

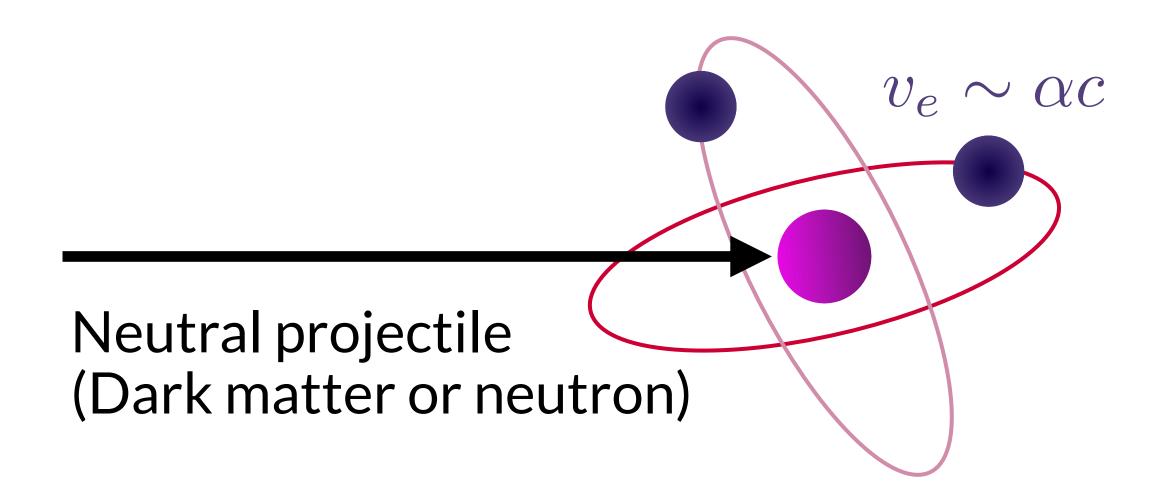
Migdal effect with neutral projectiles

Christopher McCabe

With the MIGDAL Collaboration and Peter Cox, Matthew Dolan and Harry Quiney (Univ. of Melbourne)

Motivation

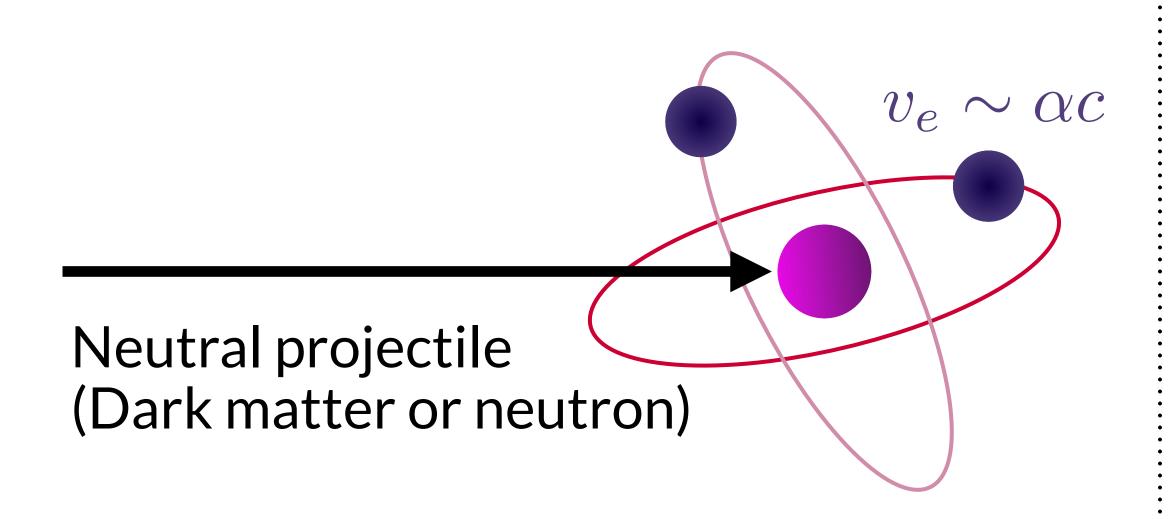
Neutral projectile scattering on atoms



Helium atom

Fine-structure constant: $\alpha = 1/137$

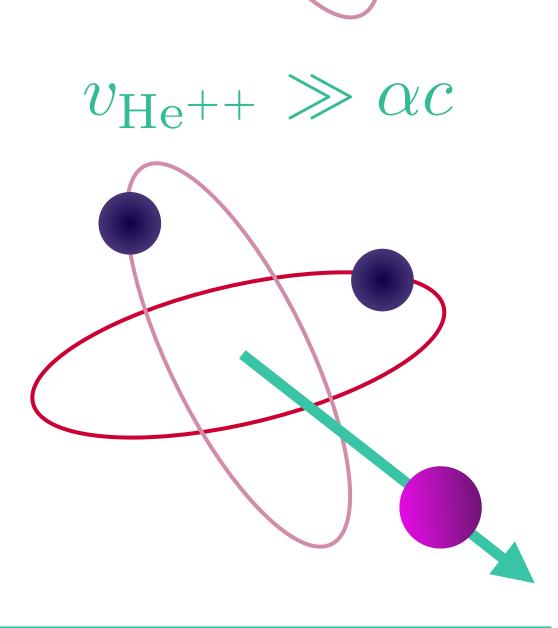
Neutral projectile scattering on atoms



Helium atom

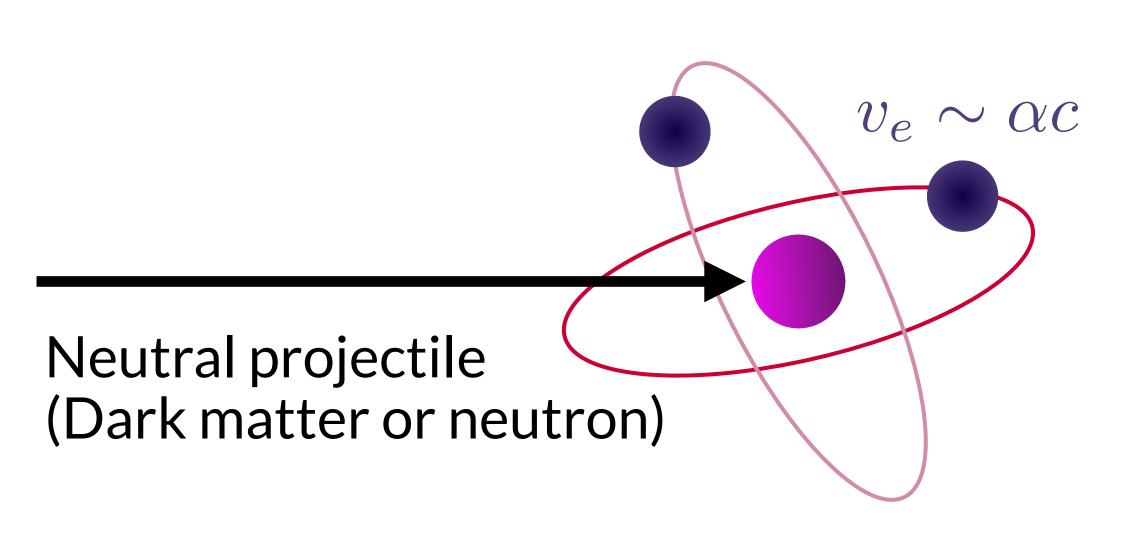
Fine-structure constant: $\alpha = 1/137$

- 1. Low speed recoil:- remain in ground state
 - ain in ground state
- 2. High speed recoil:- double ionisation(electrons 'left behind')



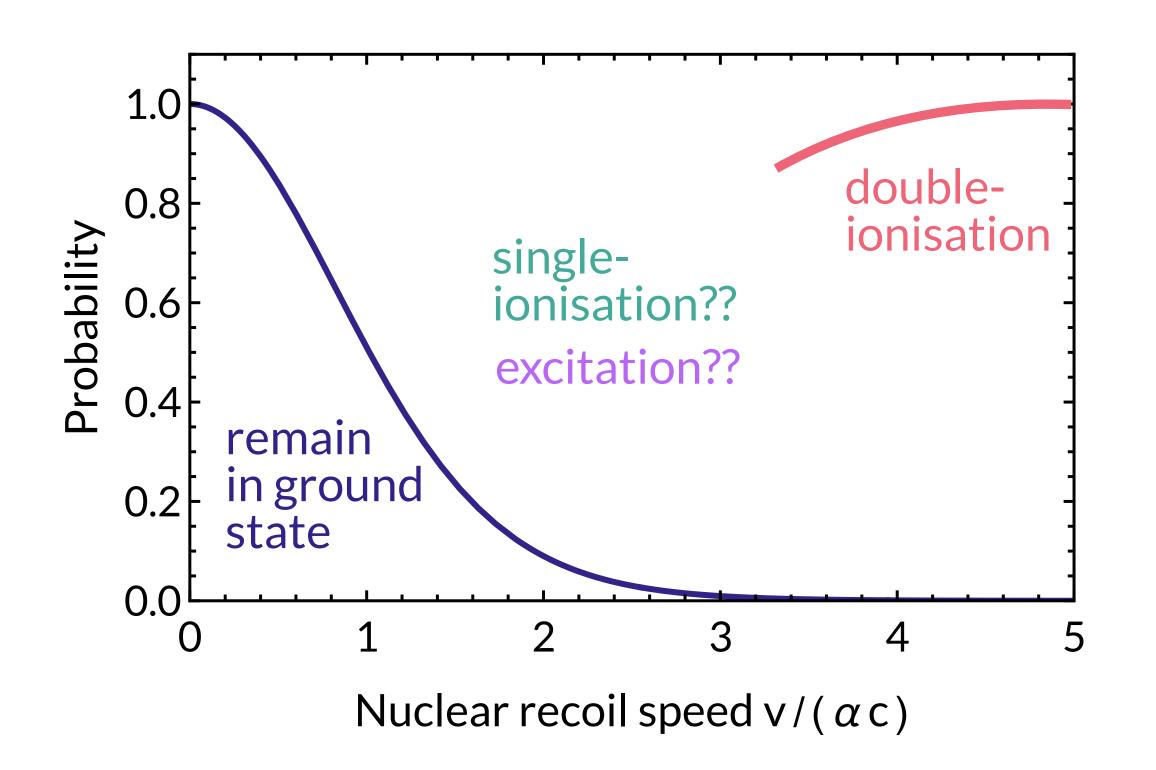
 $v_{\mathrm{He}^{++}} \ll \alpha c$

Neutral projectile scattering on atoms



Helium atom

Fine-structure constant: $\alpha = 1/137$



[*In the rest of this talk c=1]

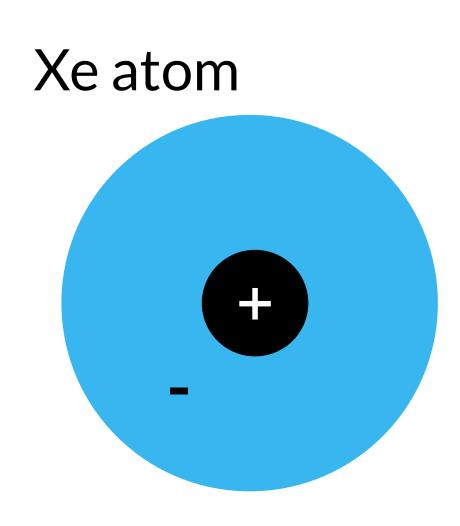
'Migdal effect'

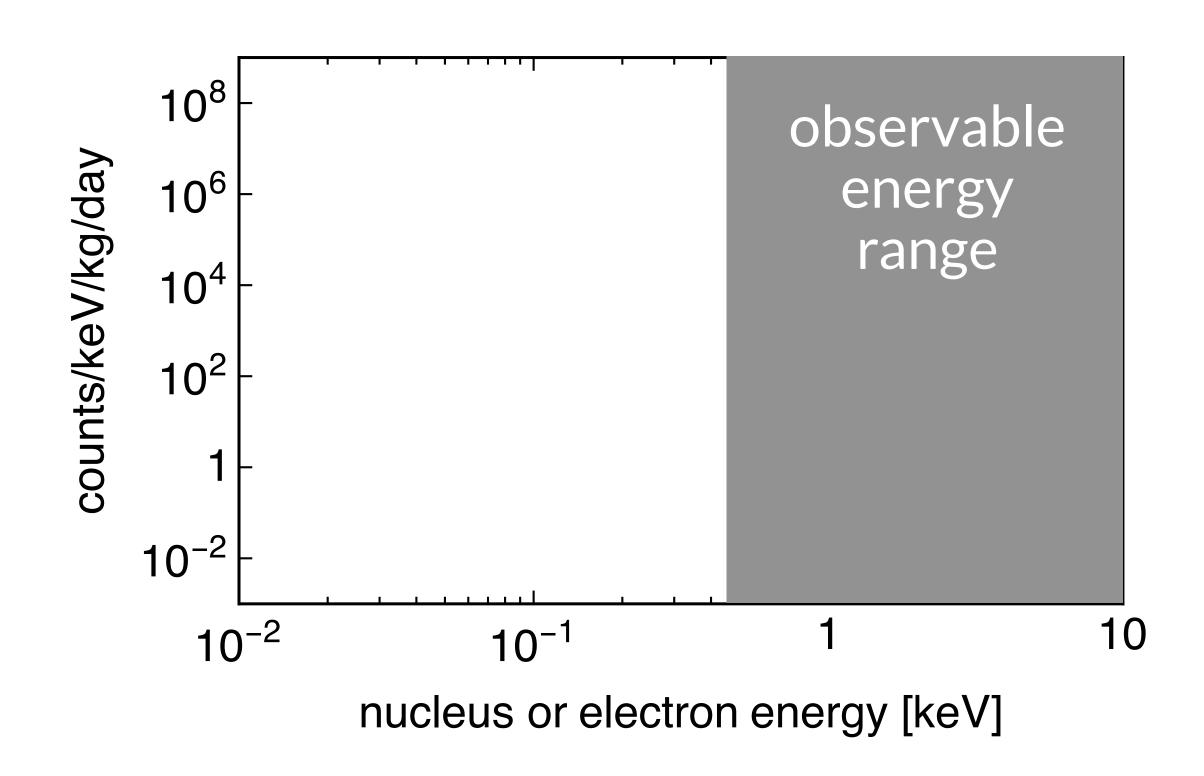
electrons and the nucleus are coupled in atoms so perturbations of the nucleus can induce electronic transitions

Transition probability depends on the speed of the recoiling nucleus

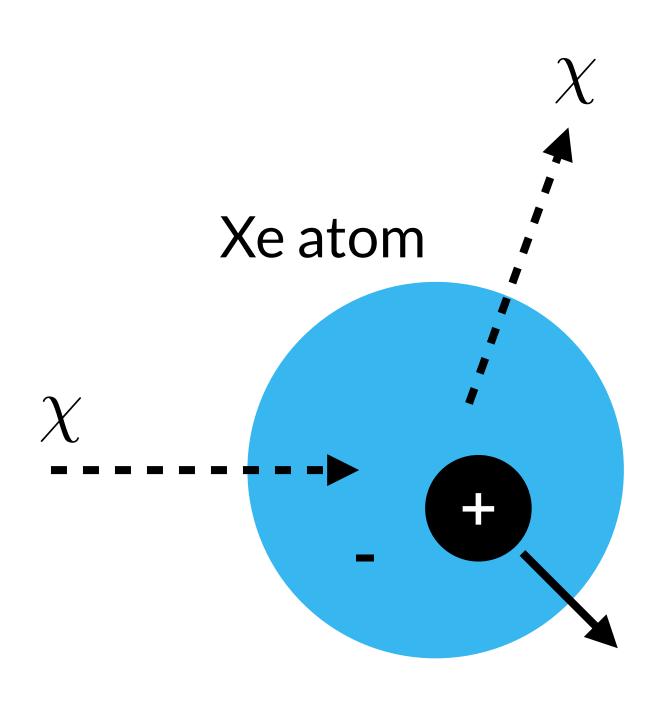
So what?

Consider DM scattering with xenon

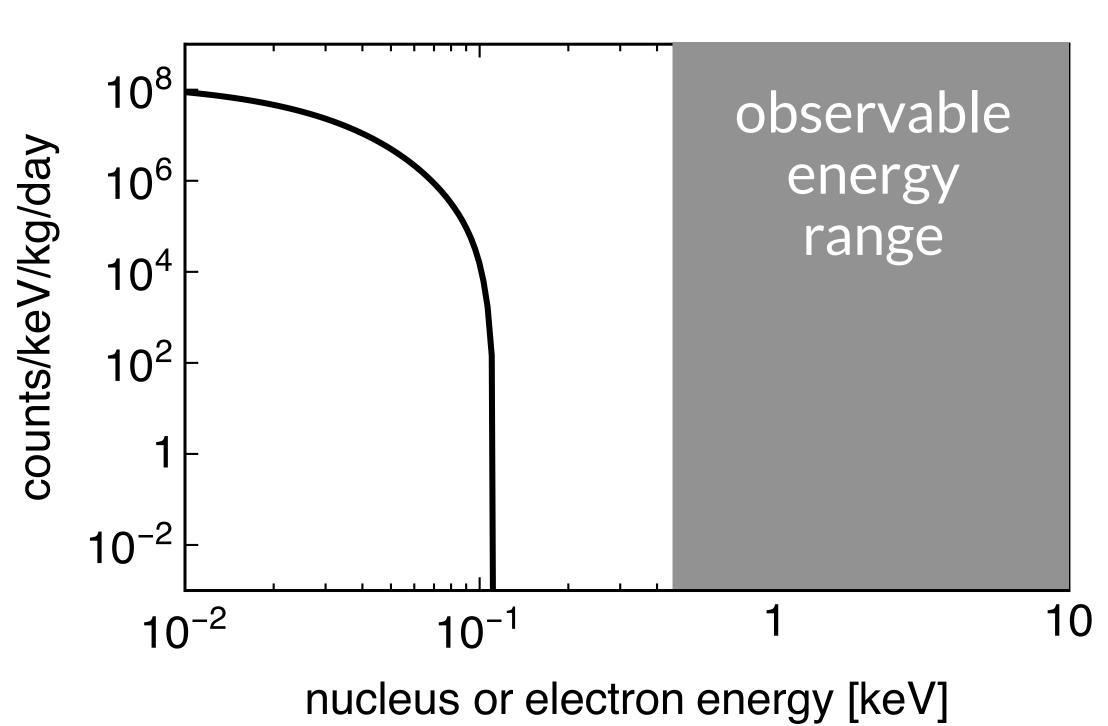




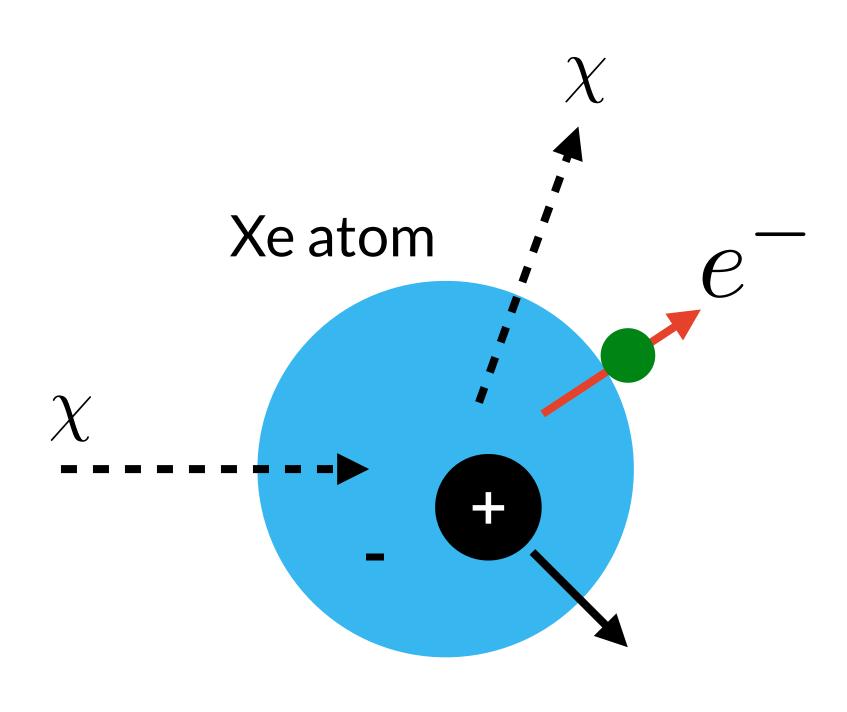
Consider DM scattering with xenon



 $m_{DM} = 1 \text{ GeV}$ 'Normal' nuclear scattering

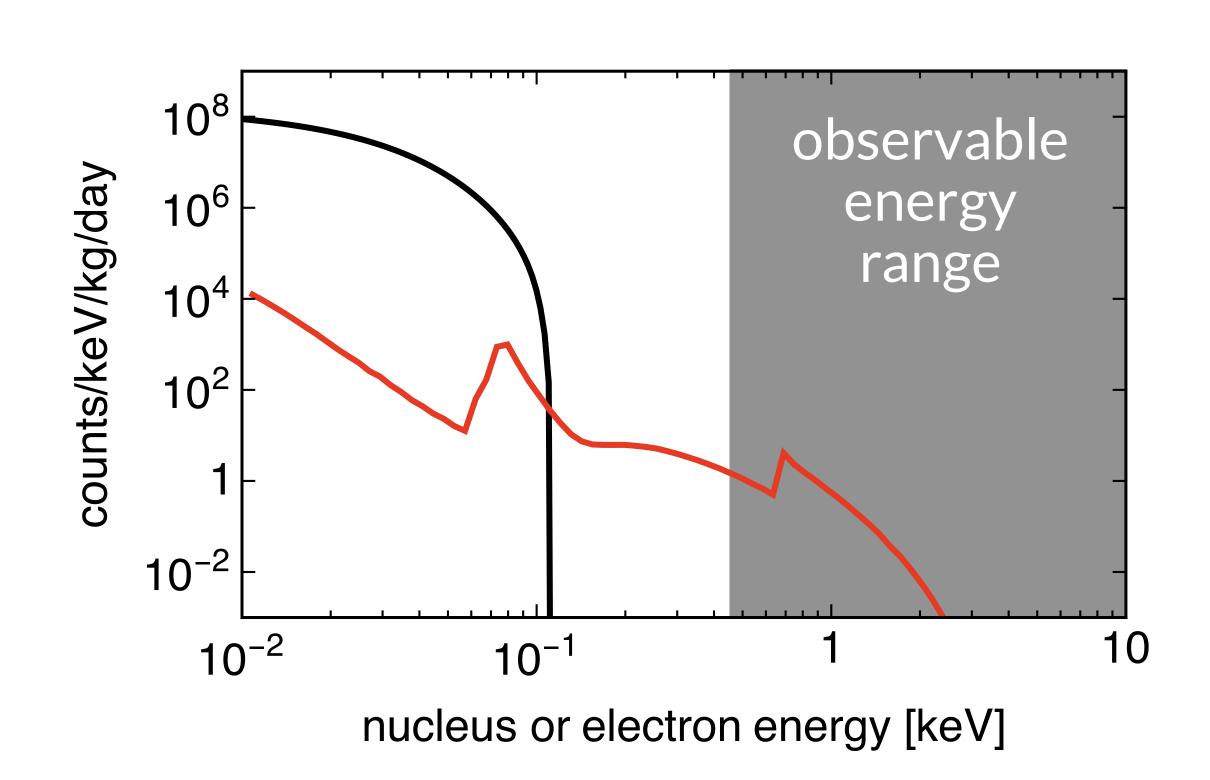


Consider DM scattering with xenon

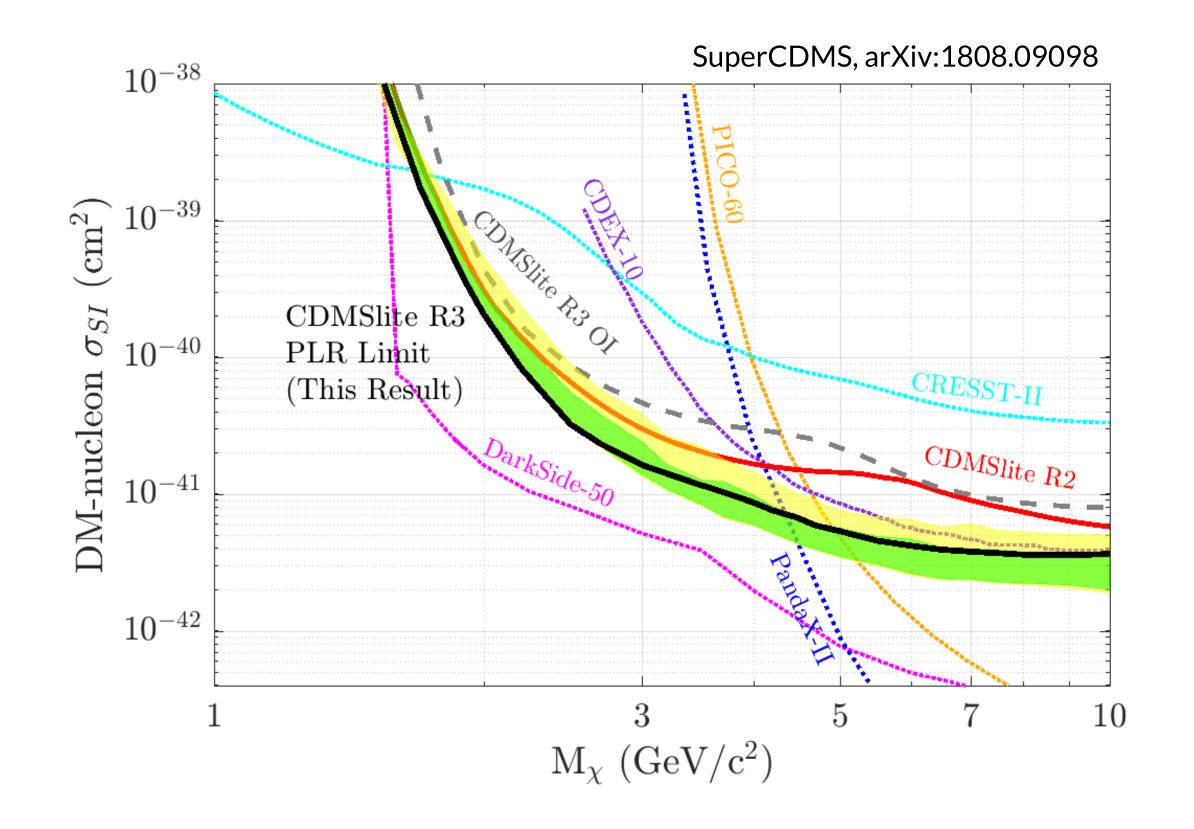


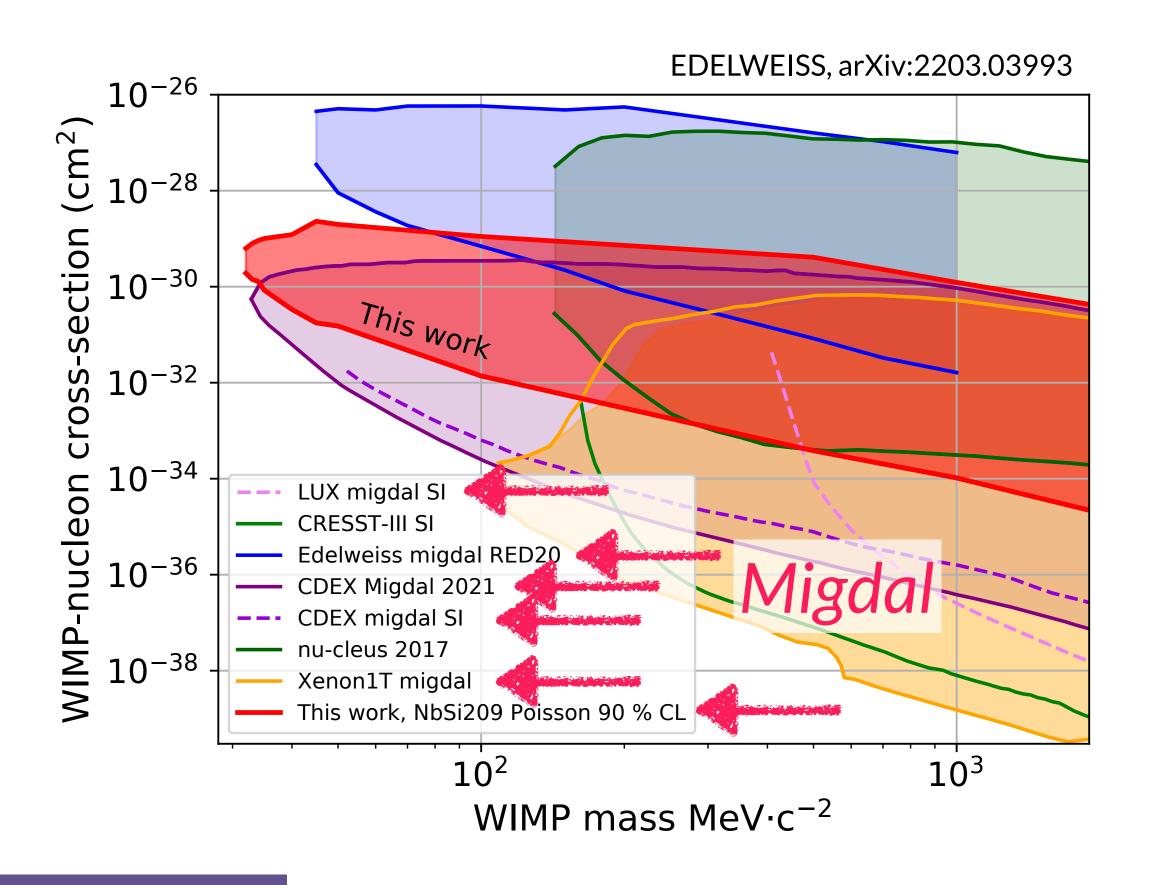
m_{DM} = 1 GeV 'Normal' nuclear scattering

+ Migdal effect (ionisation of 1 electron)



Sub-GeV searches increasingly dominated by Migdal





Pre-2018 No Migdal limits Migdal effect in dark matter direct detection experiments, lbe et al arXiv:1707.07258

Today Dominated by Migdal

Is there evidence for the Migdal effect?

Evidence? Yes, but...

дачей большой

передачей бо

Пои малых

вылетает из энергиях отд

оболочках.

При яд

При ст

A.B. Migdal's papers date back

Predicted effect in:

- 1. α , β decay
- 2. Neutral scattering

Вероятность такой нонизации может быть очень просто рассчитана. Так вероятность такой нонизации может быть очень просто рассчитана. Так как интерессы случай борших экергий отдачи и, следовательно, больших скерестей пастеры. Стеренных периодов. Следовательно, измежение скорости ядра происходит резко неадиабатически, так что У — функция электронов—не может измениться M.S. Rapaport, F. Asaro and I. Pearlman K-shell electron shake-off accompanying alpha decay, PRC 11, 1740-1745 (1975)

M.S. Rapaport, F. Asaro and I. Pearlman L- and M-shell electron shake-off accompanying alpha decay, PRC 11, 1746-1754 (1975) C. Couratin et al., First Measurement of Pure Electron Shakeoff in the β Decay of Trapped 6He+Ions, PRL 108, 243201 (2012)

вередаче внергии Р много меньше размеров электронных осолочек, то жаро можно считать не сместившимся за время удара.

вация, обусловленная магнитным и специфическим ядерным взаимодействием нейтрона с электроном, крайне мала — соответствующее сечение в первом

 $W = \left| \left[\overline{\Psi}_1 e^{i \mathbf{r} \cdot \mathbf{r}_i} \Psi_0 d \mathbf{r}_1 \dots d \mathbf{r}_f \right]^2,$

случае порядка 10⁻²⁸ см², во втором — порядка 10⁻³⁶ см²).

Для получения вероятности возбуждения или нонизации нужно исходную Ф-функцию атома разложить по собственным функциям движущегося идра. Можно поступить несколько иначе, в именно перейти к системе координат, в которой ядро покоится; тогда собственными функциями вадачи будут обыч-

Effect has not been observed by the neutral projectiles

Действительно, множитель e представляет собой Ф-функцию центра инерции оболочки, который в старой системе координат покоился, а в новой движется со скоростью у, равной по величине и противоположной по направлению скорости ядра.

Пусть конечное состояние атома в рассматриваемой системе координат дается функцией $\Phi_1(r_1, r_2 \dots r_\ell)$. Так как ядро за время удара не сместилось, то координаты влектронов в Ψ_1 отсчитаны от той же точки, что и в Ψ_0 . Вероятиссть перехода в конечное состояние дается выражением:

$$W = \left| \int \overline{\Psi}_1 e^{iq \, \mathbf{r} \, \mathbf{r}_i} \, \Psi_0 \, d\mathbf{r}_1 \dots d\mathbf{r}_f \right|^2, \tag{1}$$

IONIZATION OF ATOMS ACCOMPANYING α- and β-DECAY By A. MIGDAL

without difficulty calculated if one makes use of the fact that the velocity of a β -electron is usually great as compared with velocities of atomic electrons. It is easily seen that in this case one can neglect the direct interaction of the β -decay electron with the atomic ones. The ionization is due to the fact that the nuclear charge is changed within a time interval which is short comparing to atomic periods. $\frac{V \sim \frac{N_c}{h^2} \sim \frac{1}{h^2} \left(\frac{a}{a} \cdot \frac{a}{v_c}\right) = \left(\frac{e}{k_c}\right)$ (the quantity $\gamma = E/mc^2$ disappears because the Lorentz contraction of the field is compensated by an increase of the latter. On the other hand, the probability of ionization by a suddens change of nuclear charge, as will be shown, is of the order of $1/Z_{eff}$. Hence the condition for the direct interaction to be small

The following estimation shows that the

JOURNAL of PHYSICS

IONIZATION OF ATOMS ACCOMPANYING α- and β-DECAY

By A. MIGDAL

(Received November 15, 1940)

The probability of ionization of the inner electron shells accompanying α- and β-decay is calculated. Also an estimation of the order of magnitude of ionization of the outer shells

I. Ionization accomanying β-decay

1. The probability of ionization of an ntom as a result of the \$-decay can be without difficulty calculated if one makes use of the fact that the velocity of a (the quantity $\gamma = E/mc^2$ disappears because β-electron is usually great as compared with velocities of atomic electrons.

It is easily seen that in this case one can neglect the direct interaction of theβ-decay electron with the atomic ones. The ionization is due to the fact that the nuclear charge is changed within a time interval which is short comparing to atomic periods.

The following estimation shows that the direct interaction can be actually neglected. The probability of an electron tran-sition due to the direct interaction is according to perturbation theory:

$$W = \frac{\left|\int\limits_{0}^{\infty} V_{\alpha t} e^{i\omega_{01}t} dt\right|^{2}}{\hbar^{2}}.$$
 (1)

5 Journal of Physics, Vol. IV. No. 5

Hence the transition probability is of the

1941

$$W \sim \frac{V^2 \tau^2}{\hbar^2} \sim \frac{1}{\hbar^2} \left(\frac{\gamma e^2}{a} \cdot \frac{a}{\gamma c} \right)^2 = \left(\frac{e^2}{\hbar c} \right)^2$$

the Lorentz contraction of the field is compensated by an increase of the latter.

On the other hand, the probability of ioniziation by a «sudden» change of nuclear charge, as will be shown, is of the order of $1/Z_{\rm eff}^2$. Hence the condition for the direct interaction to be small

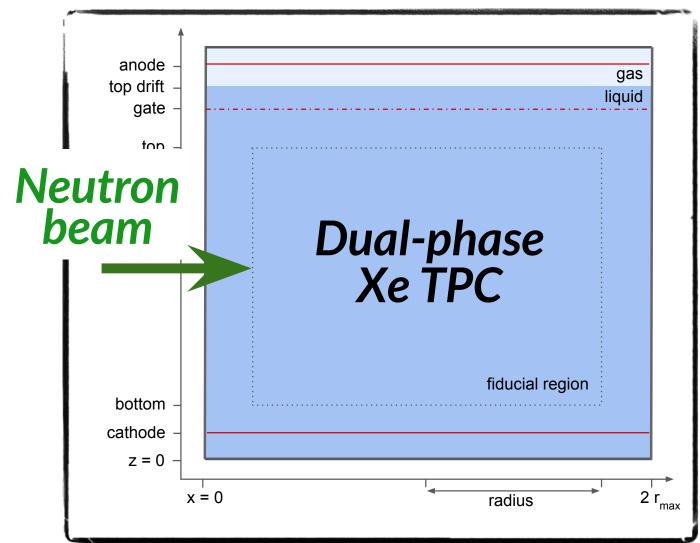
$$\left(\frac{Z_{\text{eff}}e^z}{\hbar c}\right)^2 \ll 1.$$
 (2)

The condition (2) has a simple meaning in the case of a K-electron, because $(Ze^2/\hbar c)^2 = (V_h/c)^2$. Therefore, the direct interaction is to be considered as a relativistic correction. The condition (2) is approximately valid even for K-electrons

2. One can calculate the probability of ionization by means of a sudden change Von is here the matrix element of the per- of the nuclear charge in the following turbation energy; $\omega_{01} = (E_1 - E_0)/\hbar$ —the fremanner. The above estimation shows that quency corresponding to the electron transi- the W-function of atomic electrons does tion; it is of the order of atomic frequencies. not change when the decay electron is The time interval \u03c4 within which the de- emitted. Therefore, the transition probabicay electron traverses electron shells is lity is equal to the square of the coeffimuch smaller than the atomic periods. cient of expansion of the W-function cor-

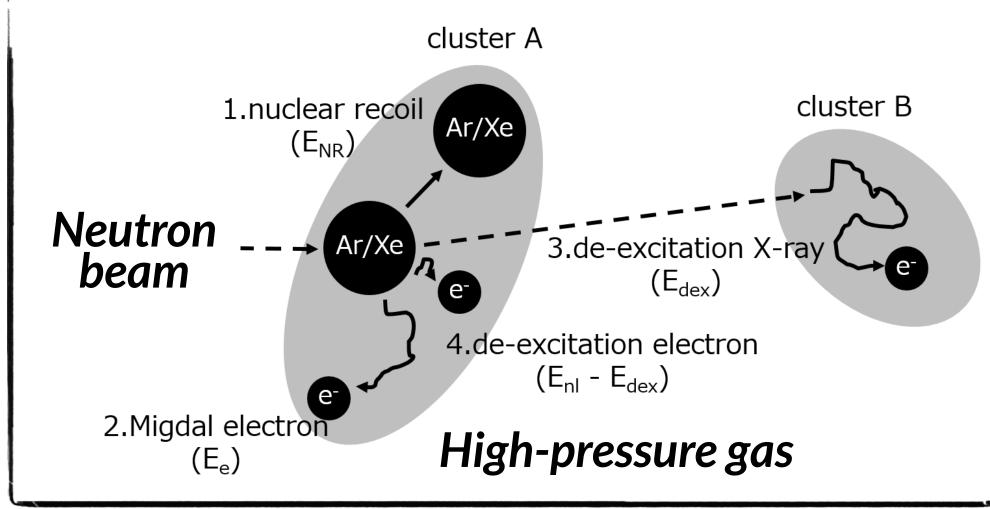
Finding evidence: Proposals with neutrons

Bell et al, arXiv:2112.08514 Xu et al, arXiv:2307.12952



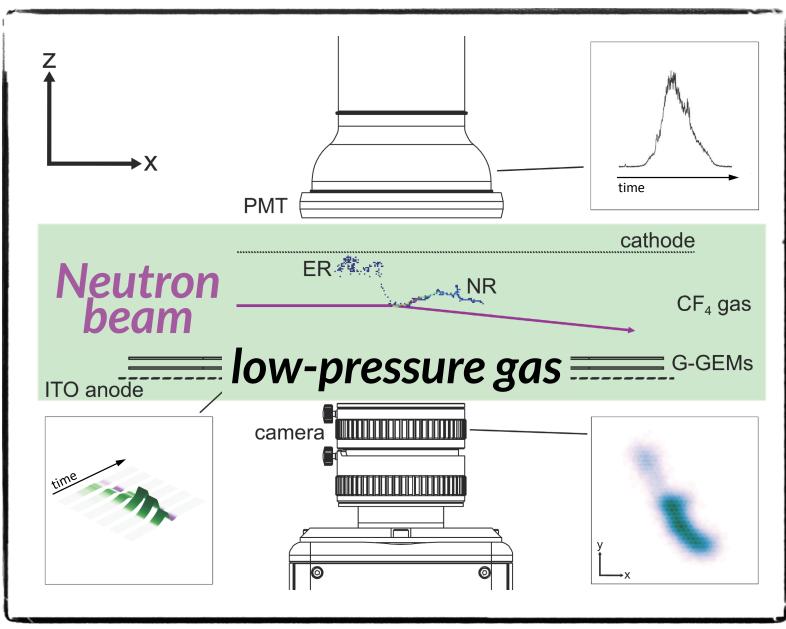
 $E_{\rm neutron} \sim 15 - 15000 \; {\rm keV}$

Nakamura et al, arXiv:2009.05939



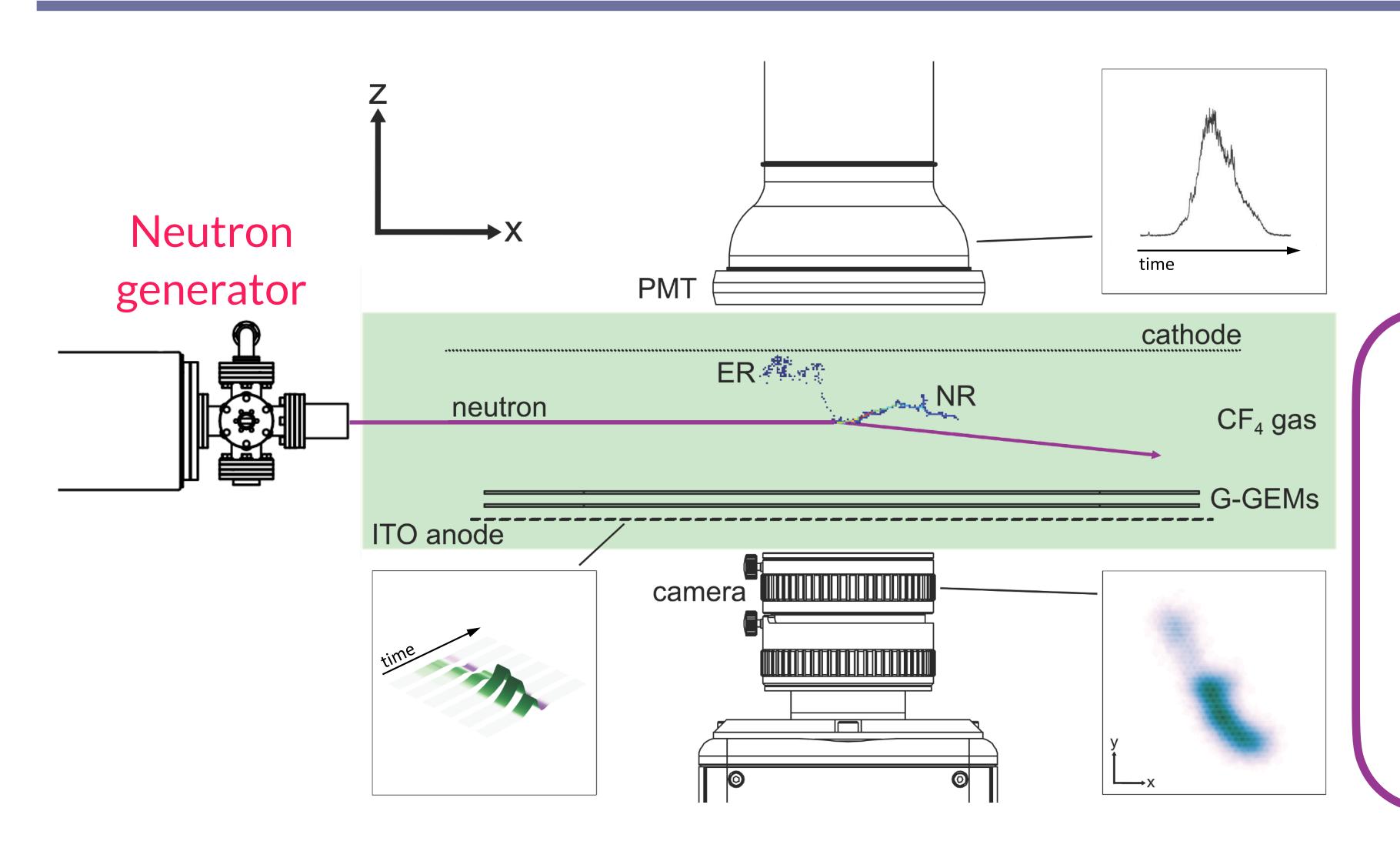
 $E_{\rm neutron} \sim 500 \ {\rm keV}$

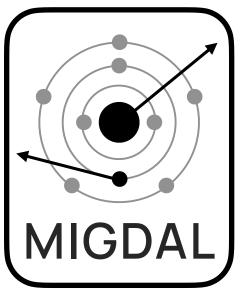
Araújo et al (MIGDAL), arXiv:2207.08284



 $E_{\rm neutron} \sim 2500 - 15000 \text{ keV}$

In the UK: MIGDAL experiment





Araújo, ..., CM, et al (MIGDAL) arXiv:2207.08284

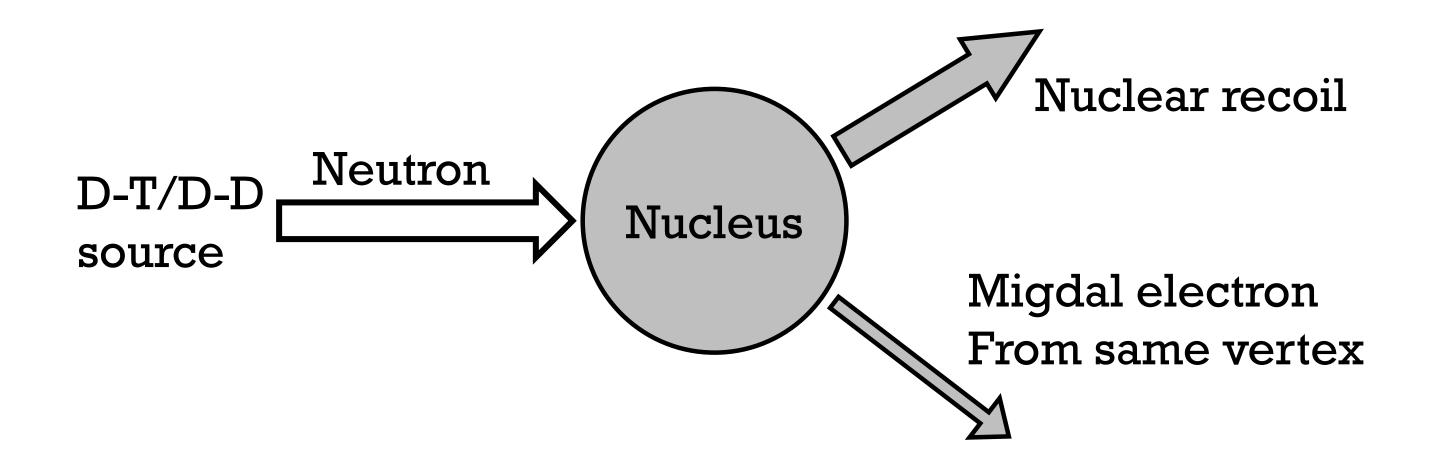
Neutron collisions give recoils with energy:

$$E_r \simeq 100 - 3000 \text{ keV}$$

[higher than dark matter regime]

MIGDAL experiment: aims

Create a dedicated experiment for the *unambiguous* observation of the Migdal effect in nuclear scattering:



We are the only experiment aiming to observe the nuclear and electron recoils emerging from a common vertex

- Phase 1: Observe the effect in CF4 in high energy recoils
- Phase 2: Observe the Migdal effect in CF4 + noble gases

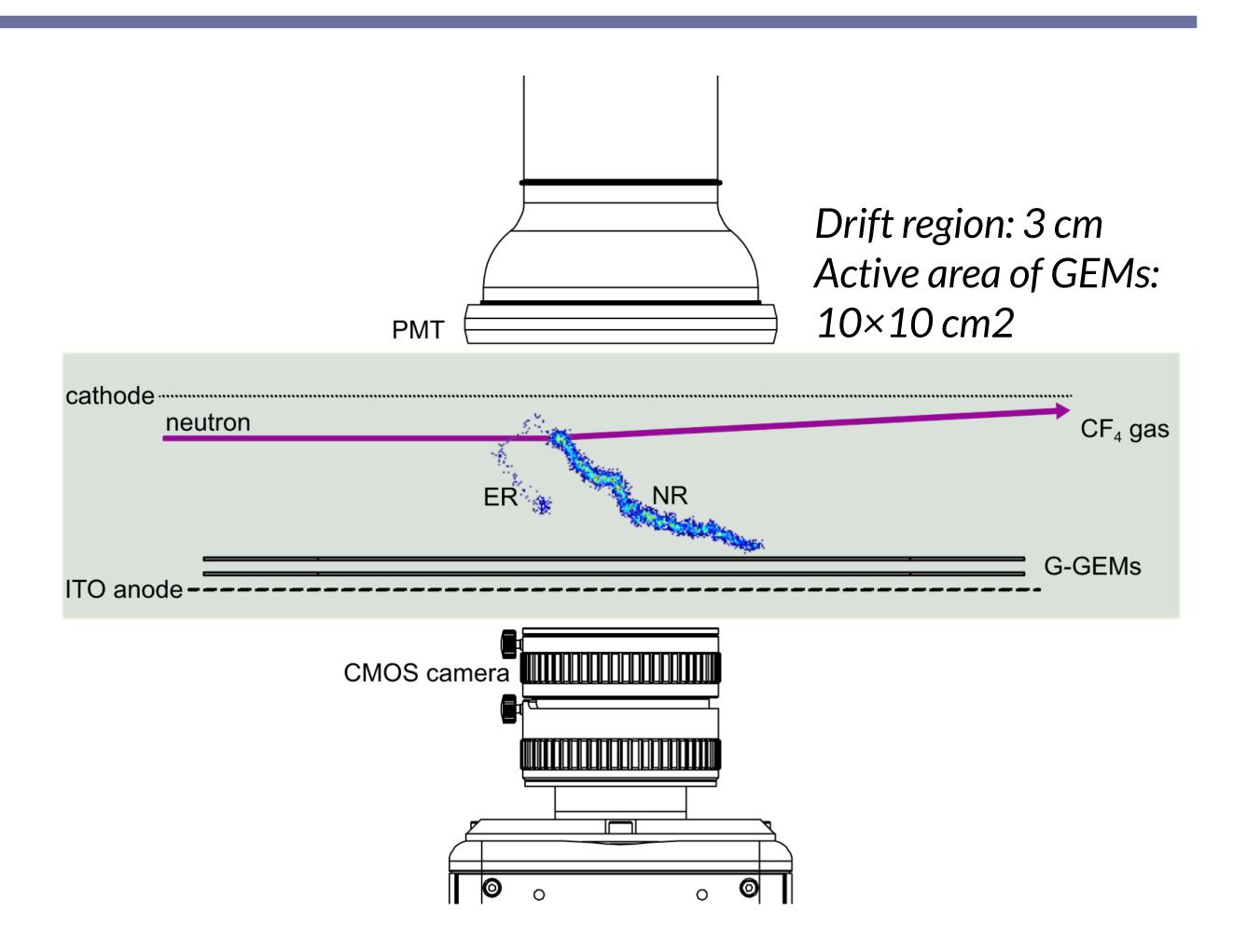
Schematic: Optical Time Projection Chamber

Camera: images GEM scintillation through viewport behind ITO anode. Readout of (x,y) plane

ITO anode: collects charge. Readout of (x,z) plane

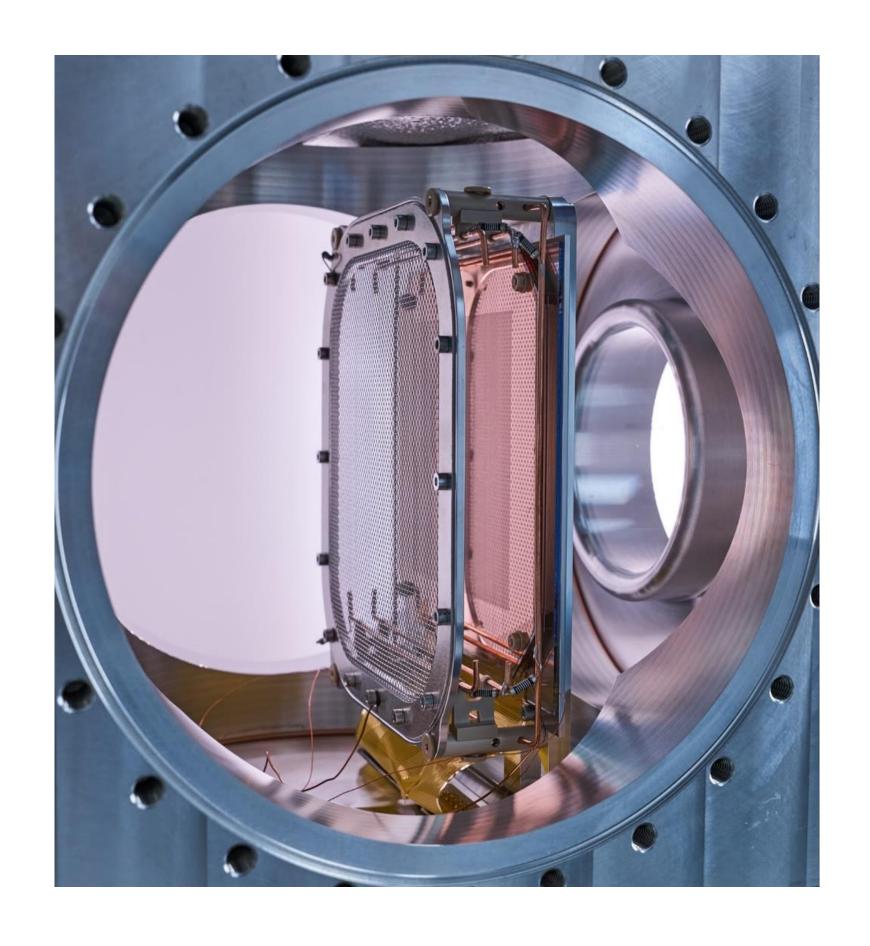
PMT: Detects primary and secondary (GEM) scintillation Readout of depth (z) coordinate

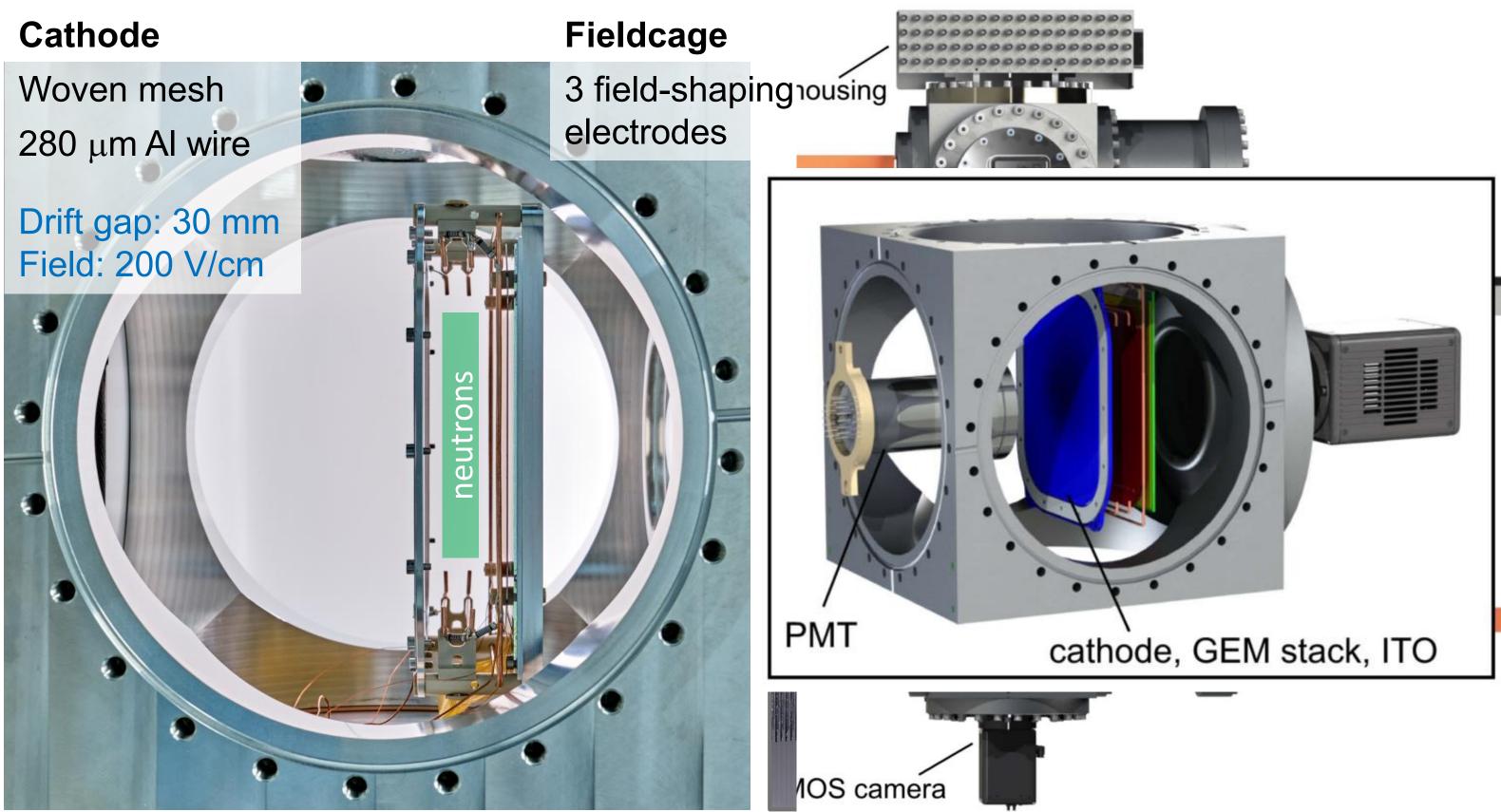
Setup allows for 3D track reconstruction



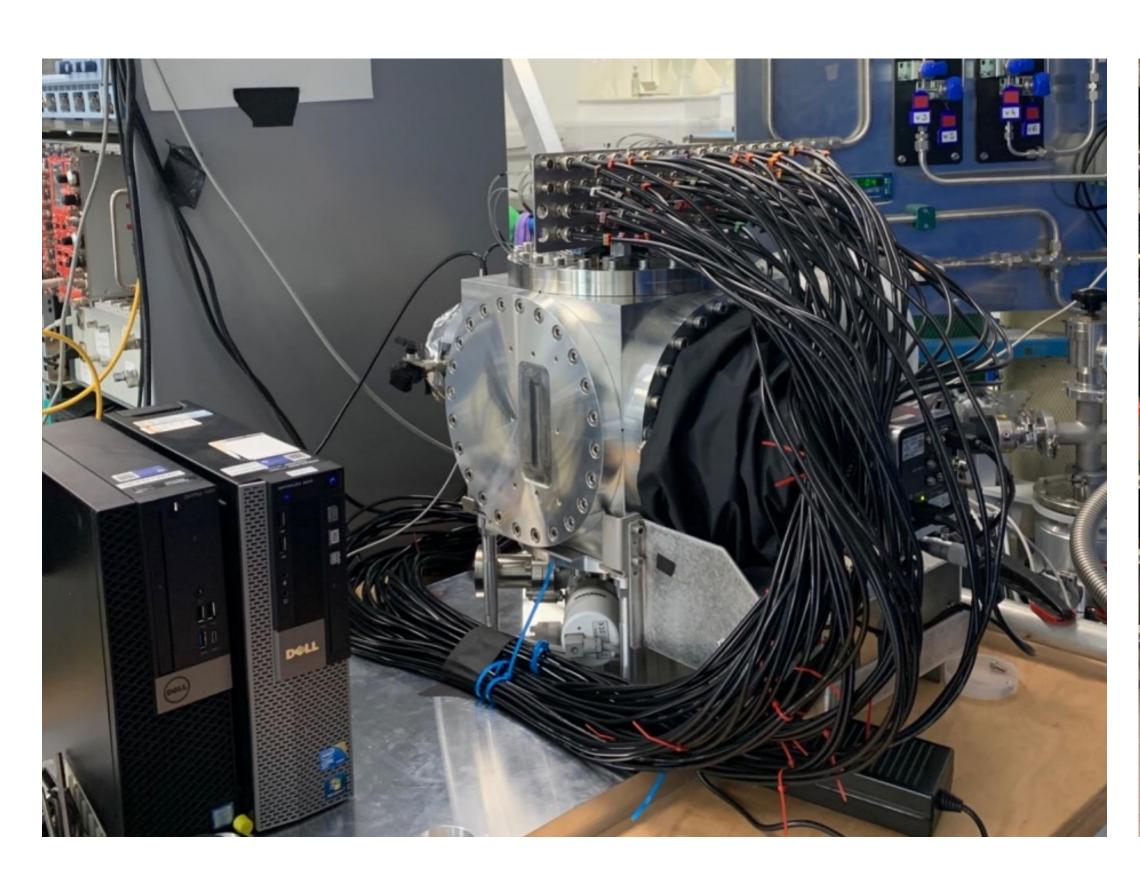
Simulated Migdal event with a 10 keV electron & 250 keV fluorine recoil. Scaled-up by a factor of 3.

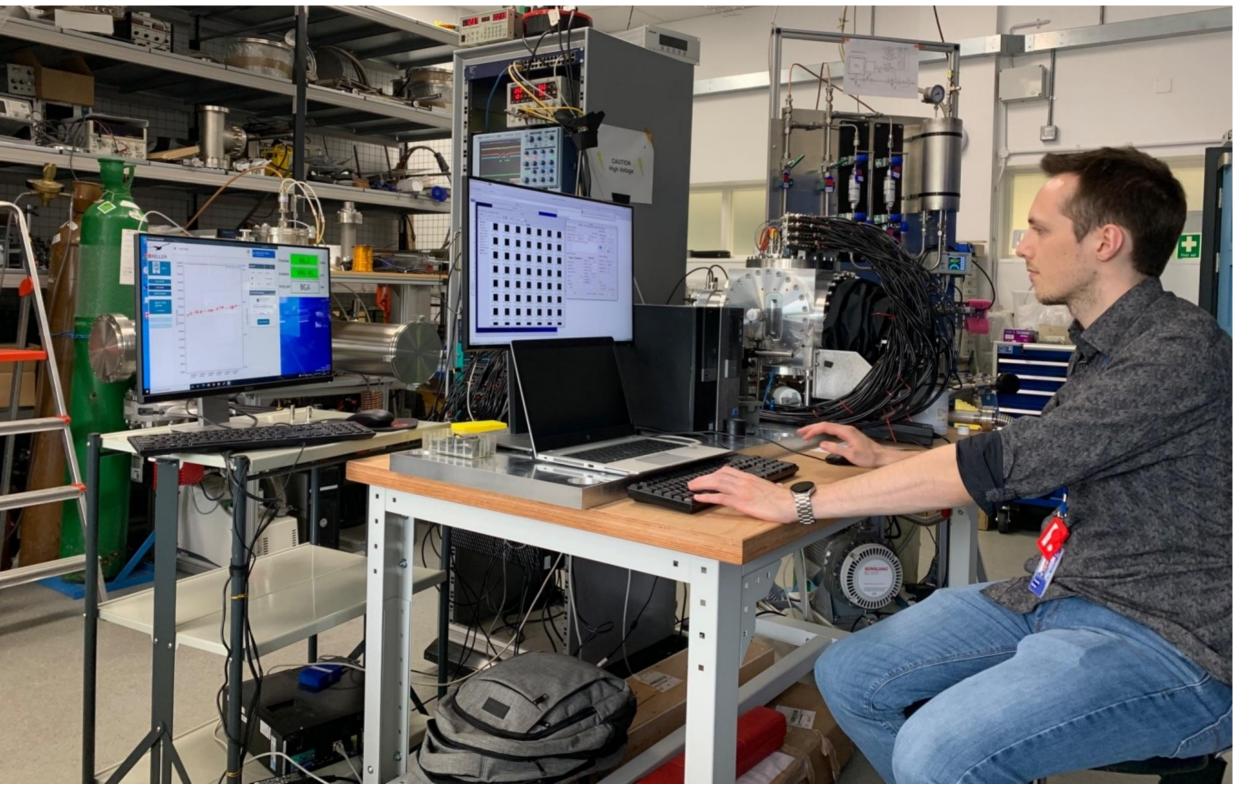
Pictures: Optical Time Projection Chamber





Pictures: Optical Time Projection Chamber

















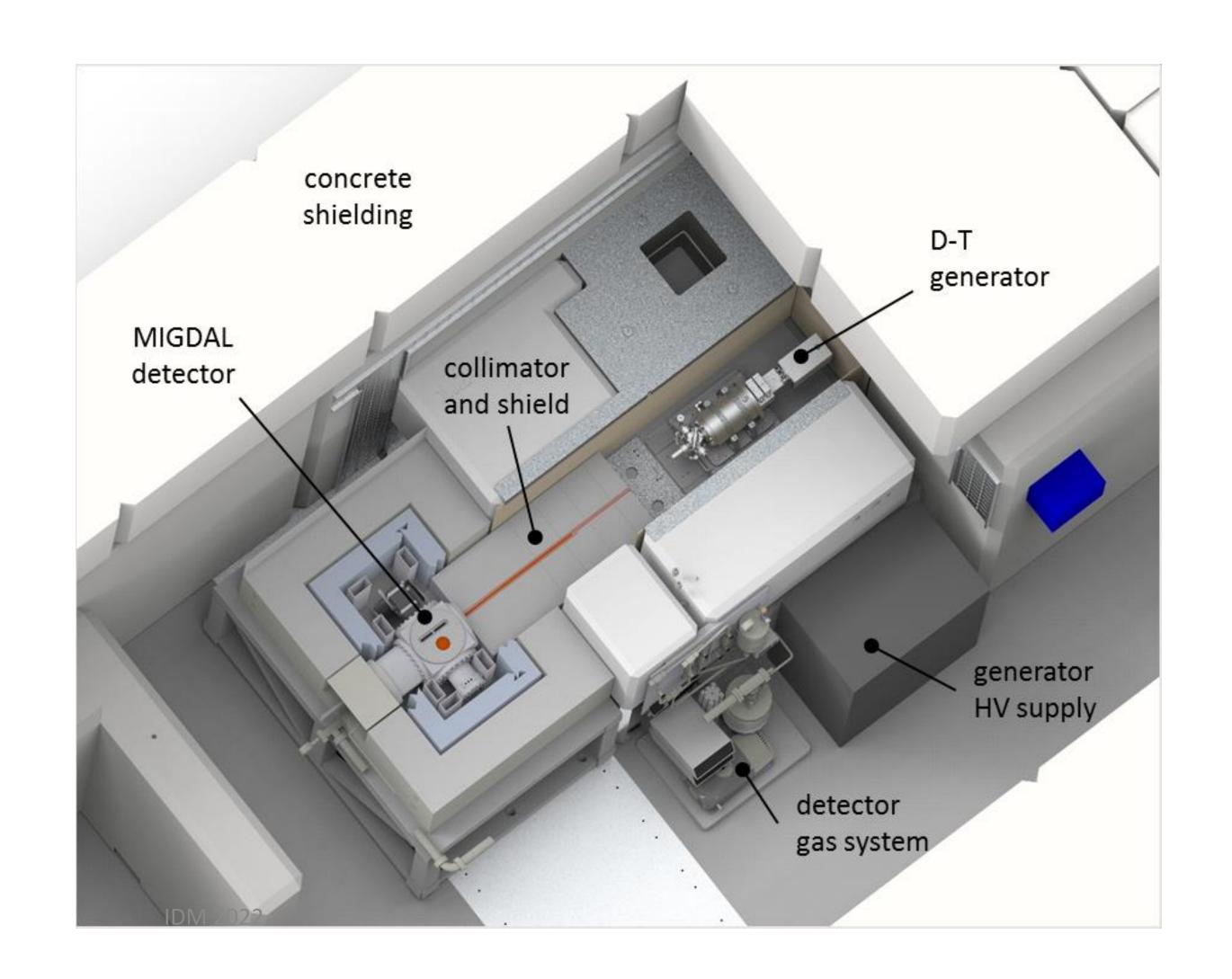
We also need some neutrons...

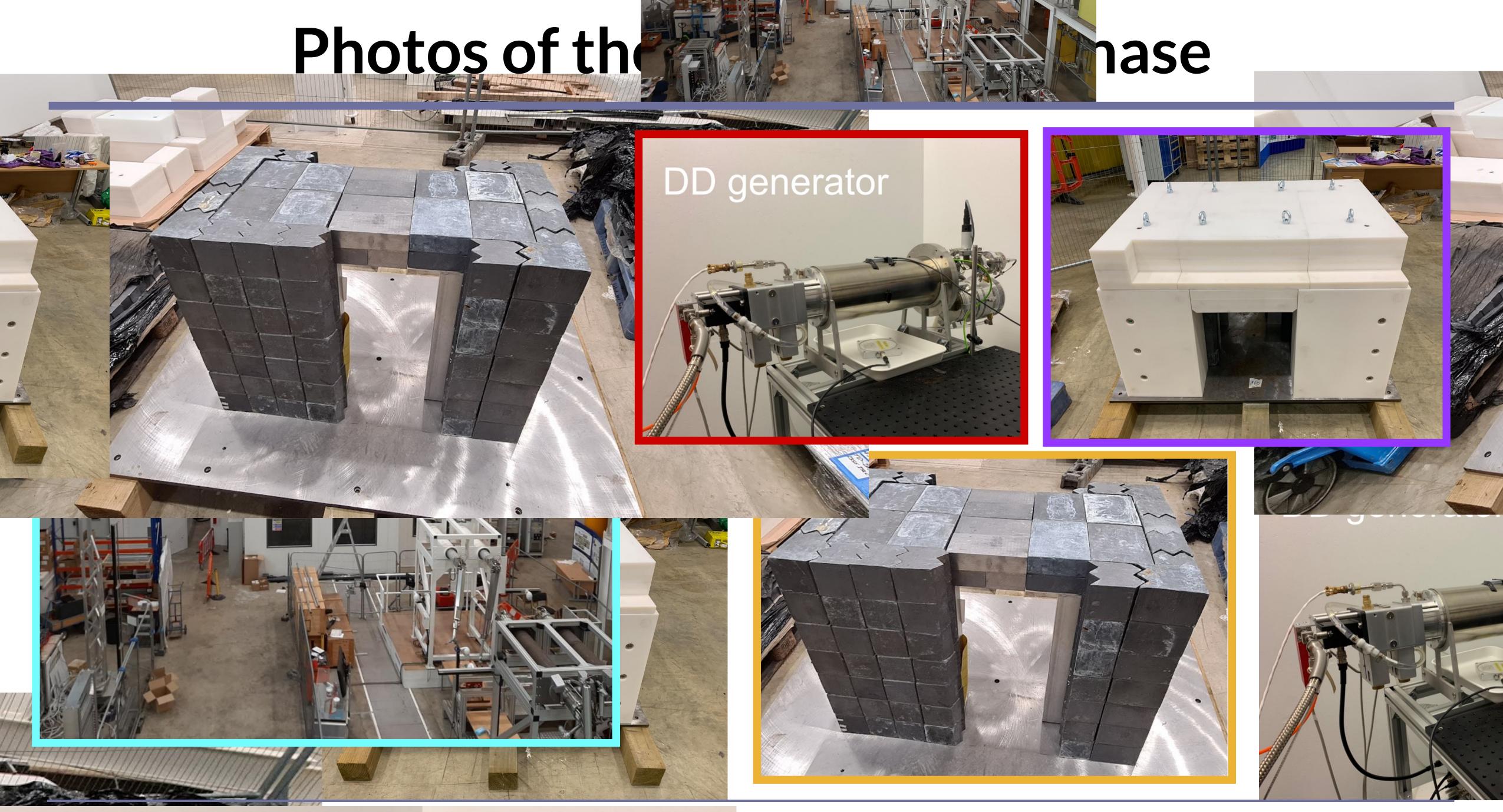
We operate at the NILE Facility at the Rutherford Appleton Laboratory, UK

D-D and D-T fusion generators installed in "shielding bunker"

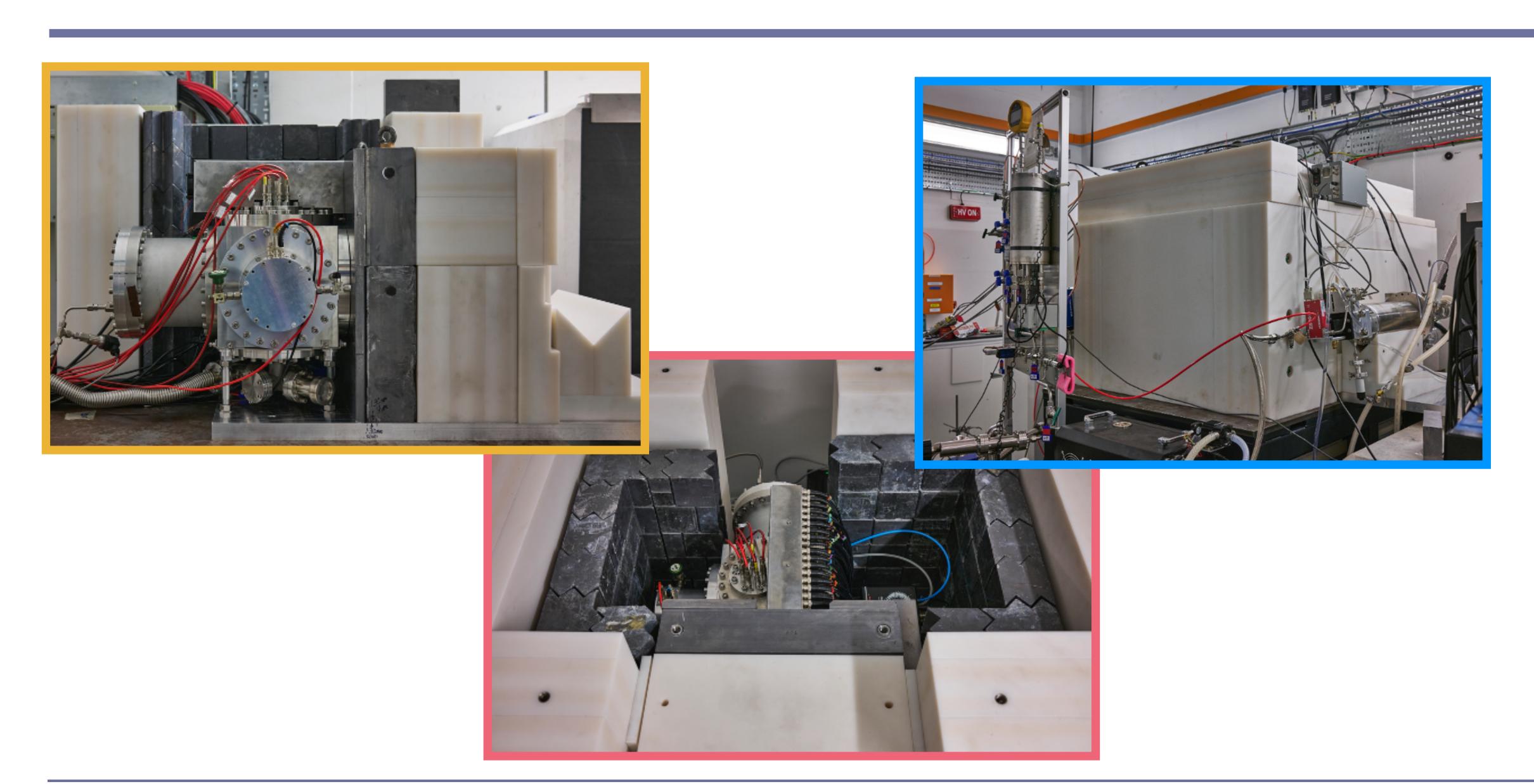
High-yield neutron generators

- D-D: 2.47 MeV (10⁹ n/s)
- D-T: 14.7 MeV (10¹⁰ n/s)





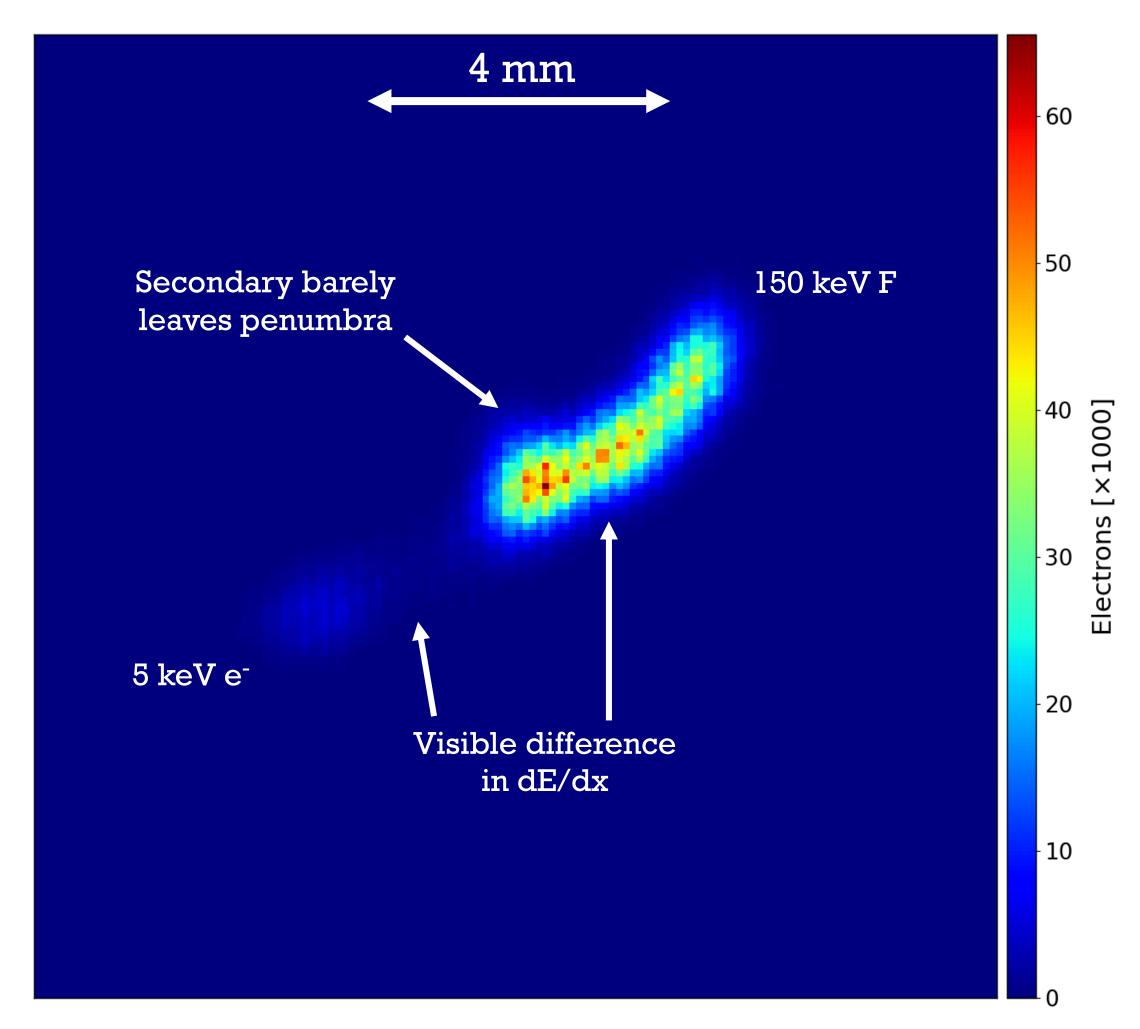
Installation in the NILE bunker



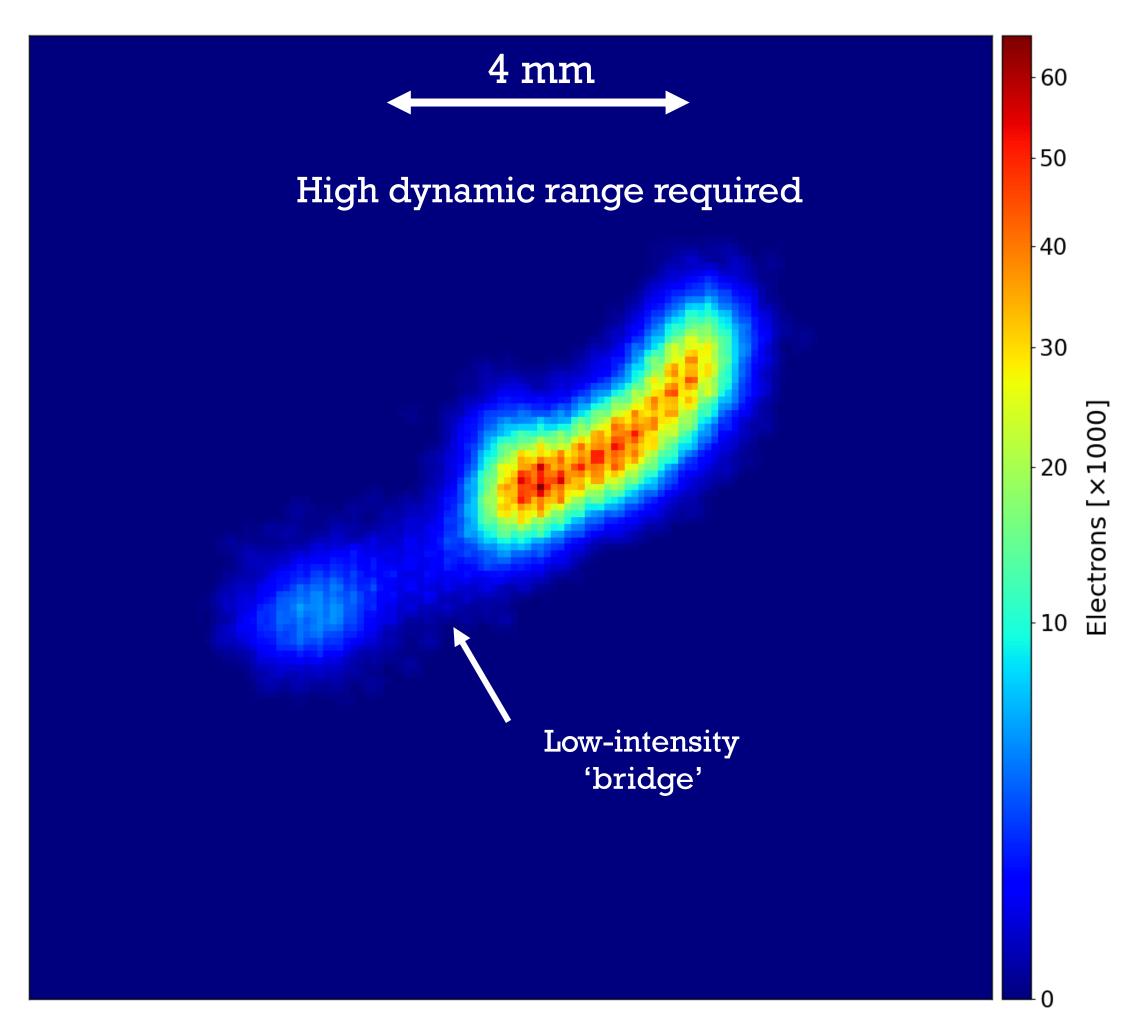
Neutrons plus OTPC gives...



Simulated camera images of Migdal event

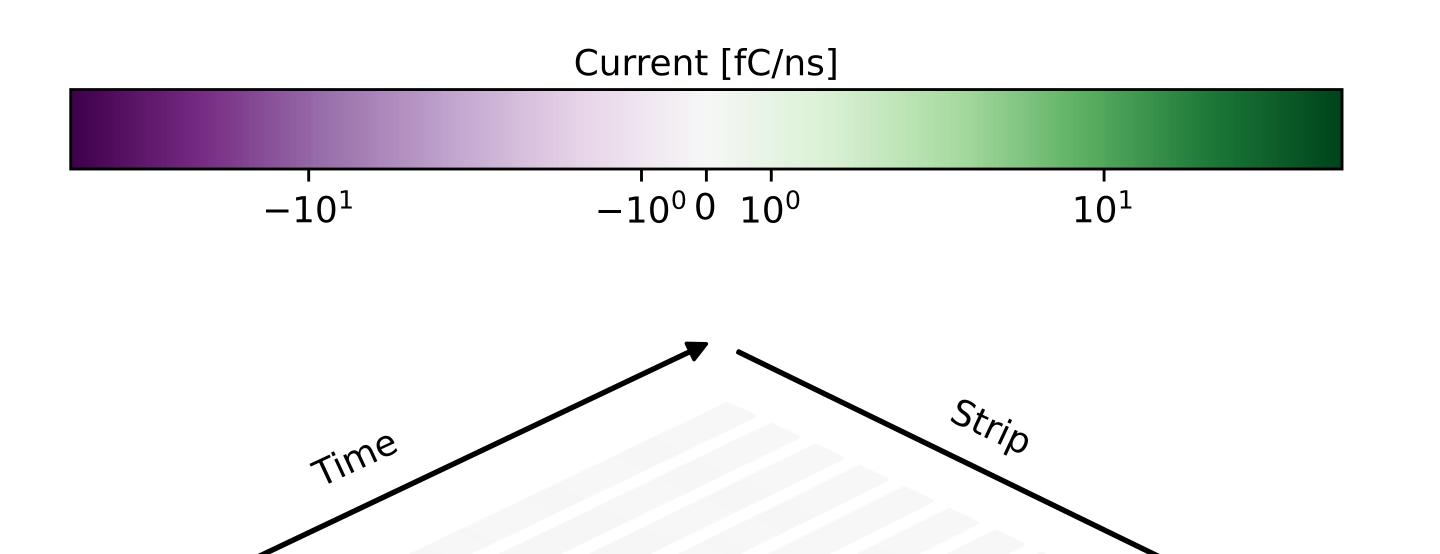


Linear-scale colour map

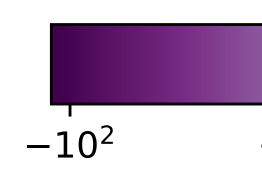


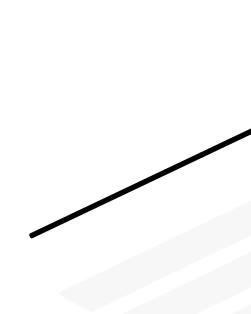
Log-scale colour map

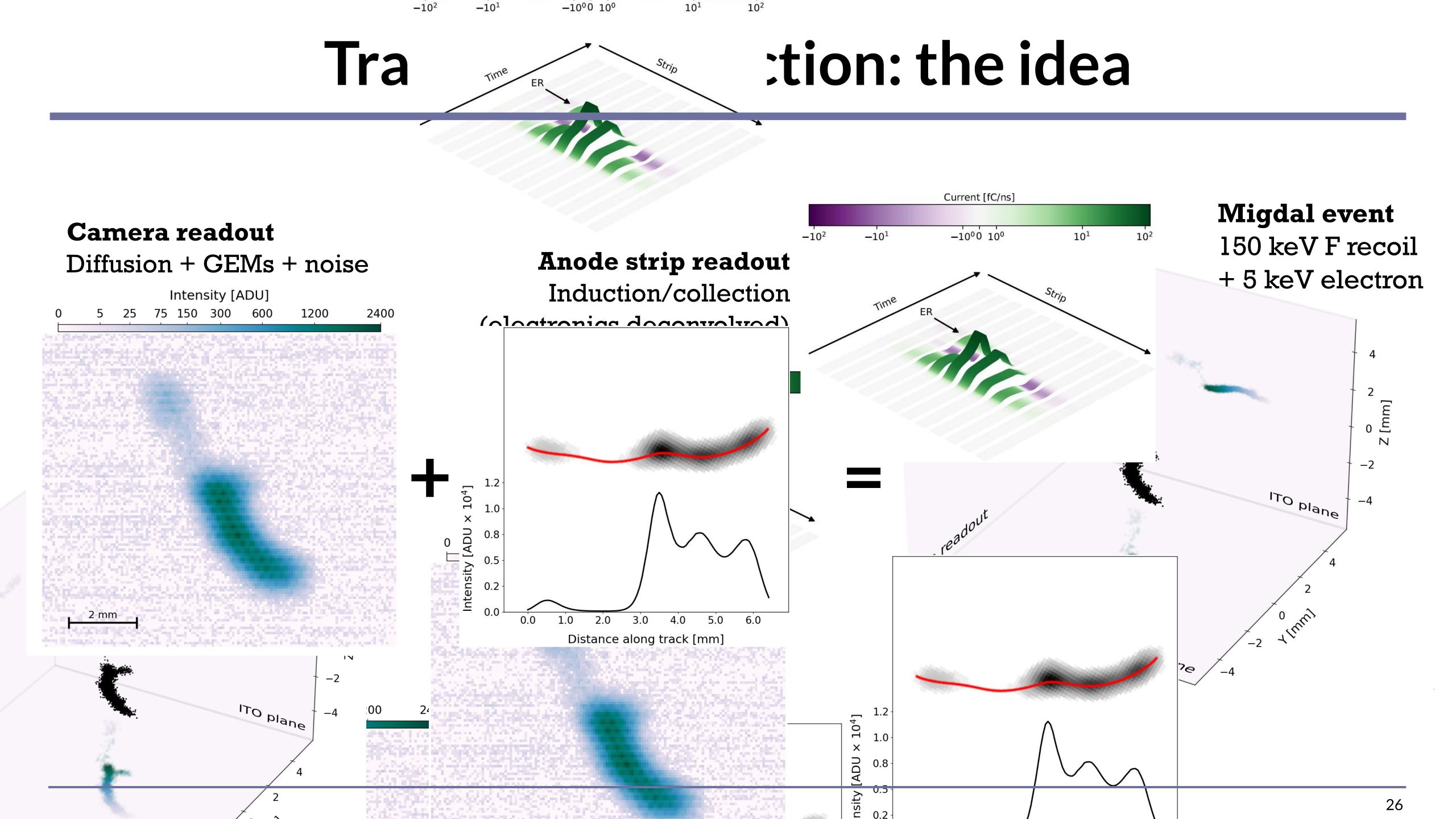
Simulated ITO signals of Migdal event



ER



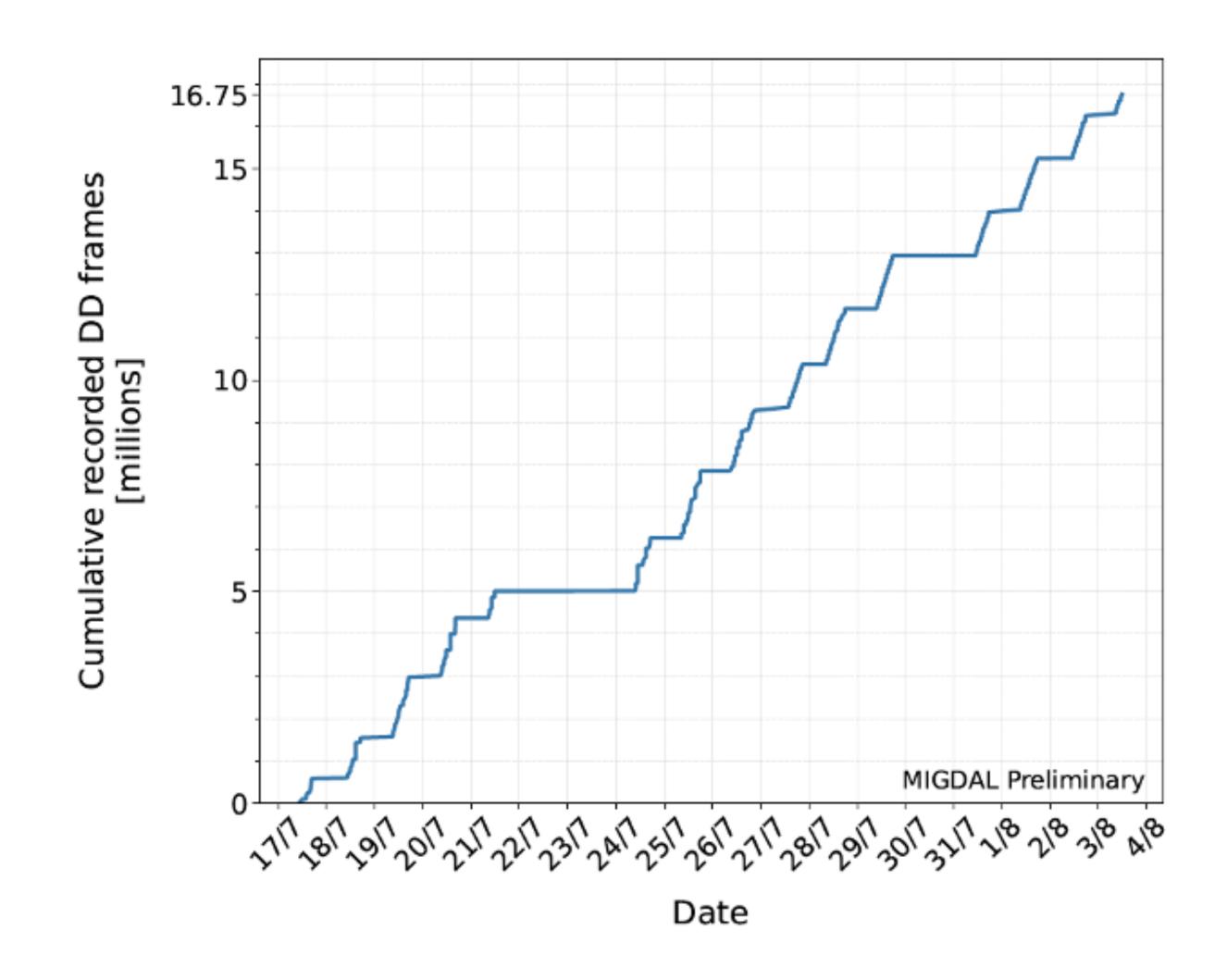




MIGDAL experiment: first science run

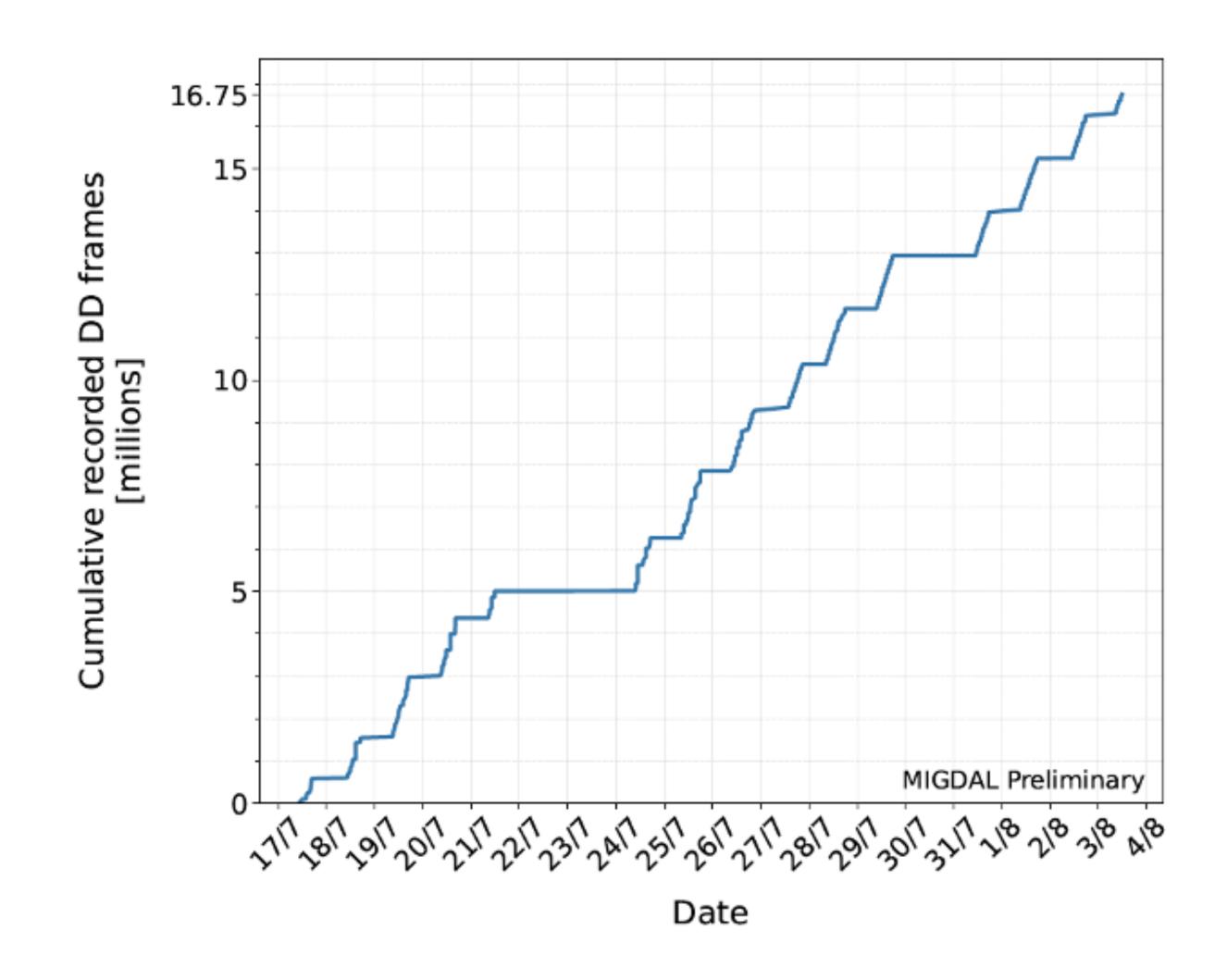
Summary of the run

- First Science run from 17July to 3 August, taken in 10 hour shifts/day
- Data taken using D-D neutron generator, with a lower NR rate than designed
- Frames taken with 20 ms exposure time, longer than planned due to problems with camera's Linux firmware

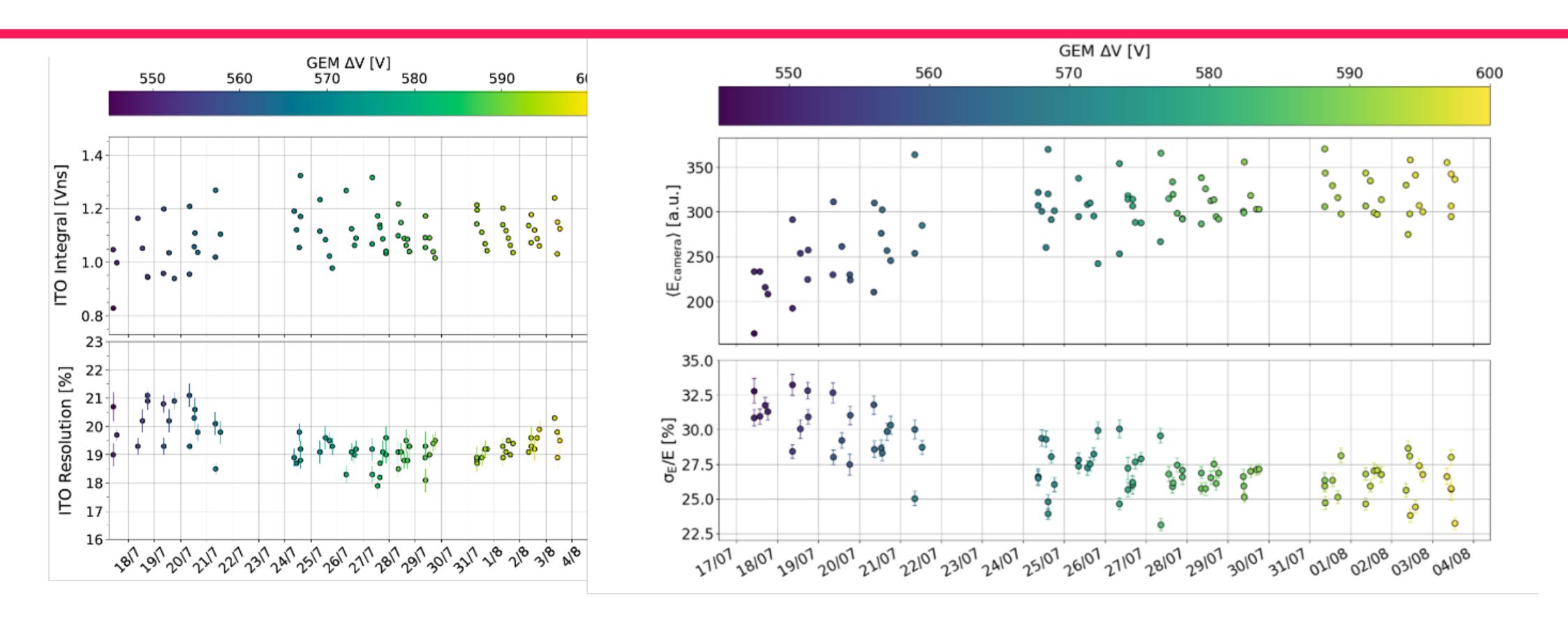


Summary of the run

- Data taking interspersed with regular calibration runs (55Fe) to monitor the gain of the detector
- Voltage across GEMs increased by a small amount each day to keep constant gain
- Total gain in GEMs tuned to a threshold required to see fully resolved ⁵⁵Fe peak
- Average spark rate ~ 7/min due to high dynamic range the detector operates at
- Half of the data is blinded

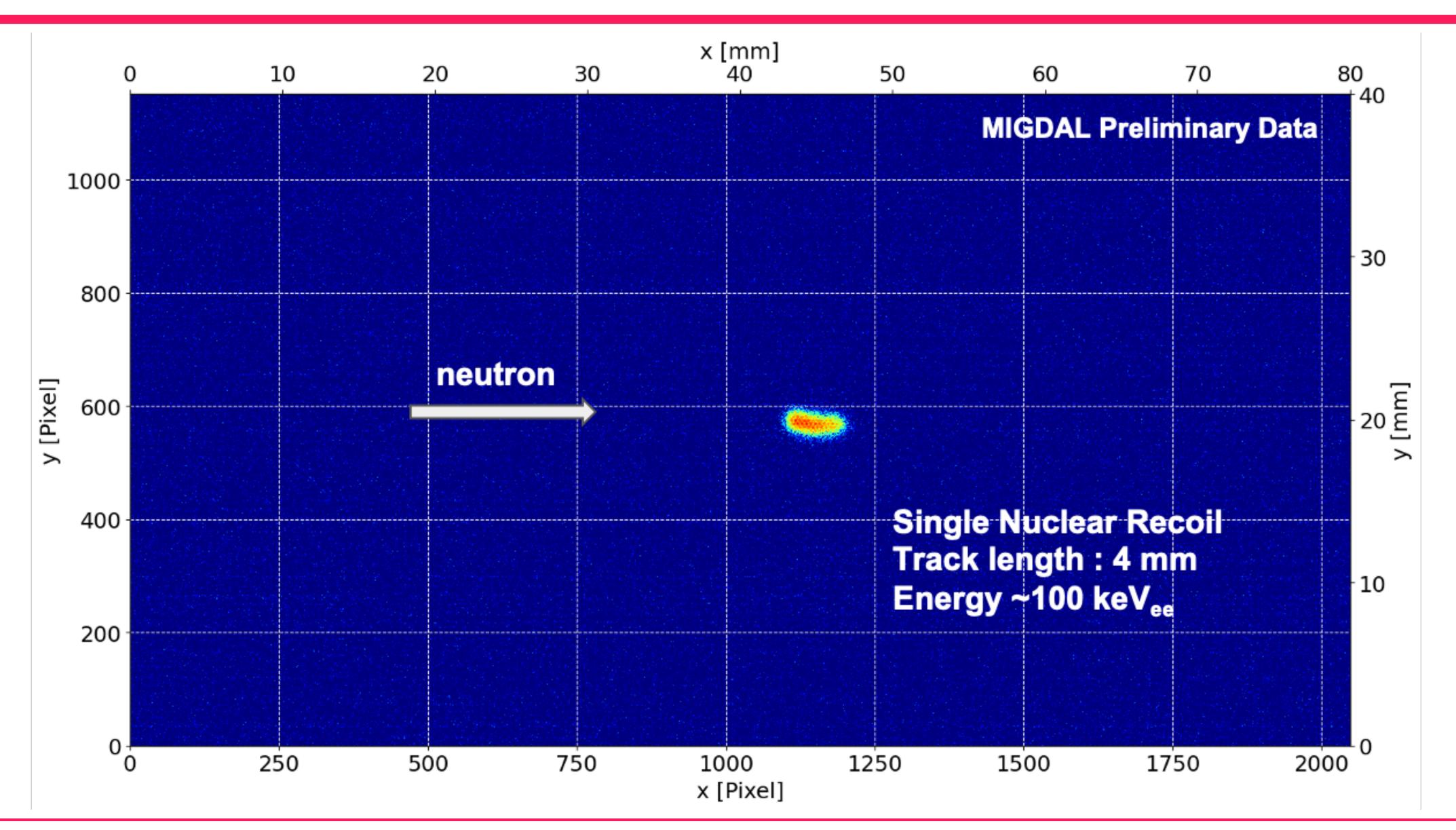


Detector calibration

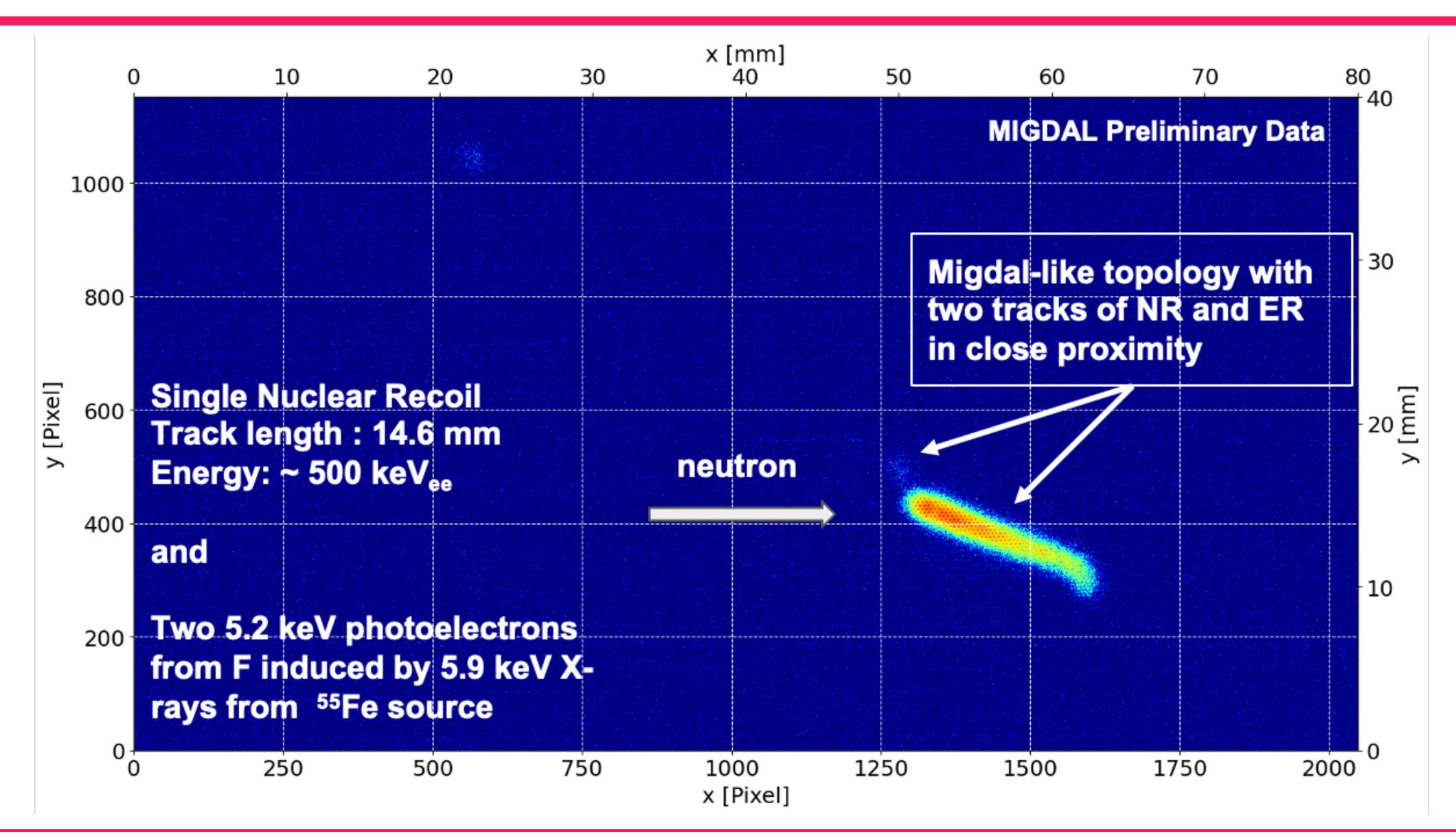


- ⁵⁵Fe calibration performed several times per day
- Energy scale is consistent over the course of the science run with ~20% variation
- Resolution in ITO ~20% and in camera ~ 25 32 % camera readout depending on the gain
- Further improvements are expected with better calibration methods

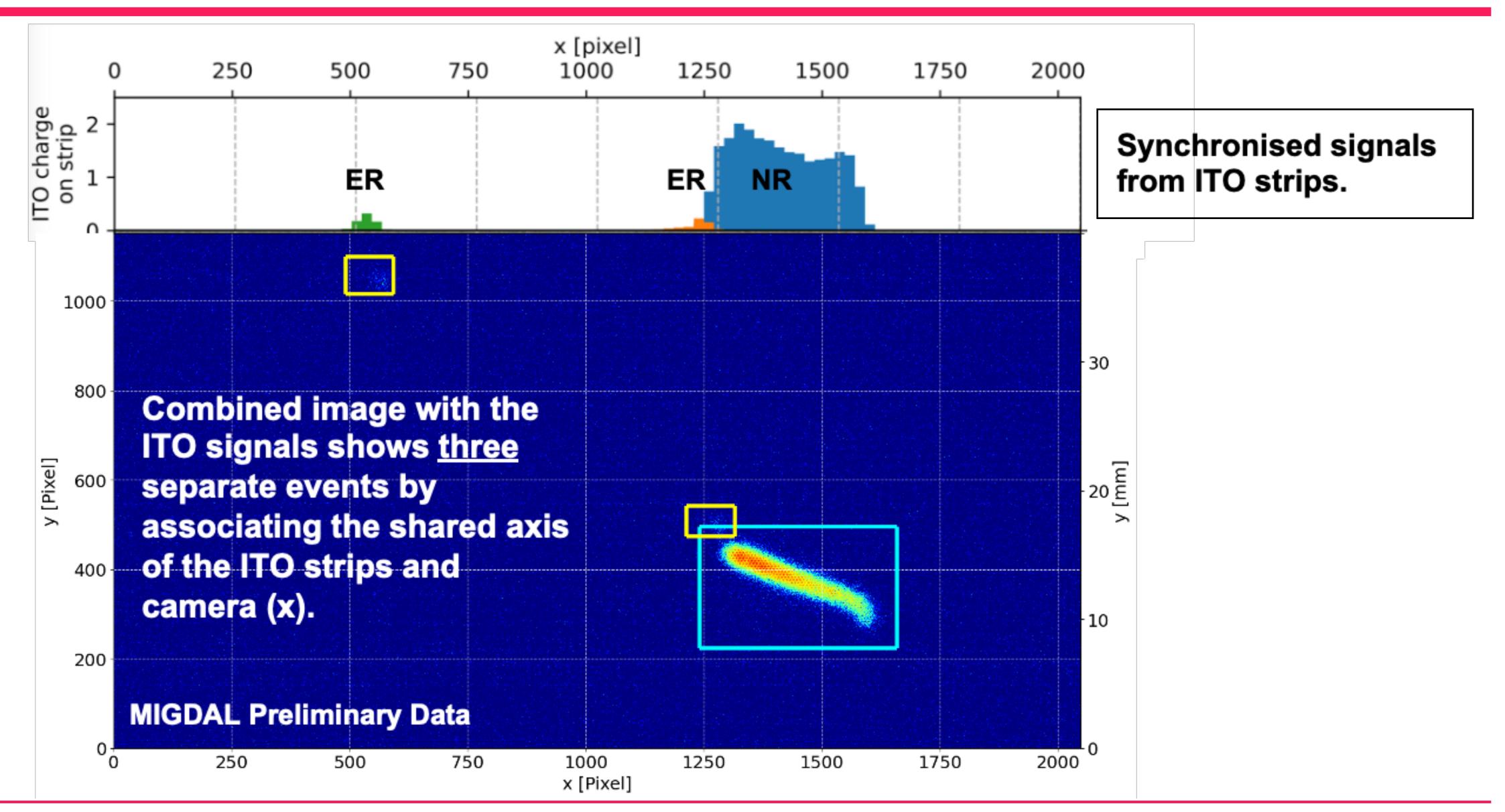
Examples of events: single nuclear recoil



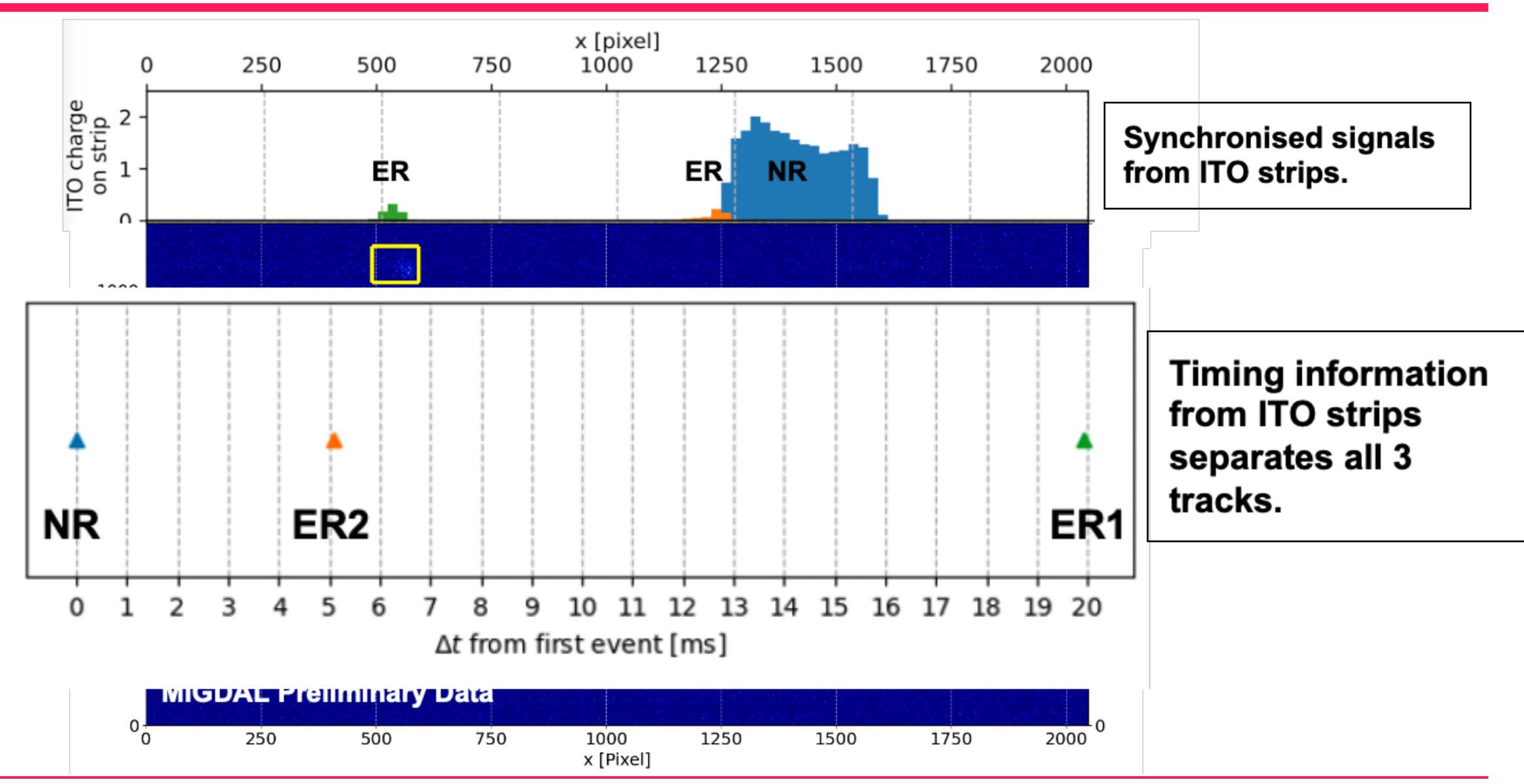
Examples of events: Migdal topology



Examples of events: Migdal topology

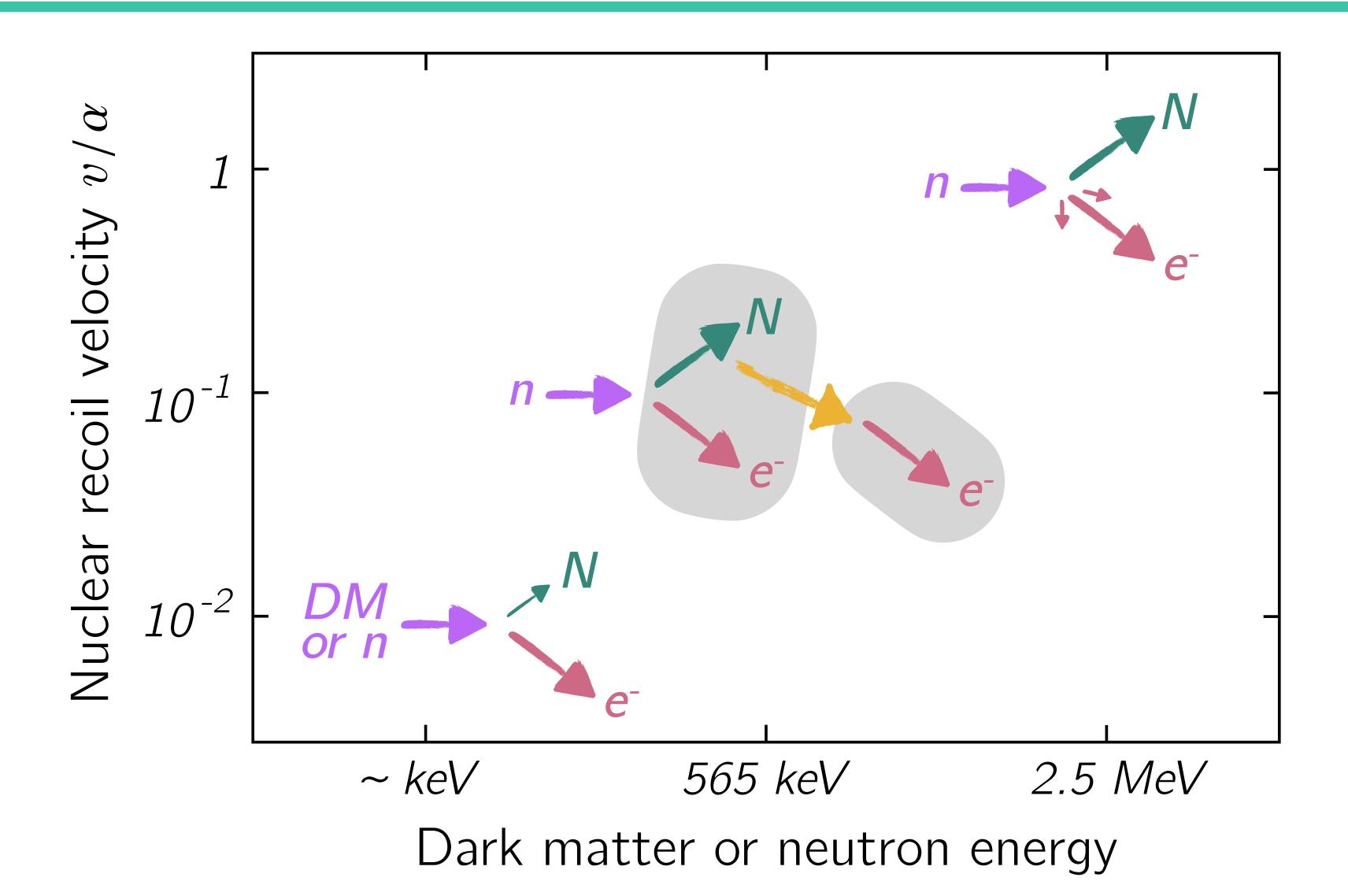


Migdal topology, but not a Migdal event



New theory work

Proposals cover orders of magnitude in v/α

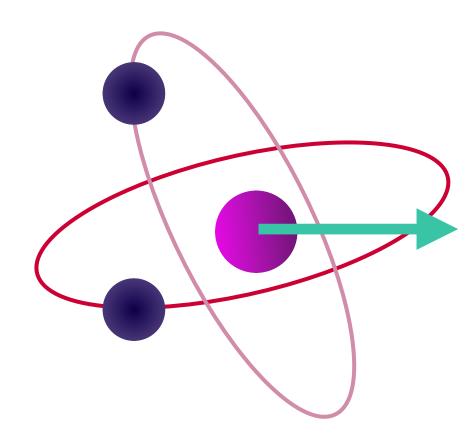


[*In this talk c=1]

Migdal transition element

A. Migdal, J. Phys. Acad. Sci. USSR 4 (1941) 449–453 (See also E. L. Feinberg, J. Phys. Acad. Sci. USSR 4 (1941) 423)

$$\langle \Psi_f^{\{k\}} | e^{im_e \mathbf{v} \cdot \sum_a \mathbf{r}_a} | \Psi_i^{\{j\}} \rangle$$



 $|\Psi_i^{\{j\}}\rangle$ describes the bound atomic-electrons wavefunction

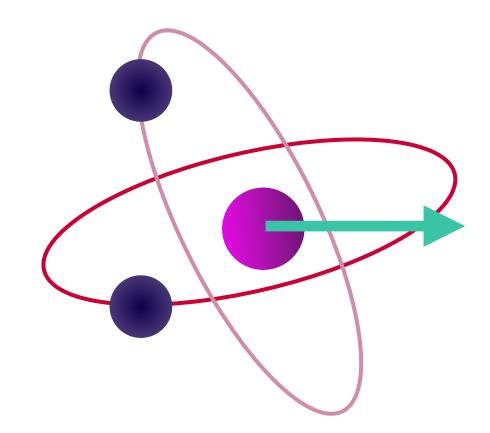
v = Nuclear recoil velocity

 $|\Psi_f^{\{k\}}
angle$ describes the final state wavefunction (excitation, ionisation, etc)

Migdal transition element

A. Migdal, J. Phys. Acad. Sci. USSR 4 (1941) 449–453 (See also E. L. Feinberg, J. Phys. Acad. Sci. USSR 4 (1941) 423)

$$\langle \Psi_f^{\{k\}} | e^{im_e \mathbf{v} \cdot \sum_a \mathbf{r}_a} | \Psi_i^{\{j\}} \rangle$$



Previous calculations made the 'dipole approximation':

$$\exp\left(im_e\mathbf{v}\cdot\sum_{a=1}^N\mathbf{r}_a\right)\approx 1+im_e\mathbf{v}\cdot\sum_{a=1}^N\mathbf{r}_a$$

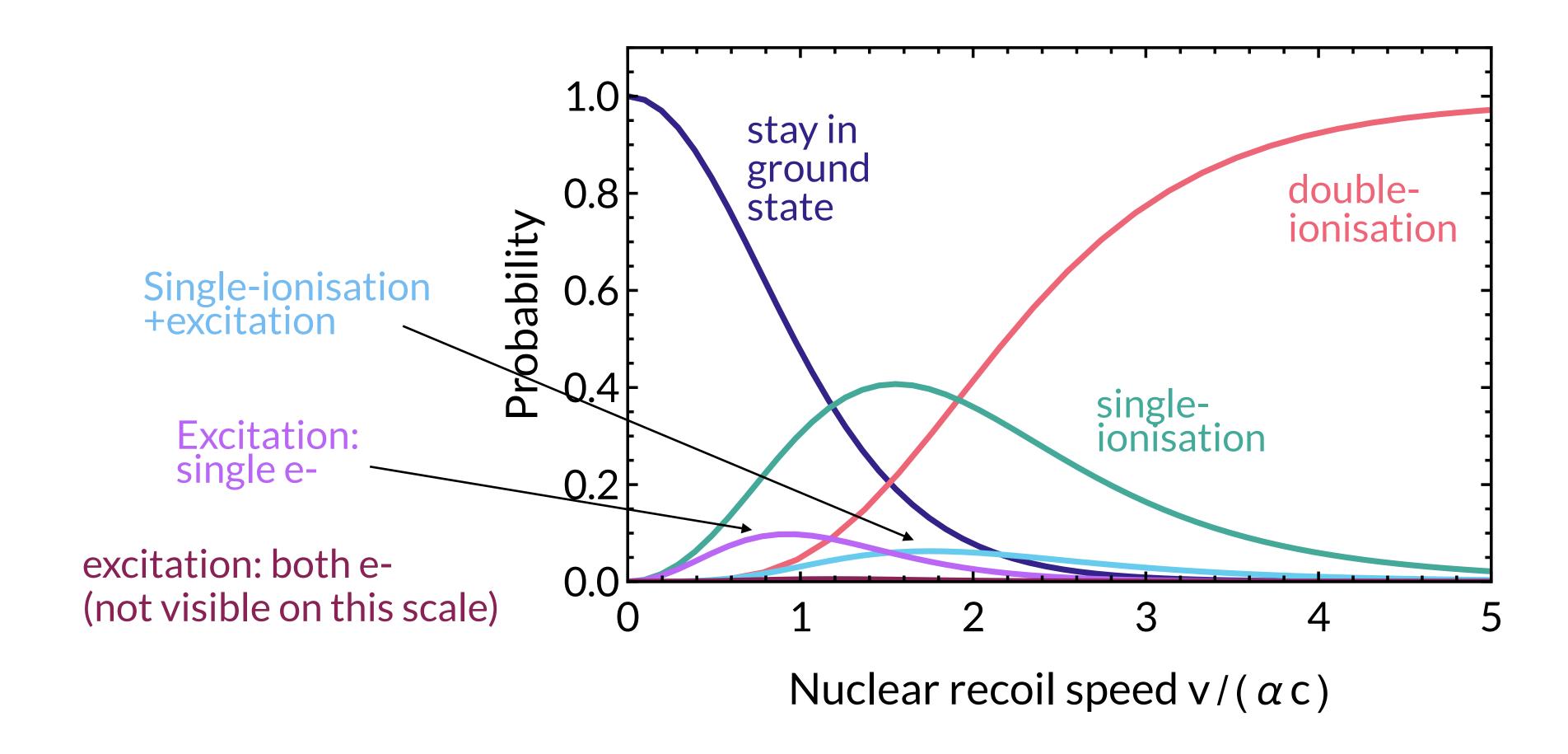
Unclear if dipole approximation holds for neutron scattering processes (high v) — and only allows for single ionisation processes to be accounted for

We keep the full exponential factor (sounds easy but lots of extra work!)

Cox, Dolan, CM, Quiney, arXiv:2208.12222

Total probability results (with the exponential factor!)

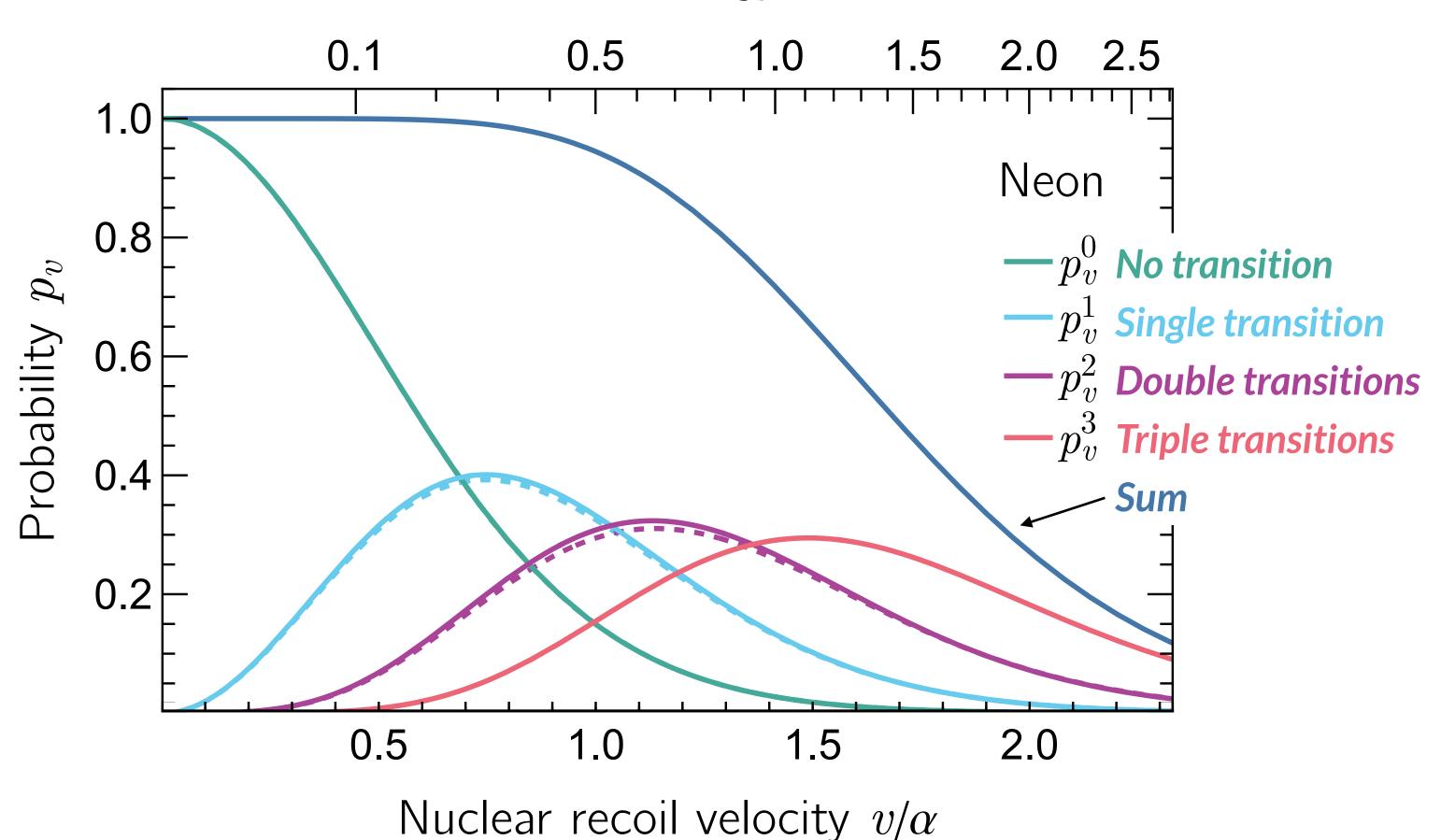
Helium results (with GRASP+RATIP)



Previous calculations could only give the single-ionisation curve for $v/lpha \ll 1$

Extending to bigger atoms (neon)

Nuclear recoil energy E_R [MeV]



Theory framework generalises straightforwardly to larger atoms...

...but there are more electrons

Probability sums to 1 to $v \simeq \alpha$ but clearly deviates beyond

At higher speeds, quads, quintics, ... will contribute but VERY difficult to calculate

Without quads, quintics, ... will we have to give up on accurate predictions at higher NR speeds?

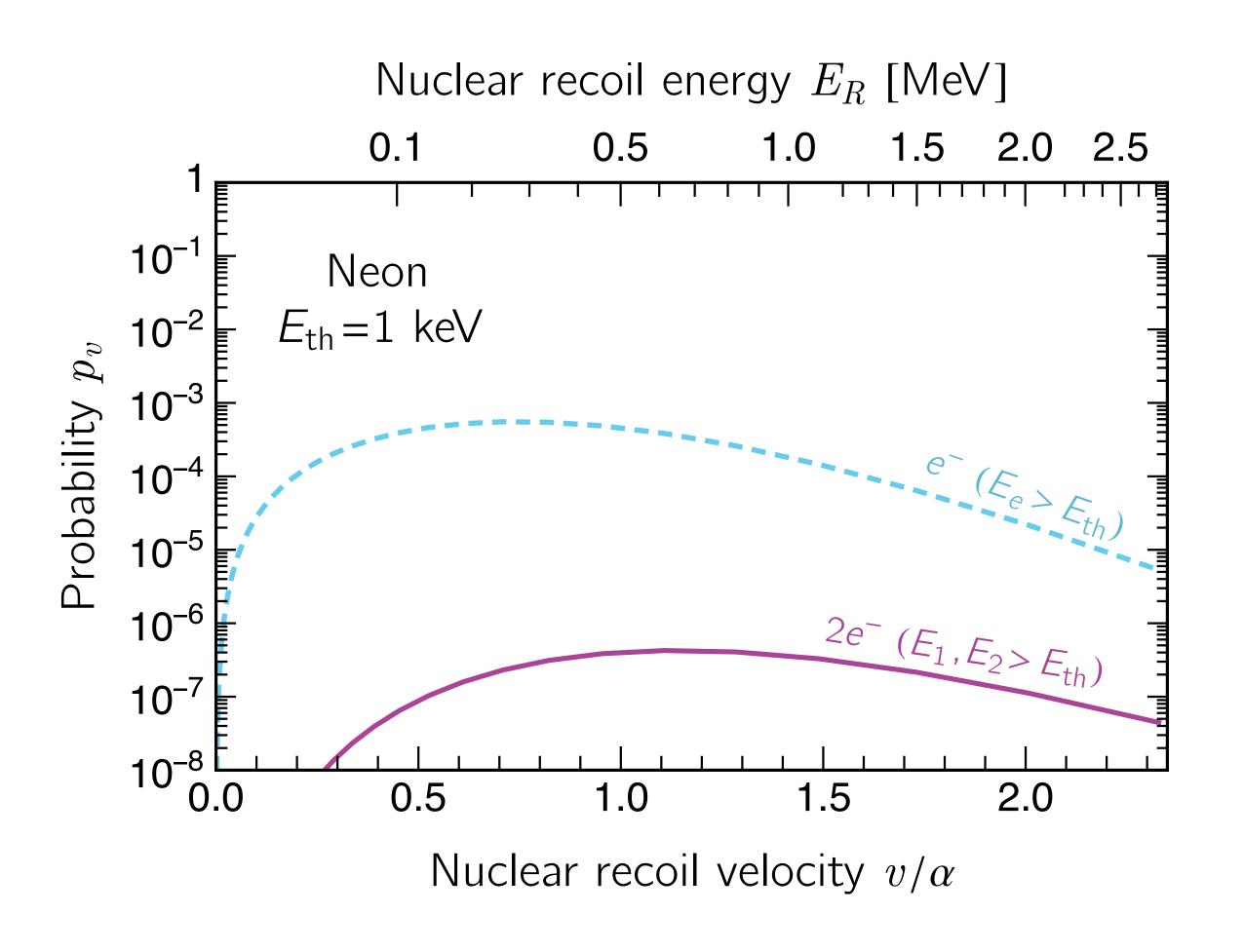
Without quads, quintics, ... will we have to give up on accurate predictions at higher NR speeds?

No! (for realistic experiments)

The impact of experimental thresholds

Realistic experiments have a **threshold** on the electron energy

Probability of two electrons above threshold is always suppressed, even at high NR speeds



The impact of experimental thresholds

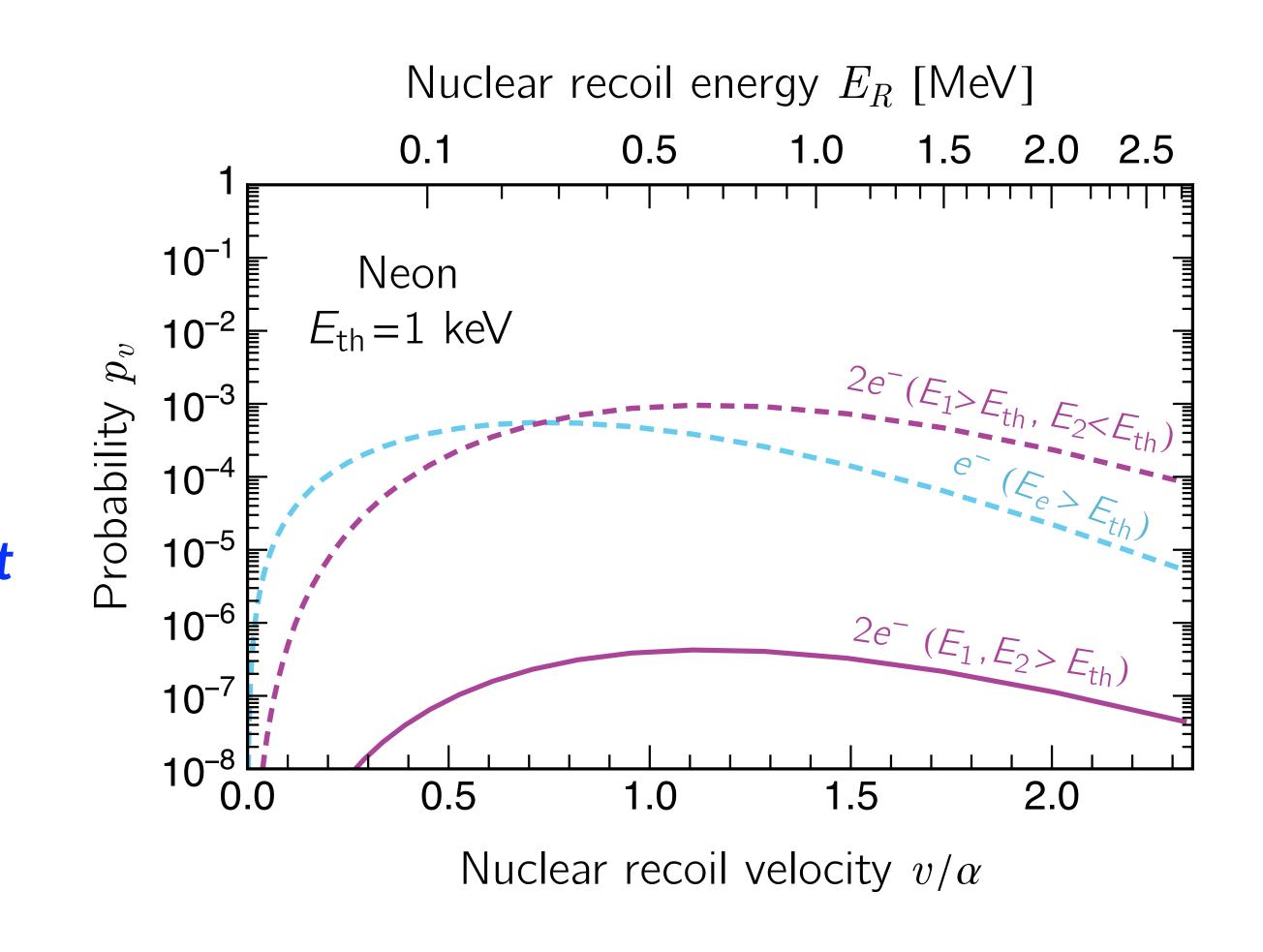
Realistic experiments have a **threshold** on the electron energy

Probability of two electrons above threshold is always suppressed, even at high NR speeds

...but what about 1 hard electron and 1 soft electron?

...Indeed, this is a large correction!

...the contributions from 1 hard, 2 soft; 1 hard 3 soft, ..., will also be important



Summing over all soft electrons

Formally, the sum over all 1-hard + N soft-electrons is

$$p_{v}(|\Psi_{i}^{\{j\}}\rangle \to |\chi_{k_{1}}X_{\text{soft}}\rangle) = \frac{1}{(N-1)!} \sum_{k_{2},\dots,k_{N}}^{E < E_{\text{th}}} \left| \left\langle \Psi_{f}^{\{k\}} \middle| e^{im_{e}\mathbf{v} \cdot \sum_{a} \mathbf{r}_{a}} \middle| \Psi_{i}^{\{j\}} \right\rangle \right|^{2}$$

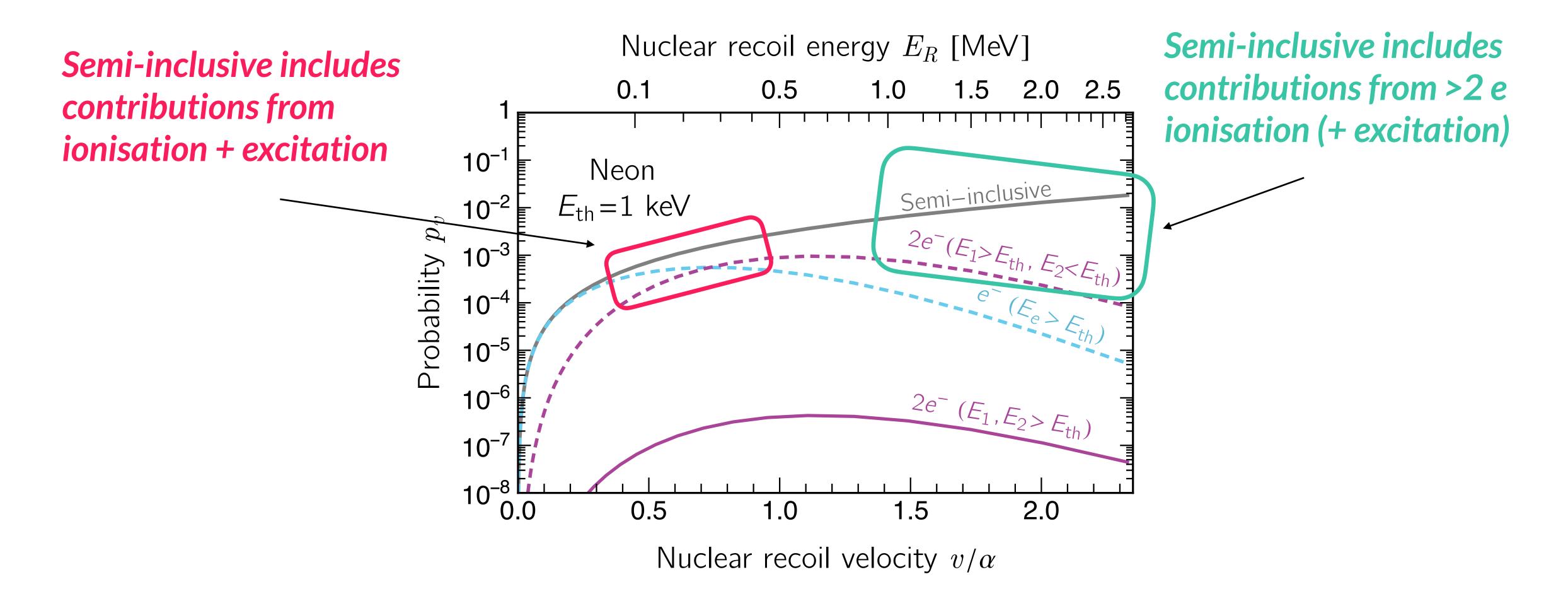
To a good approximation*, this can be manipulated into the compact expression (which is straightforward to calculate numerically)

$$p_v(|\Psi_i^{\{j\}}\rangle \to |\chi_{k_1} X_{\text{soft}}\rangle) = \sum_{\alpha=1}^N \left| \langle \chi_{k_1} | e^{im_e \mathbf{v} \cdot \mathbf{r}} | \psi_{j_\alpha} \rangle \right|^2$$

We call this the 'semi-inclusive probability'

*Valid approximation if $v/\alpha \lesssim 8.6 \sqrt{(E_{\rm th}/1~{\rm keV})}$

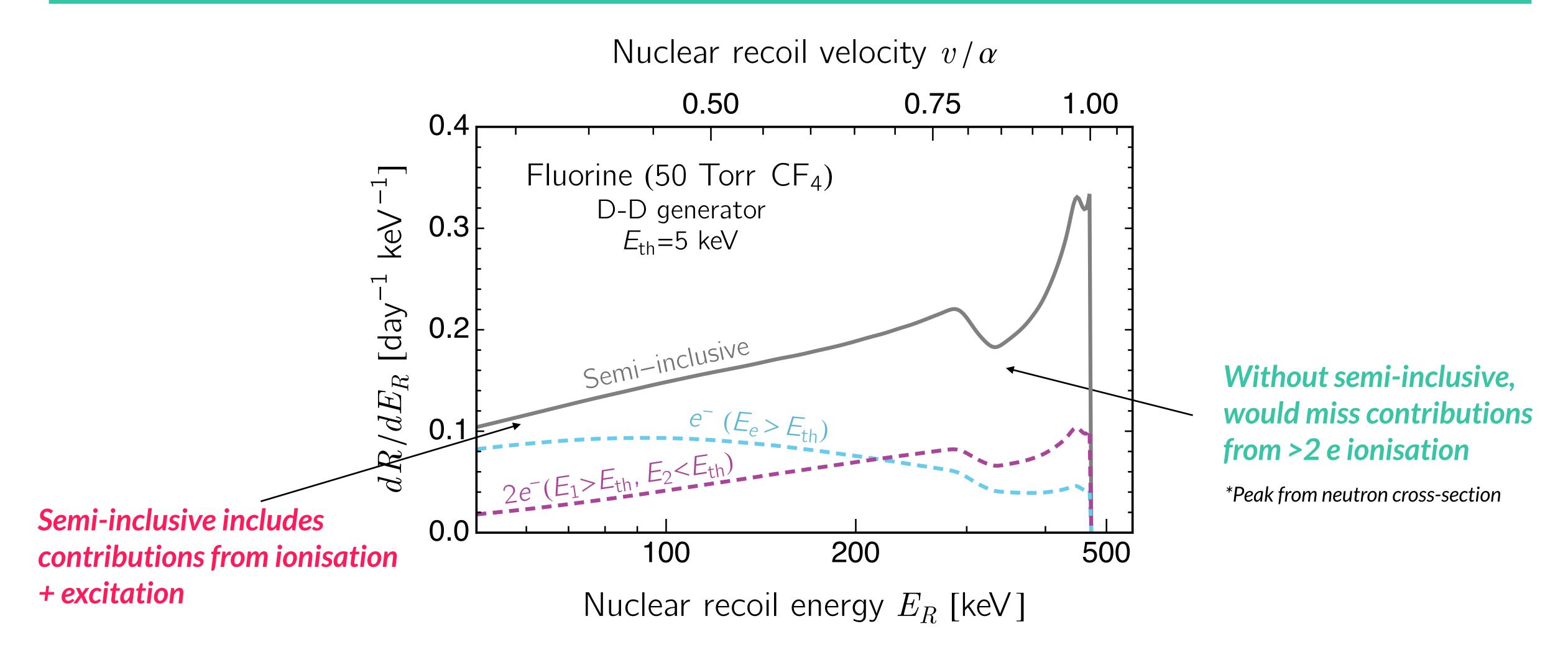
Semi-inclusive probability



Semi-inclusive probability gives accurate rates even at high NR-speeds

Application to neutron scattering [high-NR speed]

Neutron scattering rates (DD = 2.47 MeV)



Semi-inclusive rate is factor 1.6 higher than single+double ionisation

Background/Signal rates per million nuclear recoils

| Composit | To 1 | D-D neutrons | | D-T neutrons | |
|--|--|--------------|-------------|--------------|-------------|
| Component | Topology | > 0.5 | 5-15 keV | > 0.5 | 5-15 keV |
| Recoil-induced δ -rays | Delta electron from NR track origin | | 0 | 541,000 | 0 |
| Particle-Induced X-ray Emission (PIXE) | | | | | |
| X-ray emission | Photoelectron near NR track origin | | 0 | 365 | 0 |
| Auger electrons | Auger electron from NR track origin | | 0 | $42,\!000$ | 0 |
| Bremsstrahlung processes [†] | | | | | |
| Quasi-Free Electron Br. (QFEB) | Photoelectron near NR track origin | 112 | ≈ 0 | 288 | ≈ 0 |
| Secondary Electron Br. (SEB) | Photoelectron near NR track origin | 115 | ≈ 0 | 279 | ≈ 0 |
| Atomic Br. (AB) | Photoelectron near NR track origin | 70 | ≈ 0 | 171 | ≈ 0 |
| Nuclear Br. (NB) | Photoelectron near NR track origin | | ≈ 0 | 0.013 | ≈ 0 |
| Photon interactions | | | | | |
| Neutron inelastic γ -rays (gas) | Compton electron near NR track origin | 1.6 | 0.47 | 0.86 | 0.25 |
| Random track coincidences | Photo-/Compton electron near NR track | | ≈ 0 | ≈ 0 | ≈ 0 |
| Gas radioactivity | | | | | |
| Trace contaminants | Electron from decay near NR track origin | 0.2 | 0.01 | 0.03 | ≈ 0 |
| Neutron activation | Electron from decay near NR track origin | | 0 | ≈ 0 | ≈ 0 |
| Secondary nuclear recoil fork | NR track fork near track origin | | ~1 | <u> </u> | ~1 |
| Total background | Sum of the above components | | 1.5 | | 1.3 |
| Migdal signal | Migdal electron from NR track origin | | 32.6 | | 84.2 |

[†] These processes were (conservatively) evaluated at the endpoint of the nuclear recoil spectra.

Summary

The Migdal effect is...

- an old effect (from 1940s) that is used for dark matter sub-GeV searches and is an active target for near-future neutron-beam experiments

In the UK...

- we are building a detection platform to characterise the effect in multiple elements relevant to dark matter experiments

On the theory side, we have...

- extended previous calculations to the high nuclear-recoil speed regime
- confirmed the accuracy of existing calculations (lbe et al) for DM searches



Thank you

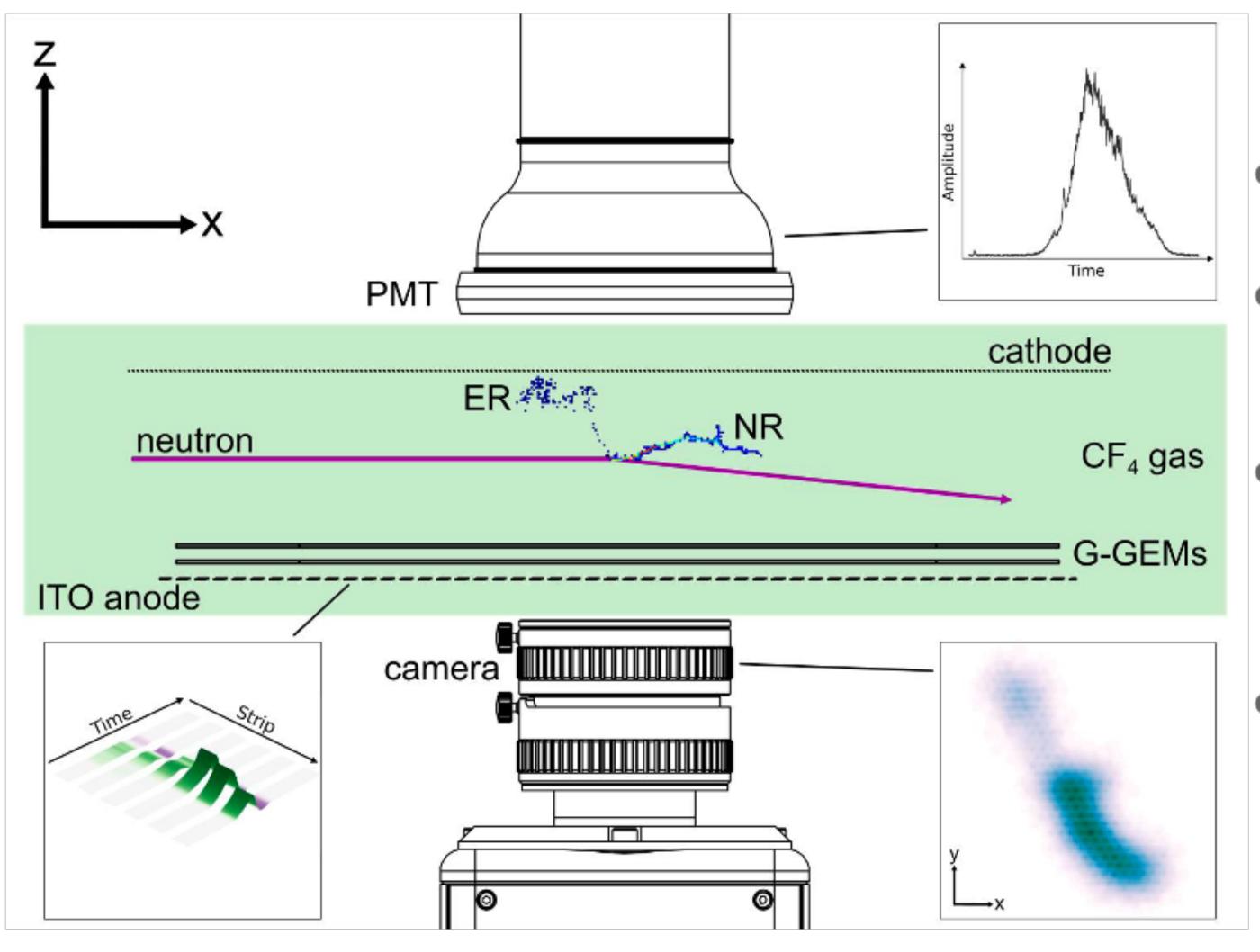
"Precise Predictions and New Insights for Atomic Ionisation from the Migdal Effect" Peter Cox, Matthew Dolan Christopher McCabe and Harry Quiney arXiv:2208.12222, PRD

Data files of probabilities available now: https://petercox.github.io/Migdal/

"The MIGDAL experiment: Measuring a rare atomic process to aid the search for dark matter" H.M. Araújo et al arXiv:2207.08284, Astroparticle

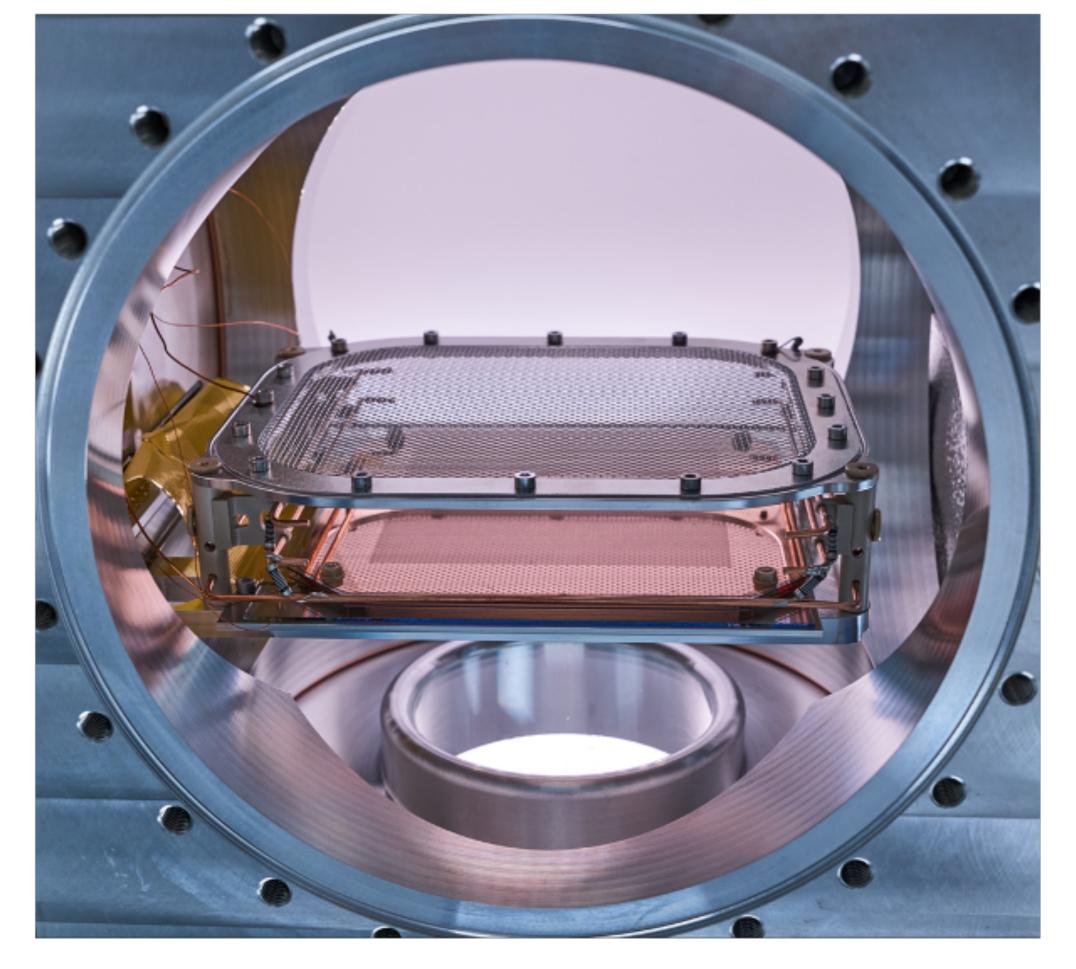
Backup

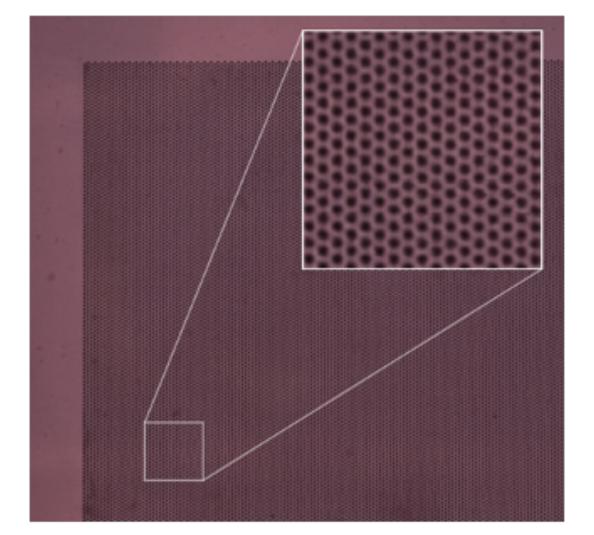
The MIGDAL experiment

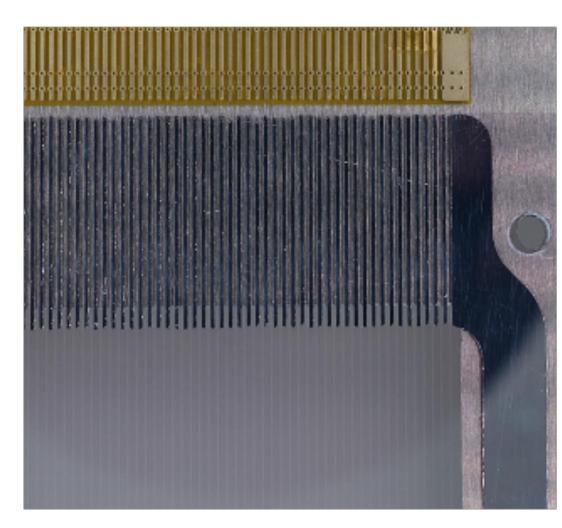


- Low-pressure gas: 50 Torr of CF₄
 - Extended particle tracks
 - Avoid gamma interactions
 - Can stably work with fraction of Ar
 - TPC Signal amplification
 - 2 x glass-GEMs (Cu + Ni cladded)
- Readout:
 - Optical : Camera + photomultiplier tube
 - Charge: GEMs + 120 ITO anode strips
- High-yield neutron generator
 - o D-D: 2.47 MeV (10⁹n/s)
 - D-T: 14.7 MeV (10¹⁰n/s)
 - Defined beam, "clear" through TPC
- Electron and nuclear recoil tracks
 - Migdal: NR+ER tracks, common vertex
 - NR and ER have very different dE/dx
 - 5 keV electron threshold
 5.9 keV X-rays from Fe-55 induce 5.2 keV photoelectrons from F for calibration at threshold.

The MIGDAL optical-TPC







Two glass GEMs one Cuand one Ni-cladded :

- thickness: 550 µm

- OD /pitch: 170/280 μm

- active area: 10x10 cm²

total gain ~10⁵

ITO strips wire bonded to readout

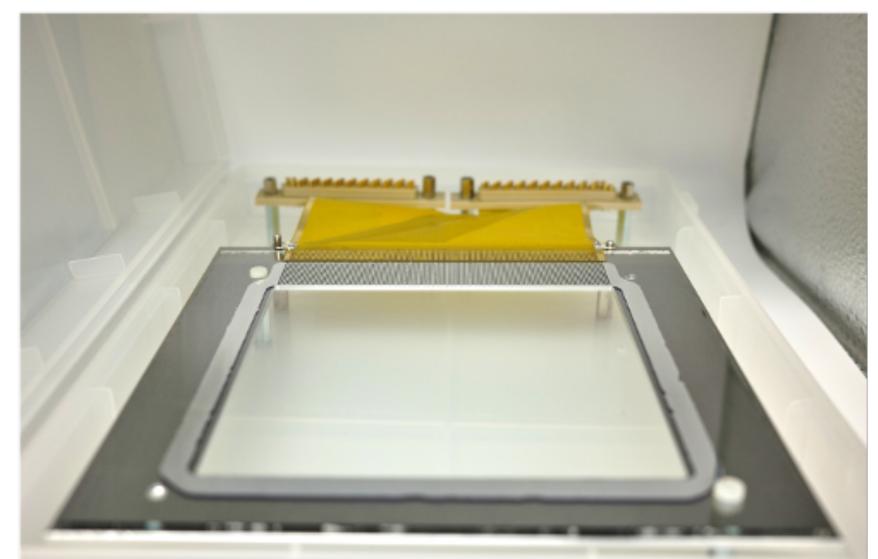
- 120 strips

width/pitch: 0.65/0.83 mm

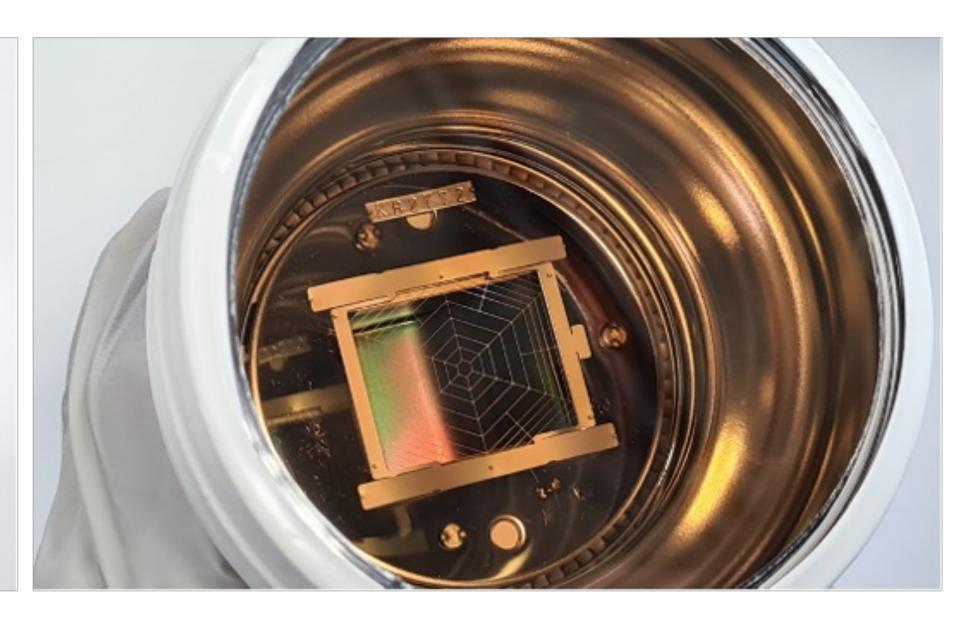
Two field shaping copper wires

- TPC inside of the central aluminium cube
- Drift gap: 3 cm between woven mesh and cascade of two glass-GEMs (E_{DRIFT}=200 V/mm for minimum electron diffusion)
- Transfer and signal induction gaps : 2 mm
- Low outgassing materials; vacuum before fill 2*10-6 mbar; signal unchanged several days after fill

Light and charge readout







ITO anode strips

Post-GEM ionisation

Readout of (x,z) plane

Pitch: 833 µm

Digitised at 2 ns/sample

(Drift velocity: 130 µm/ns)

qCMOS camera (Hamamatsu ORCA - QUEST)

Detects GEM scintillation through glass viewport behind ITO anode

Readout of (x,y) plane

Exposure: 8.33 ms/frame

(continuous)

Px scale: 39 µm (2×2 binning)

Lens: EHD-25085-C; 25mm f/0.85

VUV PMT (Hamamatsu R11410)

Detects primary and secondary (GEM) scintillation
Absolute depth (z) coordinate

Digitional at 2 pa/sample (Trige

Digitised at 2 ns/sample [Trigger]

Migdal effect for neutral atoms

Transition matrix element first found by A. Migdal: $\langle \Psi_f^{\{k\}}|e^{im_e\mathbf{v}\cdot\sum_a\mathbf{r}_a}|\Psi_i^{\{j\}}\rangle$

A. Migdal, J. Phys. Acad. Sci. USSR 4 (1941) 449–453 (See also E. L. Feinberg, J. Phys. Acad. Sci. USSR 4 (1941) 423)

Key-point: When initial/final state wavefunctions expressed as anti-symmetric products of single-electron wavefunctions [e.g. as in Hartree-Fock], this can be expressed as:

$$\left\langle \Psi_f^{\{k\}} \left| e^{im_e \mathbf{v} \cdot \sum_a \mathbf{r}_a} \left| \Psi_i^{\{j\}} \right. \right\rangle = \det(M)$$
 where $M_{ba} = \left\langle \chi_{k_b} \left| \exp(im_e \mathbf{v} \cdot \mathbf{r}) \middle| \psi_{j_a} \right. \right\rangle$ J. D. Talman and $M_{ba} = \left\langle \chi_{k_b} \left| \exp(im_e \mathbf{v} \cdot \mathbf{r}) \middle| \psi_{j_a} \right. \right\rangle$ Phys. Rev. A73, 0

J. D. Talman and A. M. Frolov, Phys. Rev. A73, 032722 (2006)

Example: Ground-state to ground-state transition in helium $\psi_{GS} = \psi_{1s}(\mathbf{r}_1, \mathbf{r}_2) \frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$

Approximate form (for illustration): $\psi_{1s}(\mathbf{r}_1, \mathbf{r}_2) = \psi_{1s}(\mathbf{r}_1)\psi_{1s}(\mathbf{r}_2) = \frac{Z_e^3}{\pi}e^{-Z_e(r_1+r_2)}$, $Z_e = \frac{27}{16}$

$$M_{12} = M_{21} = 0$$

$$M_{11} = M_{22} = \left(1 + \frac{m_e^2 v^2}{4Z_e^2}\right)^{-2}$$

$$P_{GS \to GS} = |\det(M)|^2 = \left[1 + (8m_e v/27)^2\right]^{-8}$$

Comparison of numerical methods

Our approach

Canonical Dirac-Hartree Fock method [Implemented in the GRASP+RATIP and BERTHA codes]

Ibe et al approach (arXiv:1707.07258)

Relativistic self-consistent mean-field [Implemented in the FAC code]

Impact: Model of atomic potential differs - expect *small* differences at low electron energies

We keep the full matrix element:

$$\det(M) = \langle \Psi_f^{\{k\}} | \exp\left(im_e \mathbf{v} \cdot \sum_{a=1}^N \mathbf{r}_a\right) | \Psi_i^{\{j\}} \rangle$$

Makes the dipole approximation:

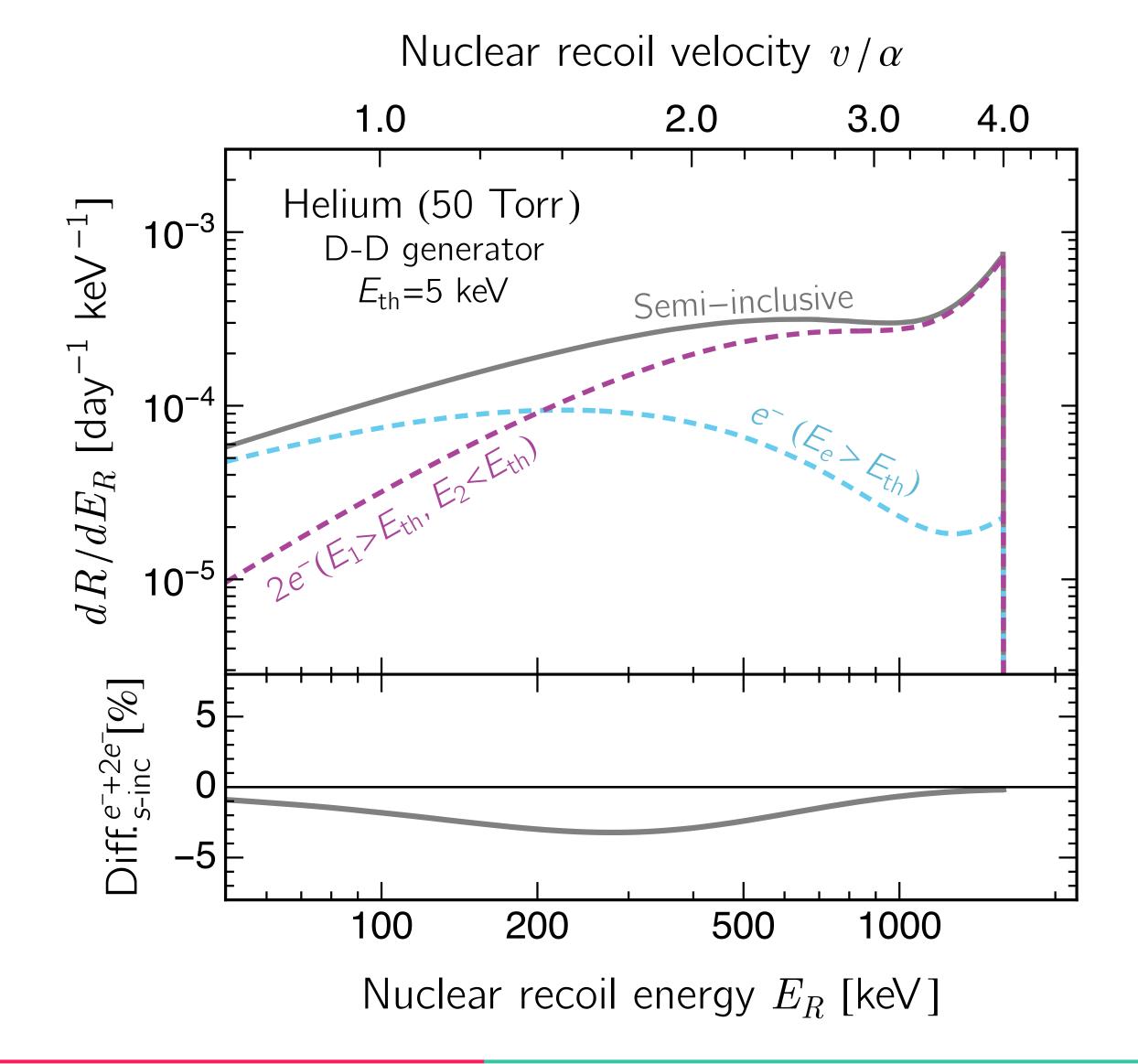
$$\exp\left(im_e\mathbf{v}\cdot\sum_{a=1}^N\mathbf{r}_a\right)\approx 1+im_e\mathbf{v}\cdot\sum_{a=1}^N\mathbf{r}_a$$

Impact: Our calculation remains valid at large nuclear speed (NR energy); and we can calculate single ionisation, double ionisation, single ionisation + excitation, ...

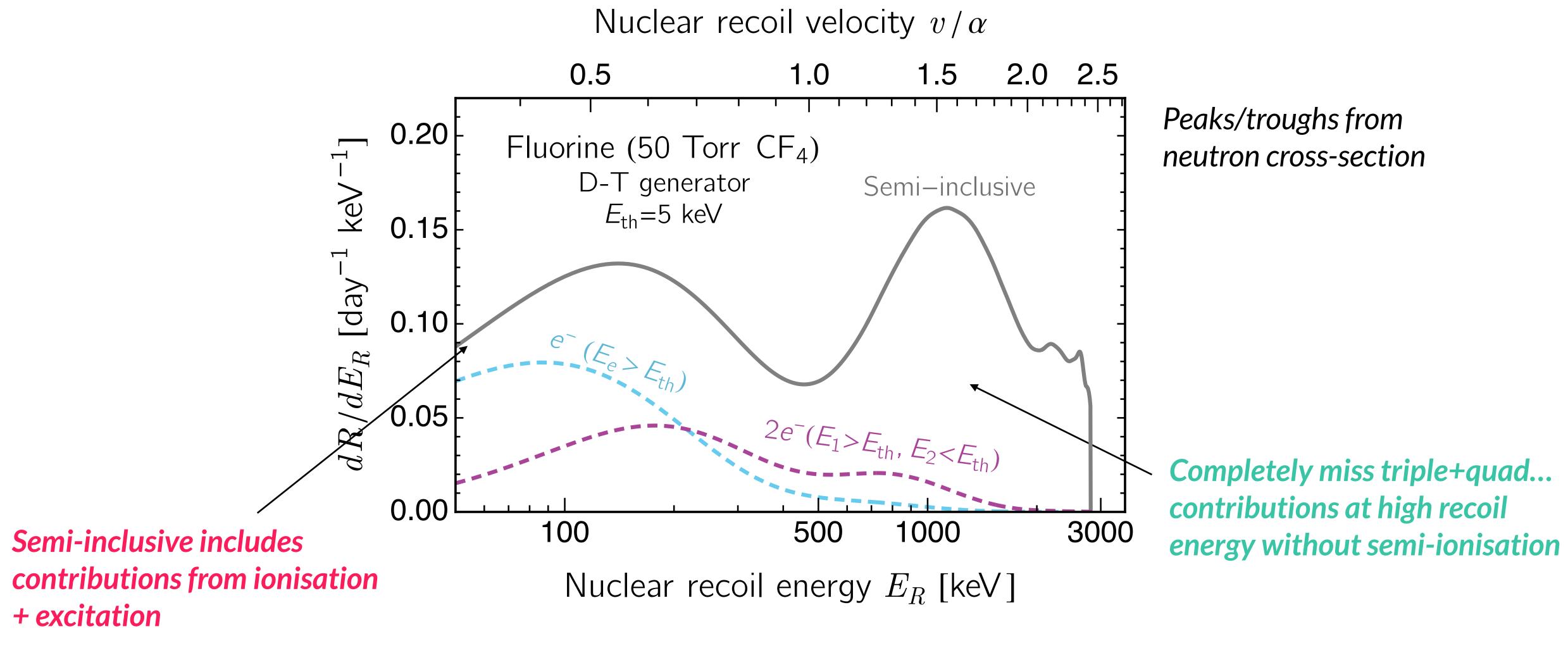
GRASP [Jonsson et al, Comput. Phys. Commun. 177, 597 (2007); Jonsson et al, Comput. Phys. Commun. 184, 2197 (2013); Froese Fischer et al, Comput. Phys. Commun. 237, 184 (2019)], RATIP [Fritzsche, Comput. Phys. Commun. 183, 1525 (2012)], BERTHA [Quiney et al, Adv. Quantum Chem. 32, 1 (1998)], FAC [Gu, Canadian Journal of Physics 86, 675 (2008)]

Helium neutron scattering rates (DD=2.47 MeV)

Helium provides a sanity check of semi-inclusive probability across all energies - it works!

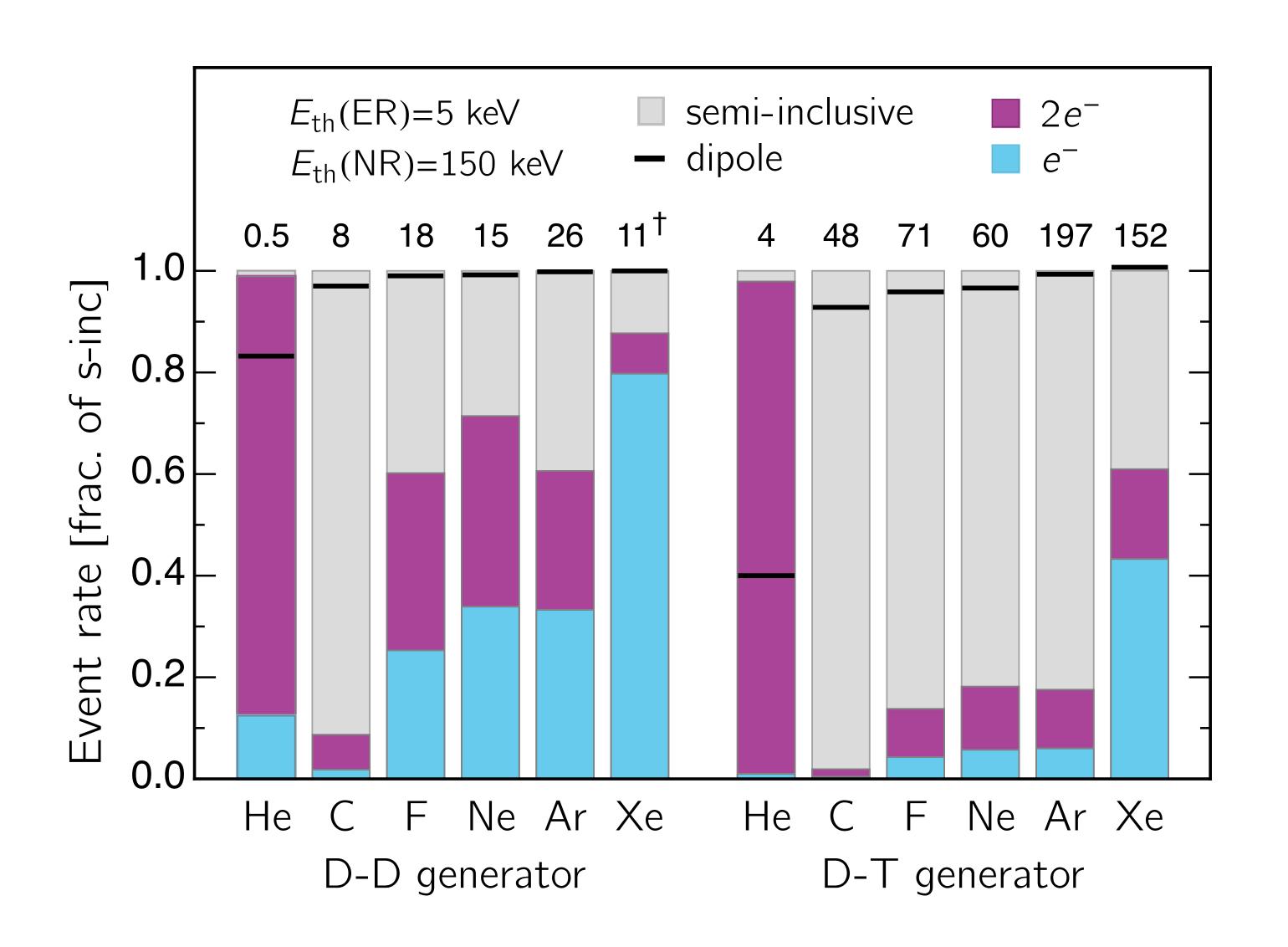


Fluorine neutron scattering rates (DT=14.7 MeV)



Semi-inclusive rate is factor 5.9 higher than single+double ionisation

Integrated rates

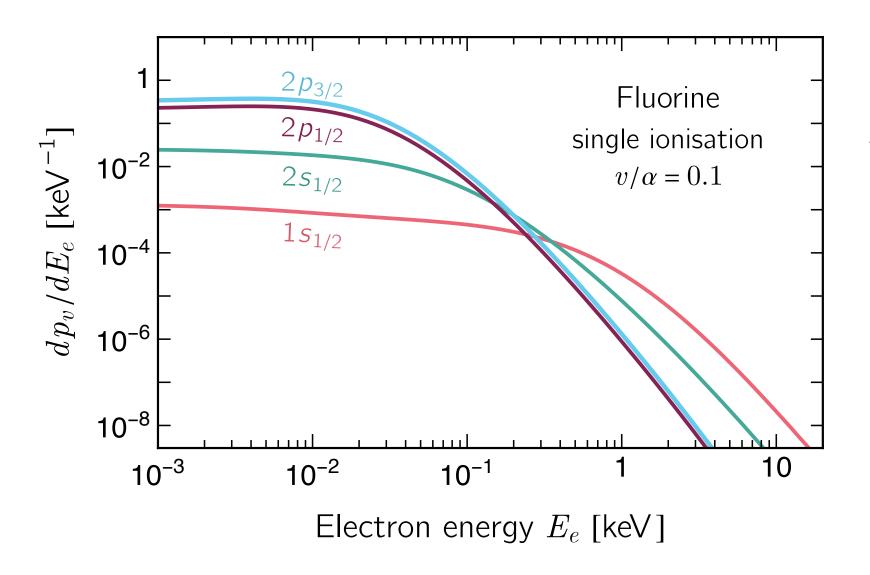


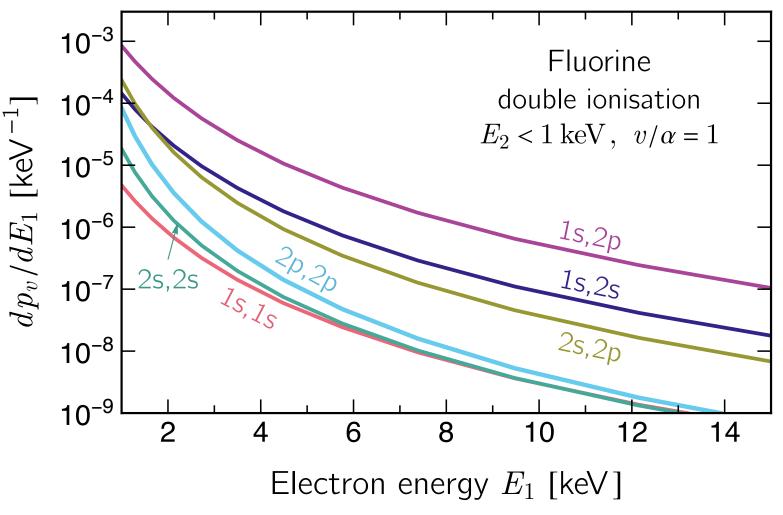
What shells are electrons ionised from?

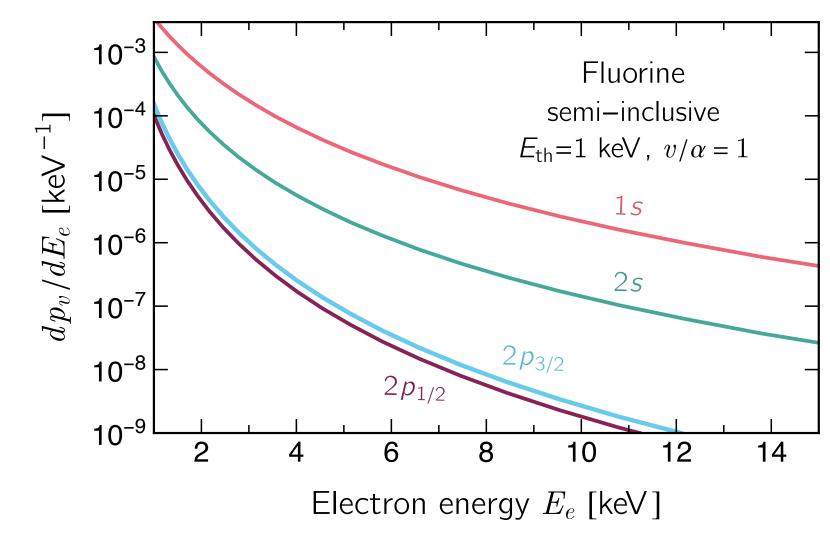
Most likely configuration for single-ionisation:
Hard electron from inner-shell
Soft-electron from valence-shell

Most likely configuration for ionisation scenario with 1 hard- and soft-electrons:

Hard-electron from inner-shell with soft-electron from valence-shell



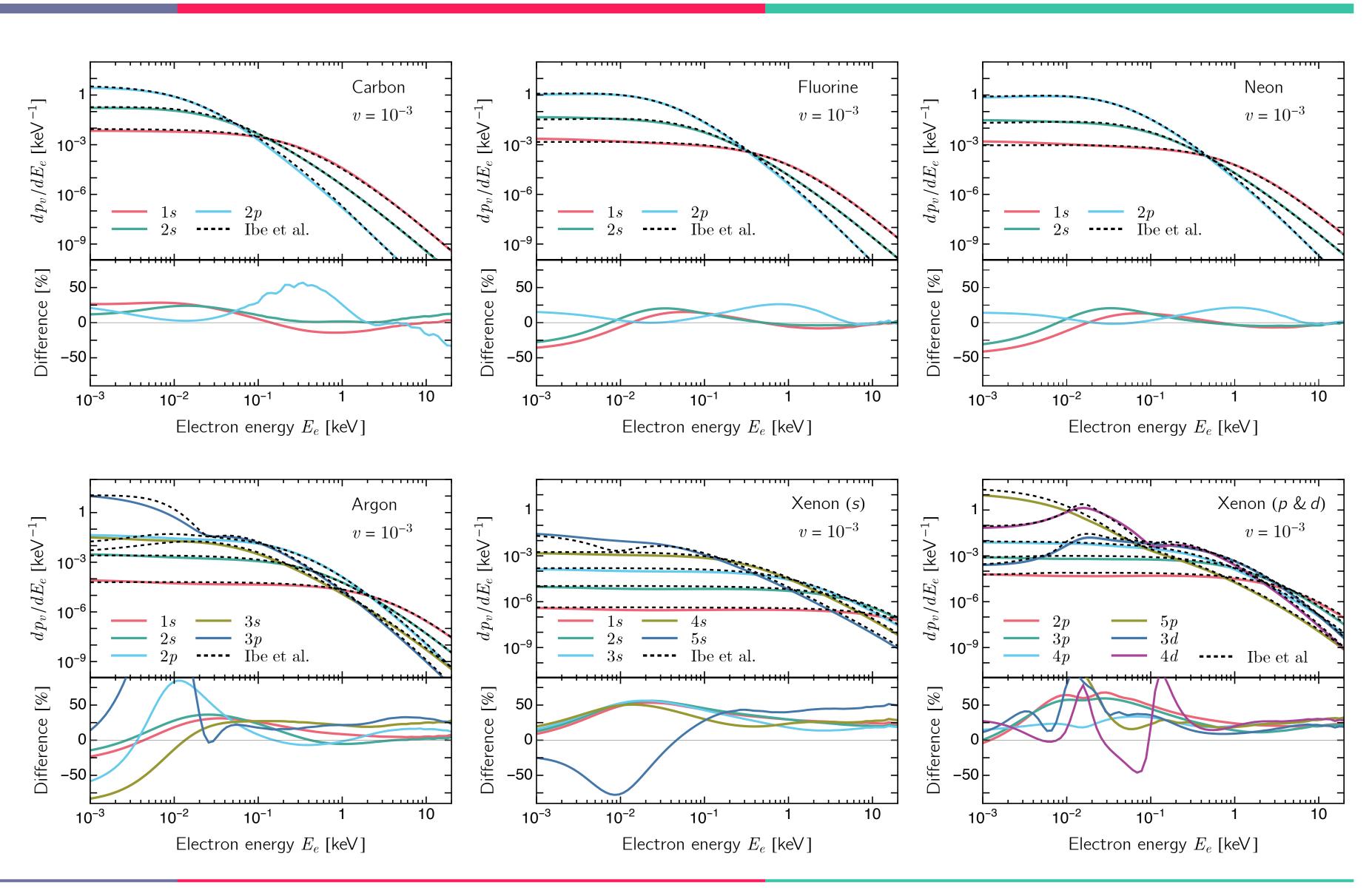




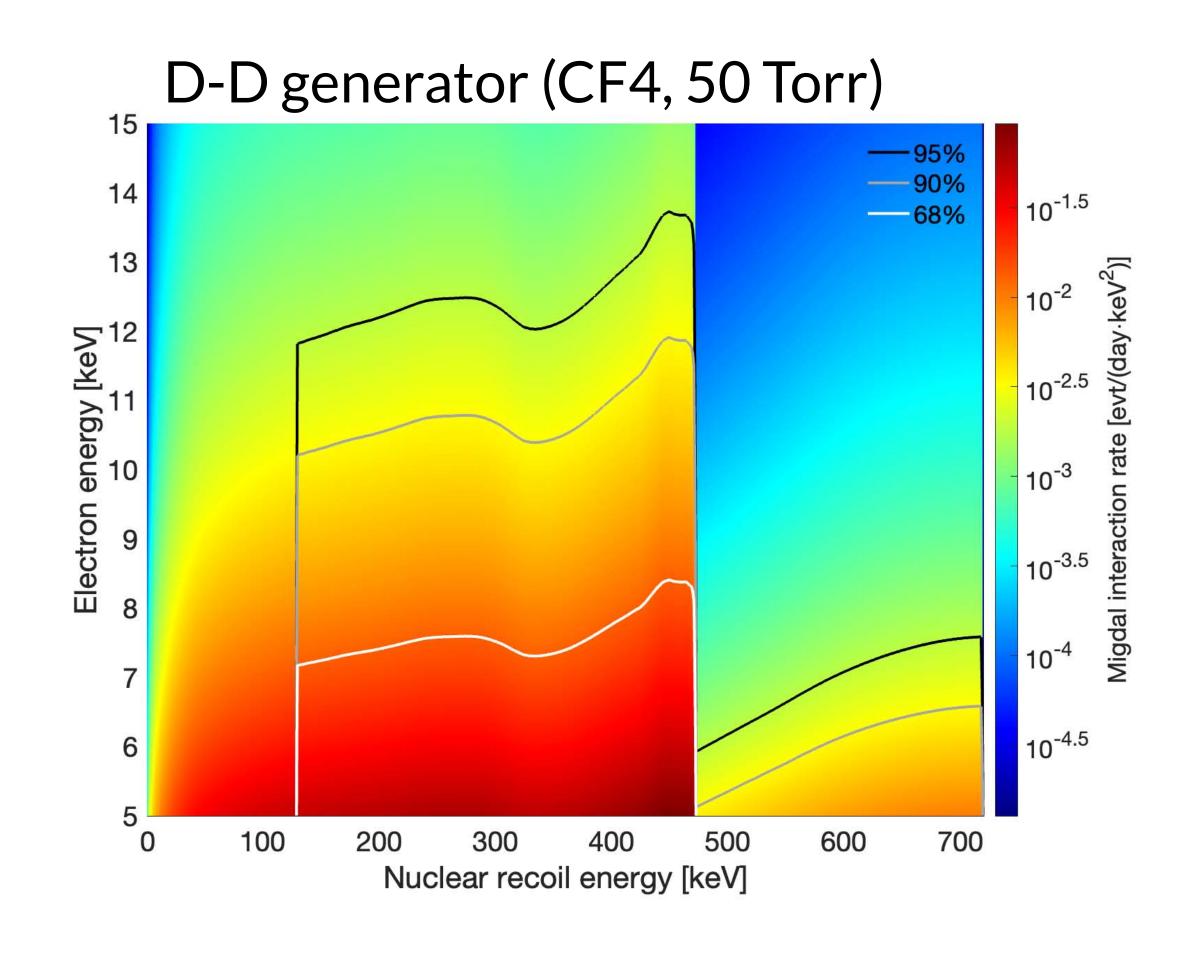
Comparison with Ibe et al

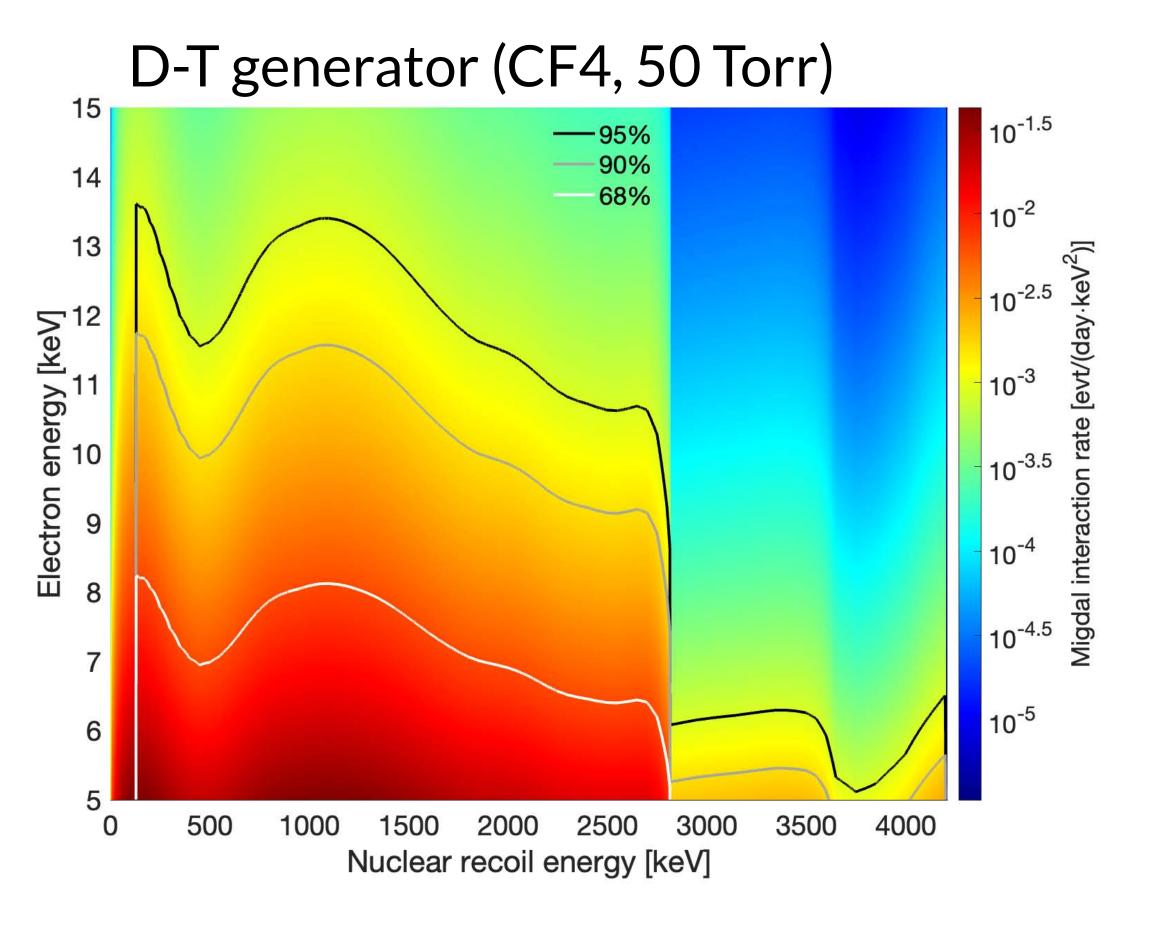
Comparison is at small v: when dipole approx is accurate

Agreement to ~25% in experimentally interesting parameter space



Event-rate map for MIGDAL experiment





Thresholds

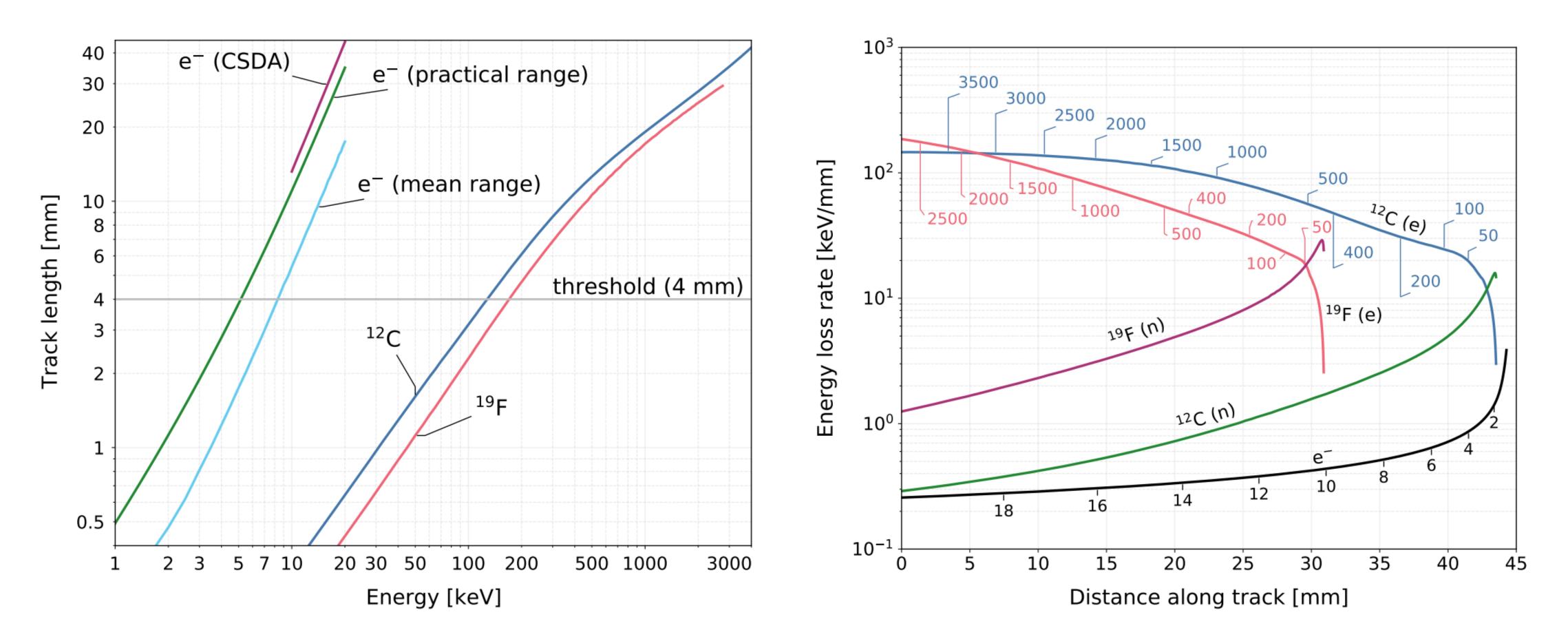


Figure 2: Left – Track length in CF₄ at 50 Torr for electrons (mean projected range calculated with Degrad [48], CSDA range with ESTAR [51], and the practical range formula from Ref. [52]), and mean projected range for carbon and fluorine ions from SRIM [49]). Right – Electronic and nuclear energy loss rates (CSDA) along carbon and fluorine ion tracks in CF₄ at 50 Torr, calculated with SRIM and electronic energy loss for 20 keV electrons obtained with ESTAR; called out values are interim particle energies (in keV) remaining at that point in the track.

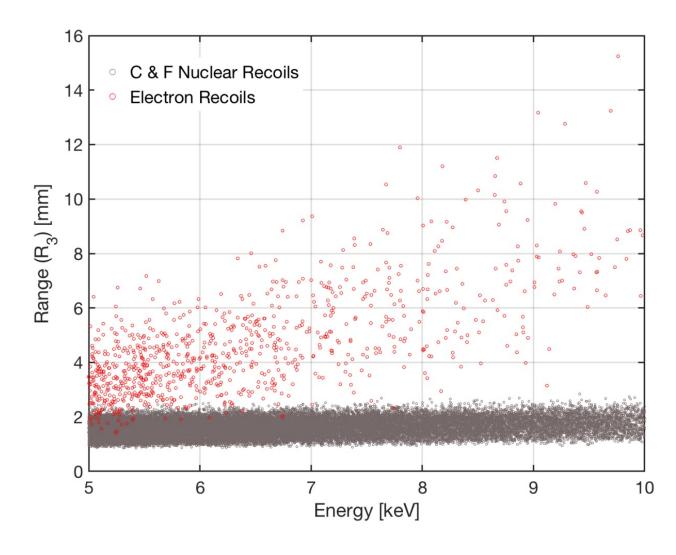
Backgrounds

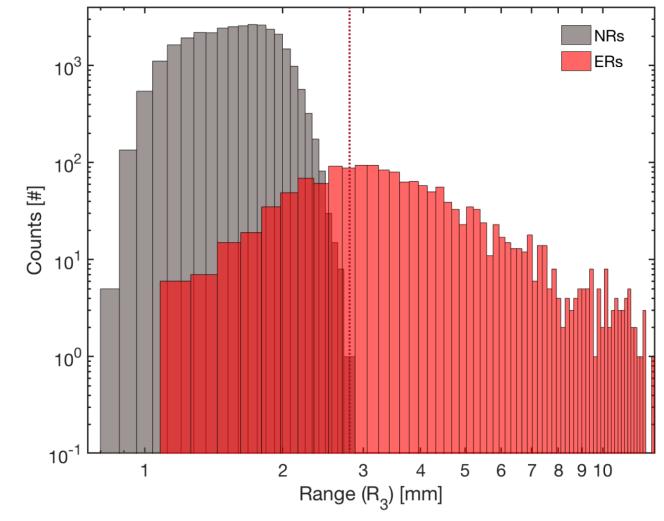
Secondary recoils per million primary ions (TRIM) created within 1 mm from the vertex in 50 Torr CF₄, when the "visible" energy of the secondary is 5–15 keVee.

| Primary ion | Secondary ion | | | |
|-----------------------|--------------------------|-------------------------|--|--|
| Fluorine | Fluorine | Carbon | | |
| 500 keV | 22,310 | 4,800 | | |
| 400 | 26,840 | 5,930 | | |
| 300 | 36,640 | 7,640 | | |
| 200 | 56,130 | 1,263 | | |
| 170 | 67,040 | 1,418 | | |
| | | | | |
| Carbon | Fluorine | Carbon | | |
| Carbon 500 keV | Fluorine 6,250 | 1,210 | | |
| | | | | |
| 500 keV | 6,250 | 1,210 | | |
| 500 keV 400 | 6,250 7,950 | 1,210 1,610 | | |
| 500 keV 400 300 | 6,250 7,950 11,380 | 1,210 1,610 2,310 | | |

~70,000
per million
(worst case)

How many of these look like 5-10 keV electrons? Simulate several thousand more tracks using full chain, analyse image and recover track lengths (R_3) Can cut down to ~1 per 70,000 secondaries, retaining 87% electron detection efficiency (i.e. ~1 per million primary recoils).





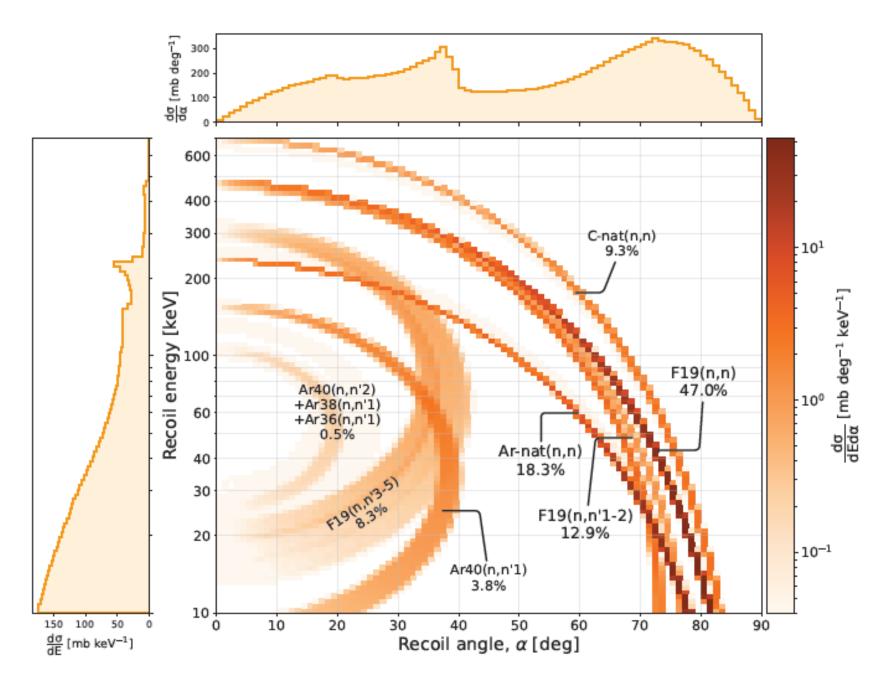
Migdal in other elements

Migdal probabilities in other elements of interest for DM searches which we aim to explore, mostly in mixtures with CF₄

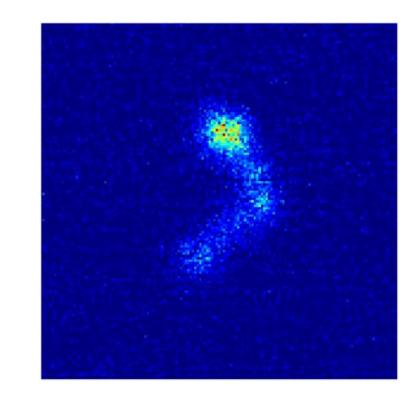
These probabilities are not too dissimilar (except for He)

Neutron scattering cross sections – total (σ_0) and bare-recoil processes (σ_s) plus Migdal probabilities for full neutron-induced NR spectrum, integrated down to zero NR threshold for electron thresholds of 0.5 keV and 5 keV (see C. McCabe's talk)

| | 2.47 MeV (D-D) | | | 14.7 MeV (D-T) | | | | |
|---------------------|-----------------|-----------------|-----------------------|--------------------------|-----------------|-----------------|--------------------------|--------------------------|
| | σ_0 , mb | σ_s , mb | P(>0.5 keV) | P(>5 keV) | σ_0 , mb | σ_s , mb | P(>0.5 keV) | P(>5 keV) |
| $^4{ m He}$ | 3,239 | 3,239 | 2.98×10^{-3} | 4.29×10^{-7} | 1,017 | 1,017 | 9.01×10^{-2} | 2.48×10^{-6} |
| $^{12}\mathrm{C}$ | 1,613 | 1,613 | 6.01×10^{-3} | 1.45×10^{-5} | 1,379 | 1,321 | 2.15×10^{-2} | 4.09×10^{-5} |
| $^{19}\mathrm{F}$ | 3,038 | 3,038 | 2.81×10^{-3} | 2.01×10^{-5} | 1,786 | 1,272 | 9.95×10^{-3} | 6.50×10^{-5} |
| $^{nat}\mathrm{Ne}$ | 2,474 | 2,465 | 2.62×10^{-3} | 2.32×10^{-5} | 1,677 | 1,055 | 8.50×10^{-3} | 6.89×10^{-5} |
| $^{nat}\mathrm{Si}$ | 3,111 | 3,111 | 2.39×10^{-3} | $2.87\!\times\! 10^{-5}$ | 1,725 | 1,150 | 1.10×10^{-2} | $1.25\!\times\! 10^{-4}$ |
| $^{40}\mathrm{Ar}$ | 5,050 | 5,050 | 2.18×10^{-3} | 2.92×10^{-5} | 2,818 | 2,754 | 6.85×10^{-3} | 8.94×10^{-5} |
| $^{nat}\mathrm{Ge}$ | 3,401 | 3,401 | 1.64×10^{-3} | 2.46×10^{-5} | 3,227 | 3,130 | 5.47×10^{-3} | 8.12×10^{-5} |
| $^{nat}{ m Kr}$ | 3,825 | 3,825 | 1.56×10^{-3} | $2.37\!\times\! 10^{-5}$ | 3,741 | 3,717 | $4.65\!\times\! 10^{-3}$ | $7.03\!\times\! 10^{-5}$ |
| $^{nat}\mathrm{Xe}$ | 5,760 | 5,760 | 7.31×10^{-4} | 1.55×10^{-5} | 4,871 | 4,861 | 2.80×10^{-3} | 5.95×10^{-5} |

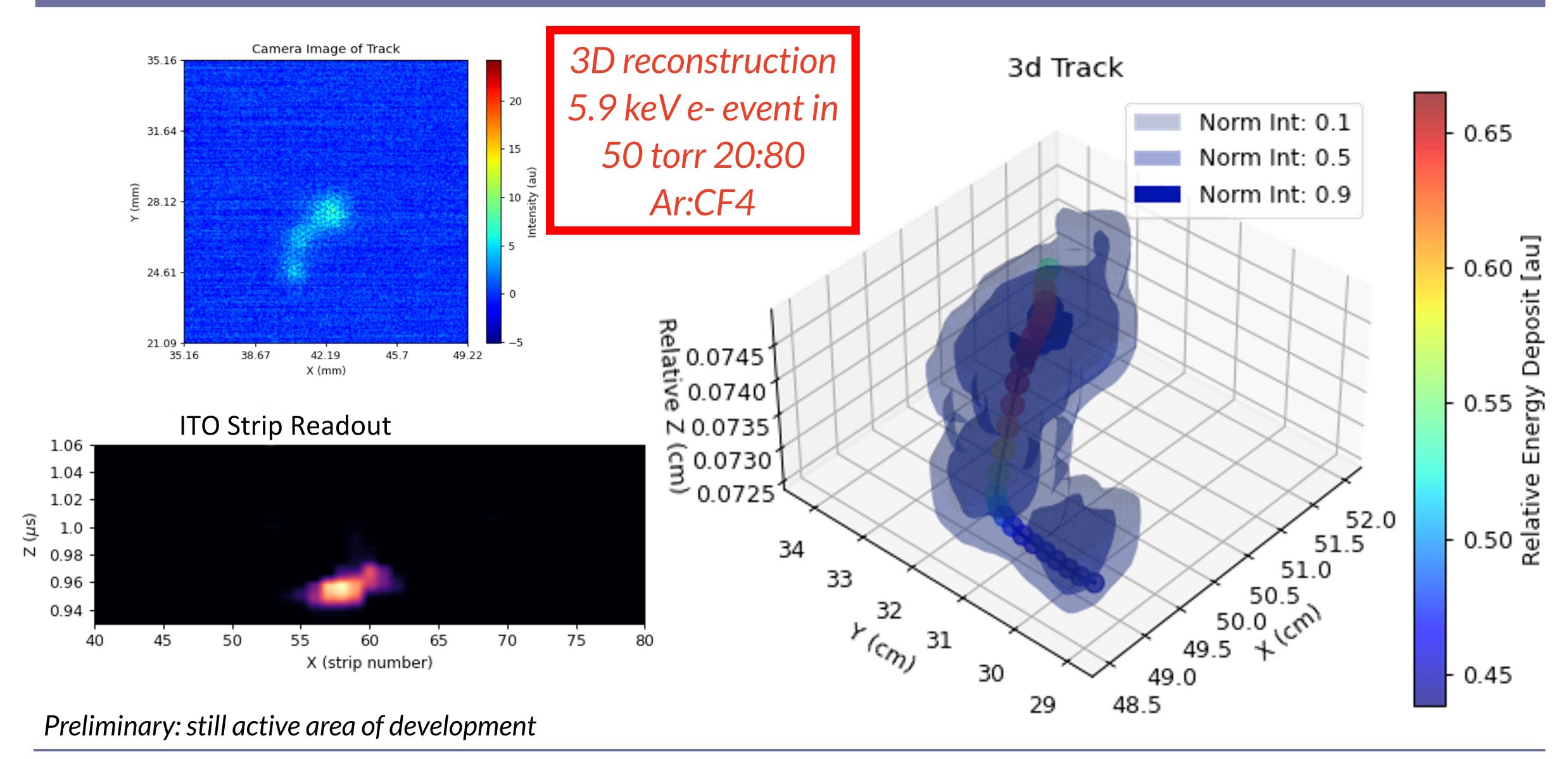


Energy-angle relations for D-D neutron scattering in 50% Ar/CF₄.



Blessing or curse?
Auger emission
in addition to
Migdal electron

Track reconstruction with real data



Neutron cross sections

