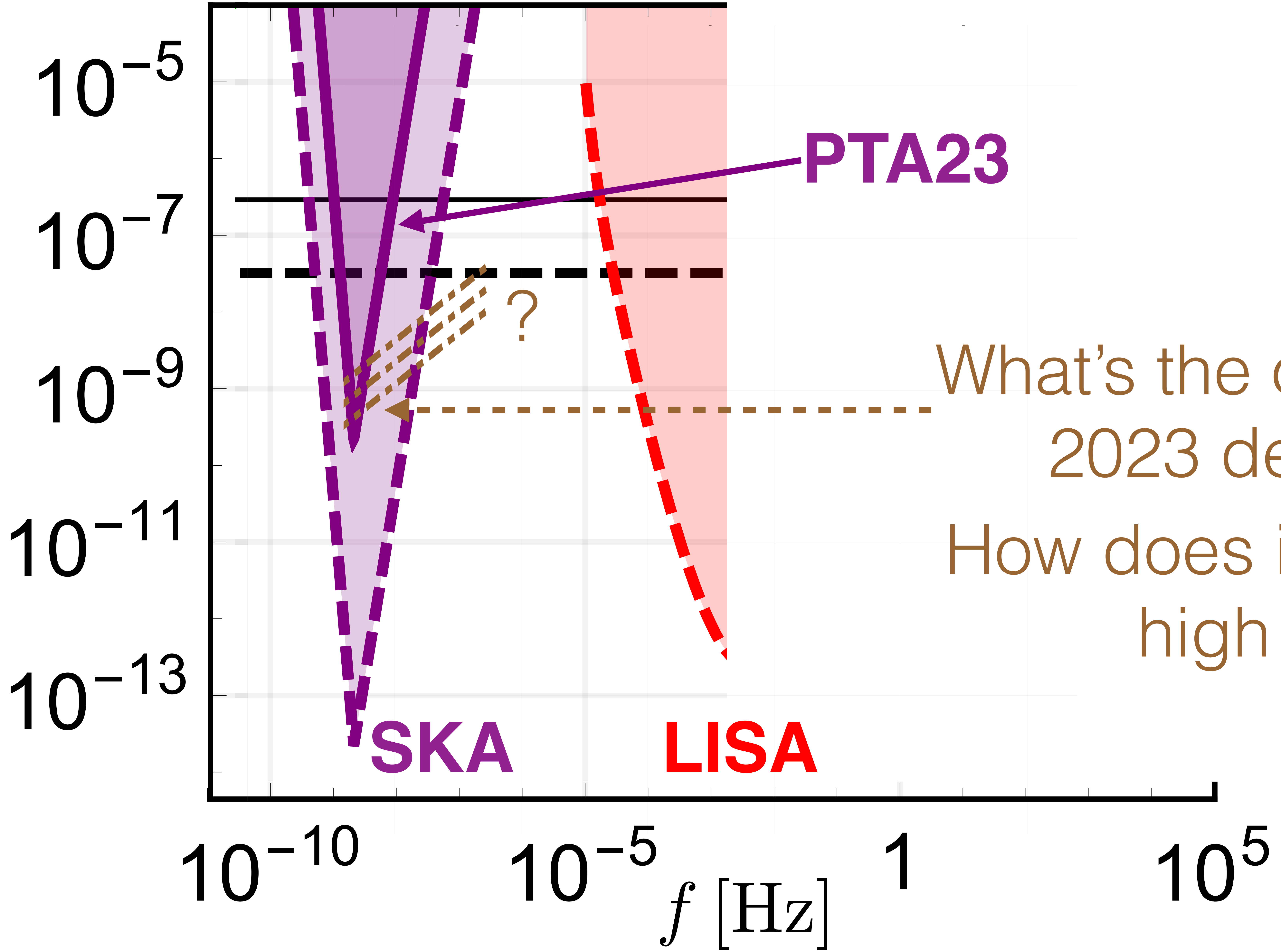
Future soundscape (maybe 2040)?



Future soundscape (maybe 2040)?

Possible **backgrounds** & ideas at μHz: a rich band

Possible **backgrounds** & ideas at μHz: a rich band



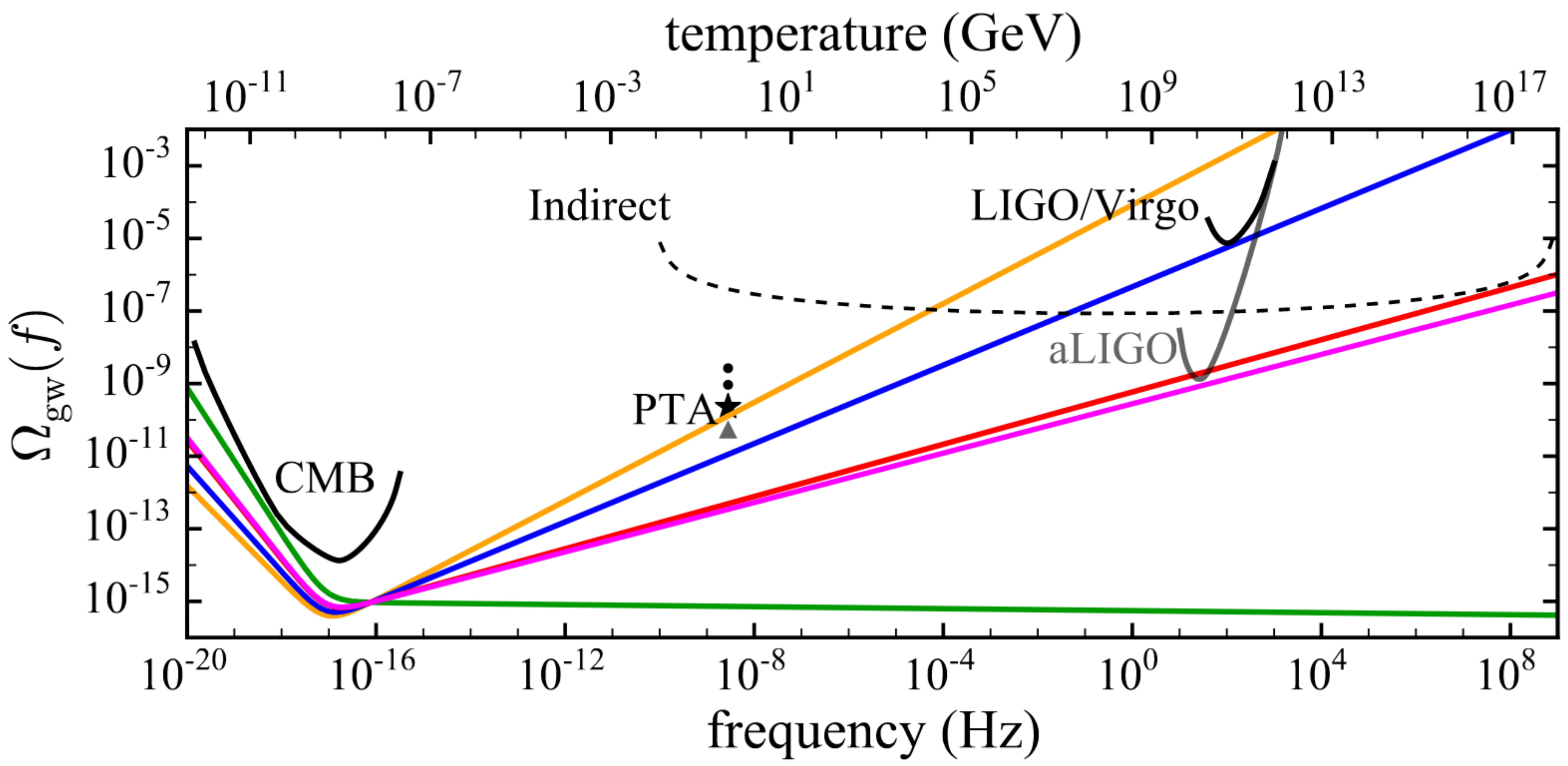
What's the origin of the 2023 detection?

How does it change at high freq?

Backgrounds from fundamental physics

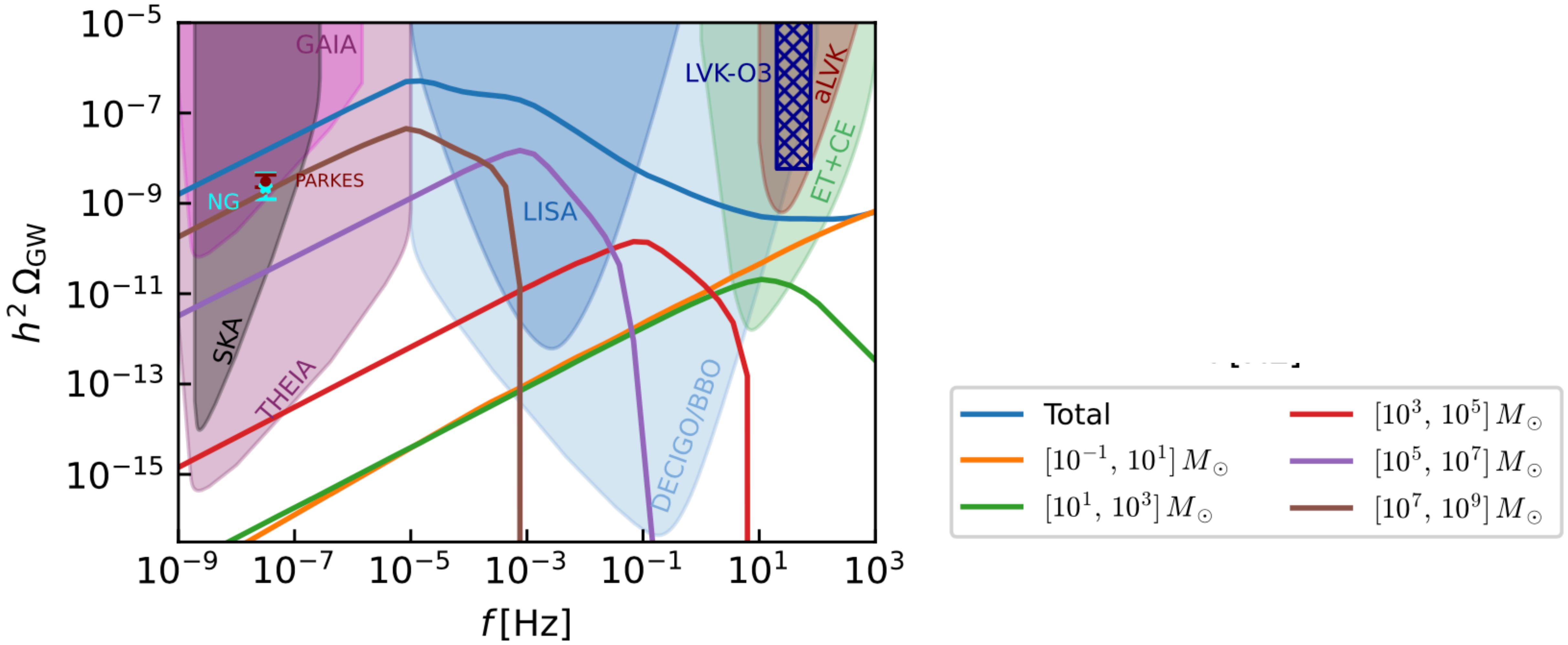
Inflation

Lasky et al PRX 6, 011035 (2016)



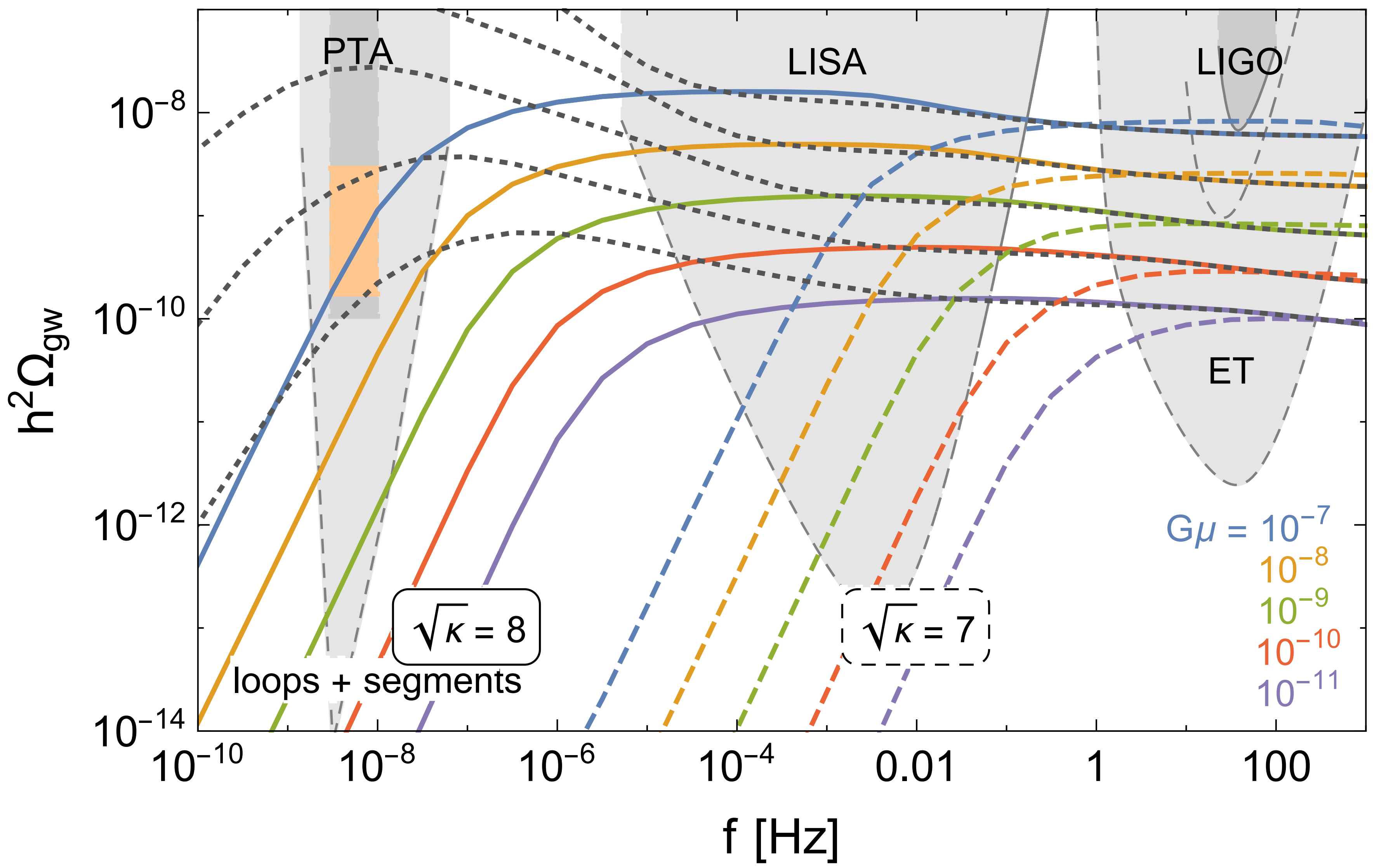
PBH

Braglia et al. JCAP 12 (2021) 12



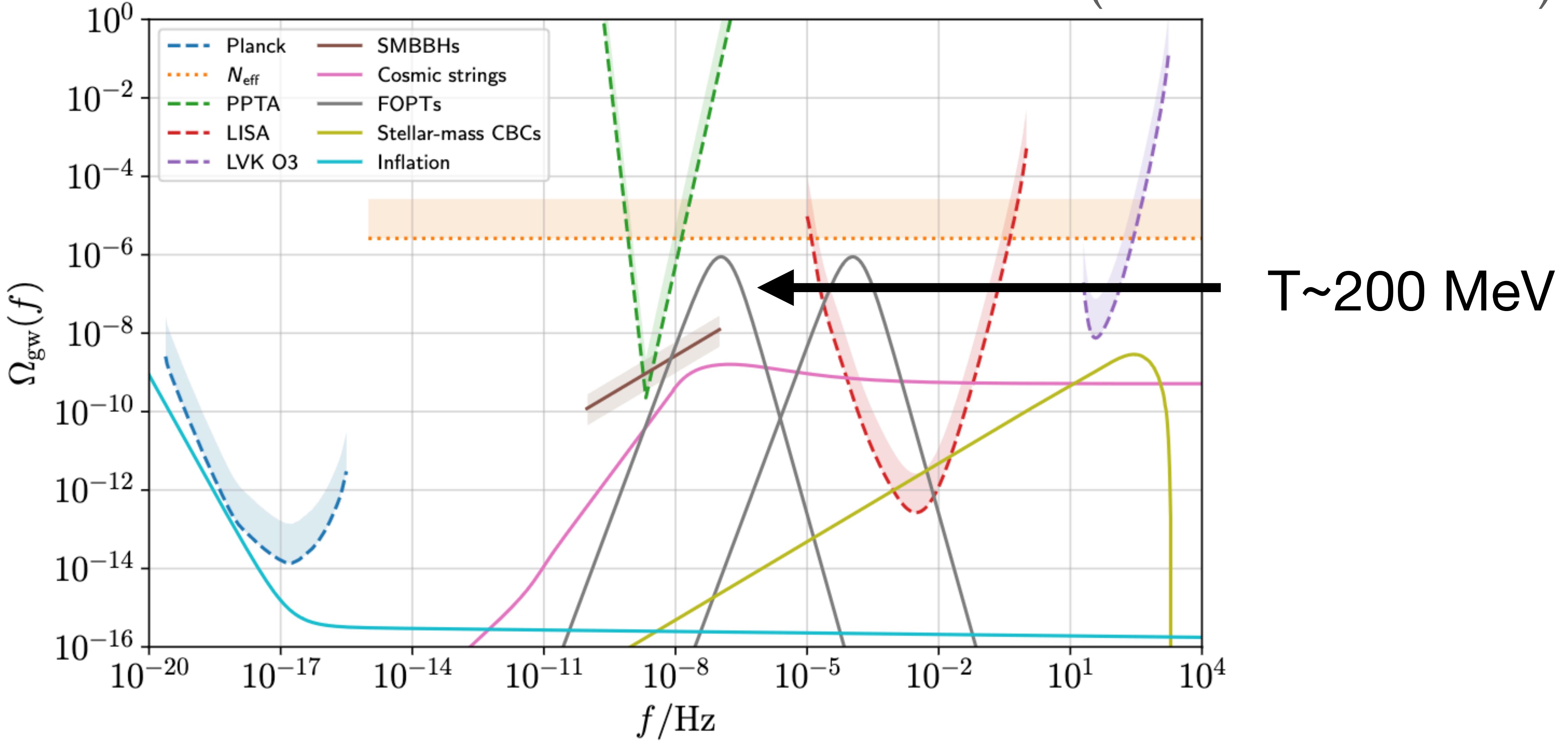
Cosmic Strings

Buchmuller et al. 2107.04578



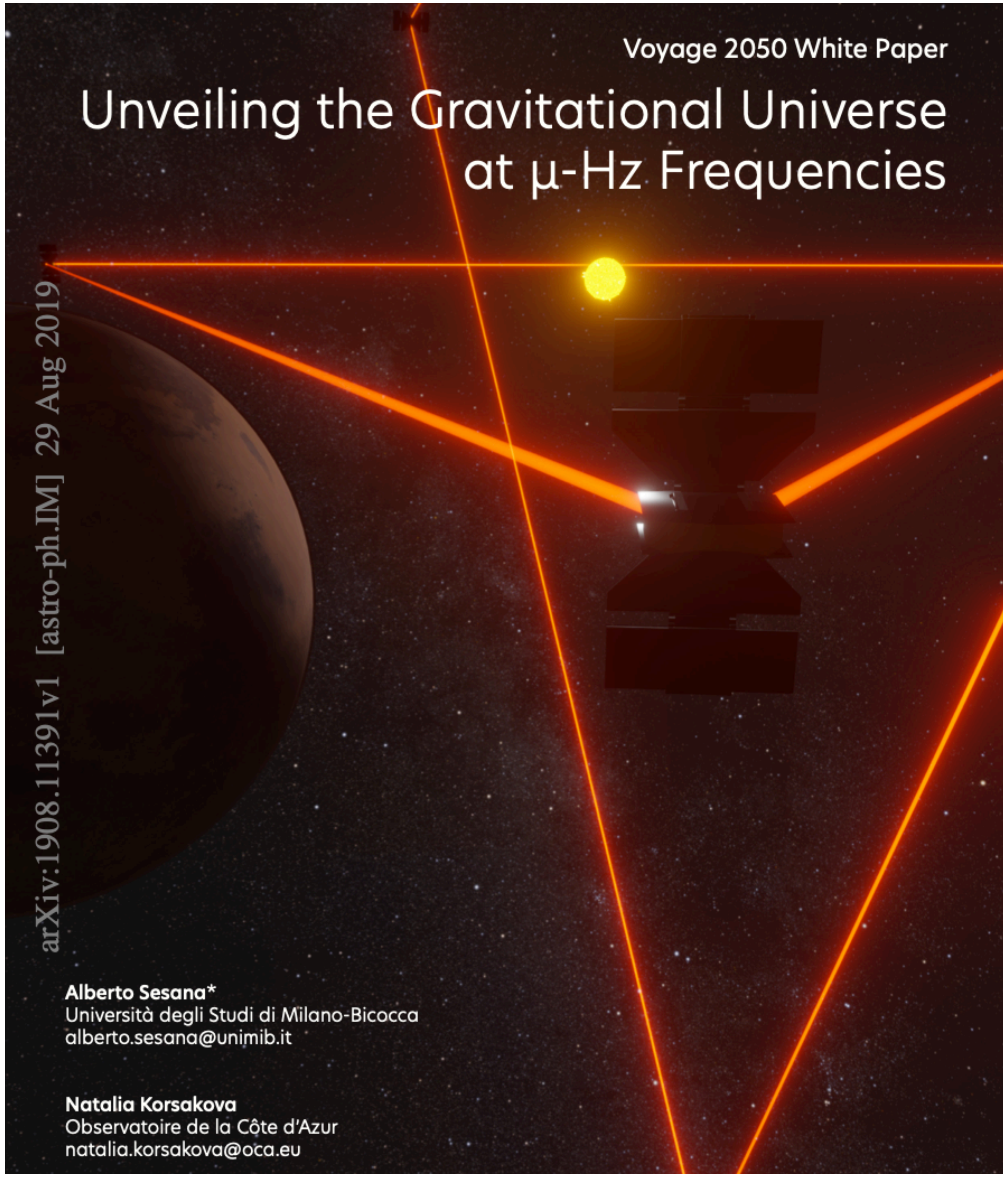
FOPT

Renzini et al 2202.00178
(based on 1512.06239)



Possible backgrounds & **ideas** at μHz: a rich band

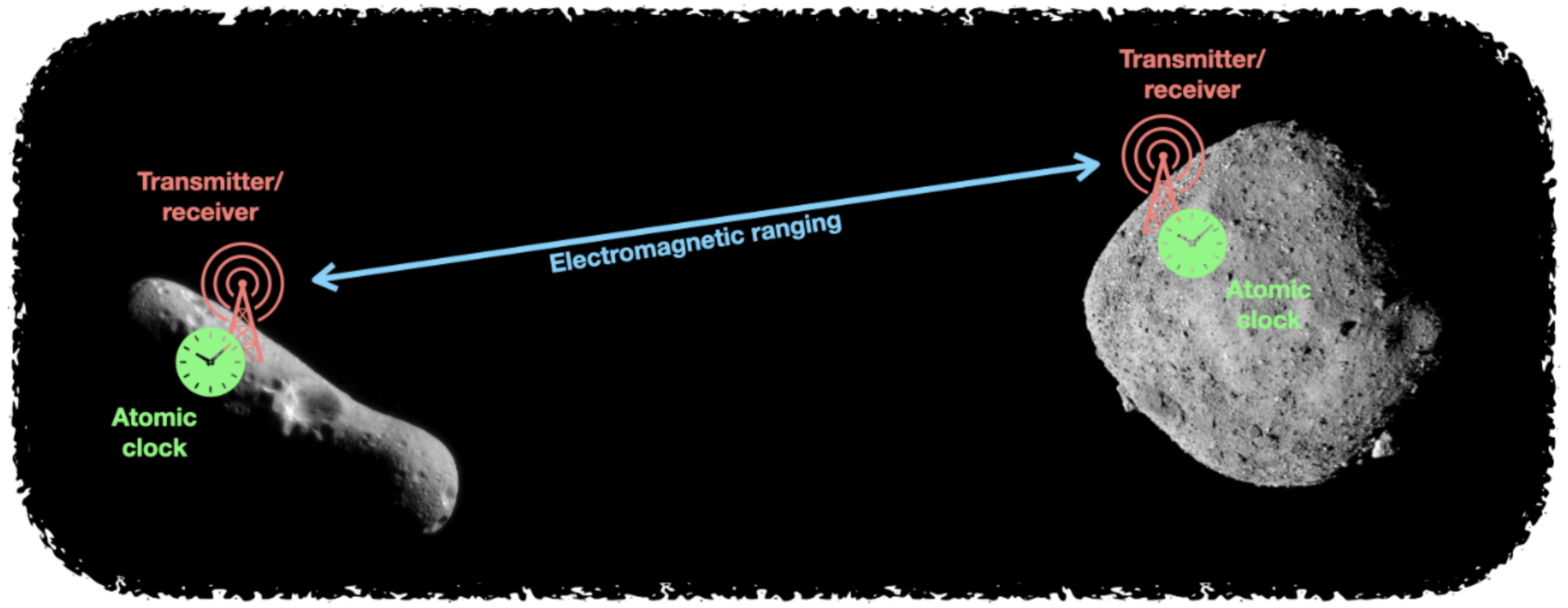
i) μAres: LISA-like concept

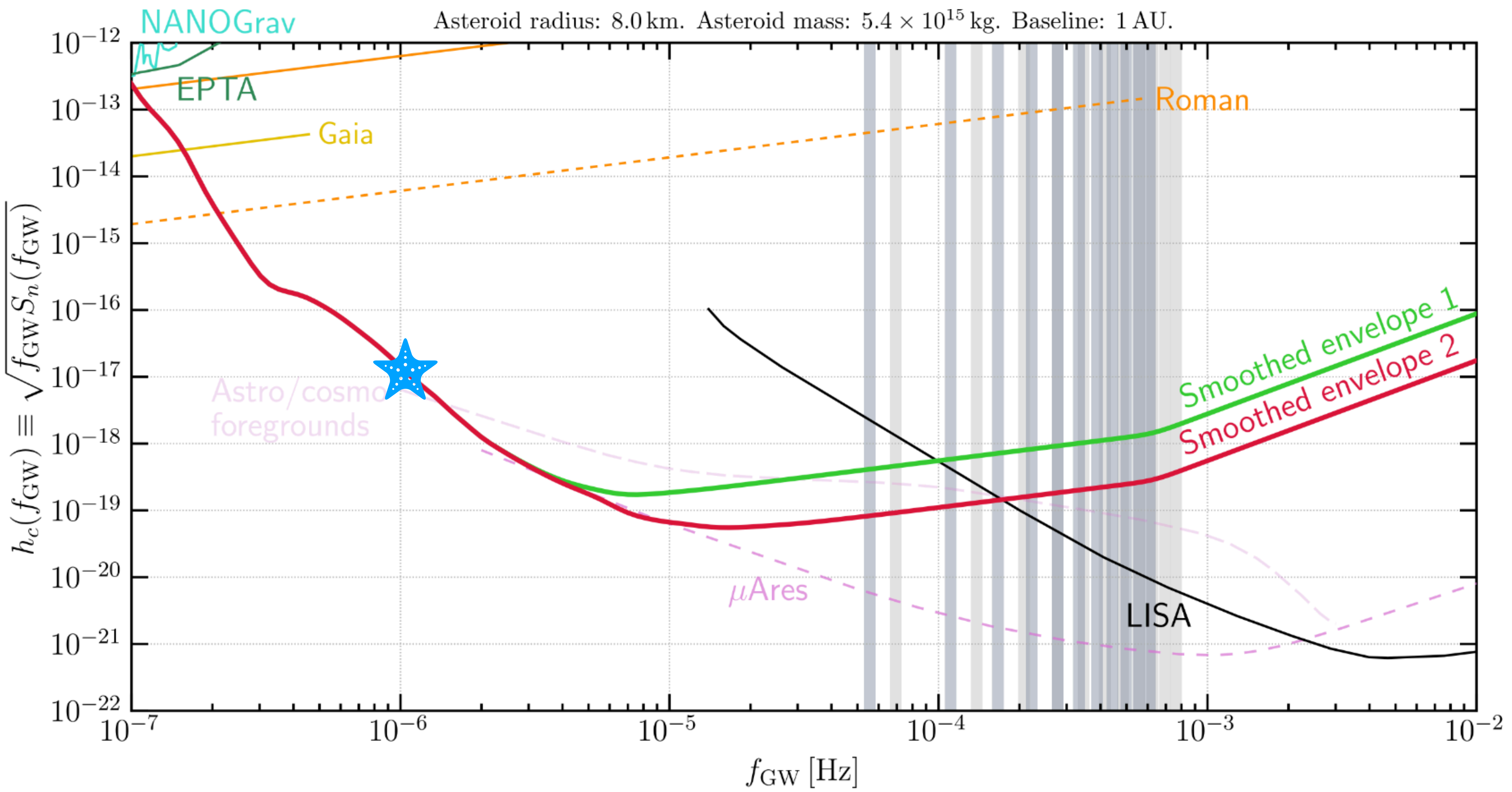


z [AU]
y [AU]
x [AU]
LISA
μAres (in plane)
μAres (out of plane)
The μAres detection landscape
Characteristic amplitude
Frequency [Hz]
SKA
LISA
μAres
~1000 inspiralling SMBHBs out to z~10
SgrA*+0.05M$_\odot$ 10^6 yr to merger
Hundreds of merging MBHBs out to z~20
~100k Galactic DWDs
SgrA*+10M$_\odot$ 10^8 yr to merger
~100 Galactic BHBs
Galactic binaries GWB
Cosmological MBHB GWB
10^{8}M$_\odot$+10^{4}M$_\odot$ IMRI @z=7
10^{7}M$_\odot$+10^{4}M$_\odot$ IMRI @z=7
10^{7}M$_\odot$+10^{3}M$_\odot$ IMRI @z=1
3×10^{5}M$_\odot$+10M$_\odot$ EMRI @z=1
>1k extragalactic BHBs

ii) Ranging of asteroids?

Fedderke et al 2112.11431





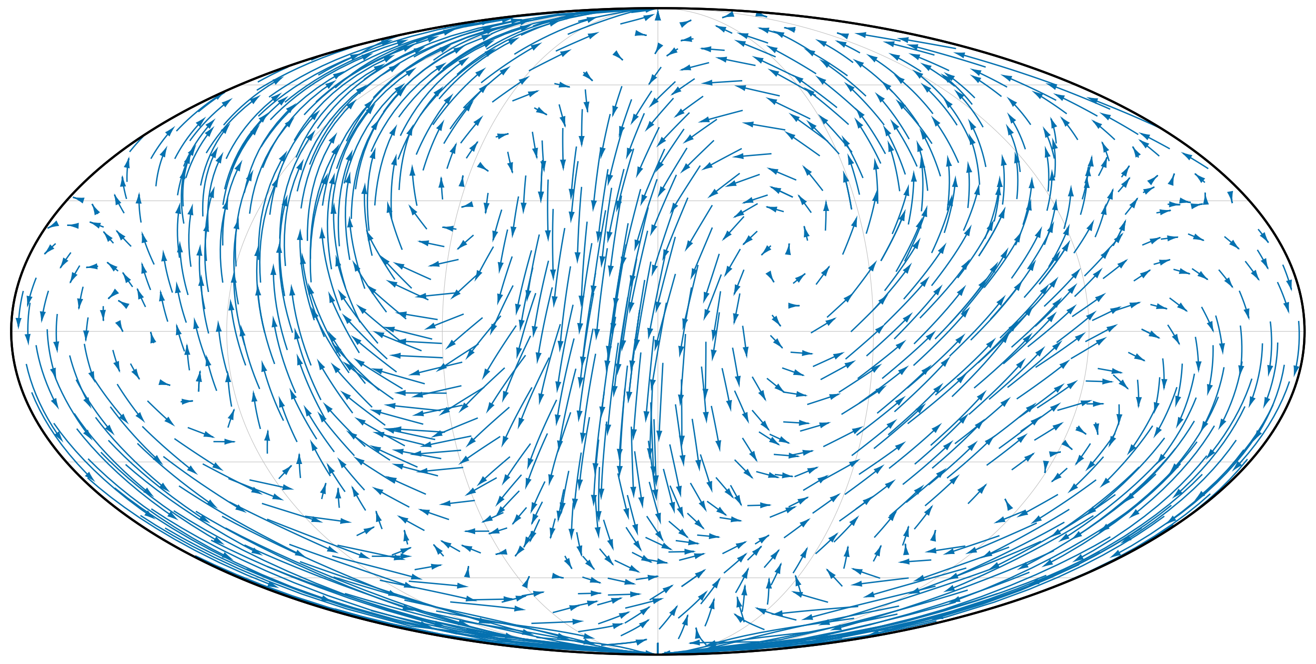
iii) Future astrometry?

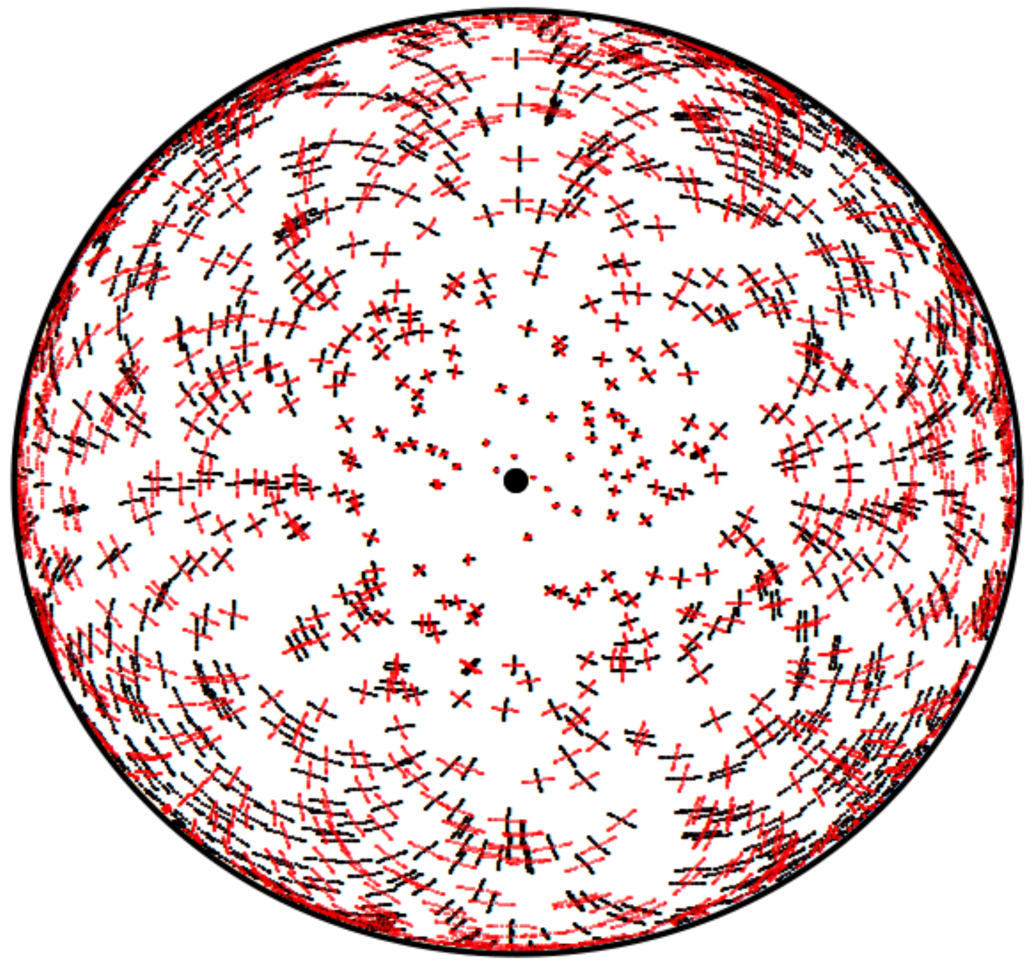
e.g. Moore et al 1707.06239
Mihaylov et al. 1804.00660

Klioner 1710.11474

Garcia-Bellido et al. 2104.04778

Monitoring many stars (GAIA or better)





Fedderke et al 2204.07677

Stellar interferometry

We evaluate the potential for gravitational-wave (GW) detection in the frequency band from 10 nHz to 1 μHz using extremely high-precision astrometry of a small number of stars

at characteristic strains around $h_c \sim 10^{-17} \times (\mu\text{Hz}/f_{\text{GW}})$. The astrometric angular precision required to see these sources is $\Delta\theta \sim h_c$ after integrating for a time $T \sim 1/f_{\text{GW}}$. We show that jitter in the photometric center of WD of this type due to starspots is bounded to be small enough to permit this high-precision, small-N approach. We discuss possible noise arising from stellar reflex motion induced by orbiting objects and show how it can be mitigated. The only plausible technology able to achieve the requisite astrometric precision is a space-based stellar interferometer. Such a future mission with few-meter-scale collecting dishes and baselines of $\mathcal{O}(100\,\text{km})$ is sufficient to achieve the target precision. This collector size is broadly in line with the collectors proposed for

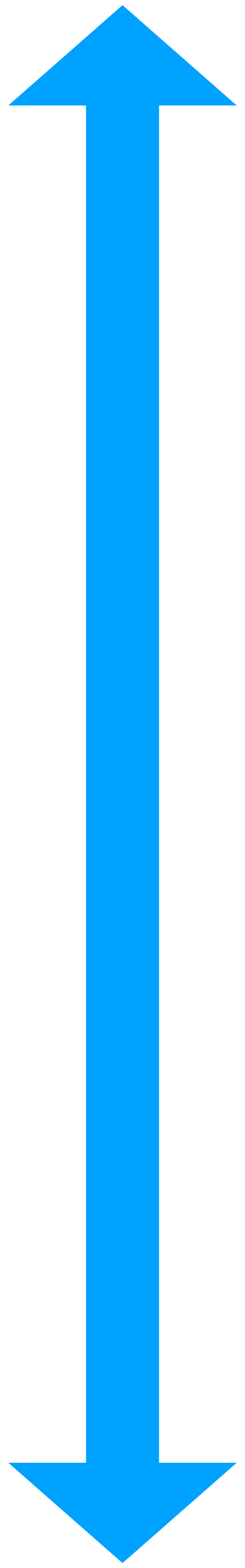
iv) Atomic interferometry in space: AEDGE

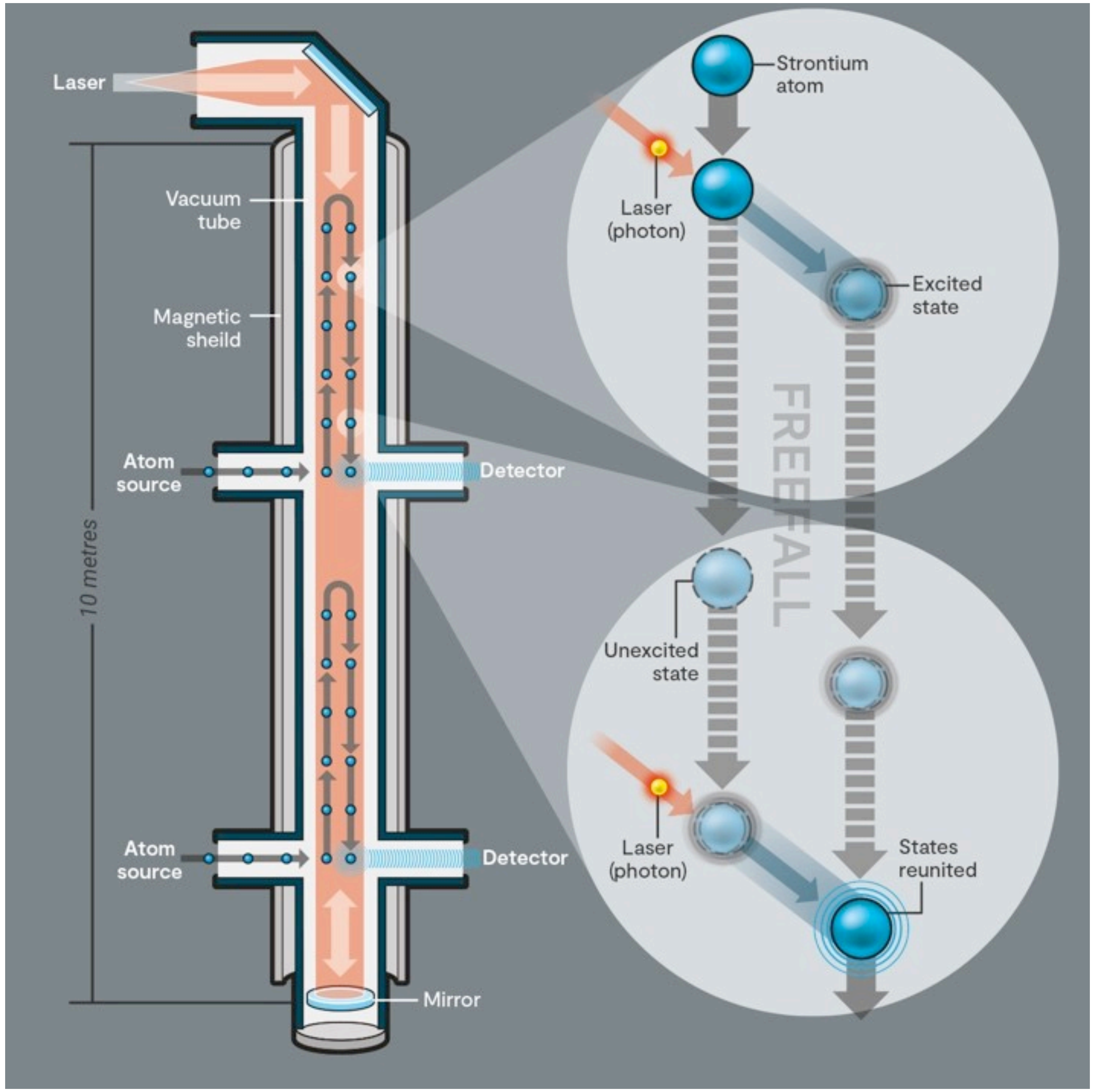
Badurina et al 2108.02468 (AION)

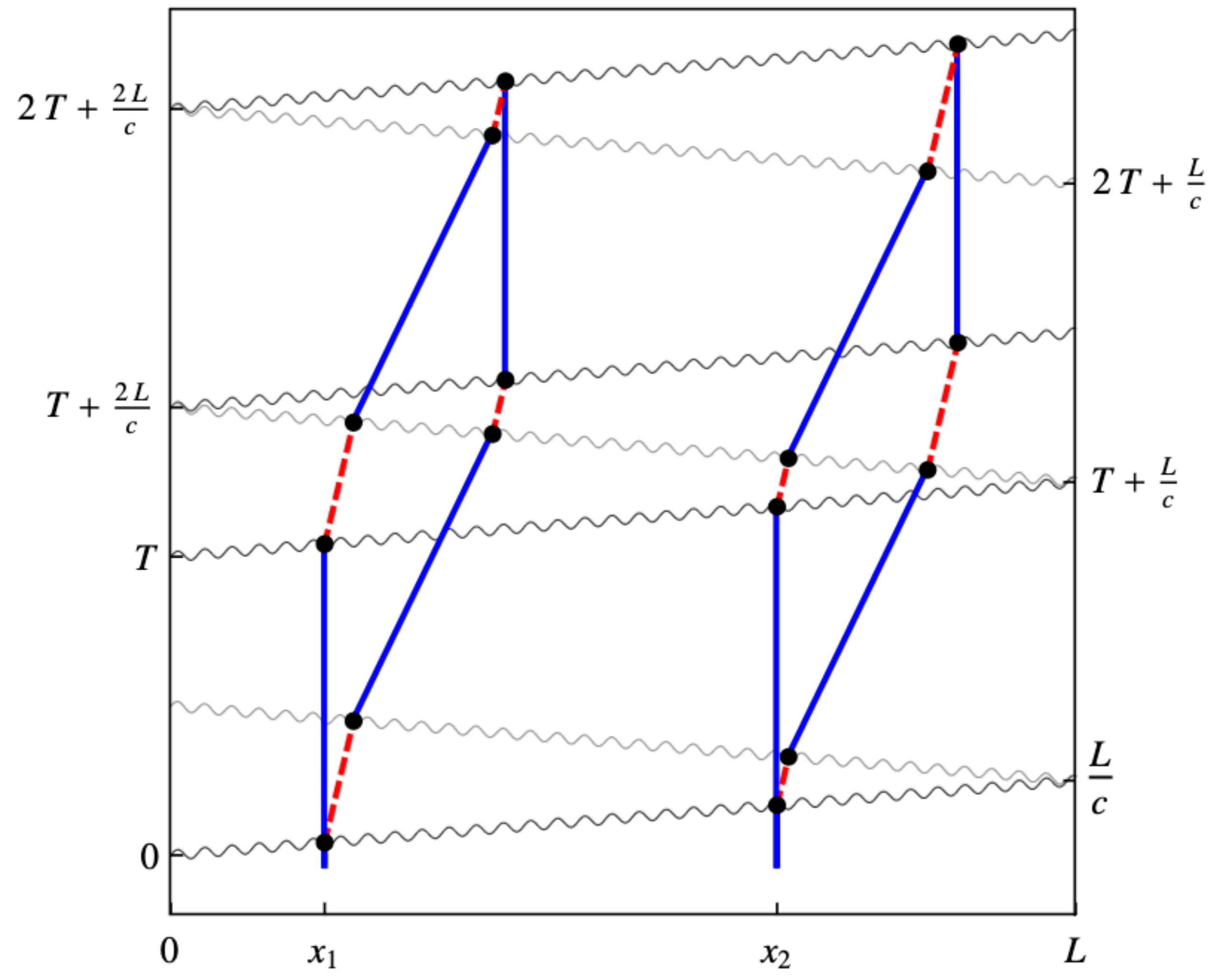
Abou El-Neaj et al 1908.00802

Graham et al 1206.0818 (MAGIS)

40000 km

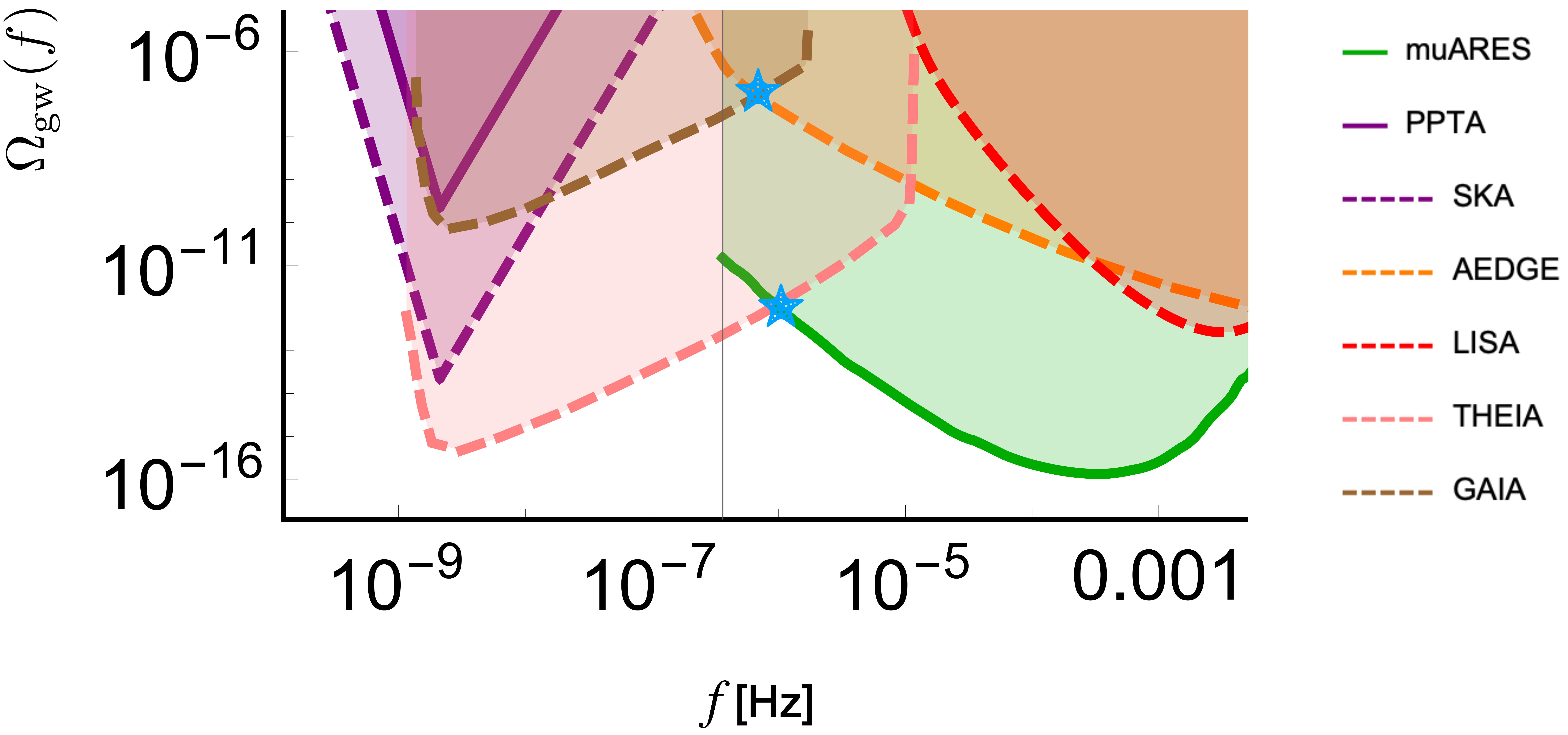




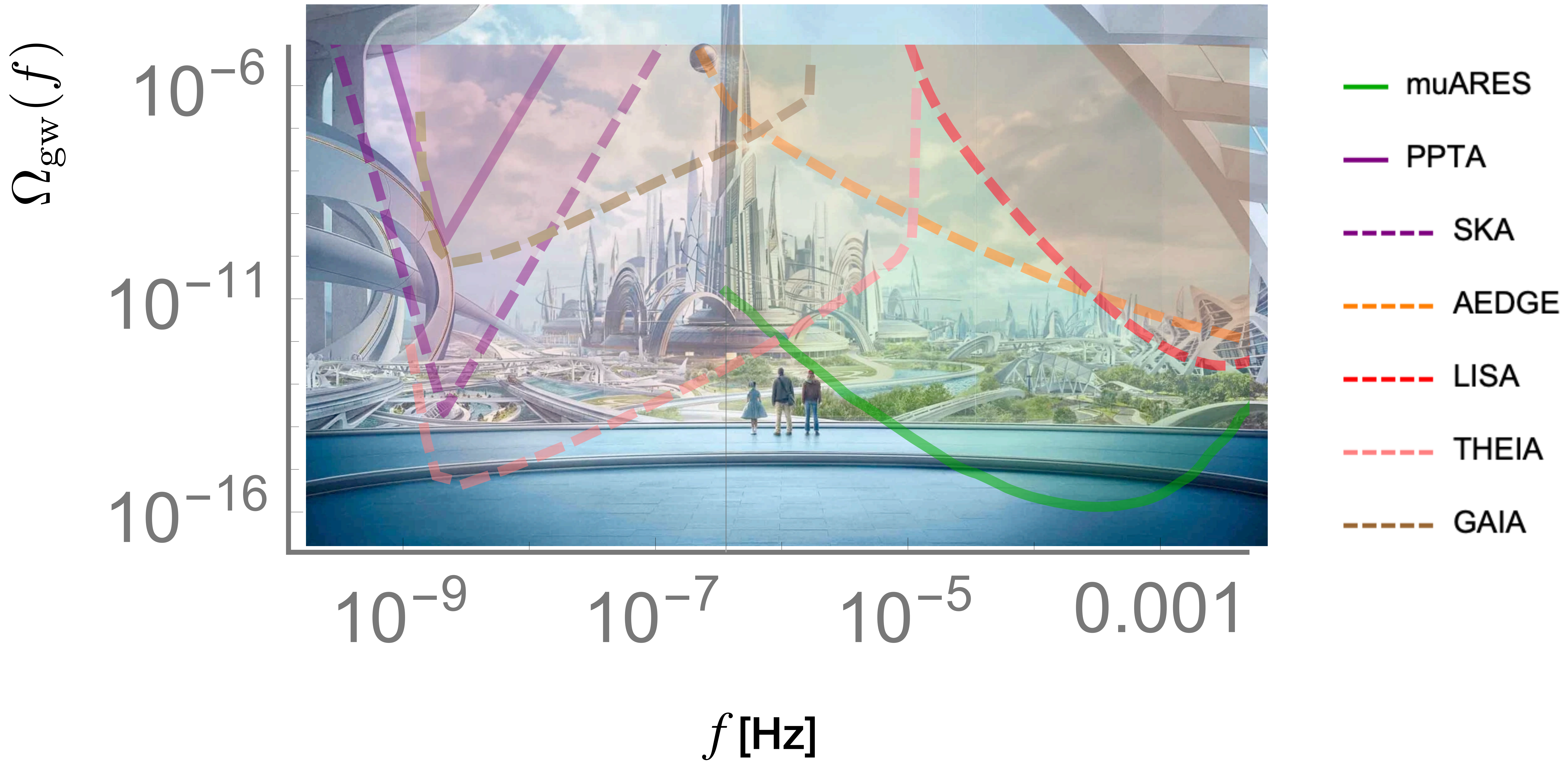


$$\Delta\phi \sim \omega L h$$

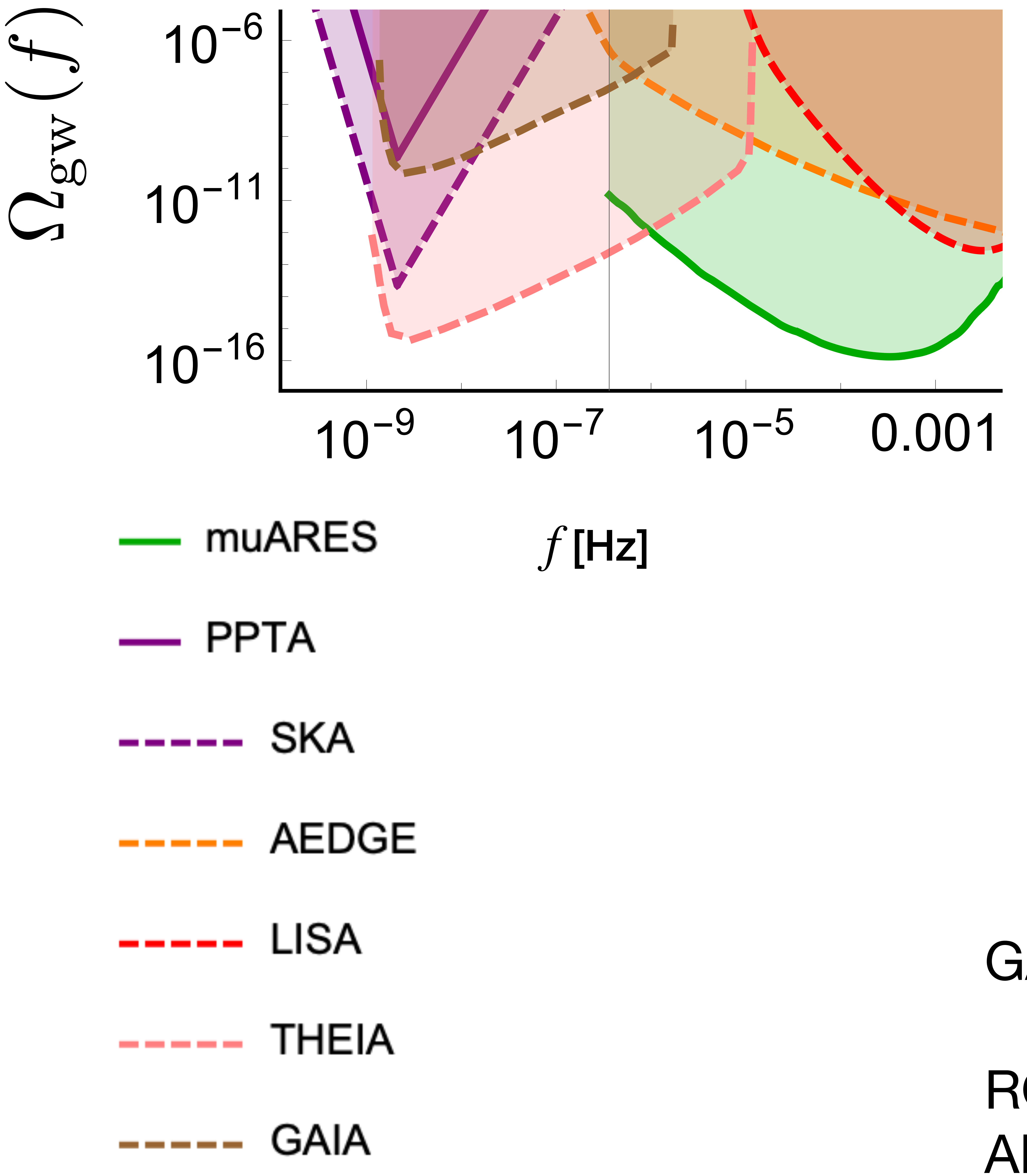
The most optimistic future…

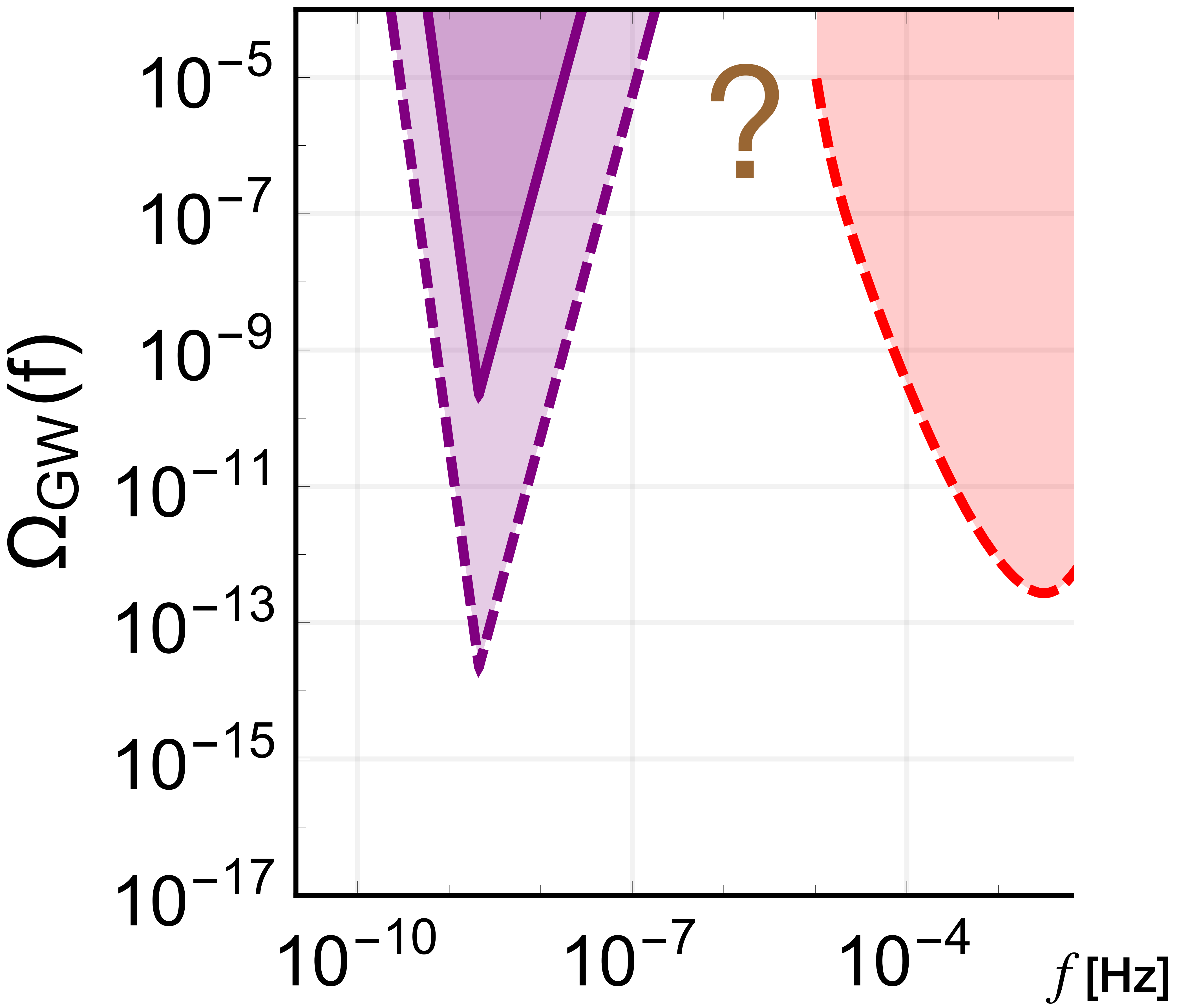


The most optimistic future…



The most optimistic future… vs 2038





GAIA DR3 Jaraba et al 2304.06350

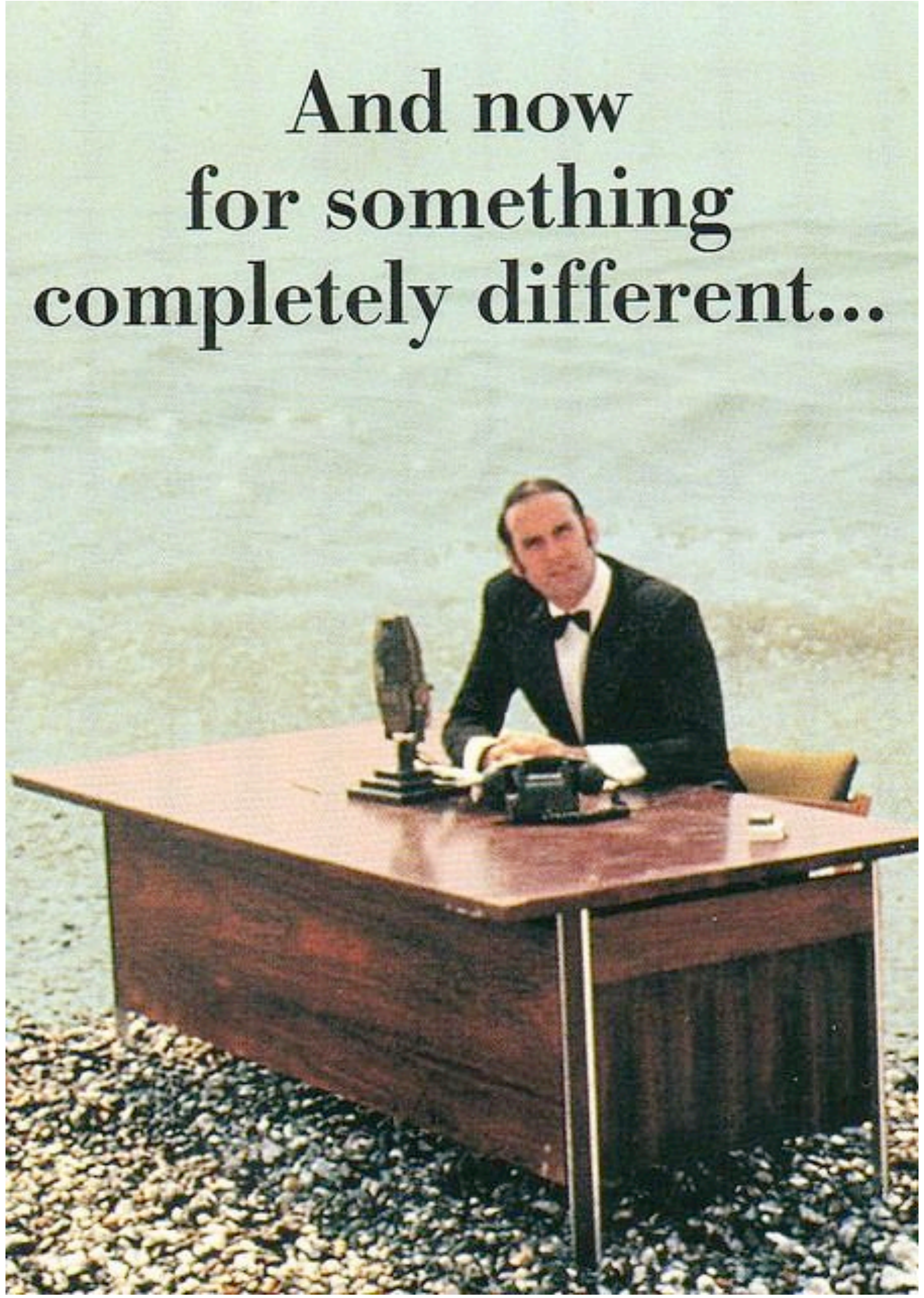
$$h_{70}^2\Omega_{GW} \lesssim 0.087 \text{ for } 4.2\times10^{-18} \text{ Hz} \lesssim f \lesssim 1.1\times10^{-8} \text{ Hz}$$

ROMAN? Wang et al 2205.07962

AION 10m/MAGIS 100m in 2025? (small interferometers)

Is this all we can do in this band?

Is this all we can do in this band?



$f \sim \mu\mathrm{Hz}$

few days

Absorption of GWs by binaries

$f \sim \mu\mathrm{Hz}$

few days

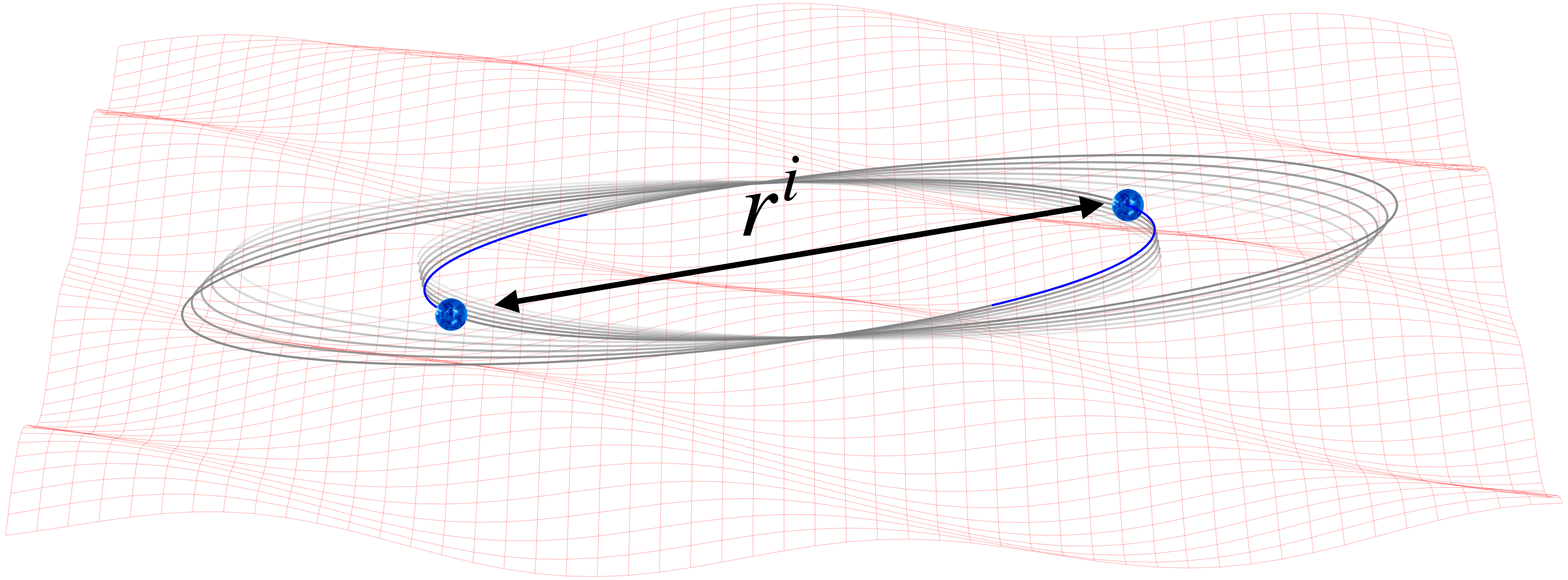
Intuitive idea (from '60s)

Influence of a GW on a binary system (e.g. non-relativistic)

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Newtonian potential

GW

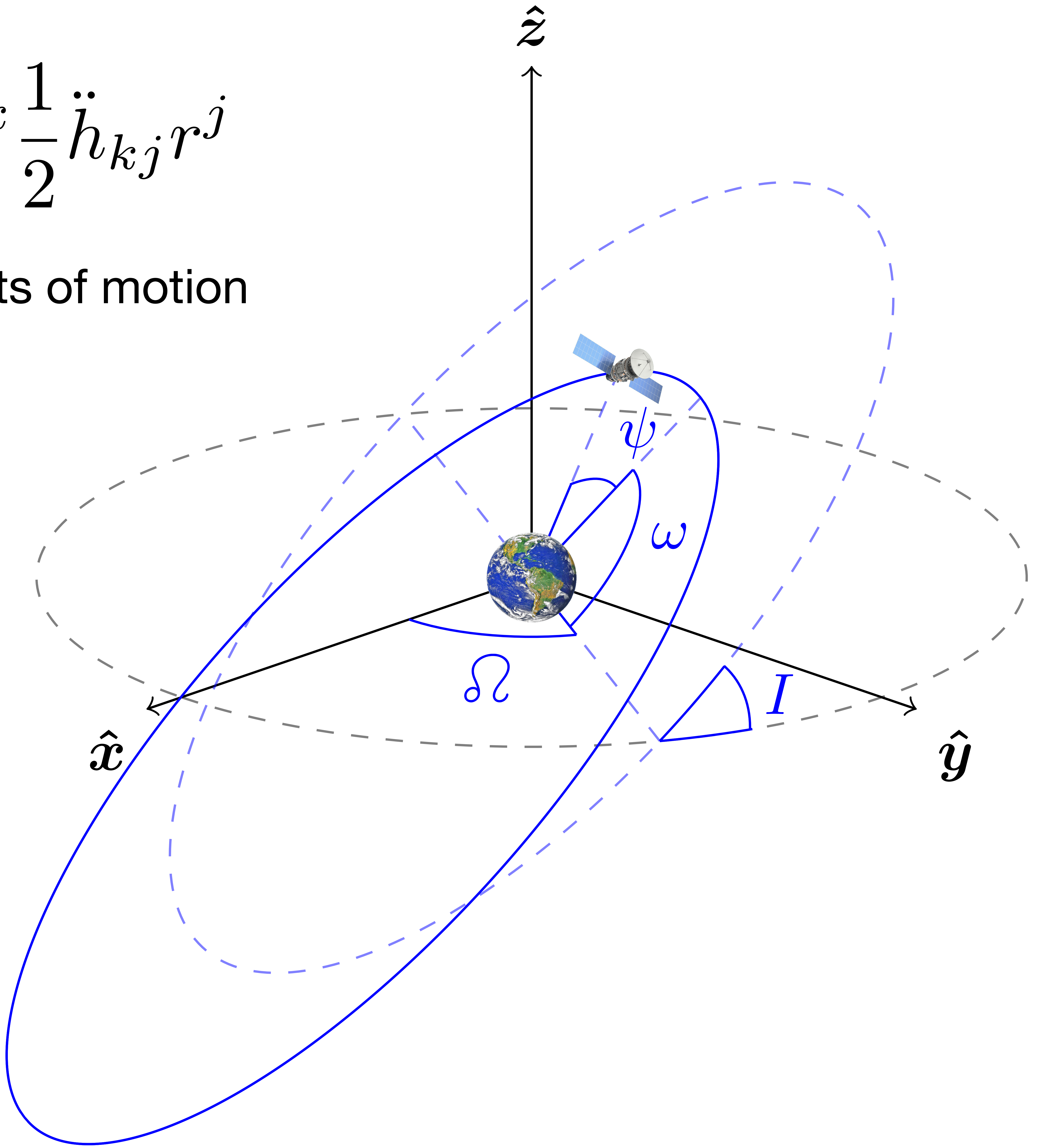


Absorption of GWs by binaries

$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

Better characterised for its 6 Newtonian constants of motion

- **period** P**, eccentricity** e:
  size and *shape* of orbit
- **inlination** I**, ascending node** Ω:
  orientation in space
- **pericentre** ω**,**
  mean anomaly at epoch ε:
  radial and angular *phases*



Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

- for generic perturbation:

$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi} + \mathcal{F}_\theta\right],$$

$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2},$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$

Absorption of GWs by binaries

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^2}\hat{\boldsymbol{r}} = \delta\ddot{\boldsymbol{r}}.$$

- for generic perturbation:

$$\delta\ddot{\boldsymbol{r}} = r(\mathcal{F}_r\hat{\boldsymbol{r}} + \mathcal{F}_\theta\hat{\boldsymbol{\theta}} + \mathcal{F}_\ell\hat{\boldsymbol{\ell}}),$$

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi} + \mathcal{F}_\theta\right],$$

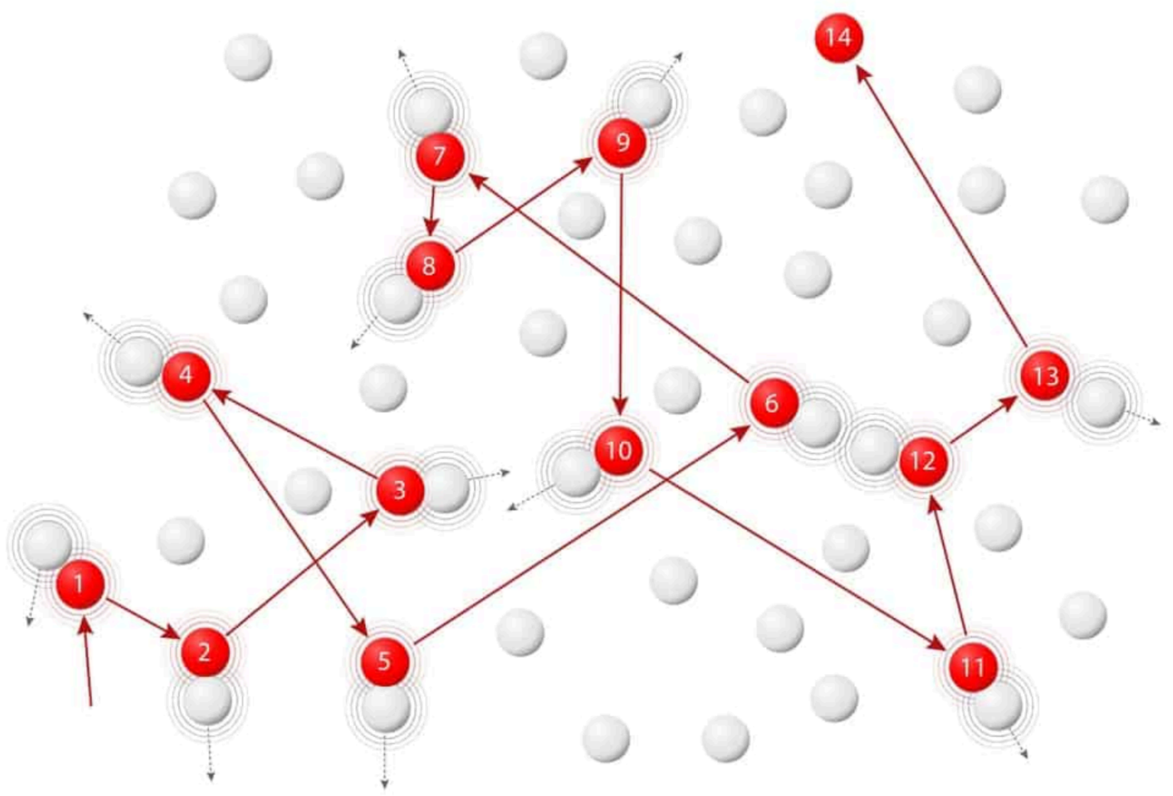
$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2},$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2},$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I},$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\dot{\Omega},$$

$$\dot{\varepsilon} = -\frac{P\gamma^4\mathcal{F}_r}{\pi(1+e\cos\psi)^2} - \gamma(\cos I\dot{\Omega} + \dot{\omega}),$$



For the SGWB… Fokker-Planck approach

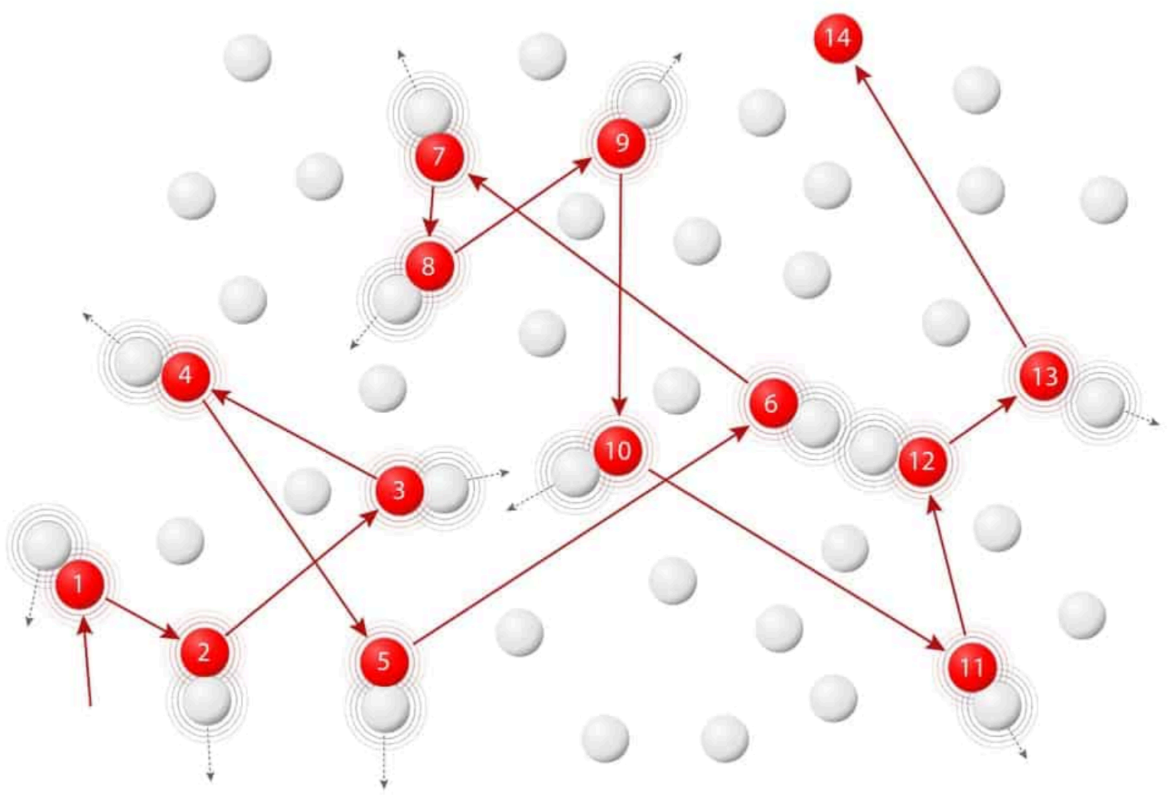
$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

deterministic

stochastic

$$\dot{X}_i(\boldsymbol{X}, t) = V_i(\boldsymbol{X}) + \Gamma_i(\boldsymbol{X}, t)$$

we move from dynamics of the variable to dynamics of the **distribution *W(X)***



$$\frac{\partial W}{\partial t} = -\partial_i \left(D_i^{(1)} W \right) + \partial_i \partial_j \left(D_{ij}^{(2)} W \right)$$

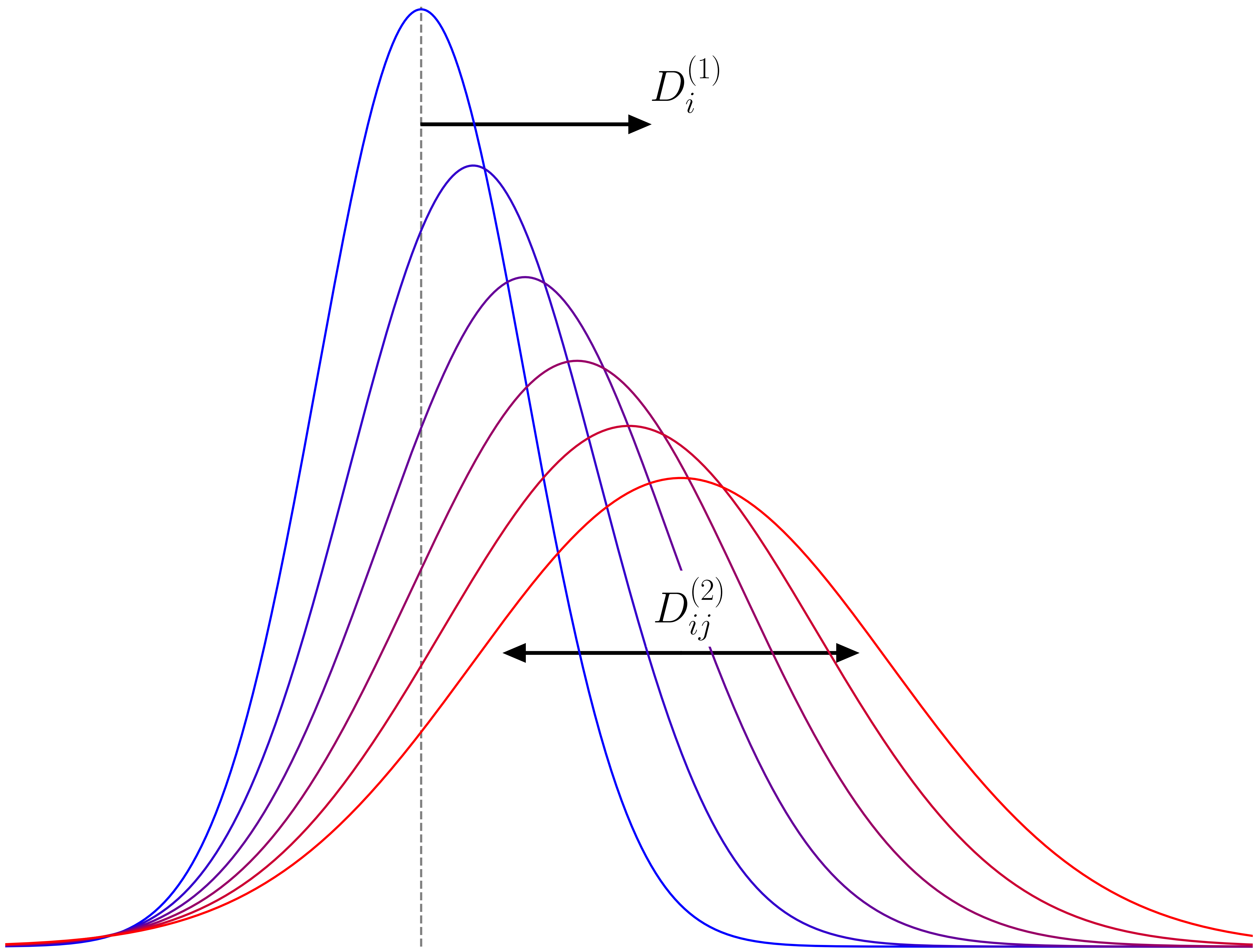
with $\partial_i \equiv \partial / \partial X_i$

$$D_i^{(1)} = V_i + \lim_{\tau \to 0} \frac{1}{\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t'} \mathrm{d}t'' \left\langle \Gamma_j \left(\boldsymbol{x}, t'' \right) \partial_j \Gamma_i \left(\boldsymbol{x}, t' \right) \right\rangle .$$

$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{2\tau} \int_t^{t+\tau} \mathrm{d}t' \int_t^{t+\tau} \mathrm{d}t'' \left\langle \Gamma_i \left(\boldsymbol{x}, t' \right) \Gamma_j \left(\boldsymbol{x}, t'' \right) \right\rangle .$$

Our approach to the problem

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\boldsymbol{X}, t)$ of orbital elements $\boldsymbol{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}\left(D_i^{(1)} W\right) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\left(D_{ij}^{(2)} W\right)$$

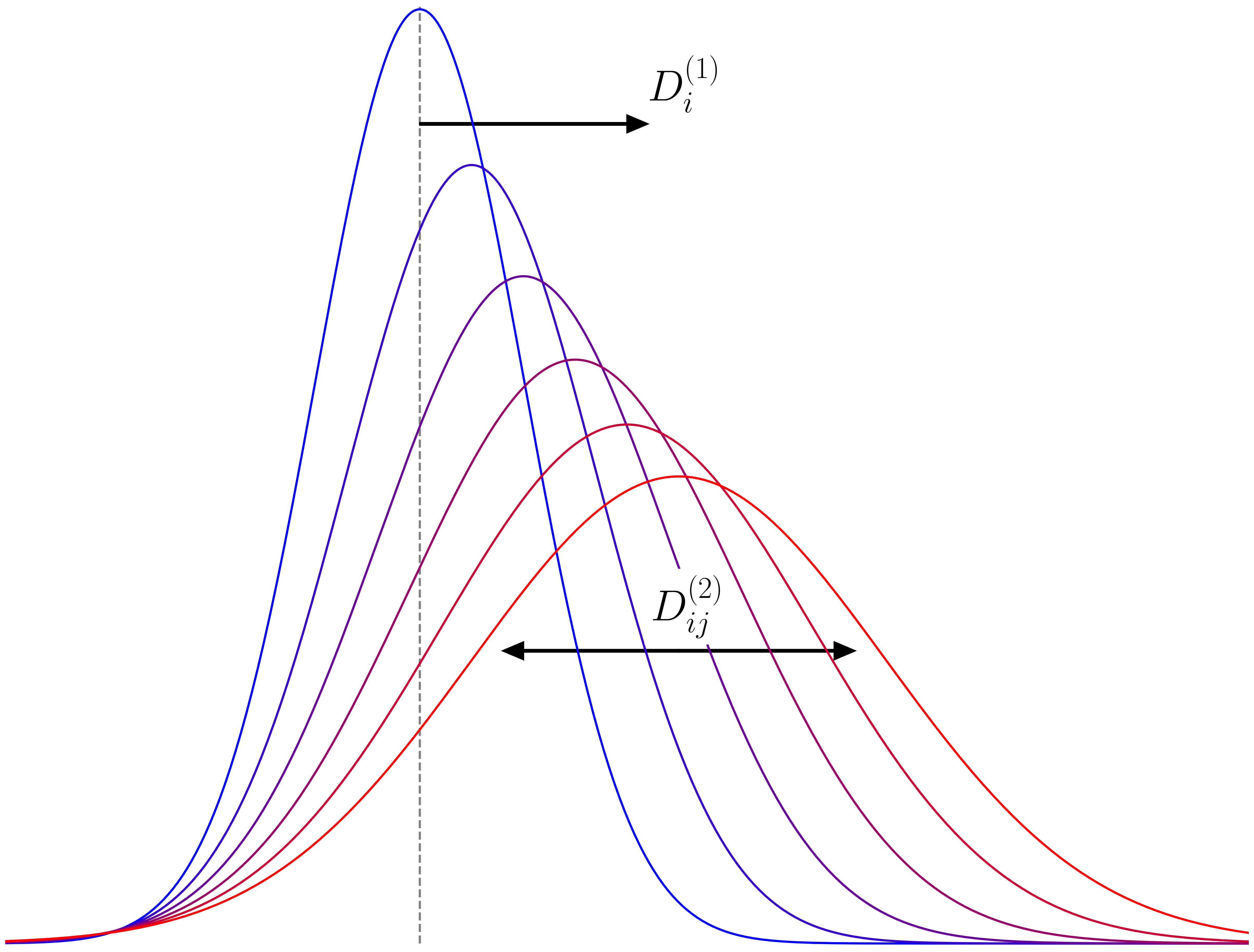
- *drift* and *diffusion* coefficients (averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X})\Omega_{\mathrm{gw}}(n/P)$$

$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X})\Omega_{\mathrm{gw}}(n/P)$$

Our approach to the problem

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\boldsymbol{X}, t)$ of orbital elements $\boldsymbol{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}\left(D_i^{(1)} W\right) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}\left(D_{ij}^{(2)} W\right)$$

- *drift* and *diffusion* coefficients

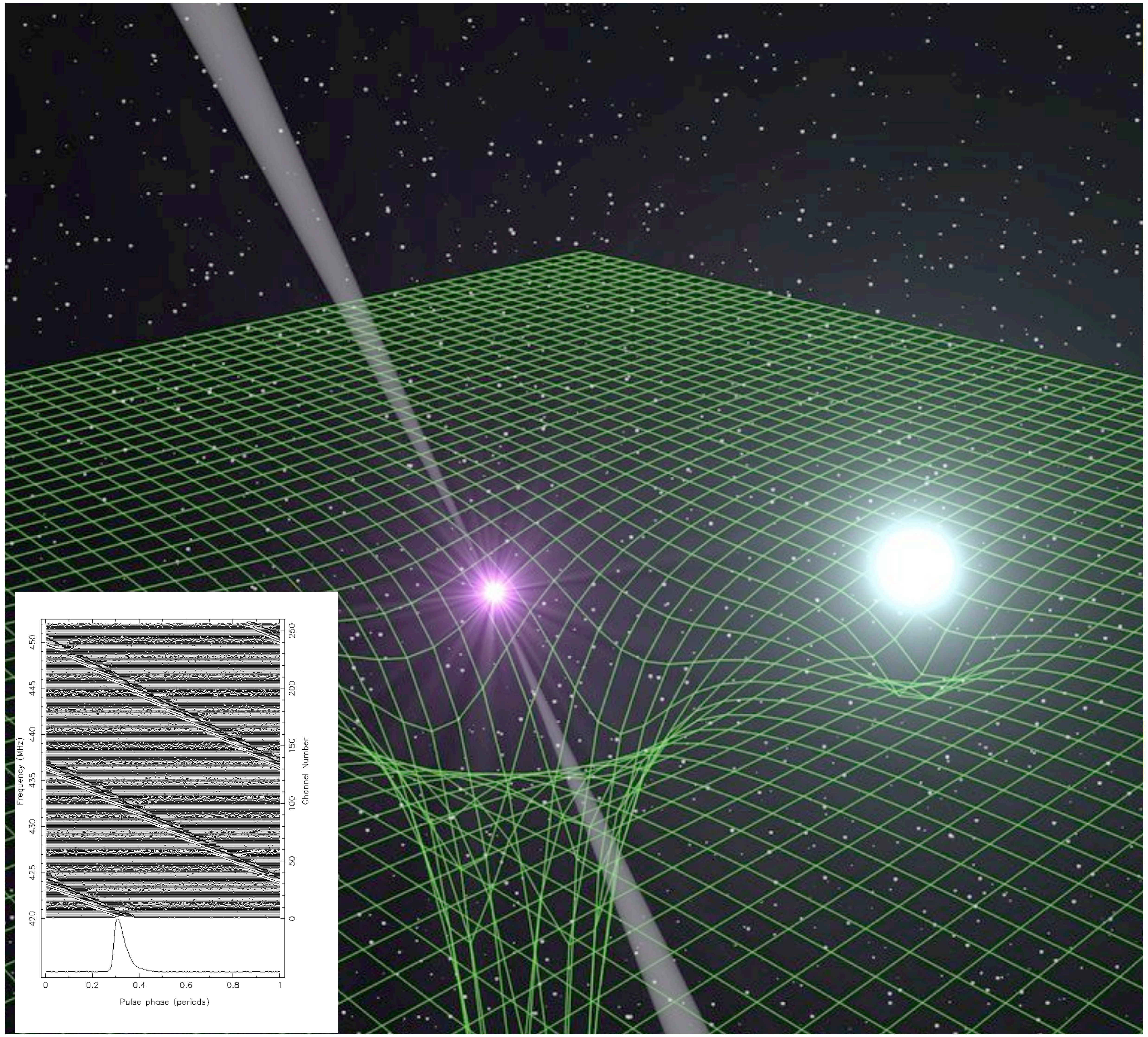
(averaged over orbits)

$$D_i^{(1)}(\boldsymbol{X}) = V_i(\boldsymbol{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\boldsymbol{X}) \Omega_{\mathrm{gw}}(n/P)$$

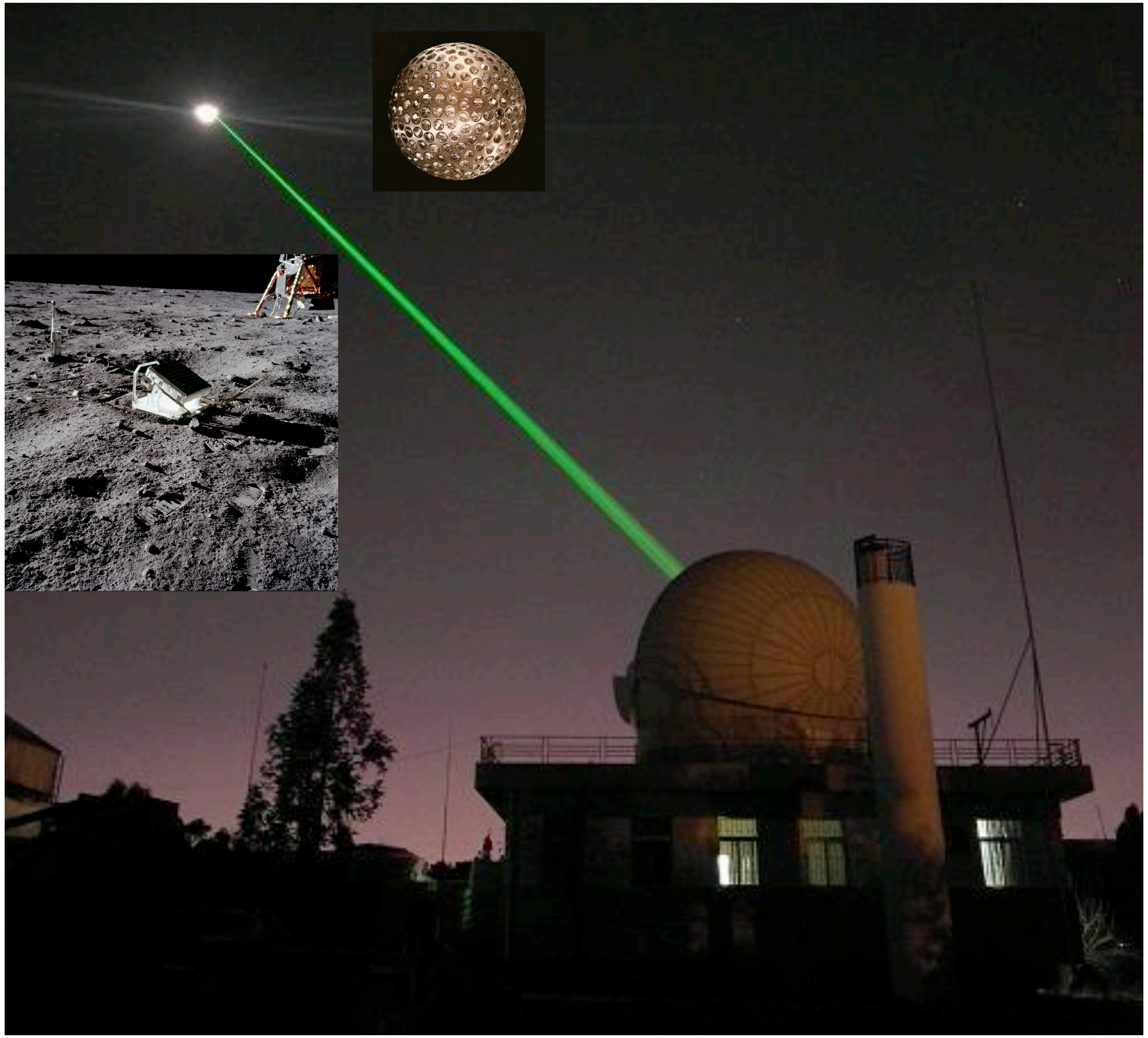
$$D_{ij}^{(2)}(\boldsymbol{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\boldsymbol{X}) \Omega_{\mathrm{gw}}(n/P)$$

Two probes

timing of binary pulsars

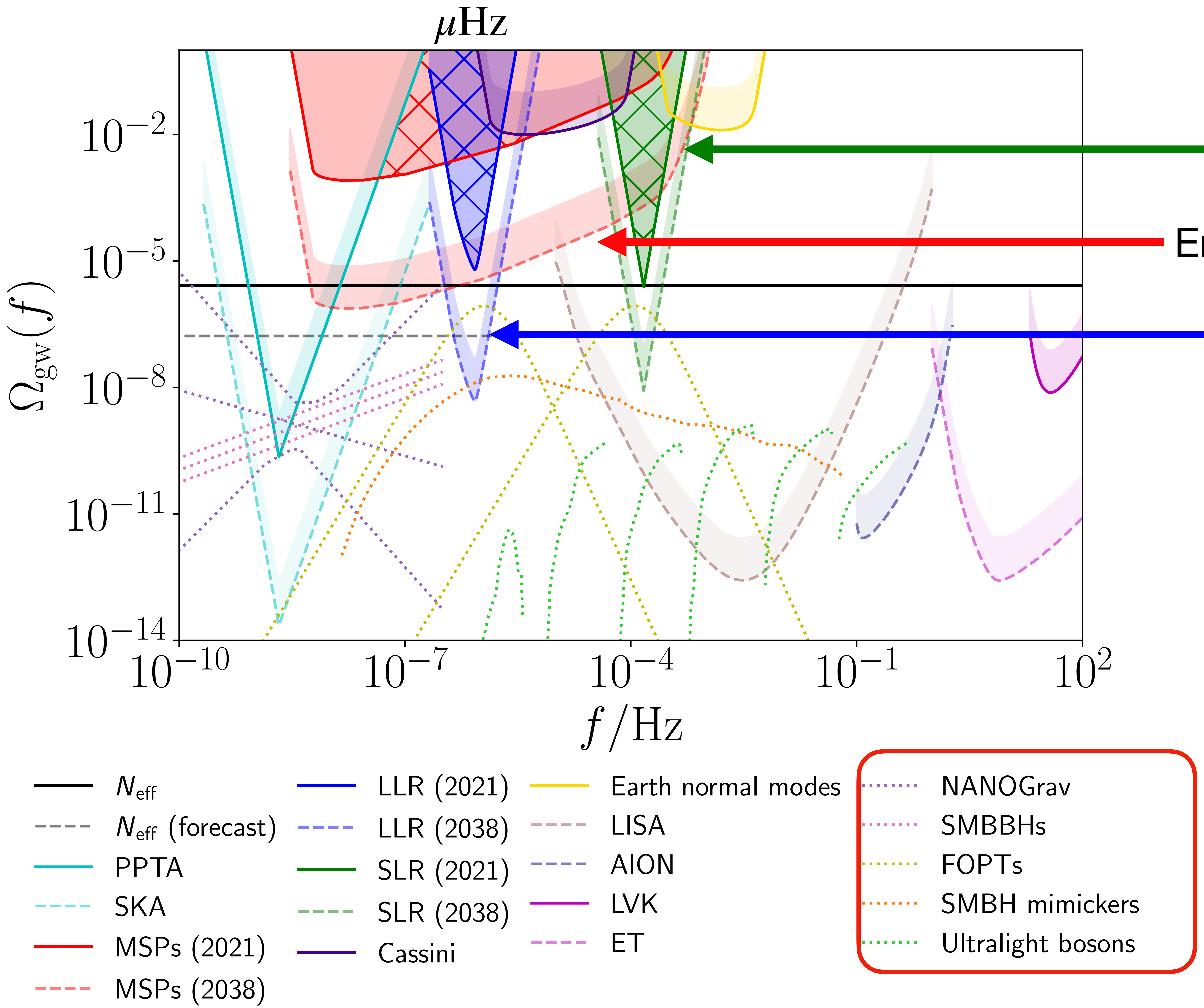


lunar and satellite laser ranging



Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

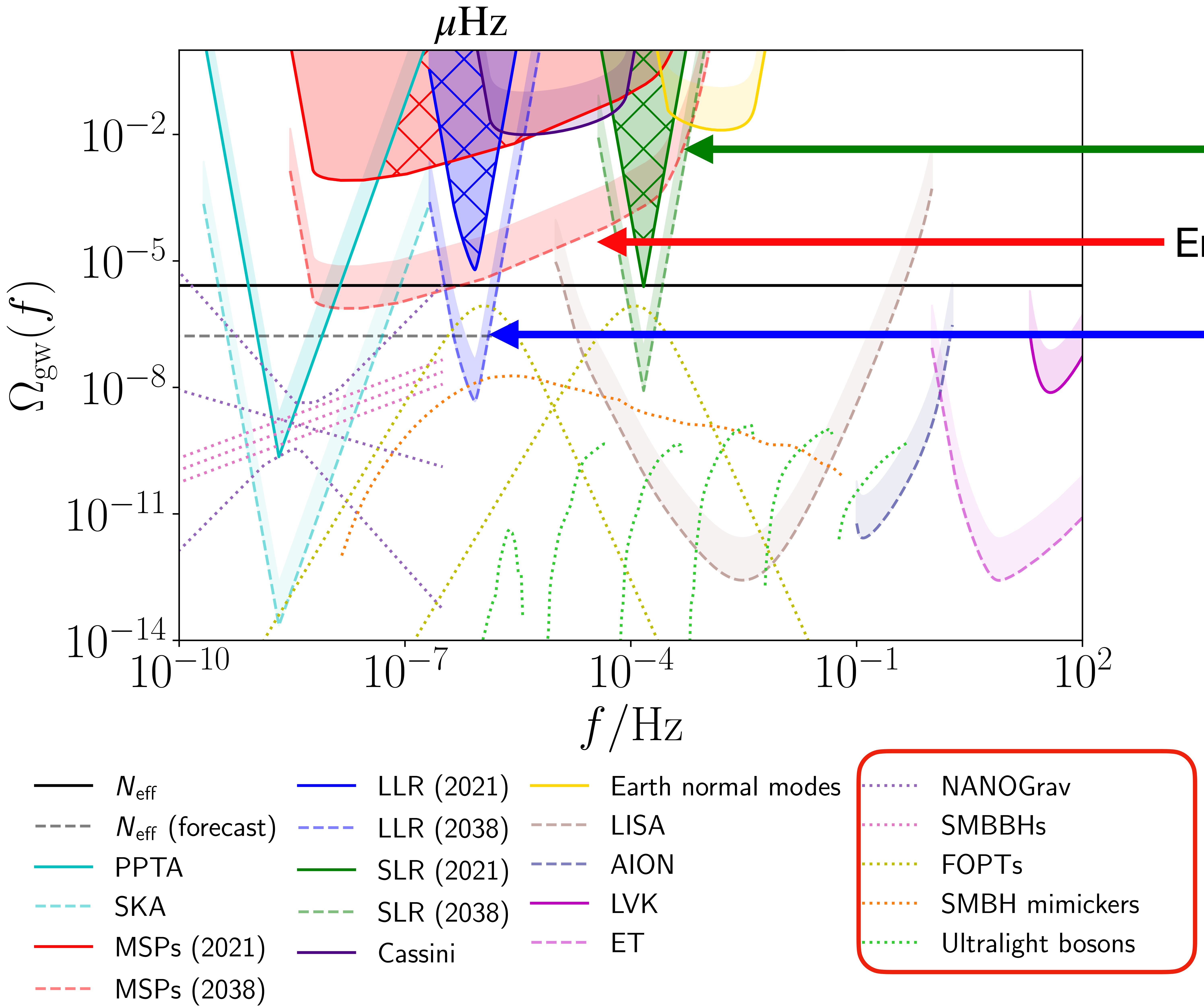
Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors ...may/will happen before 2030!)

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Murphy 1309.6294

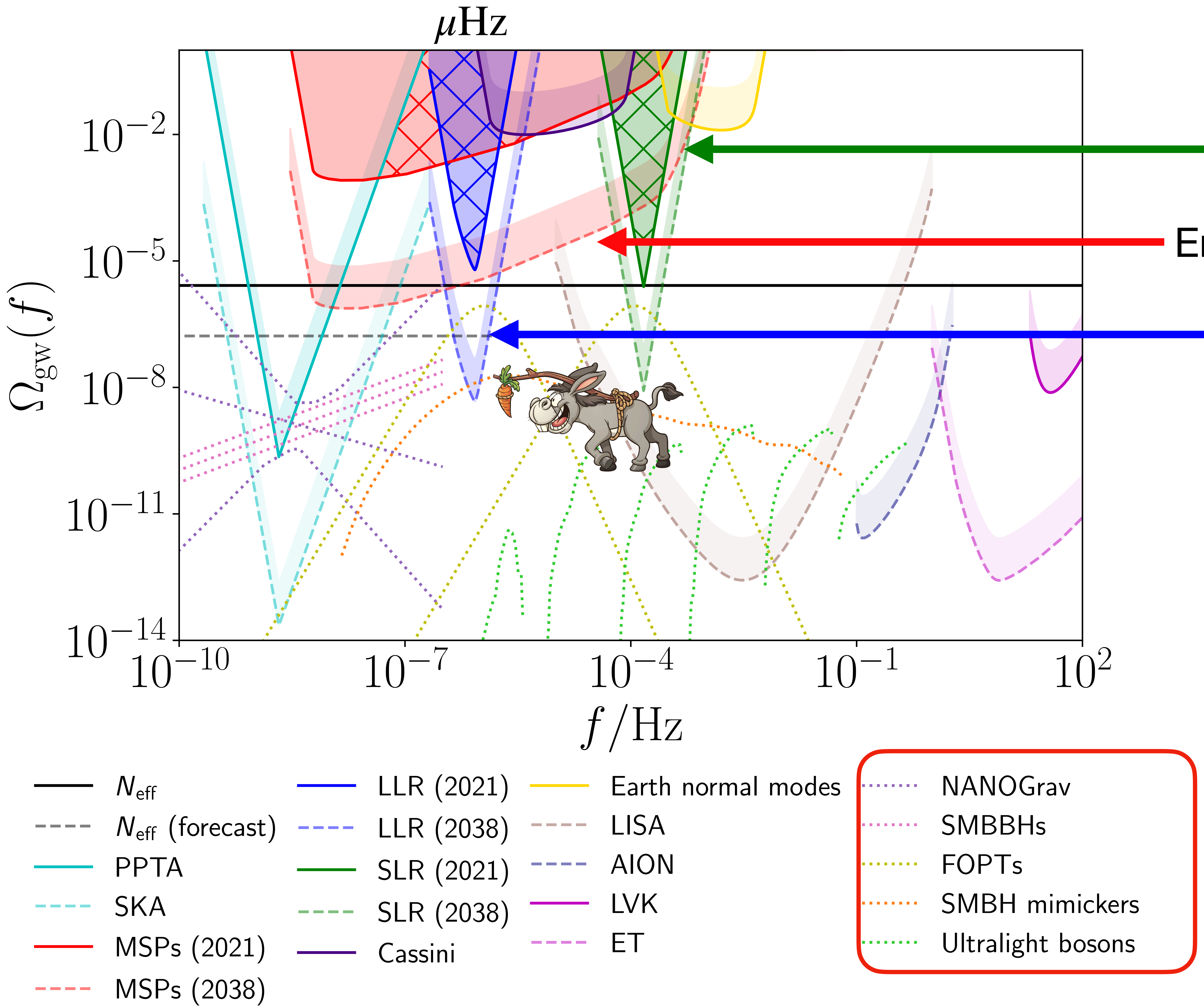
in 2050

$\Omega \lesssim 3 \times 10^{-9}$ at $f \sim \mu$Hz

Possible backgrounds

Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Satellites (*P*~hours)

Envelope of pulsars (*P*~hours - 100 days)

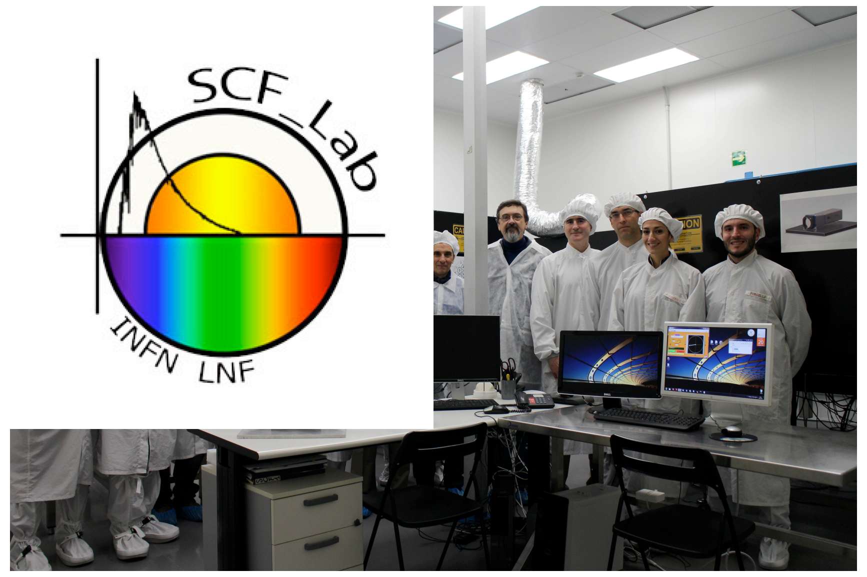
Lunar laser ranging (*P*~month)

(2038 line requires replacing the mirrors …may/will happen before 2030!)

Murphy 1309.6294

in 2050

$$\Omega \lesssim 3 \times 10^{-9} \quad \text{at} \quad f \sim \mu\text{Hz}$$



Help to perform the
real analysis welcome!

Miró, 1937.
“Help Spain” poster calling
for international help in
to support the democratic republic
in the Spanish Civil war.

μHz GWs

- The μHz band is very rich for **astrophysical** and **cosmological** sources

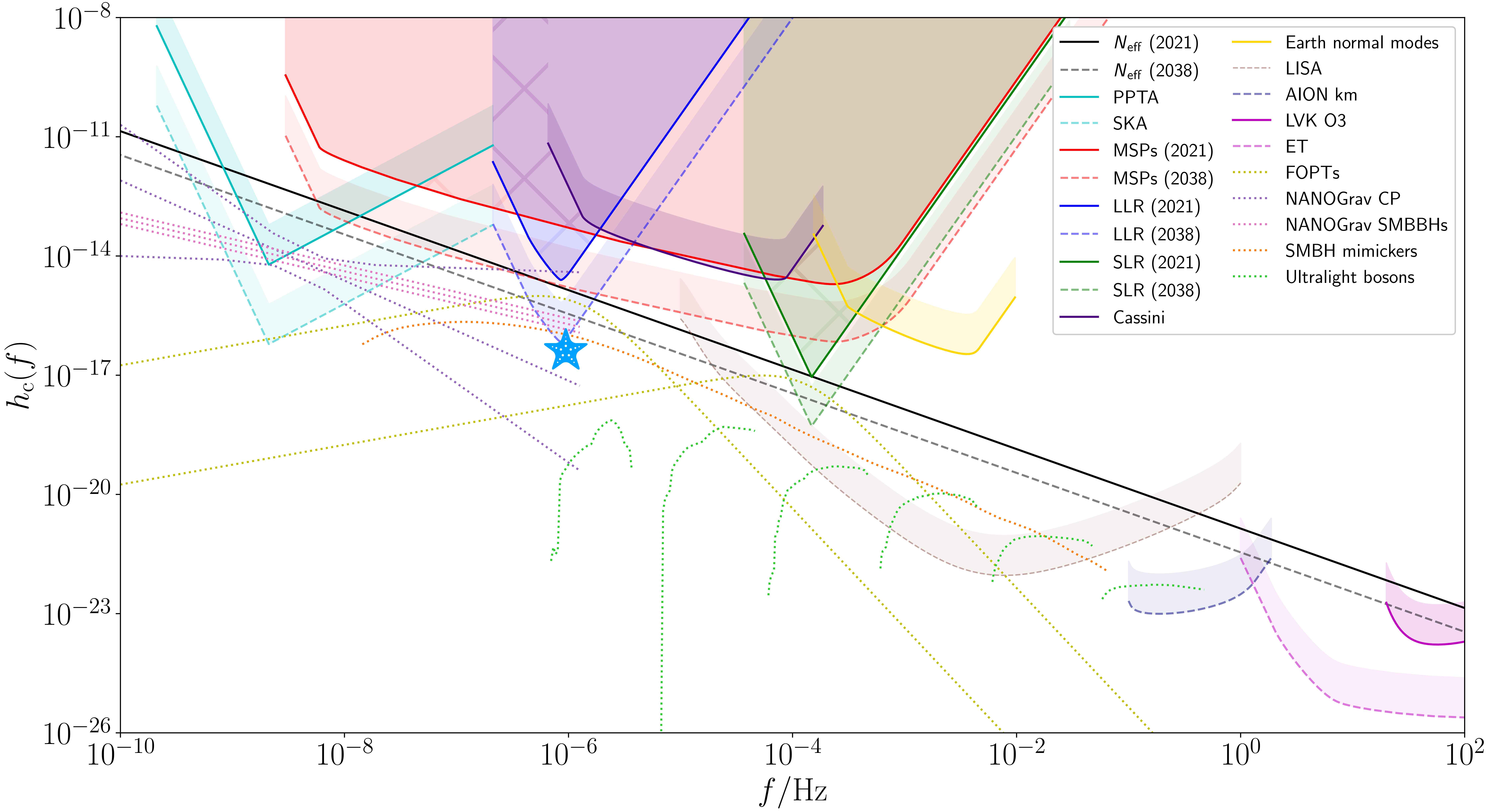
- There are **ideas** of how to access it, though **most** of them are **futuristic**

- The resonant **absorption of GWs by binaries** (LLR/SLR/pulsars) may give a handle at level (in 2038)

$$\Omega_{\rm gw} \geq 4.8 \times 10^{-9} \quad f = 0.85\,\mu{\rm Hz} \qquad\qquad \Omega_{\rm gw} \geq 8.3 \times 10^{-9} \quad f = 0.15\,{\rm mHz}$$

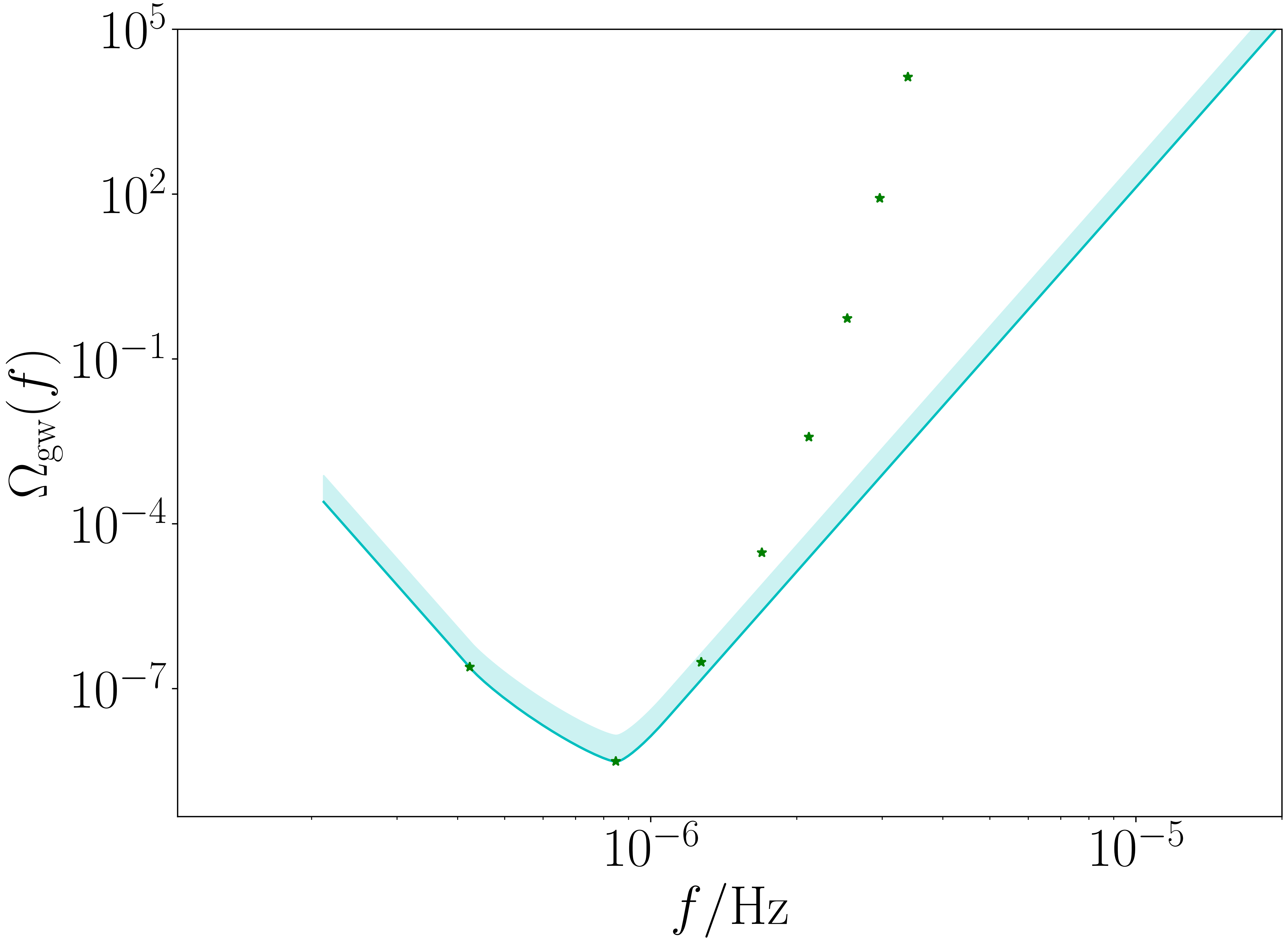
- **Future plans:** use LLR, SLR, pulsar ***data*** (w/ SYRTE, SCF_Lab, MPIfRA, Nanograv people…): we need/welcome new hands.
  Find ***other resonant effects*** (many possible, which frequencies?)

Characteristic strain

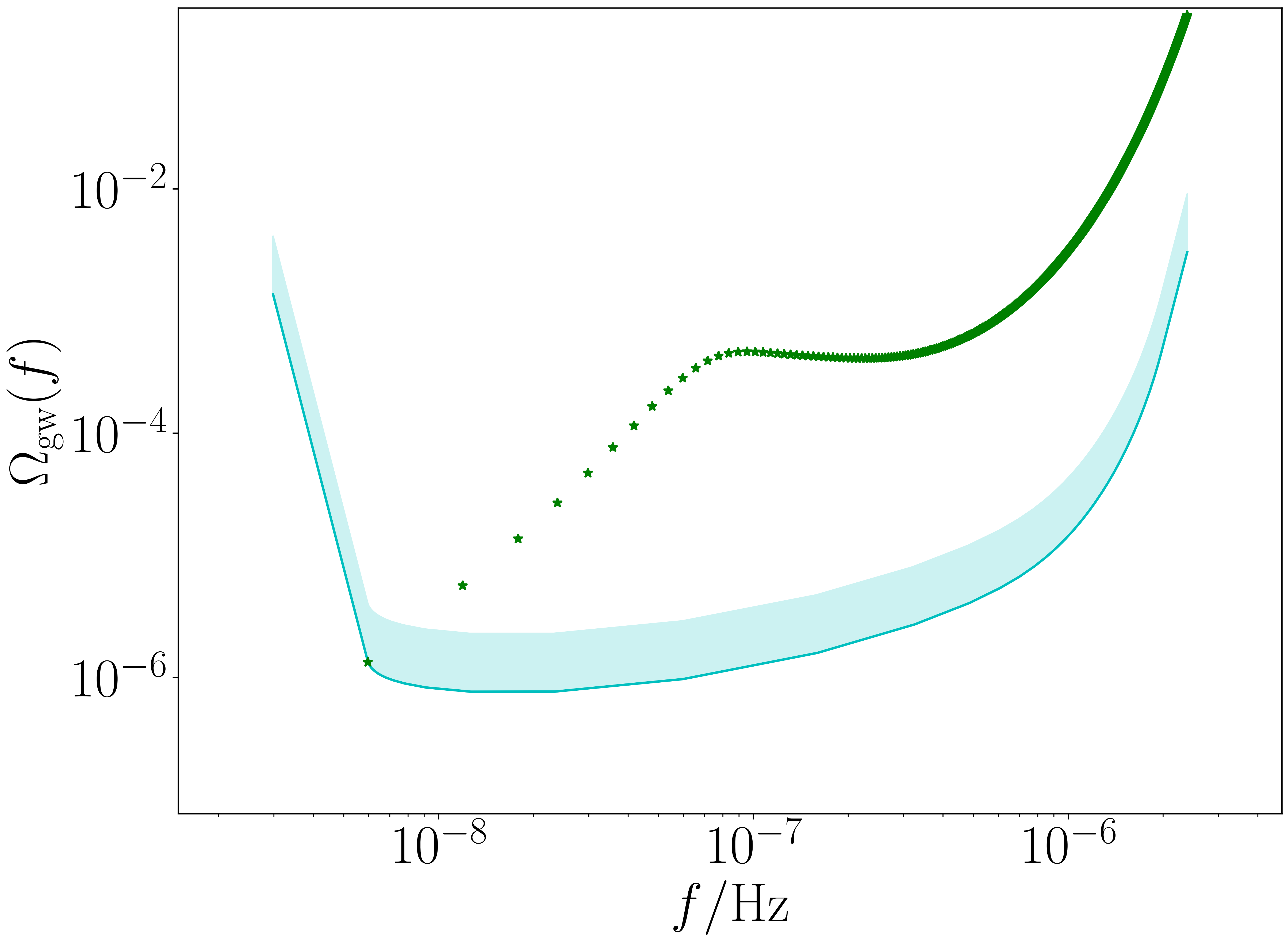


Power-law sensitivity vs. monochromatic sensitivity

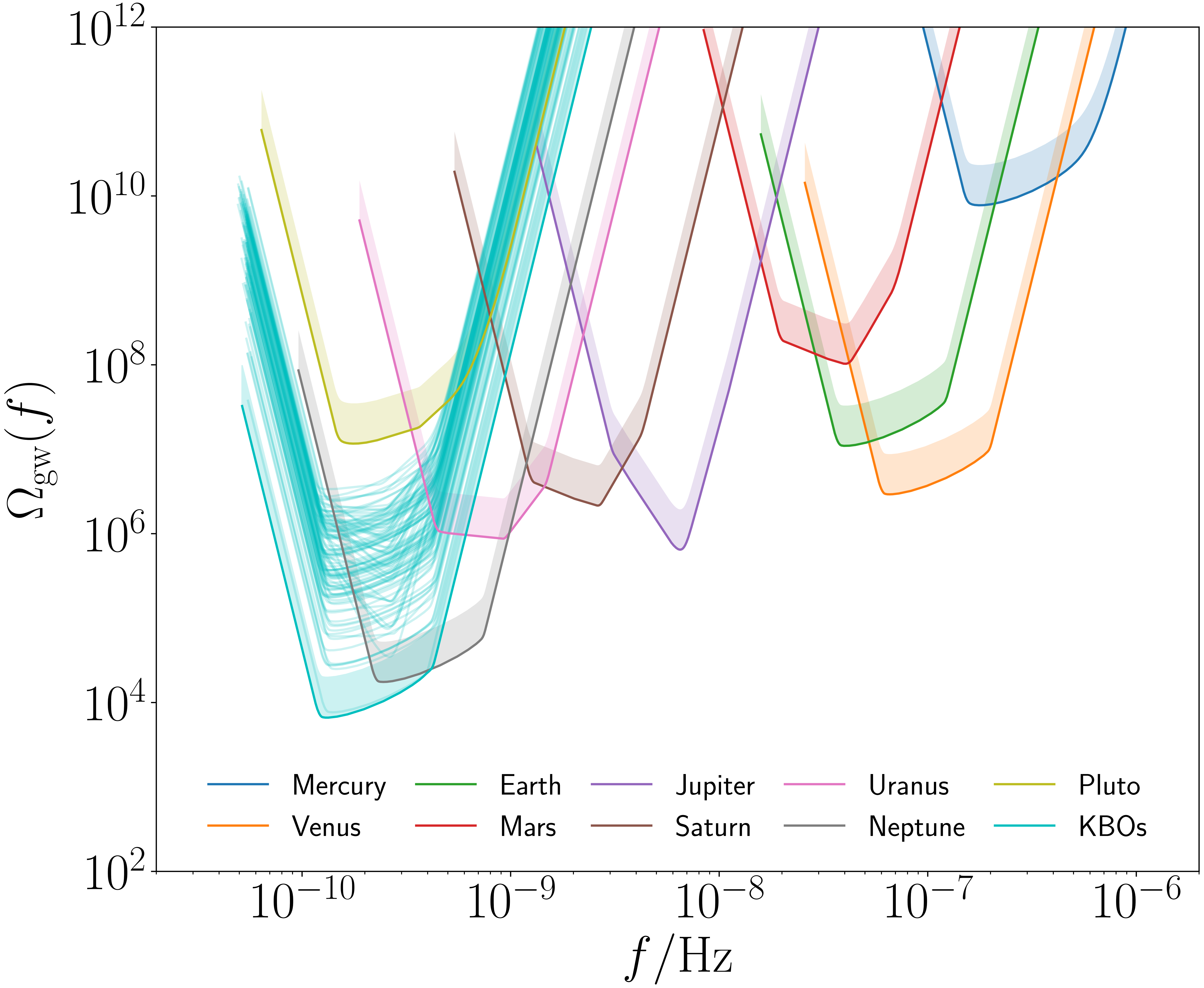
lunar laser ranging, $e \approx 0.055$



pulsar timing (J1638-4725), $e \approx 0.955$



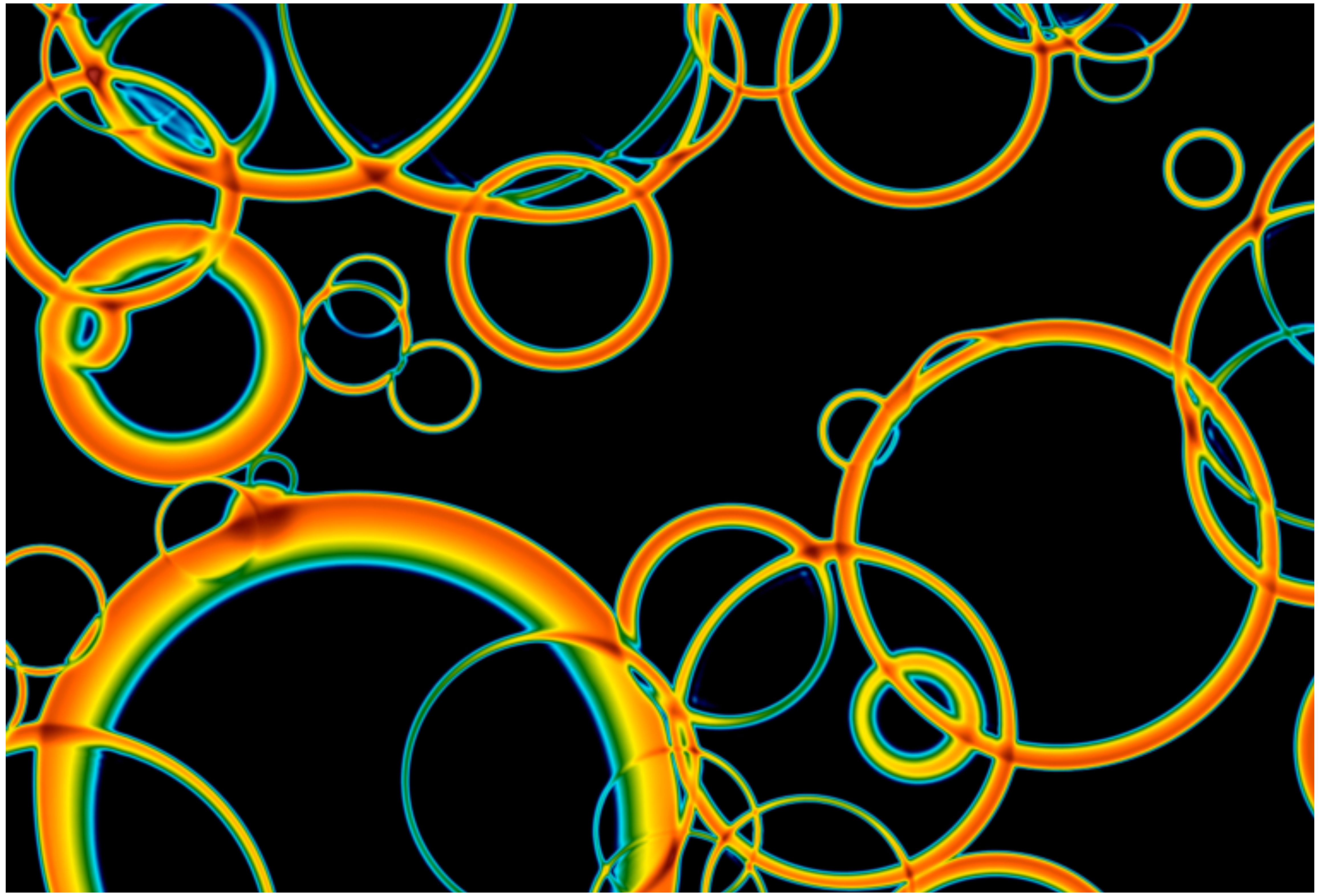
Solar system bounds



E.g. FOPT with peaks at μHz

- four parameters:
  - temperature T_*
  - strength α
  - rate β/H_*
  - bubble-wall velocity v_w
- peak frequency

$$f_* \approx 19\,\mu\text{Hz} \times \frac{T_*}{100\,\text{GeV}}\,\frac{\beta/H_*}{v_w}$$



Complementarity of probes for FOPT