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SUPERFLUID HELIUM AS A TARGET FOR LIGHT DM PARTICLES

BASED ON:

ACANFORA, ESPOSITO, ADP, EPJC79 (2019) 7, 549
CAPUTO, ESPOSITO, ADP, PRD100 (2019) 11, 116007
CAPUTO, ESPOSITO, GEOFFRAY, ADP, SUN, PLB802 (2020) 11, 135258
CAPUTO, ESPOSITO, PICCININI, ADP, ROSSI, PRD103 (2021) 5, 055017

LANDAU ARGUMENT

Let $\boldsymbol{K'}$ be the rest frame of the superfluid, flowing with velocity $\boldsymbol{V}$ with respect to the capillary, which defines the frame $\boldsymbol{K}$.

Describe the effect of friction as the creation of an excitation of energy $\varepsilon = c_s p$ in $\boldsymbol{K'}$. This must correspond, in $\boldsymbol{K}$, to a negative energy variation (friction is expected to decrease the kinetic energy of the fluid in the capillary):

$$E = \underbrace{\varepsilon + \boldsymbol{V} \cdot \boldsymbol{p}}_{\Delta E < 0} + \frac{1}{2} M V^2$$

$$V > \frac{\epsilon}{p} = c_s$$

otherwise the flow is frictionless

$V \leq c_s \Rightarrow$ frictionless flow

LANDAU ARGUMENT

The energy of the superfluid flowing in pipe is $1/2\, MV^2$. Creating an excitation of momentum $\boldsymbol{p}$, the velocity of the superfluid mass decreases

$$MV = MV' + p$$

This is possible if energy energy is available to make the excitation

$$\frac{1}{2}MV^2 \geq \frac{1}{2}M(V')^2 + \varepsilon$$

Eliminating V' we are left with

$$V \geq \frac{\varepsilon}{p} + \frac{p}{2M} \geq \frac{\varepsilon}{p} \Rightarrow V \leq c_s \Rightarrow \text{frictionless flow}$$

PAUCITY OF EXCITATIONS

The main aspect of the physics of superfluidity is the paucity of gapless excitation. The number density of excitations is

$$n(E)\,dE \propto d^3k$$

Therefore

$$n(E) \propto \frac{k^2}{dE/dk} = \begin{cases} \sim k^2 \sim E^2 & \text{linear dispersion relation} \\ \sim k \sim \sqrt{E} & \text{quadratic dispersion relation} \end{cases}$$

So in the limit $E \to 0$ there are way *less* excitations in the first case. This qualitatively means that there are *few modes* that a superfluid flowing in a can can loose momentum to: frictionless flow.

SYSTEM OF REPELLING BOSONS

$$\mathscr{L} = -(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2, \qquad \lambda > 0$$

In the non relativistic limit, taking $\Phi(t, \mathbf{x}) = \dfrac{1}{\sqrt{2m}}\, e^{-imt}\, \varphi(t, \mathbf{x})$

and neglecting $\ddot{\varphi}$ and $\dot{\varphi}^2$

$$\mathscr{L} = i\varphi^\dagger \frac{\partial}{\partial t}\varphi - \frac{1}{2m}\nabla_i \varphi^\dagger \nabla_i \varphi - \frac{\lambda}{4m^2}(\varphi^\dagger \varphi)^2$$

Require a finite density ($[\varphi] = E^{3/2}$)

$$\mathscr{L} = i\varphi^\dagger \frac{\partial}{\partial t}\varphi - \frac{1}{2m}\nabla_i \varphi^\dagger \nabla_i \varphi - \frac{\lambda}{4m^2}(\varphi^\dagger \varphi - \bar{\rho})^2$$

In such a way the potential term (bottle bottom) forces $\varphi \sim \bar{\rho}^{1/2}$

FINITE DENSITY

Since φ is complex, in presence of finite density we can use the representation $\varphi(x) = \sqrt{\rho(x)}\, e^{i\theta(x)}$, where ρ is the #density

$$\mathscr{L} = \underbrace{\frac{i}{2}\frac{d}{dt}\rho}_{\text{total der.}} - \rho\frac{\partial}{\partial t}\theta - \frac{1}{2m}\left(\rho(\nabla_i\theta)^2 + \frac{1}{4\rho}(\nabla_i\rho)^2\right) - \frac{\lambda}{4m^2}(\rho - \bar{\rho})^2$$

$$\frac{\partial\mathscr{L}}{\partial\dot{\theta}} = -\rho \text{ can. conjugate of } \theta$$

An isolated system w/ a definite number of particles does not have a definite phase θ and viceversa. If two condensates come in contact and exchange particles, then they have a definite relative phase: *phase coherence over space giving interference fringes (with space period h/p_{rel})*

FINITE DENSITY

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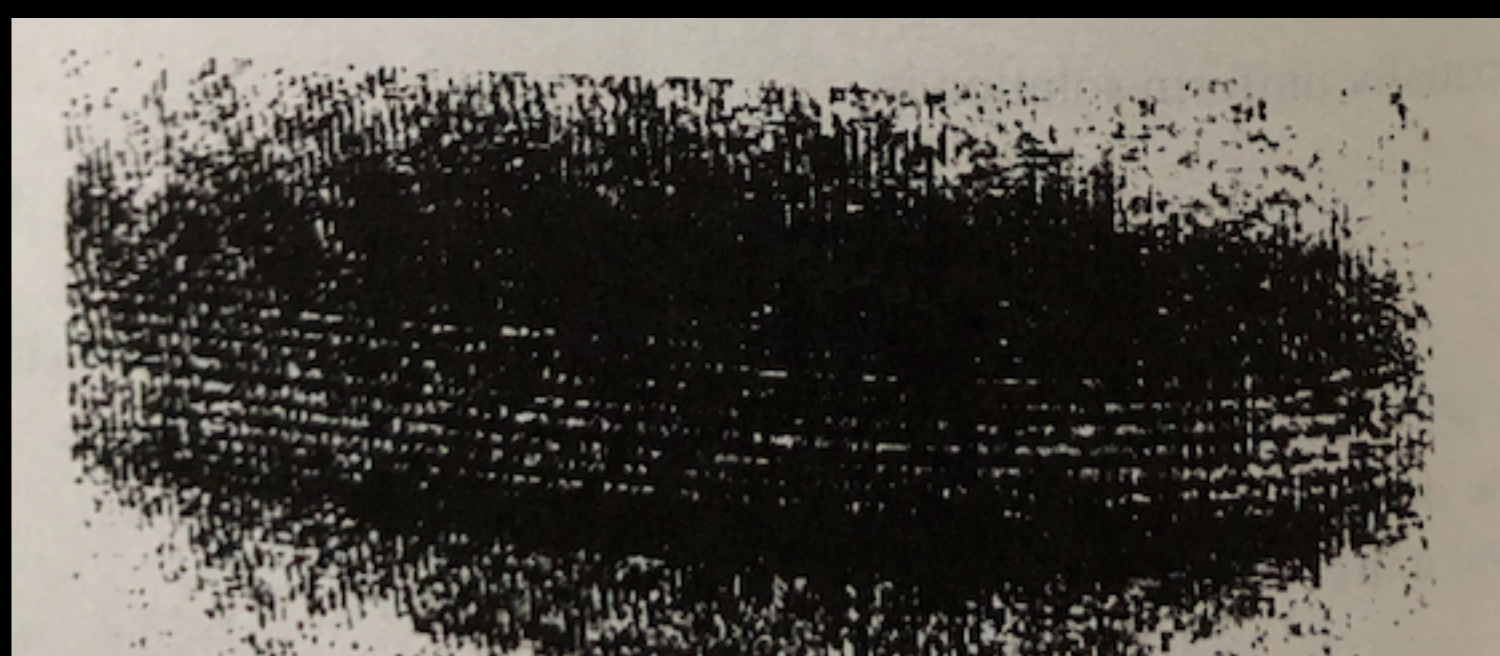


Ketterle at MIT

DENSITY FLUCTUATIONS

The next step is to introduce $\sqrt{\rho(x)} = \sqrt{\bar\rho} + \eta(x)$

$$\mathscr{L} = -\frac{\bar\rho}{2m}(\nabla_i\theta)^2 \underbrace{-\eta\left(\frac{\lambda}{m^2}\bar\rho - \frac{1}{2m}\nabla_i^2\right)\eta}_{-\frac{1}{2}\eta\cdot K\cdot\eta} \underbrace{-\eta\, 2\sqrt{\bar\rho}\,\partial_0\theta}_{\eta\cdot J}$$

do the Gaussian integral on the fluctuation $\boldsymbol{\eta(x)}$ – integrate out small fluctuation in the density

$$\mathscr{L} = -\frac{\bar\rho}{2m}(\nabla_i\theta)^2 + \underbrace{\bar\rho\,(\partial_0\theta)\left(\frac{1}{\frac{\lambda}{m^2}\bar\rho - \cancel{\frac{1}{2m}\nabla_i^2}}\right)(\partial_0\theta)}_{+\frac{1}{2}J\cdot K^{-1}\cdot J} + \ldots$$

and consider small wave-numbers $k \ll \sqrt{\frac{2\lambda\bar\rho}{m}}$, very long wavelength

THE LINEAR DISPERSION FROM QFT

$$\boxed{\mathscr{L} = \frac{(\partial_0\theta)^2}{\lambda/m^2} - \frac{\bar{\rho}}{2m}(\nabla_i\theta)^2}$$

from this write the equation of motion

$$\frac{2}{\lambda/m^2}\partial_0^2\theta - \frac{\bar{\rho}}{m}\partial^2\theta = 0$$

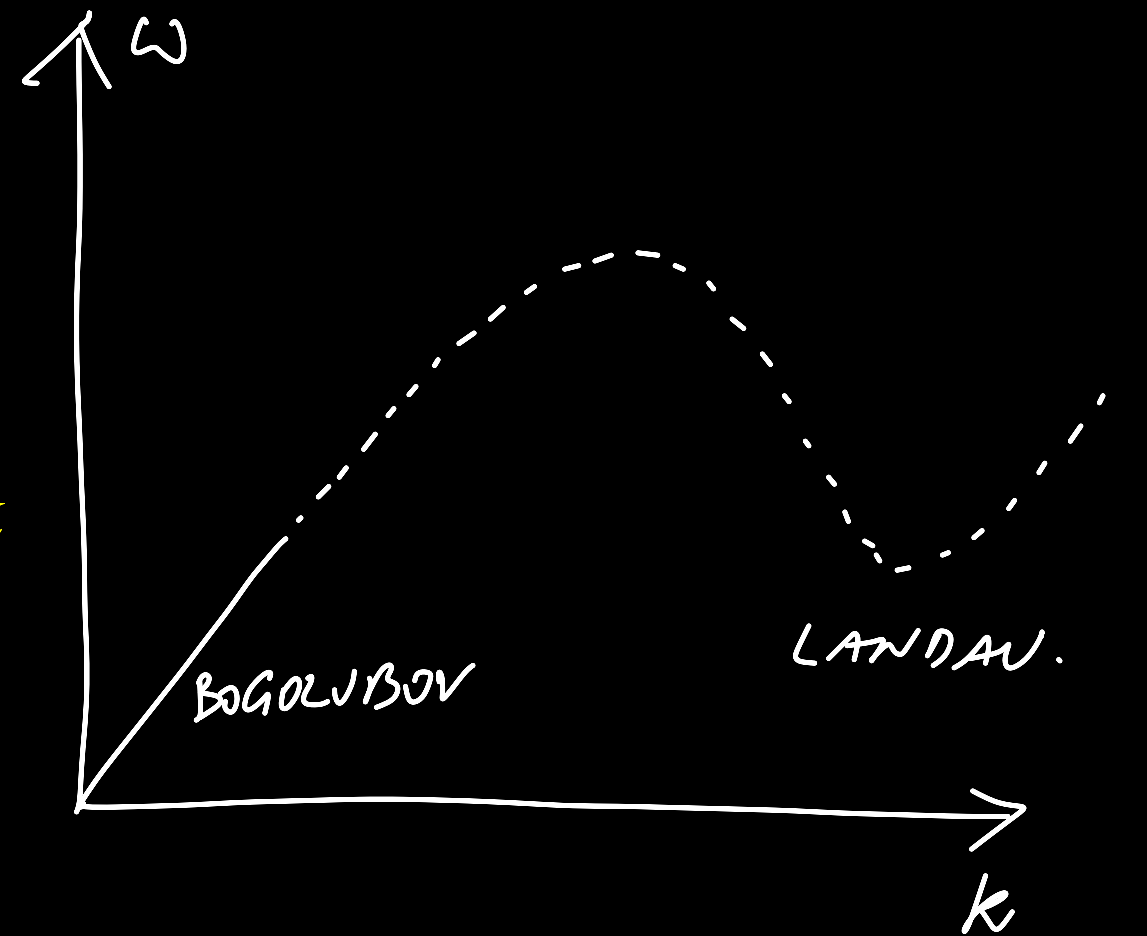
and seek a solution in the form $\theta(x) \sim e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$ getting

$$-\frac{2\omega^2 m^2}{\lambda} + \frac{\bar{\rho}k^2}{m} = 0$$

or

$$\omega = \underbrace{\sqrt{\frac{\lambda\bar{\rho}}{2m^3}}}_{c_s}\, k$$ "gapless mode"

$$\omega = \underbrace{\sqrt{\frac{\lambda\bar{\rho}}{2m^3}}}_{c_s}\, k$$



The sound velocity grows with repulsion and density. In superfluid helium it is $c_s = 248$ m/s.

BACK TO THE RELATIVISTIC FORMALISM

In the previous Lagrangian scale/adjust the $\boldsymbol{x_\mu}$ coordinates to write

$$\mathscr{L} = \frac{(\partial_0\theta)^2}{\lambda/m^2} - \frac{1}{2m/\bar{\rho}}(\nabla_i\theta)^2 = \frac{m^2}{\lambda}((\partial_0\theta)^2 - c_s^2(\nabla_i\theta)^2)$$

$$t \to c_s t$$

$$\mathscr{L} = -\frac{(c_s m)^2}{\lambda}(\partial_\mu\theta\,\partial^\mu\theta) = -\xi^2\,(\partial_\mu\theta\,\partial^\mu\theta)$$

Consider the case $\theta(x) = \theta(t)$. In place of $\boldsymbol{\theta}$ we may use the new phase $\psi = m\,t \simeq \mu\,t$ which represents a family of solutions of the equations of motion; $\mathscr{L} = -\,\partial_\mu\psi\,\partial^\mu\psi$.

SSB

The relativistic and non-relativistic lagrangians we discussed so far have a U(1)-global symmetry which, in $\mathscr{L} = -\xi^2 (\partial_\mu \theta \, \partial^\mu \theta)$, is represented by translations $\theta(x) \rightarrow \theta(x) + \alpha$.

This internal symmetry is SB: in the condensate $\langle N \rangle \neq 0$, at finite density.

CHEMICAL POTENTIAL

At finite density it is customary to look for the ground state as the state which minimizes $\widetilde{H} \equiv H - \mu N$ where N is the U(1) conserved charge/particle number and μ a Lagrange multiplier, which can be interpreted as chemical potential.

Heisenberg operators evolve in time with H, the true Hamiltonian.

The internal symmetry generated by N is spontaneously broken. The vacuum we are seeking is $\widetilde{H}\,|0\rangle = 0$ so can't be eigenstate neither of N nor of H: *if N is spontaneously broken also H is.*

LOCAL CHEMICAL POTENTIAL

The implementation of a chemical potential goes through the introduction of a time-dependent phase in the U(1) direction.

$$\boxed{\psi(x) = \mu\, t + c_s\, \mu\, \frac{1}{\sqrt{\mu \bar{\rho}}}\, \pi(x)}$$

$$X \equiv \sqrt{-\partial_\mu \psi\, \partial^\mu \psi}$$

In place of $\mathscr{L}(x)$ use $P(X) - P$ from pressure. In the absence fluctuations, $X = \mu$, and $P(\mu)$ defines the equation of state.
($[\pi] = E, \;\; [\psi] = [\theta] = E^0, \;\; [X] = E$)

A. Nicolis, 1108.2513 [hep-th]

A. Nicolis, R. Penco, F. Piazza, R. Rattazzi, JHEP (2015)

A. Nicolis, R. Penco, Phys. Rev. B97, 134516 (2018)

EXPANSION OF $P(X)$ UP TO $\pi(x)^3$

$$\boxed{\mathscr{L}_{\text{He}} = \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\nabla\pi)^2 + \frac{g_1}{2}\,\dot{\pi}(\nabla\pi)^2 + \frac{g_2}{3!}\,\dot{\pi}^3}$$

The last two terms vanish for $q_\mu \to 0$

$$g_1 = -\sqrt{\frac{m_{\text{He}}c_s^2}{\bar{\rho}}}\frac{1}{m_{\text{He}}} \qquad g_2 = \left(\frac{m_{\text{He}}c_s^2}{\bar{\rho}}\right)^{3/2}\bar{\rho}''(\mu)$$

$$P'(\mu) = \bar{\rho}(\mu) \qquad [P'] = E^3 \qquad [P''] = E^2 \qquad [P'''] = E$$

$$c_s^2 = \frac{P'(\mu)}{\mu P''(\mu)};$$

In the NR limit $\mu \simeq m_{\text{He}}$

RANGE OF VALIDITY OF EFT

The linear dispersion relation featured by the field theory holds true for phonons with momenta $|\boldsymbol{p}| < 1$ keV, which means $\boldsymbol{\varepsilon} < 1$ meV. Above this point linearity is lost: the EFT needs higher dimensional operators and predictive power decreases.

DM COUPLING TO HELIUM

Introduce a mediator ϕ assuming spin independent interactions

$$\mathscr{L}_I = g_\chi m_\chi \phi |\chi|^2 + g_{\rm He}\, \phi\, \rho$$

where ρ is the Helium number density, acquiring a vev $\bar{\rho}$ in the condensate. In its passage through helium the DM particle has an effective mass lighter than the vacuum value m_χ

$$m_\chi^2 - \underbrace{\left(\frac{g_\chi g_{\rm He}}{m_\phi^2}\right)}_{G_\chi} m_\chi \underbrace{\bar{\rho}(\mu)}_{P'(\mu)} \equiv m_\chi^2(\mu)$$

where $\mu \simeq m_{\rm He}$ in the NR limit.

DM COUPLING TO HELIUM

The floating effect of DM in its passage in Helium, is translated into an effective mass in

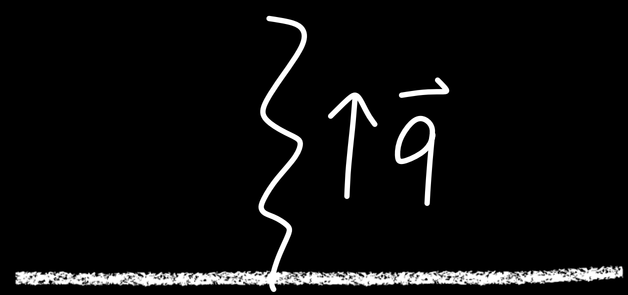
$$\mathscr{L}_I = -|\partial\chi|^2 - m^2(X)|\chi|^2$$

where we have promoted $\boldsymbol{\mu}$ to the local chemical potential $\boldsymbol{X}$, and an expansion in powers of $\boldsymbol{\pi(x)}$ leads to

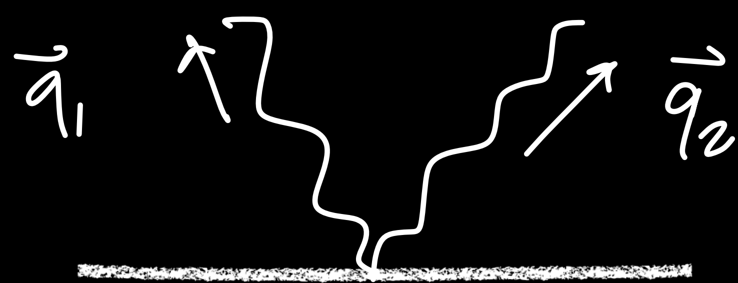
$$\boxed{\mathscr{L}_I = \mathscr{G}_1\,\dot{\pi}|\chi|^2 + \frac{\mathscr{G}_2}{2}(\nabla\pi)^2|\chi|^2 + \frac{\mathscr{G}_3}{2}\dot{\pi}^2|\chi|^2}$$

$$\mathscr{G}_1 \simeq G_\chi m_\chi \sqrt{\frac{\bar{n}}{m_{\mathrm{He}} c_s^2}} \qquad \mathscr{G}_2 = -G_\chi m_\chi \frac{1}{m_{\mathrm{He}}} \qquad \mathscr{G}_3 = G_\chi m_\chi \frac{m_{\mathrm{He}} c_s^2}{\bar{\rho}} \bar{\rho}''$$

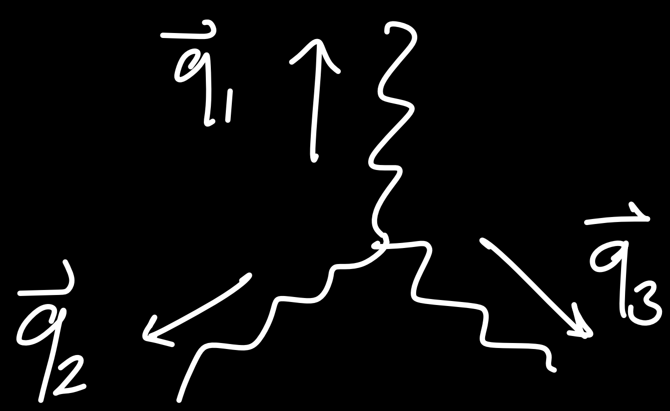
FEYNMAN RULES



$$i\mathscr{G}_1\,\omega$$



$$-(\mathscr{G}_3\,\omega_1\omega_2 + \mathscr{G}_2\,\mathbf{q}_1\cdot\mathbf{q}_2)$$



$$-ig_1\big(\underbrace{\omega_1\,\mathbf{q}_2\cdot\mathbf{q}_3}_{(123)} + (213) + (312)\big) - ig_2\,\omega_1\omega_2\omega_3$$

FEYNMAN RULES

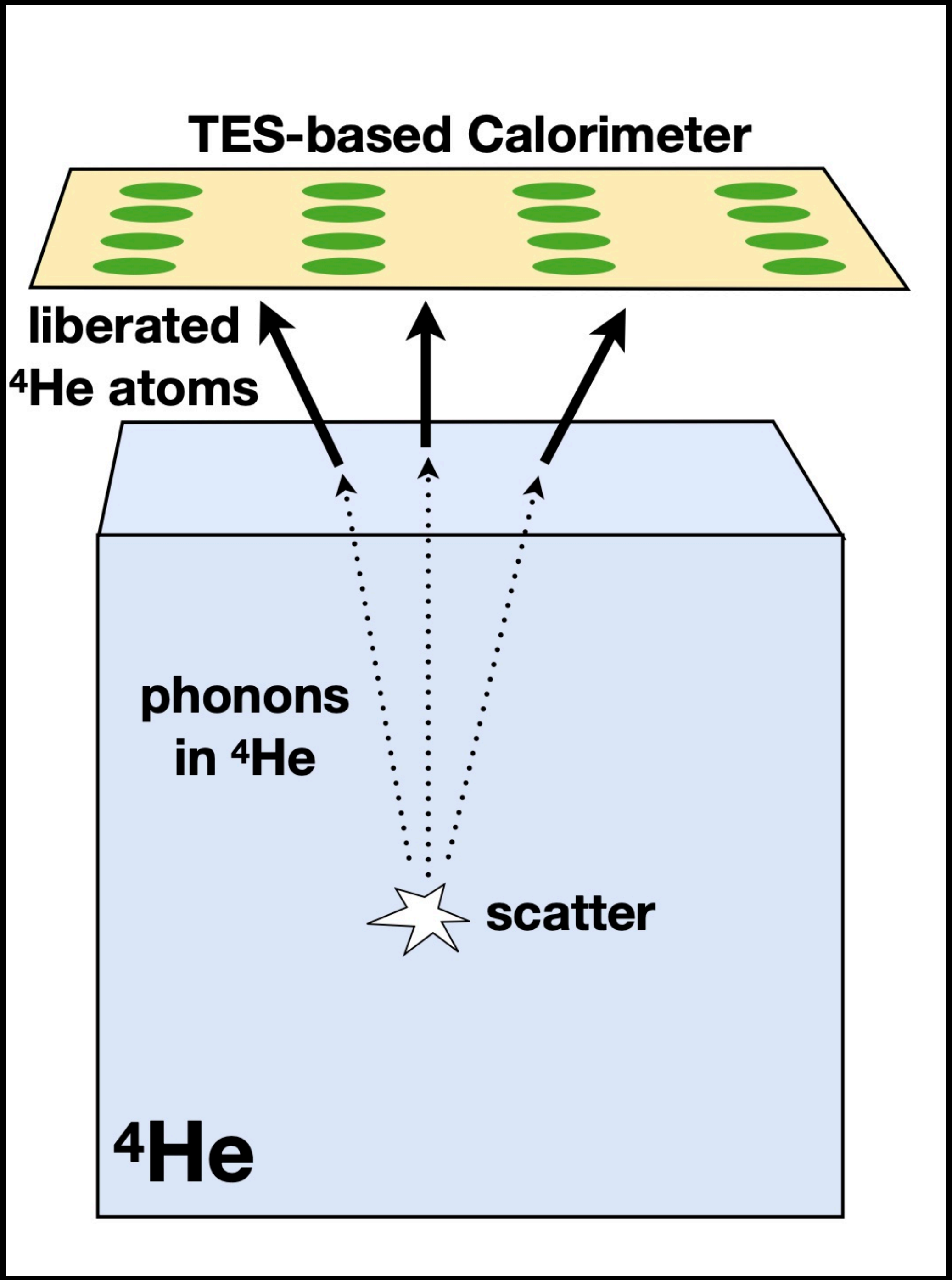
$$\Delta_\pi = \frac{i}{\omega^2 - c_s^2 \mathbf{q}^2 + i\epsilon}$$

phonon propagator

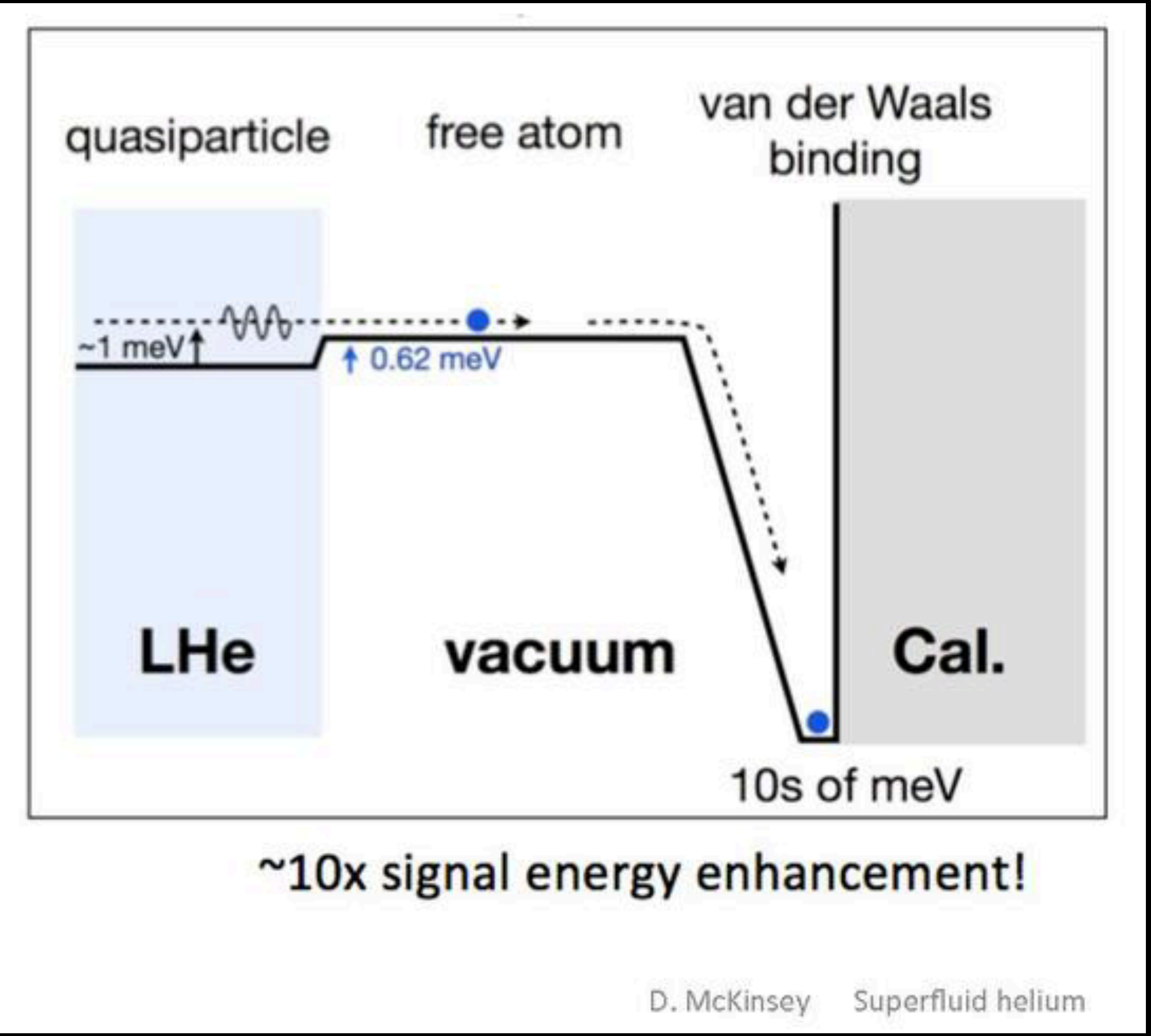
We will require that momenta flowing in $\boldsymbol{\Delta_\pi}$ do not exceed the cutoff of the EFT, $|\mathbf{q}| < 1$ keV – this removes the $\theta_{12} = 0$ divergence.

ACOUSTIC SCINTILLATORS

Quantum evaporation – angle wrt normal to the surface $\lesssim 25^\circ$



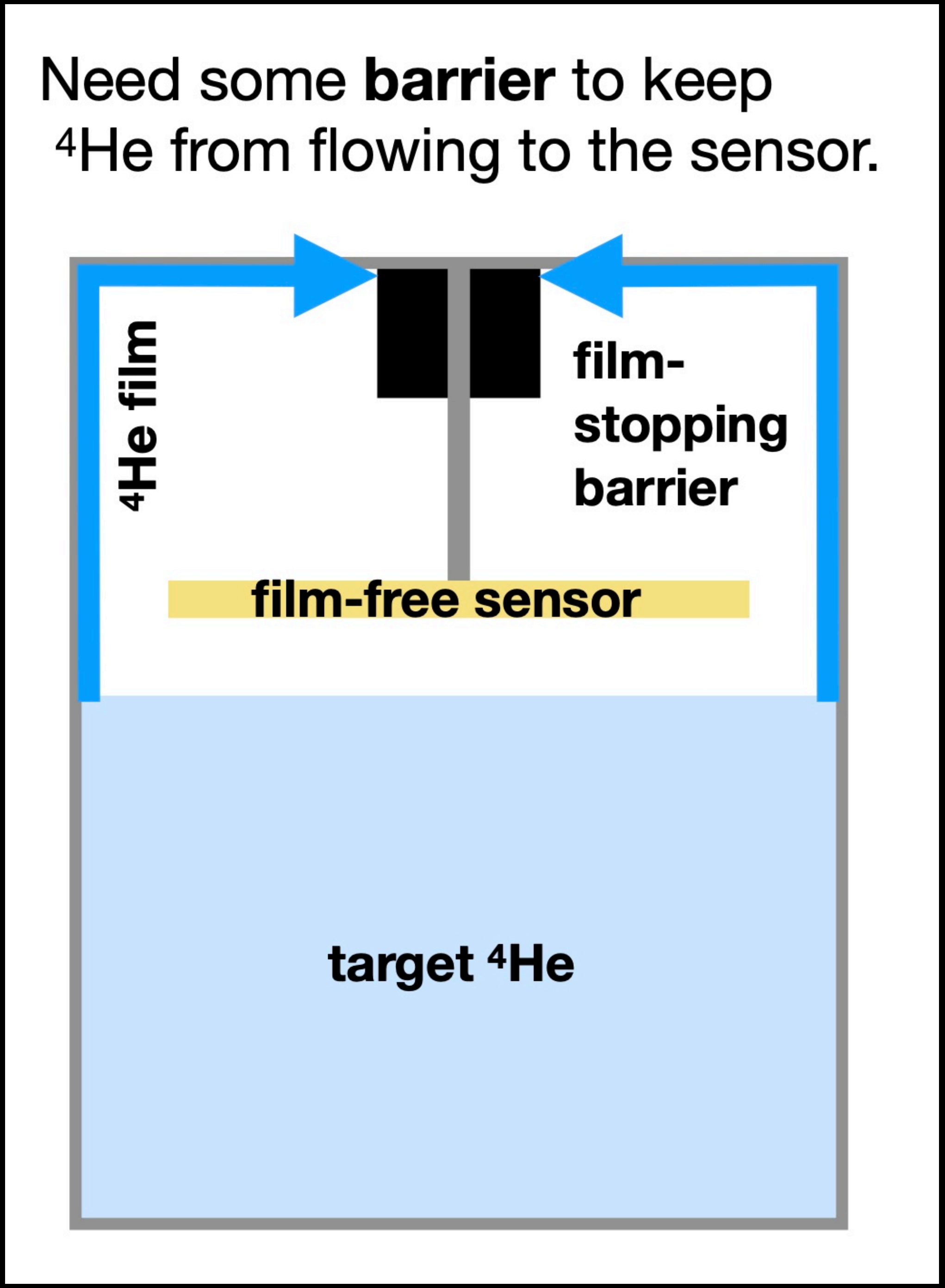
ACOUSTIC SCINTILLATORS



Helium sticks more strongly to any surface than it does to itself

ACOUSTIC SCINTILLATORS

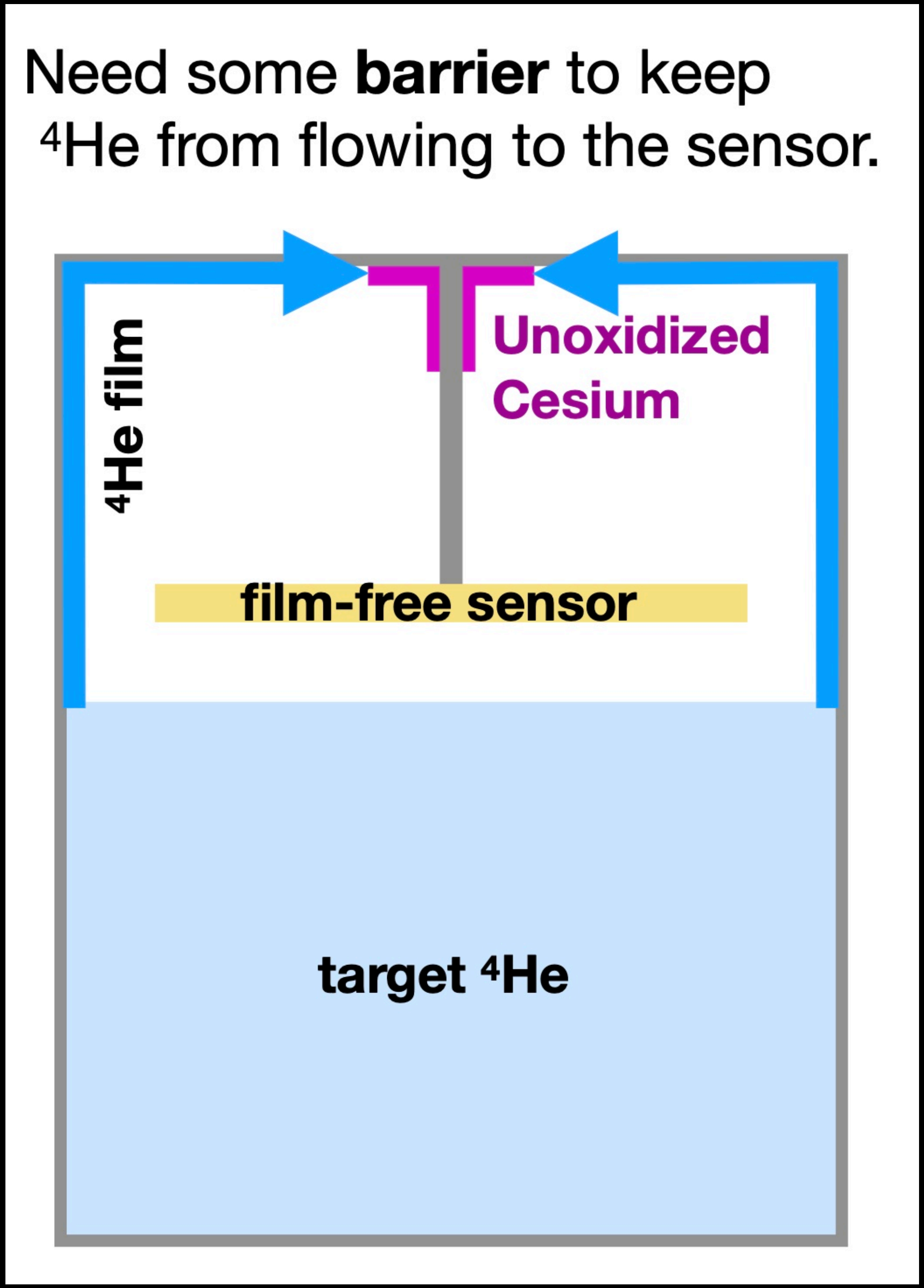
If helium reaches it, the sensor is gone.



See S. Hertel et al. arXiv:2307.11877

ACOUSTIC SCINTILLATORS

There are very few surfaces that superfluid He does not wet.



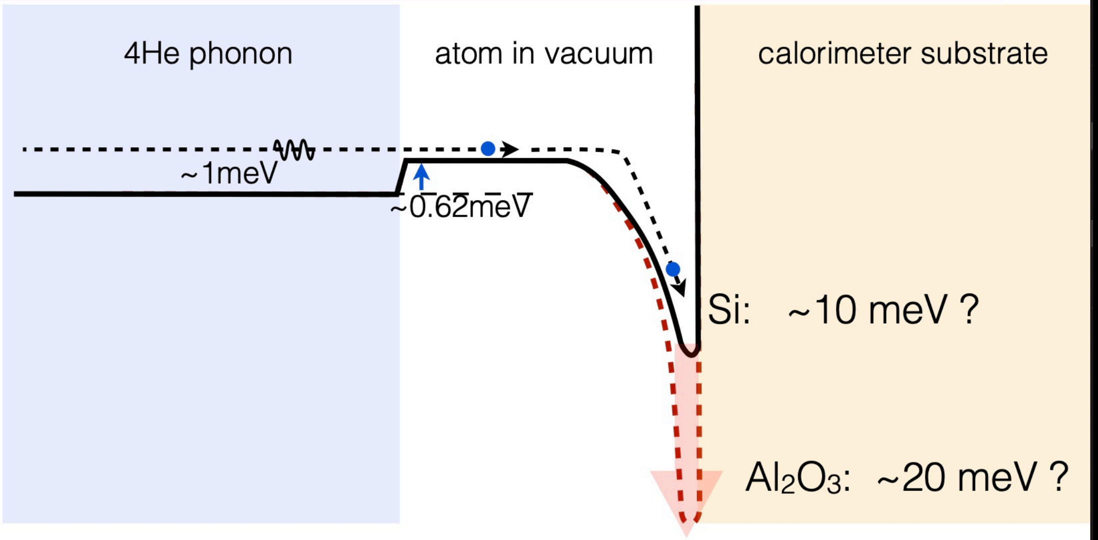
See S. Hertel et al. arXiv:2307.11877

Next Steps: Pushing the Gain

1. Higher van der Waals gain per atom

- Depends on the calorimeter surface
- ~10meV/atom is *typical* of many surfaces
- Expect higher energies from polar lattices/surfaces

→ Near-term plan to test Al_2O_3 calorimeter
Expect 20-30meV/atom (based on condensed matter sim)
(Improvement by factor of a few)



See S. Hertel @ TAUP 23

ONE-PHONON EMISSION

The recoil $\boldsymbol{q}$ is transformed into a phonon of energy $\boldsymbol{c_s q}$

$$m_\chi + \frac{\boldsymbol{p}_\chi^2}{2m_\chi} = m_\chi + \frac{(p'_\chi)^2}{2m_\chi} + c_s q$$

$$\mathbf{p}_\chi = \mathbf{p}'_\chi + \mathbf{q}$$

Let $\boldsymbol{\alpha}$ be the angle between $\boldsymbol{p}_\chi$ and $\boldsymbol{q}$. Since $\boldsymbol{p_\chi = m_\chi v_\chi}$ we have

$$\cos\alpha = \frac{q}{2m_\chi v_\chi} + \frac{c_s}{v_\chi}$$

$$\varepsilon_{\max} = c_s q_{\max} = 2m_\chi c_s (v_\chi - c_s) \simeq 2m_\chi c_s v_\chi \simeq 2m_\chi \times 10^{-9} > 0.62 \text{ meV}$$

need $\boldsymbol{m_\chi \approx 300}$ keV

TWO PHONON EMISSION

In calorimetric techniques, the minimum readable energy deposit is estimated to be 1 meV. By *stopping* the DM particle in medium

$$\frac{1}{2}m_\chi \times 10^{-6} \geq 1 \text{ meV}$$

or

$$m_\chi \geq 1 \text{ keV}$$

In principle we can probe dark matter masses much lower than 300 keV, and this can be done, not stopping the DM particle, but considering two-phonon ($\boldsymbol{q}_1, \boldsymbol{q}_2$) emission processes. Kinematically

$$|\mathbf{q}_1 + \mathbf{q}_2| \lesssim 2m_\chi v_\chi$$

Consider $m_\chi = 1$ keV so that (way below the EFT cutoff)

$$|\mathbf{q}_1 + \mathbf{q}_2| \simeq 2 \text{ eV}$$

TWO-PHONON EMISSION

Consider $m_\chi = 1$ keV so that $|\mathbf{q}_1 + \mathbf{q}_2| \simeq 2$ eV. On the other hand we need $c_s q_1 + c_s q_2 \simeq 10^{-6} \times (q_1 + q_2) \geq 1$ meV, so that must be $q_{1,2} \approx 1$ keV: can be done w/ *back-to-back* 3-momenta $\mathbf{q}_1, \mathbf{q}_2$.

The presence of the medium breaks boost invariance and the rate must be computed directly in the LAB frame.

TWO-PHONON EMISSION

$$\Gamma(\chi \to \chi + 2\pi) = \frac{1}{8(2\pi)^4 c_s^5 E} \int_{\mathscr{R}} \frac{|\mathscr{M}|^2}{\sqrt{1-\mathscr{A}^2}} \frac{\omega_2}{p_\chi} \, d\omega_1 \, d\omega_2 \, d\theta_{12} \, d\theta_2$$

$$\mathscr{A}(\theta_{12}, \theta_2, \omega_1, \omega_2) = \frac{1}{\sin\theta_{12} \sin\theta_2} \left(\cos\theta_{12} \cos\theta_2 + \frac{\omega_2}{\omega_1} \cos\theta_2 - \frac{\omega_2}{c_s p_\chi} \cos\theta_{12} - \frac{\omega_1^2 + \omega_2^2}{2\omega_1 c_s p_\chi} \right)$$

The integration region $\mathscr{R}$ is defined by $|\mathscr{A}| \leq 1$. In the limit $m_\chi \to 0$

$$\mathscr{A} \sim -\frac{1}{p_\chi} \frac{(\mathbf{q}_1 + \mathbf{q}_2)^2}{|\mathbf{q}_1 \times \mathbf{q}_2|} \sim - \underbrace{\frac{1}{p_\chi}}_{\text{large}} \times \underbrace{\frac{1 + \cos\theta_{12}}{\sin\theta_{12}}}_{\text{small if } \theta_{12} \to \pi}$$

TWO-PHONON EMISSION

$$\mathscr{A}(\theta_{12},\theta_2,\omega_1,\omega_2) = \frac{1}{\sin\theta_{12}\sin\theta_2}\left(\cos\theta_{12}\cos\theta_2 + \frac{\omega_2}{\omega_1}\cos\theta_2 - \frac{\omega_2}{c_s p_\chi}\cos\theta_{12} - \frac{\omega_1^2+\omega_2^2}{2\omega_1 c_s p_\chi}\right)$$

Taking $\omega \simeq 1$ meV and $c_s p_\chi \simeq 10^{-9} \times m_\chi$

$$\frac{\omega}{c_s p_\chi} \simeq 1 \quad \text{for} \quad m_\chi \simeq 1 \text{ MeV}$$

For higher values of the masses the yellow terms become less important and phase space (with no cuts) allows all 2π configurations.

But we have cuts! $|\mathbf{q}_1 + \mathbf{q}_2| < 1$ keV. Larger p_χ hit this cut more often.

Taking $q_1 = q_2 = \omega/c_s$ and $\omega \sim 1$ meV we get

$$\theta_{12} > 2\pi/3$$

TWO-PHONON EMISSION

The lighter dark particle mass we want to probe, the more back-to-back is the two-phonon emission. Highly recoiling phonons are, in any case, preferred (EFT cuts).

Now we will see that the more back-to-back is the two-phonon emission, the smaller is the matrix element $\mathscr{M}$.

(a) (b)

TWO-PHONON EMISSION

$$\mathscr{M}_a = i^1(-1)(\mathscr{G}_3\,\omega_1\omega_2 + \mathscr{G}_2\,\mathbf{q}_1\cdot\mathbf{q}_2)$$

$$\mathscr{M}_b = i^2(i\mathscr{G}_1\,\omega)\times\frac{i}{\omega^2 - c_s^2\mathbf{q}^2}\times\Big(ig_1\big(\underbrace{\omega_1\,\mathbf{q}_2\cdot\mathbf{q}}_{(12\diamond)} + (21\diamond) + (\diamond 12)\big) + ig_2\,\omega\,\omega_1\omega_2\Big)$$

change one of the momenta out → in

From the couplings determined in the effective theory we observe that

$$\mathscr{G}_1\,g_2 = \mathscr{G}_3 \quad \text{and} \quad \mathscr{G}_1\,g_1 = \mathscr{G}_2$$

$$\mathscr{M}_b = \omega\times\frac{i}{\omega^2 - c_s^2\mathbf{q}^2}\times\Big(\mathscr{G}_2\big(\underbrace{\omega_1\,\mathbf{q}_2\cdot\mathbf{q}}_{(12\diamond)} + (21\diamond) + (\diamond 12)\big) + \mathscr{G}_3\,\omega\,\omega_1\omega_2\Big)$$

In the limit $\mathbf{q}\to 0$ (corresponding to the back-to-back case) there is a (perfect) cancellation of $\mathscr{M}_a$ and $\mathscr{M}_b$.

TWO-PHONON EMISSION

The relations found in the effective theory

$$\mathcal{G}_1 g_2 = \mathcal{G}_3 \quad \text{and} \quad \mathcal{G}_1 g_1 = \mathcal{G}_2$$

are a manifestation of the conservation of the $\boldsymbol{J^\mu(x)}$ current associated to the U(1) symmetry of the superfluid

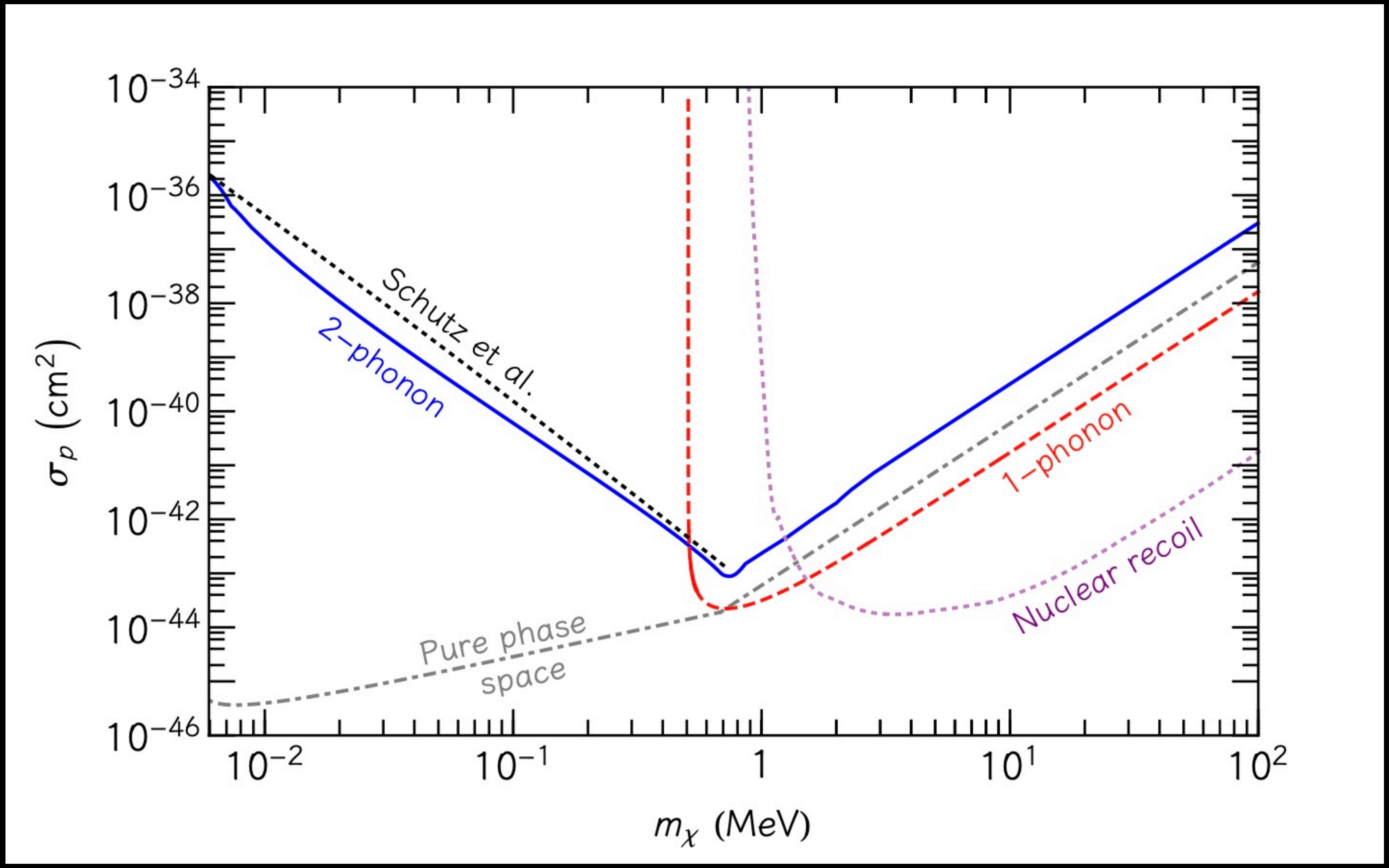
$$q_\mu \langle \pi(q_1)\pi(q_2) \,|\, J^\mu(x) \,|\, 0 \rangle = \omega \, \langle \pi(q_1)\pi(q_2) \,|\, \rho(x) \,|\, 0 \rangle + \boldsymbol{q} \cdot \langle \pi(q_1)\pi(q_2) \,|\, \boldsymbol{J} \,|\, 0 \rangle$$

If $\boldsymbol{q} \to 0$, the second term vanishes and, to ensure the conservation of the current, one needs $\boldsymbol{\omega} \, \langle \pi(q_1)\pi(q_2) \,|\, \rho(x) \,|\, 0 \rangle = 0$ Given that $\boldsymbol{\omega} \neq 0$ – the latter corresponds to $\mathcal{M}_a + \mathcal{M}_b \to 0.$

Argument due to A. Esposito

THE EXCLUSION PLOT

The previous discussion explains the rise below 1 MeV observed by other means by Schutz & Zurek PRL2016, 117

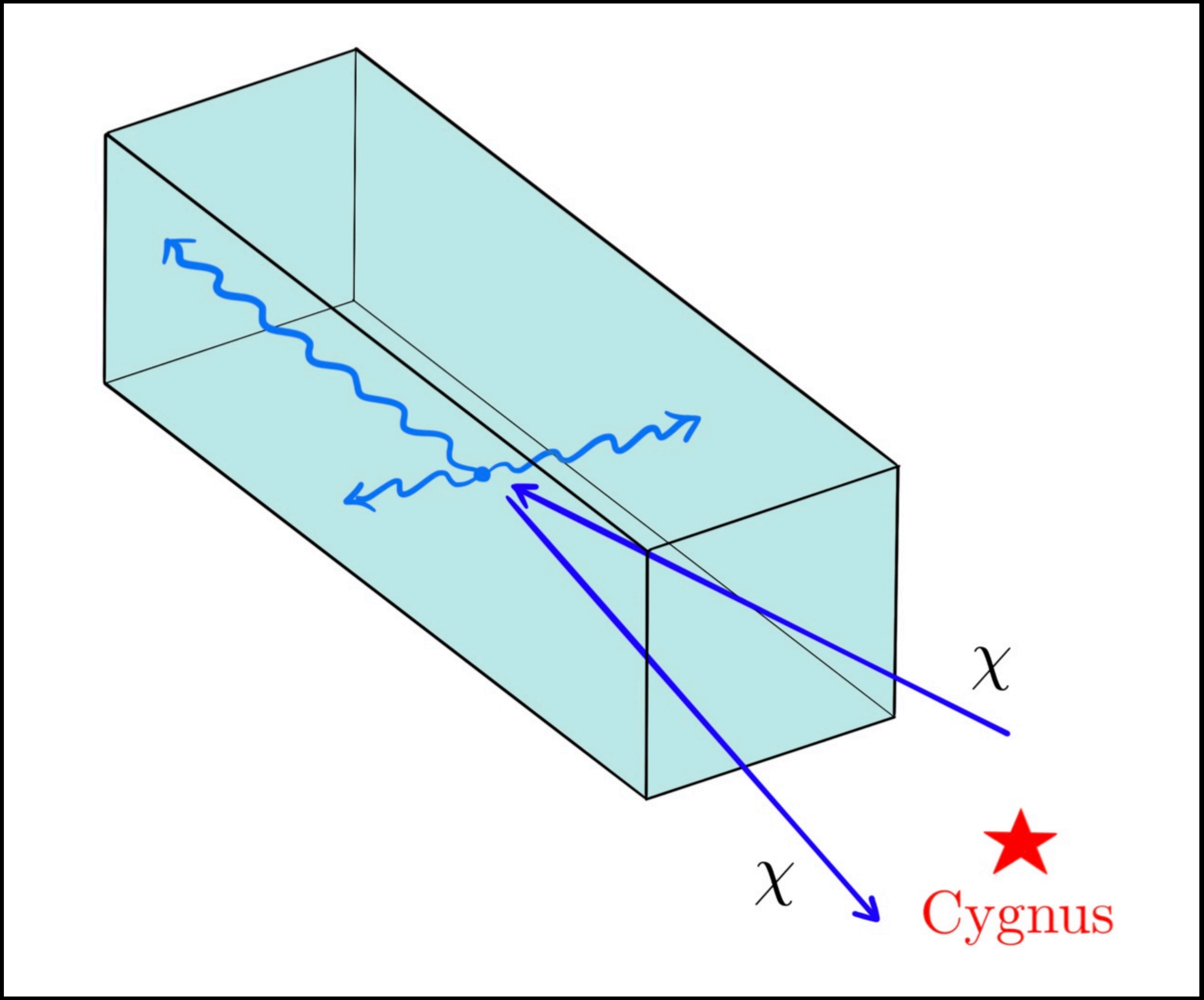


Excluded region corresponding to 3/evts/Kg/year @ zero bckg. Impose total energy released >1meV

See also S. Knapen, T. Lin, K.M. Zurek, PRD95 (2017) 056019

THREE-PHONON EMISSION

Cygnus Shaped Events @ $m_\chi \approx 500$ keV



For a good fraction of the events, DM releases most of its $\boldsymbol{p}$ to the fwd $\boldsymbol{\pi}$.

A. Caputo, A. Esposito, F. Piccinini, ADP, G. Rossi, PRD103 (2021) 5, 055017

CYGNUS EVENTS

Such 3π events are suppressed wrt 2π events *but*

- The two back-to-back phonons may be used as a trigger to look for the third, forward phonon, which turns out to be strongly correlated with the direction of the incoming DM
- this allows in principle background rejection, vertex reconstruction (remove multiple scatterings due to other sources e.g. neutrons) and directionality!

You simply need the perfect detector of phonons in Superfluid He….

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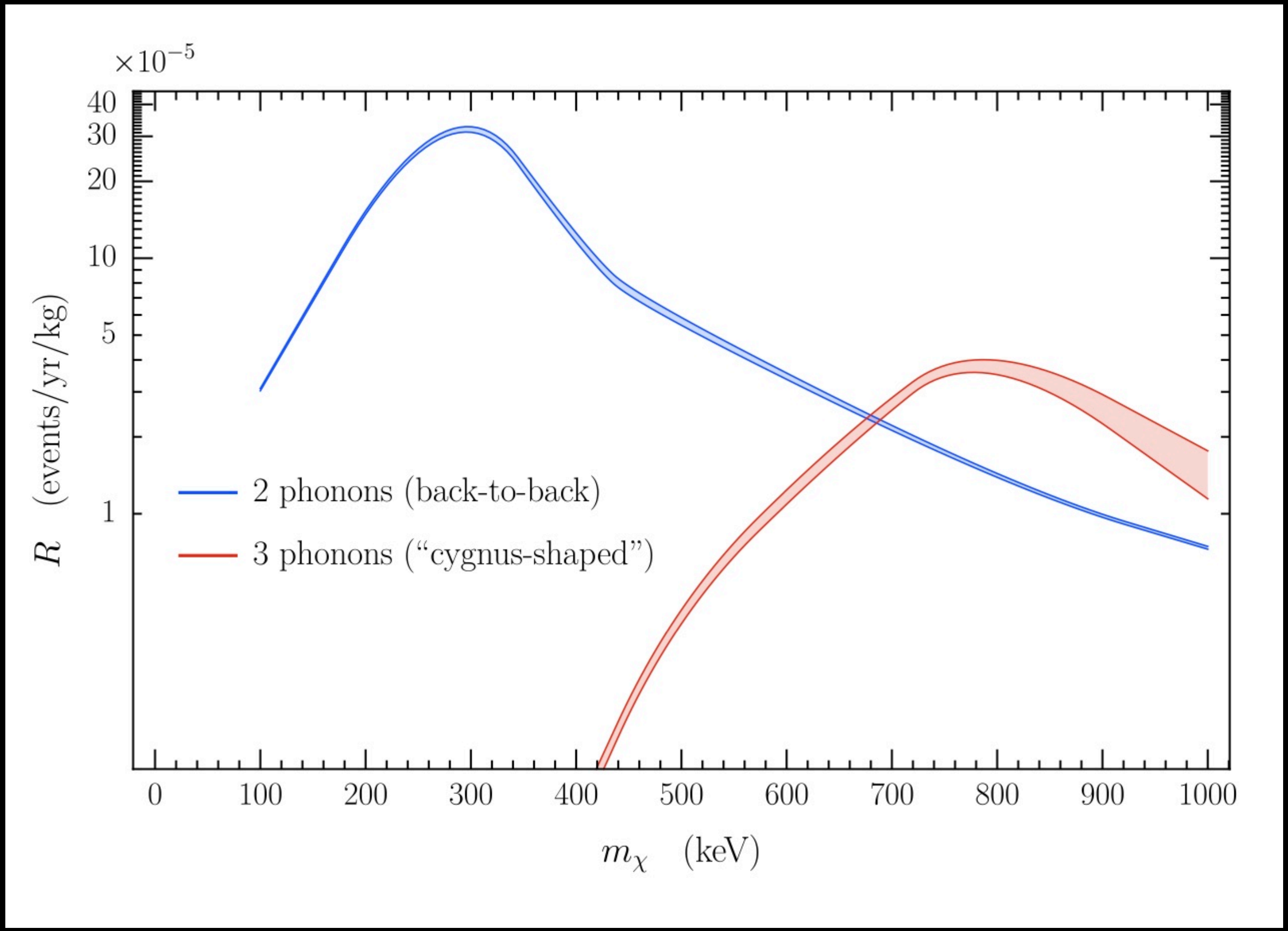
CYGNUS EVENTS

$$R = \frac{\rho_\chi(= 0.3 \text{ GeV/cm}^3)}{m_\chi \bar{n} m_{\text{He}}} \Gamma_{3\pi}$$



$\sigma_n = 10^{-42}$ cm^2

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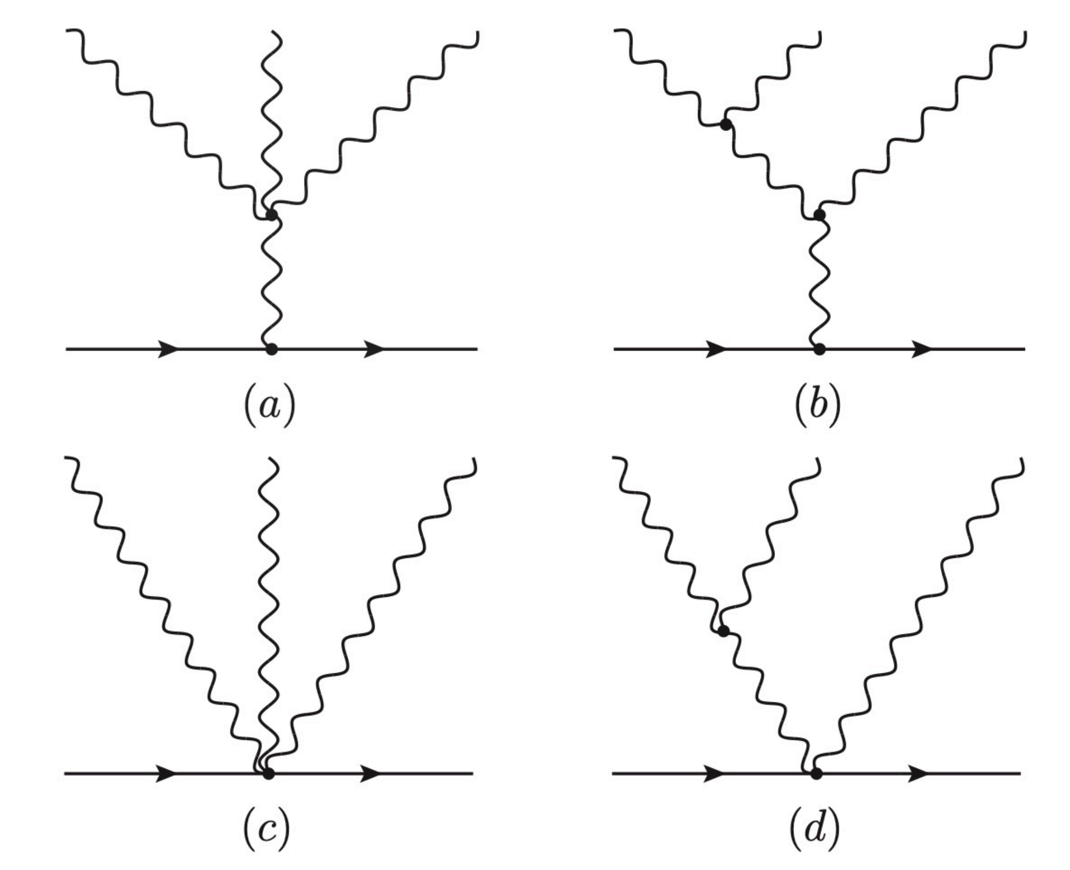
THREE-PHONON EMISSION

The quartic phonon couplings $(\lambda_1, \lambda_2, \lambda_3)$ have to be worked out from the EFT, as well as the couplings (γ_1, γ_2) responsible for DM-3π.



For $\boldsymbol{q} \to 0$, $\mathscr{M}_a + \mathscr{M}_c \to 0$ and $\mathscr{M}_b + \mathscr{M}_d \to 0$

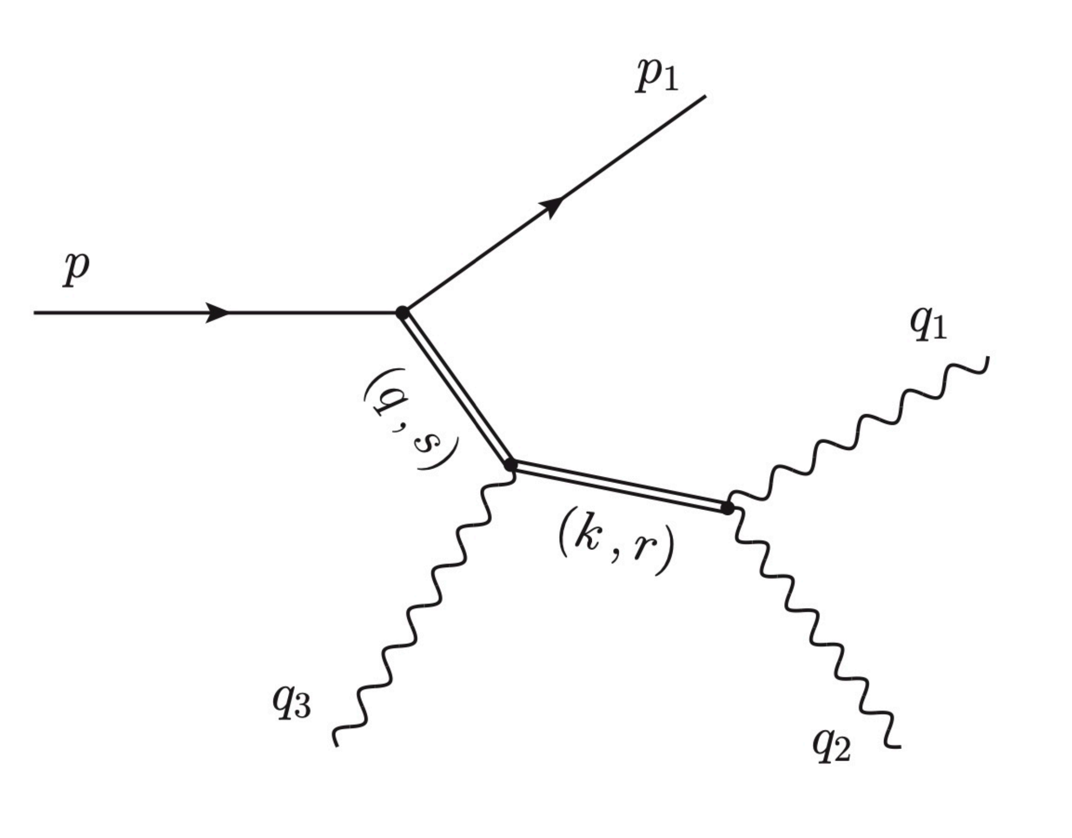
THREE-PHONON EMISSION

The $\mathbf{1} \to \mathbf{4}$ process is factorized into three $\mathbf{1} \to \mathbf{2}$ processes by introducing two fictitious space-like 4-momenta $\boldsymbol{q}, \boldsymbol{k}$.

In its passage the DM particle releases a *space-like momentum* and the superfluid reacts producing two *space-like phonons* $\boldsymbol{q_\mu = (\mathbf{q}, c_s q)}$, with $\boldsymbol{c_s = 10^{-6}}$. In the case of the two-phonon emission an analytic formula can be determined.

THREE-PHONON EMISSION

The $1 \to 4$ process is factorized into three $1 \to 2$ processes by introducing two fictitious space-like 4-momenta $\boldsymbol{q}, \boldsymbol{k}$.



Calculations including M.E. are done numerically; we checked numerical *phase space volumes* against analytic calculations.

Set M.E. = 1

$$\Phi(\chi \to \chi + 2\pi) = \frac{1}{8(2\pi)^4 c_s^5 E} \int_{\mathscr{R}} \frac{1}{\sqrt{1 - \mathscr{A}^2}} \frac{\omega_2}{p_\chi} \, d\omega_1 \, d\omega_2 \, d\theta_{12} \, d\theta_2$$

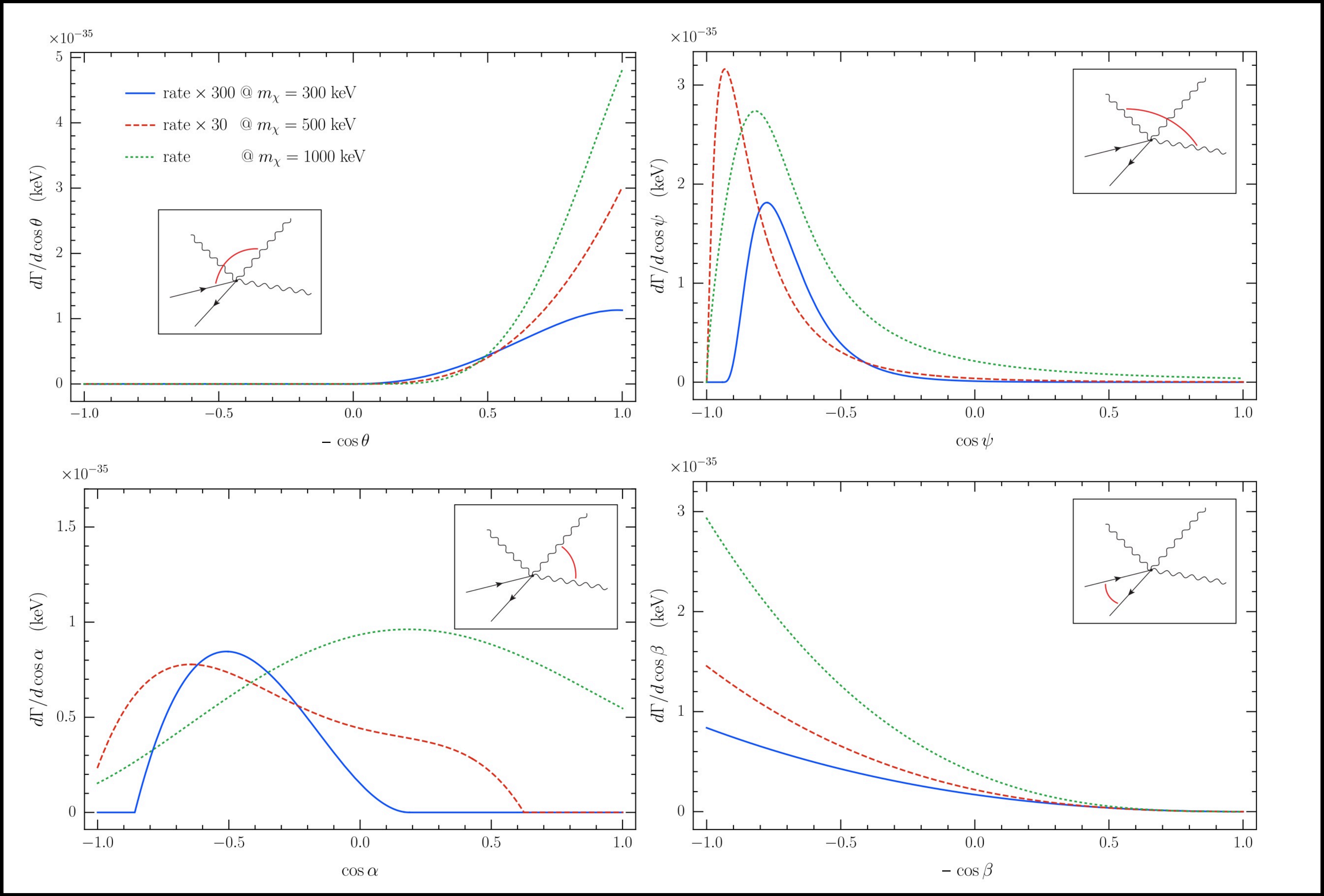
$$\mathscr{A}(\theta_{12}, \theta_2, \omega_1, \omega_2) = \frac{1}{\sin\theta_{12}\sin\theta_2}(\cos\theta_{12}\cos\theta_2 + \frac{\omega_2}{\omega_1}\cos\theta_2 - \frac{\omega_2}{c_s p_\chi}\cos\theta_{12} - \frac{\omega_1^2 + \omega_2^2}{2\omega_1 c_s p_\chi})$$

$$\Phi(\chi \to \chi + 2\pi) = \frac{1}{2(p_\chi / c_s)} I_3$$

$$I_3 = \frac{p_\chi^3}{96\pi^3 c_s^3 m_\chi}$$

In the $\mathbf{3\pi}$ case the PS volume is $I_4 = \dfrac{p_\chi^7}{53760\pi^5 c_s^6 m_\chi^3}$

CYGNUS EVENTS



A. Caputo, A. Esposito, F. Piccinini, ADP, G. Rossi, PRD103 (2021) 5, 055017

SOME MORE TECHNICAL CONCLUSIONS

- There are recent good experimental reasons to talk again about Superfluid He as a light DM target.
- The role of NR-EFT is vary promising. We have to recall that there exist a consolidated standard approach in condensed matter theory to do this sort of calculations – see original papers.
- The other excitations in superfluid Helium (rotons, vortices) have not been mentioned.

BACKUP

BACK TO THE RELATIVISTIC FORMALISM

In the previous Lagrangian scale/adjust the $\boldsymbol{x_i}$ *coordinates to write*

$$\mathscr{L} = -\xi^2 \left(\partial_\mu \theta \, \partial^\mu \theta\right)$$

with the constraint $\theta(x) = \theta(x) + 2\pi$. This corresponds to

$$\mathscr{L} = -(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) \quad \text{with} \quad \Phi^\dagger \Phi = \xi^2 \text{ or } \Phi = \xi e^{i\theta(x)}$$

which in turn corresponds to the strong coupling $\lambda \to \infty$ of

$$\mathscr{L} = -(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) - \lambda(\Phi^\dagger \Phi - \xi^2)^2$$

climbing the wall of the bottle bottom is very inconvenient.

CHEMICAL POTENTIAL

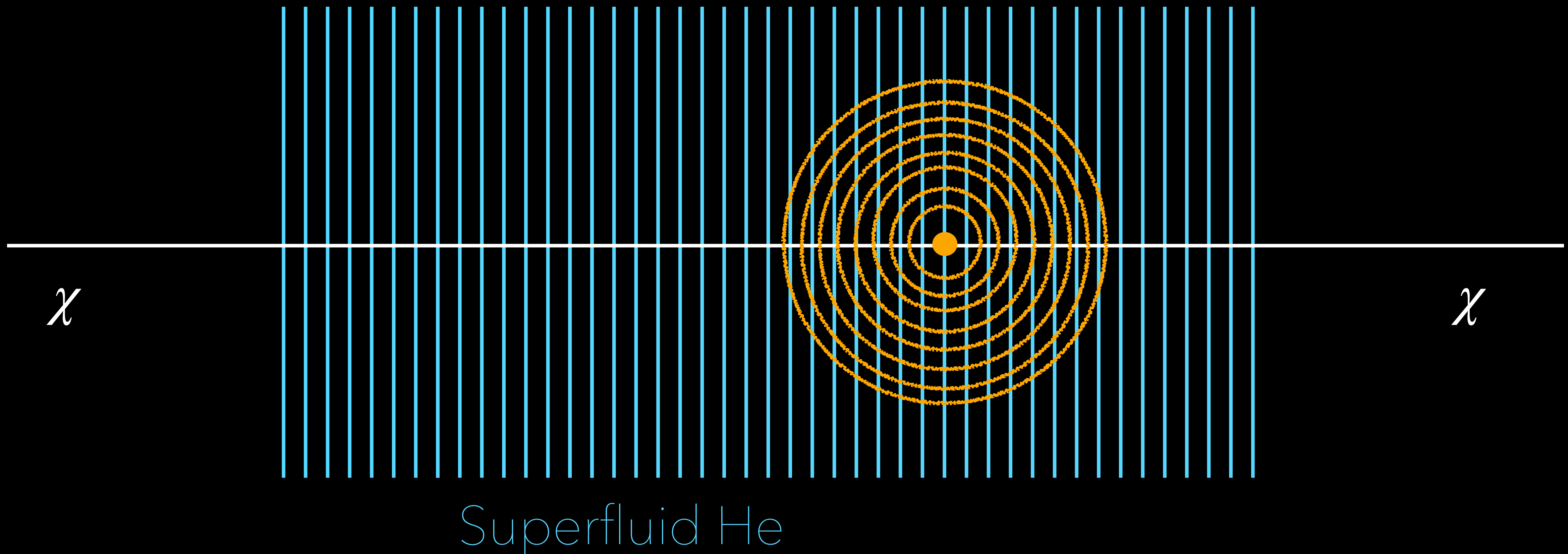
The implementation of a chemical potential goes through the introduction of a time-dependent phase in the U(1) direction.

This is like searching for the ground state of $\boldsymbol{H}$ evolving in time along the U(1) direction.

The time-dependent phase $\psi(t)$ is propto μ so $\psi(t) = \mu\, t$ – these are solutions of the equations of motion parametrized by μ.

A variation in μ corresponds to exciting a configuration $\delta\mu\, t = \pi(x)$ of the Goldstone boson.

ACOUSTIC SCINTILLATOR



The DM particle interacts in one point only, if it interacts at all.
The velocity $v_\chi \simeq 220$ Km/s is much larger than $c_s = 248$ m/s
(but waves do not build a Cerenkov cone (with $\tan\theta = c_s/v_\chi$))

EXPANSION OF $P(X)$ UP TO $\pi(x)^3$

$$\boxed{\mathscr{L}_{\mathrm{He}} = \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\nabla\pi)^2 + \frac{g_1}{2}\,\dot{\pi}(\nabla\pi)^2 + \frac{g_2}{3!}\,\dot{\pi}^3}$$

The last two terms vanish for $q_\mu \to 0$

Indeed, in general, the sum of all graphs with three external, zero 4-momentum Goldstone boson lines, vanishes

$$\sum_{\ell,m,n} \underbrace{\frac{\partial^3 V(\phi)}{\partial\phi_\ell \partial\phi_m \partial\phi_n}}_{\sum(\text{1PI diag. w/ ext. lines } \ell,m,n)} (t_a\bar{\phi})_\ell (t_b\bar{\phi})_m (t_c\bar{\phi})_n = 0$$

$$t_{nm}\bar{\phi}_m \neq 0 \qquad SB$$

EXPANSION OF $P(X)$ UP TO $\pi(x)^3$

$$\boxed{\mathscr{L}_{\text{He}} = \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\nabla\pi)^2 + \frac{g_1}{2}\,\dot{\pi}(\nabla\pi)^2 + \frac{g_2}{3!}\,\dot{\pi}^3}$$

The last two terms vanish for $q_\mu \to 0$

In general processes, to leading order in small Goldstone boson energies, low energy Goldstone bosons are not emitted from external low energy Goldstone boson lines.

CHEMICAL POTENTIAL

The chemical potential $\boldsymbol{\mu}$ can be different from zero only if the total number of particles is conserved. At very low temperature $\langle N_{\boldsymbol{p}} \rangle$ is sharply peaked at $\boldsymbol{p}$ such that $\boldsymbol{E(p) \simeq \mu}$.

$$\langle N_{\boldsymbol{p}} \rangle = \frac{1}{\exp[(E(\boldsymbol{p}) - \mu)/KT] - 1}$$

In BEC one has a macroscopic number of particles with energy $\boldsymbol{\mu}$. Sort of BEC in liquid helium.

STRONG COUPLING FROM NR LAGRANGIAN

Conversely if we start from the NR Lagrangian

$$\mathscr{L} = i\varphi^\dagger \frac{\partial}{\partial t}\varphi - \frac{1}{2m}\nabla_i \varphi^\dagger \nabla_i \varphi - \frac{\lambda}{4m^2}(\varphi^\dagger\varphi - \bar{\rho})^2$$

and take $\lambda \to \infty$ and $\bar{\rho} = \text{const}.$ we get

$$\mathscr{L} = i\varphi^\dagger \frac{\partial}{\partial t}\varphi - \frac{1}{2m}\nabla_i \varphi^\dagger \nabla_i \varphi \quad \text{with} \quad \varphi^\dagger\varphi = \bar{\rho} \text{ i.e. } \varphi = \sqrt{\bar{\rho}}e^{i\theta}$$

dropping the total derivative $\bar{\rho}\partial_0\theta$

$$\mathscr{L} = -\frac{\bar{\rho}}{2m}(\nabla_i \theta)^2$$

with eq. of motion

$$\Delta\theta = 0$$

The only way for it to be zero everywhere in the superfluid is $\theta =$ const.; not a Goldstone.

(recall $c_s = \sqrt{\frac{\lambda\bar{\rho}}{2m^3}}$ so the limit $\lambda \to \infty$ does not work.)