Dark Matter Misalignment Through the Higgs Portal

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Particle Avenues in the Dark Universe Arena

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Motivation and Overview

- Ultra-light scalar dark matter ($10^{-22}\mathrm{eV} \lesssim m_\phi \lesssim \mathrm{keV}$), generically produced via the misalignment mechanism, is a theoretically well-motivated and phenomenologically distinctive scenario.

- A minimal model realization consists of a scalar field coupled through the super-renormalizabe Higgs portal.
[Piazza, Pospelov '10]

- The cosmology of this scenario is rich and distinctive, involving the dynamical misalignment of the scalar field during the radiation era through two competing mechanisms: *thermal misalignment* and *VEV misalignment*.

- Under certain conditions, the DM relic abundance is insensitive to initial conditions and thus controlled by the DM mass and Higgs portal coupling. This leads to a relic density target that can be compared with experimental tests.

Outline

- Review of the standard misalignment mechanism
- The super-renormalizable Higgs portal model
- Cosmology
    - Scalar effective potential
    - Higgs field and electroweak phase transition (EWPT)
    - Sources of misalignment and initial conditions
    - Scalar field dynamics
    - Results for the DM relic abundance
- Experimental and observational tests

The standard misalignment mechanism

[Preskill, Wise, Wilczek; Abbott, Sikivie; Dine, Fischler, '83]

- Consider a massive scalar field in early universe:

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

- Initially, the scalar field is held up by Hubble friction at its initial field value ϕ_i

- Scalar oscillations commence when the Hubble rate falls below the scalar mass

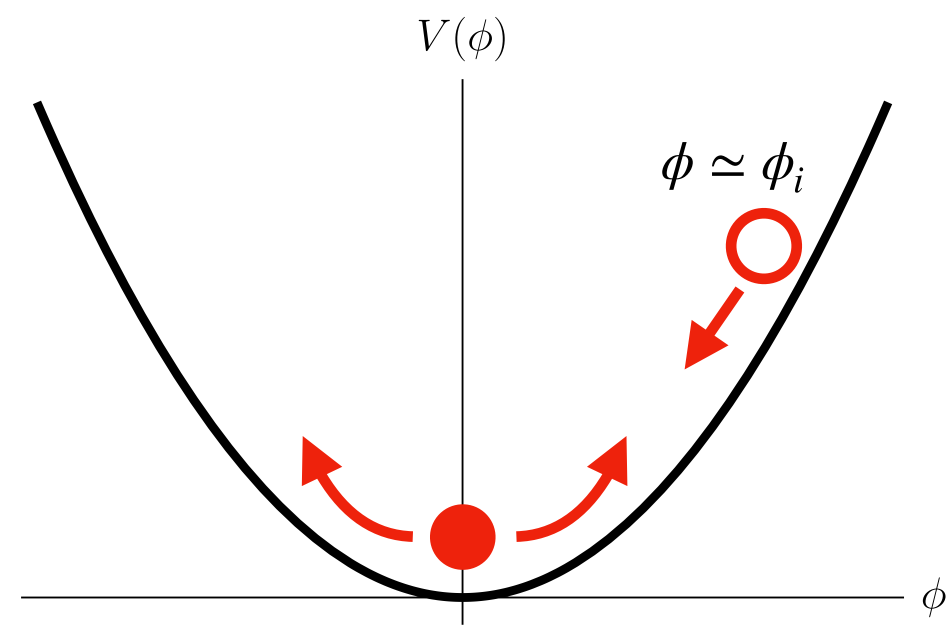
- The oscillating scalar forms a pressureless, non-relativistic fluid and is thus a good DM candidate

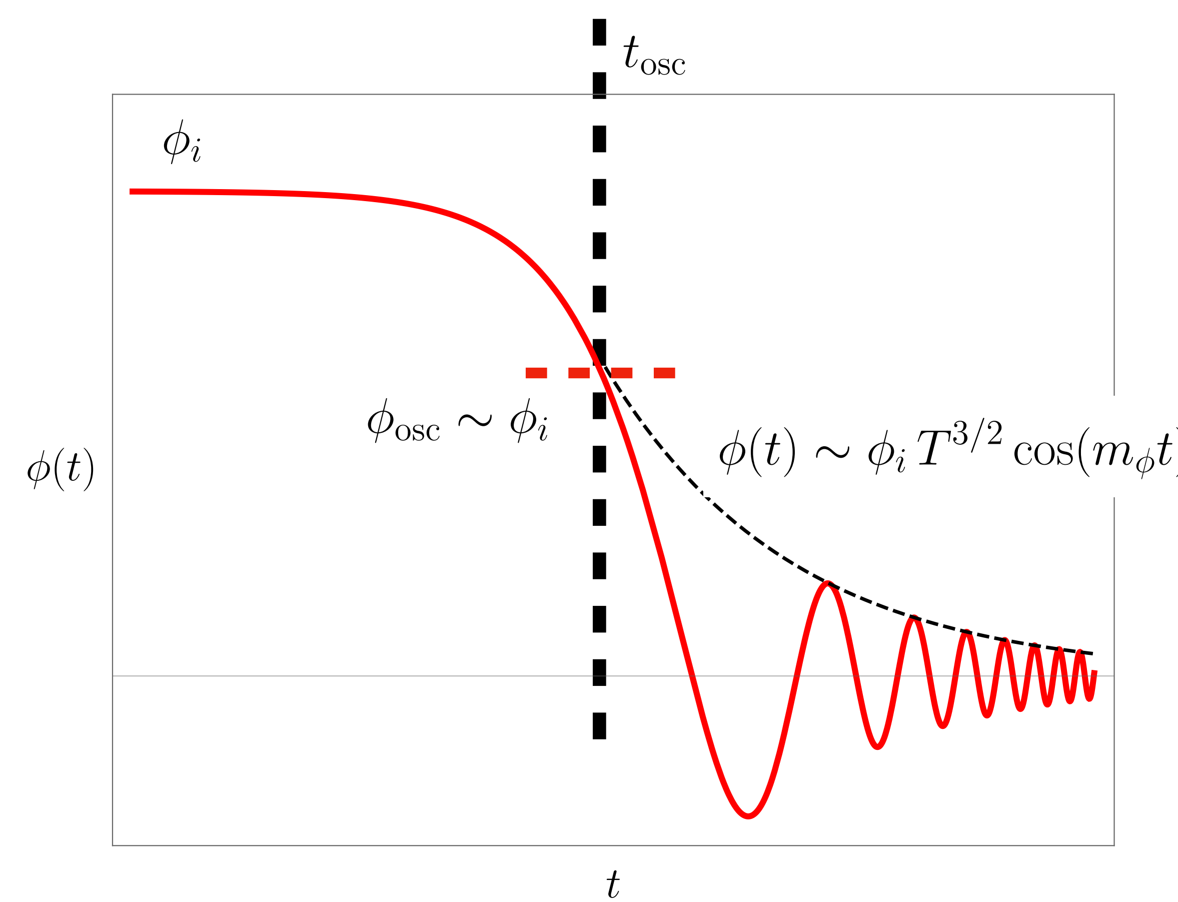
$$\rho_\phi = \frac{1}{2} m_\phi \langle \phi^2(t) \rangle \sim a(t)^{-3} \sim t^{-3/2} \sim T^3$$

- Relic abundance estimate

$$\Omega_\phi \big|_0 = \frac{\rho_{\phi,0}}{\rho_{c,0}} \simeq \frac{\frac{1}{2} m_\phi^2 \phi_{\rm osc}^2 (T_0/T_{\rm osc})^3 (g_{*S}^0/g_{*S}^{\rm osc})}{\rho_{c,0}}$$

$$\approx 0.2 \left(\frac{m_\phi}{10^{-11}\,{\rm eV}}\right)^{1/2} \left(\frac{\phi_i/M_{\rm pl}}{10^{-4}}\right)^2$$





Super-renormalizable Higgs portal model

[Piazza, Pospelov '10]

- Add a real scalar singlet ϕ to the SM, with scalar potential (note $H^T = (0, h/\sqrt{2})$)

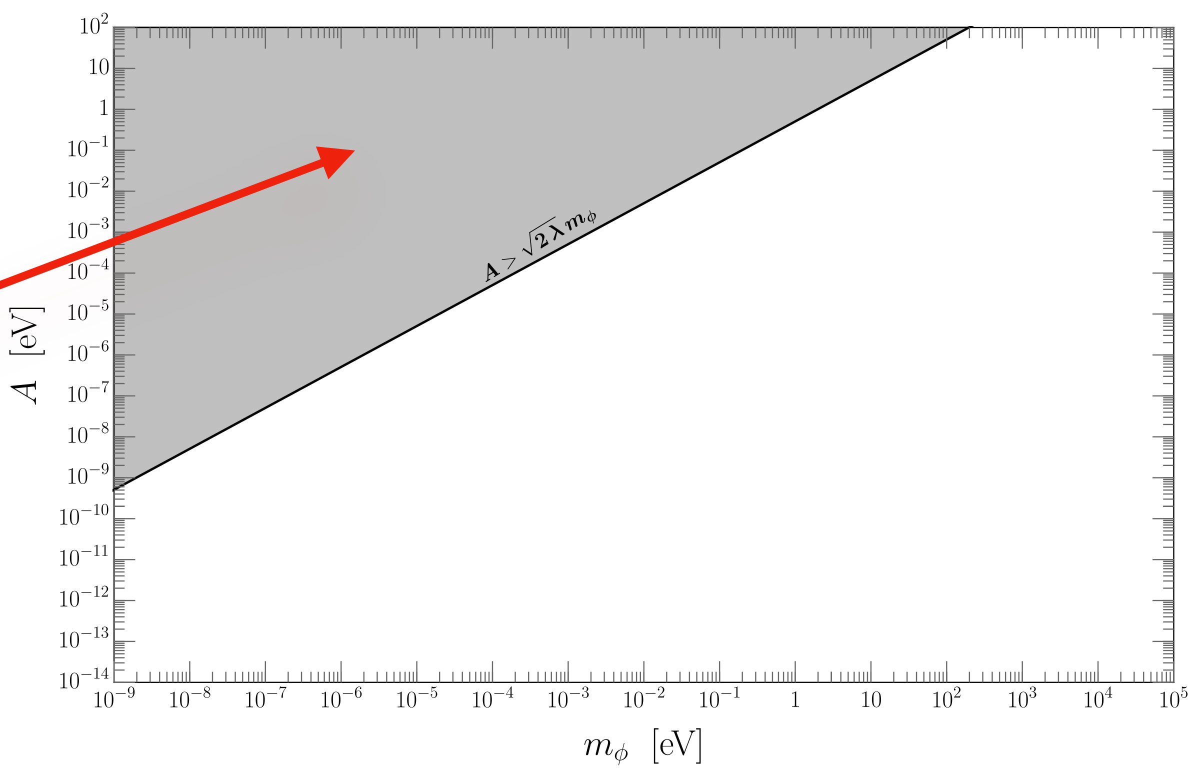
$$V_0(\phi, h) = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}A\,\phi\,h^2$$

- Two parameters: scalar mass m_ϕ and dimensionful Higgs portal coupling A

- Scalar vaccum expectation values (VEVs):

$$v^2 = \frac{\mu^2}{\lambda - A^2/2m_\phi^2}\,, \qquad \phi_0 = -\frac{Av^2}{2m_\phi^2}\,,$$

- Electroweak vacuum condition: $\frac{A^2}{2\,\lambda\, m_\phi^2} < 1$



- We will generally be interested in feeble couplings, $A \ll m_\phi$

Cosmology overview

- Our study starts in the radiation era at high temperatures, $T \gg v$
- The feeble coupling of the scalar to the Higgs leads to non-trivial dynamical evolution of ϕ during the radiation era through two effects:

  [Piazza, Pospelov '10]

  - ***Thermal misalignment*** $\hat{\phi}_T$ **:** The scalar experiences a finite temperature potential and is driven towards its high temperature minimum at large field values.

    [BB, Ghalsasi '21]
    [for related work see also Buchmuller, Hamaguchi, Lebedev, Ratz '04; Lillard, Ratz, Tait, Trojanowski '18; Chun '21; Cheek Osinski, Roszkowski, Trojanowski '22]

  - ***VEV misalignment*** $\hat{\phi}_V$ **:** During the electroweak phase transition the Higgs VEV turns on and induces a shift in the ϕ VEV.

    [see also Arkani-Hamed, Tito D'Agnolo, Kim '20]

- We study two choices for the initial conditions
  - $\phi_i = \phi_0$: the scalar begins at its zero temperature VEV
  - $\phi_i = 0$: $|\phi_i|$ is significantly different than $|\phi_0|$.

Scalar effective potential

- At high temperatures, $T \gg v$, the scalar fields experience an effective potential with the following contributions

$$V_{\text{eff}}(\phi, h, T) = V_0(\phi, h) + V_{\text{CW}}(\phi, h) + V_T(\phi, h, T)$$

Tree-level | Coleman-Weinberg | Finite-Temperature

- Of particular importance is the finite temperature effective potential

[See, e.g., M. Quiros, 9901312]

$$V_T(\phi, h, T) \supset \frac{1}{2\pi^2} T^4 J_B\left[\frac{m_h^2(\phi, h, T)}{T^2}\right] + \frac{3}{2\pi^2} T^4 J_B\left[\frac{m_\chi^2(\phi, h, T)}{T^2}\right] + \quad \cdots$$

- The ϕ-dependent masses of the Higgs and Nambu-Goldstone bosons are

$$m_{0,h}^2(\phi, h) = -\mu^2 + 3\,\lambda\, h^2 + A\,\phi\,,$$

$$m_{0,\chi}^2(\phi, h) = -\mu^2 + \lambda\, h^2 + A\,\phi\,,$$

- The functions $J_{B,F}$ are defined as $J_{B,F}(w^2) = \int_0^\infty dx\, x^2\, \log\left[1 \mp \exp\left(-\sqrt{x^2 + w^2}\right)\right]$

Scalar field evolution and relic abundance

- We find it convenient to work with the following dimensionless variables:

$$y \equiv \frac{T}{\mu}\,, \qquad \hat{\phi} \equiv \frac{\phi}{M_{\rm pl}}\,, \qquad \hat{h} \equiv \frac{h}{\mu}\,, \qquad \kappa \equiv \frac{m_\phi M_{\rm pl}}{\mu^2}\,, \qquad \beta \equiv \frac{A M_{\rm pl}}{\mu^2}\,.$$

$$\eta_i(\hat{\phi}, \hat{h}, y) \equiv m_i^2(\phi, h, T)/T$$

- In terms of these dimensionless variables, the scalar equation of motion reads (prime denotes derivative wrt. $y = T/\mu$)

$$\hat{\phi}'' + \frac{1}{\gamma^2 y^6}\left[\kappa^2\hat{\phi} + \frac{\beta\hat{h}^2}{2} + \frac{\beta y^2}{2\pi^2}\left(J_B'[\eta_h] + 3J_B'[\eta_\chi]\right)\right] = 0$$

- As the universe expands, the Hubble parameter decreases until it eventually falls below the effective ϕ mass, marking the onset of scalar oscillations:

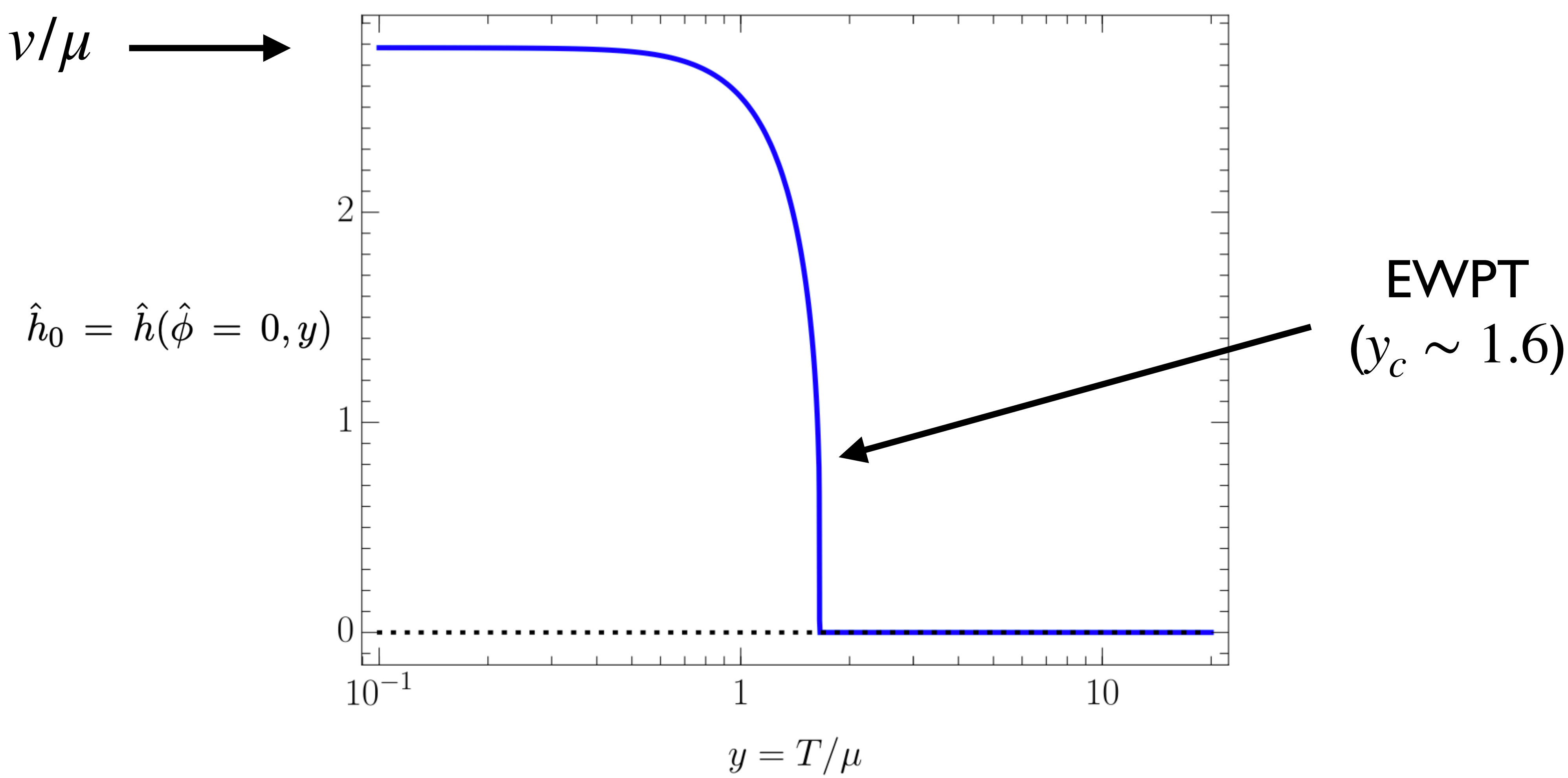
$$[3H(y_{\rm osc})]^2 = m_\phi^2(y_{\rm osc}) \simeq m_\phi^2 \quad \Rightarrow \quad y_{\rm osc} = \frac{T_{\rm osc}}{\mu} = \sqrt{\frac{\kappa}{3\gamma}}$$

- We estimate the oscillation amplitude, $\phi_{\rm osc} = \phi(y_{\rm osc})$, and from here we can estimate the relic abundance

Higgs field and EWPT

- The electroweak phase transition (EWPT) is a smooth crossover characterized by the critical temperature $T_c \sim \mathcal{O}(v)$, $\quad [y_c = T_c/\mu \sim \mathcal{O}(1)]$.

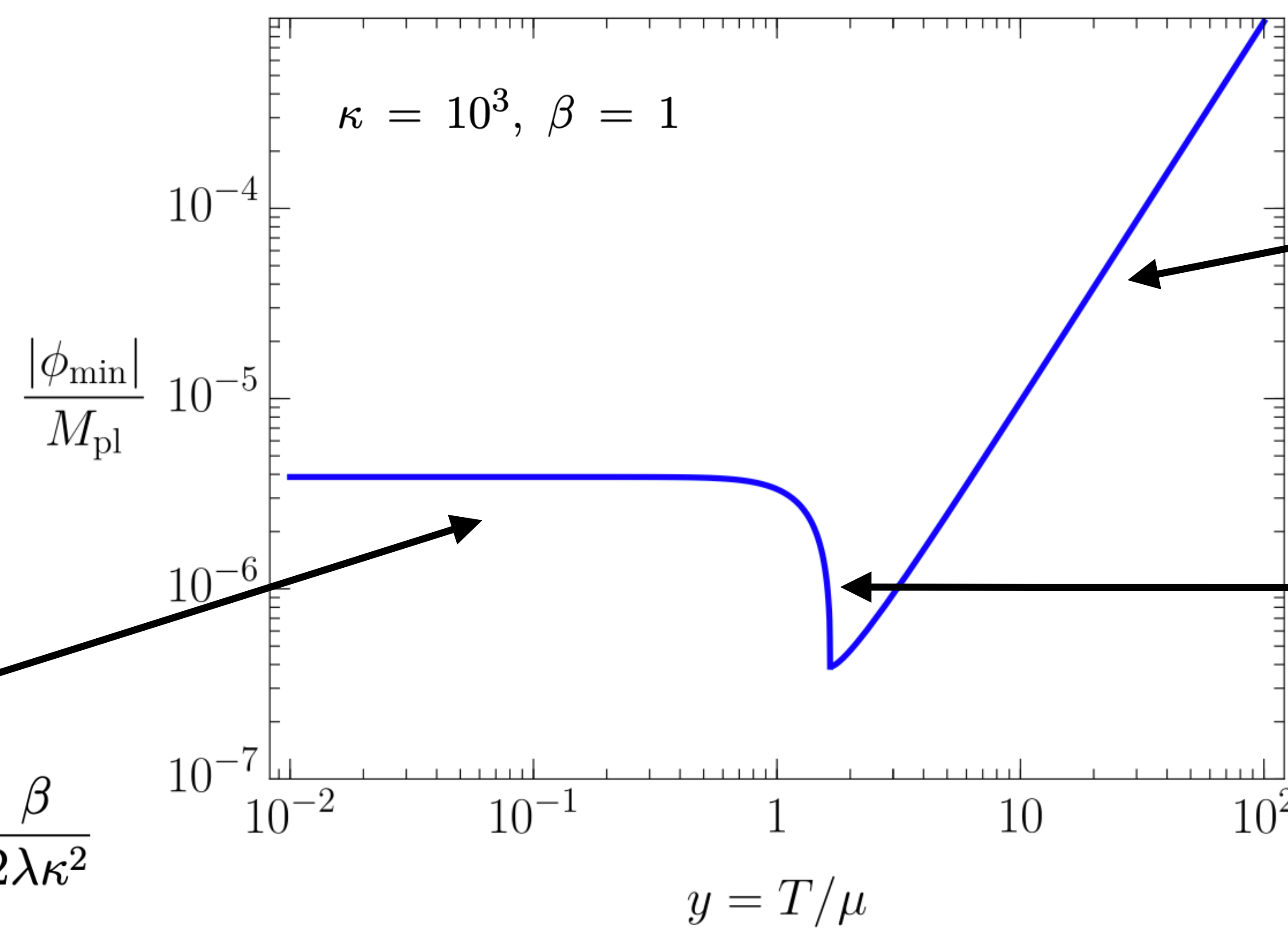
- Assume the Higgs field tracks its potential minimum throughout the EWPT. Thus, the evolution of $\hat{h}(\phi, y)$ is determined by the minimization condition $\partial \hat{V}_{\text{eff}}/\partial \hat{h} = 0$



Temperature-dependent ϕ VEV

- Much insight can be gained by studying the shape of the potential and the ϕ VEV as a function of temperature. The ϕ VEV is obtained from the condition, $\partial \hat{V}_{\rm eff}/\partial \hat{\phi} = 0$,

$$\hat{\phi}_{\rm min} = -\frac{\beta}{2\kappa^2}\left[\hat{h}^2 + \frac{y^2}{\pi^2}\left(J_B'[\eta_h] + 3J_B'[\eta_\chi]\right)\right]$$



High temperature, $|\hat{\phi}| \propto y^2$

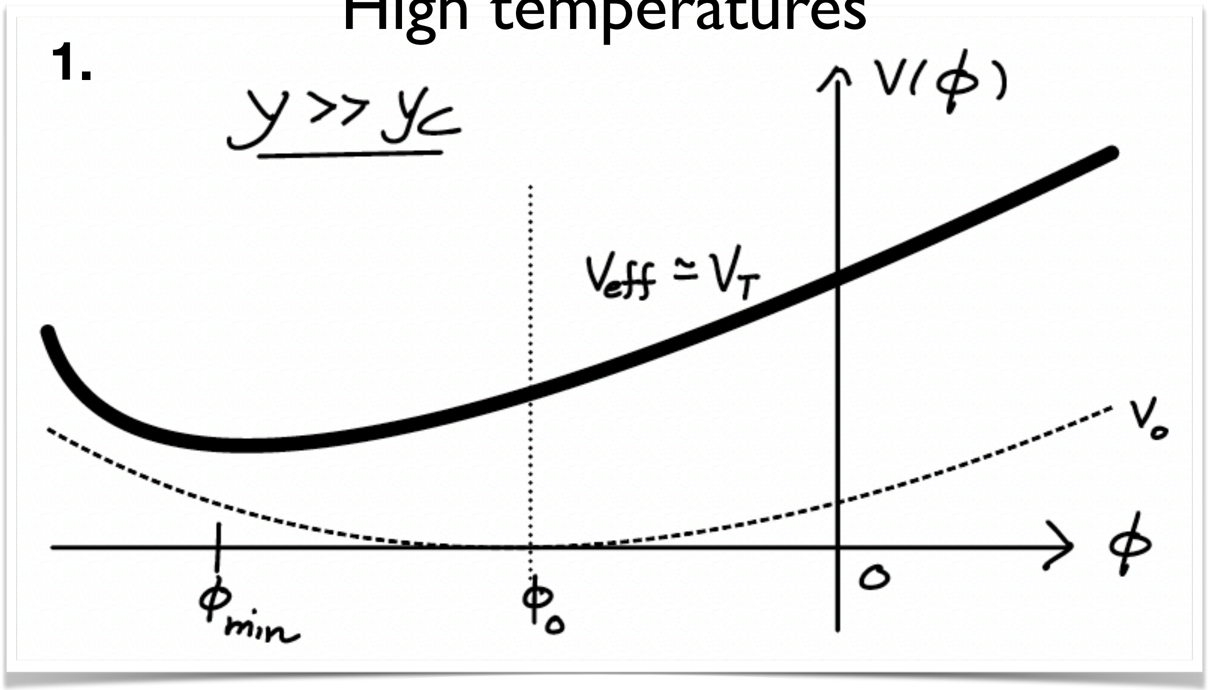
EWPT

Low temperature, $|\hat{\phi}| = \hat{\phi}_0$

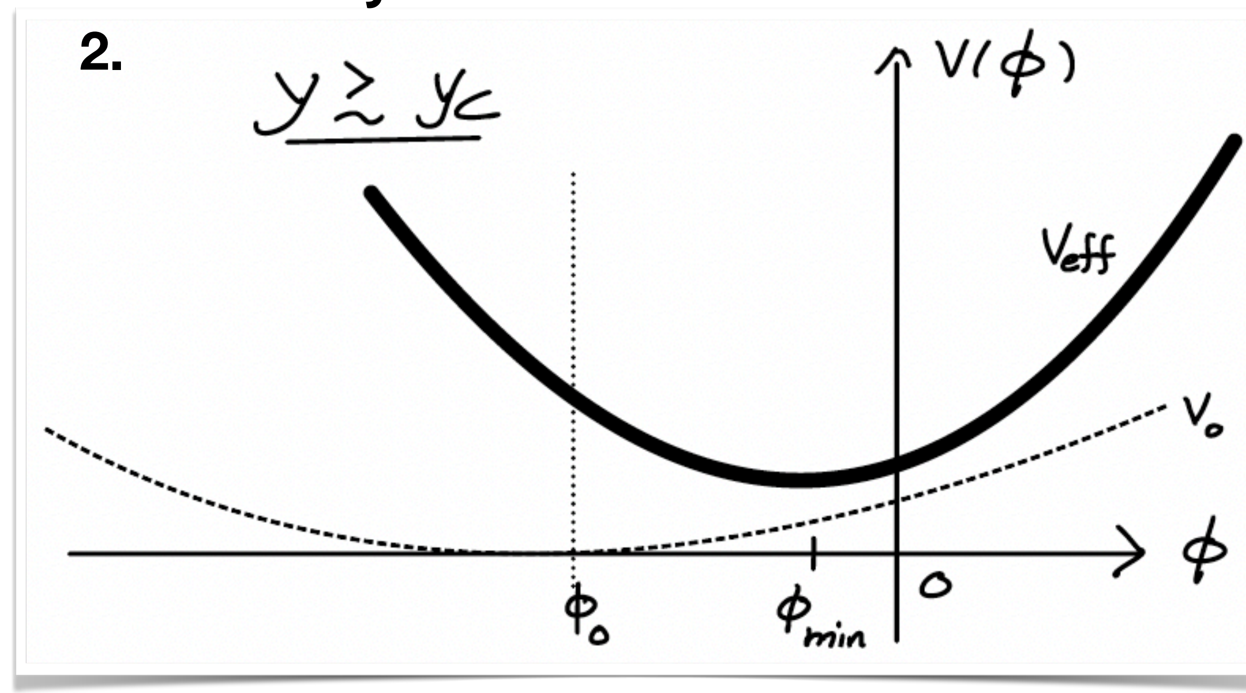
$$\hat{\phi}_0 = -\frac{\beta}{2\lambda\kappa^2 - \beta^2} \simeq -\frac{\beta}{2\lambda\kappa^2}$$

Temperature-dependent ϕ potential

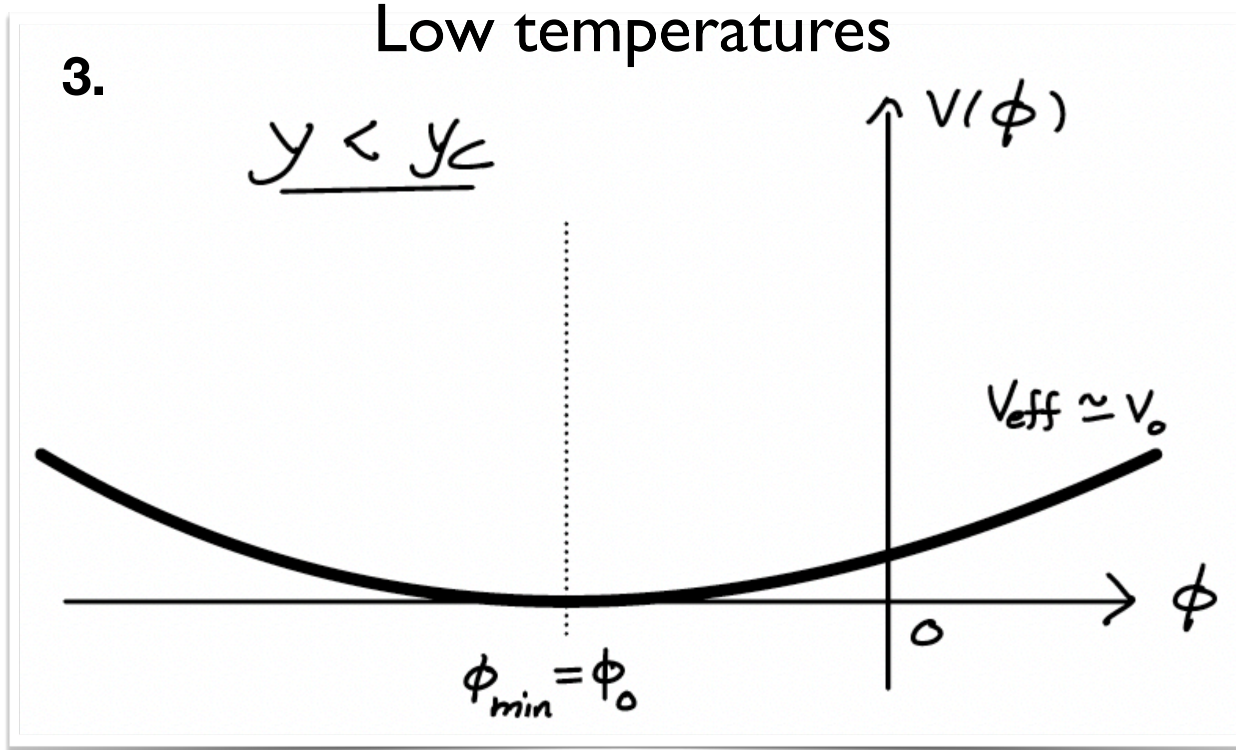
High temperatures



Just above the EWPT

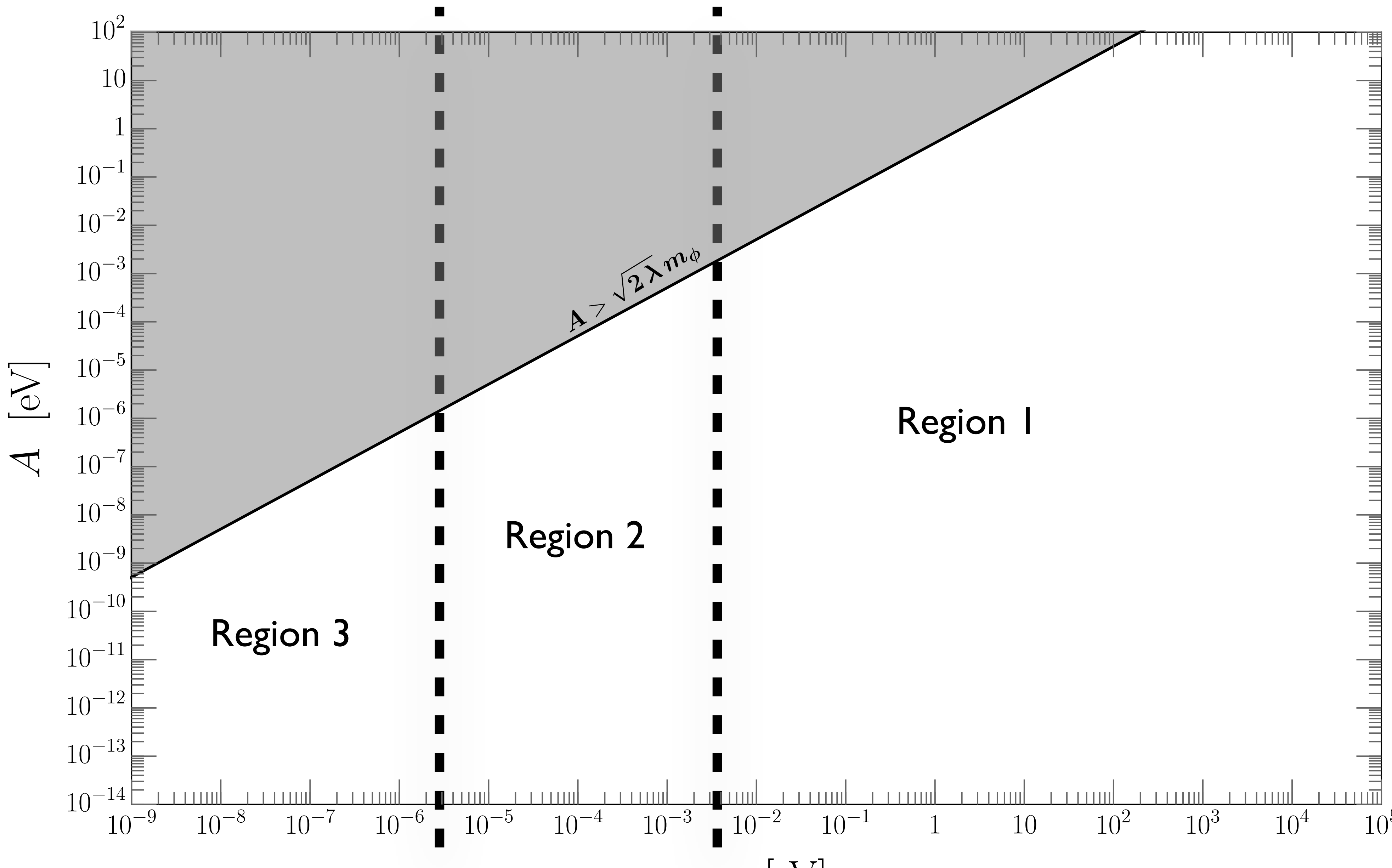


Low temperatures

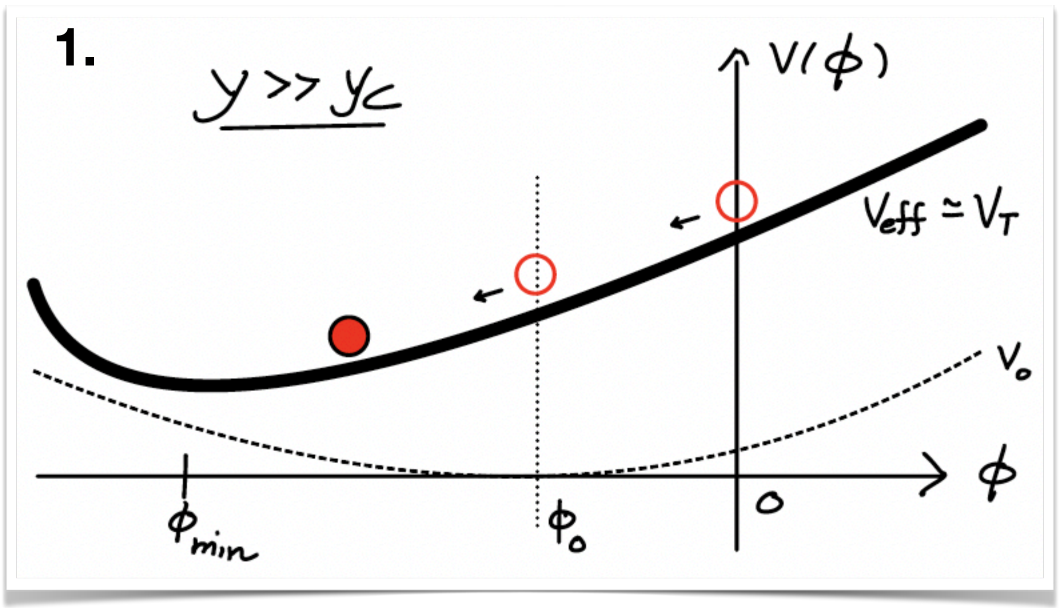


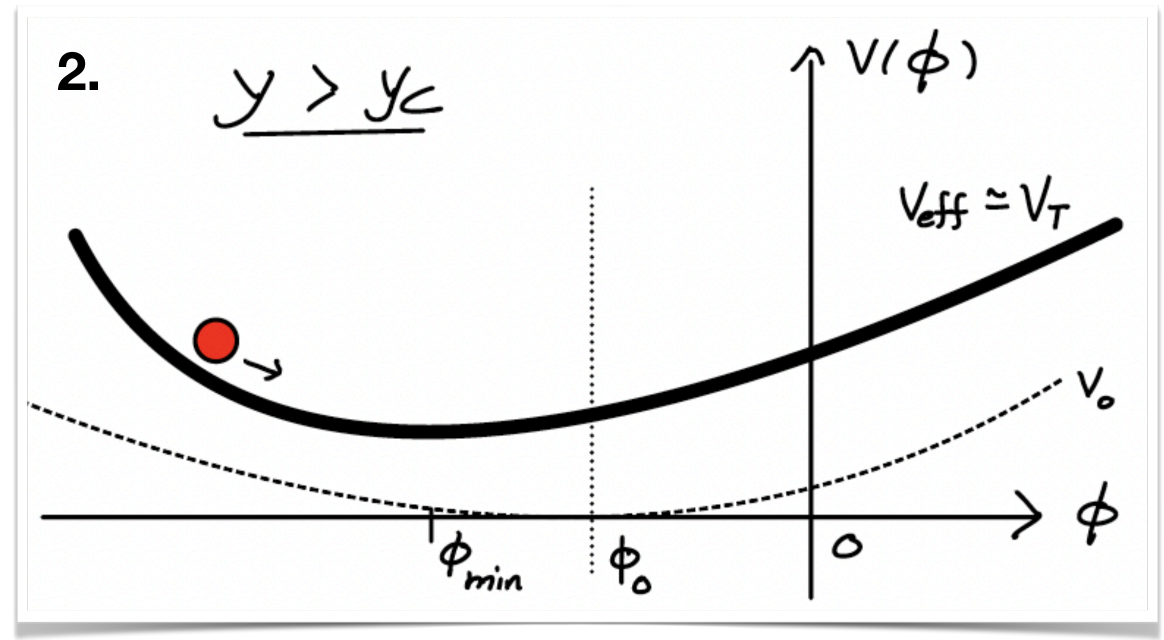
Analytic estimate of relic abundance

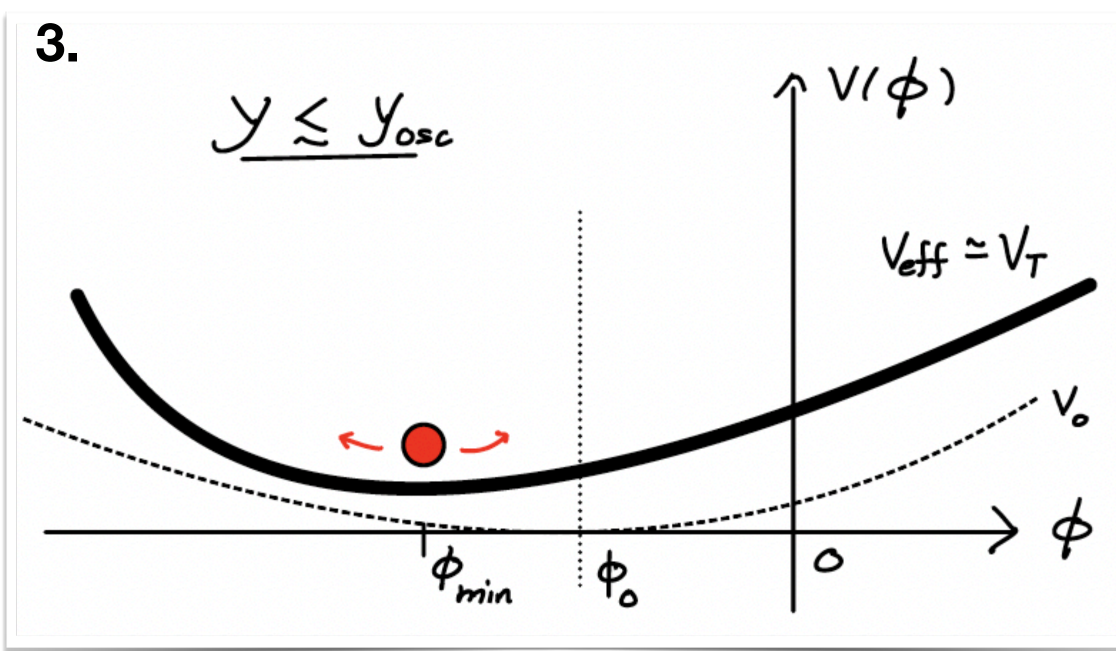
- The scalar evolution exhibits qualitatively distinct behavior depending on its mass:
  - (i) thermal misalignment dominates at large masses, (ii) VEV misalignment is important at small masses, (iii) both are relevant at intermediate masses

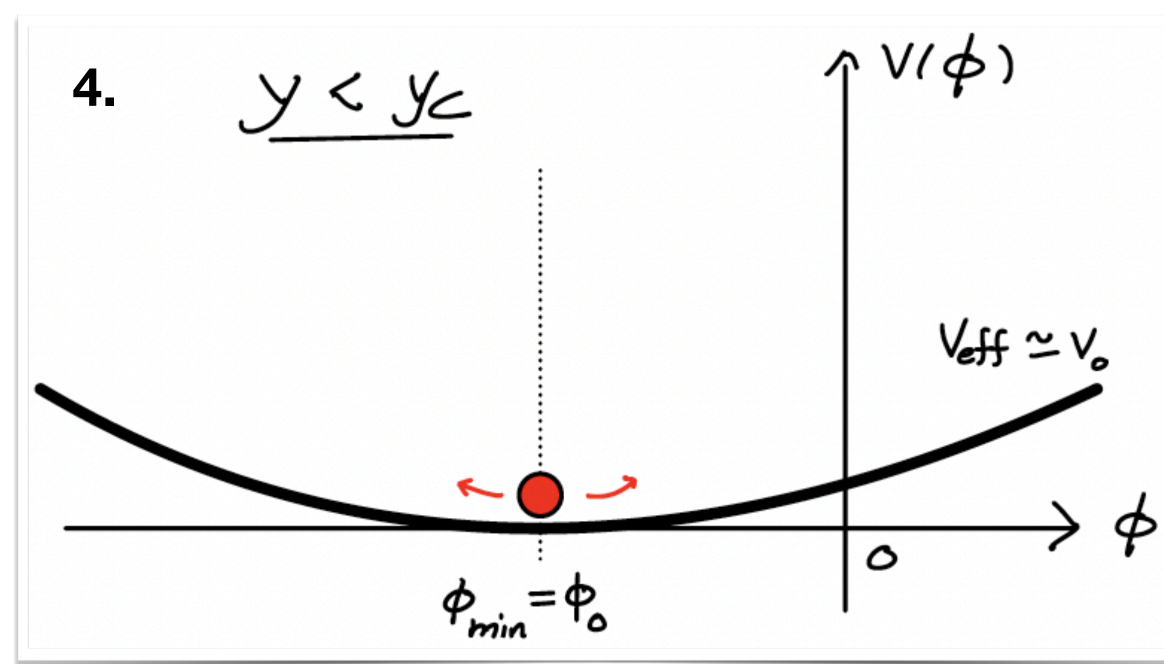


Region I (higher masses)









Region 1, Analytic Estimate

- At high temperatures, $y \gg y_c$, the scalar undergoes thermal misalignment. The approximate equation of motion and solution is

$$\hat{\phi}''(y) + \frac{\beta}{\pi^2\gamma^2 y^4} = 0 \quad \Rightarrow \quad \hat{\phi}(y) = \underbrace{-\frac{\beta}{6\pi^2\gamma^2 y^2}}_{\text{thermal misalignment}} + \hat{\phi}_i$$

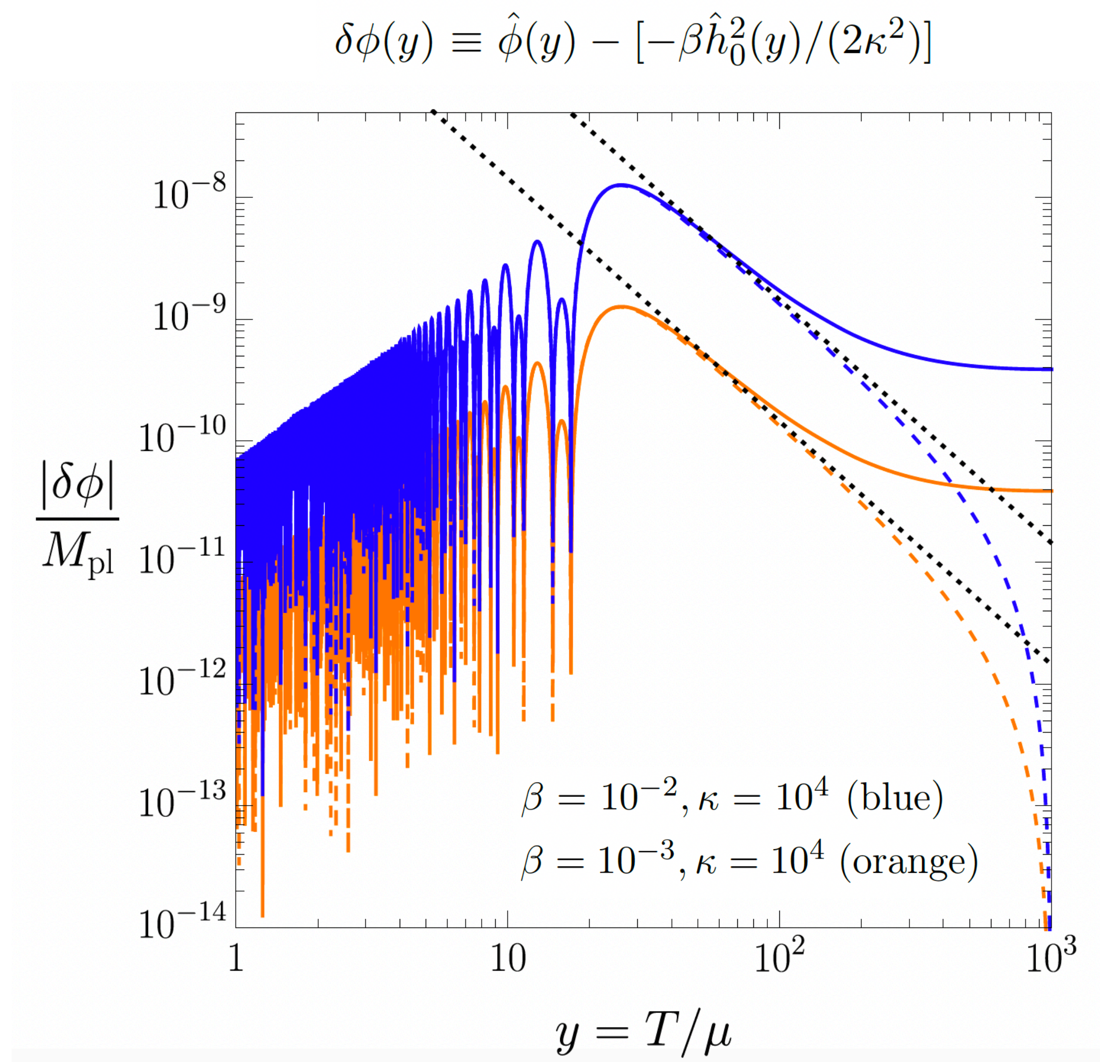
- Oscillations start at high temperatures. Using $y_{\rm osc} = \sqrt{\kappa/3\gamma}$, we estimate the oscillation amplitude to be

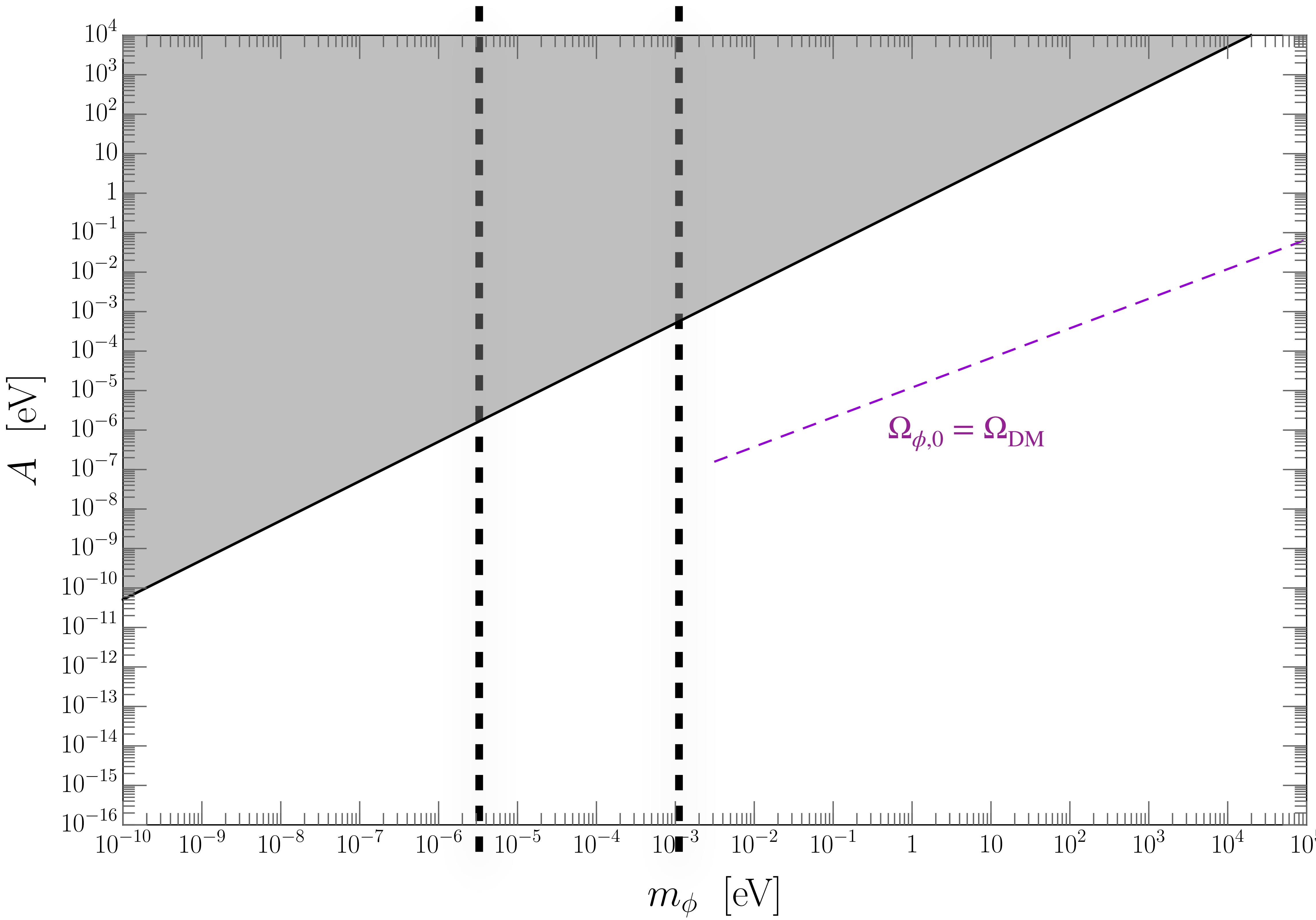
$$\hat{\phi}_{\rm osc} = \hat{\phi}(y_{\rm osc}) = -\frac{\beta}{2\pi^2\gamma\kappa} + \hat{\phi}_i$$

- Scalar relic abundance:

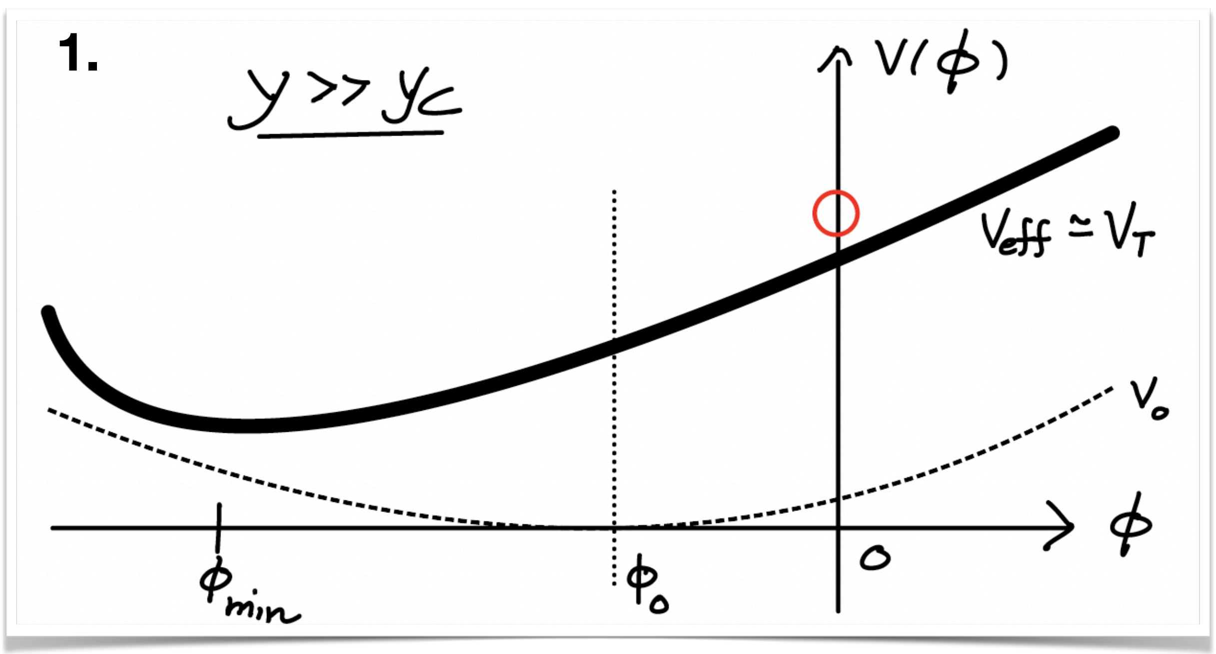
$$\Omega_\phi\big|_0 = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{\frac{1}{2}m_\phi^2\phi_{\rm osc}^2(y_0/y_{\rm osc})^3(g_{*S}^0/g_{*S}^{\rm osc})}{\rho_{c,0}}$$

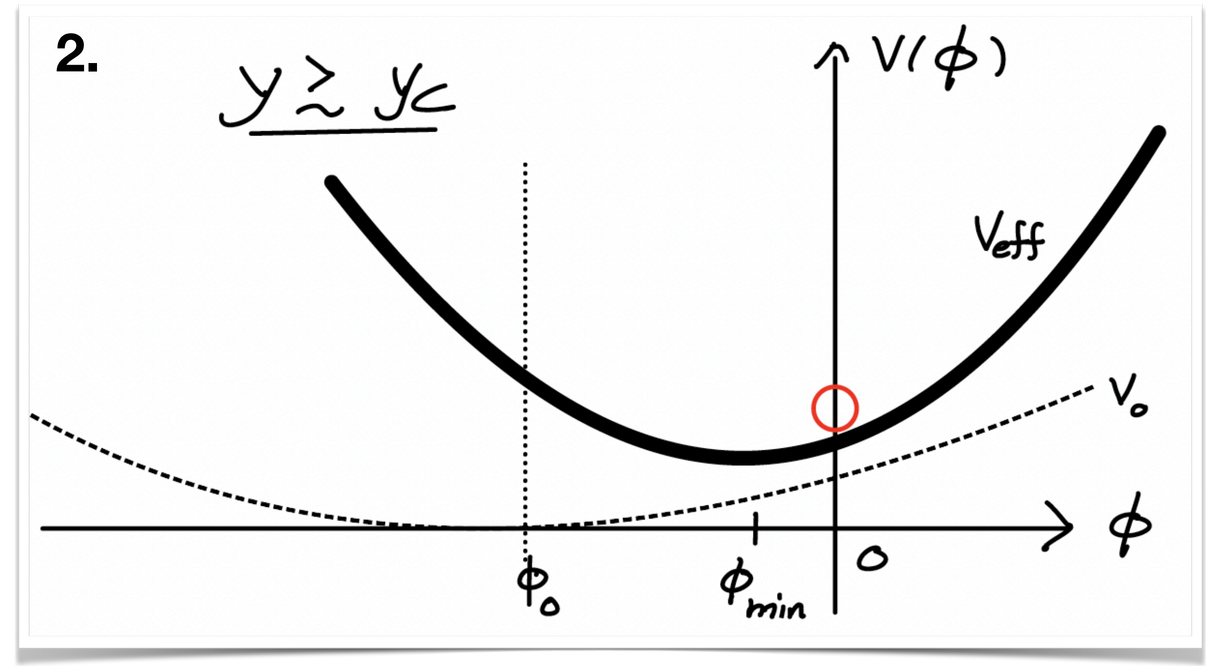
$$\simeq 0.26\left(\frac{\beta}{0.05}\right)^2\left(\frac{1000}{\kappa}\right)^{3/2}$$

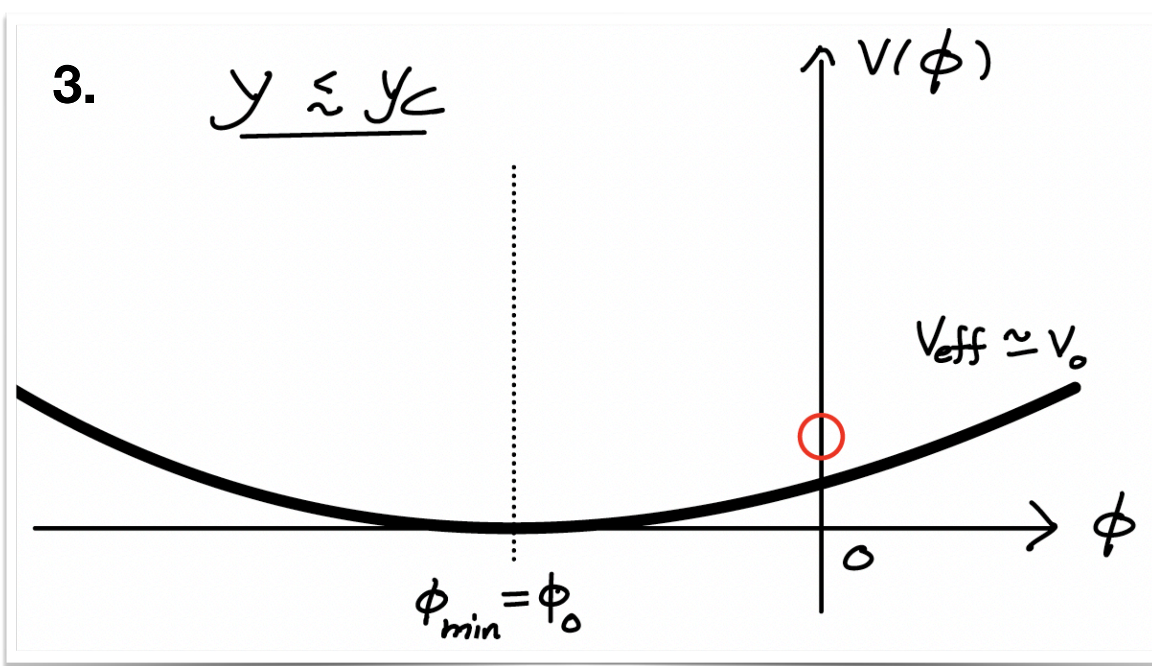


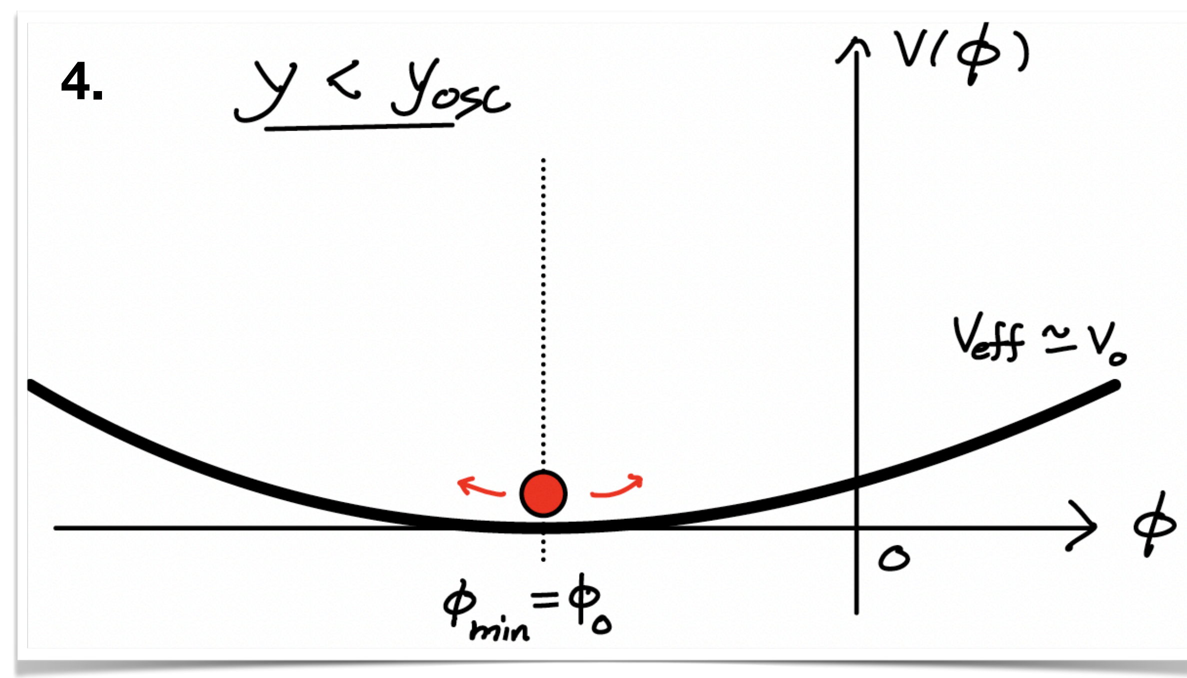

A [eV]
m_ϕ [eV]
$\Omega_{\phi,0} = \Omega_{\rm DM}$

Region 3 (lower masses), $\hat{\phi}_i = 0$



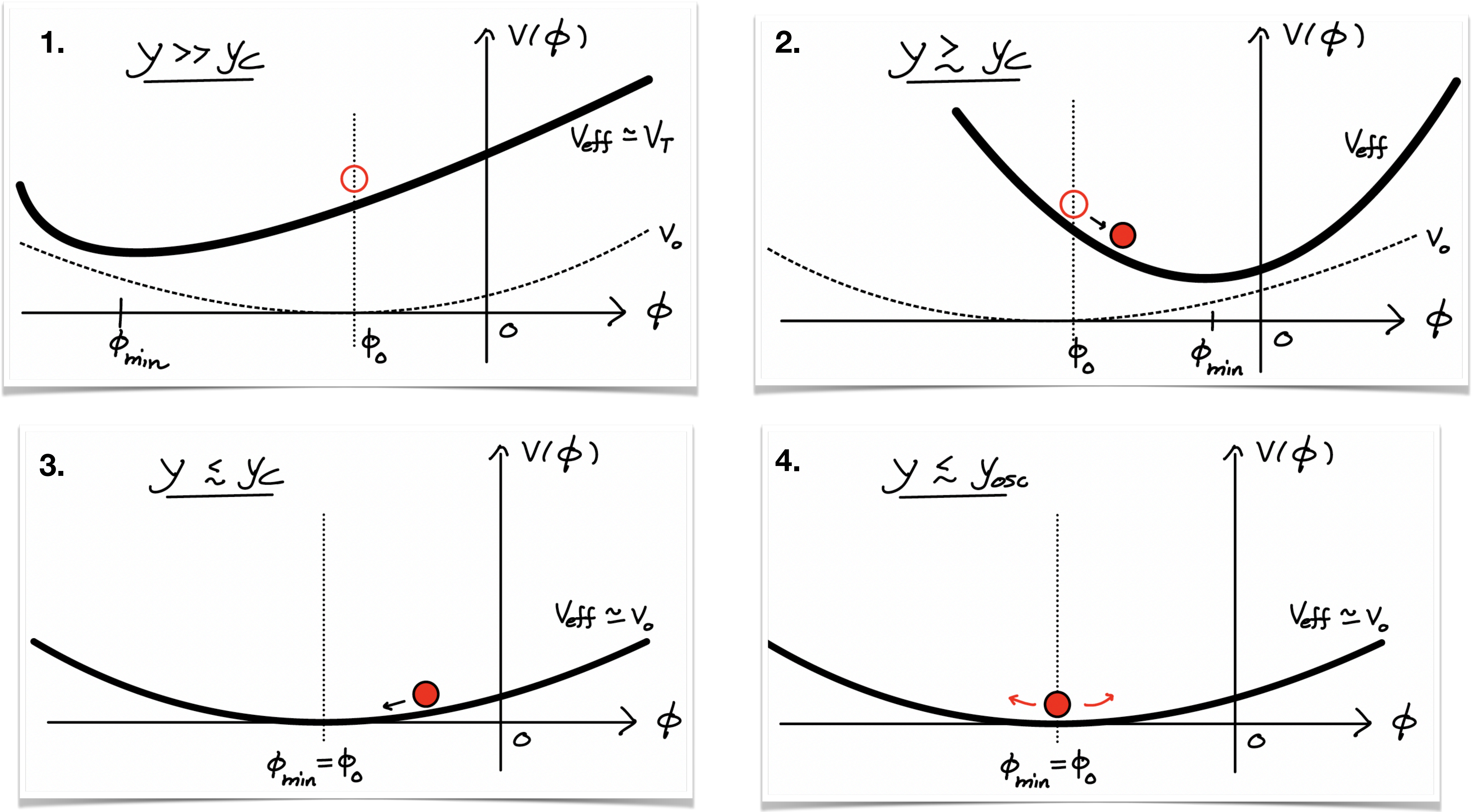




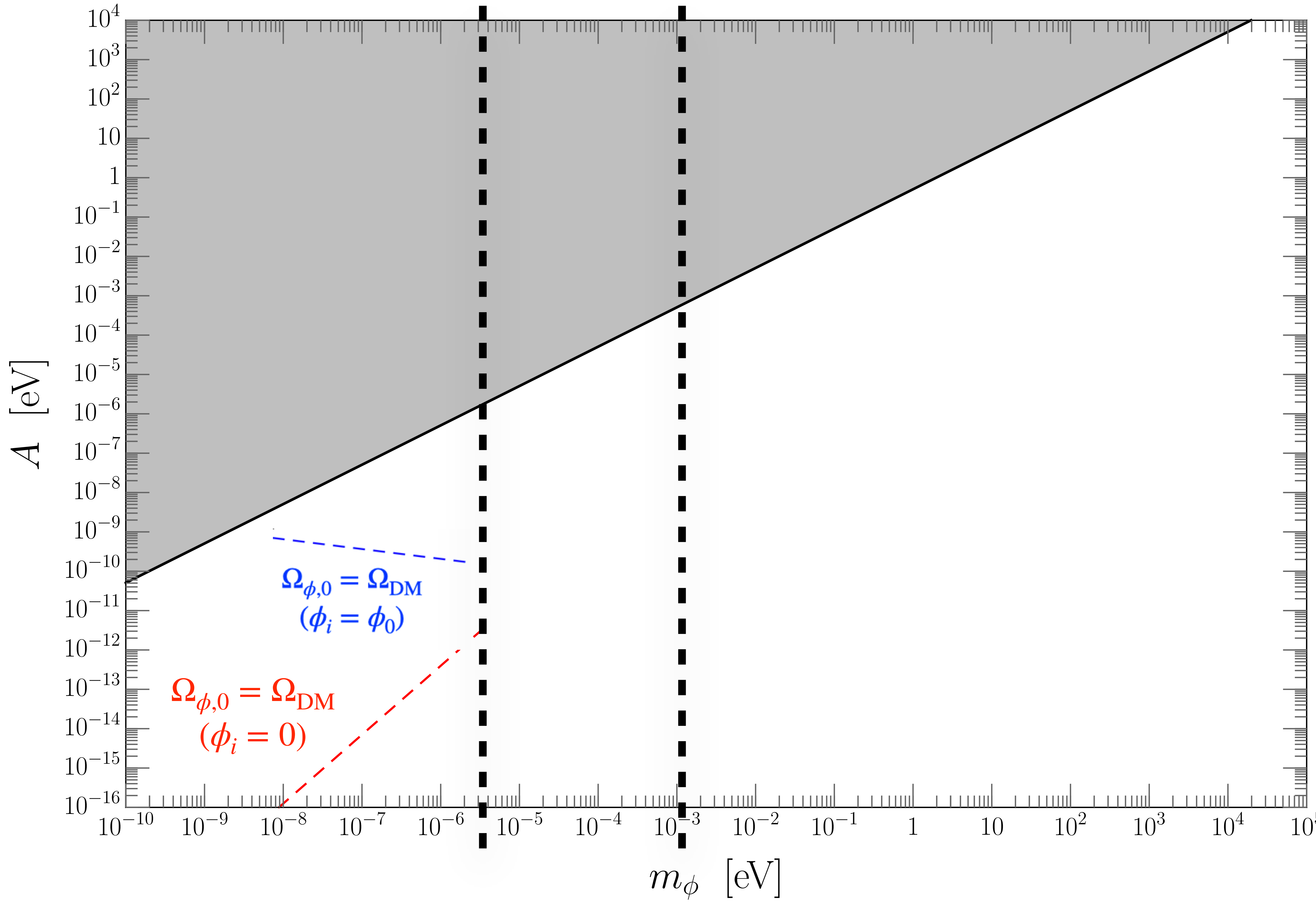


- VEV misalignment — $\phi_V \simeq \phi_0$, requires only small coupling
- See Backup Slides for analytic estimate of relic abundance

Region 3 (lower masses), $\hat{\phi}_i = \hat{\phi}_0$



- VEV misalignment induces displacement from zero temperature minimum
- See Backup Slides for analytic estimate of relic abundance

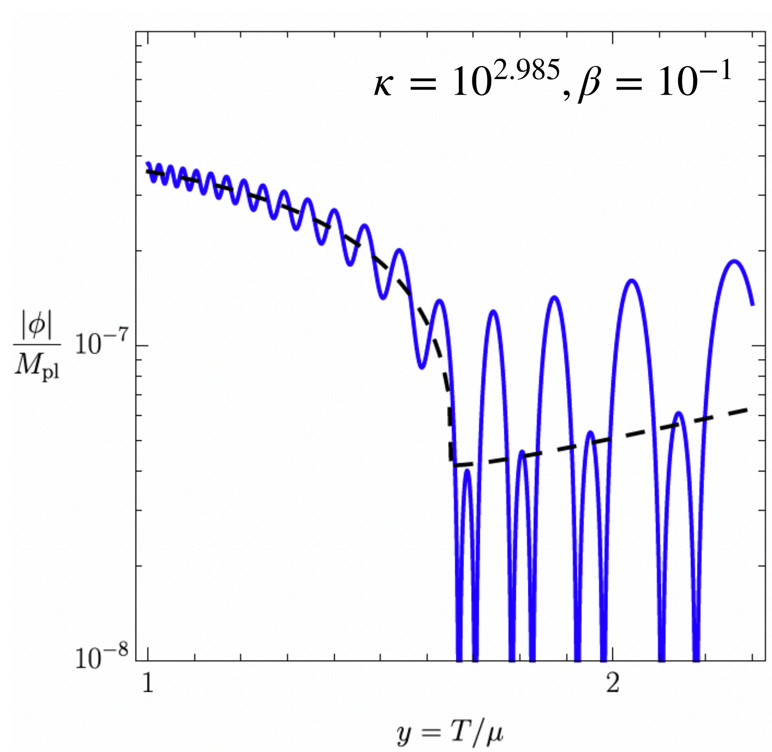


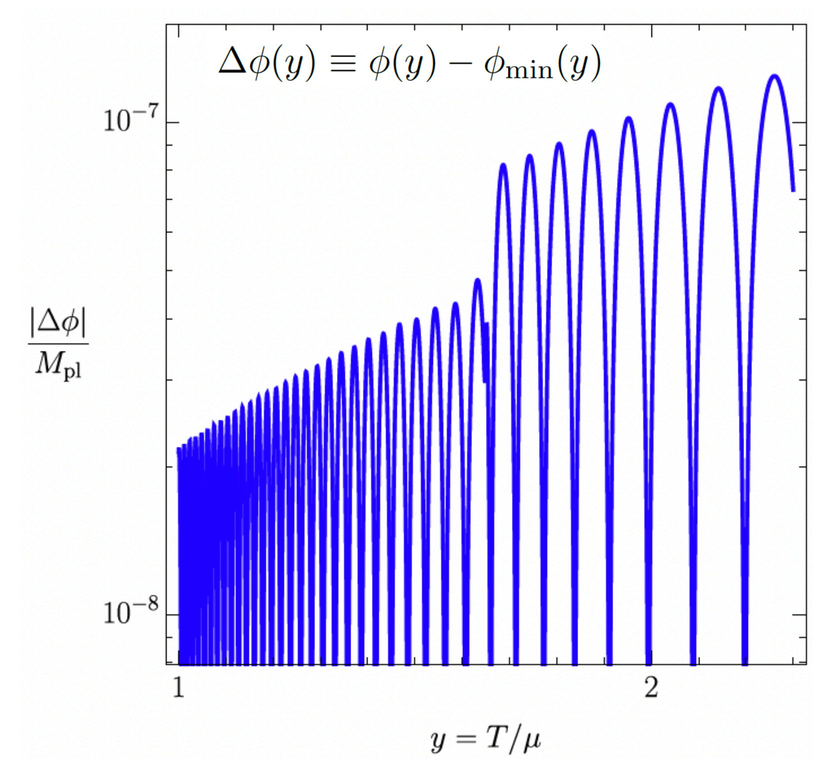
- See Backup Slides for analytic estimates of relic abundance

Region 2

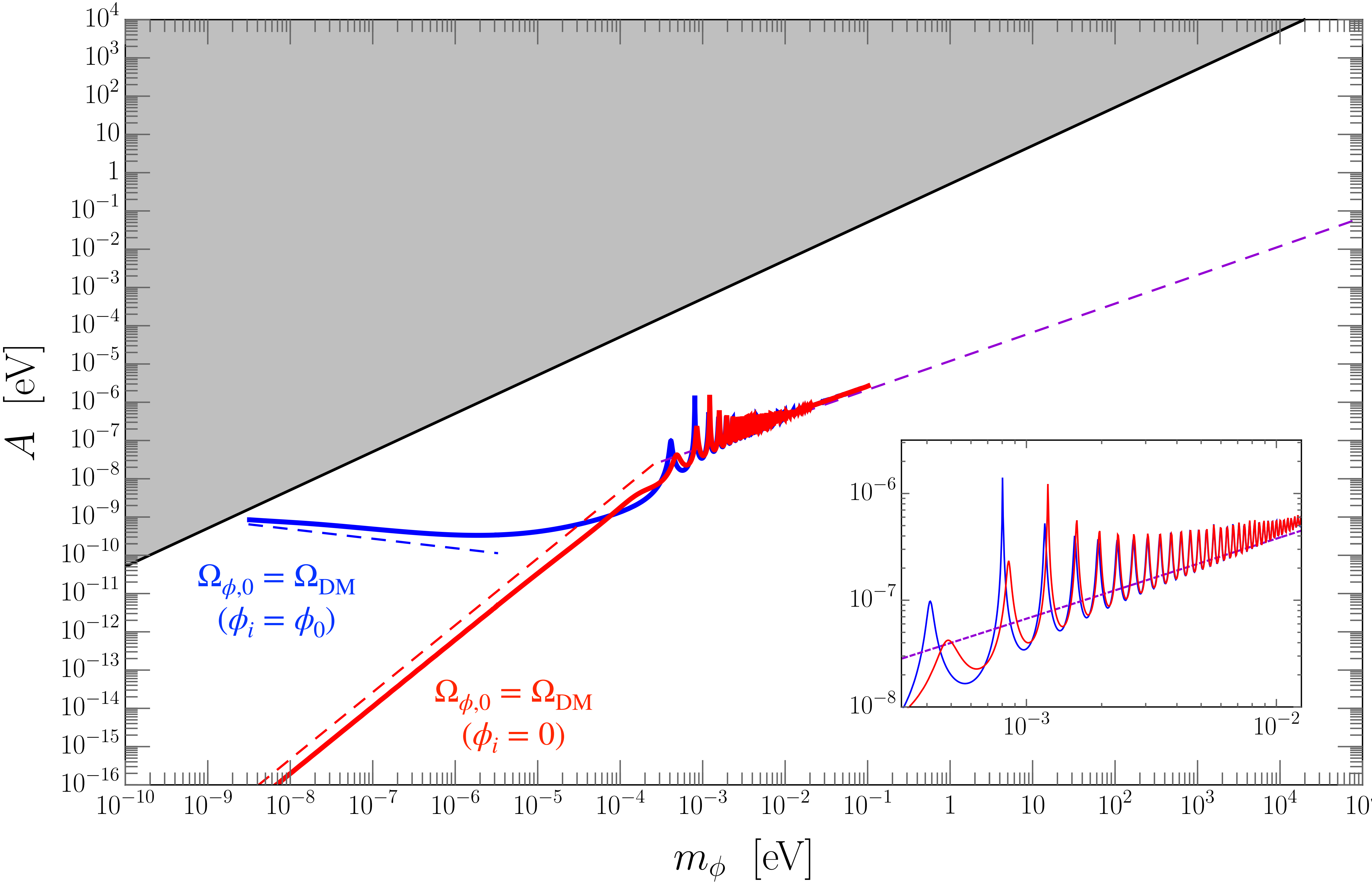
- In Region 2, the scalar evolution is the result of a competition between thermal misalignment and VEV misalignment.
- Initially, thermal misalignment occurs at high temperatures and oscillations begin before the EWPT
- At the EWPT, the Higgs field rapidly moves from the origin towards $h \to v$, simultaneously inducing a shift in the ϕ VEV towards its zero-temperature value.
- This acts as a step-like forcing term in the scalar equation of motion, causing a suppression or enhancement in the oscillation amplitude

- In the example at right, the scalar field is near its oscillation maximum as the shift in the ϕ VEV occurs.





Relic abundance results



Experimental and observational probes

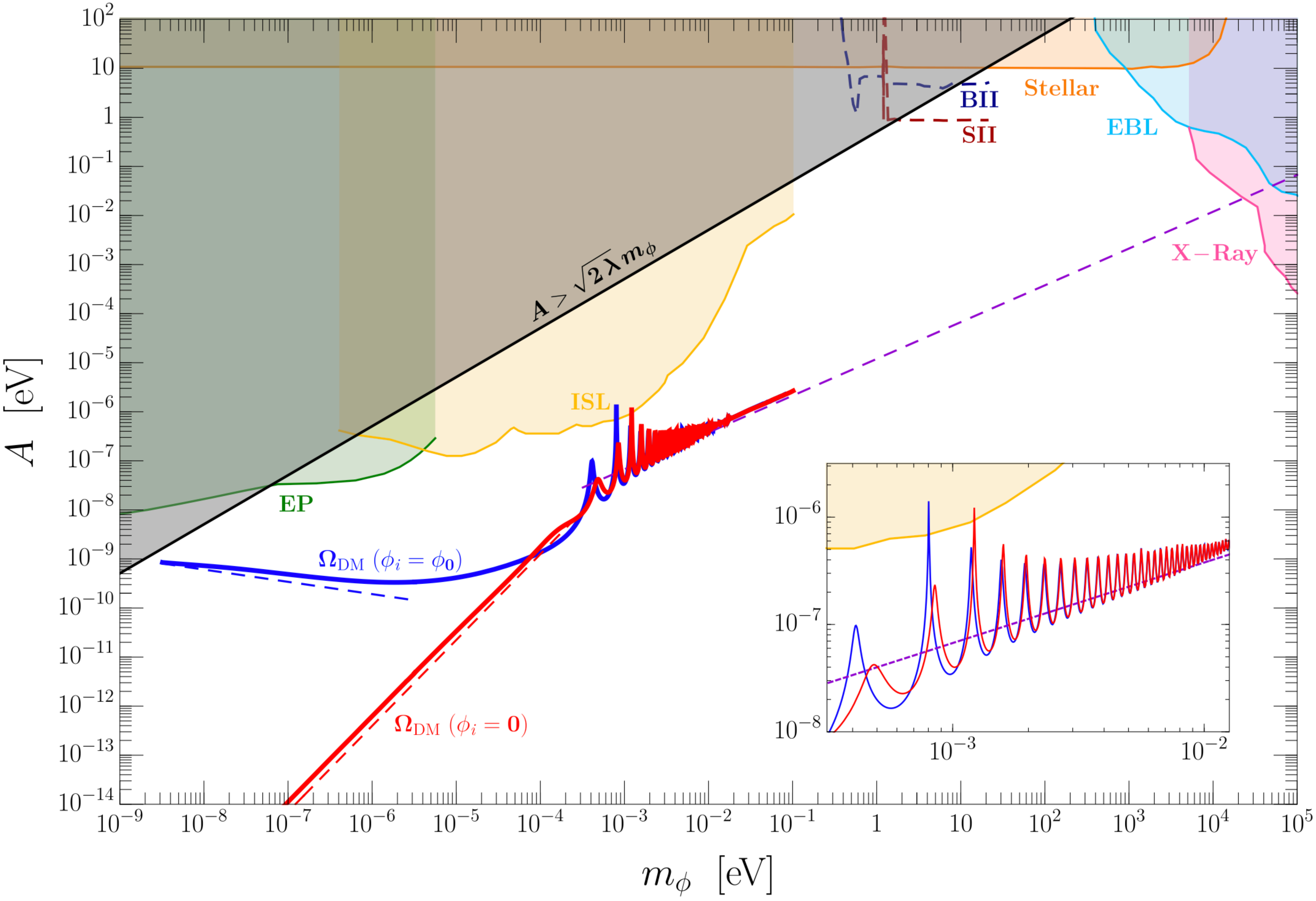
- Equivalence principle / inverse square law tests
  [Piazza, Pospelov, '10]
  [Graham, Kaplan, Mardon, Rajendran, Terrano '16]

- Stellar cooling
  [Hardy, Lasenby, '16]

- Extragalactic background light and X-rays
  [Cadamuro, Redondo, '11]
  [Flacke, Frugiuele, Fuchs, Gupta, Perez, '17]
  [Essig, Kuflik, McDermott, Volansky, Zurek, '11]
  [Fradette, Pospelov, Pradler, Ritz, '18]

- Resonant absorption in molecules
  [Arvanitaki, McDermott, Van Tilburg '17]

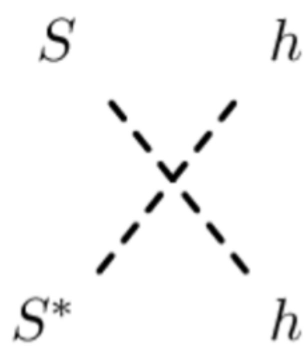
- At lower masses: atomic & nuclear clocks, atom interferometers, black hole superradiance, Lyman-alpha, …

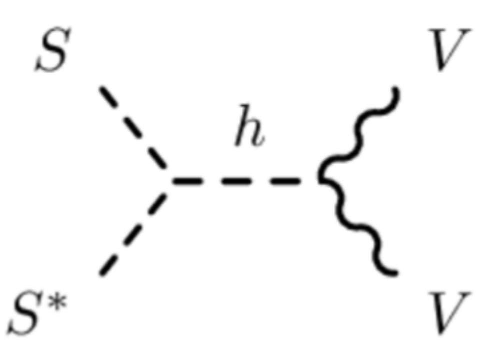


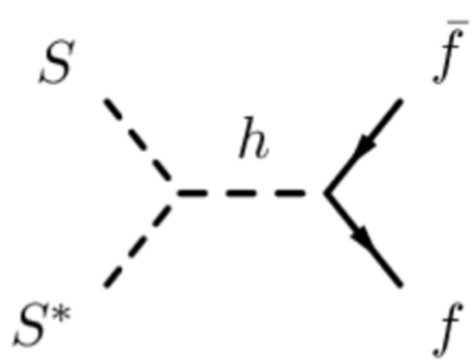
$A > \sqrt{2}\lambda m_\phi$
Stellar
BII
SII
EBL
X−Ray
ISL
EP
Ω_{DM} ($\phi_i = \phi_0$)
Ω_{DM} ($\phi_i = 0$)
A [eV]
m_ϕ [eV]

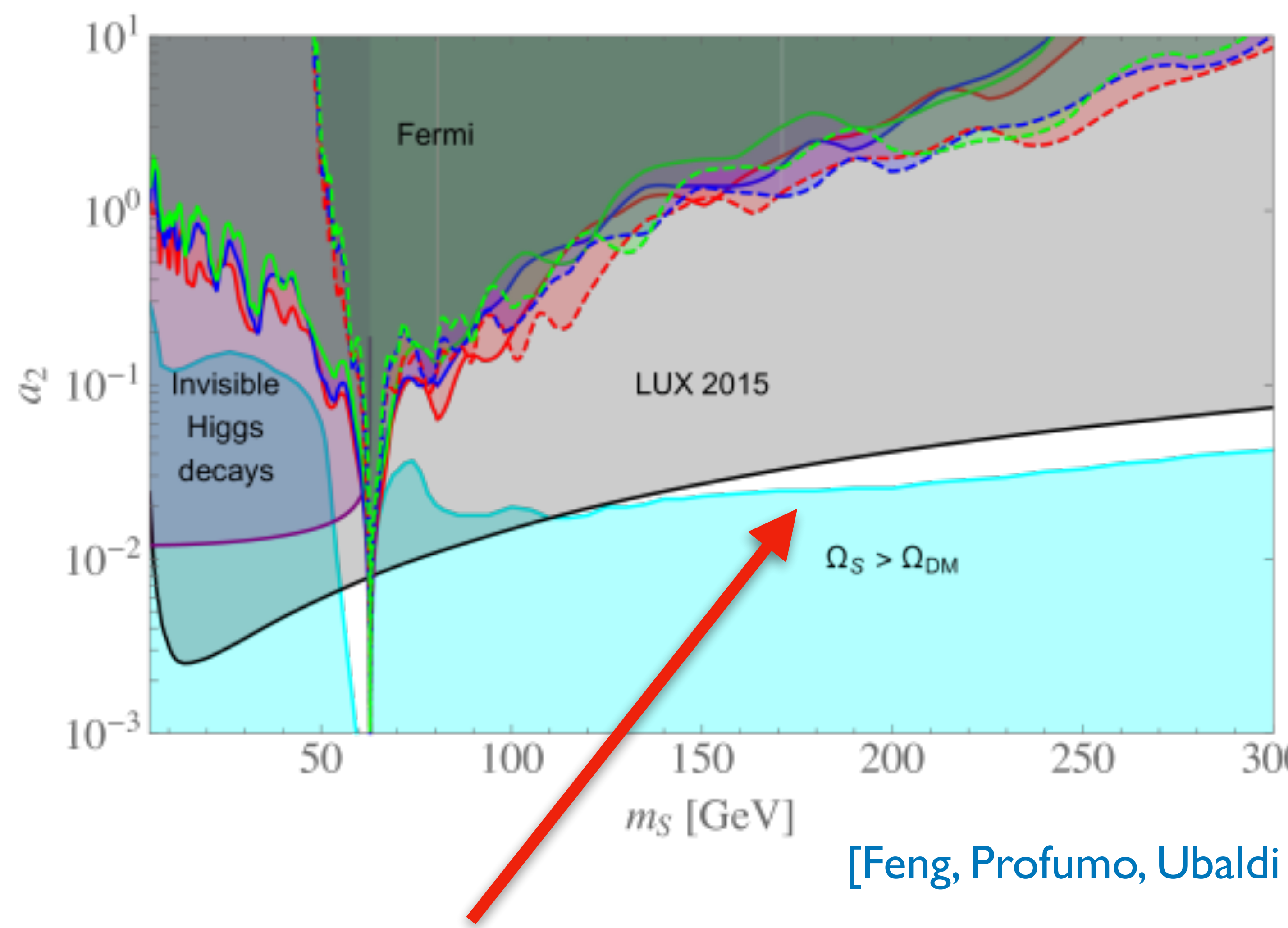
Comparison to WIMP scalar singlet DM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{b_2}{2} S^2 - \frac{b_4}{4} S^4 - a_2 S^2 H^\dagger H \,,$$









[Feng, Profumo, Ubaldi ’14]

Relic Density Target

Outlook

- Ultralight bosons represent a well-motivated and phenomenologically distinctive class of DM models.
- We have studied the cosmology of a light scalar coupled through the super-renormalizable Higgs portal.
- The cosmology of this scenario is rich and distinctive, involving the dynamical misalignment of the scalar field during the radiation era through two competing mechanisms: *thermal misalignment* and *VEV misalignment.*
- Under certain conditions, a relic density target can be defined which is not insensitive to initial conditions.
- New ideas are needed to probe much of the cosmologically interesting regions of parameter space.

Backup

Region 3, $\hat{\phi}_i = 0$ — Analytic Estimate

- Thermal misalignment is negligible. The scalar is held up by Hubble friction at its initial value, $\phi = \phi_i = 0$.

- After the EWPT, the scalar VEV rapidly transitions to its zero temperature minimum, generating misalignment.

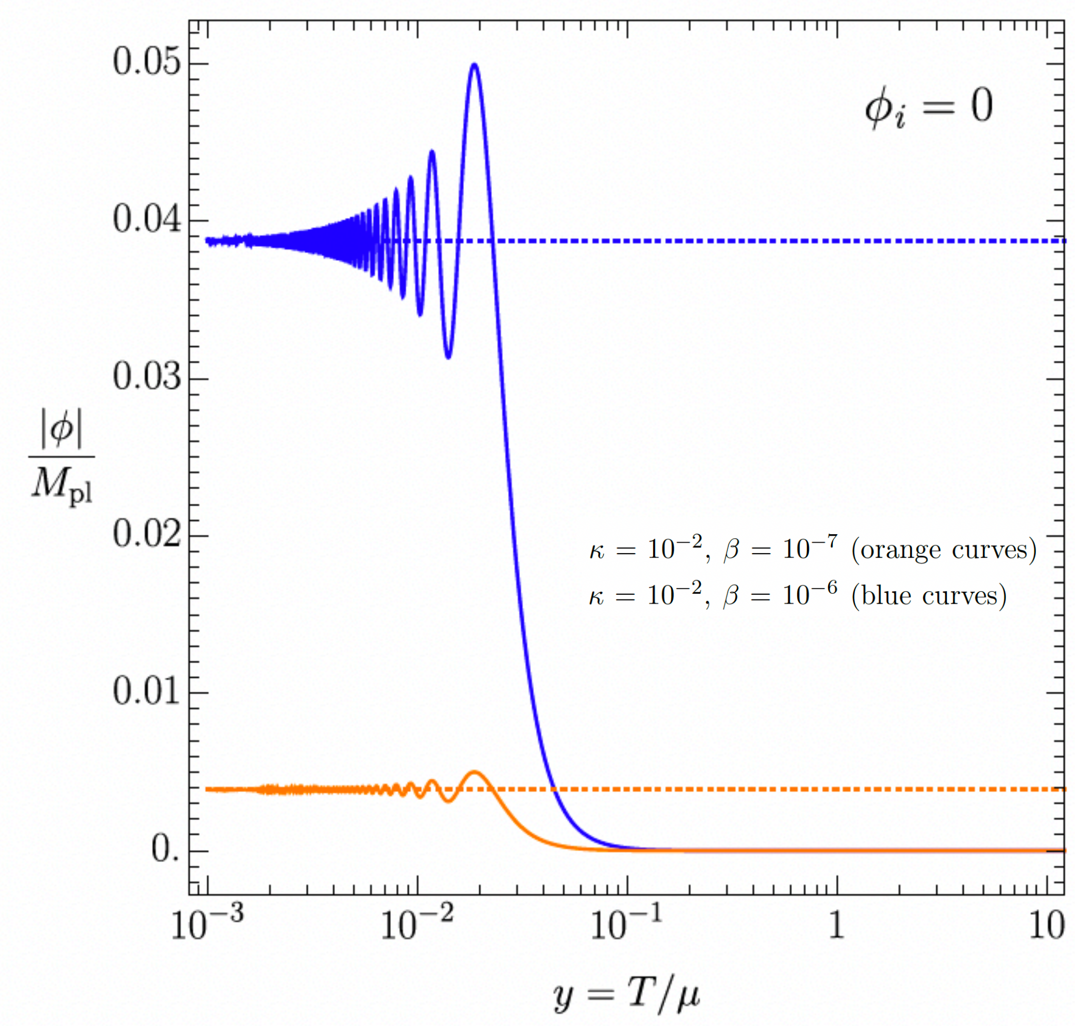
- Eventually, oscillations begin, with amplitude given by the VEV misalignment

$$\hat{\phi}(y_{\rm osc}) \simeq \hat{\phi}_0 \simeq -\beta/(2\lambda\kappa^2) \qquad y_{\rm osc} \simeq \sqrt{\frac{\kappa}{3\gamma}}$$

- Scalar relic abundance:

$$\Omega_\phi\big|_0 = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{\frac{1}{2}m_\phi^2\phi_{\rm osc}^2(y_0/y_{\rm osc})^3(g_{*S}^0/g_{*S}^{\rm osc})}{\rho_{c,0}}$$

$$\simeq 0.26\left(\frac{\beta}{3\times10^{-10}}\right)^2\left(\frac{10^{-2}}{\kappa}\right)^{7/2}$$



Region 3, $\hat{\phi}_i = \hat{\phi}_0$ — Analytic Estimate

- Thermal misalignment is negligible. As the temperature approaches the EW scale, the scalar mass term dominates. The equation of motion and solution is

$$\hat{\phi}''(y) + \frac{1}{\gamma^2 y^6}\left(\kappa^2 \hat{\phi}\right) = 0 \quad \Rightarrow \quad \hat{\phi}''(y) - \frac{\beta}{2\gamma^2\lambda}\frac{1}{y^6} = 0 \quad \Rightarrow \quad \hat{\phi}(y) = \frac{1}{y^4}\frac{\beta}{40\gamma^2\lambda} + \hat{\phi}_0$$

- At $y \sim 1$, the Higgs is Boltzmann suppressed and Hubble friction dominates. The trajectory asymptotes to a maximum value, which gives the oscillation amplitude:

$$\hat{\phi}(y_{\rm osc}) \simeq 5\beta/(40\gamma^2\lambda) \qquad y_{\rm osc} \simeq \sqrt{\frac{\kappa}{3\gamma}}$$

- Scalar relic abundance:

$$\Omega_\phi\big|_0 = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{\frac{1}{2} m_\phi^2 \phi_{\rm osc}^2 (y_0/y_{\rm osc})^3 (g_{*S}^0/g_{*S}^{\rm osc})}{\rho_{c,0}}$$

$$\simeq 0.26 \left(\frac{\beta}{10^{-4}}\right)^2 \left(\frac{\kappa}{4\times 10^{-2}}\right)^{1/2}$$

$\Delta\phi(y) \equiv \phi(y) - \phi_0$