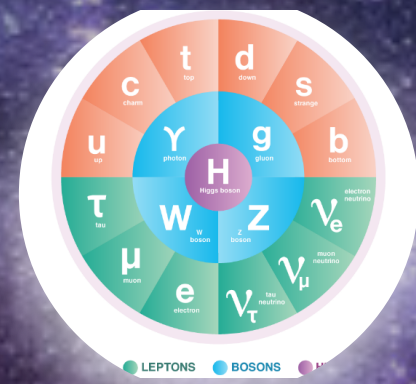


WAVES IN A BOX: RESONANT CAVITIES FOR AXION AND GW DETECTION

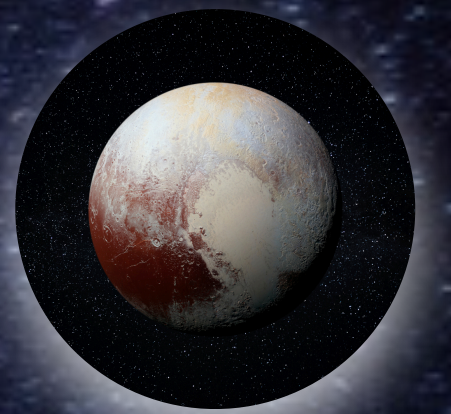


Raffaele Tito D'Agnolo - IPhT Saclay

DARK MATTER MASS



Person



Neutrino

Higgs



Pluto

Self-coupling

50 O.M.

Mass

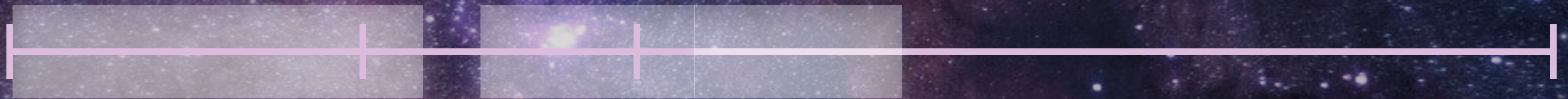
50 O.M.

80 O.M.

Couplings to ordinary matter



DARK MATTER MASS



Neutrino

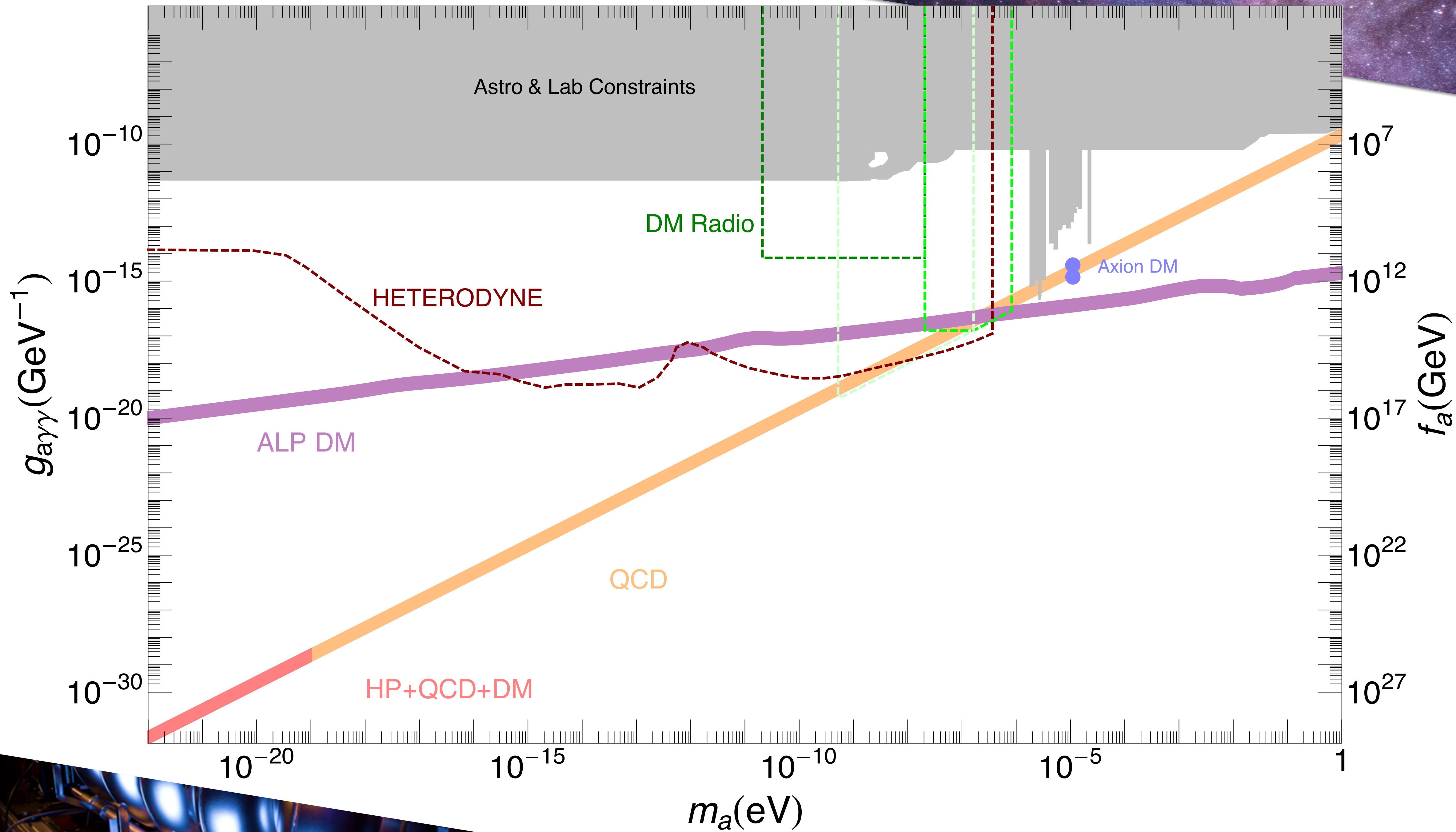
Higgs

Theory Spotlight

DARK MATTER MASS



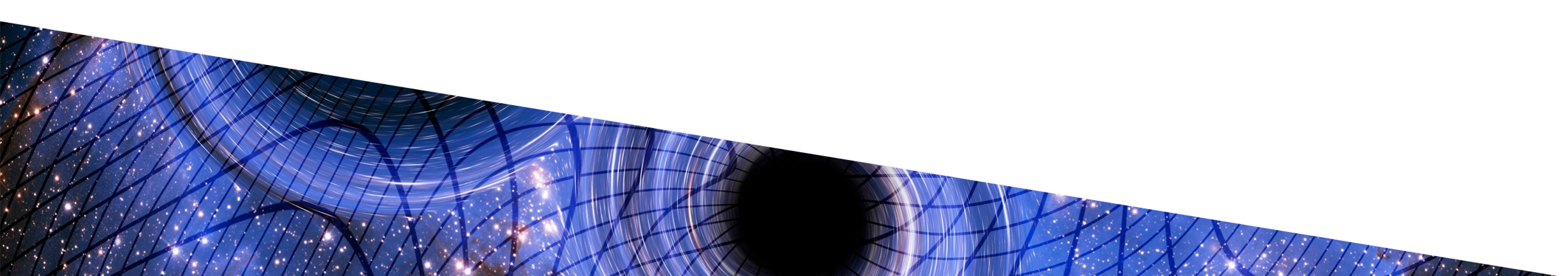
Theory Spotlight





ω_g

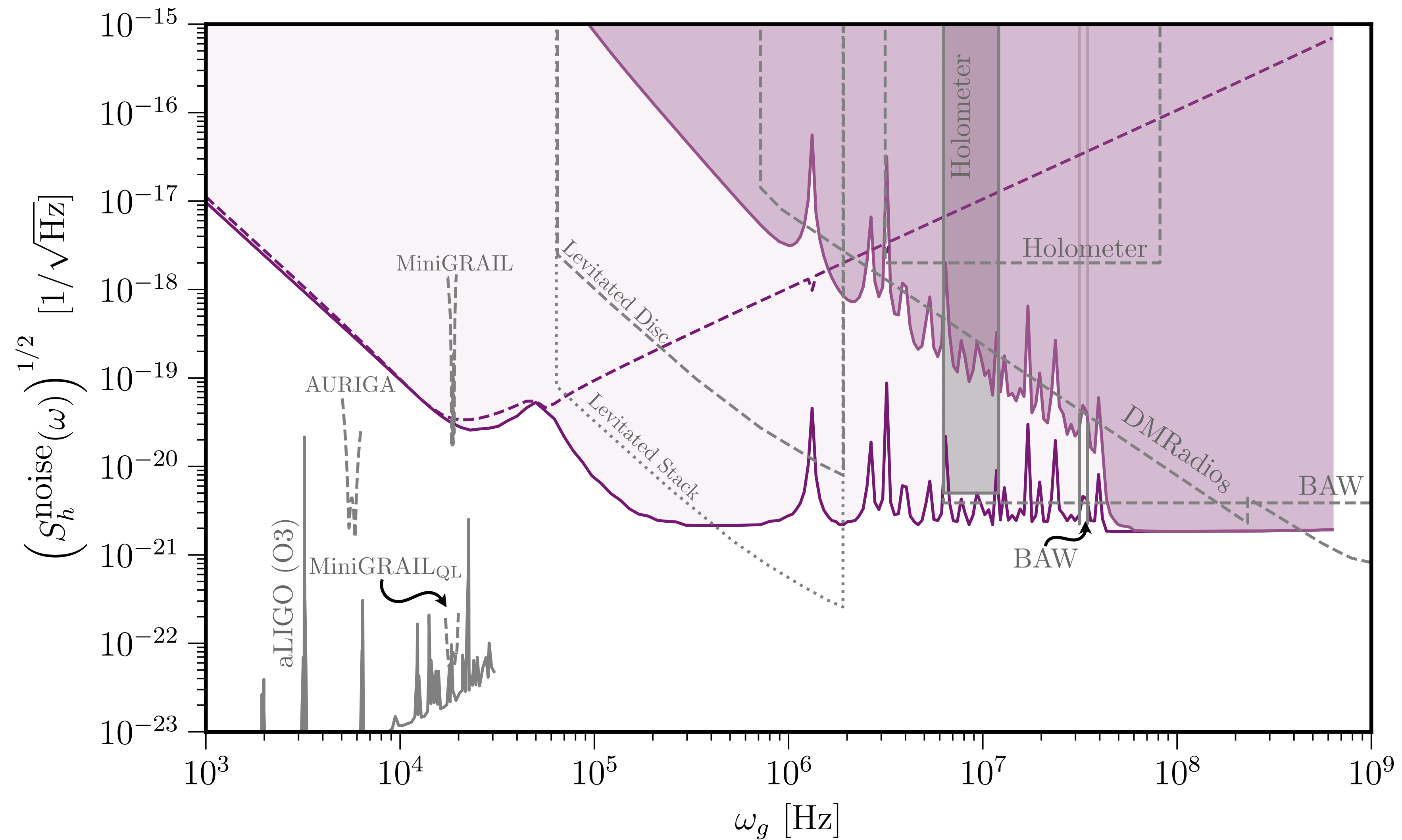
$$\lambda(T_*) < \frac{1}{H(T_*)} \quad \text{Causality}$$



$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)} \quad \text{Causality}$$

$$\omega_0(T_*) = \omega(T_*) \frac{a(T_*)}{a_0} \gtrsim \boxed{100 \text{ MHz}} \left(\frac{T_*}{10^{15} \text{ GeV}} \right) \left(\frac{g_*(T_*)}{100} \right)^{1/6}$$



Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel, Wentzel
 arXiv:2303.01518

The image features a diagonal split background. The upper right portion shows a vibrant view of the Milky Way galaxy, with its characteristic band of stars and interstellar dust in shades of purple, blue, and white. The lower left portion shows a close-up of a complex scientific instrument, likely the ALPS (Axion-Like Particle Search) detector, with several large, polished, metallic, bowl-shaped components arranged in a row. The text "ALPS DETECTION" is centered across the diagonal boundary in a bold, black, sans-serif font with a white outline.

ALPS DETECTION

Dark Matter Particles in a de Broglie Volume **Today**

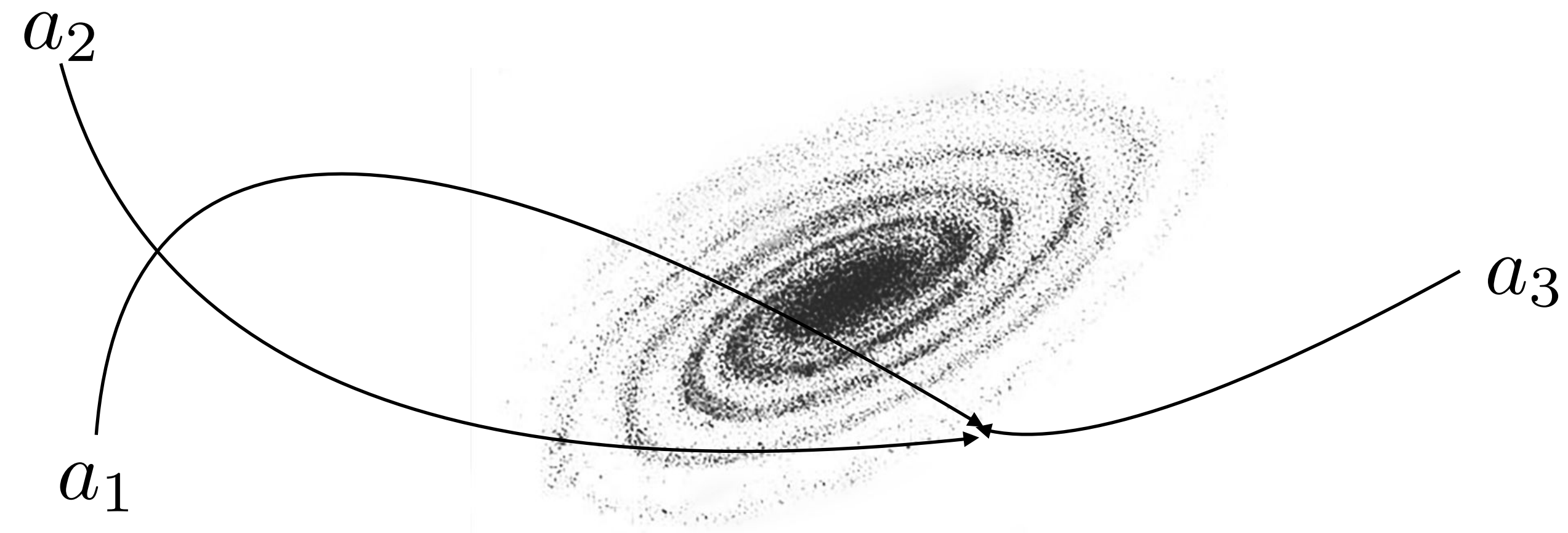
Galaxy: $N_{\text{DM}} \simeq 10^3 \left(\frac{\text{eV}}{m_{\text{DM}}} \right)$

Universe: $N_{\text{DM}} \simeq 10^{-3} \left(\frac{\text{eV}}{m_{\text{DM}}} \right)$

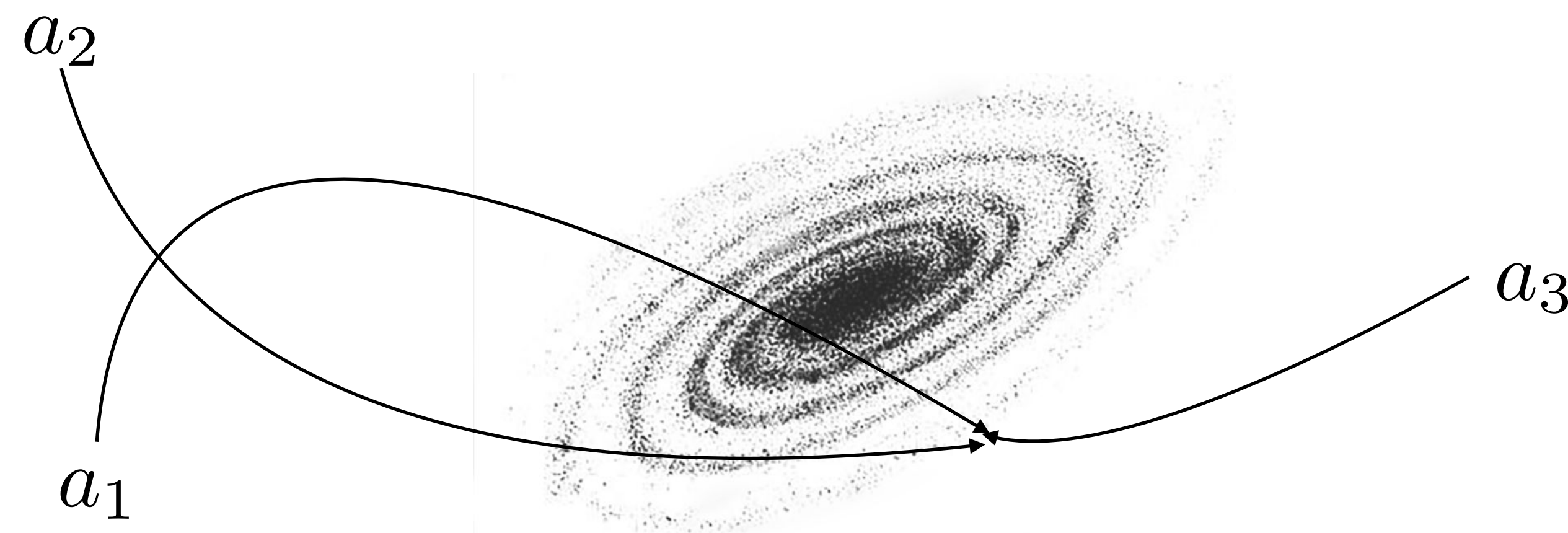
$$a(t) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \cos(\omega_a t + \phi)$$

ALP DARK MATTER IN THE LAB

In each experimental bin we are **summing over a multitude of plane waves** with different phases



In each experimental bin we are **summing over a multitude of plane waves** with different phases



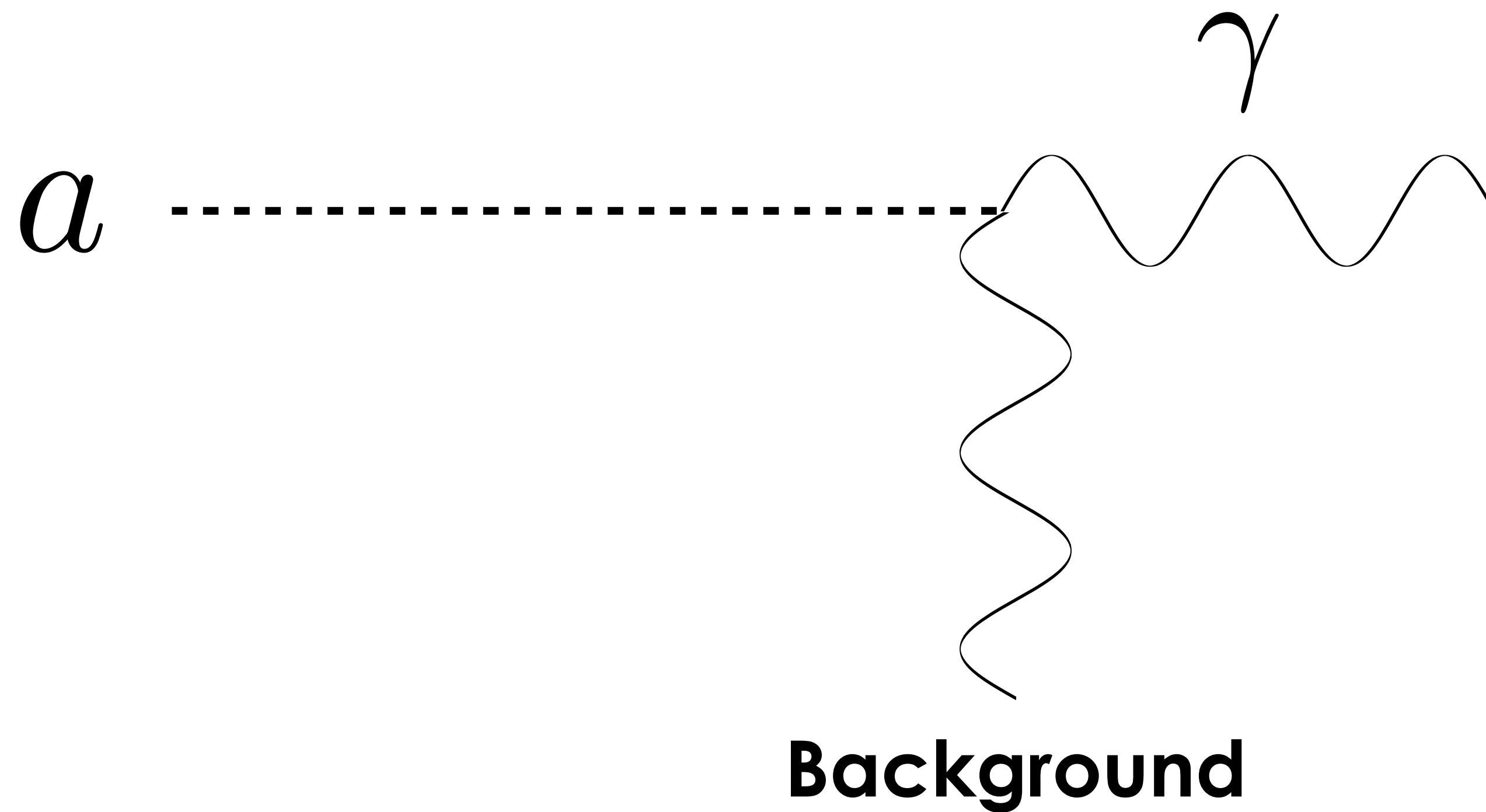
$$a(t) = a_0 \left[\cos \left(m_a \left(1 + \frac{v_1^2}{2} \right) t + \phi_1 \right) + \cos \left(m_a \left(1 + \frac{v_2^2}{2} \right) t + \phi_2 \right) + \dots \right]$$

$$\simeq a_0 \cos(m_a t + \phi) [\cos(\delta\omega_a t + \phi') + \dots]$$

$$\delta\omega_a \simeq \frac{1}{m_a \langle v_{\text{DM}}^2 \rangle} \simeq \frac{10^6}{m_a}$$

Effectively: very **slow modulation** of an approximately **monochromatic field**

ALP DARK MATTER DETECTION

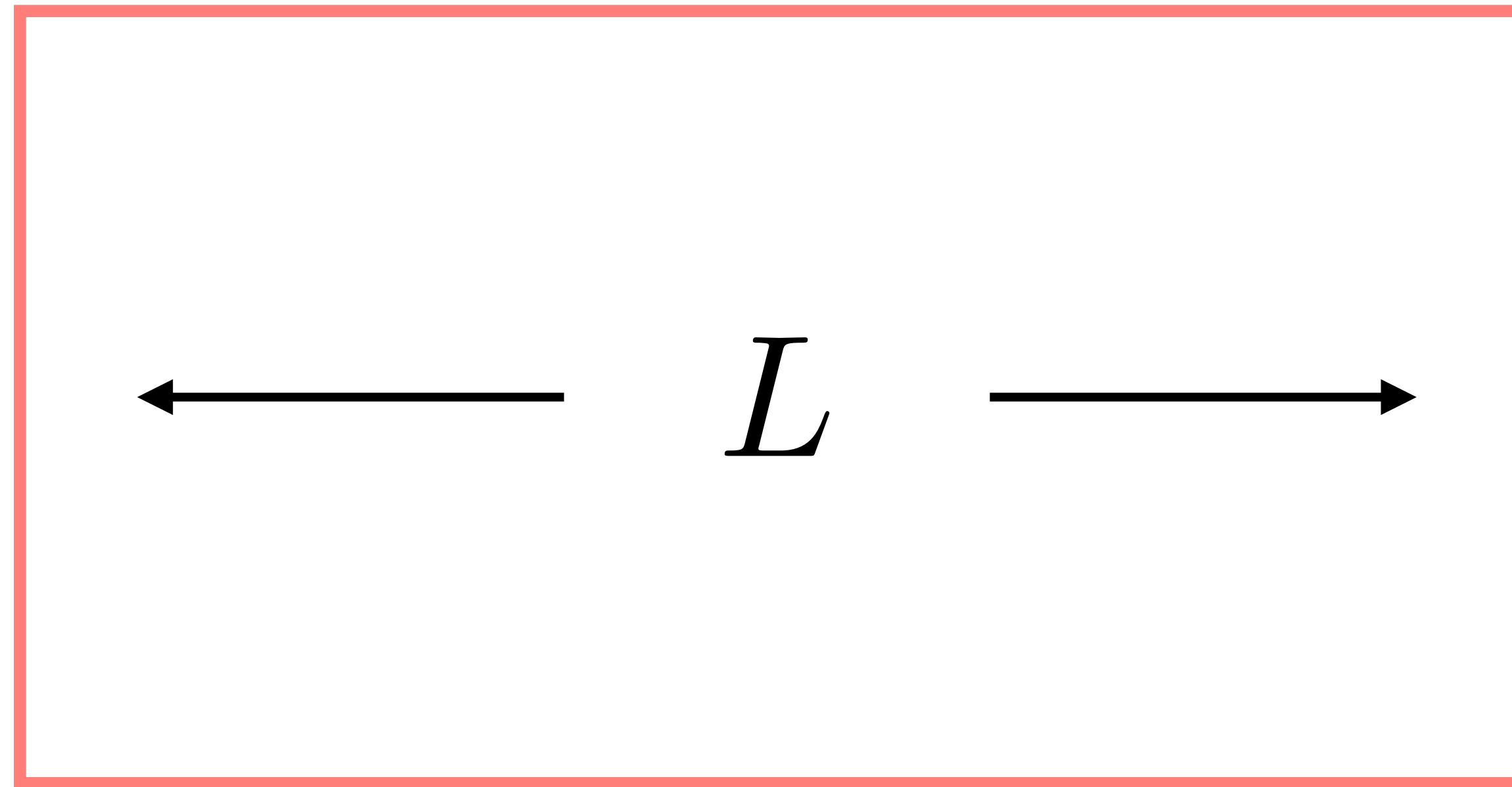


$$\sim \frac{a}{f_a} E_{\text{bkg}} \simeq 10^{-21} E_{\text{bkg}}$$

but you know exactly the waveform
and the signal is always there

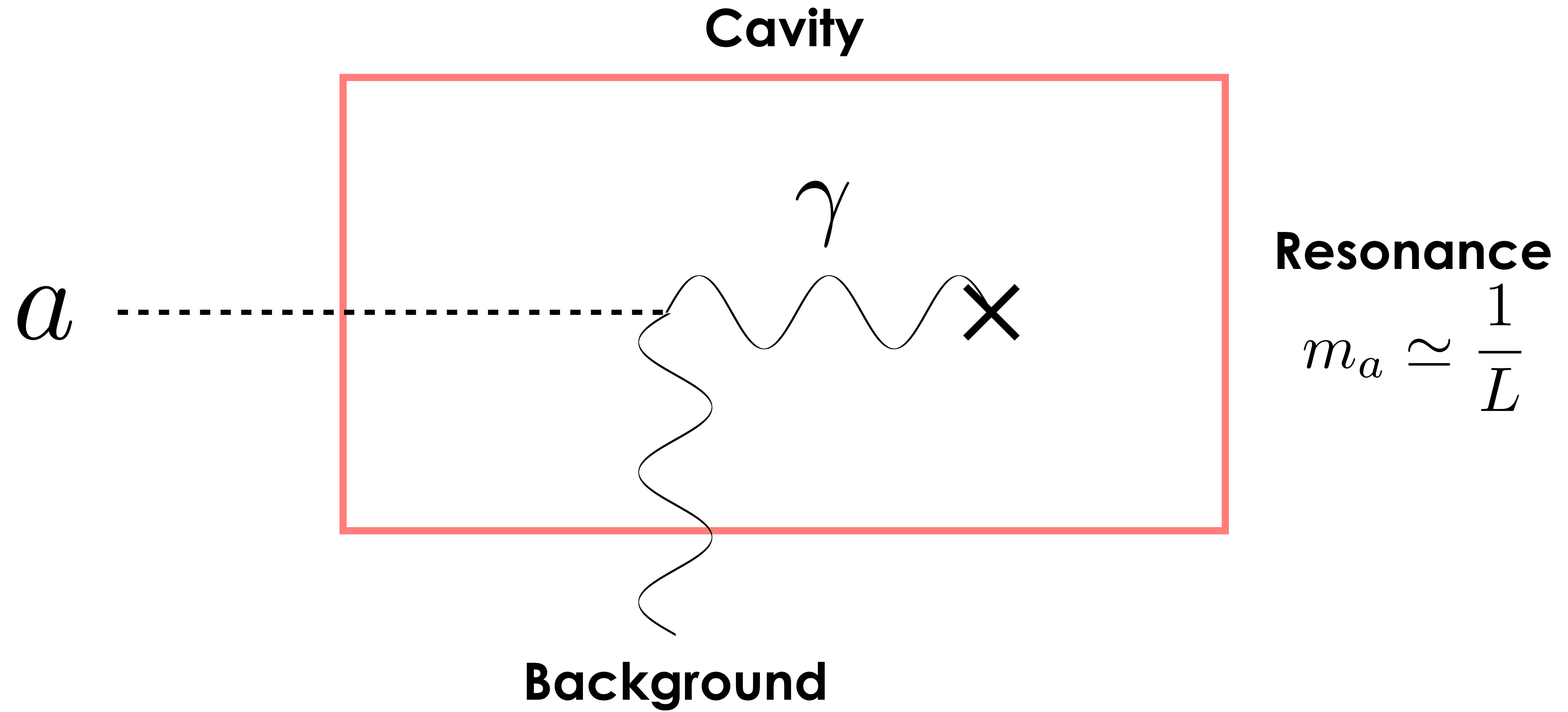
AXION DARK MATTER DETECTION

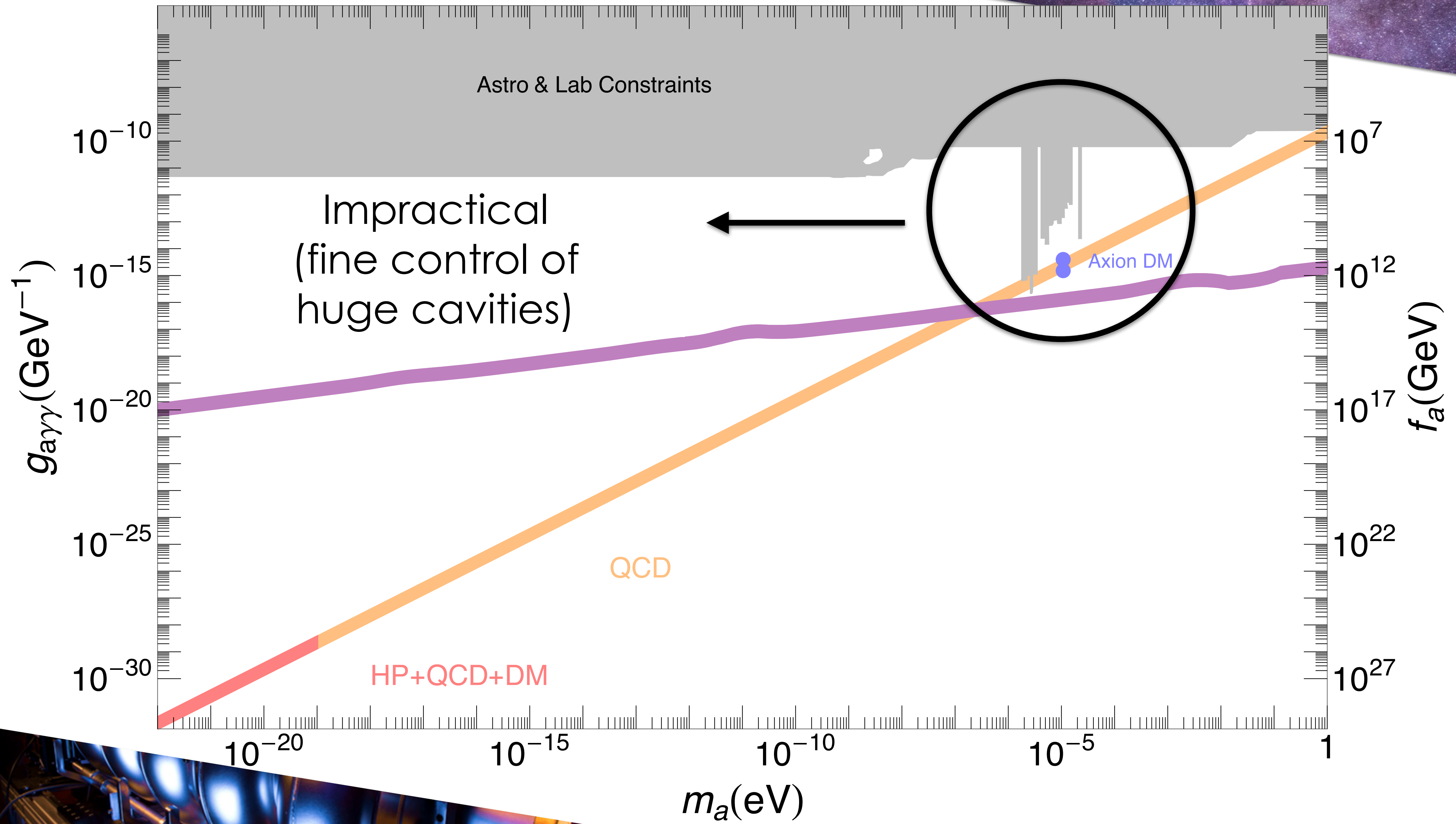
Cavity



$$m_\gamma \simeq \frac{1}{L}$$

AXION DARK MATTER DETECTION





Cavity:

$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

Cavity:

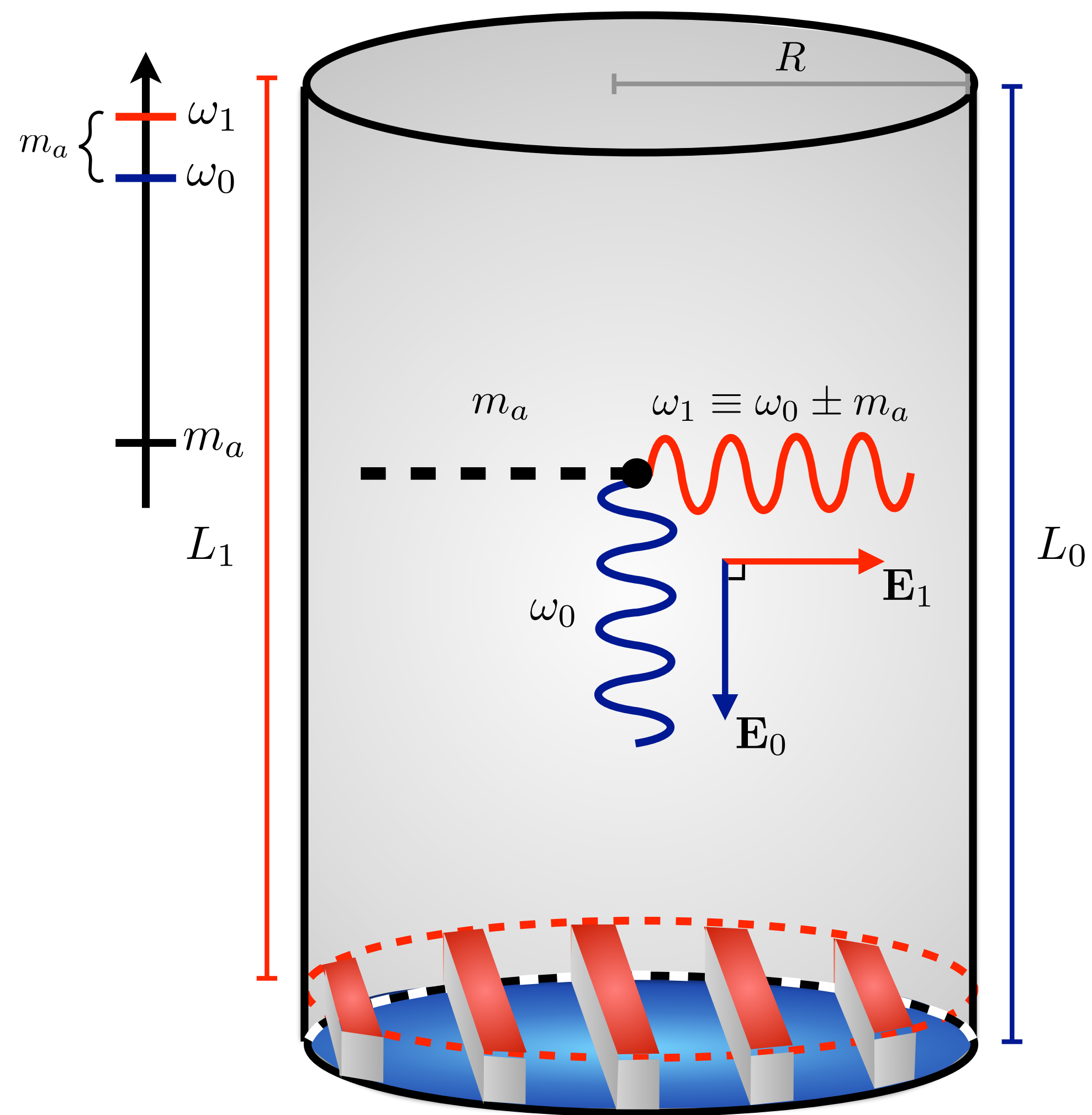
$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

$$\omega_1 \simeq m_a \quad \partial_t (\mathbf{B}) \simeq 0$$

$$\left(\partial_t^2 + \frac{m_a}{Q_1} \partial_t + m_a^2 \right) \mathbf{E}_1 = g_{a\gamma\gamma} \mathbf{B} \sqrt{\rho_{\text{DM}}} m_a \cos m_a t$$

HETERODYNE DETECTION

[Berlin, RTD, S. Ellis, C. Nantista, J. Nielson, P. Schuster, S. Tantawi, N. Toro, K. Zhou '19]



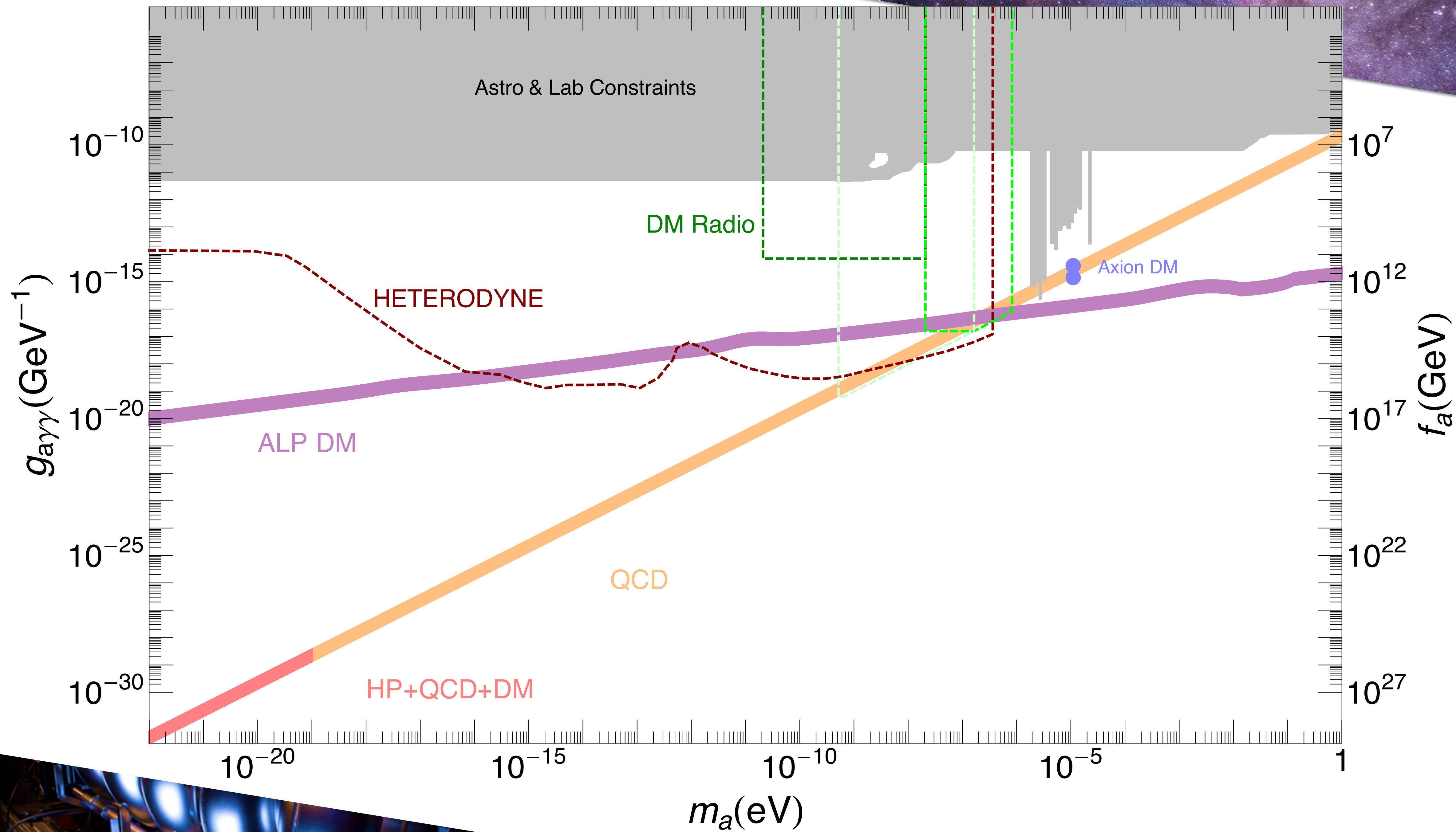
HETERODYNE DETECTION

[Berlin, RTD, S. Ellis, C. Nantista, J. Nielson, P. Schuster, S. Tantawi, N. Toro, K. Zhou '19]

$$\sum_n \left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) \mathbf{E}_n = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a)$$

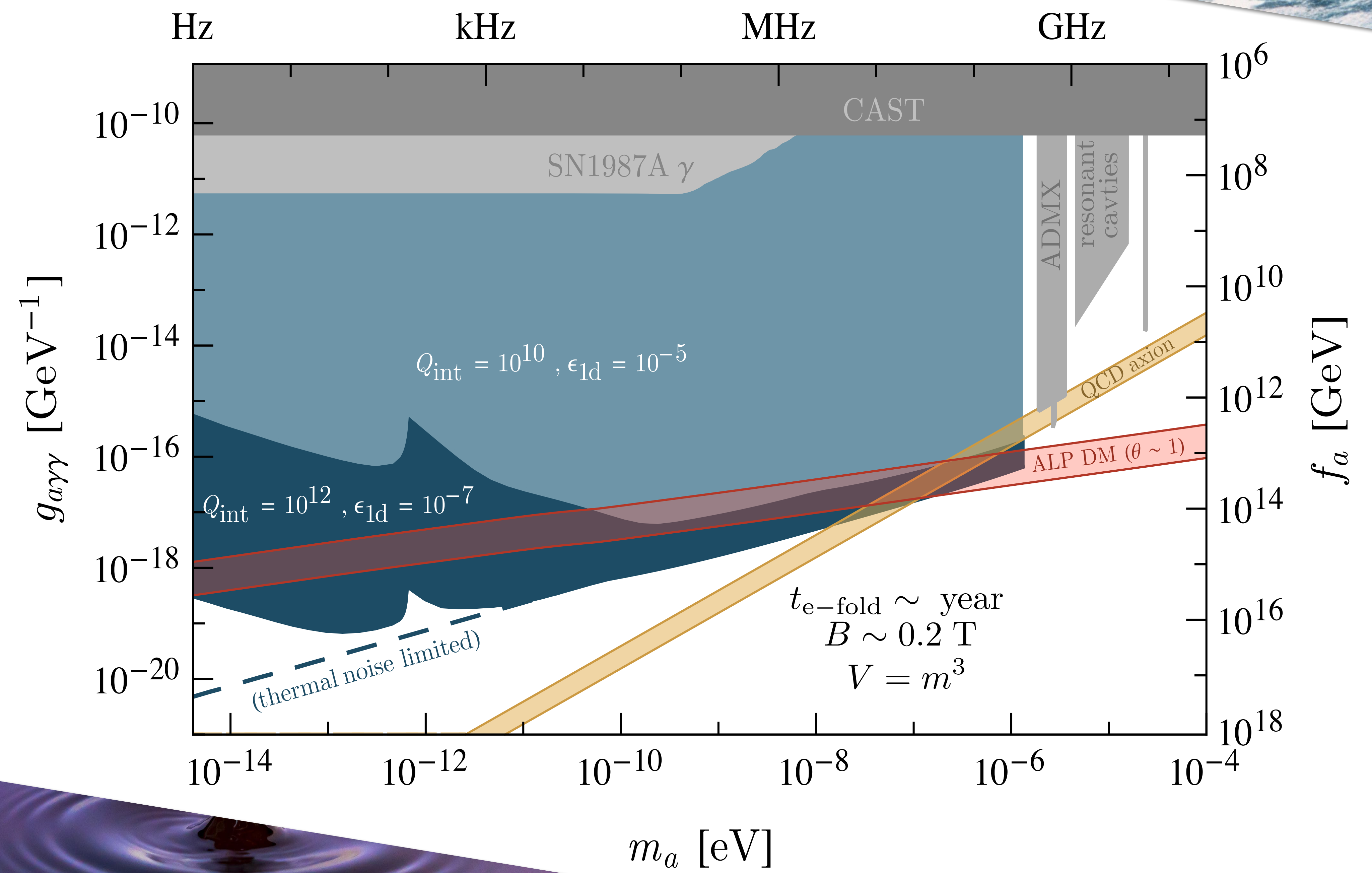
$$\partial_t (\mathbf{B}) \simeq i\omega_0 \mathbf{B} \quad \omega_1 \simeq \omega_0 + m_a$$

$$\partial_t J_{\text{eff}} = g_{a\gamma\gamma} \partial_t (\mathbf{B} \partial_t a) \propto \omega_0 m_a \gg m_a^2$$



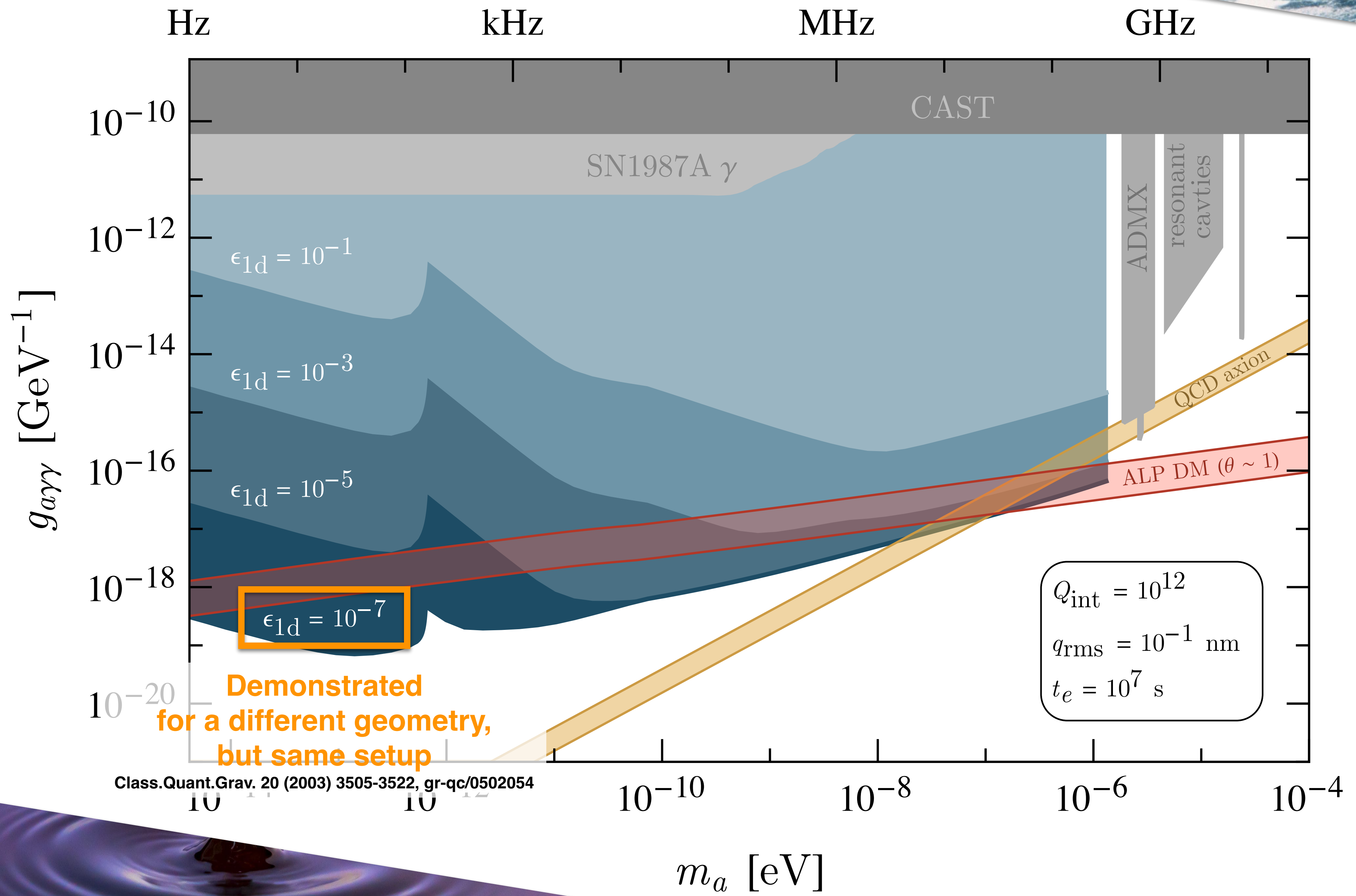
RESONANT

frequency = $m_a/2\pi$



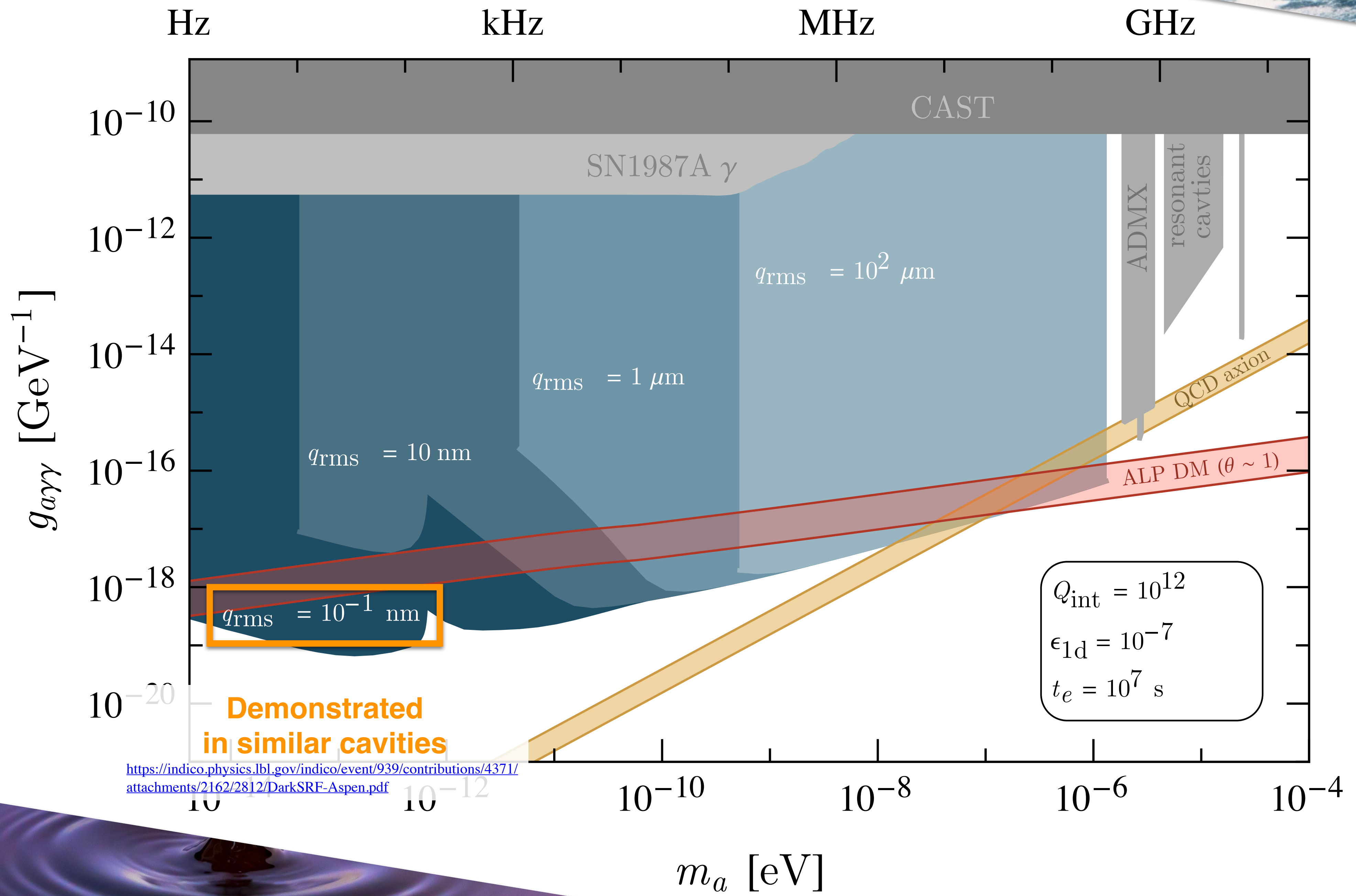
RESONANT

$$\text{frequency} = m_a / 2\pi$$



RESONANT

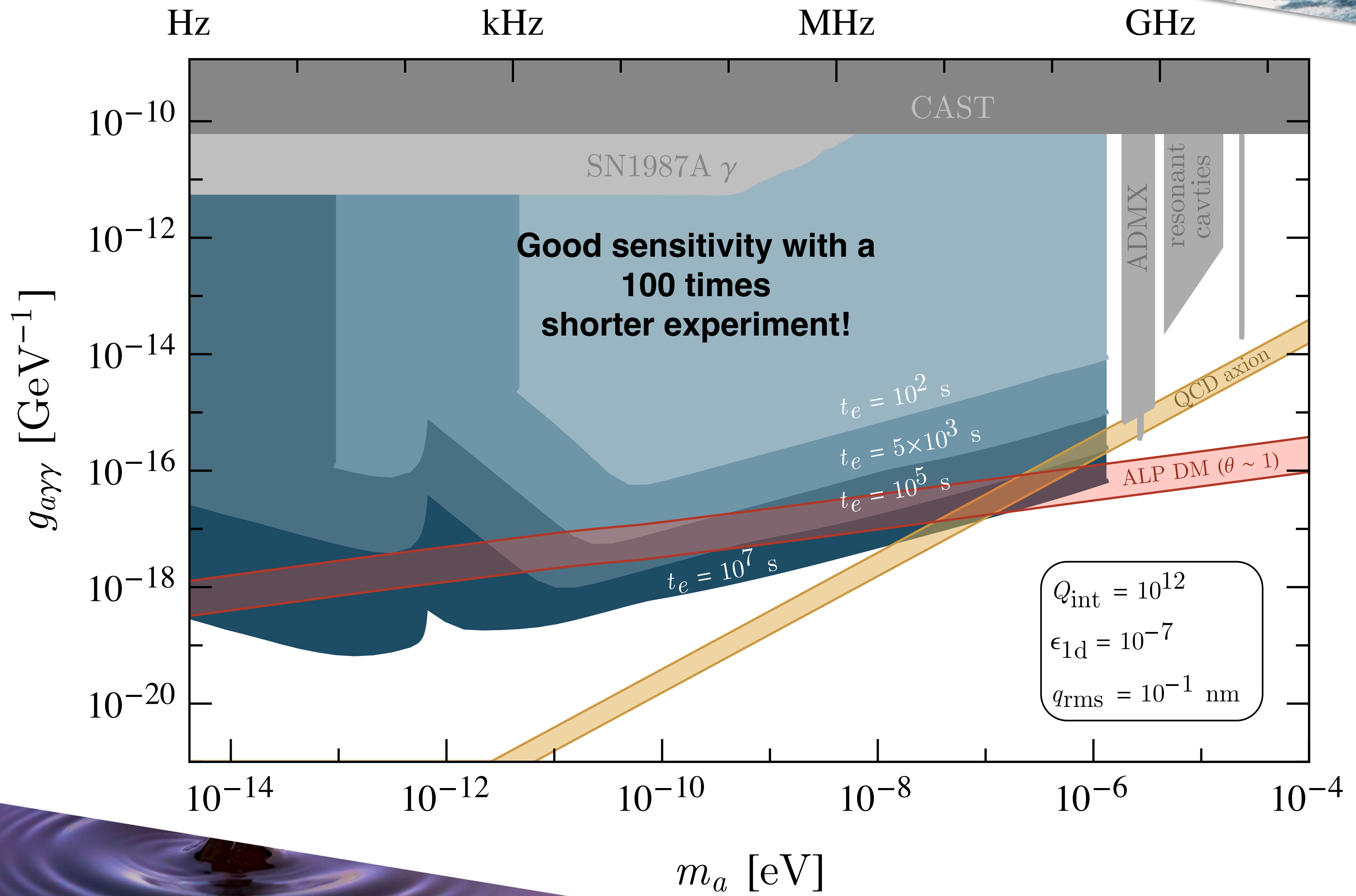
$$\text{frequency} = m_a / 2\pi$$



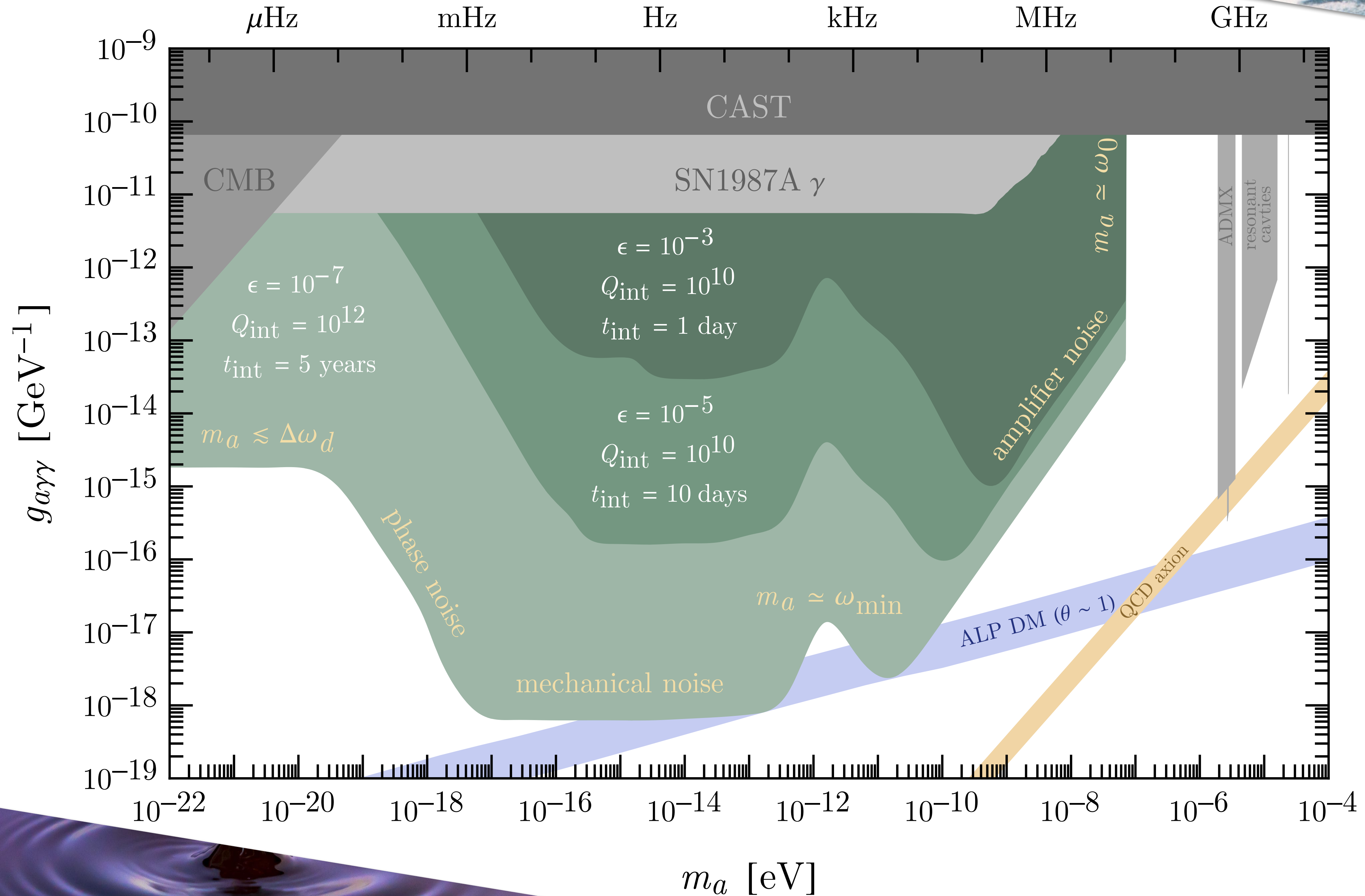
<https://indico.physics.lbl.gov/indico/event/939/contributions/4371/attachments/2162/2812/DarkSRF-Aspen.pdf>

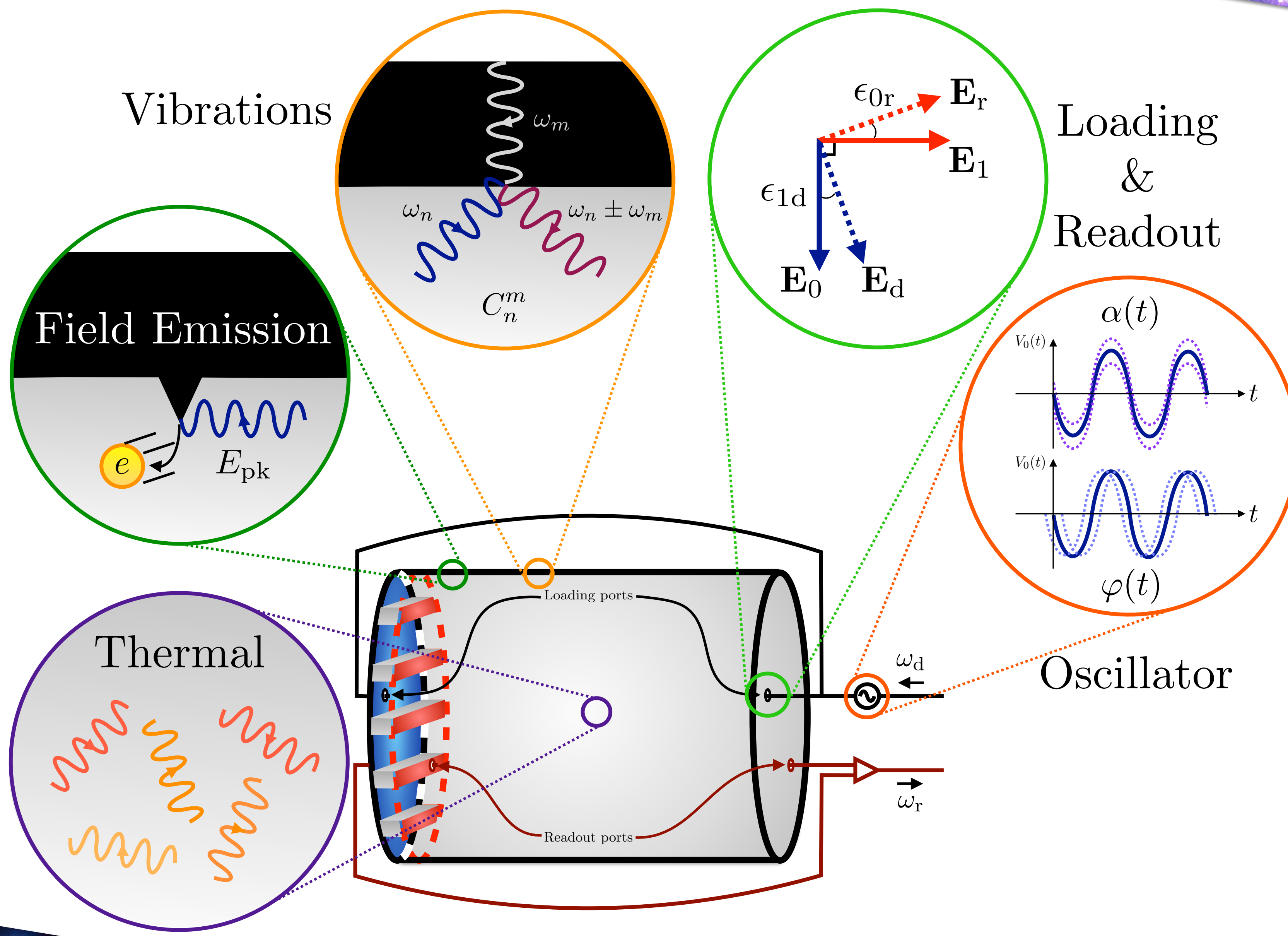
RESONANT

$$\text{frequency} = m_a / 2\pi$$



A. Berlin, RTD, S. Ellis, K. Zhou 2007.15656





LEAKAGE NOISE

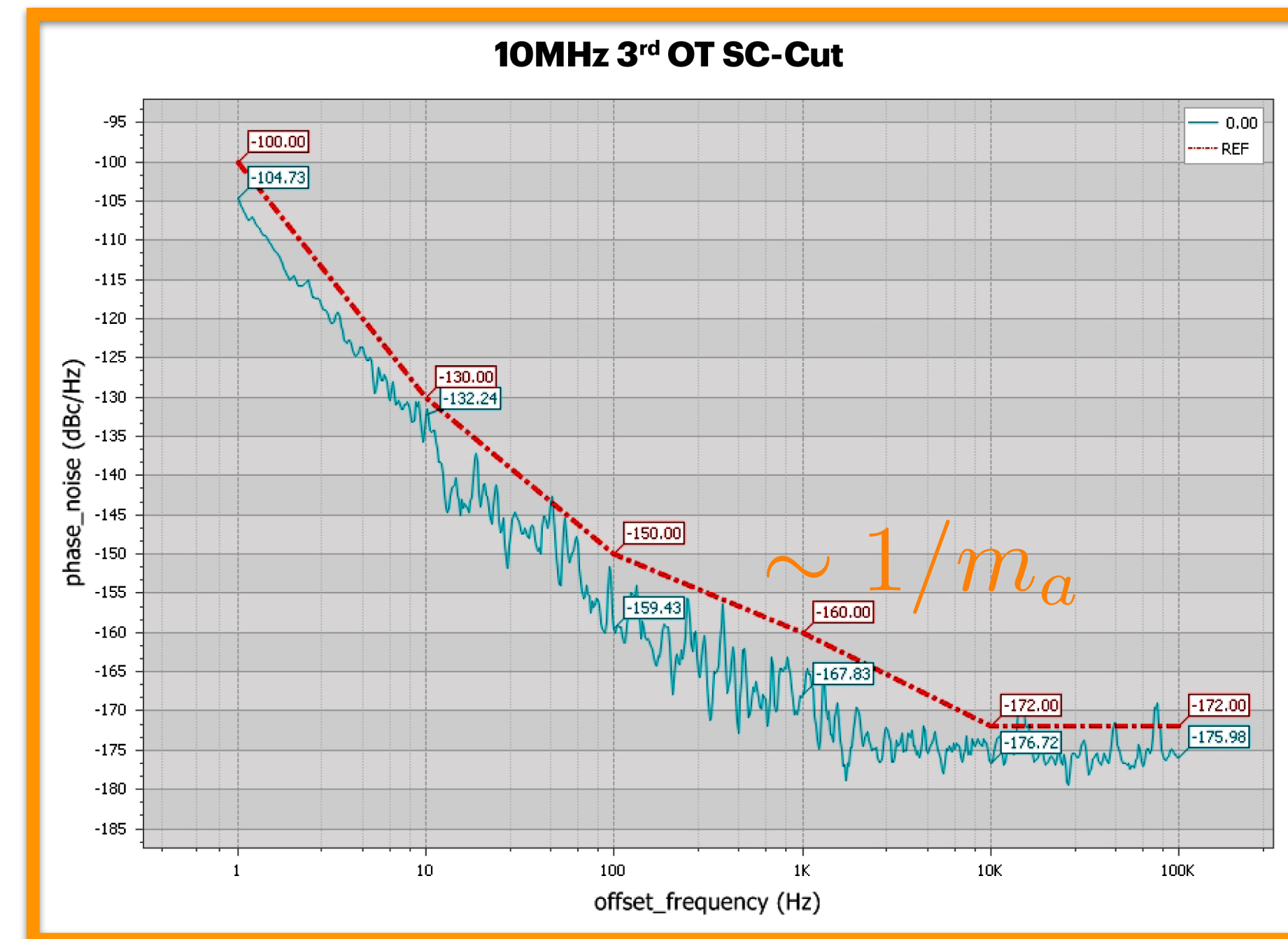
$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_{\phi}(\omega - \omega_0) \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$

Cavity Response

LEAKAGE NOISE

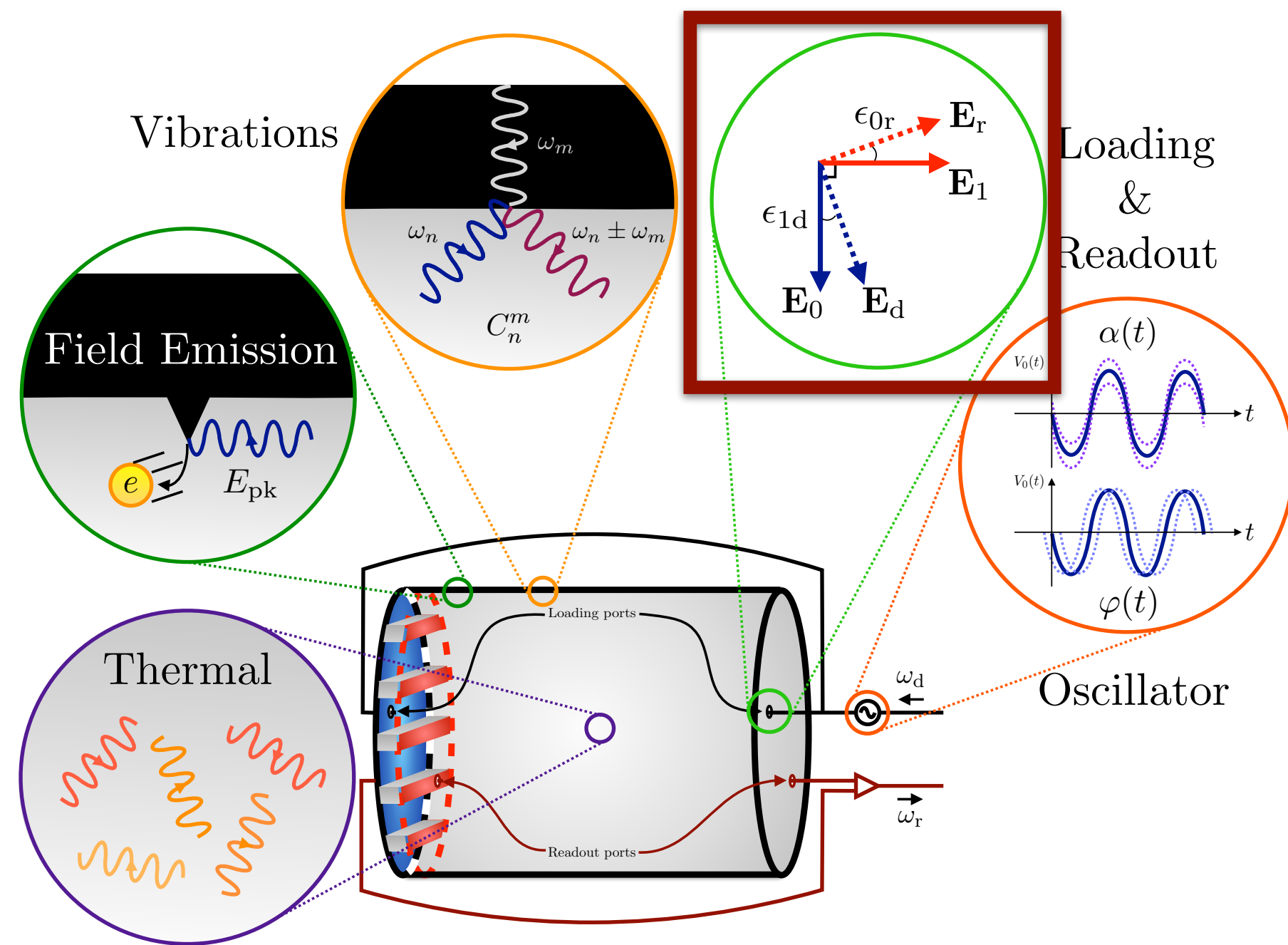
$$S_{\text{phase}}(\omega) \approx \frac{1}{2} \epsilon_{1d}^2 \boxed{S_{\phi}(\omega - \omega_0)} \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$

$\sim 1/m_a$



LEAKAGE NOISE

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_{\phi}(\omega - \omega_0) \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$

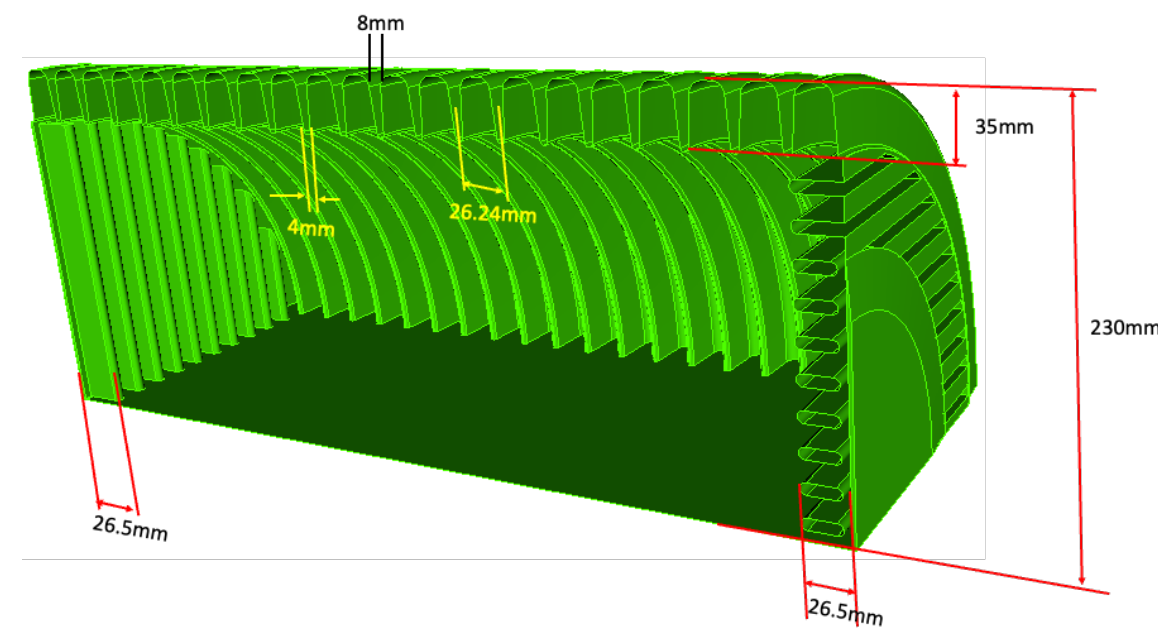


**From MAGO
and other similar cavities**

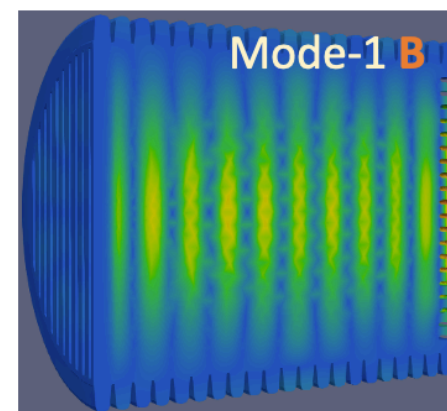
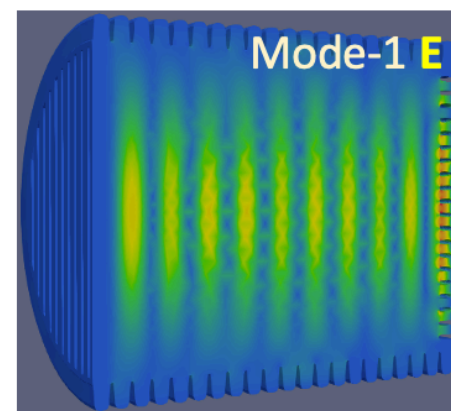
TWO PROTOTYPES [~ 1 YEAR]



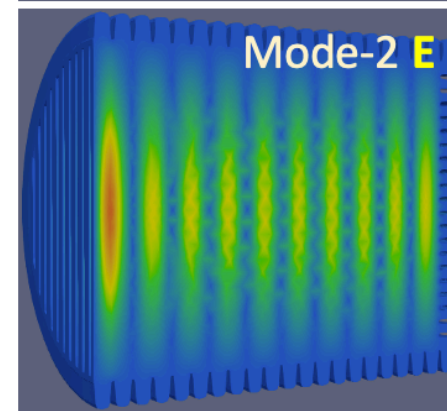
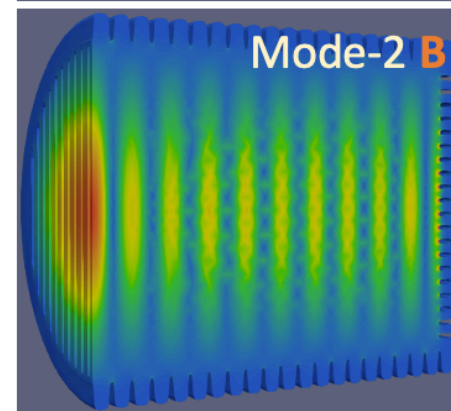
LDRD [only internal documents]



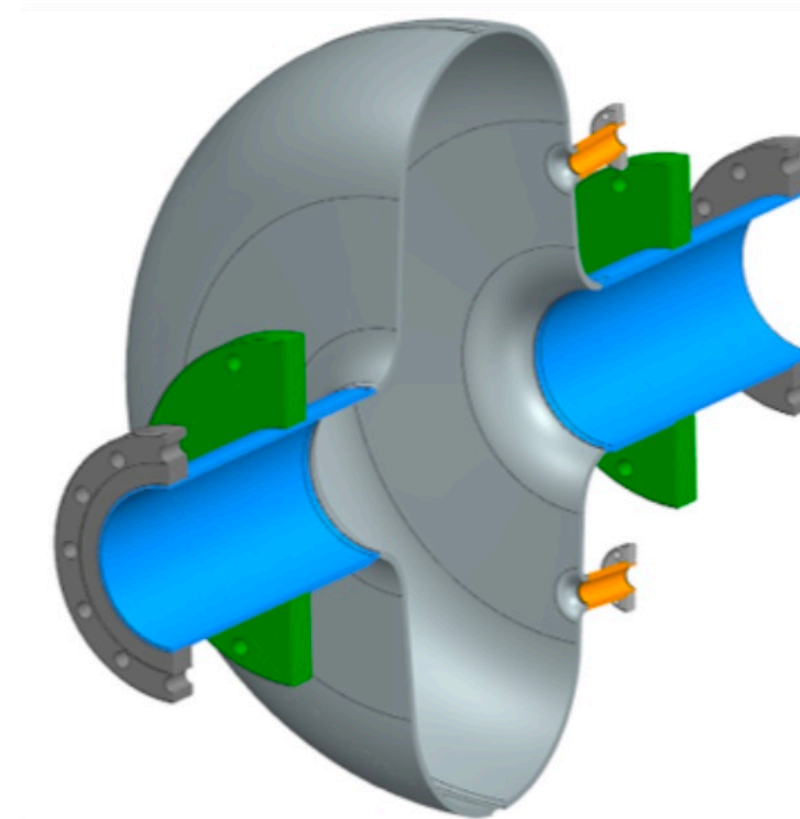
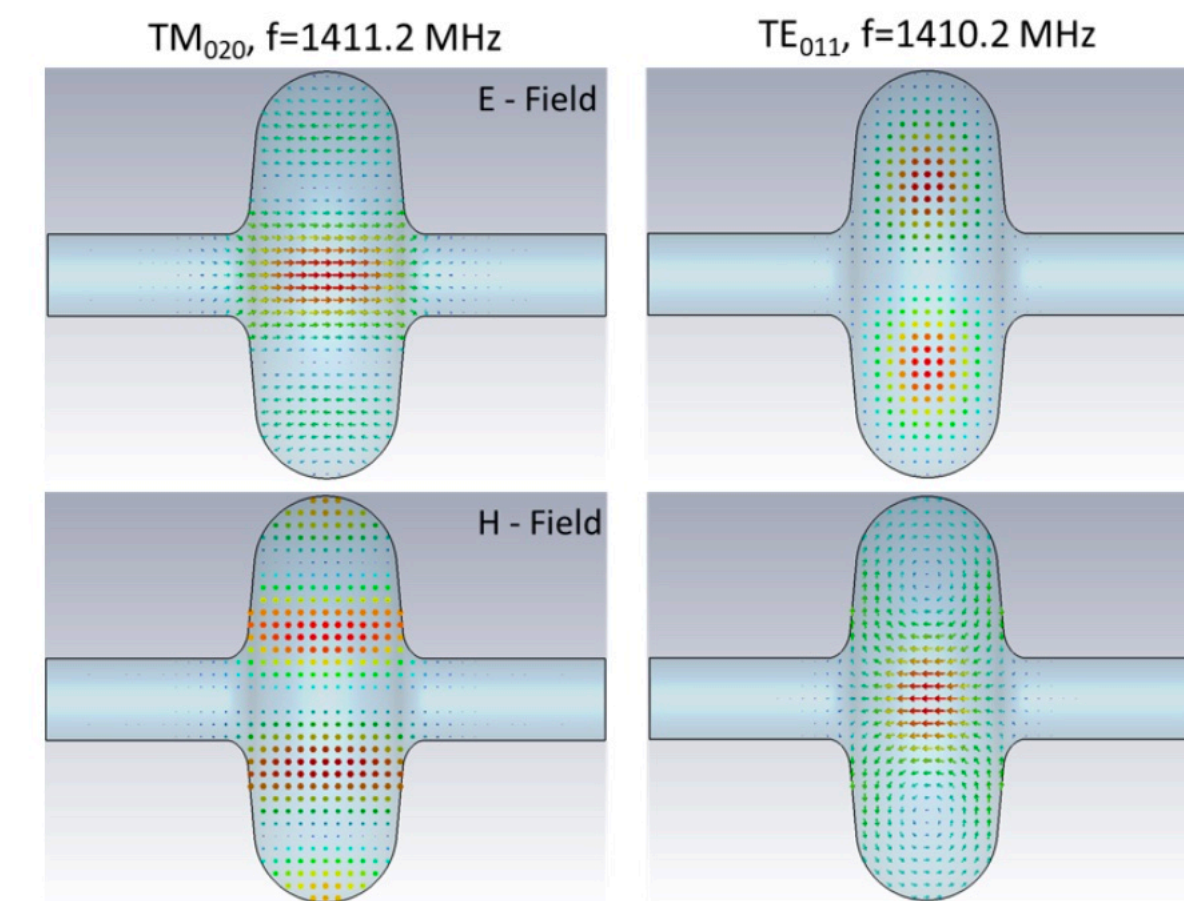
HE11
polarization-1 (E,B)



HE11
polarization-2 (B,E)




arXiv:2207.11346



GRAVITATIONAL WAVES




$$S \supset -\frac{1}{2} \int d^4x j_{\text{eff}}^\mu A_\mu$$

GW

$$j_{\text{eff}}^\mu = \partial_\nu \left(\frac{1}{2} h \underline{F^{\mu\nu}} + h^\nu_\alpha \underline{F^{\alpha\mu}} - h^\mu_\alpha \underline{F^{\alpha\nu}} \right)$$

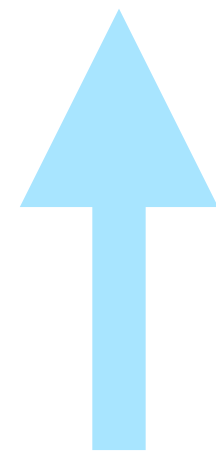
Axion

$$j_{\text{eff}}^\mu = \epsilon^{\mu\nu\rho\sigma} \partial_\nu (a \underline{F_{\rho\sigma}})$$

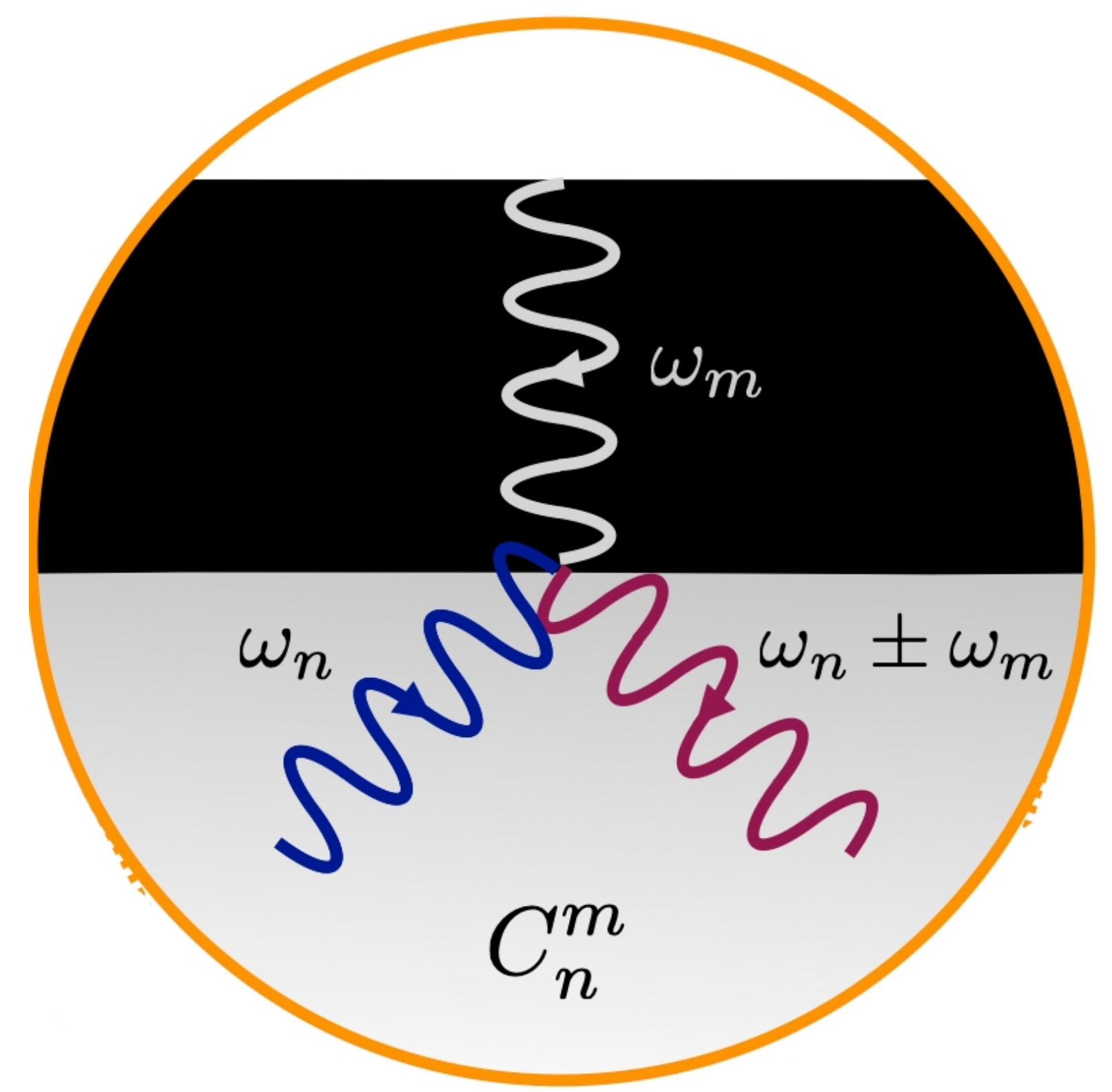
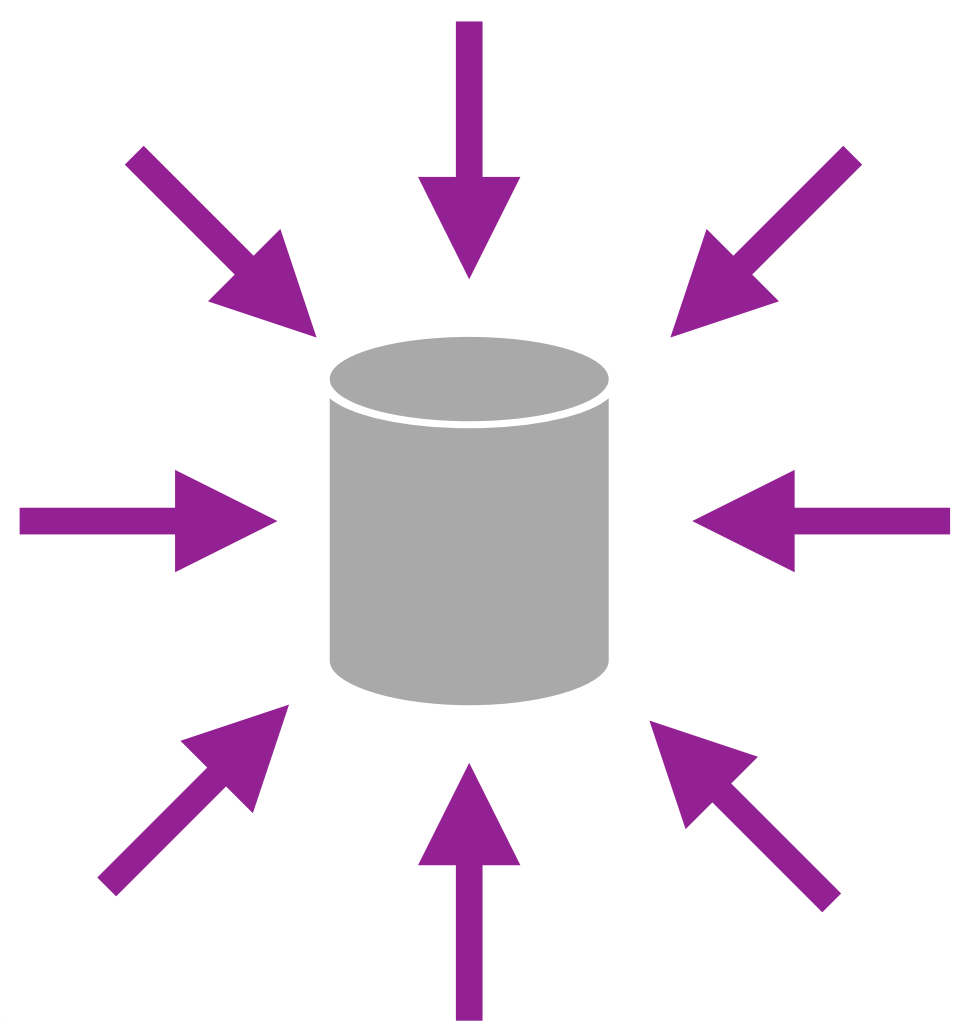
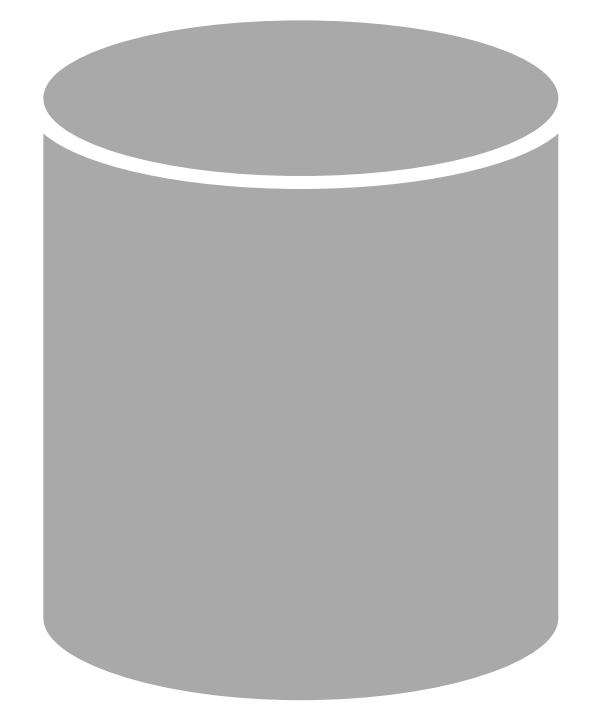
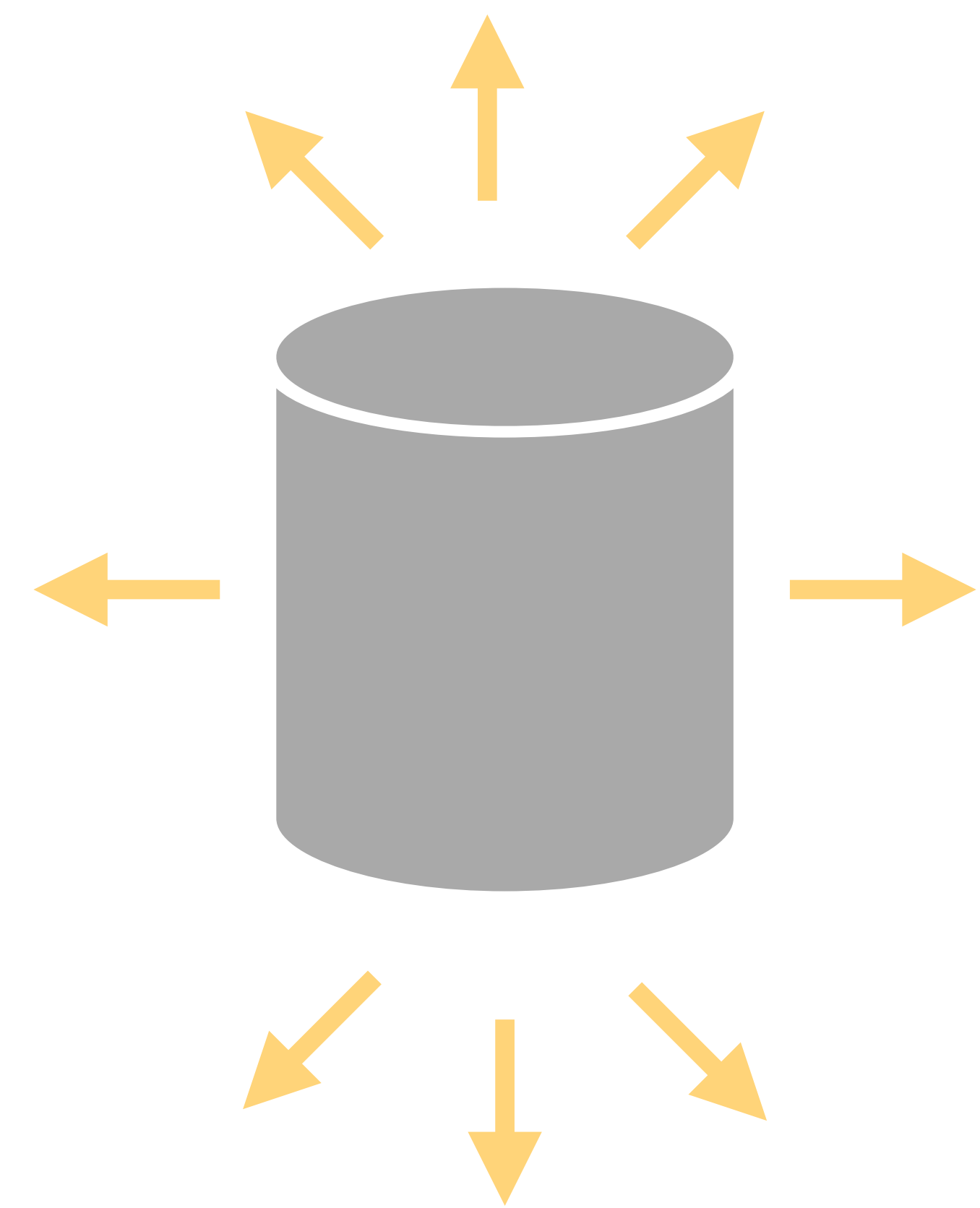
depends on the background field in the laboratory



$$E_{\text{sig}} \sim \frac{\partial_t j_{\text{eff}}}{\omega_1^2}$$



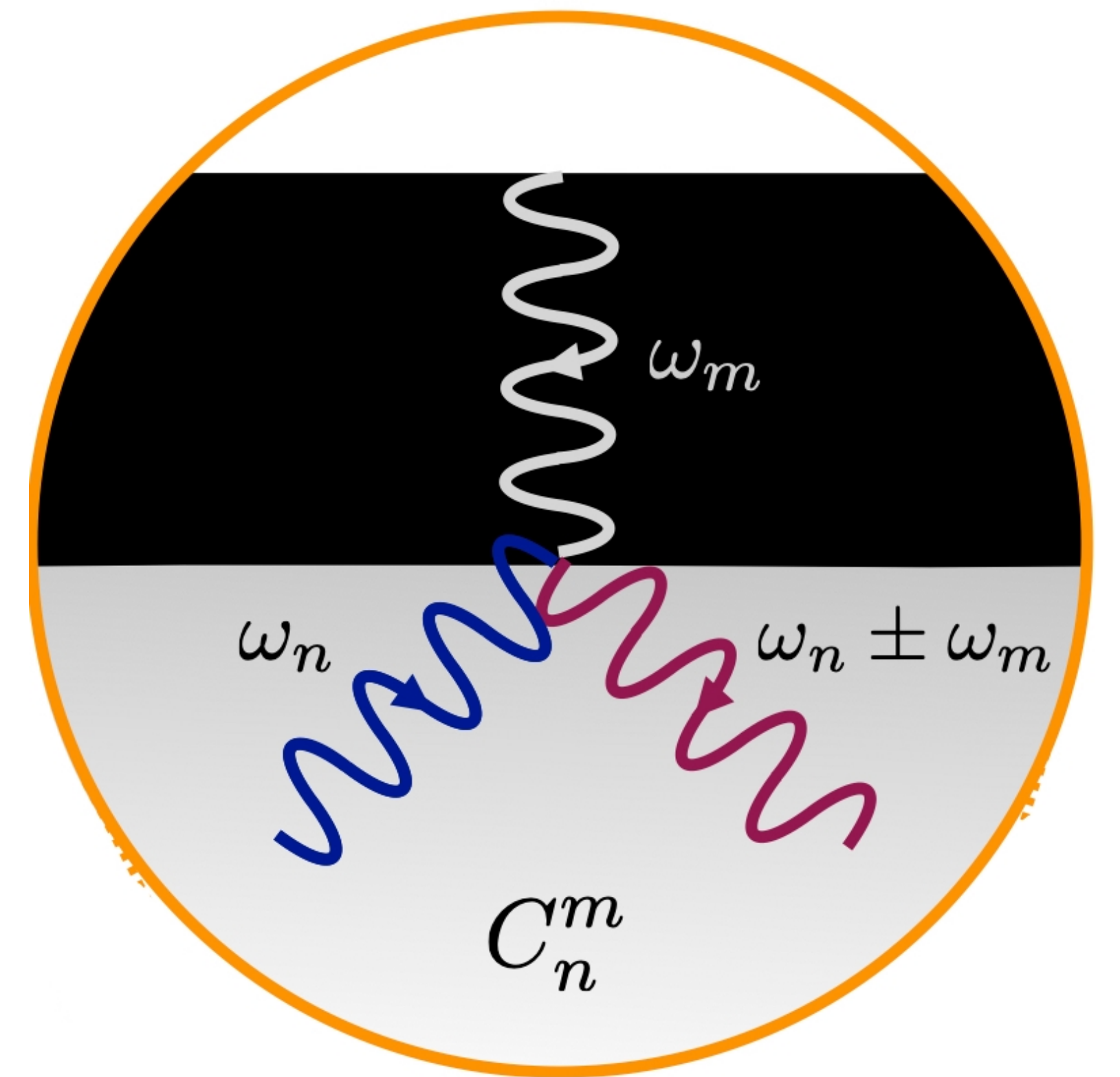
$$\partial_t j_{\text{eff}} \sim \partial_t^2 (RB_0) \sim \omega_g^2 (hB_0) (\omega_g L_{\text{cav}})^2$$



Mechanical Mode

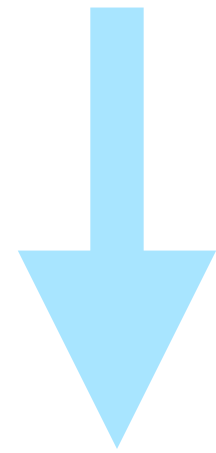
$$\left(\partial_t^2 + \frac{\omega_m}{Q} \partial_t + \omega_m^2 \right) \underline{u_m} = \frac{F_m}{M_{\text{cav}}}$$

MECHANICAL



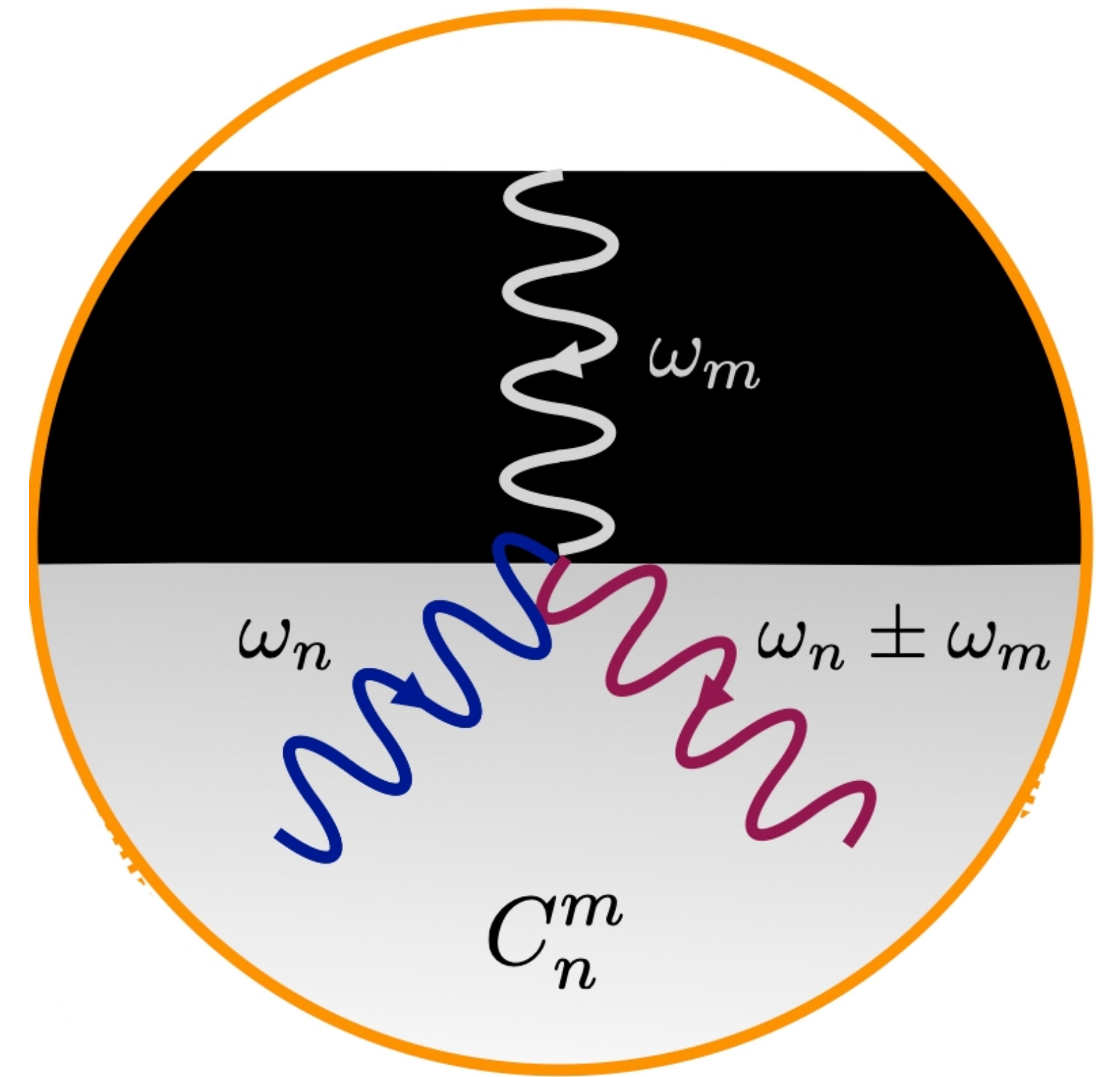
Mechanical Mode

$$\left(\partial_t^2 + \frac{\omega_m}{Q} \partial_t + \omega_m^2 \right) \underline{u_m} = \frac{F_m}{M_{\text{cav}}}$$



$$u_m \sim Q \frac{\omega_g^2}{\omega_m^2} (hL_{\text{cav}})$$

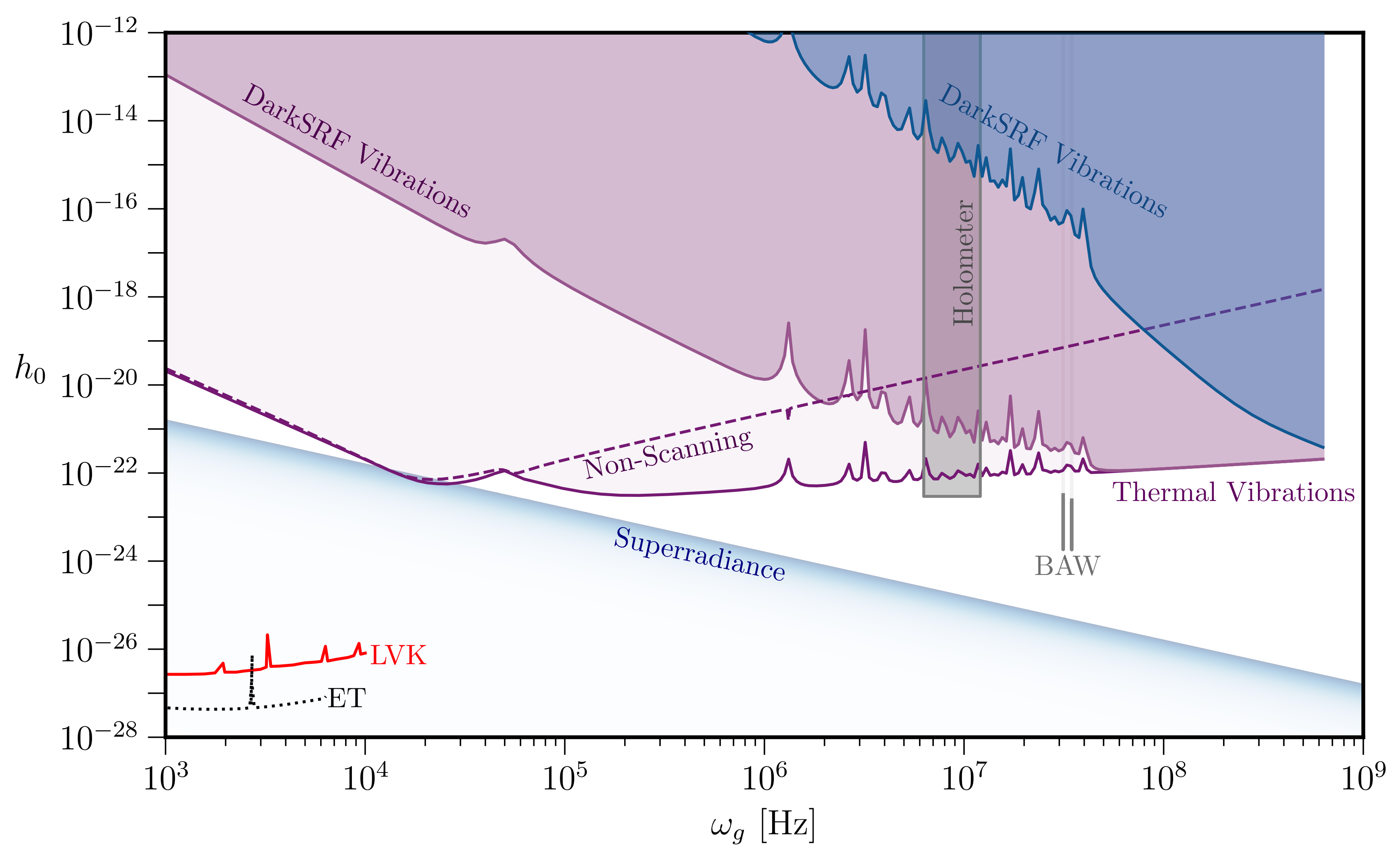
MECHANICAL



MECHANICAL VS ELECTROMAGNETIC RESONANCE

$$\omega_m \approx \frac{c_s}{L} \ll \omega_{\text{em}} \approx \frac{c}{L}$$

MONOCHROMATIC SIGNAL



Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel, Wentzel '23
arXiv:2303.01518

EXISTING PROTOTYPE



MAGO '05

R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme,* R. Parodi, A. Podest`a, and R. Vaccarone

INFN and Universita` degli Studi di Genova, Genova, Italy

Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito

CERN, Geneva, Switzerland

R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto

INFN, Napoli, and Universita` degli Studi del Sannio, Benevento, Italy

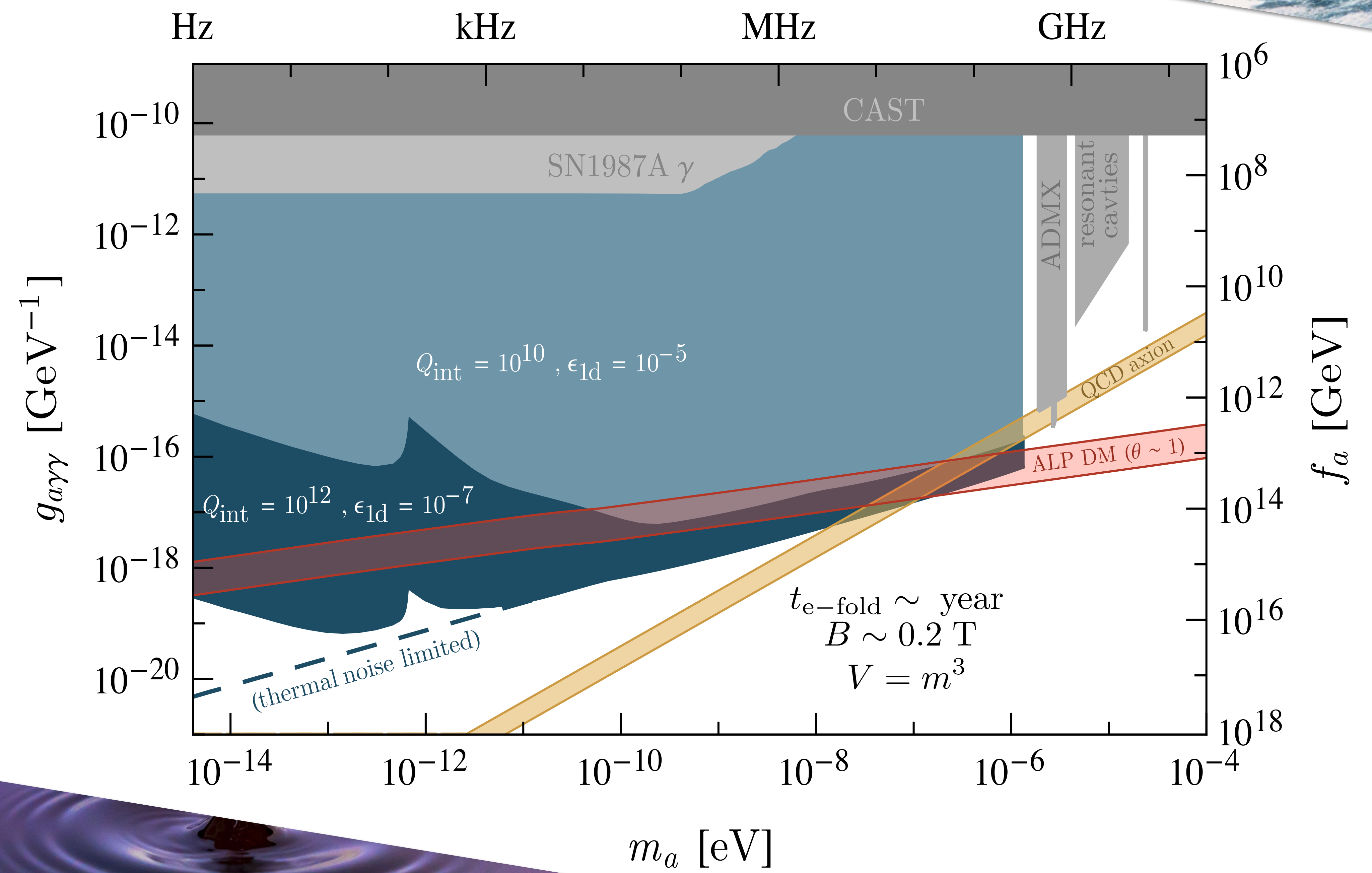
E. Picasso

INFN and Scuola Normale Superiore, Pisa, Italy and CERN, Geneva, Switzerland

BACKUP

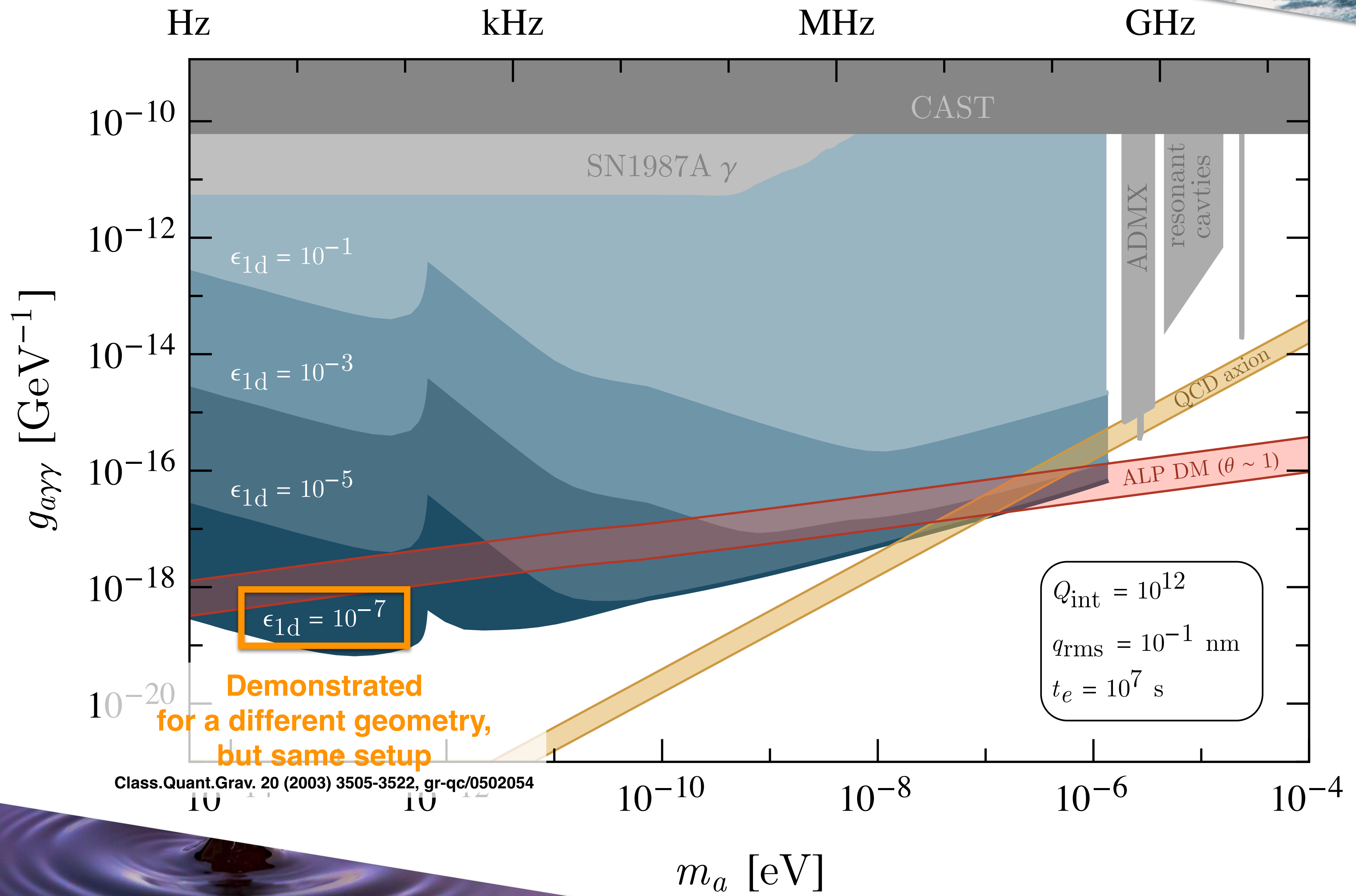
RESONANT

frequency = $m_a/2\pi$



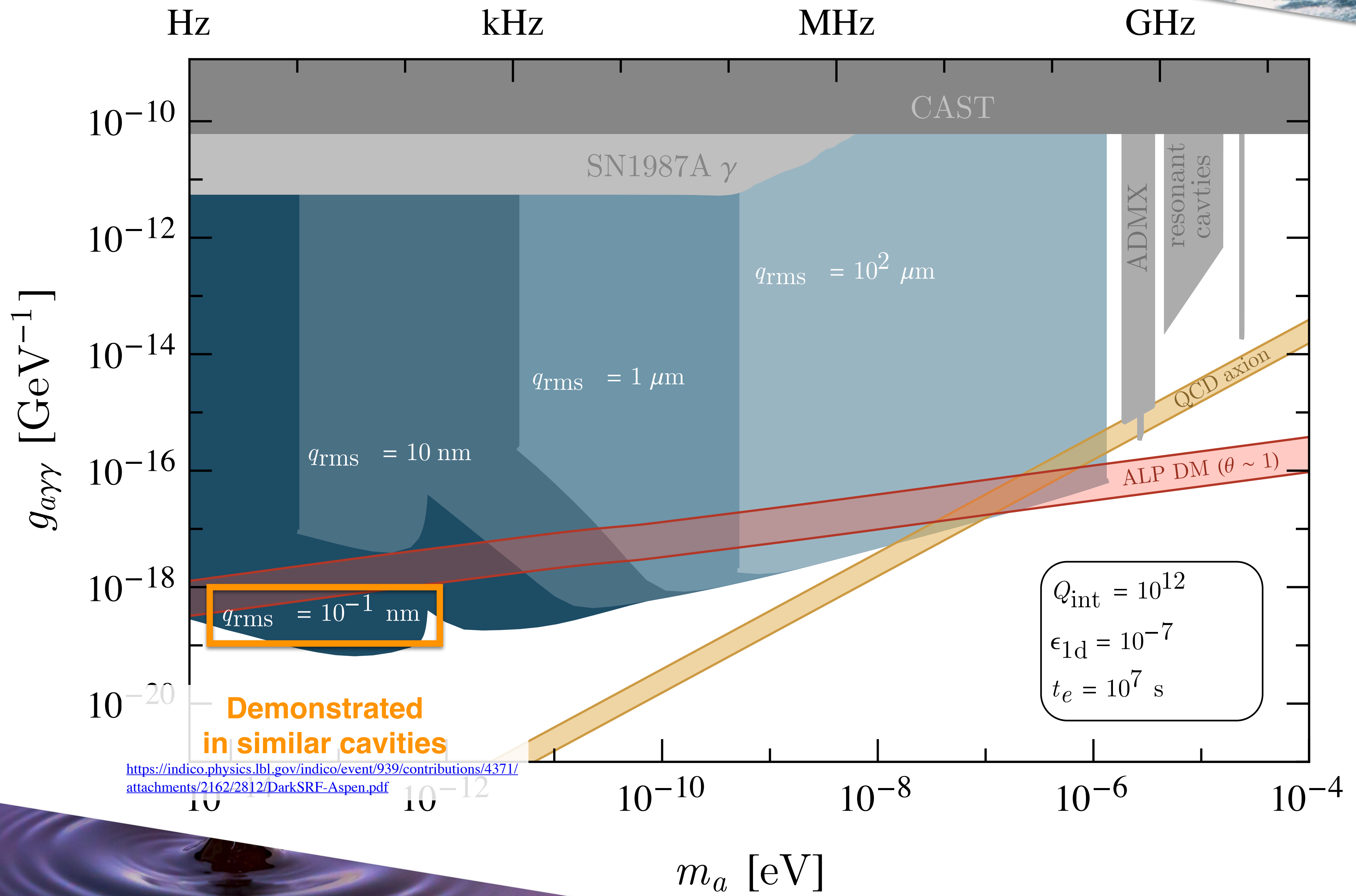
RESONANT

$$\text{frequency} = m_a / 2\pi$$



RESONANT

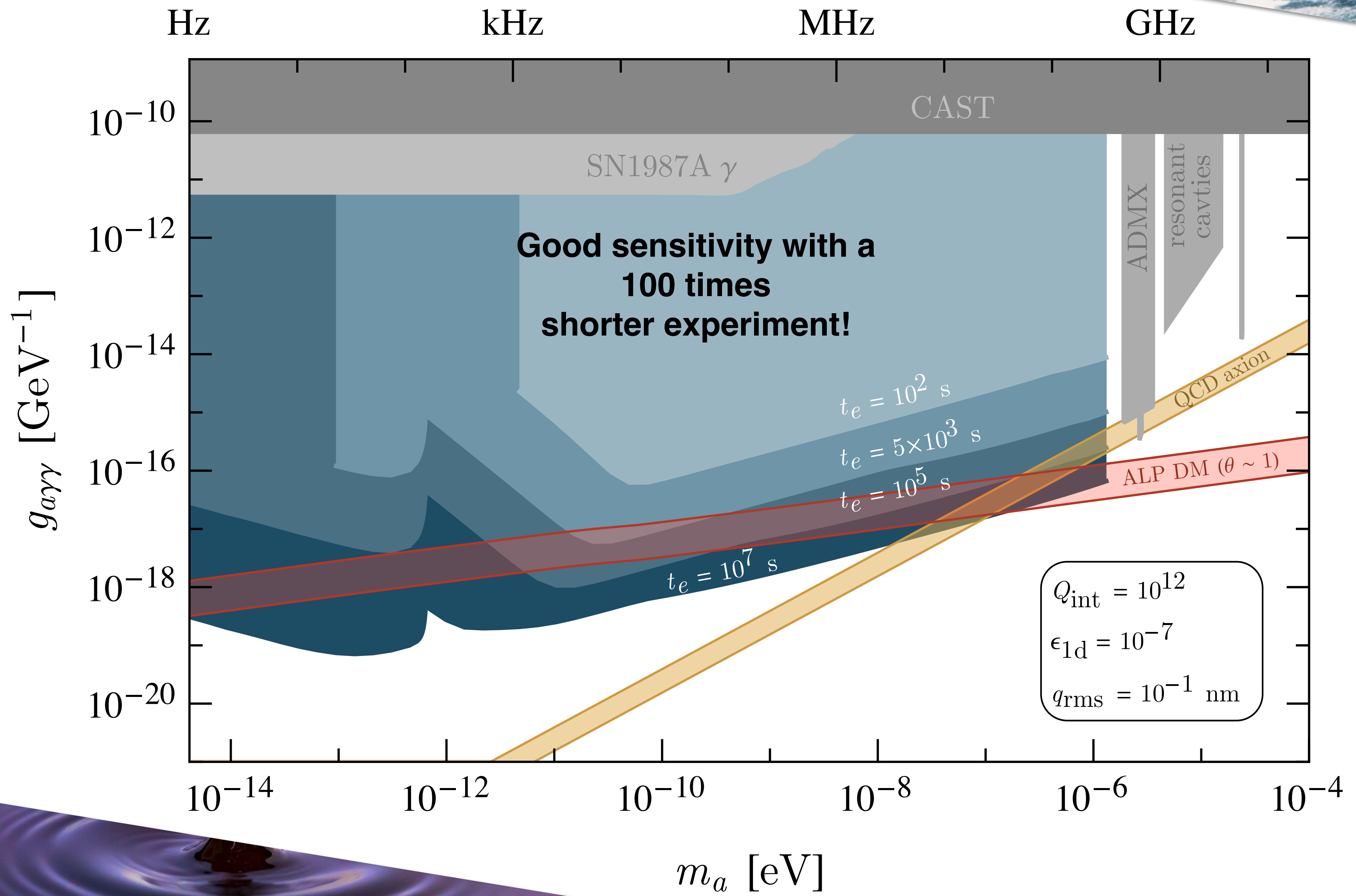
$$\text{frequency} = m_a / 2\pi$$

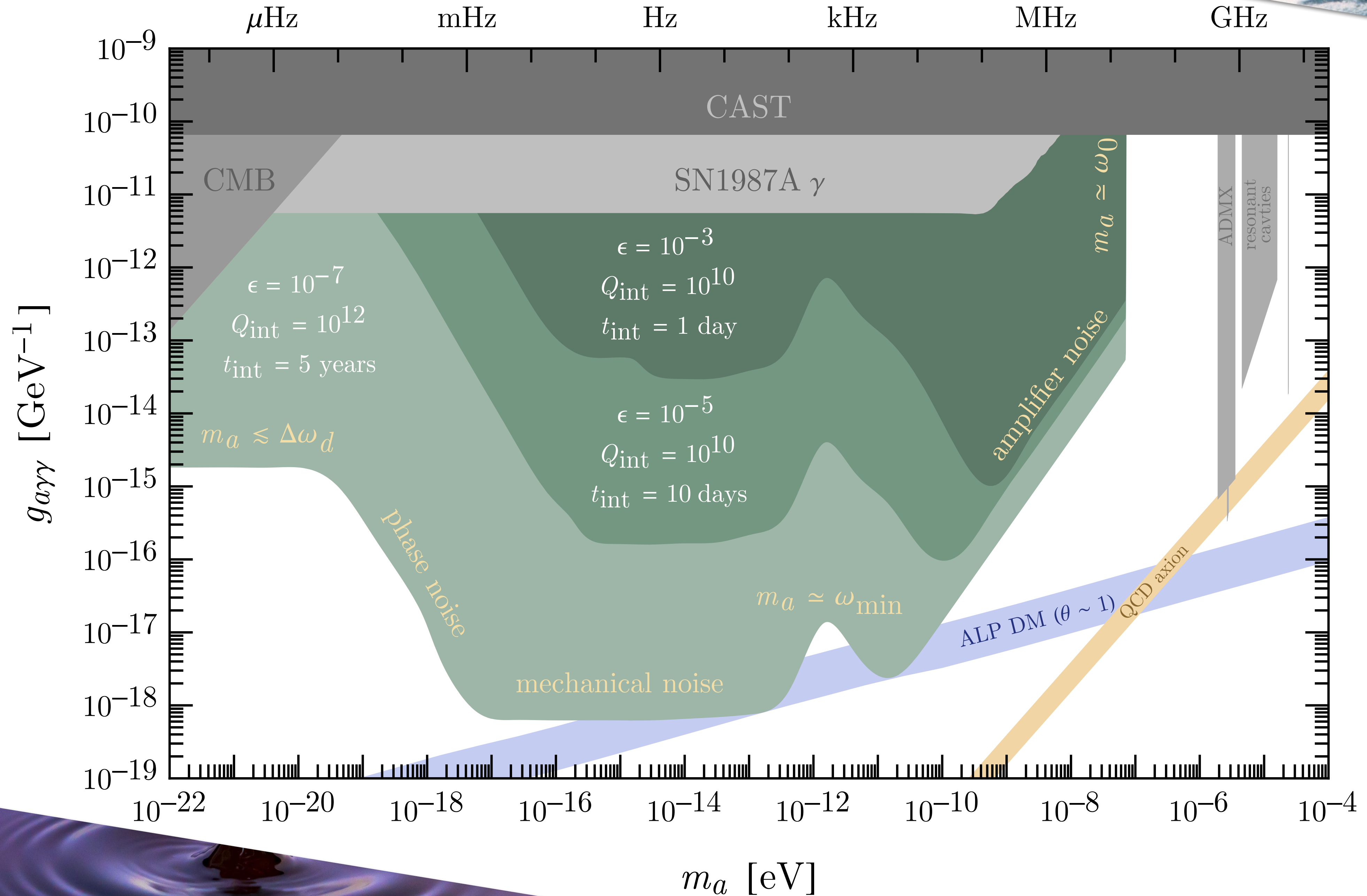


<https://indico.physics.lbl.gov/indico/event/939/contributions/4371/attachments/2162/2812/DarkSRF-Aspen.pdf>

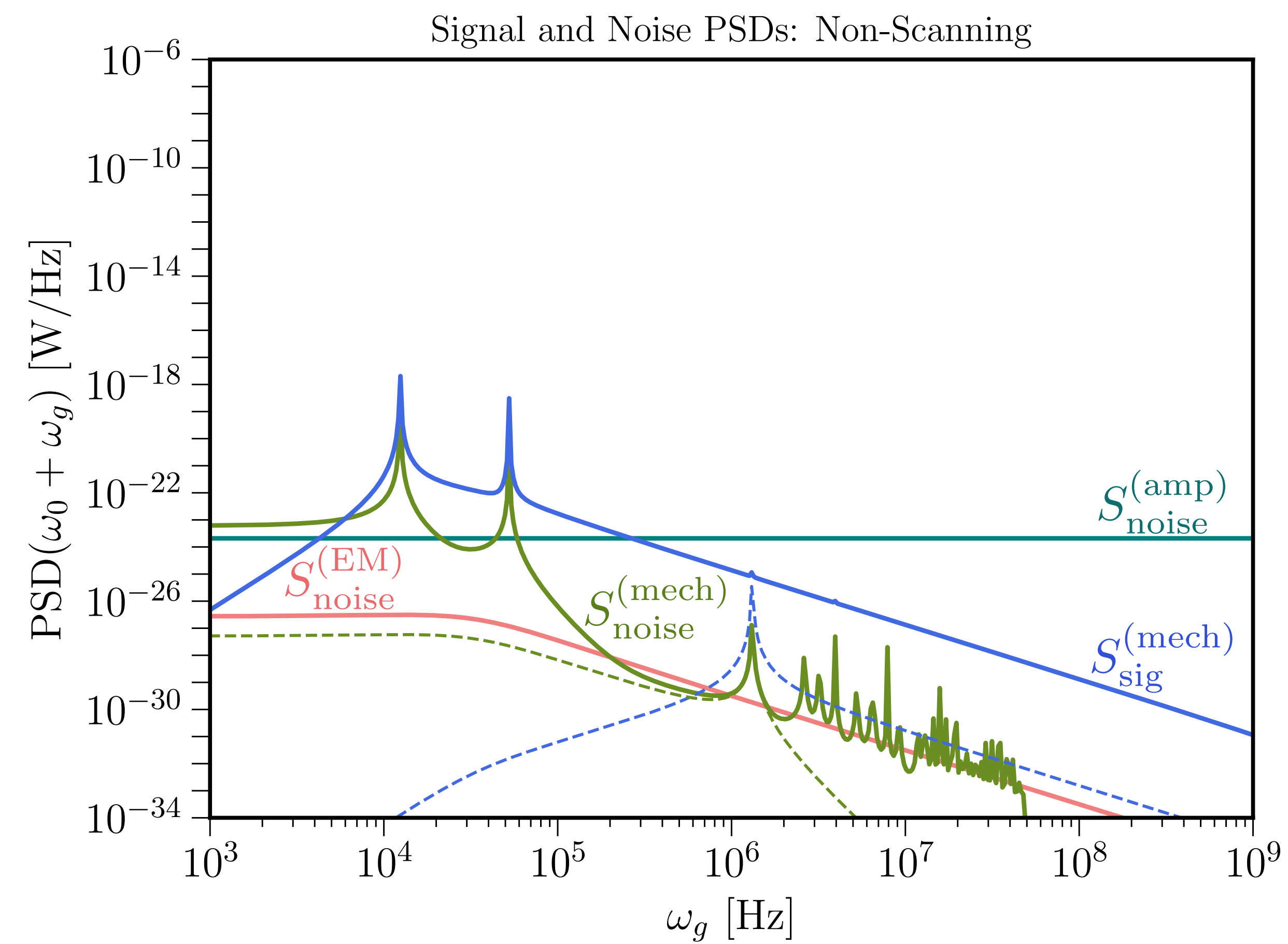
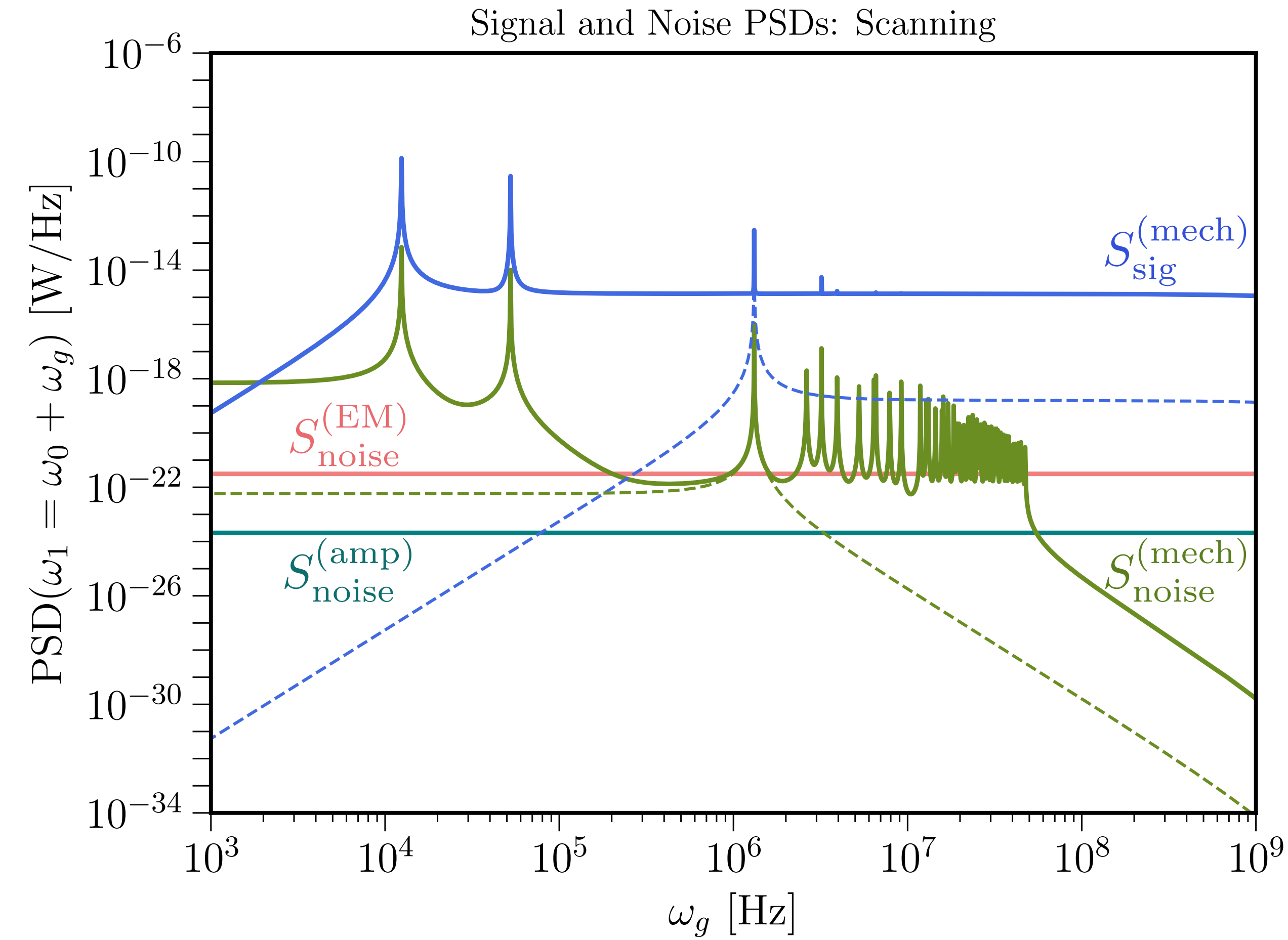
RESONANT

$$\text{frequency} = m_a / 2\pi$$





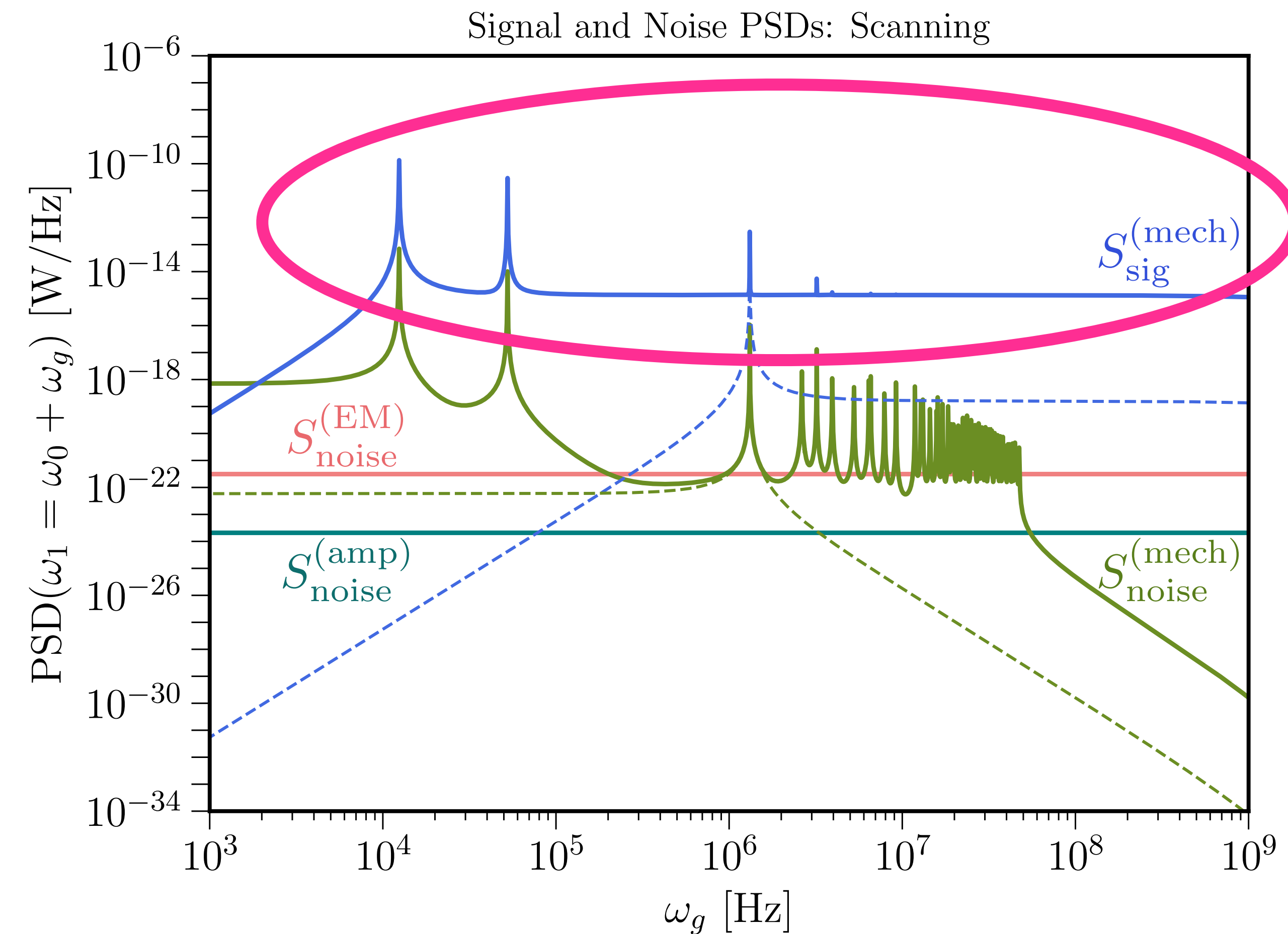
NOISE



Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel, Wentzel '23

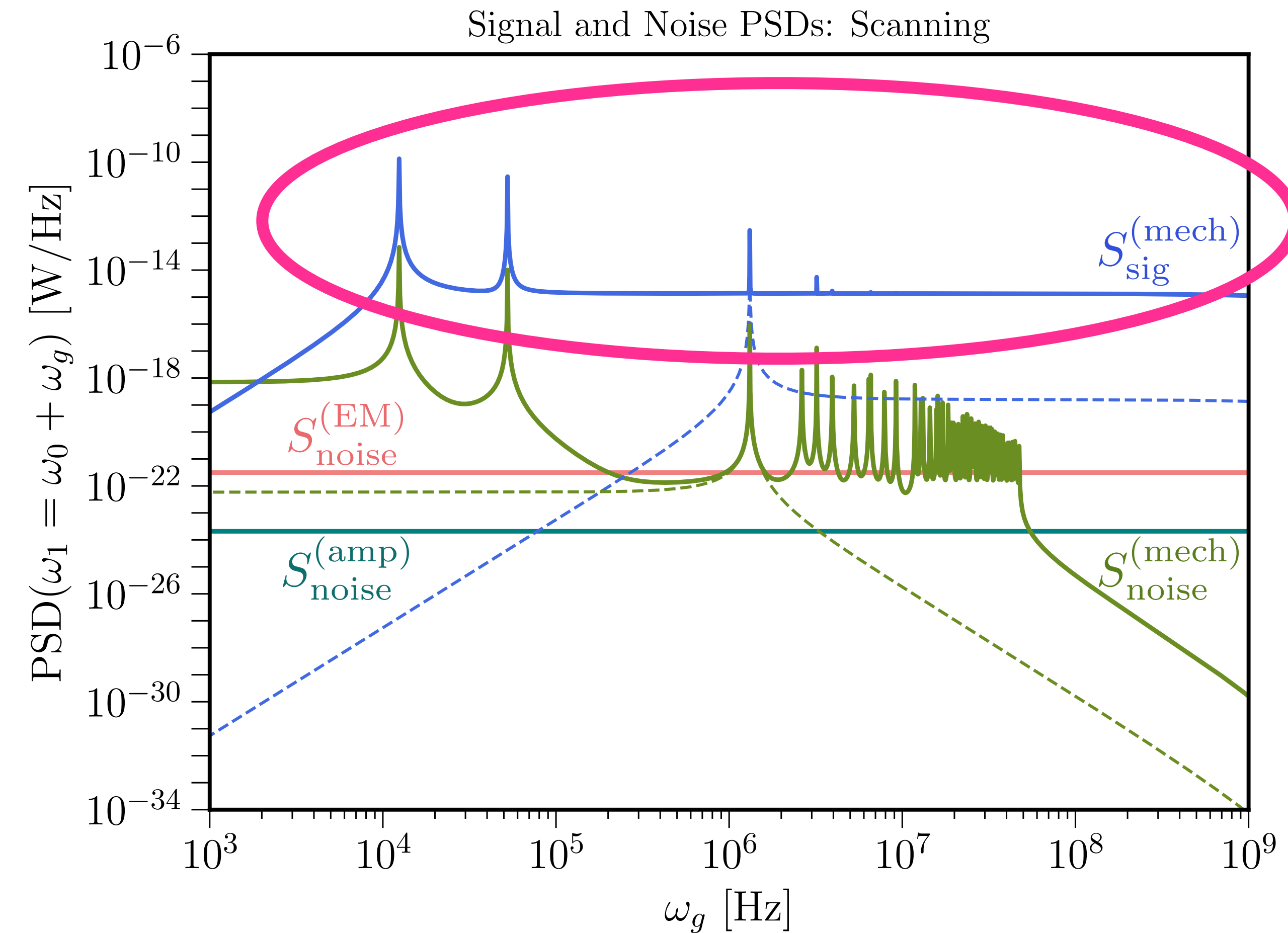
arXiv:2303.01518

RESONANT

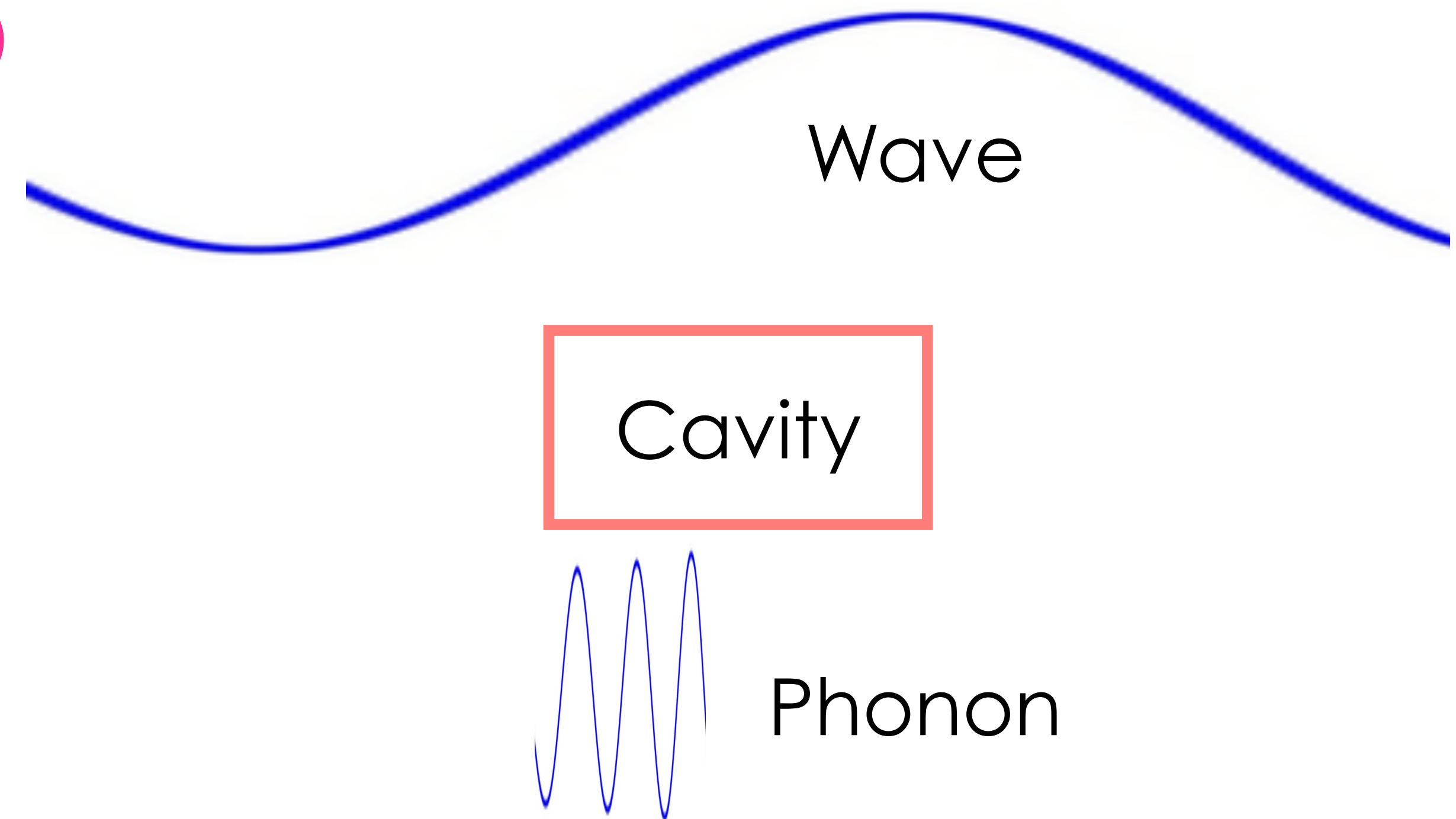


SIGNAL DOMINATED BY THE FIRST FEW MECHANICAL RESONANCES

RESONANT



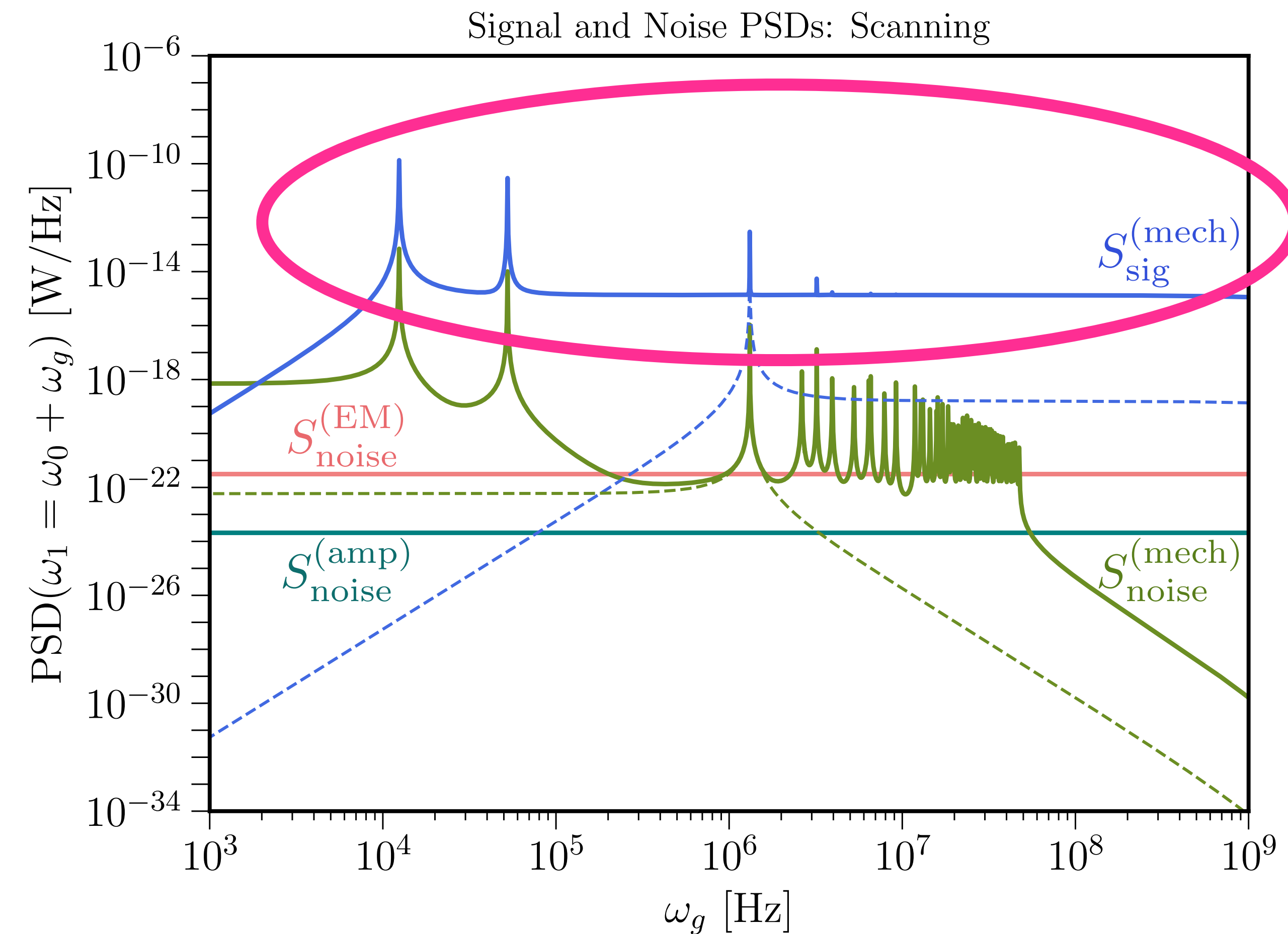
SIGNAL DOMINATED BY THE FIRST FEW MECHANICAL RESONANCES



Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel, Wentzel '23

arXiv:2303.01518

RESONANT

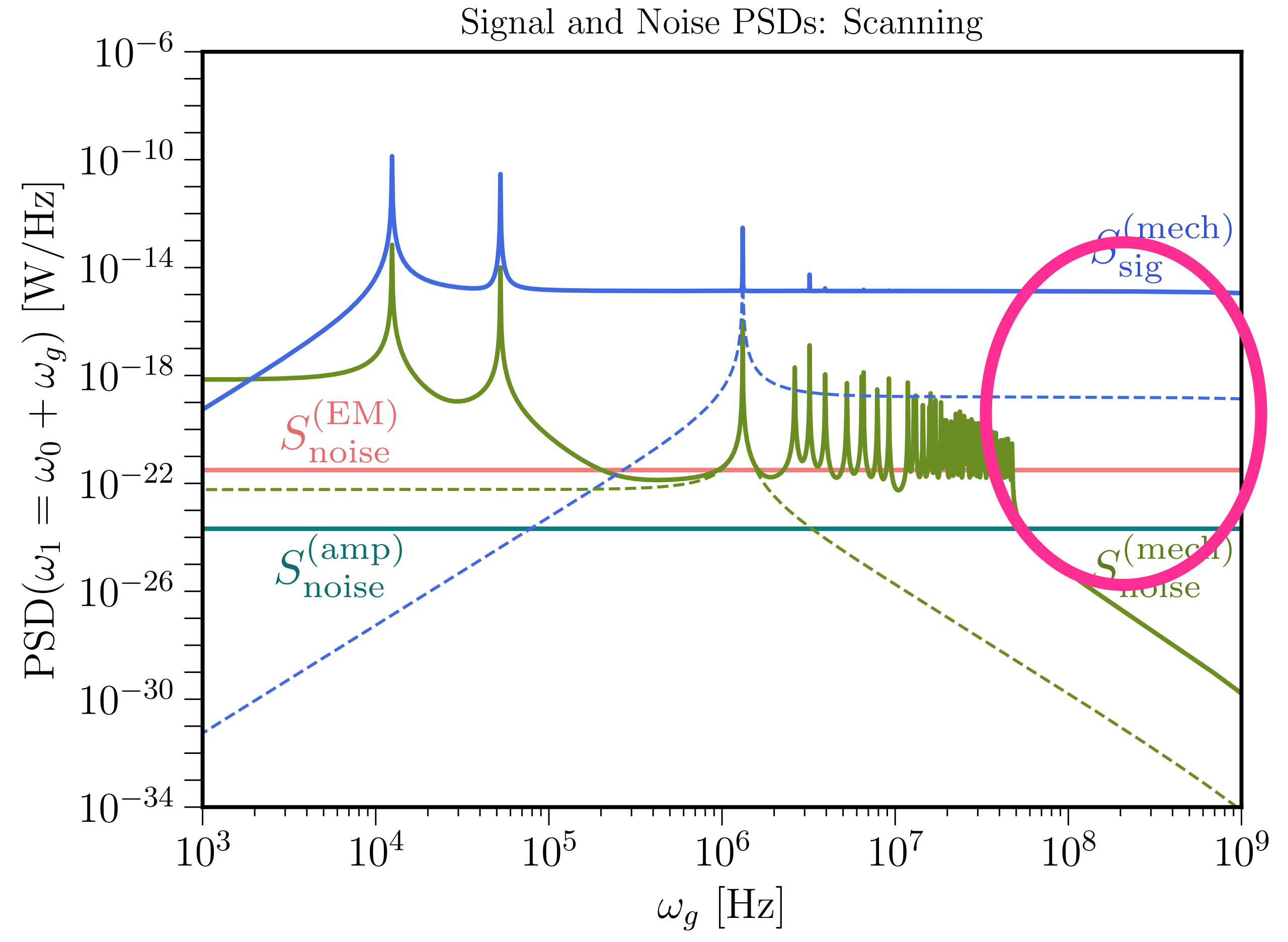


SIGNAL DOMINATED BY THE FIRST FEW MECHANICAL RESONANCES

$$\eta_{\text{mech}}^g \sim \frac{1}{\omega_m^2}$$

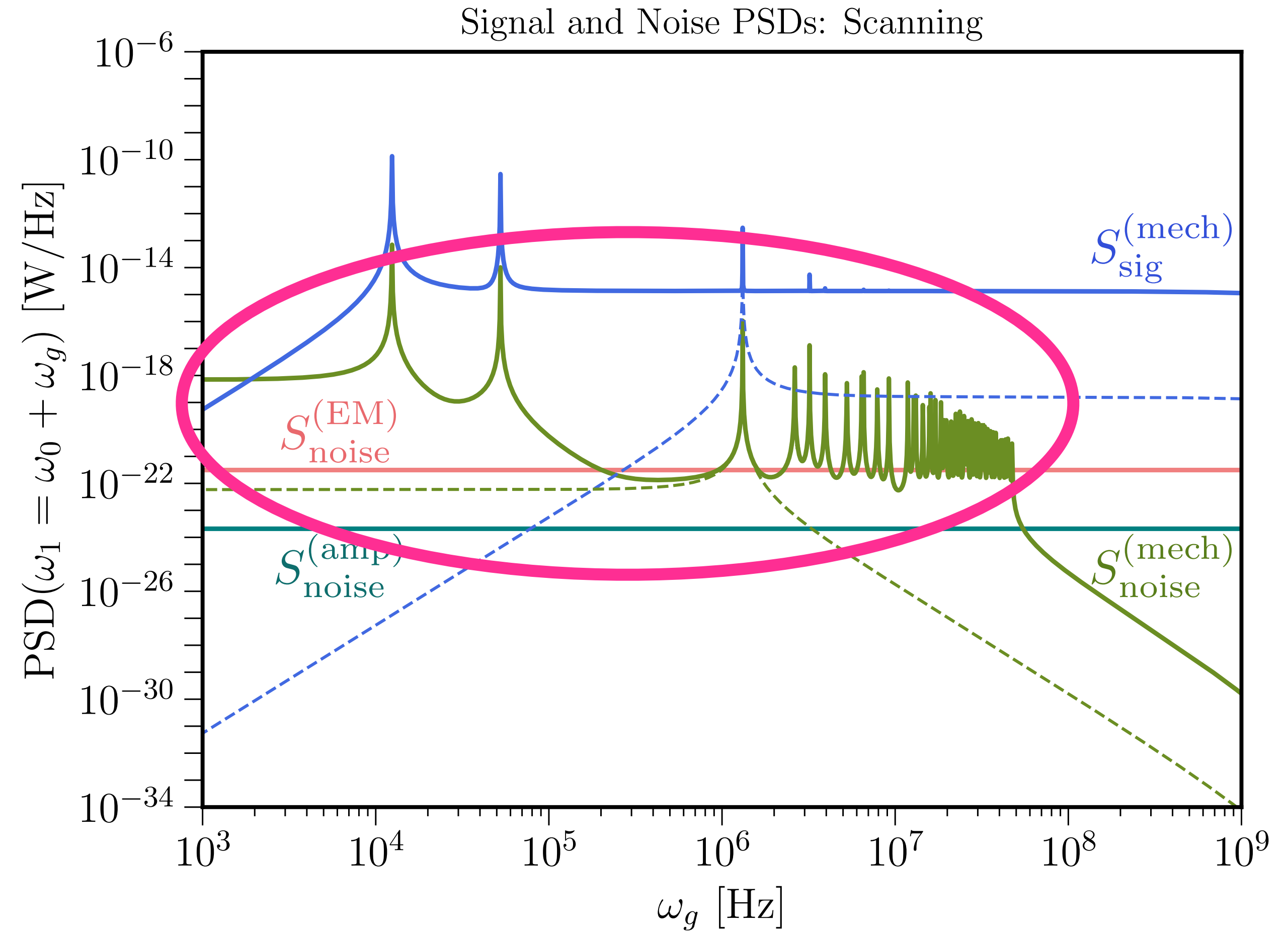
$$Q_m \sim \frac{1}{\omega_m}$$

RESONANT



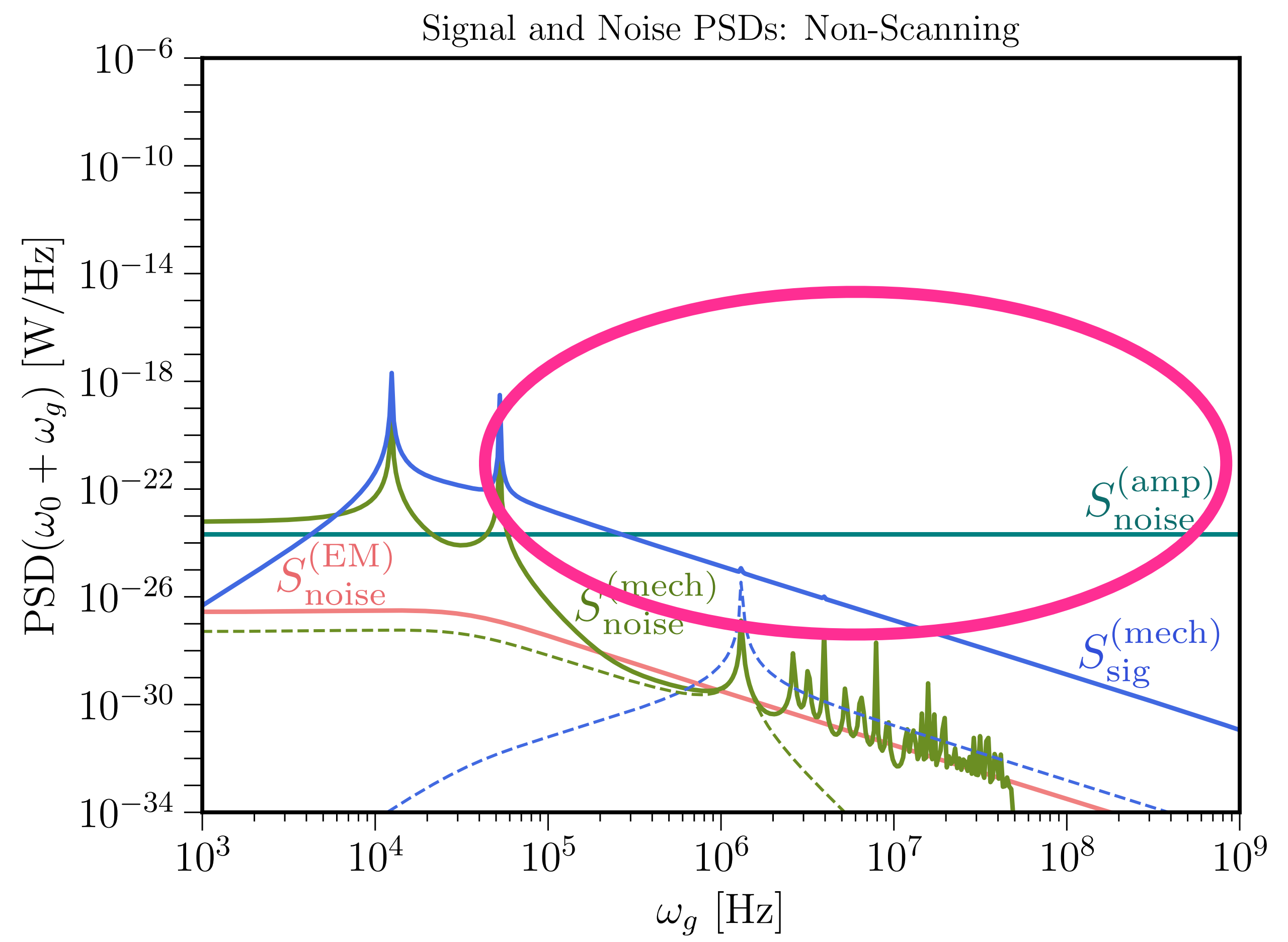
$$\text{SNR}_{\text{EM noise}} \simeq \frac{1}{64} \sqrt{\frac{\pi t_{\text{int}}}{2 \Delta \omega_{\text{osc}}}} Q_{\text{int}}^2 |\eta_{\text{mech}}^g|^2 |\eta_{\text{mech}}^{\text{EM}}|^2 \frac{P_{\text{in}}}{T} h_0^2$$

RESONANT




$$\text{SNR}_{\text{mech noise}} \approx \frac{1}{4} \sqrt{\frac{\pi^3 t_{\text{int}}}{2 \Delta \omega_{\text{osc}}}} |\eta_{\text{mech}}^g(\omega_p^{\text{sig}})|^2 \frac{|\eta_{\text{mech}}^{\text{EM}}(\omega_p^{\text{sig}})|^2}{|\eta_{\text{mech}}^{\text{EM}}(\omega_p^{\text{noise}})|^2} \frac{M_{\text{cav}}^2 V_{\text{cav}}^{2/3}}{S_{F_p}(\omega_g)} h_0^2 \omega_g^4$$


BROADBAND



$$\text{SNR}_{\text{amp noise}}^{(\text{broadband})} \approx \frac{1}{64} \sqrt{\frac{\pi t_{\text{int}}}{2 \Delta\omega_{\text{osc}}}} \frac{Q_{\text{int}}}{Q_{\text{cpl}}} |\eta_{\text{mech}}^g|^2 |\eta_{\text{mech}}^{\text{EM}}|^2 \frac{P_{\text{in}}}{T} h_0^2 \frac{\omega_0}{\omega_g^2}$$



Proper Detector Frame (PDF) Vs Transverse Traceless (TT) Frame
(Fermi Normal Coordinates)

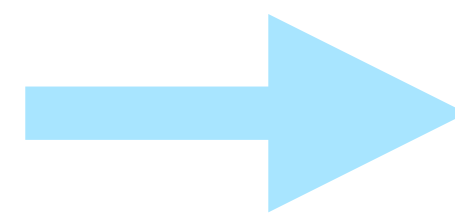


TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

$$t_{\text{TT}} \simeq t - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t}, \quad x_{\text{TT}} \simeq x - \frac{1}{2} x (1 - i\omega_g z) h_+ e^{i\omega_g t}$$
$$y_{\text{TT}} \simeq y + \frac{1}{2} y (1 - i\omega_g z) h_+ e^{i\omega_g t}, \quad z_{\text{TT}} \simeq z - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t}$$

TT gauge = comoving with the wave
 $(t_{\text{TT}}, x_{\text{TT}}, y_{\text{TT}}, z_{\text{TT}})$

Theorem: $j_{\text{eff},\text{TT}}^{\mu} = 0$



Wrong Conclusion

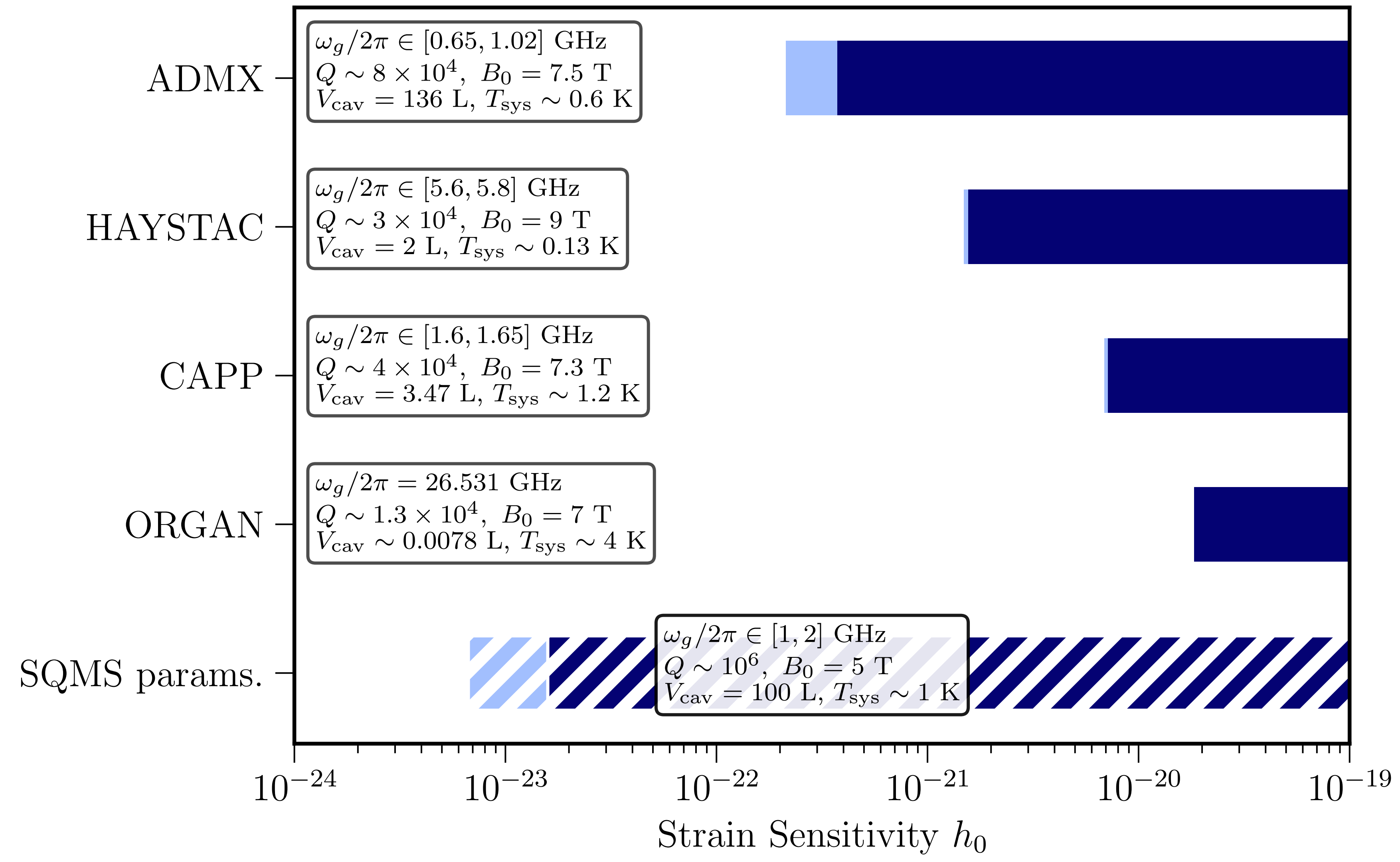
No signal

Doubly Wrong:

1. Impossible to prepare a uniform B-field in the TT frame
2. Even if you could do it, there would still be a signal (wire moving)

GRAVITATIONAL WAVE DETECTION

Projected Sensitivities of Axion Experiments



SUPERRADIANCE

$$\omega_g \simeq 2m_a \simeq \text{GHz} \left(\frac{10^{-4} M_\odot}{M_{\text{BH}}} \right)$$

$$h = 10^{-24} \left(\frac{\Delta a_*}{0.1} \right) \left(\frac{1 \text{ kpc}}{D} \right) \left(\frac{M_b}{10^{-1} M_\odot} \right) \left(\frac{\alpha}{0.2} \right)^7$$

PRIMORDIAL BHs

$$\frac{N_{\text{cycles}}}{Q} \simeq \left(\frac{\text{GHz}}{f_g} \right)^{5/3} \left(\frac{10^{-9} M_{\odot}}{m_{\text{PBH}}} \right)^{5/3} \quad Q = 10^6$$

$$h \simeq 10^{-26} \left(\frac{1 \text{ pc}}{D} \right) \left(\frac{m_{\text{PBH}}}{10^{-9} M_{\odot}} \right)^{5/3} \left(\frac{\omega_g}{\text{GHz}} \right)^{2/3}$$

If they are DM and are all in binaries (1 year)

$$D \simeq 10^{-3} \text{ pc}$$

If they are DM

$$D \simeq 10 \text{ pc} \left(\frac{m_{\text{PBH}}}{10^{-9} M_{\odot}} \right)$$

POWER FROM THE SUN

Power

$$\frac{dP}{dE} \sim \text{const.}$$

$$\frac{d\Gamma}{dE} \sim \frac{1}{E}$$

Number of
gravitons

$$\Gamma \sim \exp[B \log(E/\Lambda)], \quad B \ll 1$$

POWER FROM THE SUN

Signal

$$P_{\text{tot}} \simeq 6 \times 10^{14} \frac{\text{erg}}{\text{s}} \simeq 10^{11} \text{ eV}^2$$

$$P_{\text{exp}} \simeq 10^{-12} \text{ eV}^2$$

Total emitted power (in gravitons)

Power reaching a m-sized
experiment on Earth (in gravitons)

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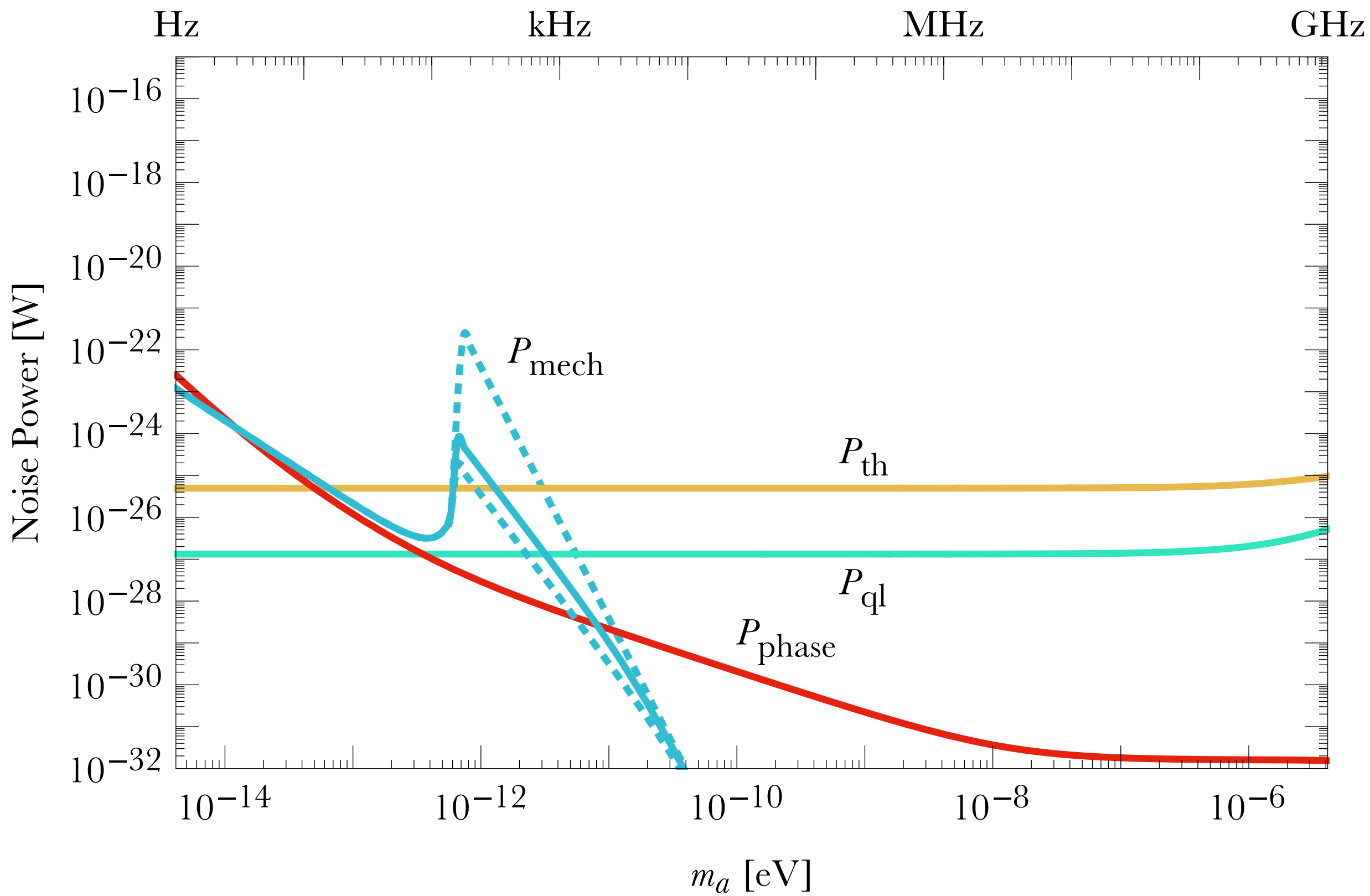
Noise

$$P_{\text{th}} \simeq T \Delta\omega \simeq \text{eV}^2 \left(\frac{T}{\text{K}} \right)$$

Thermal Noise in the bandwidth of the signal (in photons)

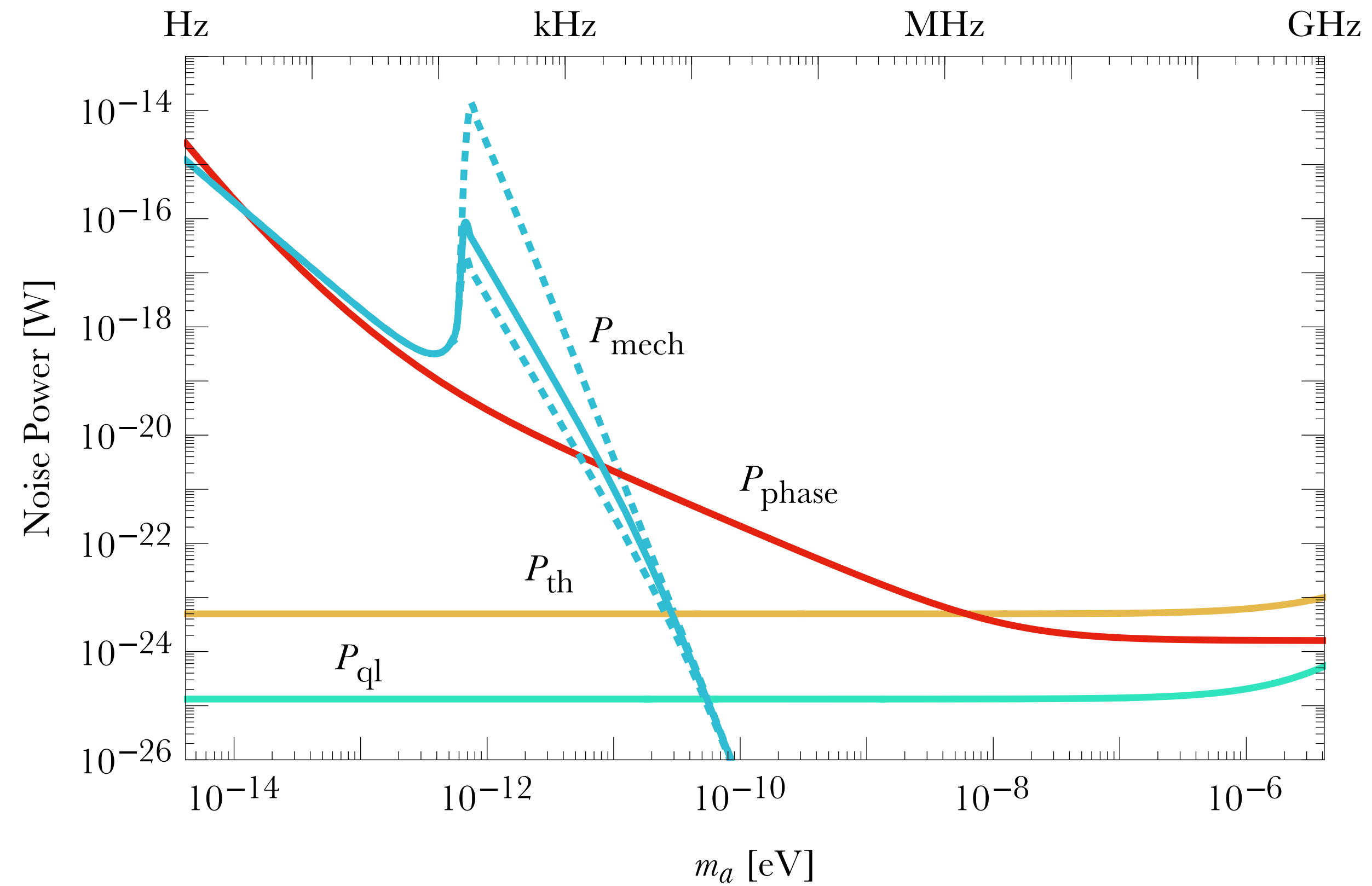
NOISE PSDs

frequency = $m_a/2\pi$



$$\epsilon_{1d} = 10^{-7}, \quad Q = 10^{12}$$

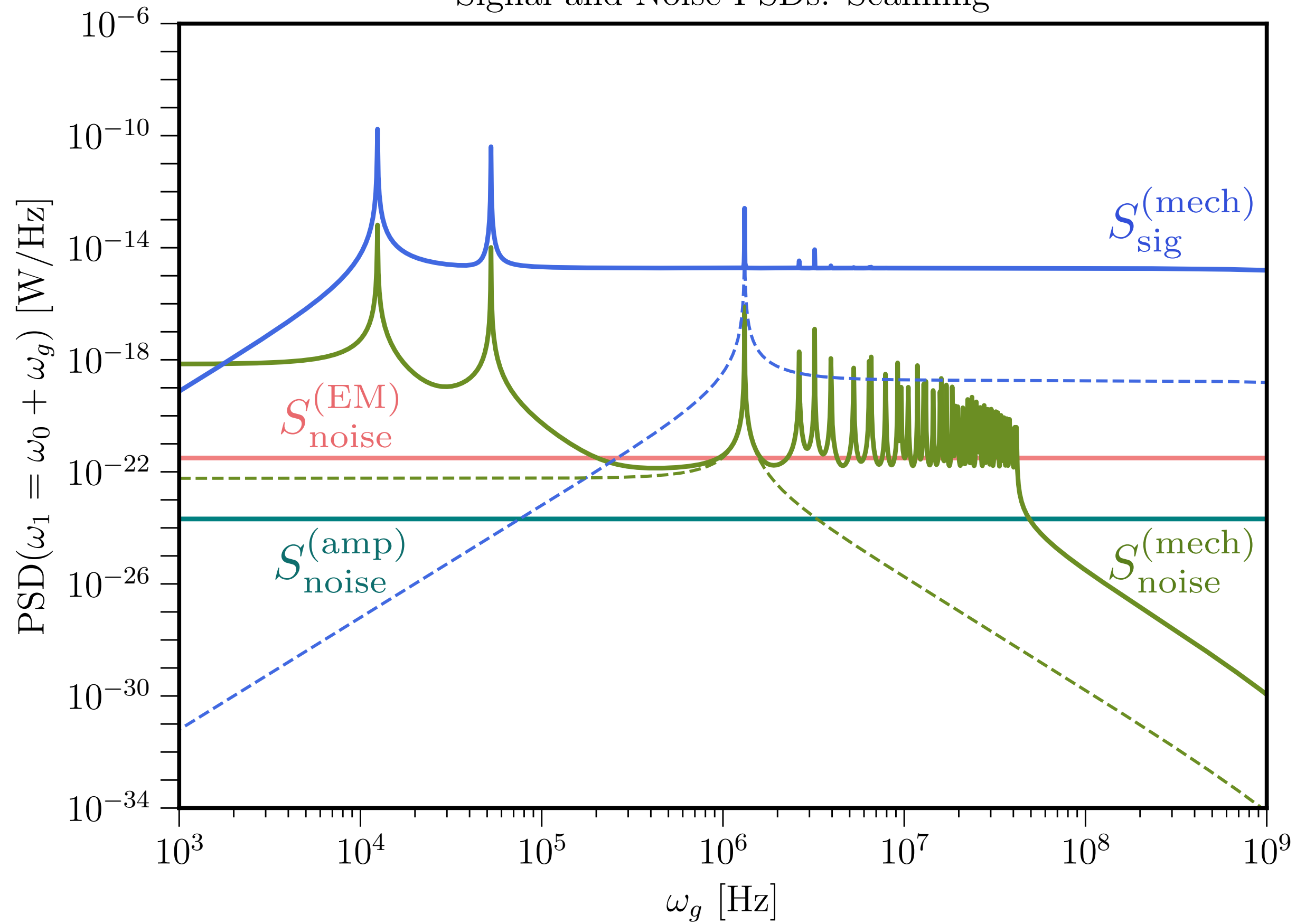
frequency = $m_a/2\pi$



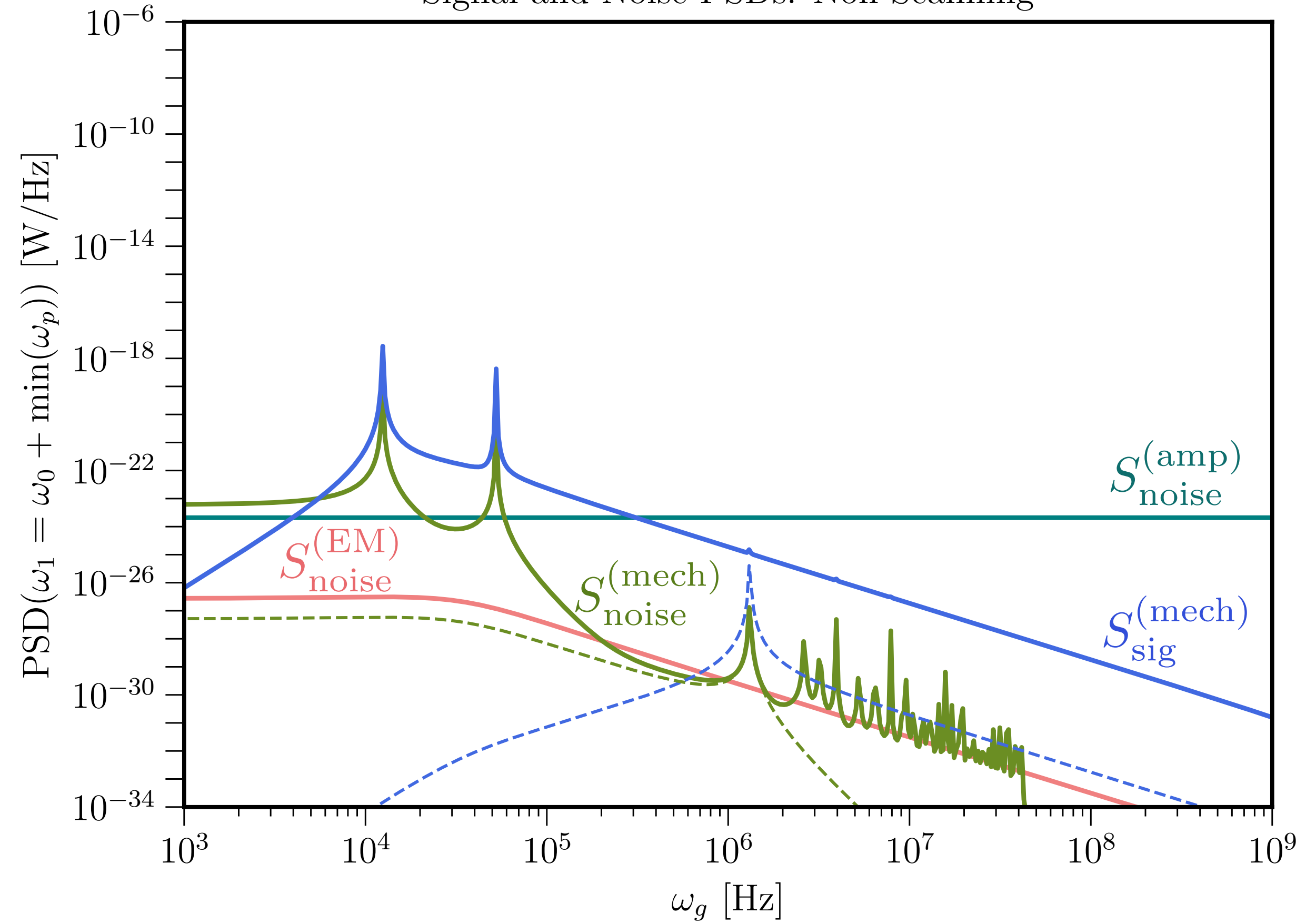
$$\epsilon_{1d} = 10^{-5}, \quad Q = 10^{10}$$

NOISE PSDs GWs

Signal and Noise PSDs: Scanning



Signal and Noise PSDs: Non-Scanning



MADMAX and LAMPOST

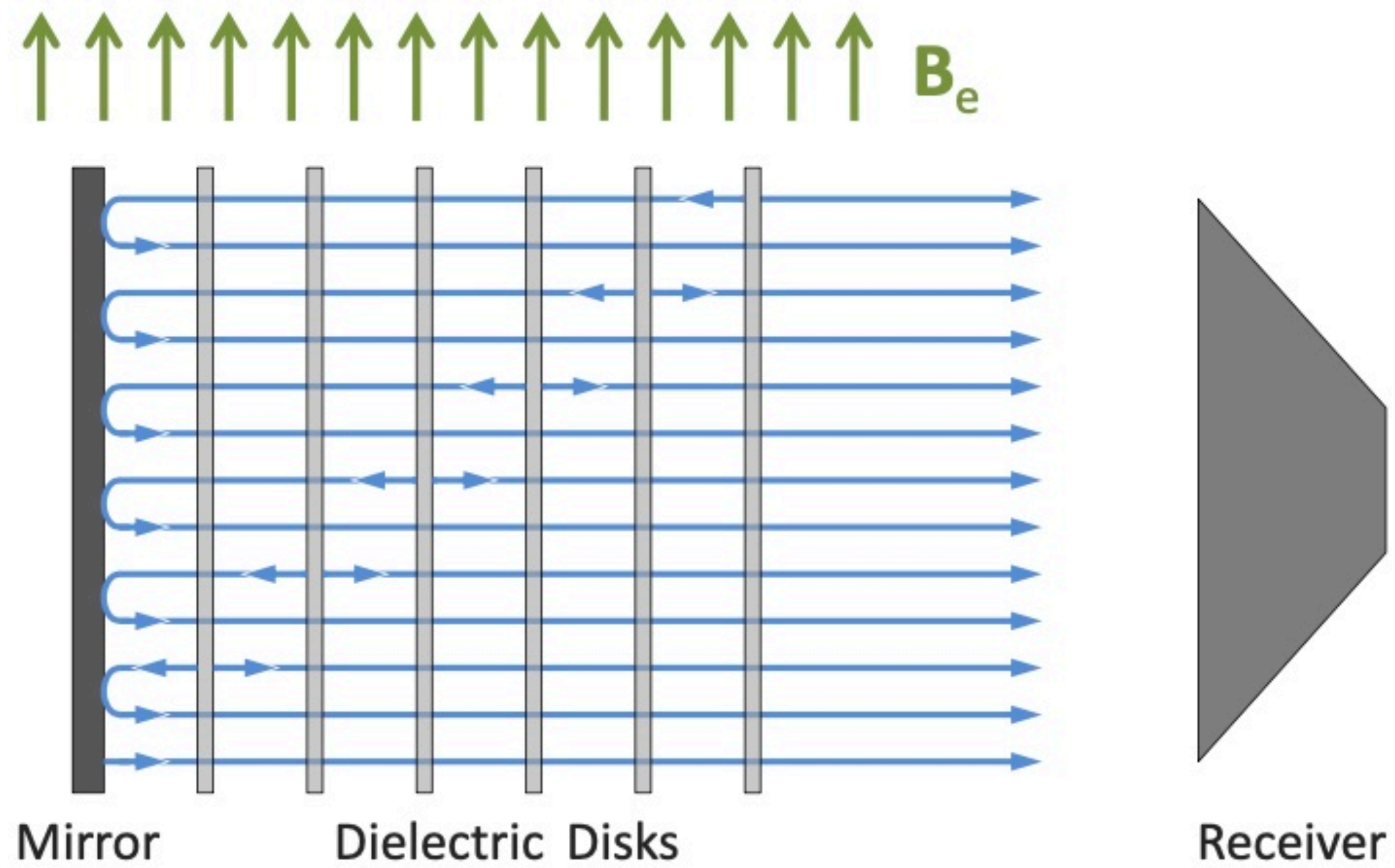
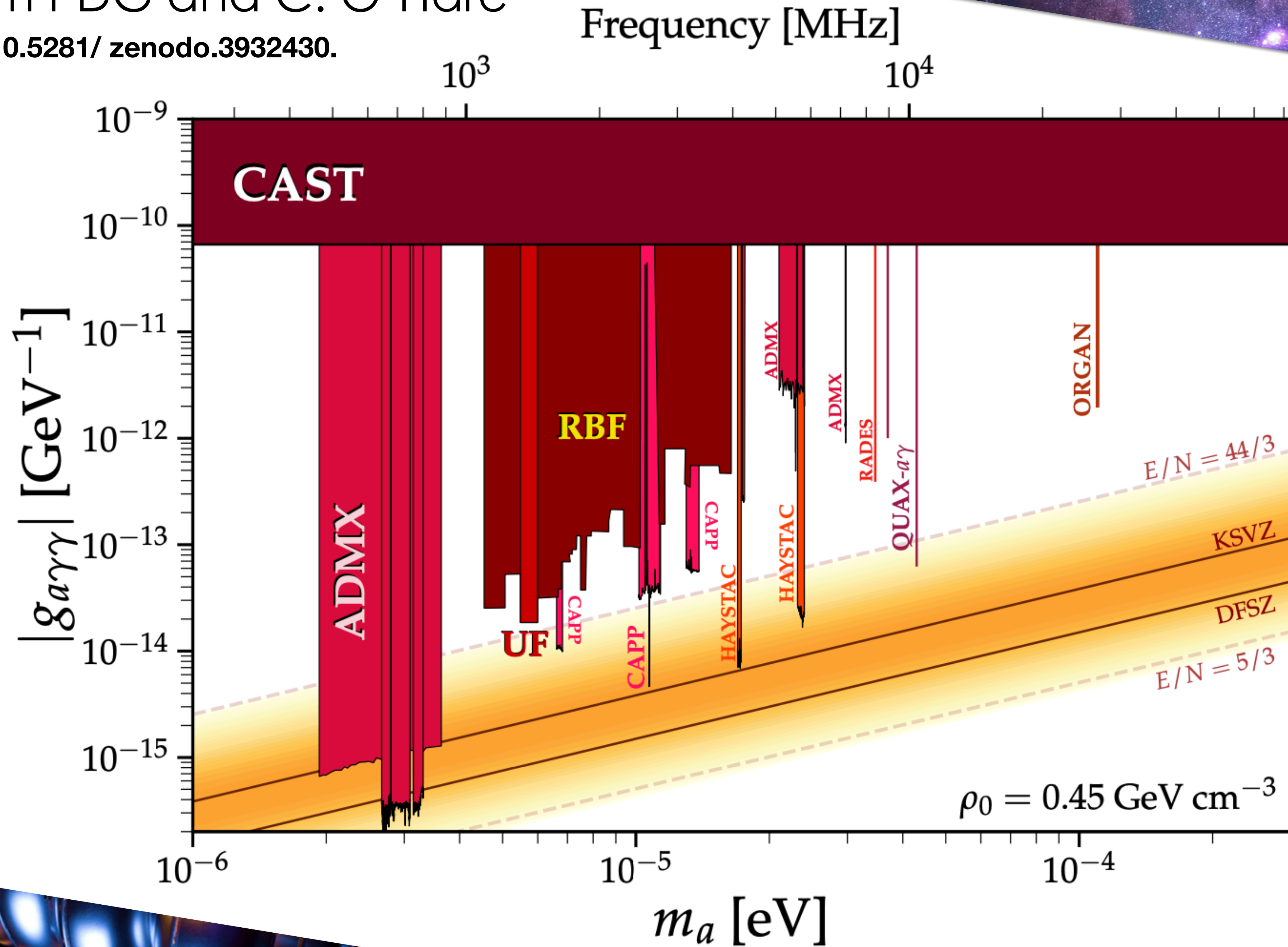


Figure from PDG and C. O'Hare

<https://doi.org/10.5281/zenodo.3932430>.



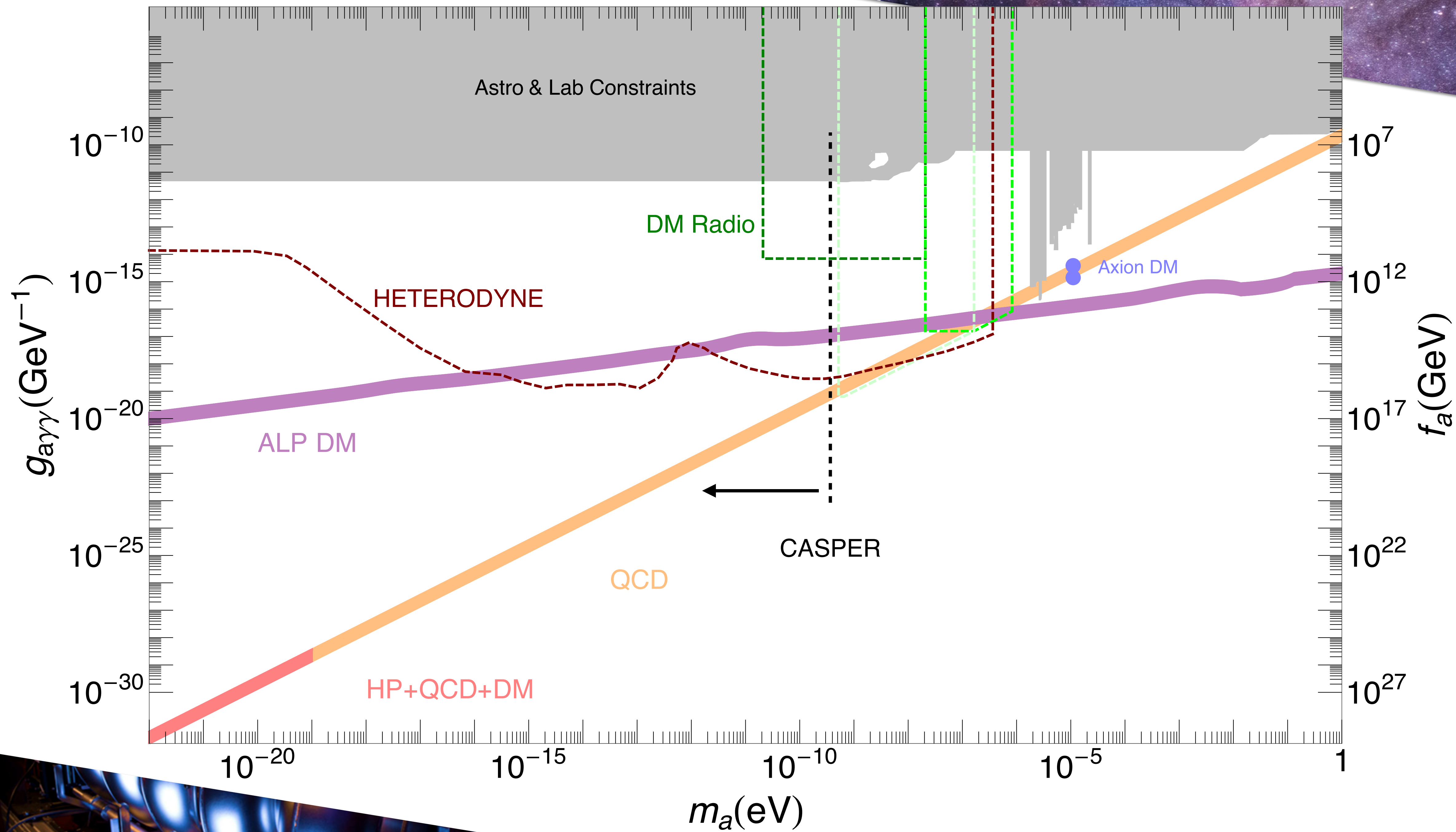
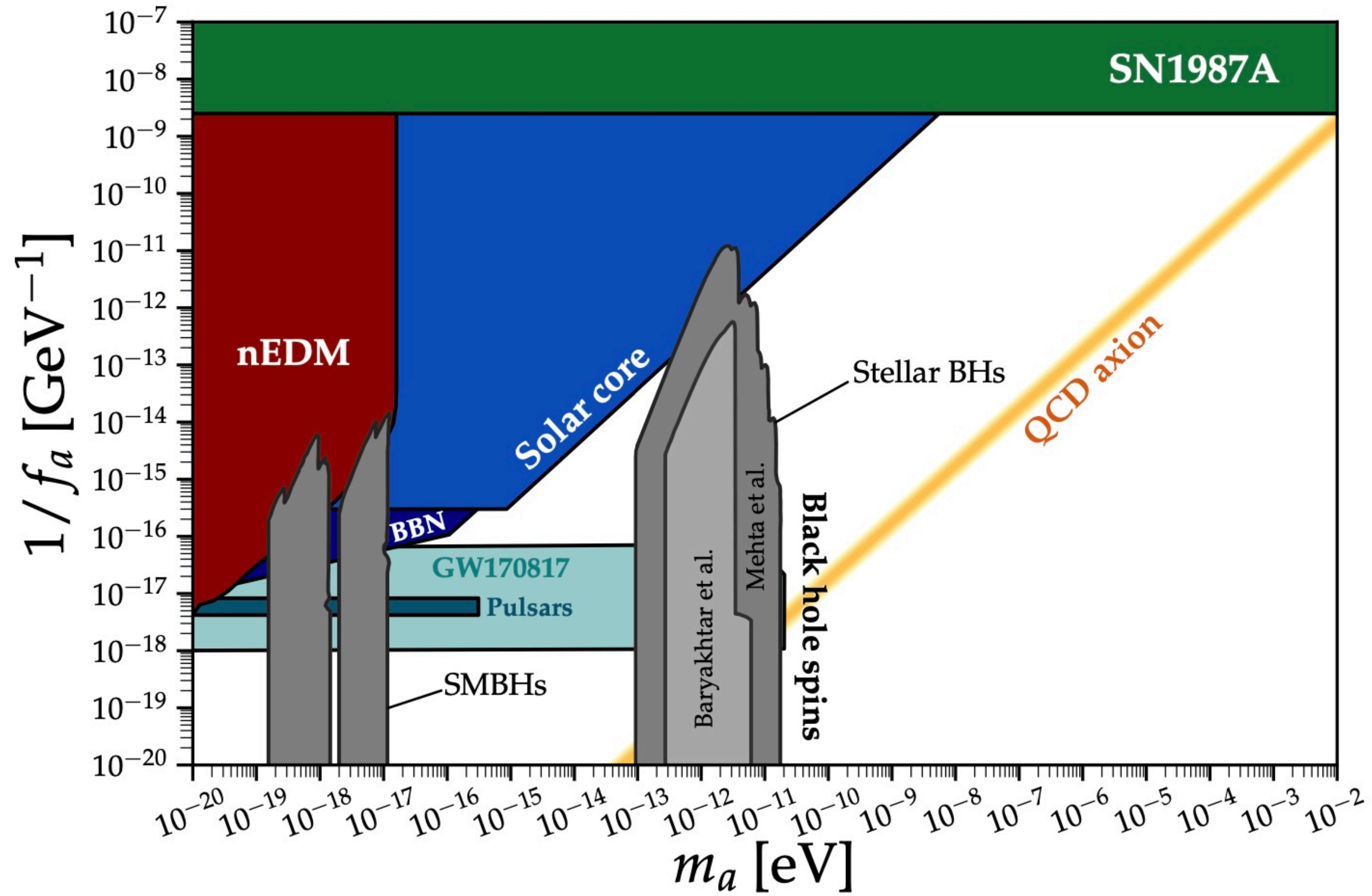


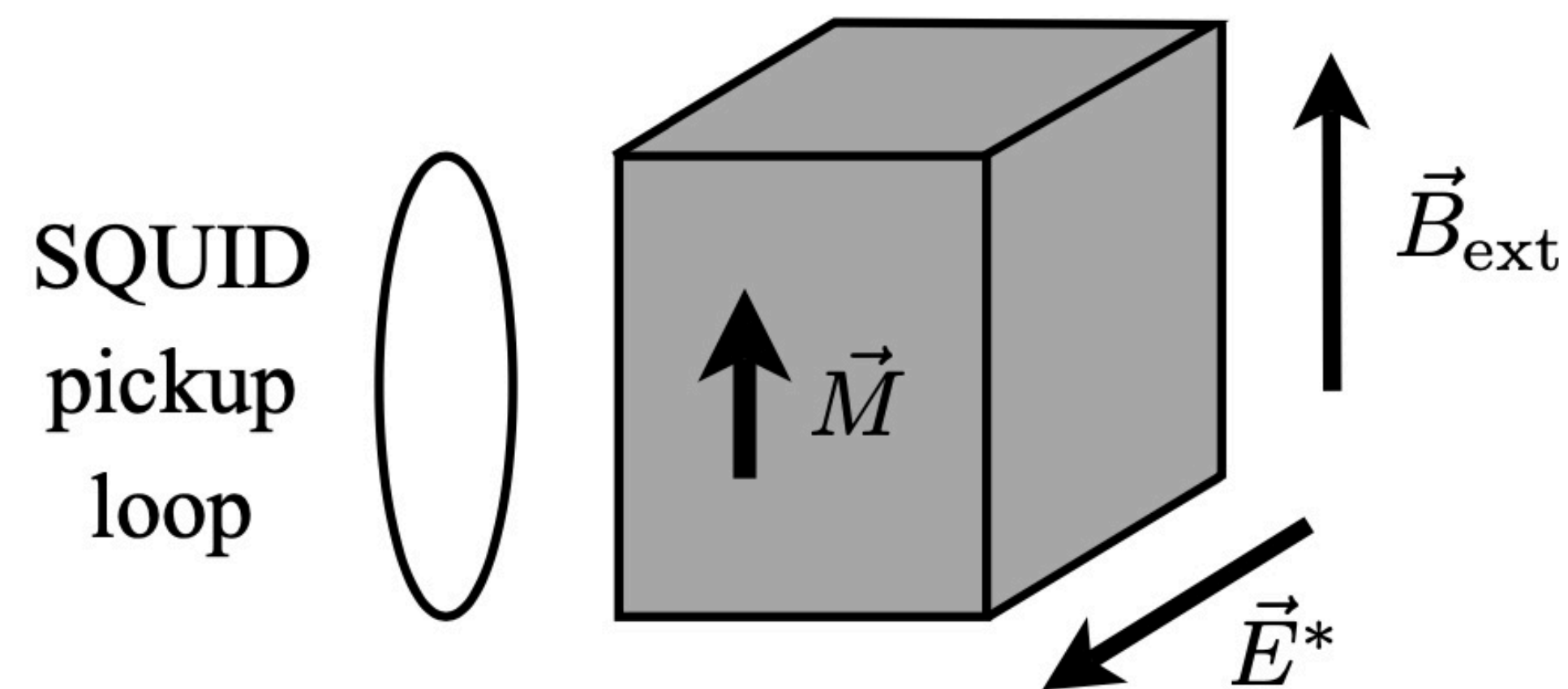
Figure from PDG and C. O'Hare

<https://doi.org/10.5281/zenodo.3932430>.



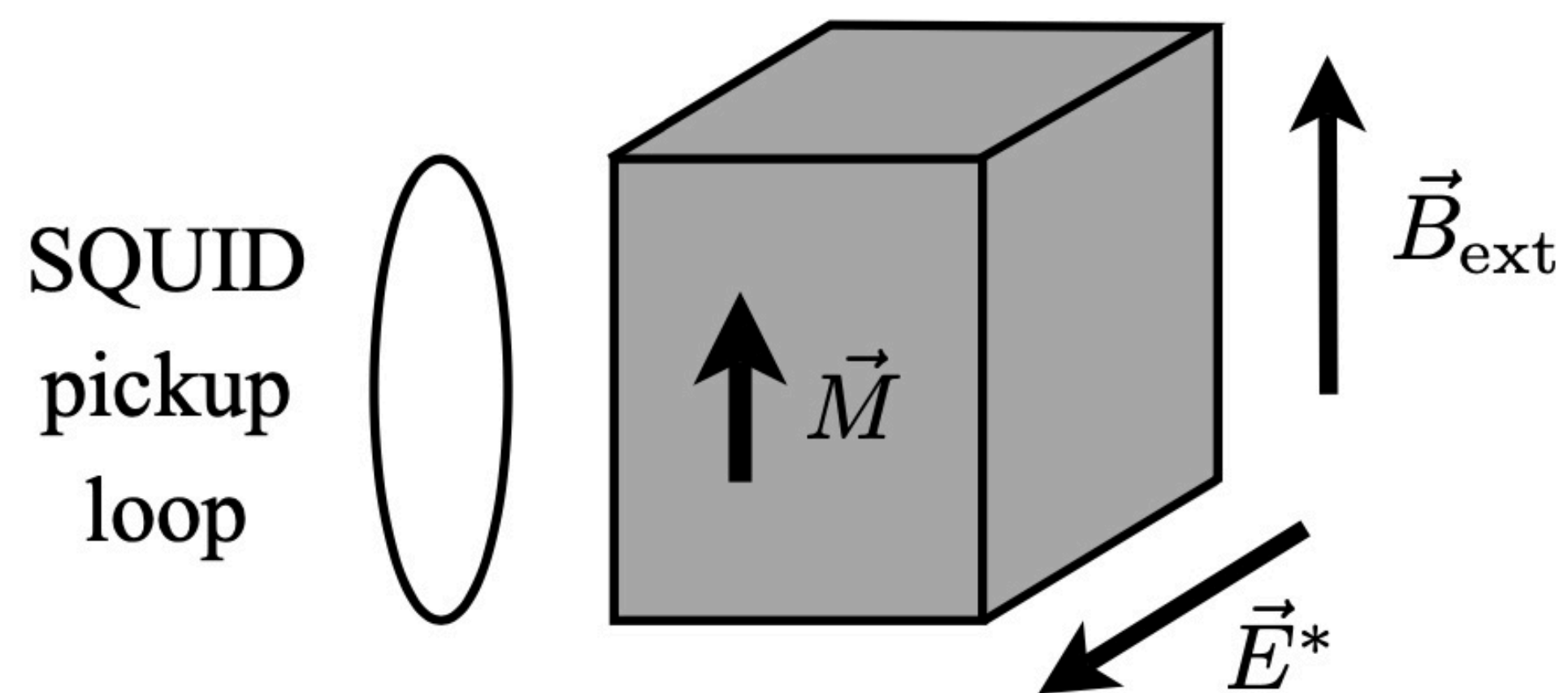
Valid for QCD Axion!

$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$



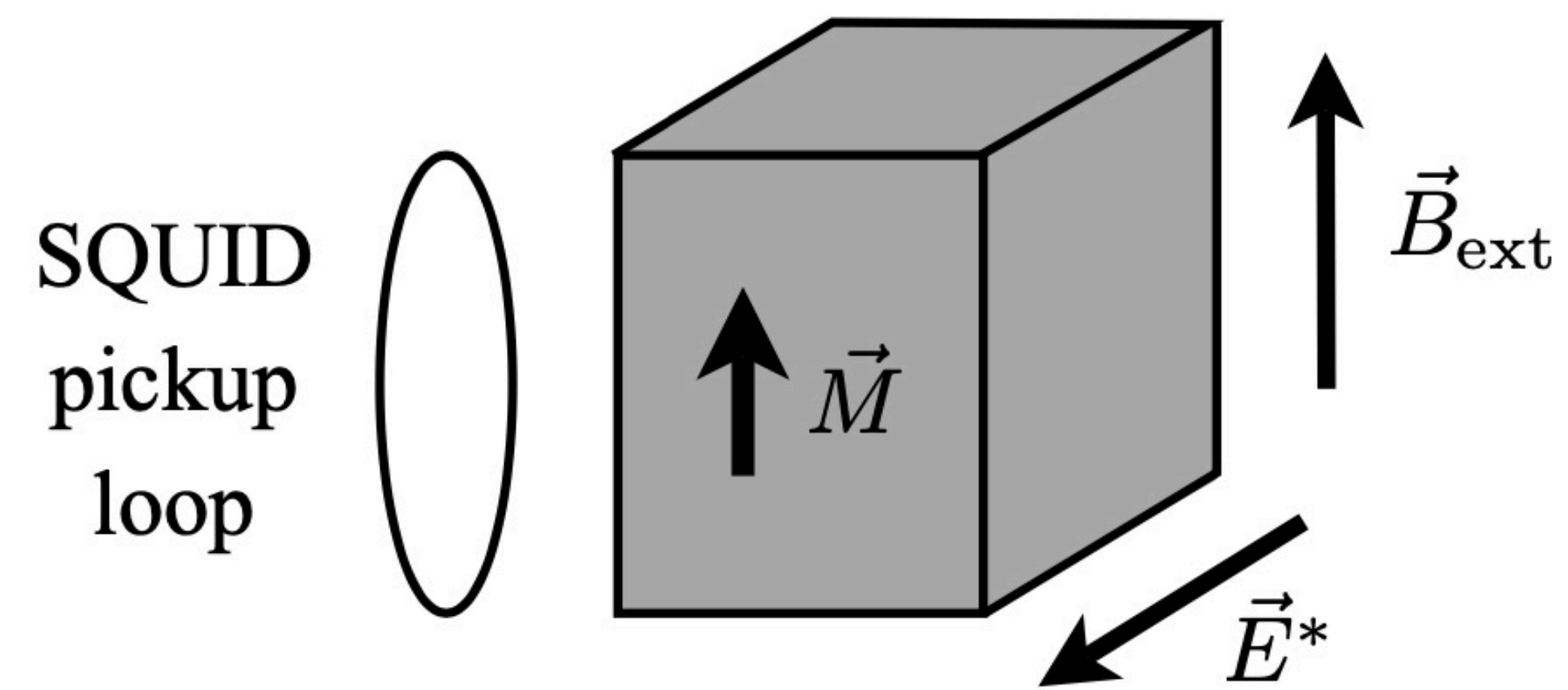
Without the axion you have a magnetisation component precessing around B (Larmor frequency)

$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$



$$M(t) \approx np\mu E^* \epsilon_S d_n \frac{\sin \left[\left(\frac{2\mu B_{\text{ext}} - m_a c^2}{\hbar} \right) t \right]}{\frac{2\mu B_{\text{ext}} - m_a c^2}{\hbar}} \sin (2\mu B_{\text{ext}} t)$$

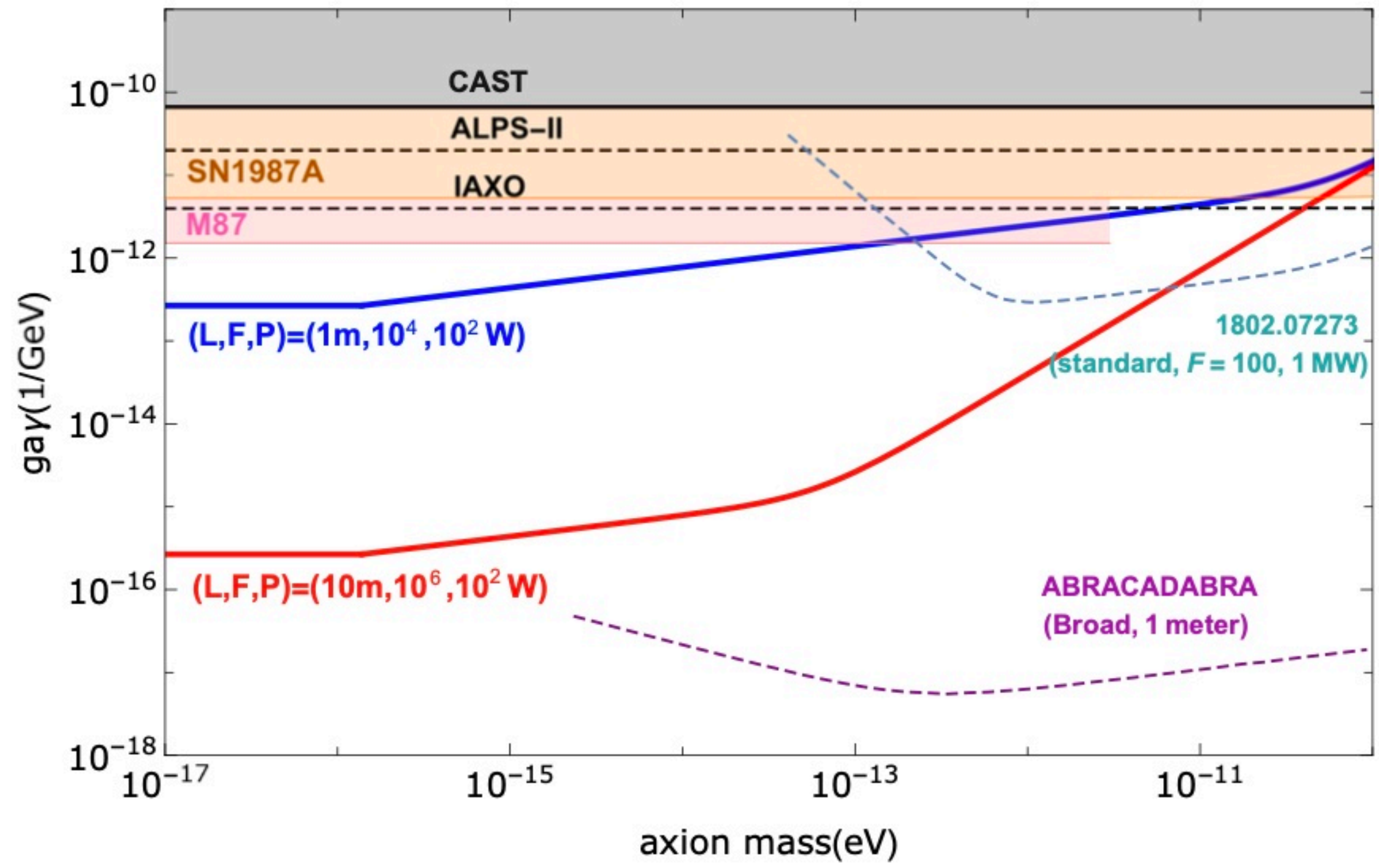
$$H \simeq \frac{a}{m_N f_a} \vec{\sigma} \cdot \vec{E}$$



	n	E^*	p	T_2	Max. B_{ext}
Phase 1	$10^{22} \frac{1}{\text{cm}^3}$	$3 \times 10^8 \frac{\text{V}}{\text{cm}}$	10^{-3}	1 ms	10 T
Phase 2	$10^{22} \frac{1}{\text{cm}^3}$	$3 \times 10^8 \frac{\text{V}}{\text{cm}}$	1	1 s	20 T

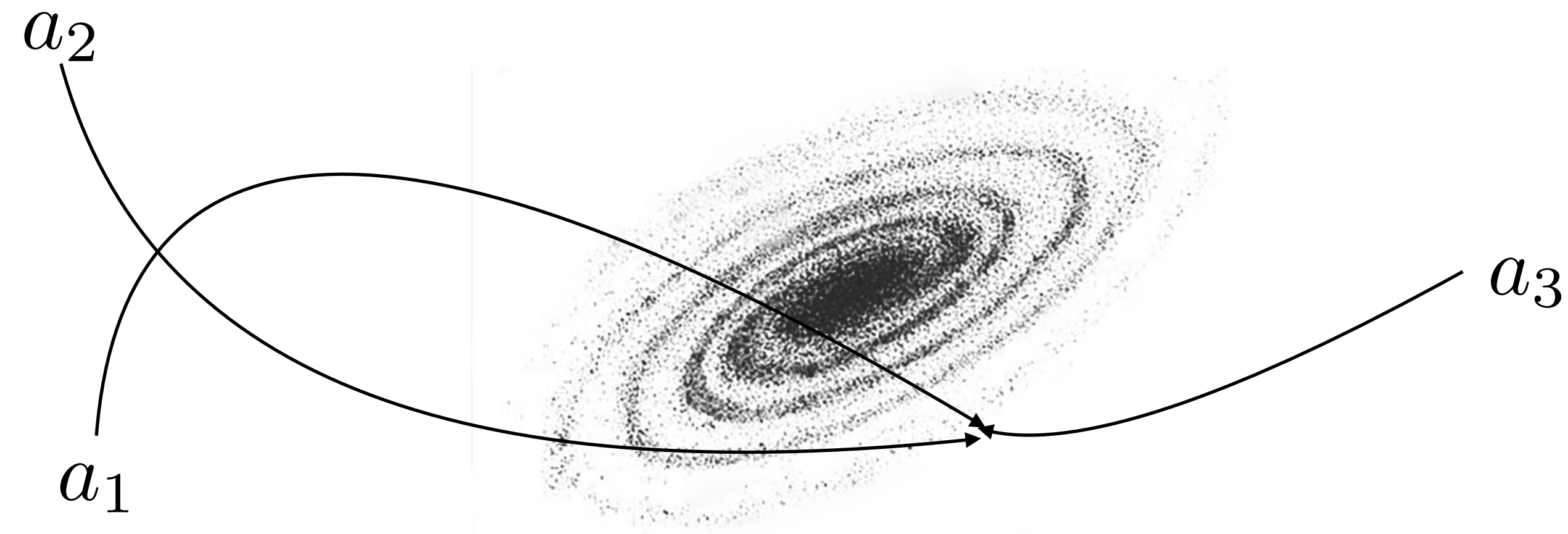
DANCE

Obata, Fujita, Michimura, '18

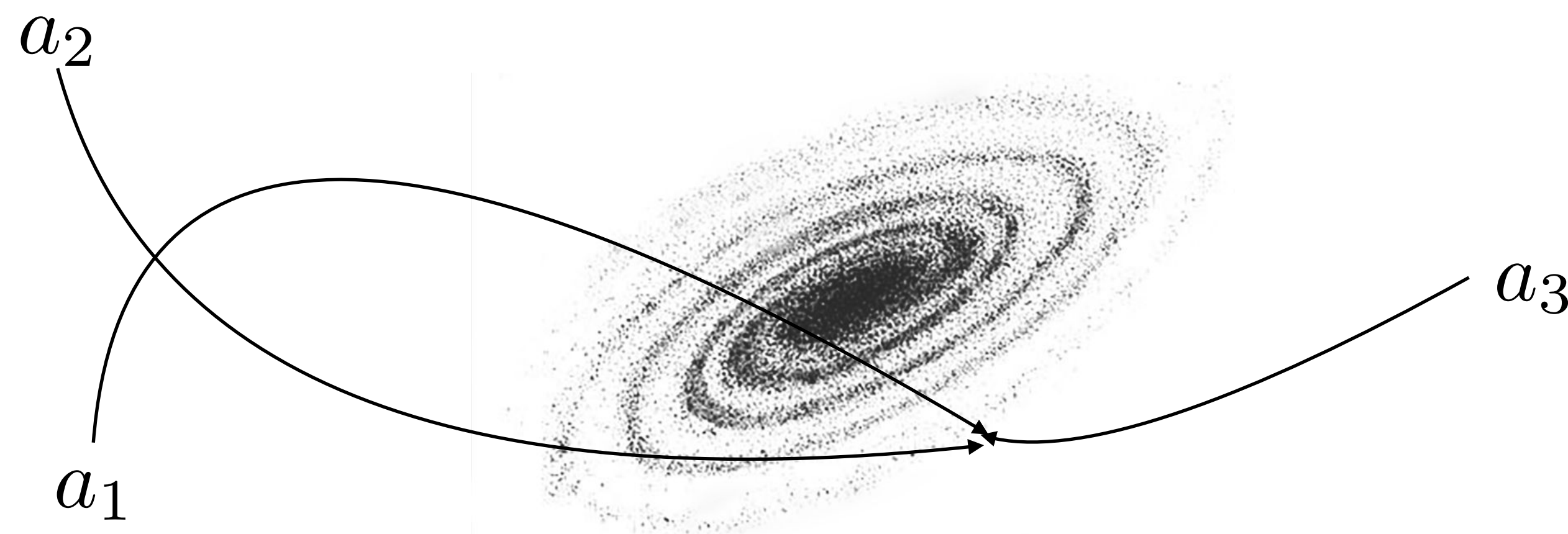


ALP DARK MATTER IN THE LAB

In each experimental bin we are **summing over a multitude of plane waves** with different phases



In each experimental bin we are **summing over a multitude of plane waves** with different phases



$$a(t) = a_0 \left[\cos \left(m_a \left(1 + \frac{v_1^2}{2} \right) t + \phi_1 \right) + \cos \left(m_a \left(1 + \frac{v_2^2}{2} \right) t + \phi_2 \right) + \dots \right]$$

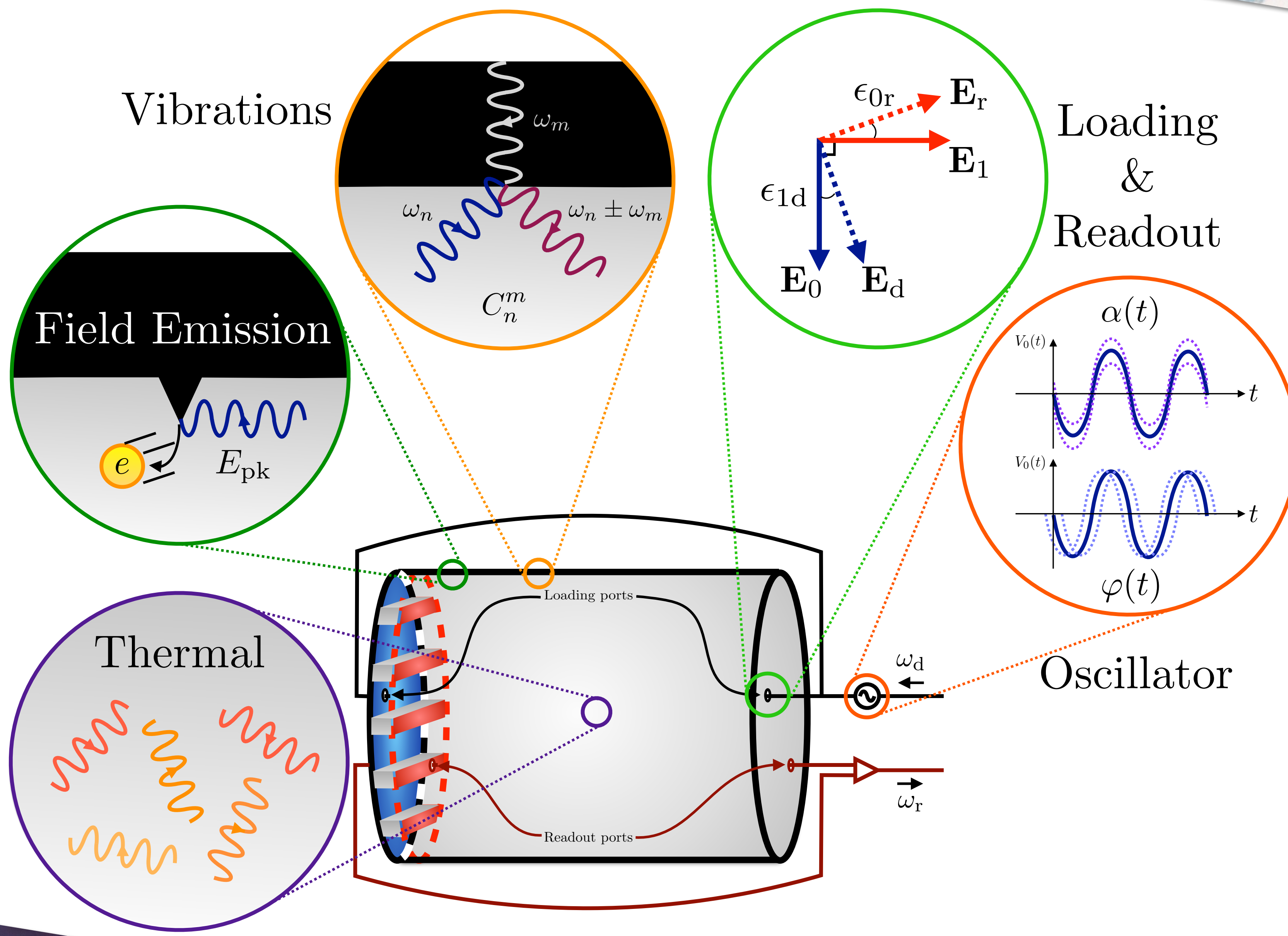
$$\simeq a_0 \cos(m_a t + \phi) [\cos(\delta\omega_a t + \phi') + \dots]$$

$$\delta\omega_a \simeq \frac{1}{m_a \langle v_{\text{DM}}^2 \rangle} \simeq \frac{10^6}{m_a}$$

Effectively: very **slow modulation** of an approximately **monochromatic field**



ULTRALIGHT
AXION-LIKE
DARK MATTER



Vibrations

Field Emission

Thermal

Loading & Readout

Oscillator

Loading ports

Readout ports

ω_d

ω_r

$\alpha(t)$

$\varphi(t)$

ω_m

ω_n

$\omega_n \pm \omega_m$

C_n^m

ϵ_{1d}

ϵ_{0r}

\mathbf{E}_r

\mathbf{E}_1

\mathbf{E}_0

\mathbf{E}_d

e

E_{pk}

$V_0(t)$

$V_0(t)$

t

t

LEAKAGE NOISE

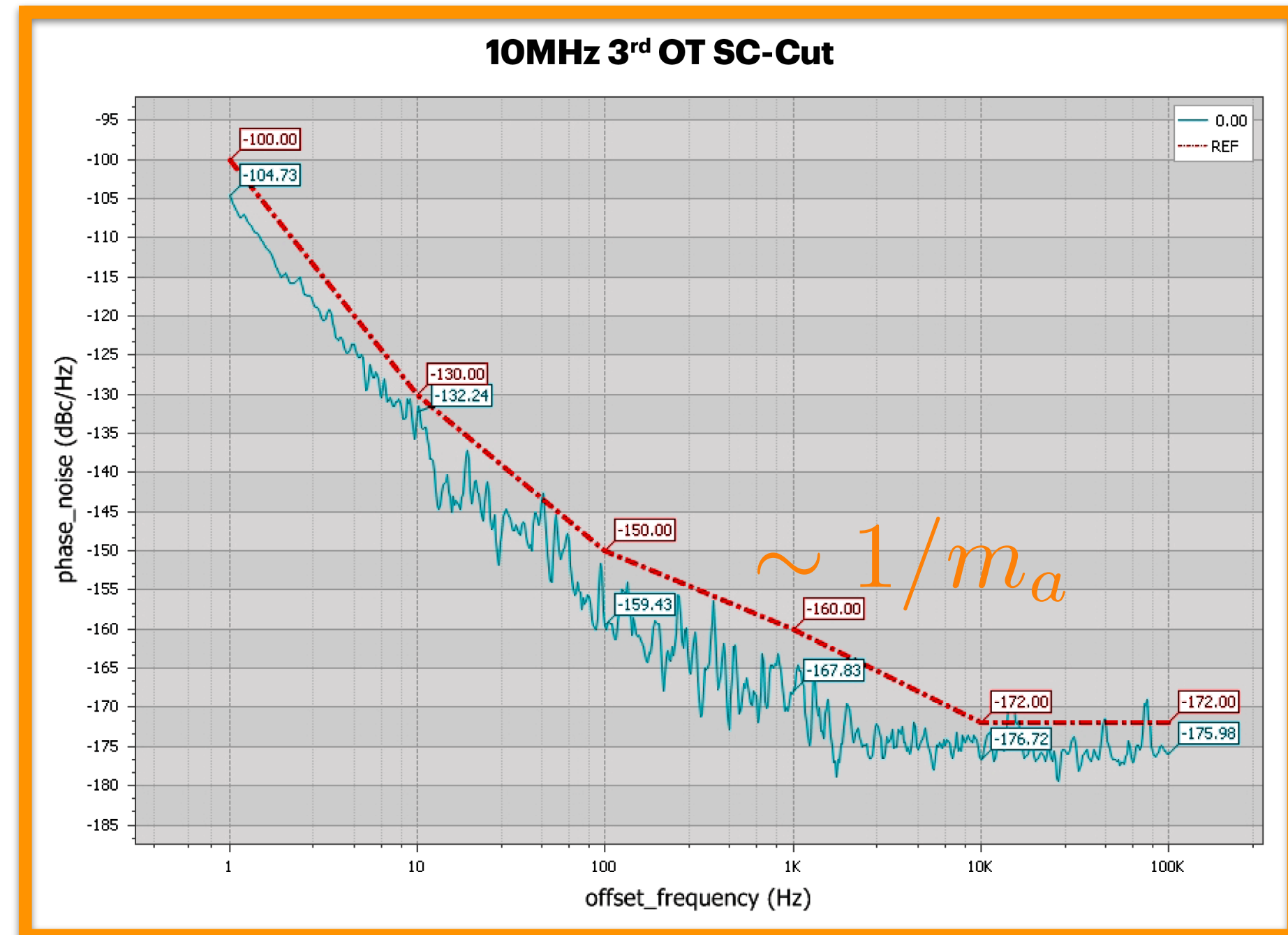
$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_{\phi}(\omega - \omega_0) \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$

Cavity Response

LEAKAGE NOISE

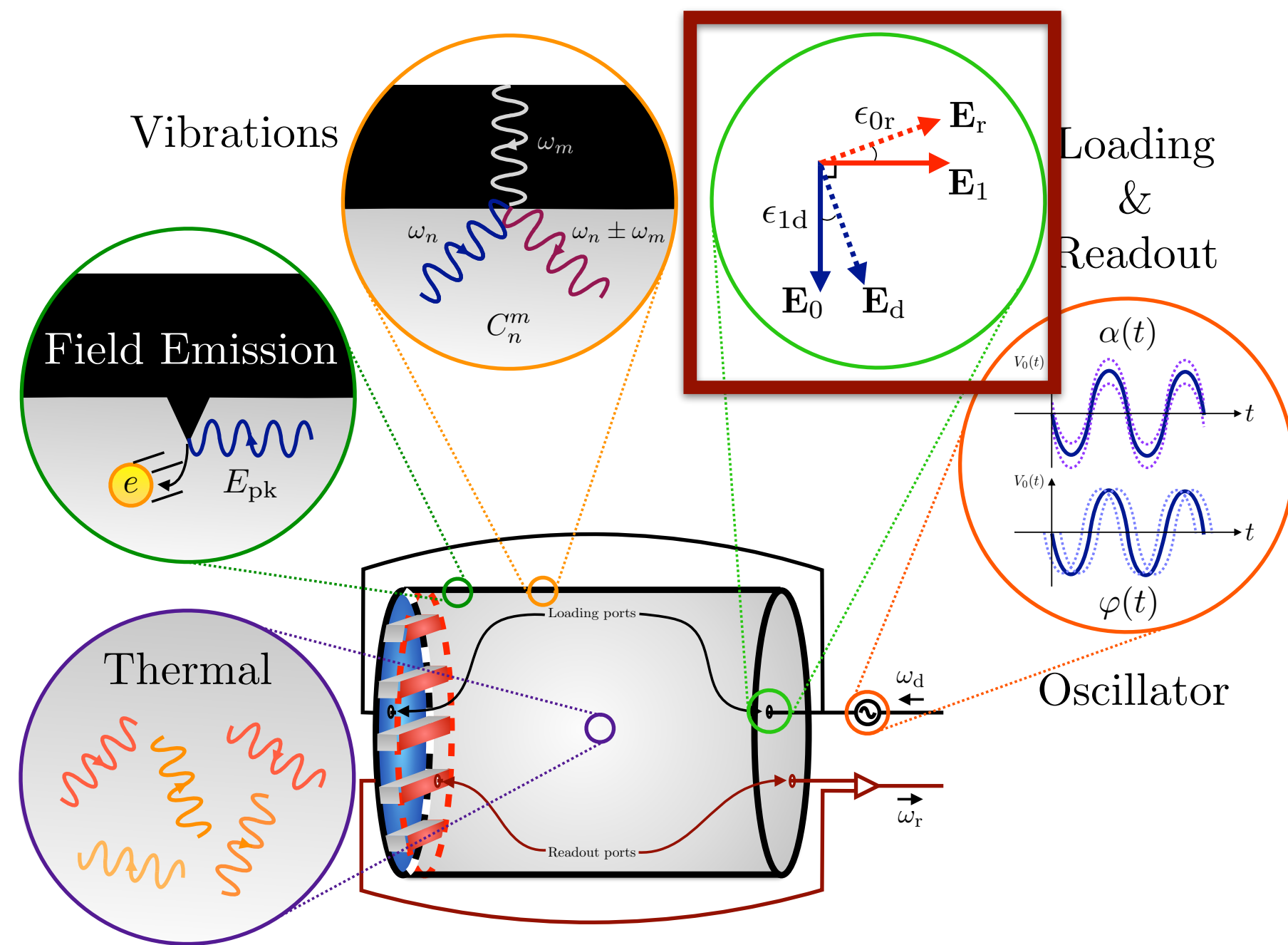
$$S_{\text{phase}}(\omega) \approx \frac{1}{2} \epsilon_{1d}^2 \boxed{S_{\phi}(\omega - \omega_0)} \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$

$\sim 1/m_a$



LEAKAGE NOISE

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_{\phi}(\omega - \omega_0) \frac{(\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2} \frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}$$



**From MAGO
and other similar cavities**