# Probing ultralight dark matter with Interferometers

**Hyungjin Kim (DESY)**

May 24, 2023
University of Padova

What can ***GW interferometers*** tell us about
the nature of dark matter (more specifically ***ultralight dark matter***)?

## *Ultralight dark matter (ULDM)*

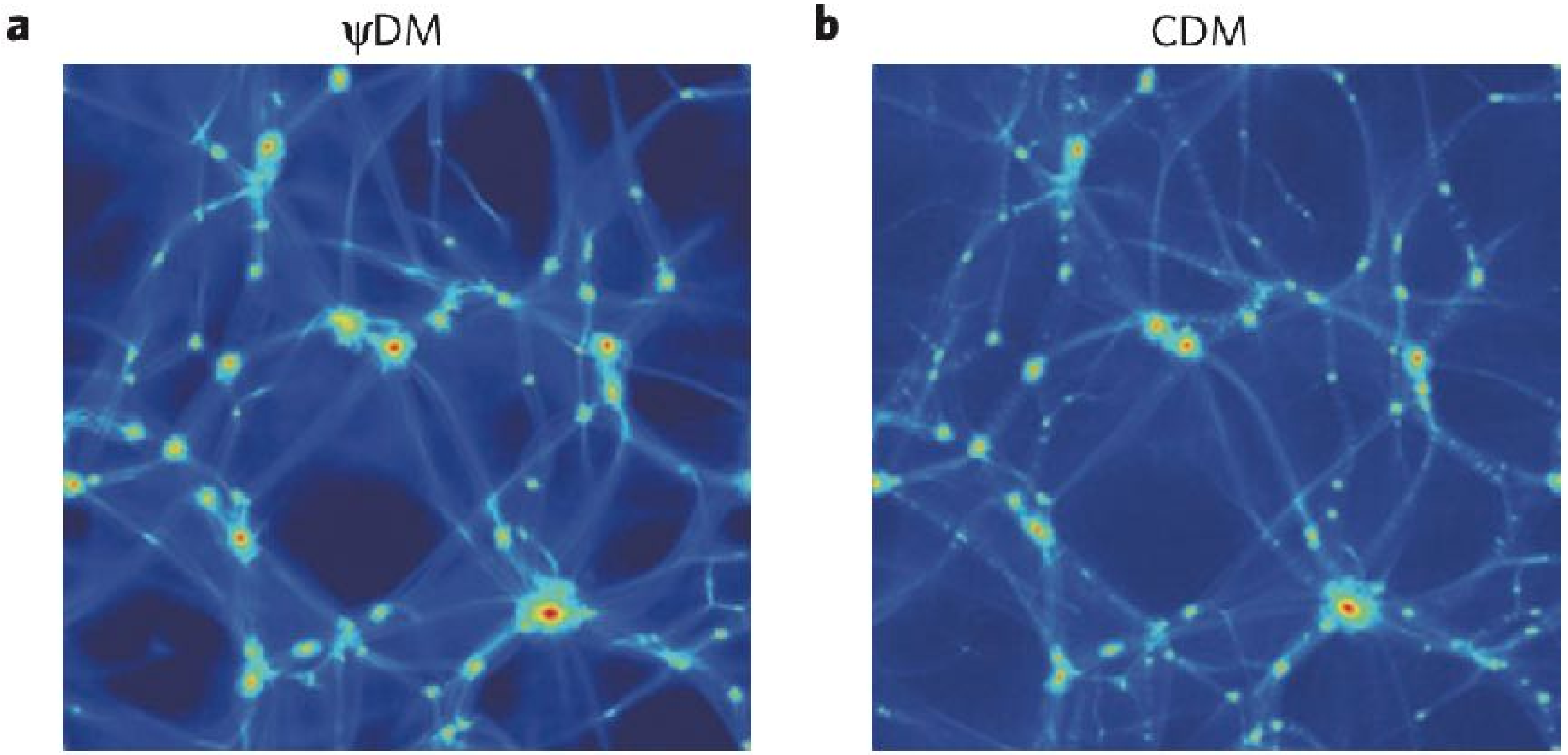
# Terminology:

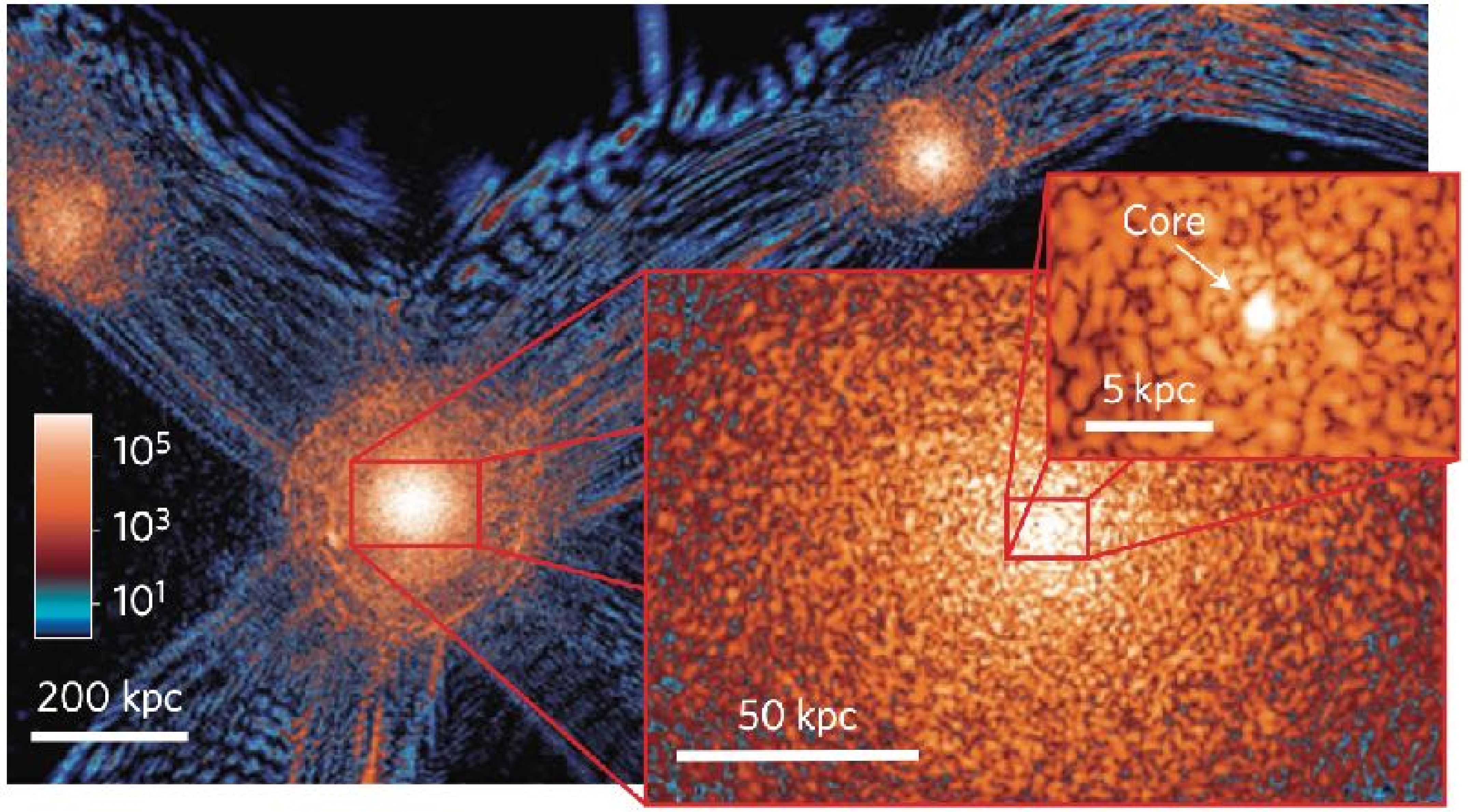
Ultralight (wave) dark matter

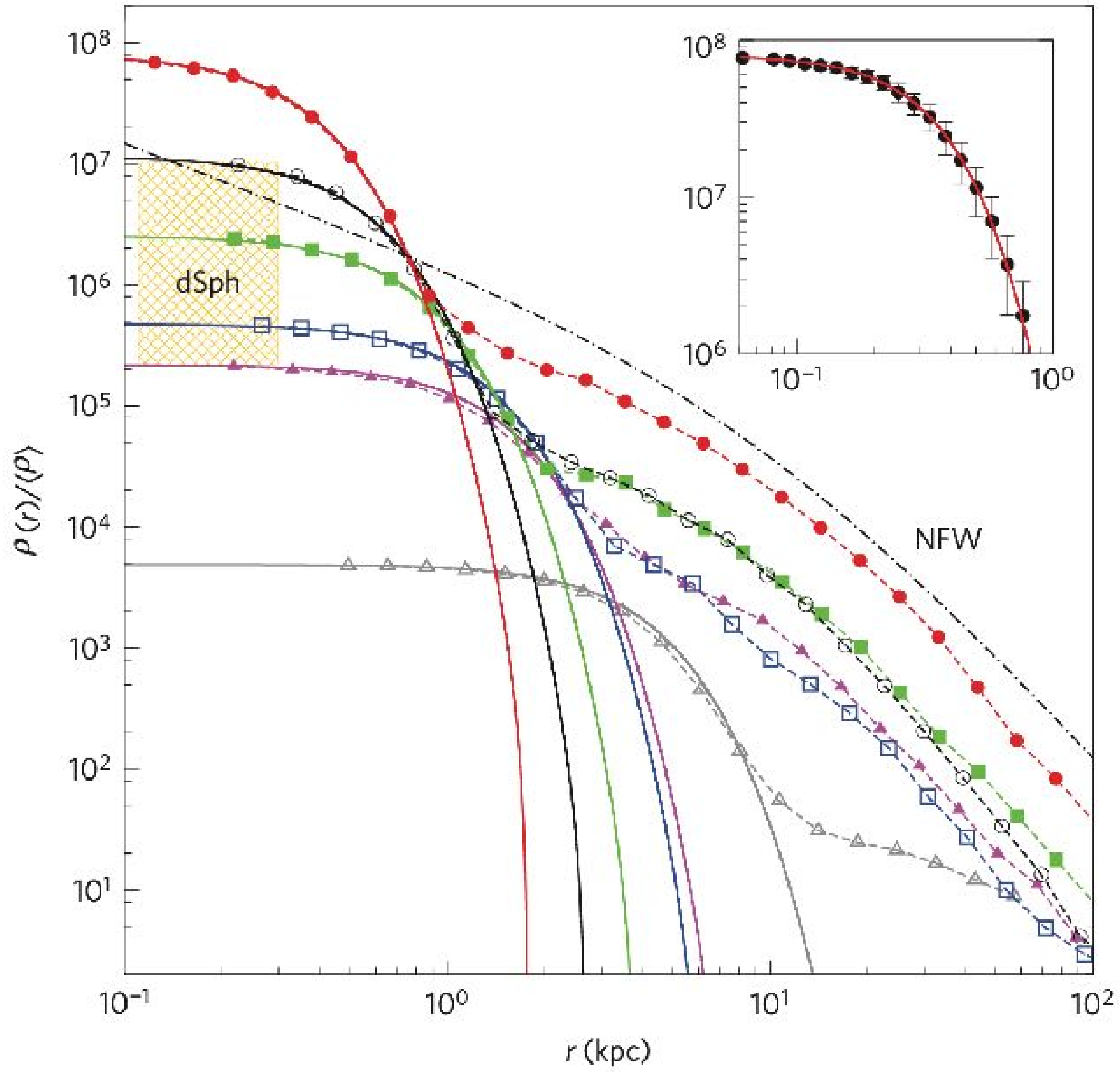
$$m \lesssim 10\,\mathrm{eV}$$

$$N_{\mathrm{occ}} \sim n_{\mathrm{dm}}\lambda^3 \sim \left(\frac{10\,\mathrm{eV}}{m}\right)^4$$

Schive, Chiueh, Broadhurst (14)

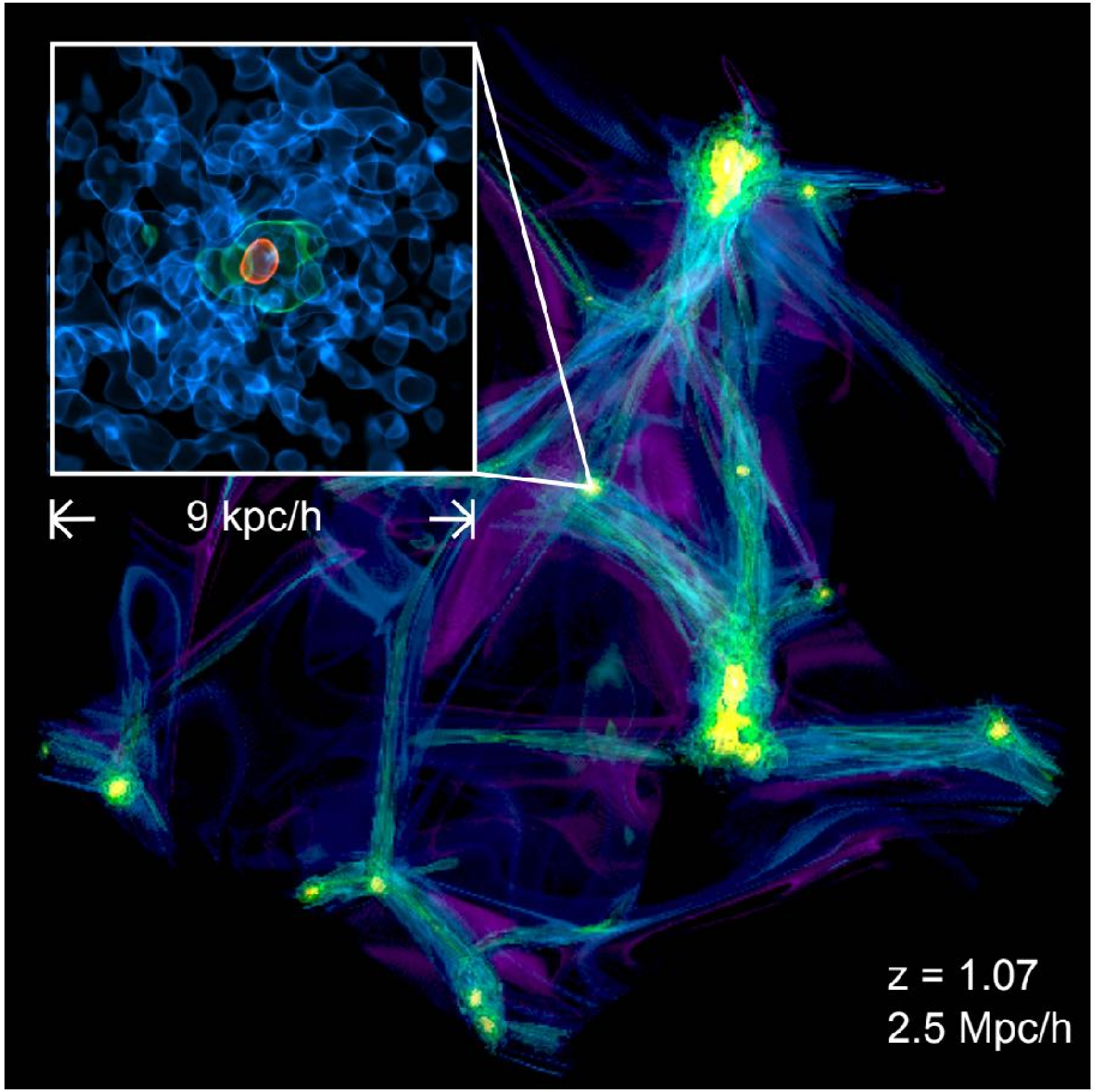


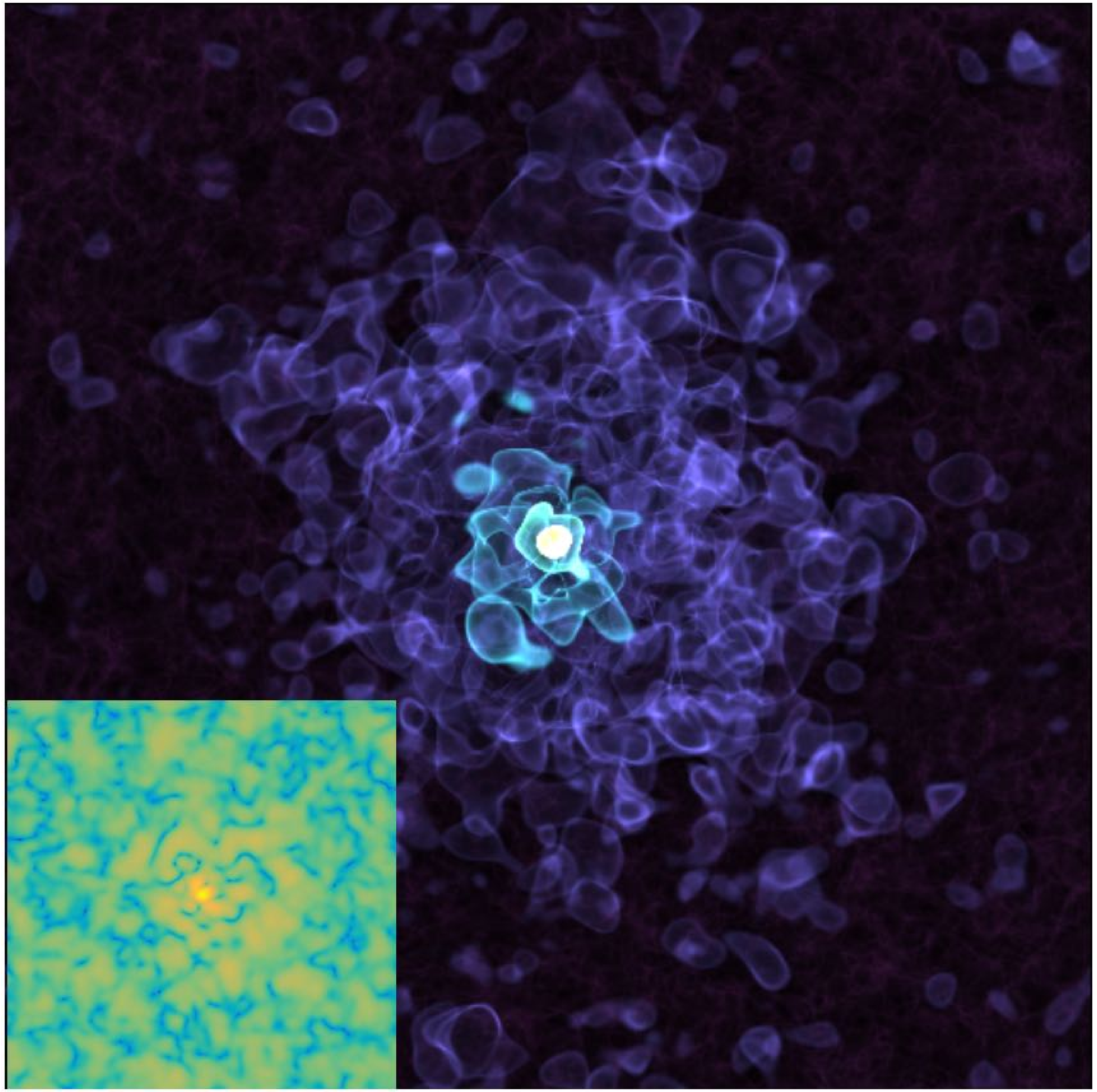

* characteristic *soliton* at the center has been observed

* small scale structures are erased




Veltmaat, Niemeyer, Schwabe (18)

Mocz et al (17)



The granule structure

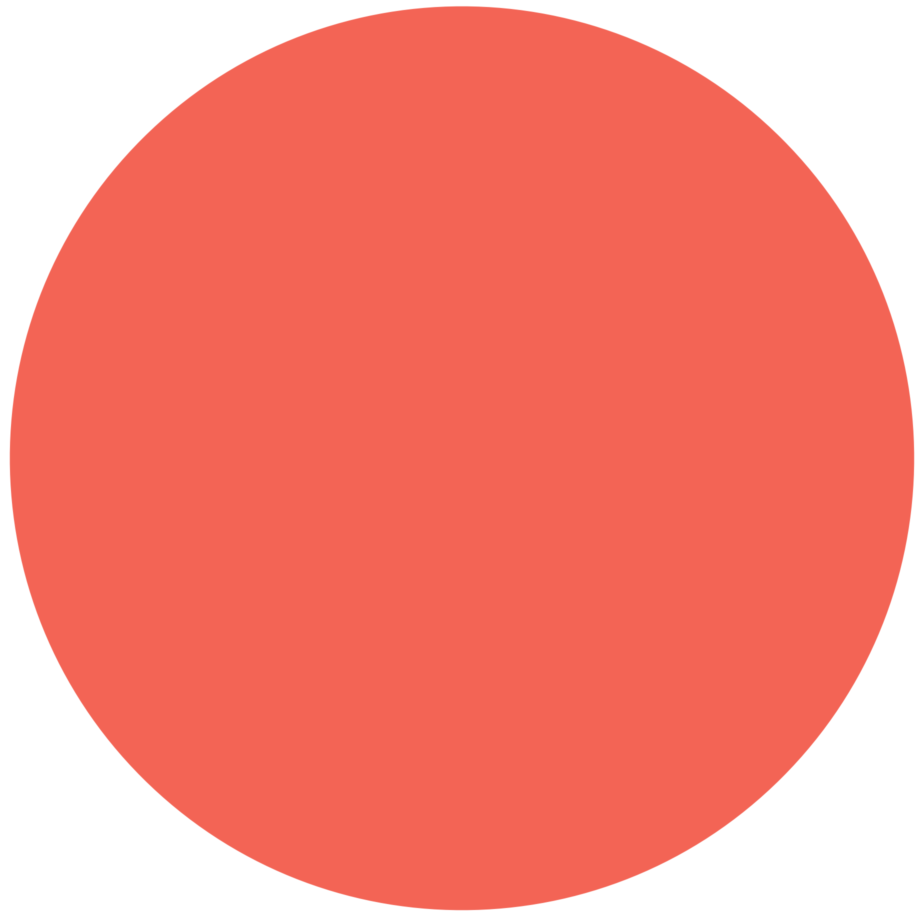
$$\frac{\delta\rho}{\rho} \sim \mathcal{O}(1)$$

over the scale of

$$\lambda = \frac{2\pi}{mv} \simeq 0.6\,\mathrm{kpc}\Big(\frac{10^{-22}\,\mathrm{eV}}{m}\Big)\Big(\frac{200\,\mathrm{km/sec}}{v}\Big)$$

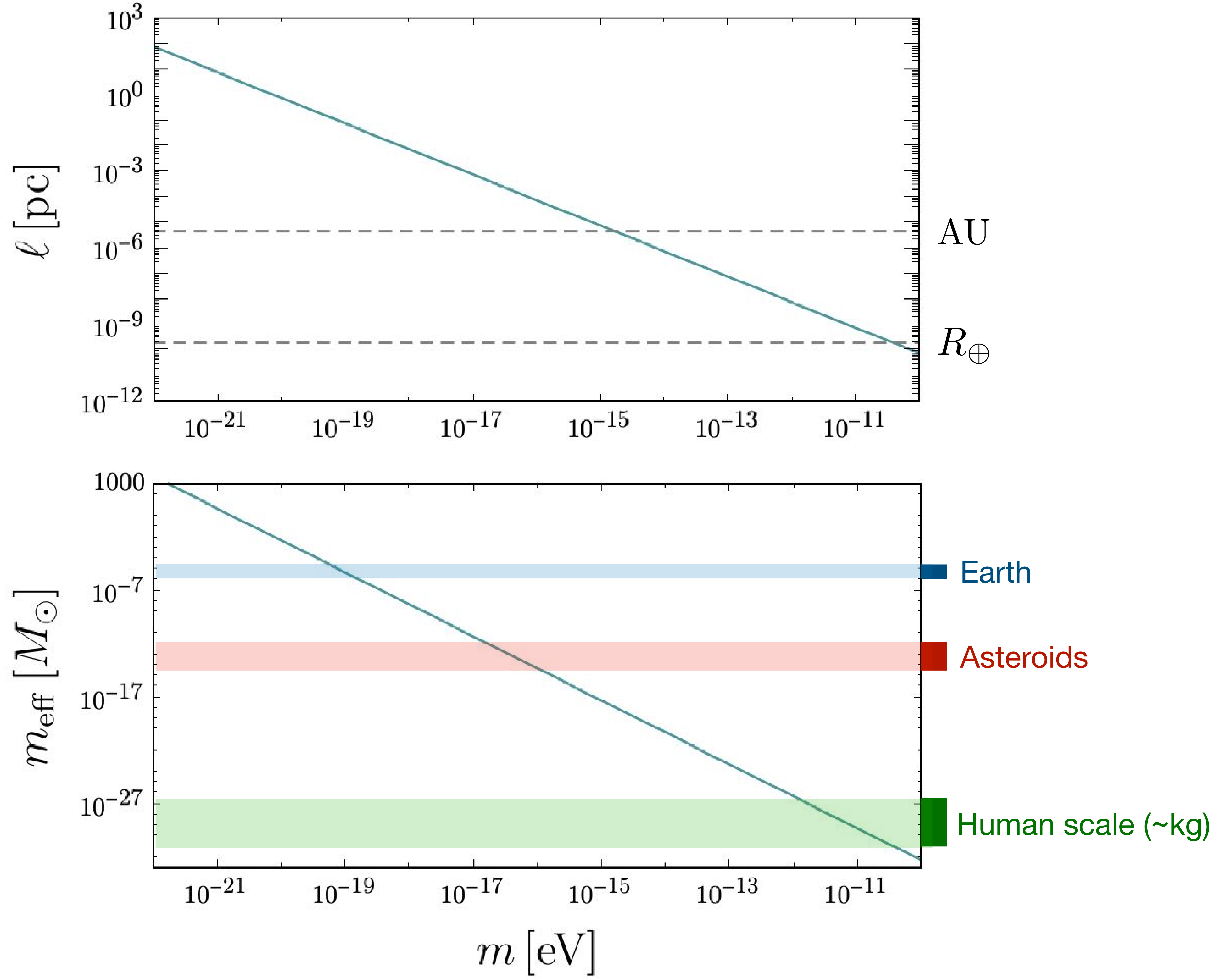
An intuitive understanding of the granule structure:

***Quasiparticle***



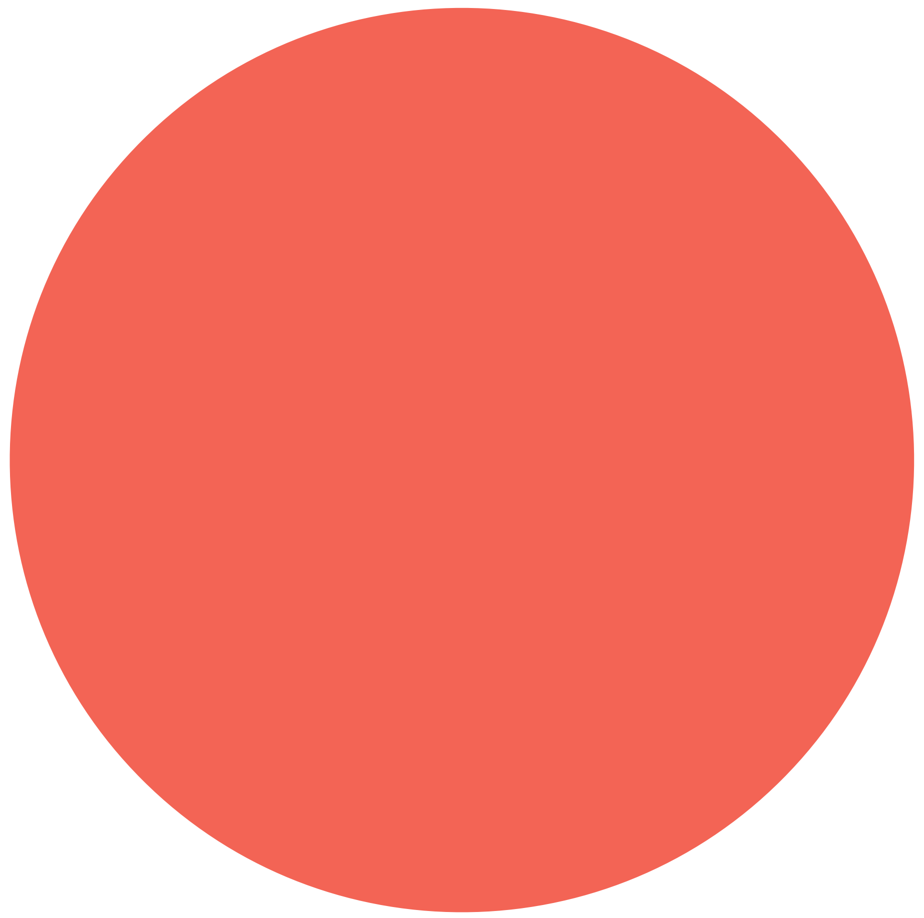
$$\ell \sim \lambda = \frac{1}{mv}$$

$$m_{\rm eff} \sim \rho_{\rm DM} \ell^3$$



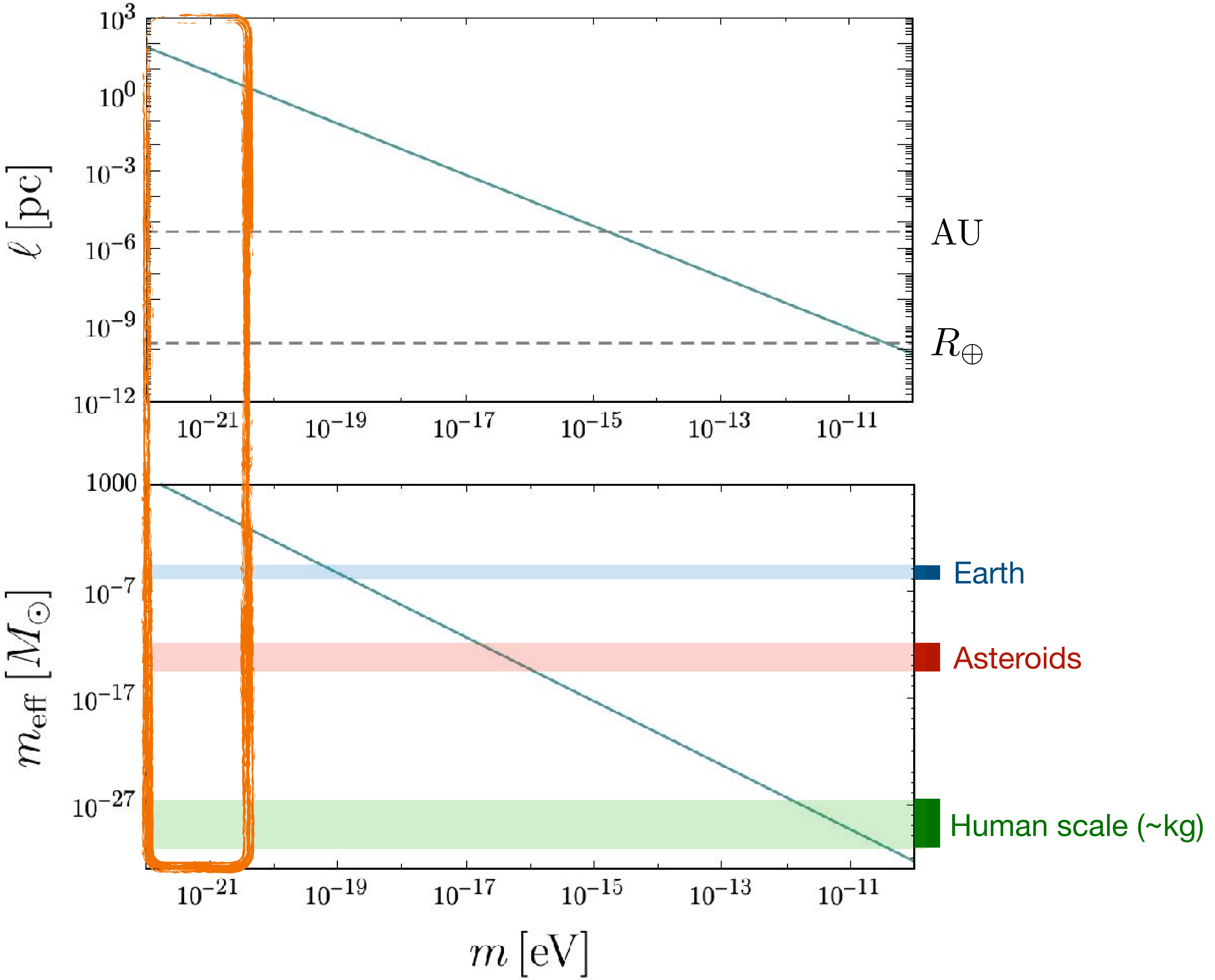
$\ell$ [pc]
$10^{3}$
$10^{0}$
$10^{-3}$
$10^{-6}$
$10^{-9}$
$10^{-12}$
AU
$R_\oplus$
$10^{-21}$
$10^{-19}$
$10^{-17}$
$10^{-15}$
$10^{-13}$
$10^{-11}$
$m_{\rm eff}\,[M_\odot]$
1000
$10^{-7}$
$10^{-17}$
$10^{-27}$
Earth
Asteroids
Human scale (~kg)
$m$ [eV]

$m \sim 10^{-22}\,\mathrm{eV}$

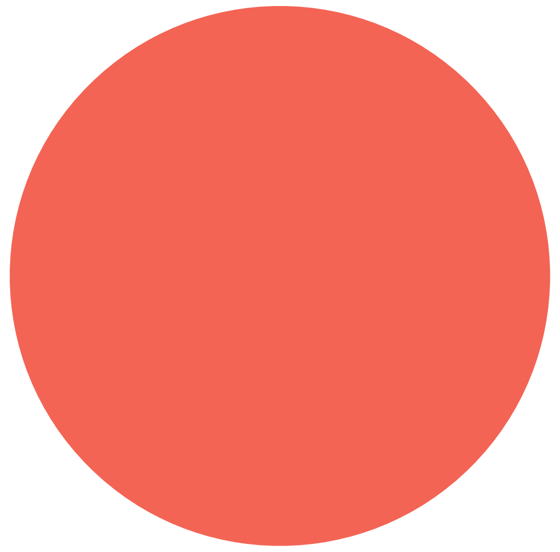


$m_{\rm eff} \sim \mathcal{O}(10^4) M_\odot$

$\ell \sim \mathcal{O}(10^2)\mathrm{pc}$

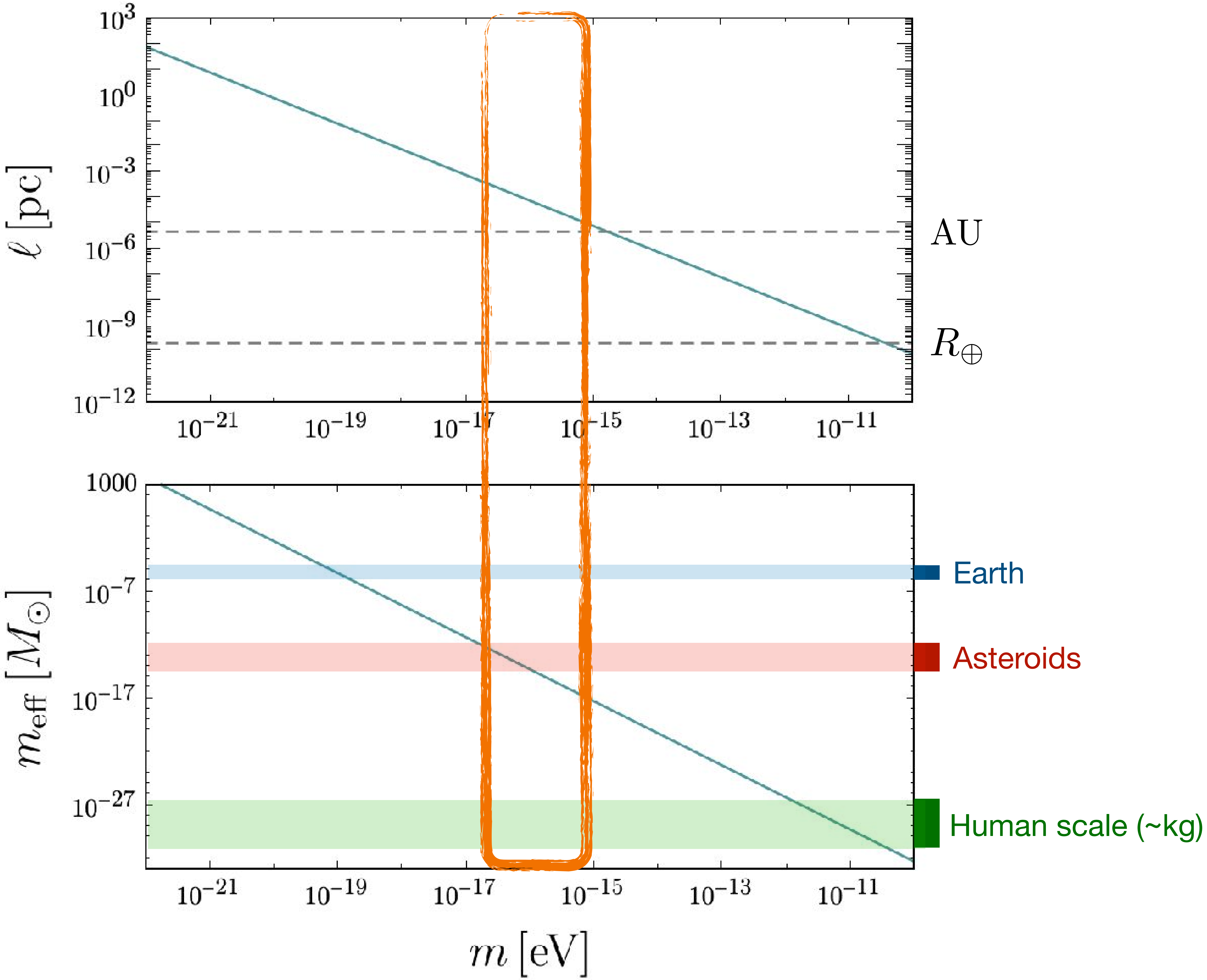


$m \sim 10^{-16}\,\mathrm{eV}$

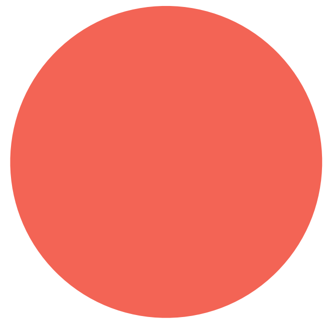


$m_{\rm eff} \sim \mathcal{O}(10^{17})\,\mathrm{kg}$

$\ell \sim \mathcal{O}(10)\mathrm{AU}$

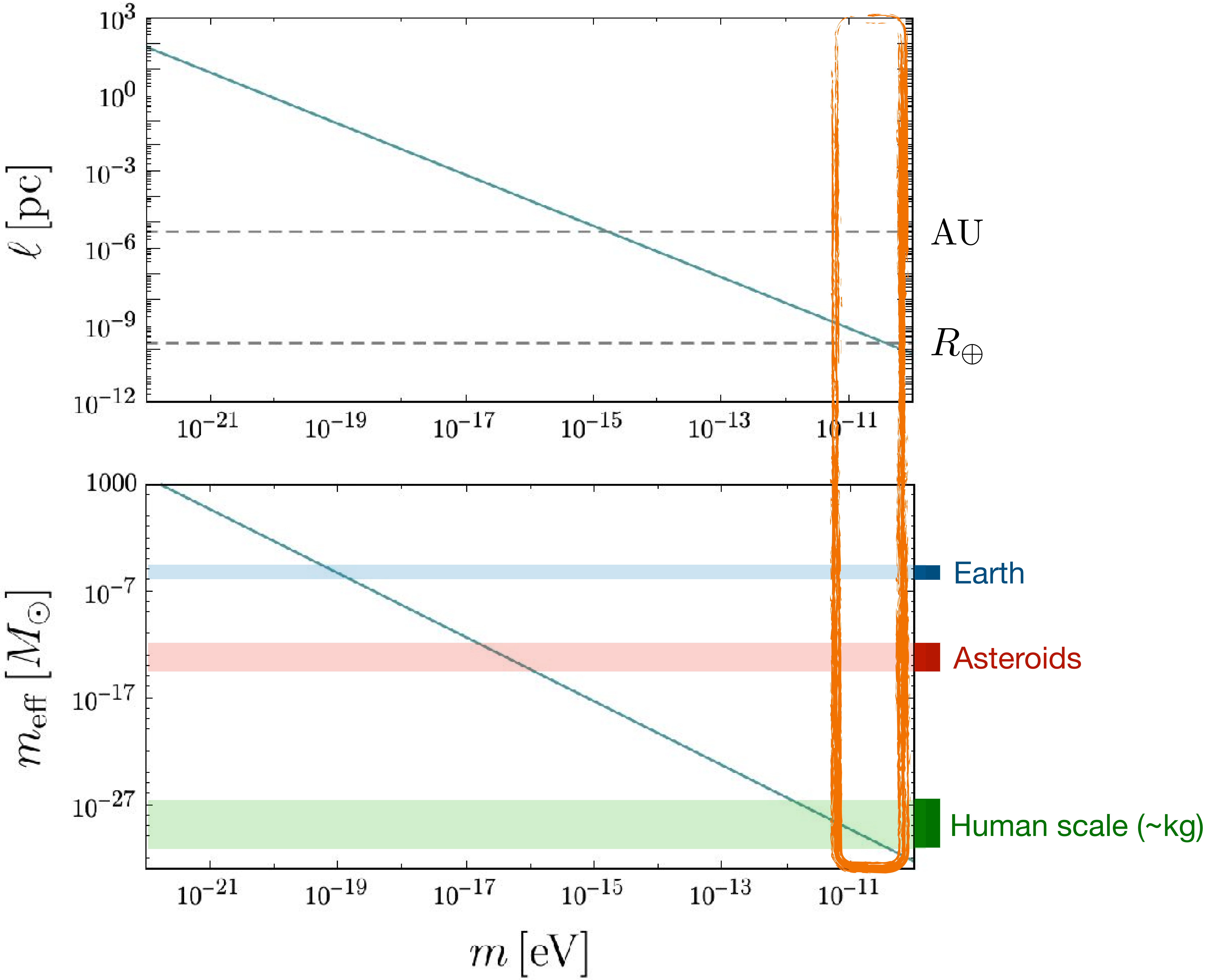


$$m \sim 10^{-11}\,\mathrm{eV}$$

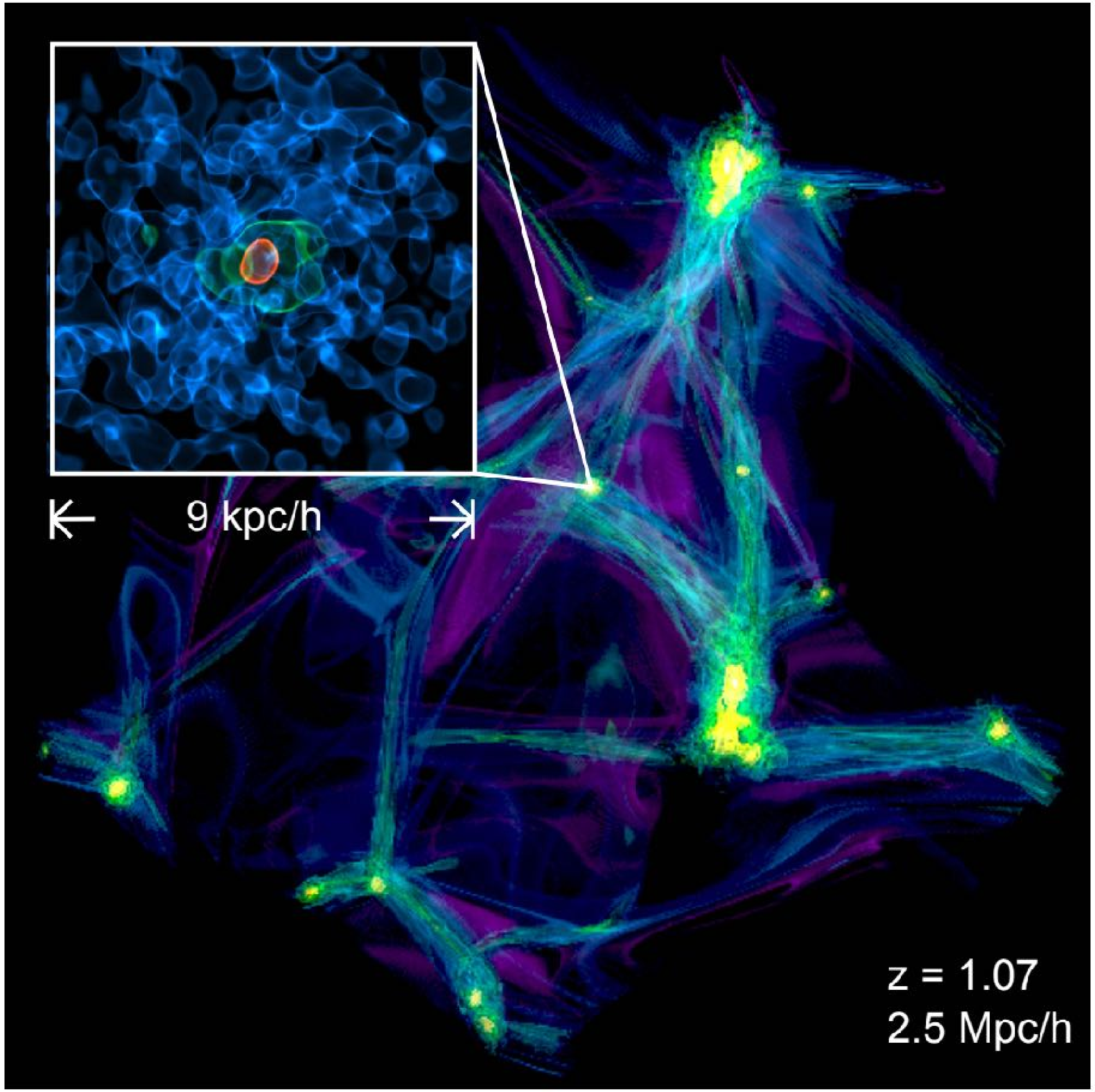


$$m_{\mathrm{eff}} \sim \mathcal{O}(10^3)\,\mathrm{kg}$$

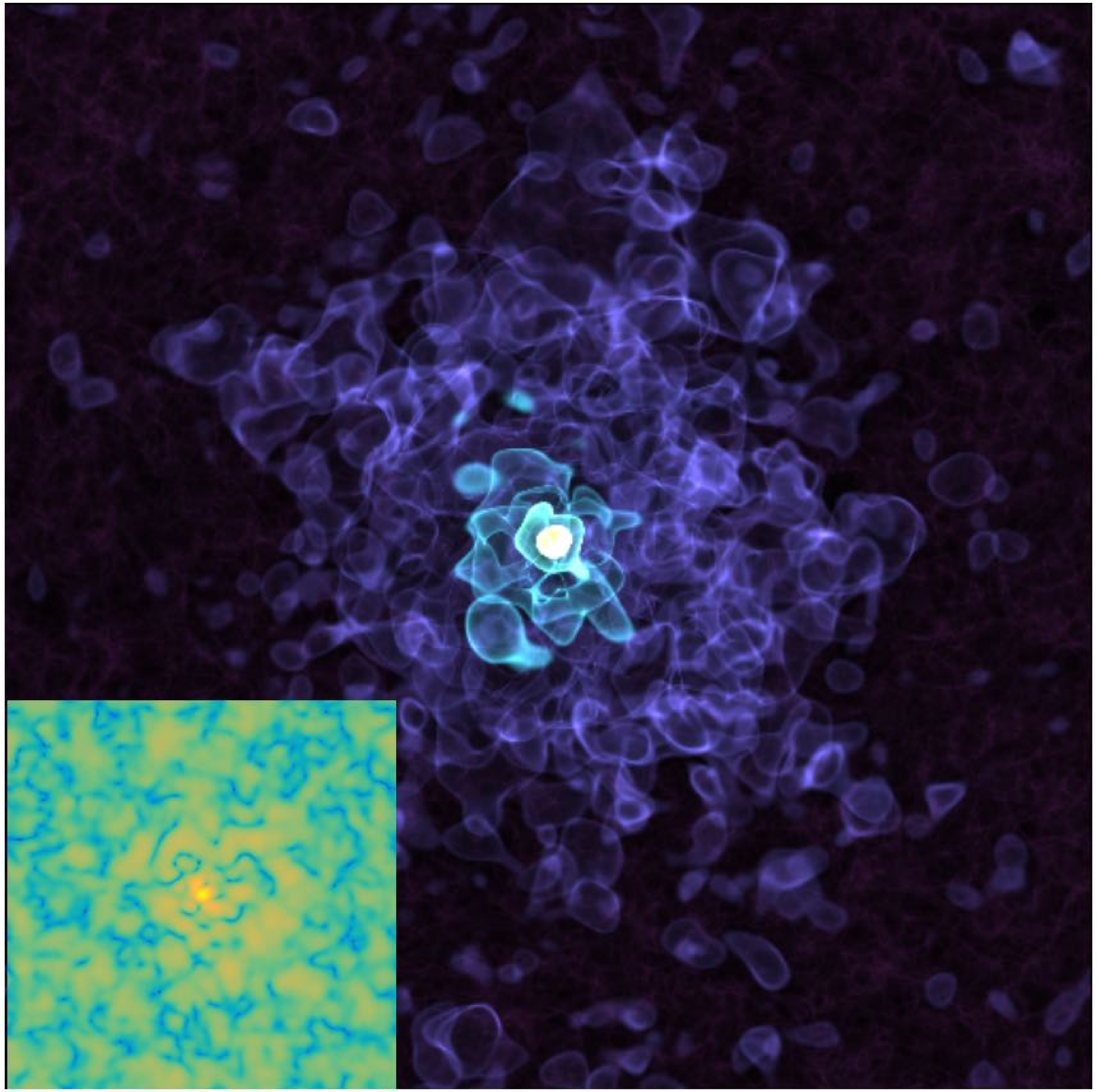
$$\ell \sim \mathcal{O}(10^3)\mathrm{km}$$



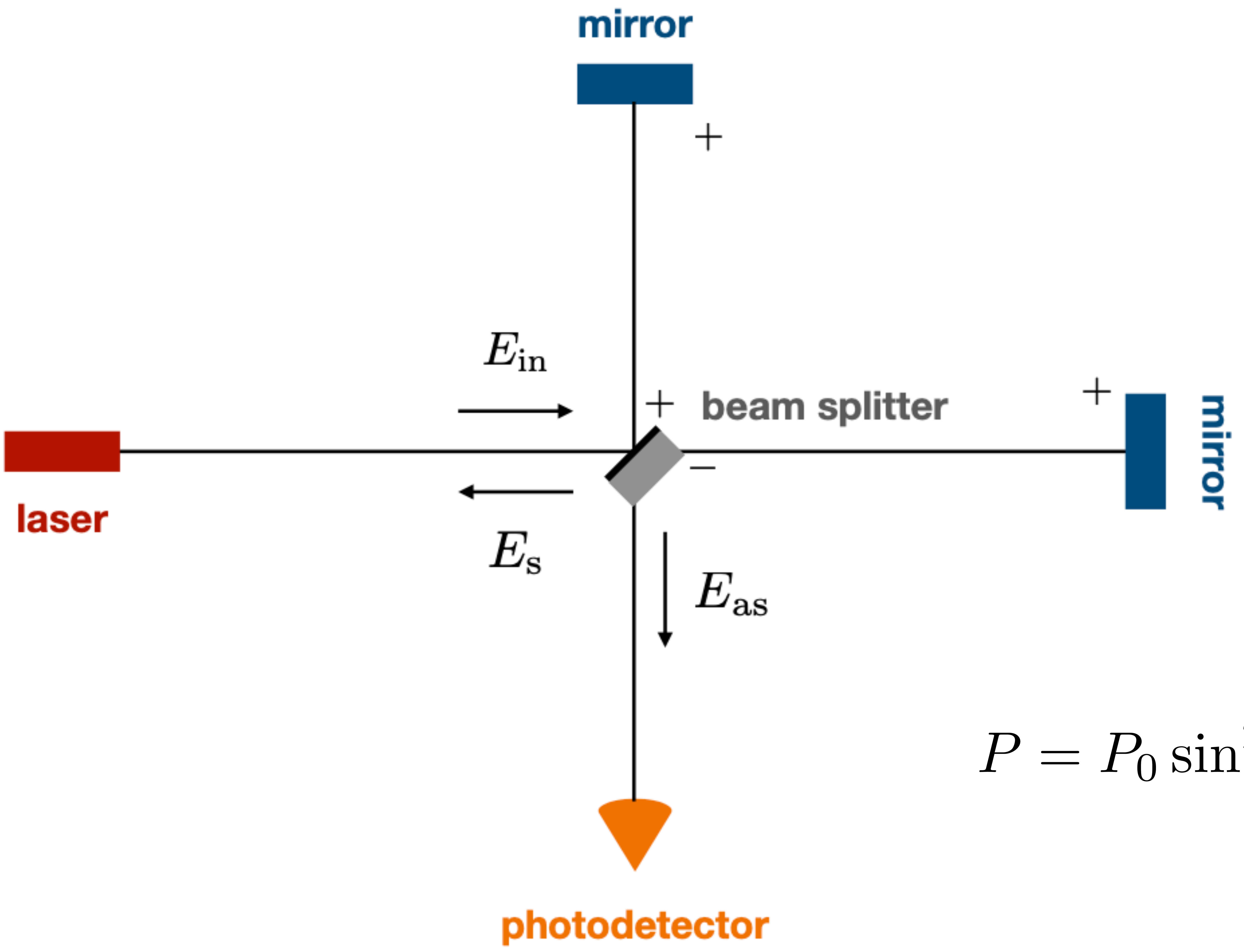
Mocz et al (17)




Veltmaat, Niemeyer, Schwabe (18)



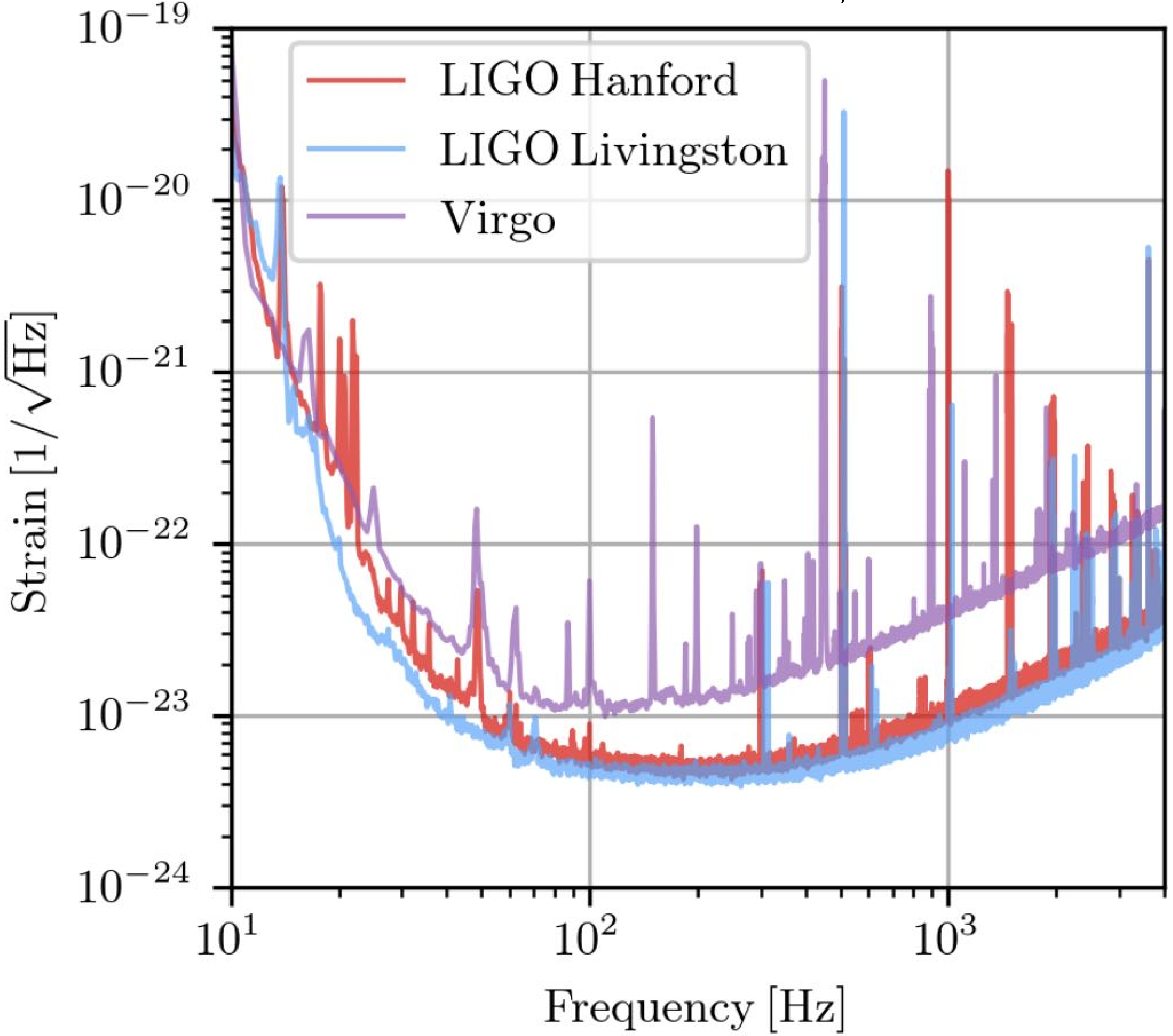
# *GW interferometers*



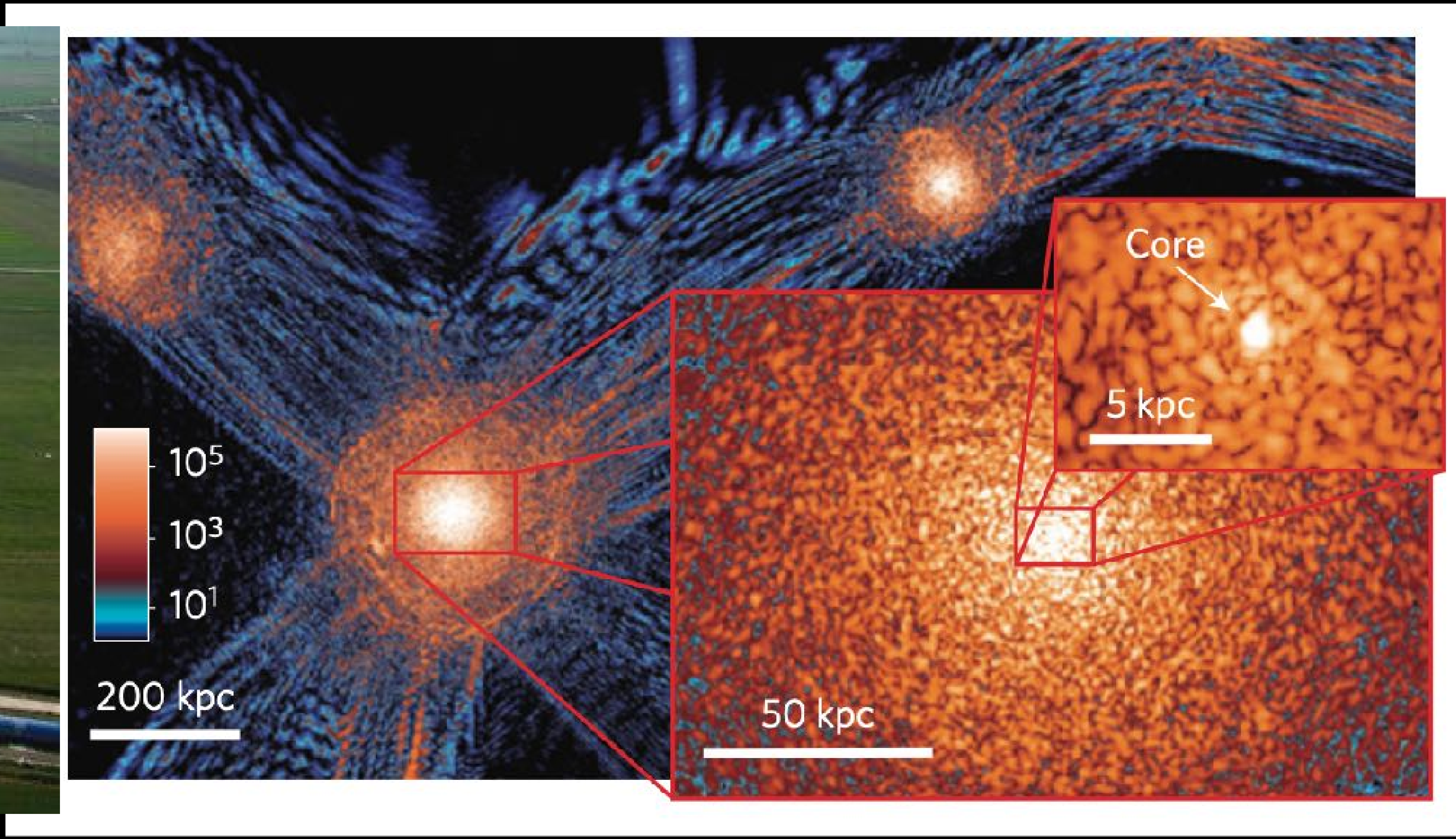

$$P = P_0 \sin^2(k\Delta L)$$

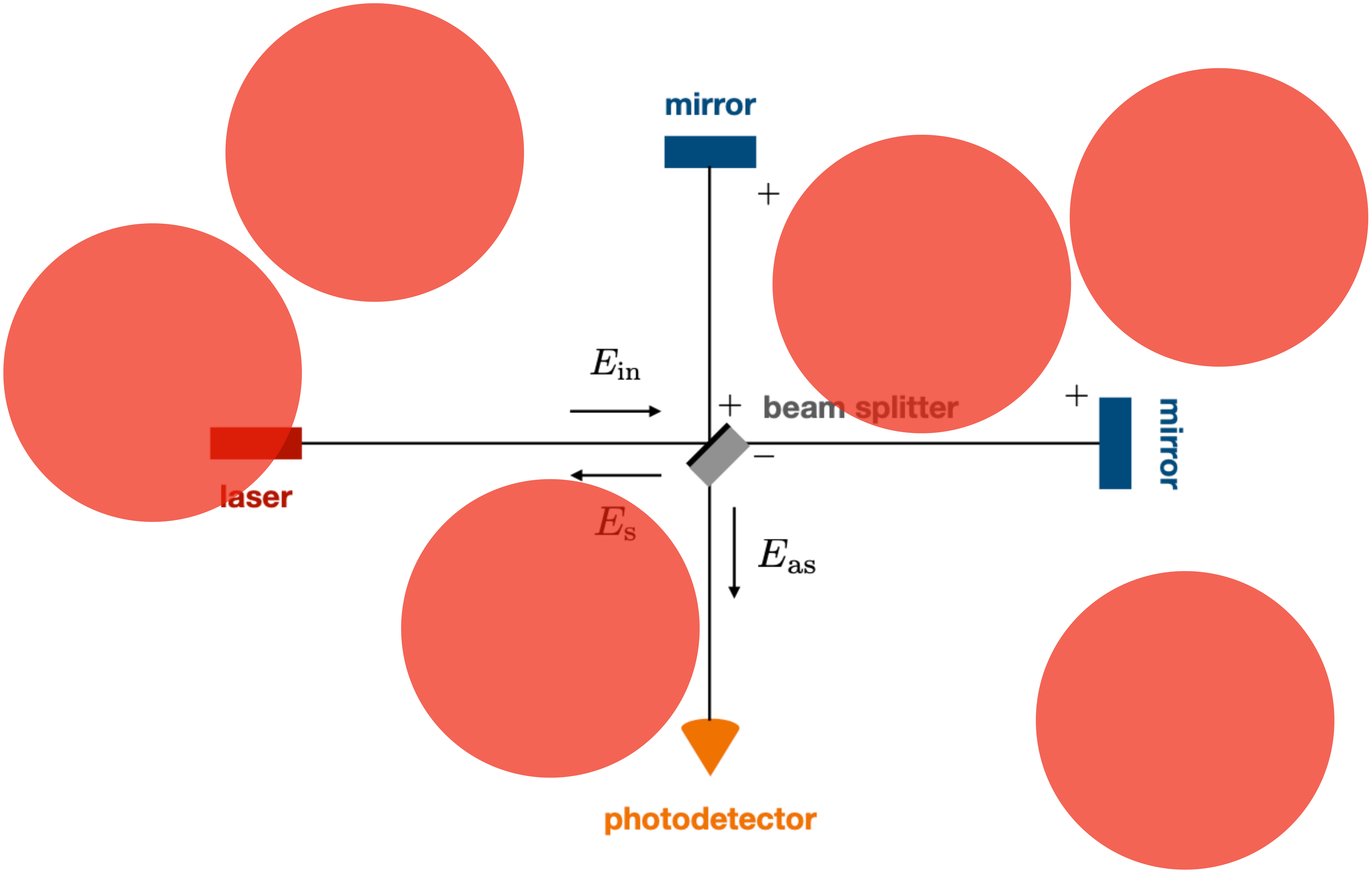
$$S_n^{1/2}(f) \sim S_{\Delta L/L}^{1/2}(f)$$

$$\langle x^2 \rangle = \int df\, S_x(f)$$



Imagine placing these sensitive GW interferometers in the sea of ultralight dark matter




mirror
$E_{in}$
beam splitter
mirror
laser
$E_s$
$E_{as}$
photodetector

Two questions:

*1. Do we have to worry about* ***ULDM-induced noise*** *in the current and future gravitational waves?*

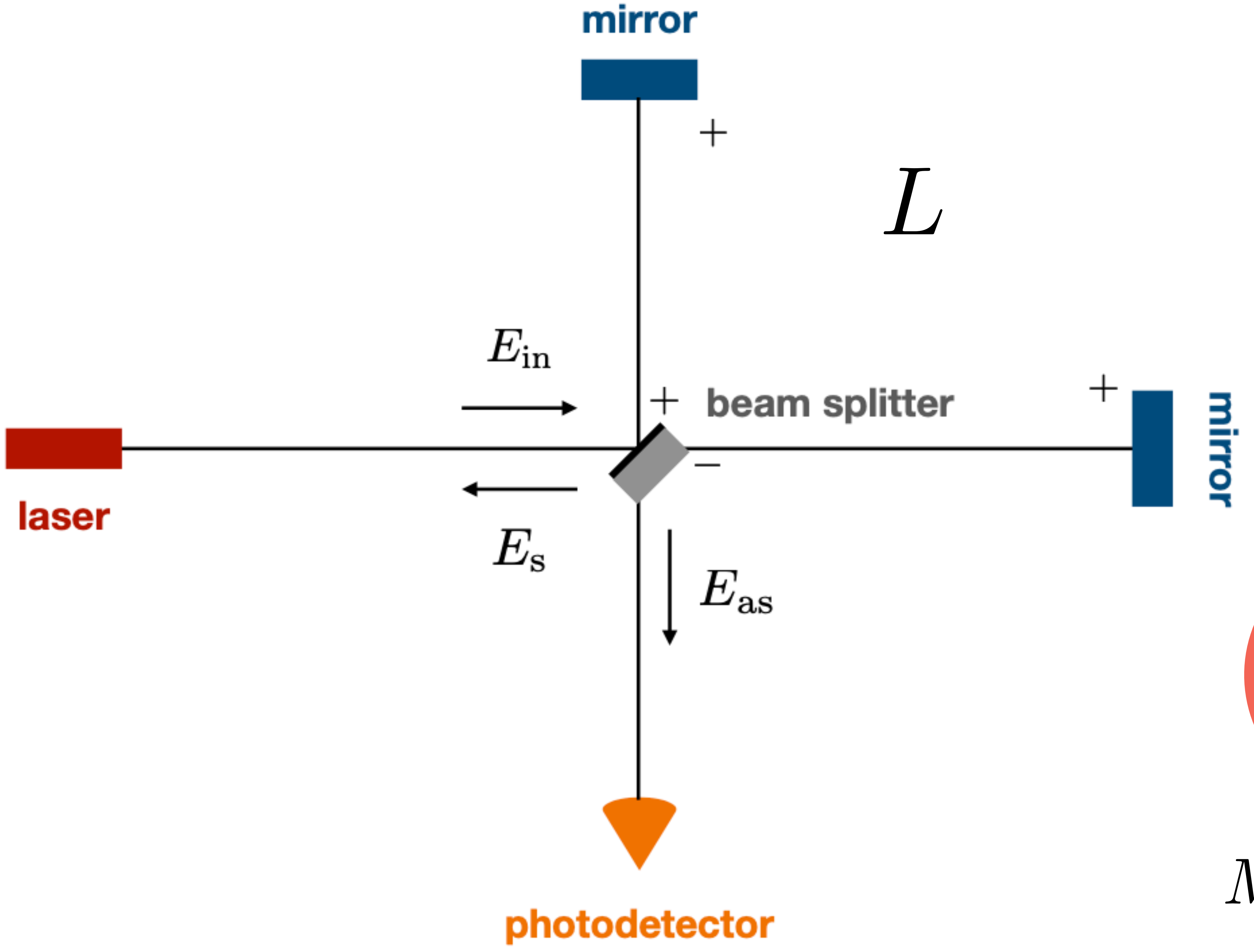
2. Can current and future GW interferometers ***probe ultralight dark matter gravitationally?***

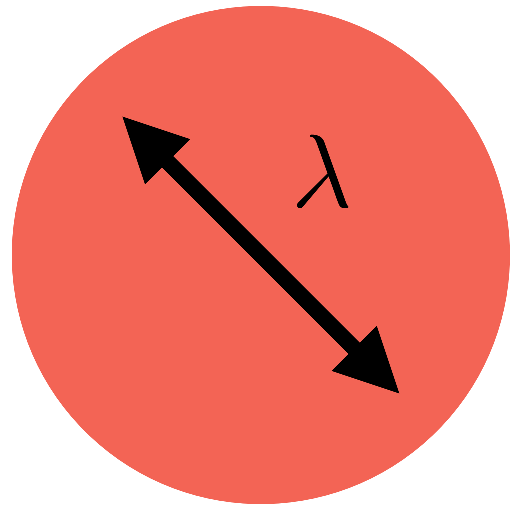
$\lambda = 1/mv$

# Back-of-envelope estimation

$$\Delta a = a_1 - a_2 = \frac{GM_{\rm eff}}{(L+\lambda)^2} - \frac{GM_{\rm eff}}{\lambda^2}$$

$$\simeq \frac{GM_{\rm eff}}{\lambda^2} = G\rho\lambda$$




$M_{\rm eff} = \rho\lambda^3$

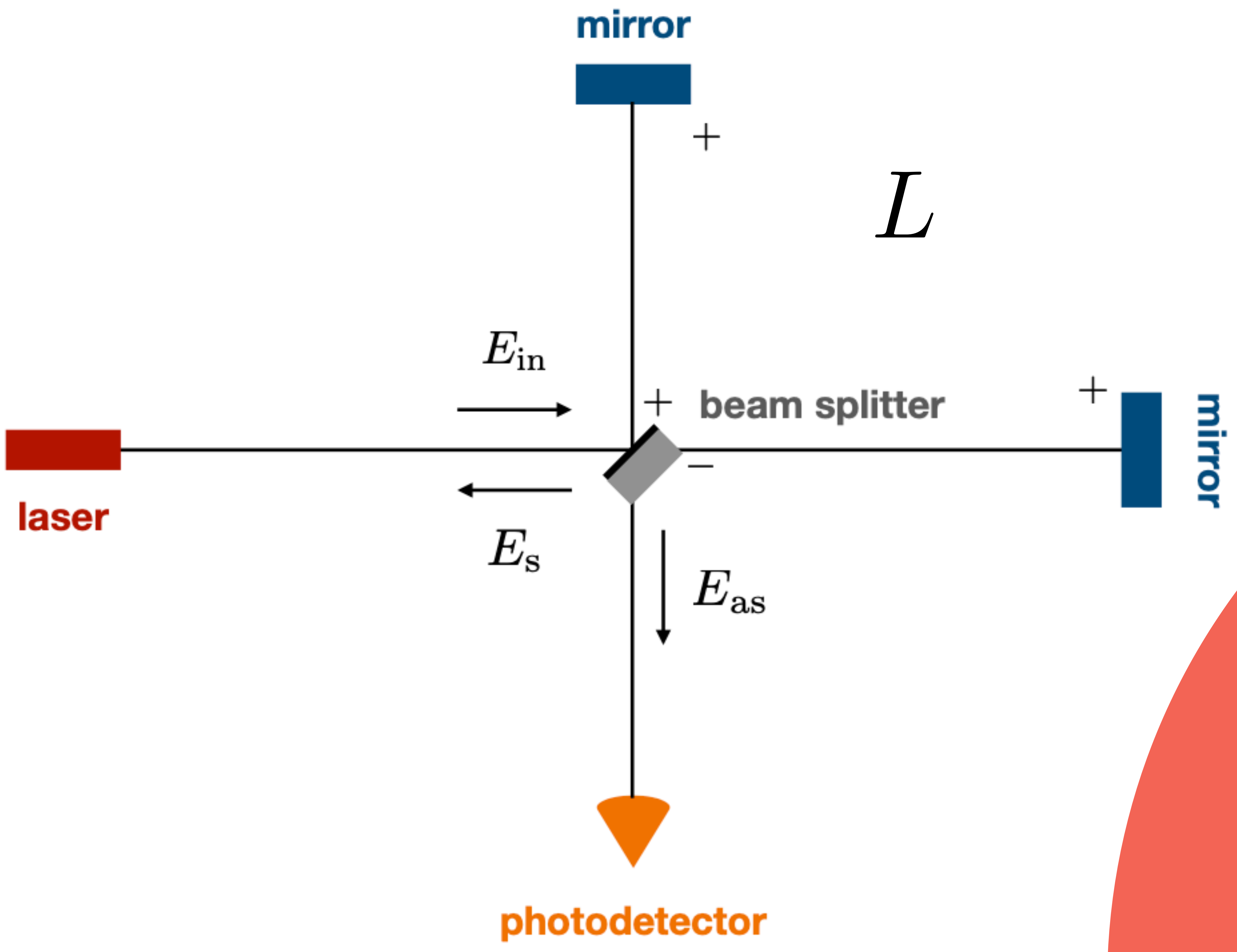
$\lambda = 1/mv$

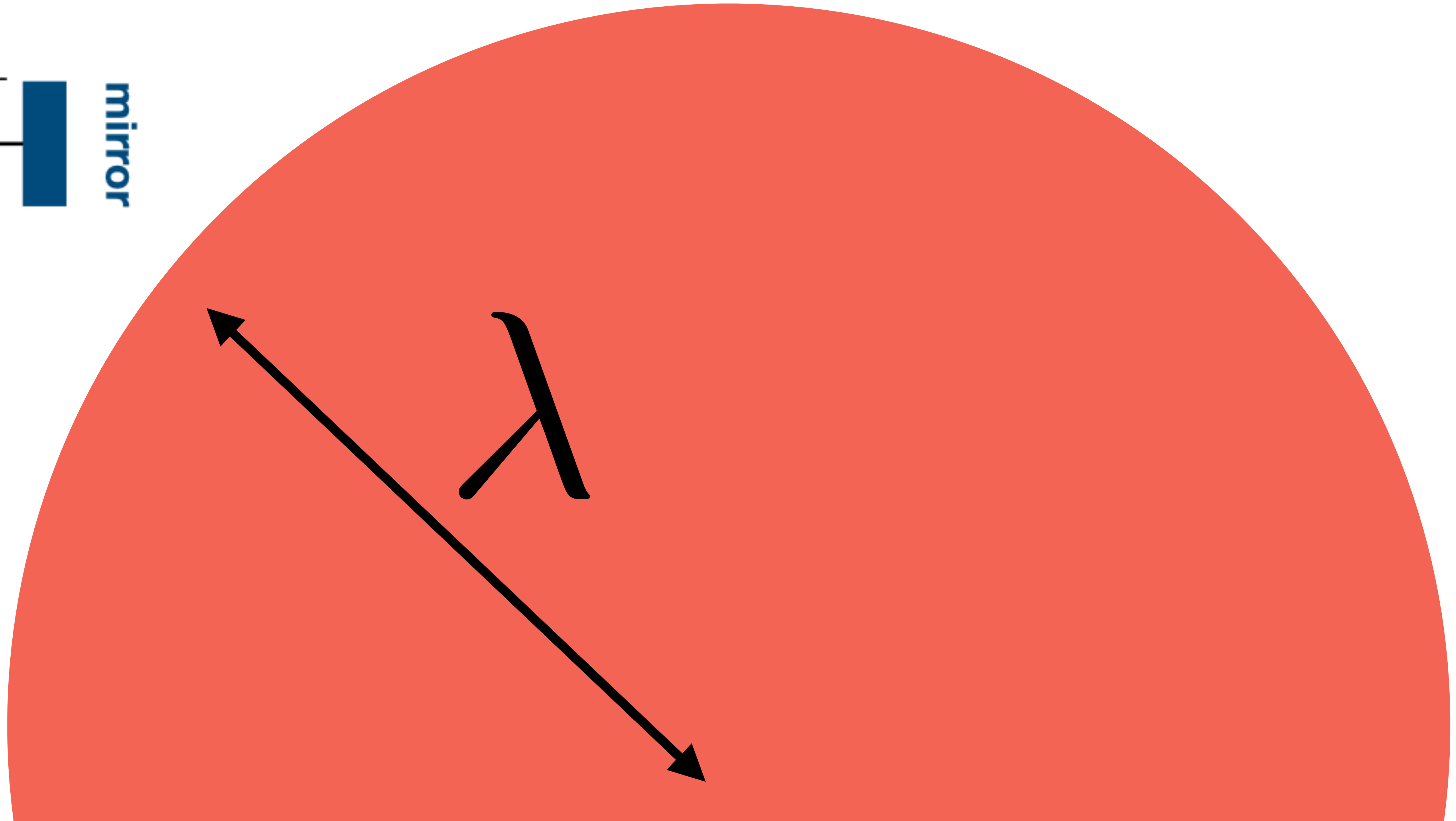
# Back-of-envelope estimation

$$\Delta a = a_1 - a_2 = \frac{GM_{\text{eff}}}{(L+\lambda)^2} - \frac{GM_{\text{eff}}}{\lambda^2}$$

$$\simeq \frac{GM_{\text{eff}}}{\lambda^2} = G\rho\lambda$$

$$\simeq \frac{GM_{\text{eff}}}{\lambda^2}\frac{L}{\lambda} = G\rho L$$




$M_{\text{eff}} = \rho\lambda^3$

# Back-of-envelope estimation

$$\Delta a \simeq G\rho L \quad \simeq 10^{-28}\,\mathrm{m\,s^{-2}}$$

$L = 4\,\mathrm{km}$

LIGO VIRGO …

$$\simeq 10^{-22}\,\mathrm{m\,s^{-2}}$$

$L = 2.5\mathrm{M}\,\mathrm{km}$

LISA

$$\simeq 10^{-20}\,\mathrm{m\,s^{-2}}$$

$L = 400\mathrm{M}\,\mathrm{km}$

μAres or Asteroid?

[Sesana et al (19)]
[Fedderke et al (21)]

$$\Delta a \sim [S_h (2\pi f)^4 L^2 \Delta f]^{1/2}$$

$\sim 10^{-14}\,\mathrm{m\,s^{-2}}$ LIGO (~50 Hz)

$\sim 10^{-16}\,\mathrm{m\,s^{-2}}$ LISA (~0.1 mHz)

$\sim 10^{-18}\,\mathrm{m\,s^{-2}}$ μAres strawman mission concept (~ μHz)

Two questions:

*1. Do we have to worry about* ***ULDM-induced noise*** *in the current and future gravitational waves?*

2. Can current and future GW interferometers ***probe ultralight dark matter gravitationally?***

*to completely answer these questions, we need detailed computation for noise power spectrum from ultralight dark matter*

# Some statistical properties of ULDM

$$\phi = \sum_i \frac{1}{\sqrt{2mV}} \left[ a_i e^{-ikx} + a_i^\dagger e^{ikx} \right]$$

**Operators**

**Wave func.**

*One can start from field theory*
*define density operator under certain assumptions*
*to completely specify the statistical properties of the scalar field* [Kim and Lenoci 21]

$$\hat{\rho} = \prod_i \hat{\rho}_i \otimes$$

$$\hat{\rho}_i = \int d^2\alpha_i \, P(\alpha_i) |\alpha_i\rangle\langle\alpha_i|$$

$$P(\alpha_i) = \frac{1}{\pi f_i} \exp\left[ -\frac{|\alpha_i|^2}{f_i} \right]$$

(~ probability distribution)

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}(\hat{\rho}\hat{\mathcal{O}})$$

$$a_i|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$$

$$\alpha_i \in \mathbb{C}$$

# Some statistical properties of ULDM

$$\phi = \sum_i \frac{1}{\sqrt{2mV}} \left[ \alpha_i e^{-ikx} + \alpha_i^* e^{ikx} \right]$$

***random #***

***Wave func.***

*More conveniently*
*we can just take operators to complex random variables*

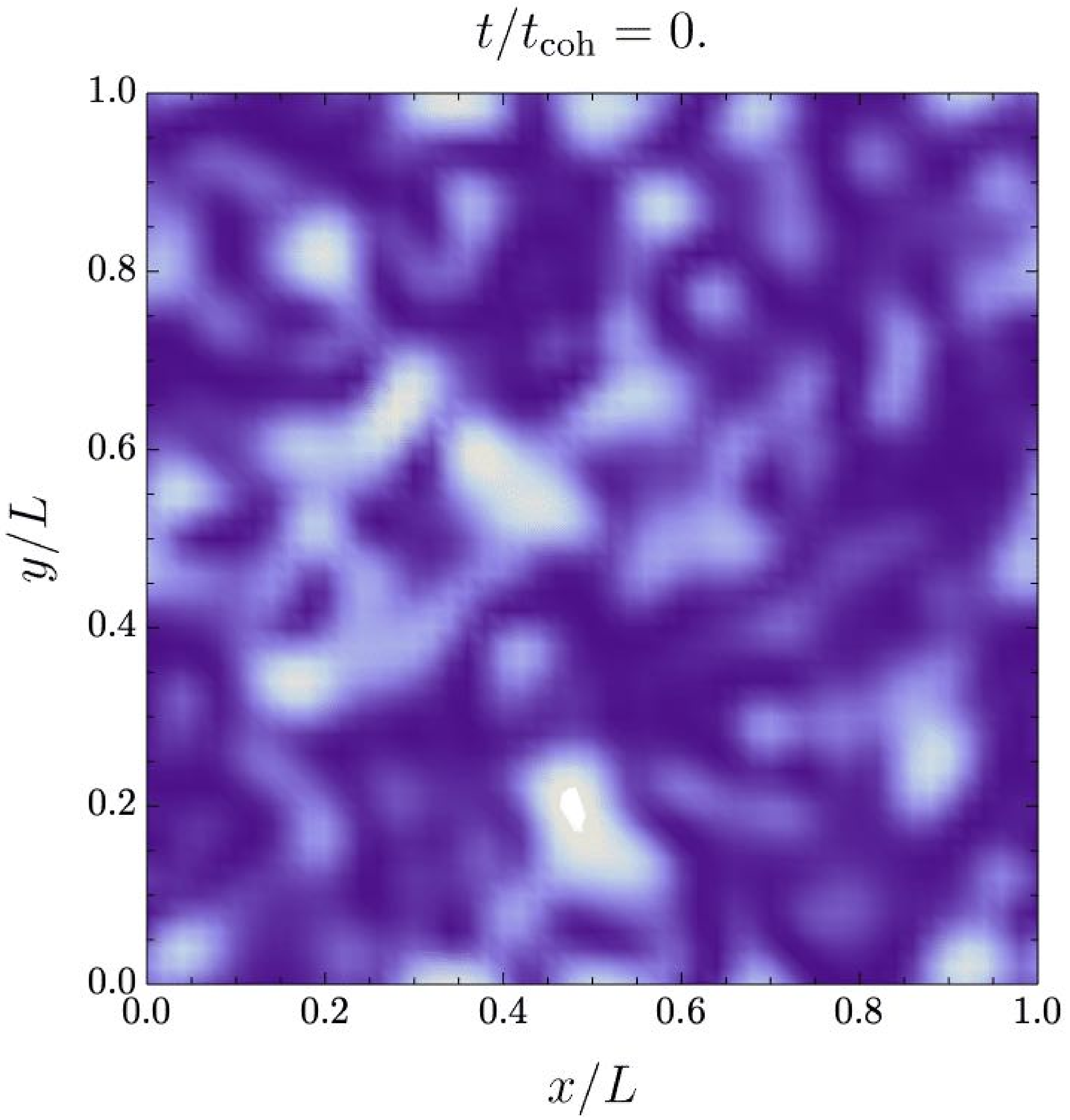
$$(a_i, a_i^\dagger) \to (\alpha_i, \alpha_i^*)$$

*where each of them is distributed according to the following p.d.f.*

$$p(\alpha_i) = \frac{1}{\pi f_i} \exp\left[ -\frac{|\alpha_i|^2}{f_i} \right]$$

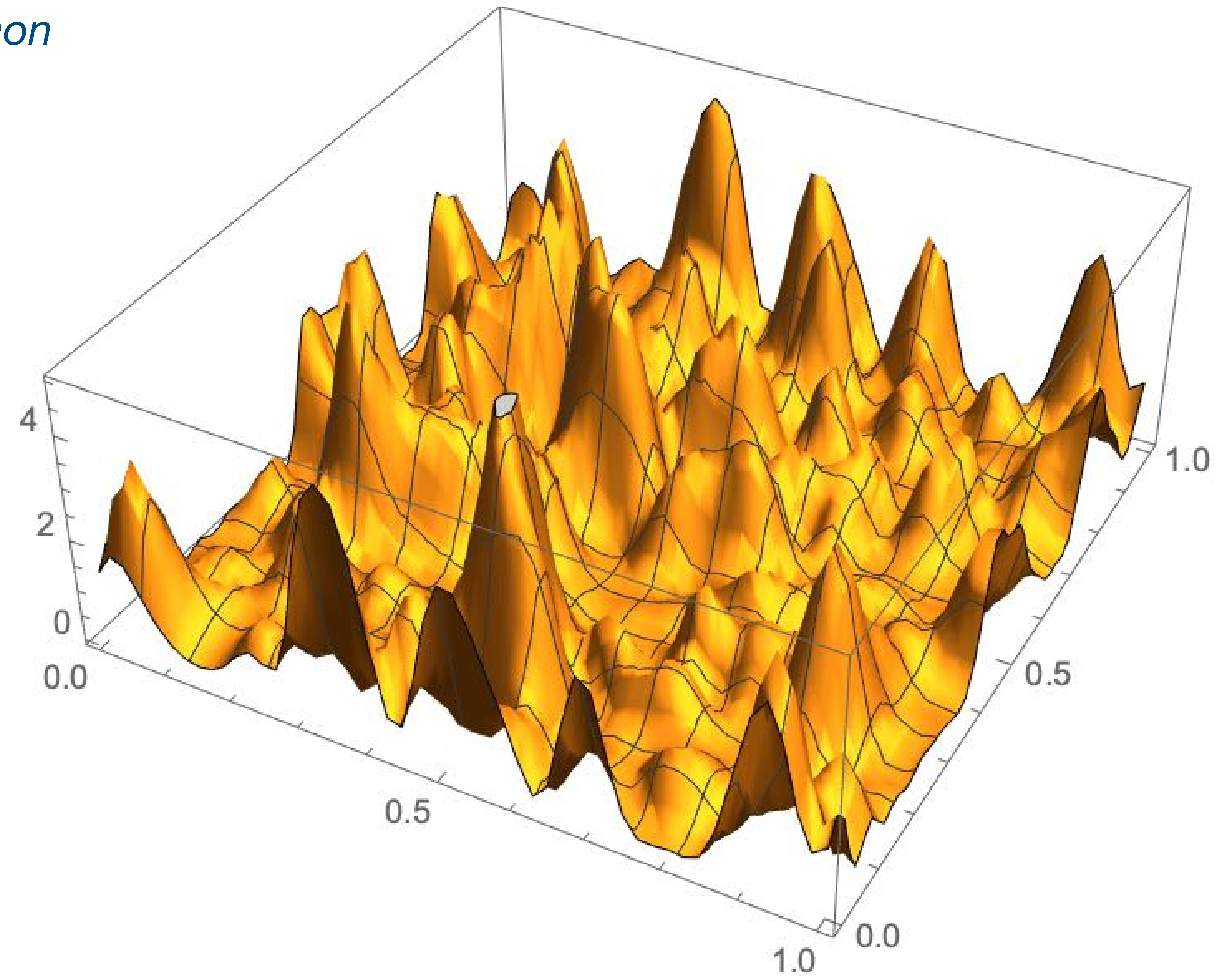
$$dP = \prod_i p(\alpha_i) d^2\alpha_i$$

*Once we obtain one particular realization of the field and compute the density of the field it would look like the following*



*an order-one density fluctuation is very common*

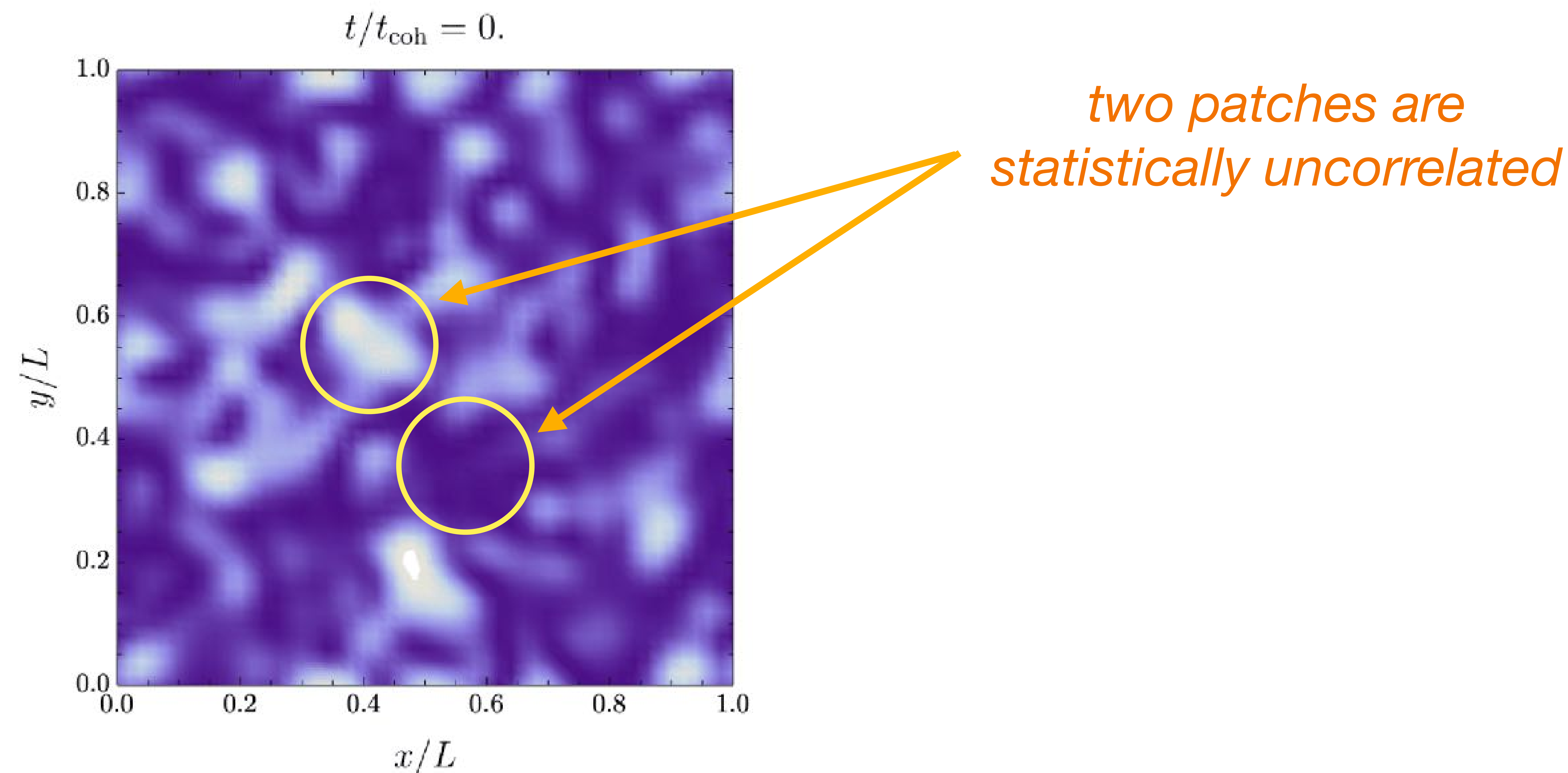
$\rho/\rho_0$



*The statistical properties of these density clumps can be analytically investigated*

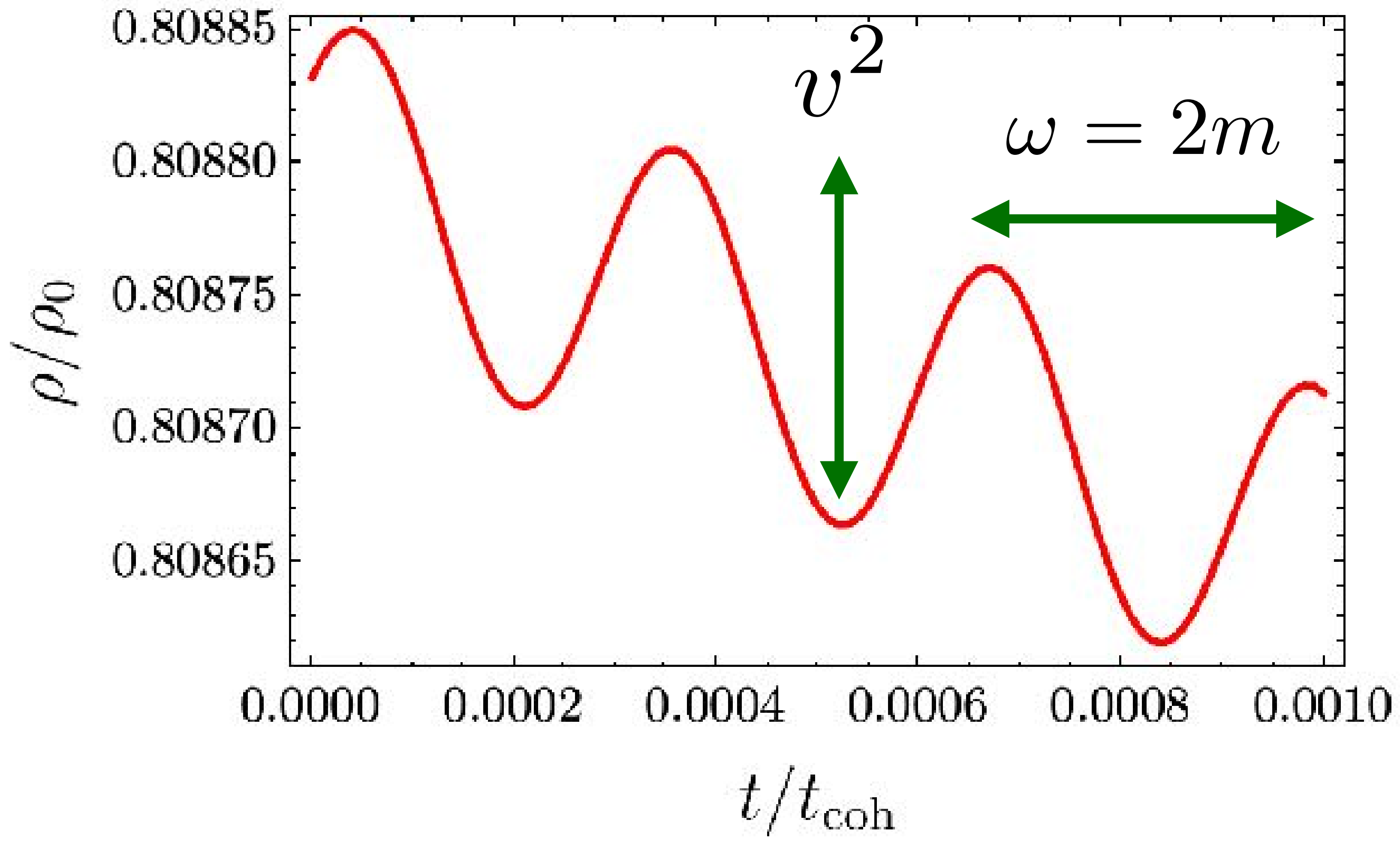
*For instance*
*the density-density correlator of space-like separation is*

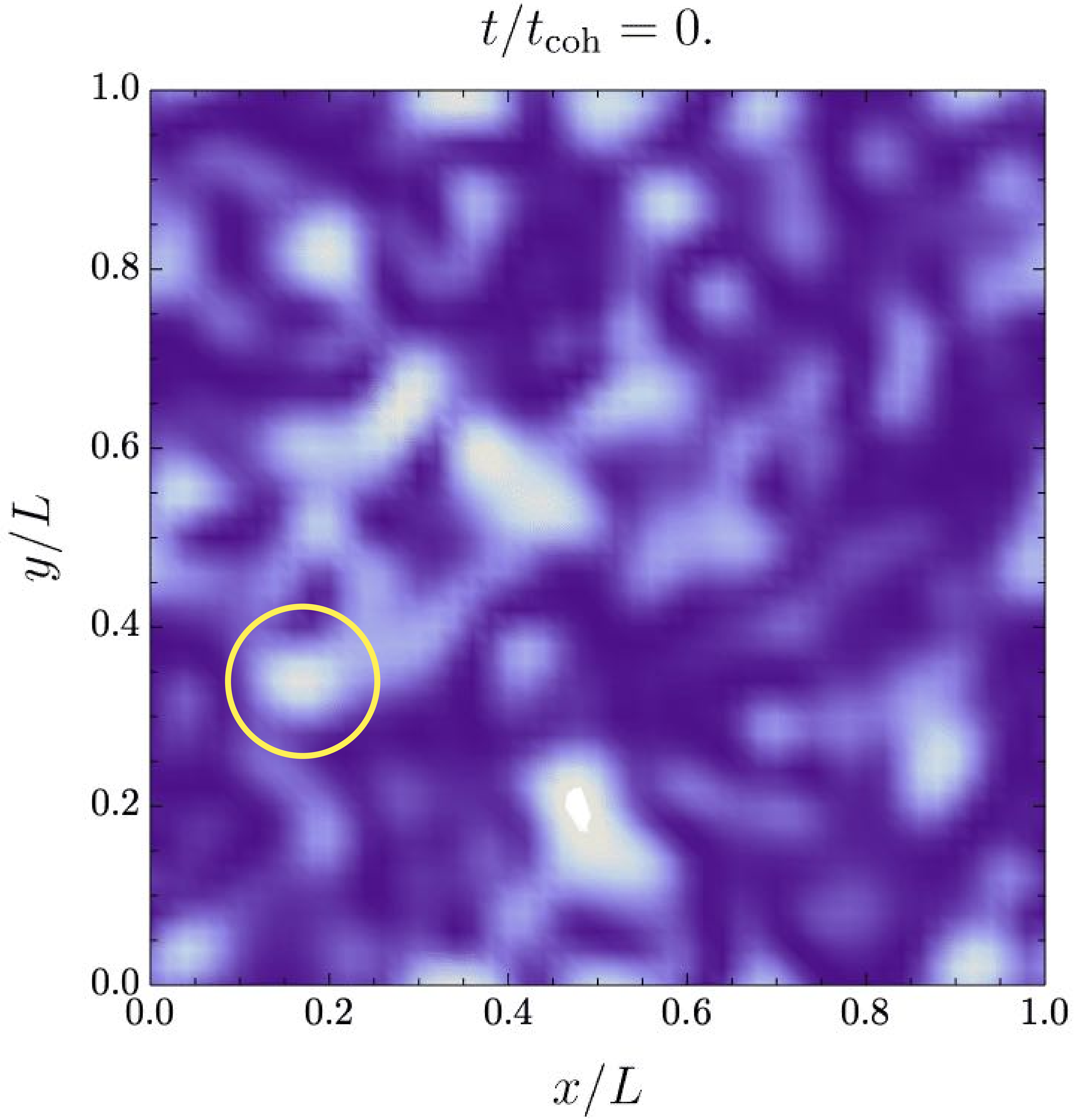
$$\langle \delta(x)\delta(y) \rangle \propto \exp\left[-|\Delta x|^2/\lambda^2\right]$$

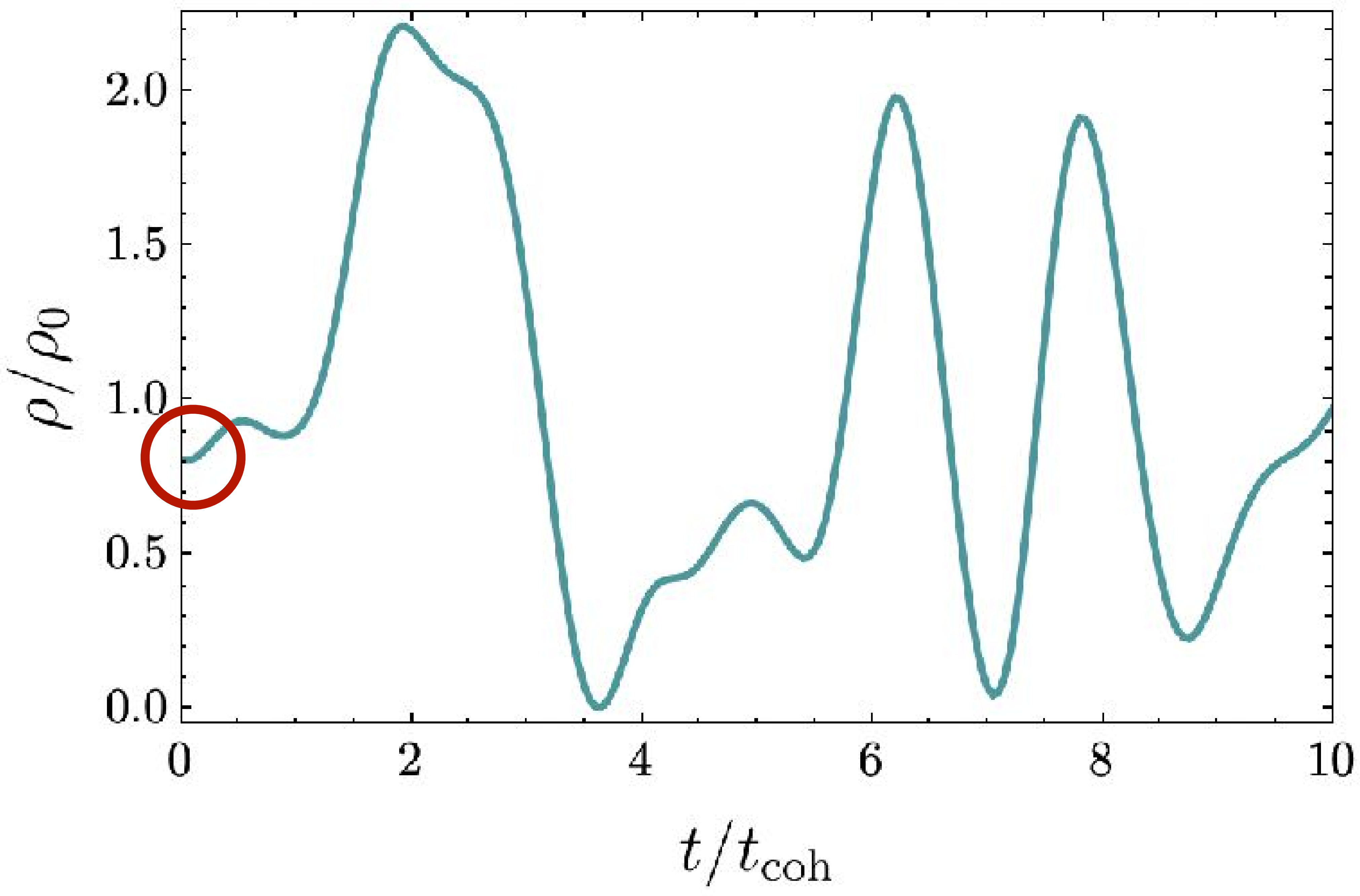


*the density-density correlator at the same position is*

$$\langle \delta(x)\delta(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$

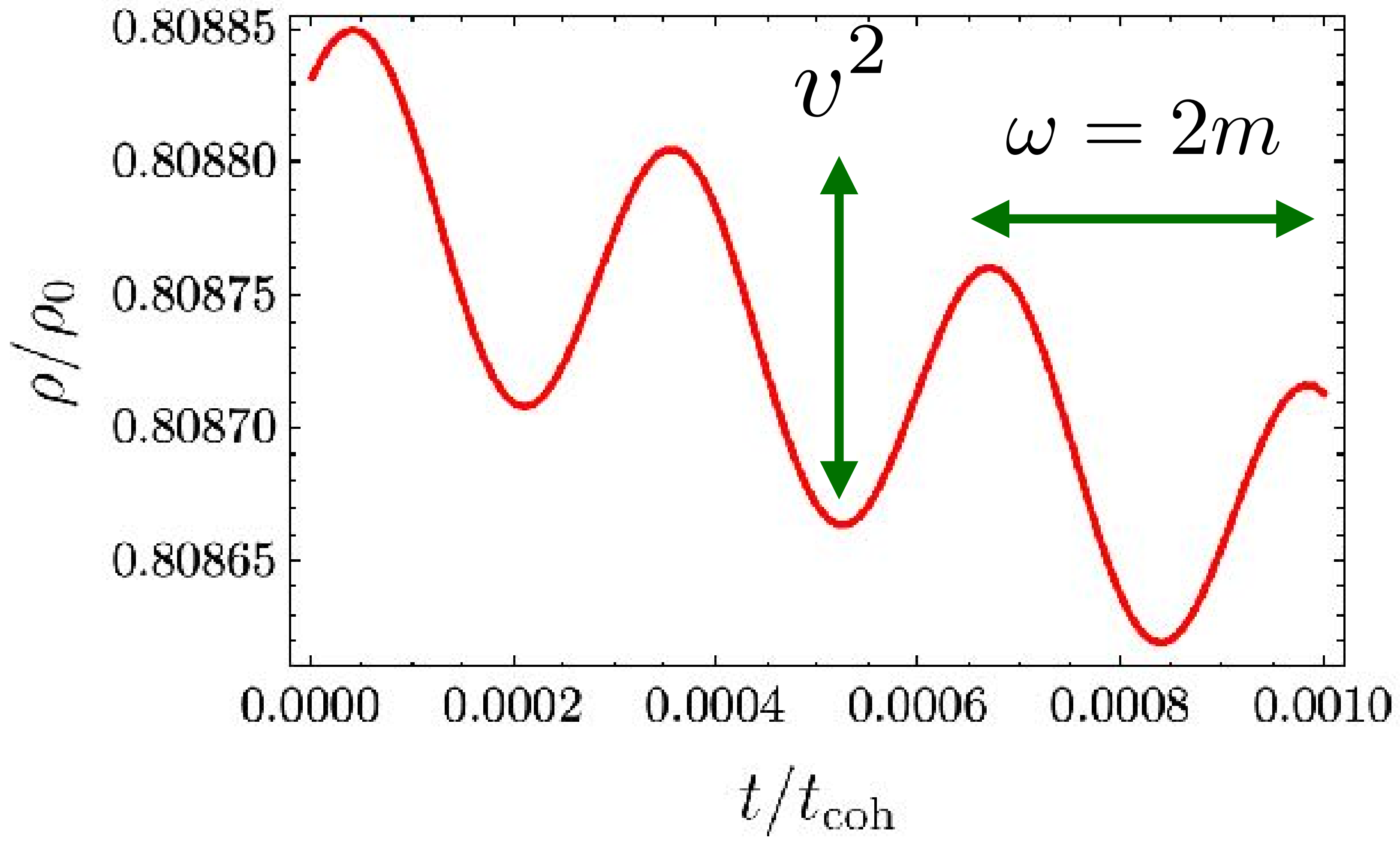


*the density-density correlator at the same position is*

$$\langle \delta(x)\delta(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$



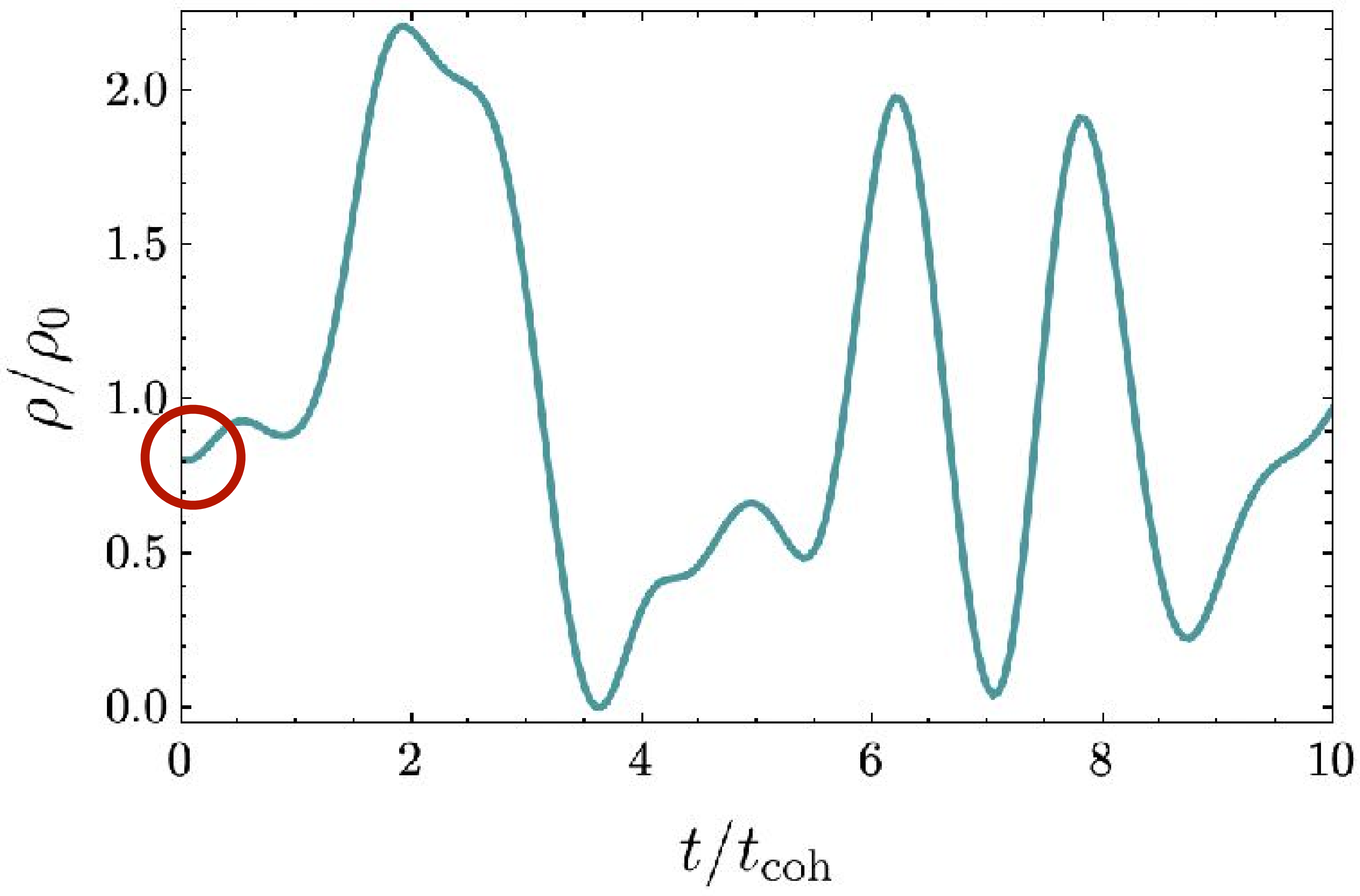

*for a single mode*

$$\phi = \phi_0 \cos(\omega t - kx)$$

*the energy density is*

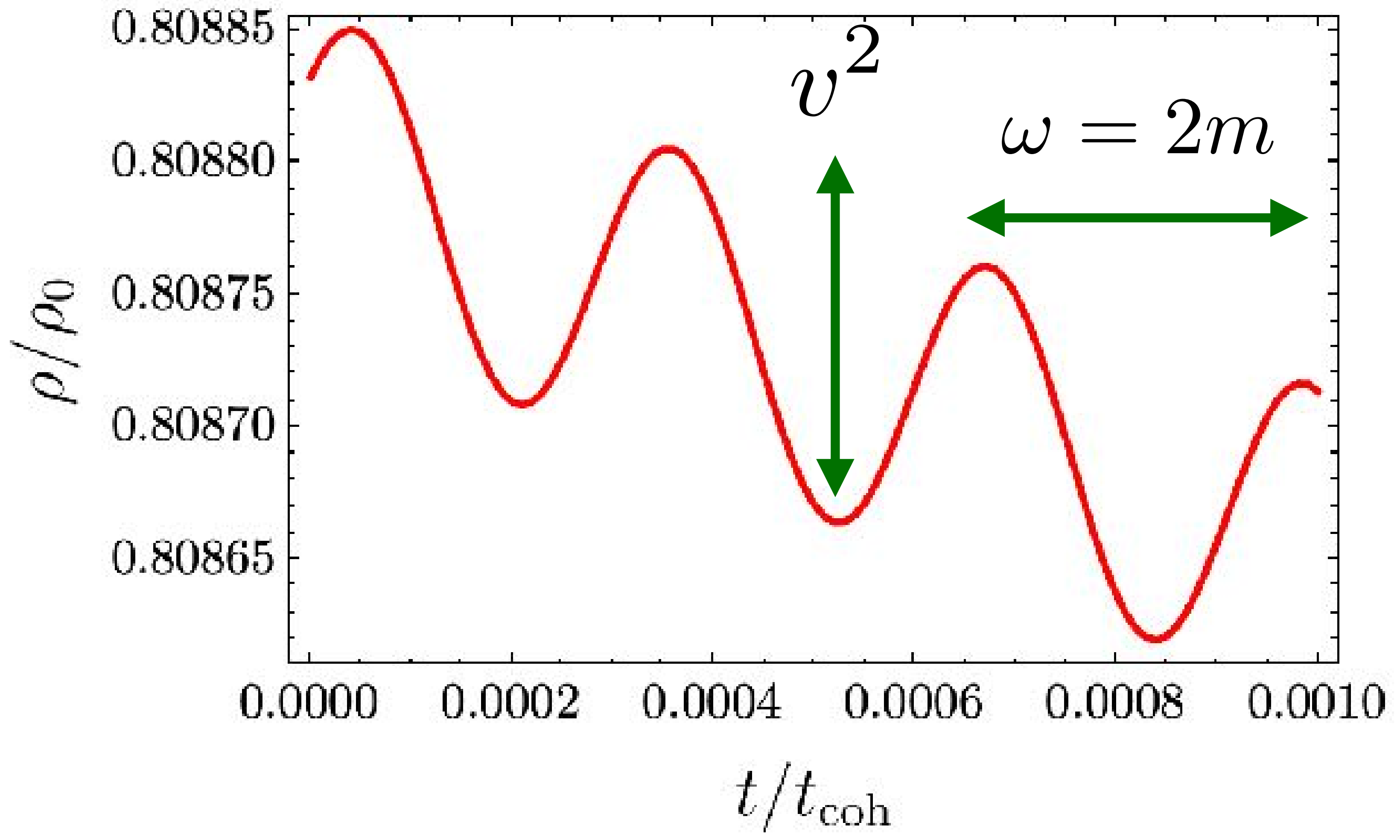
$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\simeq \rho_0[1 - v^2 \cos(2(mt - kx))]$$

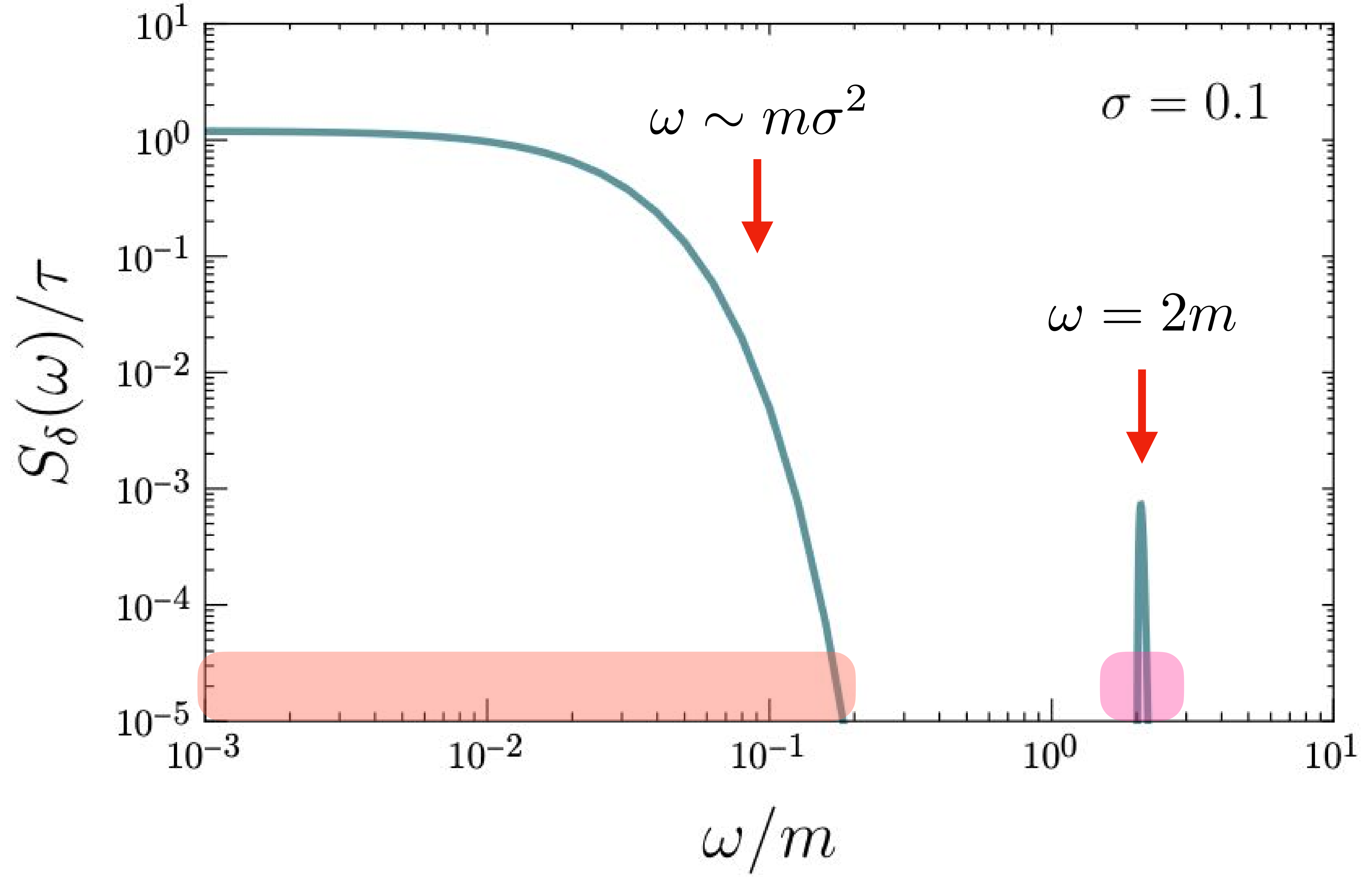


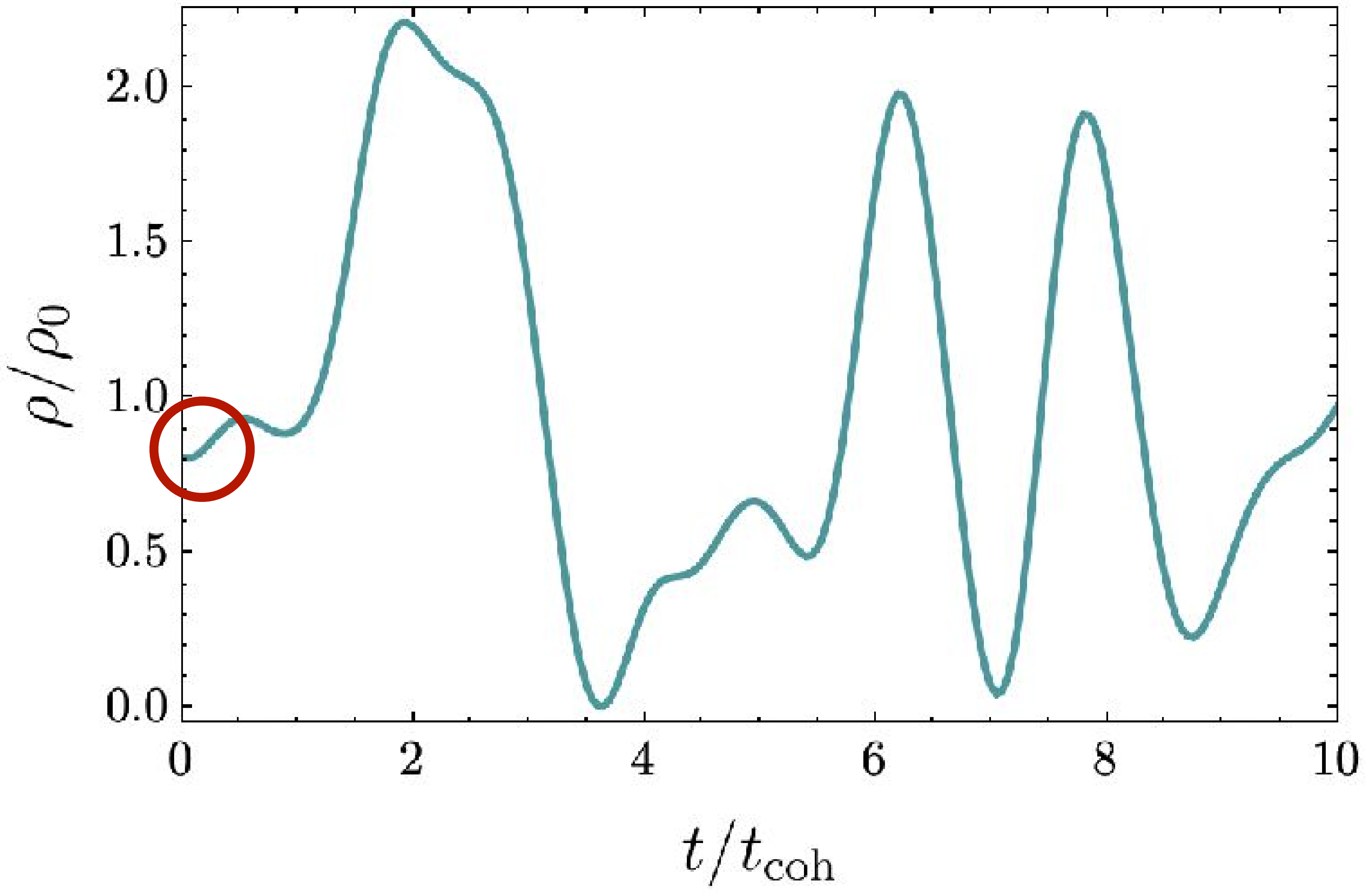
*the density-density correlator at the same position is*

$$\langle \delta(x)\delta(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$

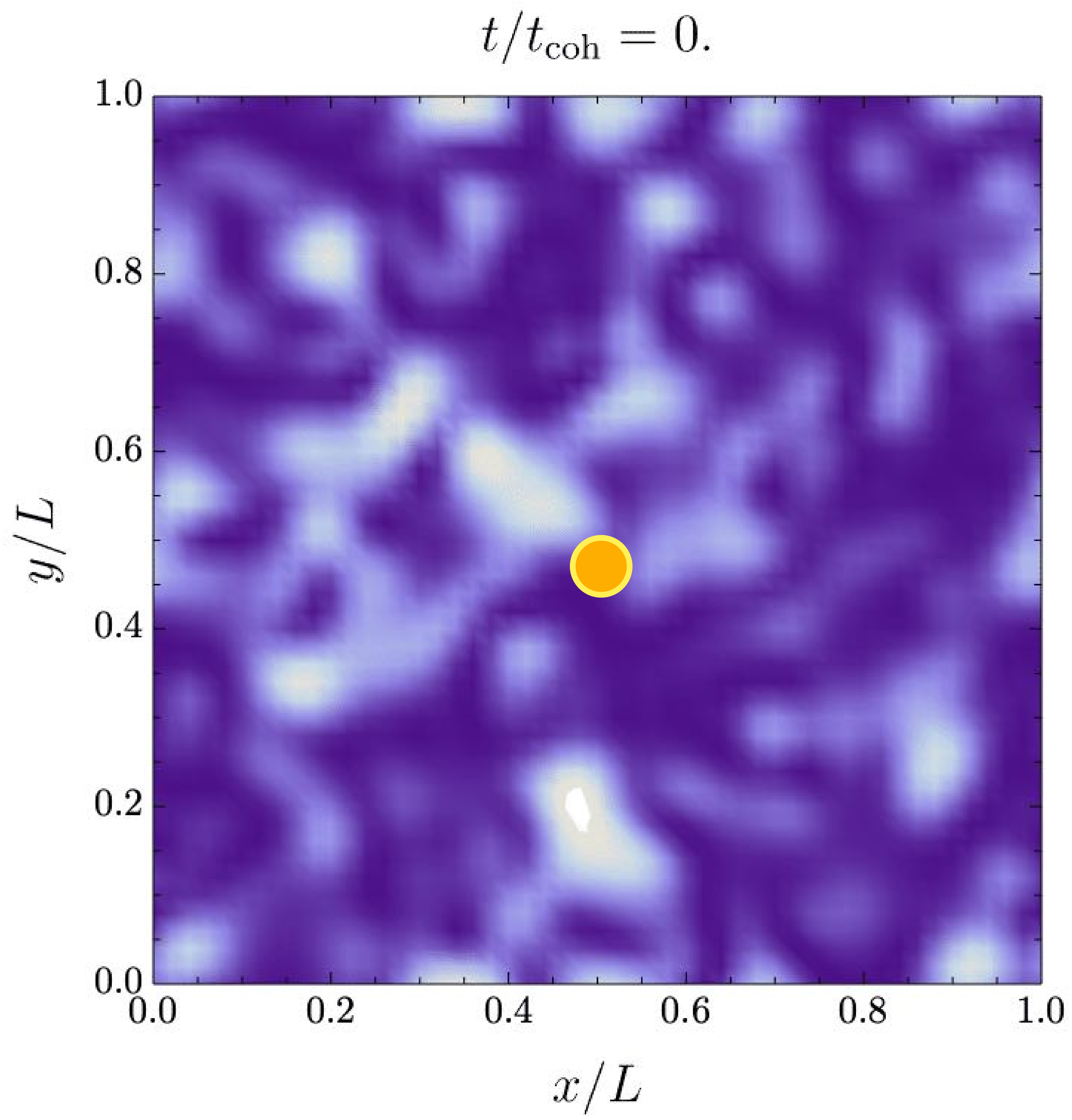



$$S_\delta(\omega) = \tau[\sigma^4 A_\delta(\omega) + B_\delta(\omega)]$$



the next step would be to investigate
how a test mass evolves in the sea of ULDM




$$x_{\rm rms}^2 = \langle x^2 \rangle = \int df\, S_x(f)$$

$$= \int df\, \frac{S_a(f)}{(2\pi f)^4}$$

how will the test mass fluctuate?

$$a = \ddot{x}$$

$$\tilde{a} = -(2\pi f)^2 \tilde{x}$$

*acceleration on test mass is related to the density contrast through Newton's equation of motion & Poisson equation*

$$a = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\rho_0\delta$$

*In Fourier space*

$$\tilde{a}(\omega, \vec{k}) = -i\vec{k}\tilde{\Phi}(\omega, \vec{k})$$

$$\tilde{\Phi}(\omega, \vec{k}) = -\frac{4\pi G\rho_0}{k^2}\tilde{\delta}(\omega, \vec{k})$$

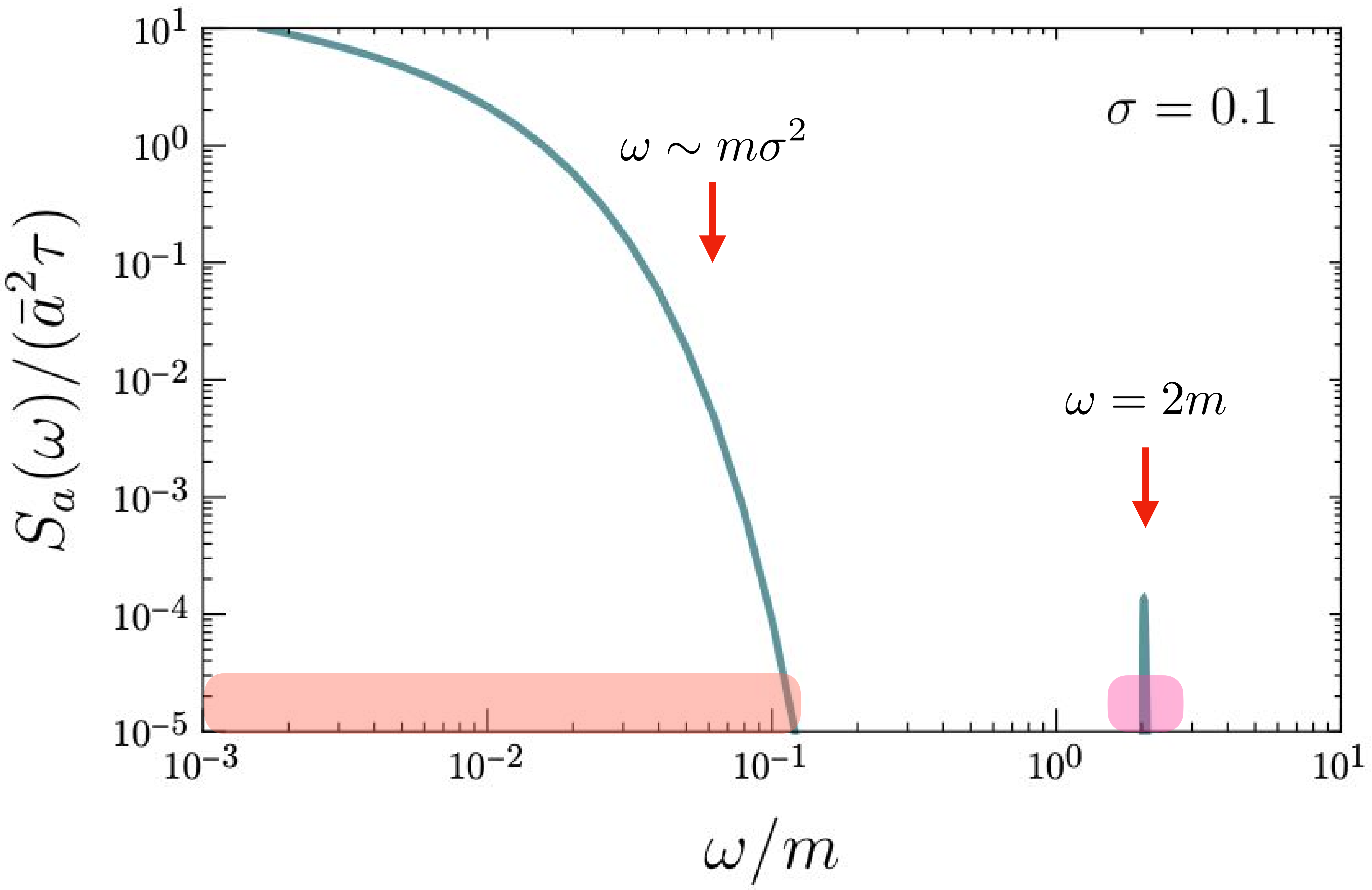
*leading to*

$$S_a(f) \sim \int \frac{d^3k}{(2\pi)^3}\left(\frac{4\pi G\rho_0}{k^2}\right)^2 k^2 P_\delta(\omega, \vec{k})$$

$$S_a(\omega) = \bar{a}^2\tau[\sigma^4 A_a(\omega) + B_a(\omega)]$$

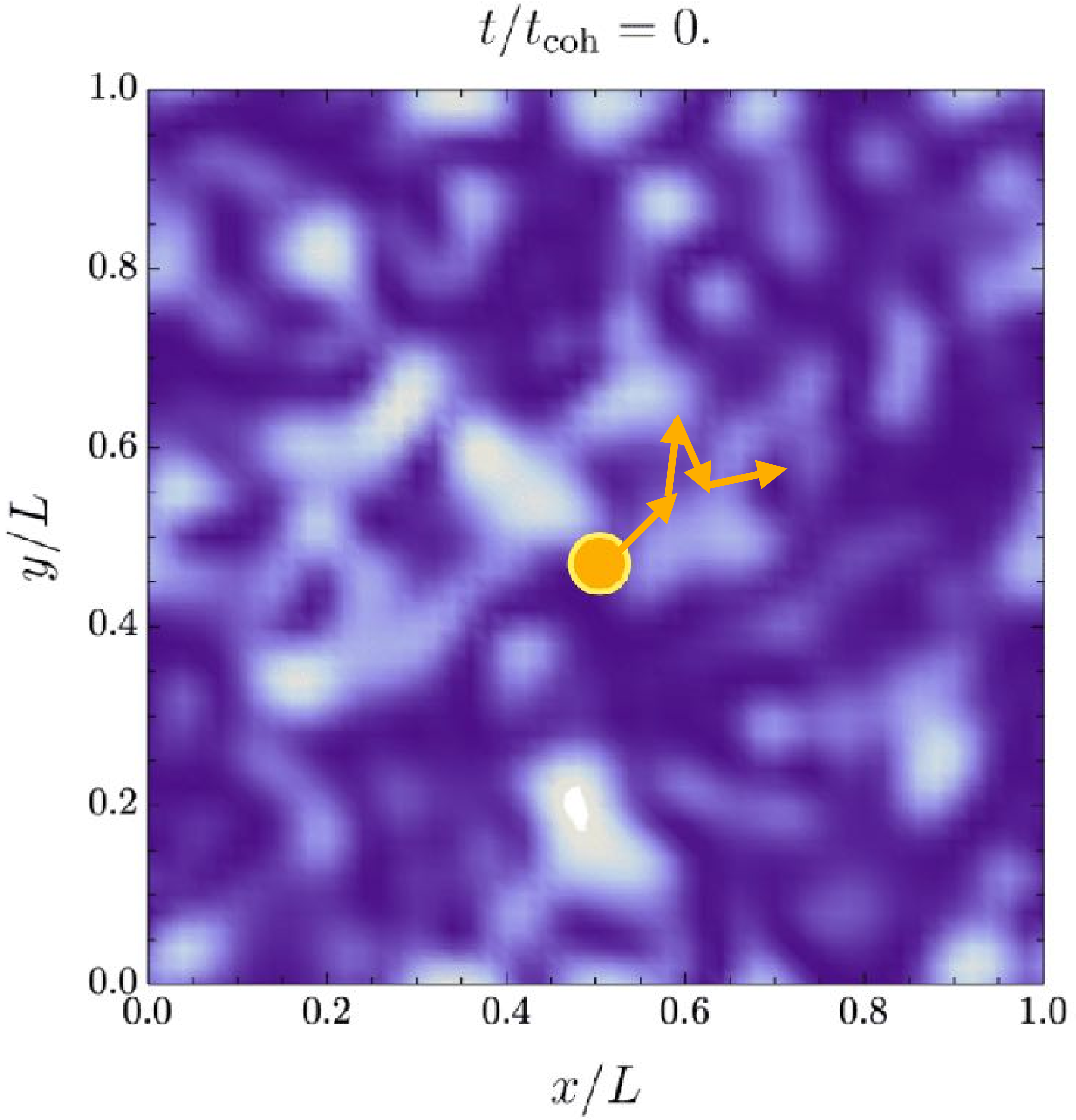
$$\bar{a} = \frac{Gm_{\text{eff}}}{\lambda^2}$$

$$\tau = \frac{1}{m\sigma^2}$$



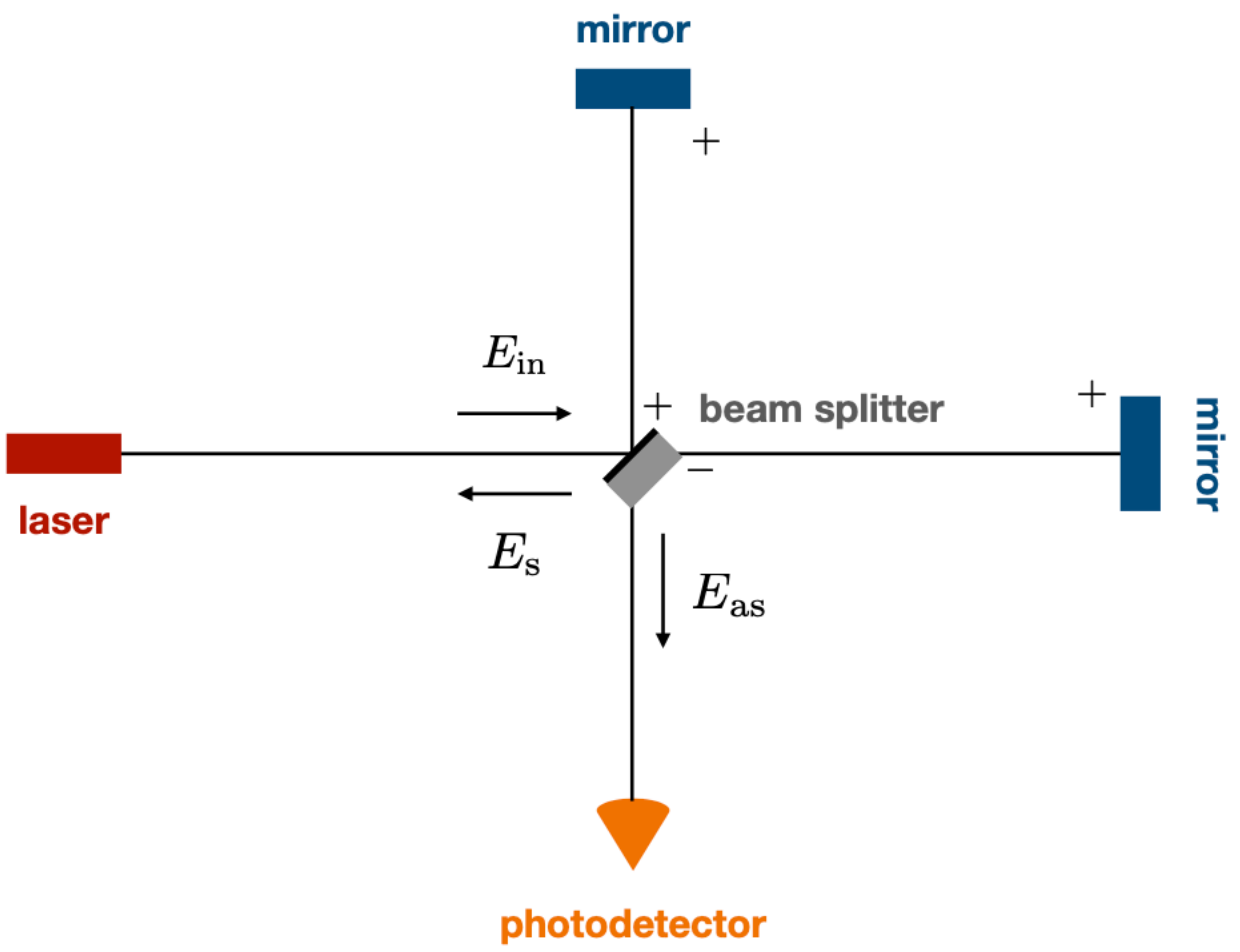

*test mass acceleration spectrum inherits its properties from δ(ω,k)*
*logarithmic divergence at small frequencies is due to long-range nature of gravitational force*

*the spectrum of acceleration allows us*
*to estimate the rms fluctuation of the position of a test mass*

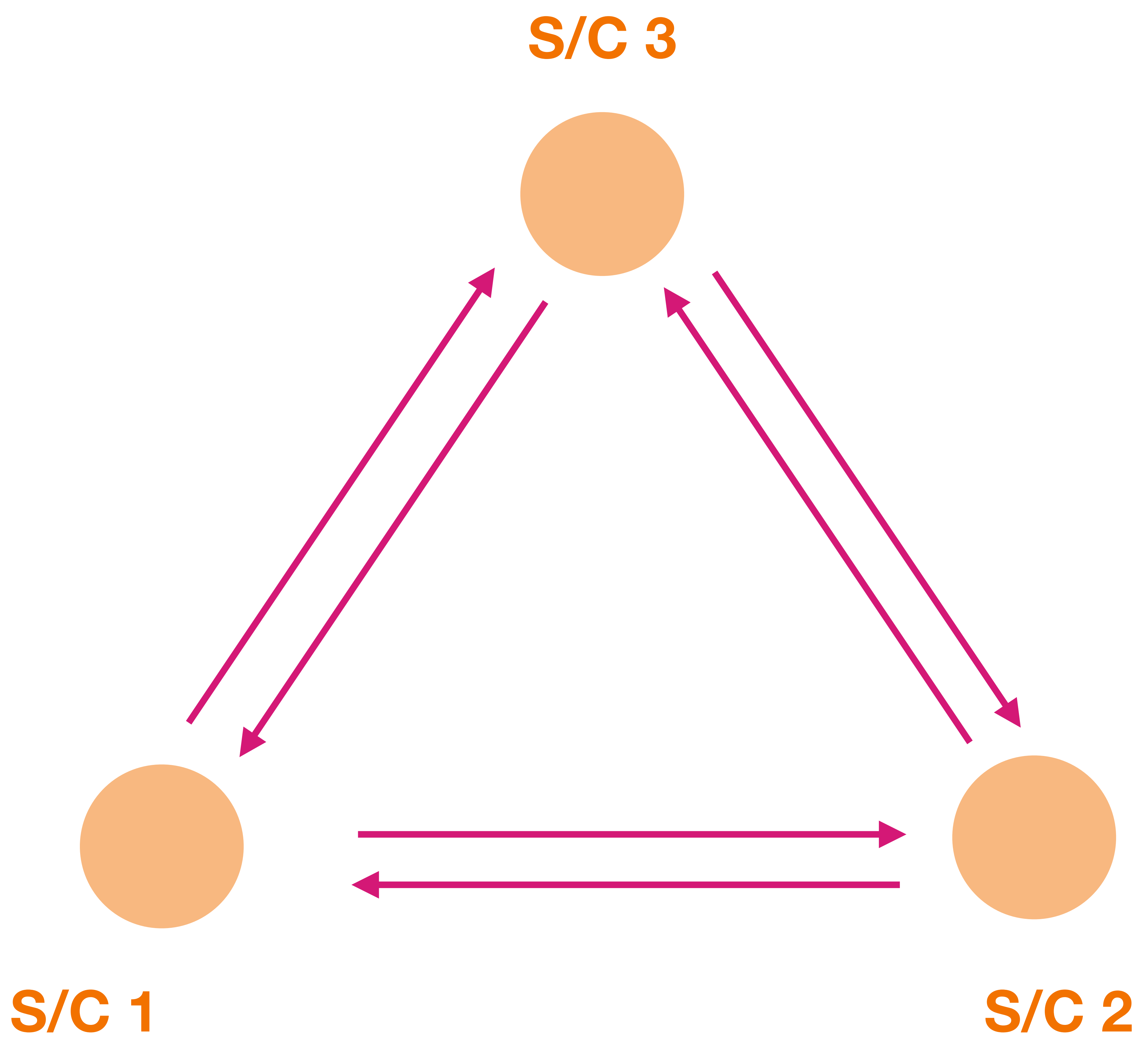



$$x^2_{\rm rms} = \int df \, \frac{S_a(f)}{(2\pi f)^4}$$

*it also constitutes a basic building block*
*to study the response of the interferometers*
*with respect to the ultralight dark matter*

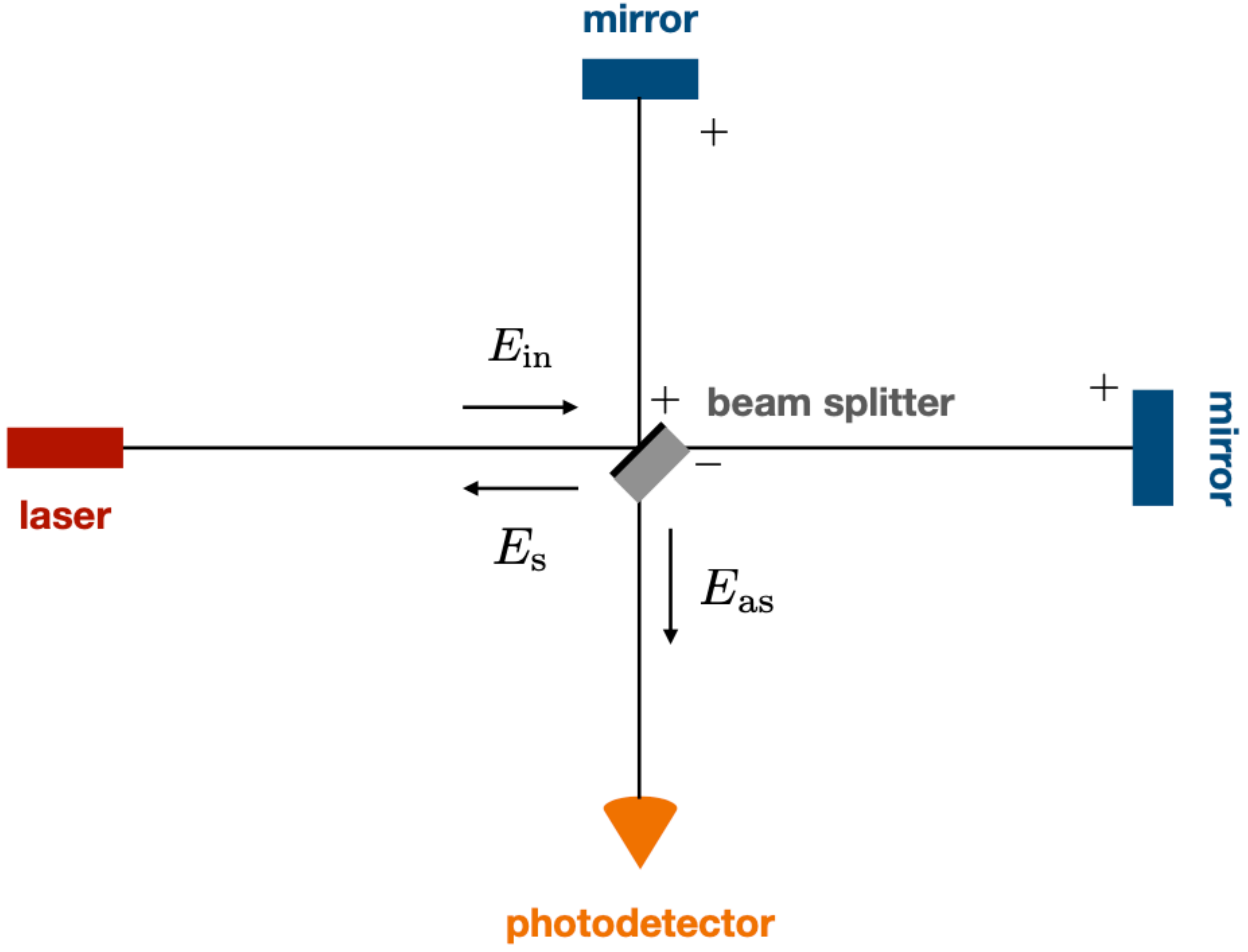



*Michelson Interferometer (LIGO, VIRGO, etc)*




*Time-delay Interferometer (LISA and etc)*

*In Michelson interferometers*
*the change of arm-length due to ULDM*
*will lead to fluctuations in output power*




$$P = P_0 \sin^2(k_L \Delta L)$$

*to see how Michelson interferometer responds to ULDM*
*let us investigate how light travels in one arm*


laser enters BS at **( t - 2L )**

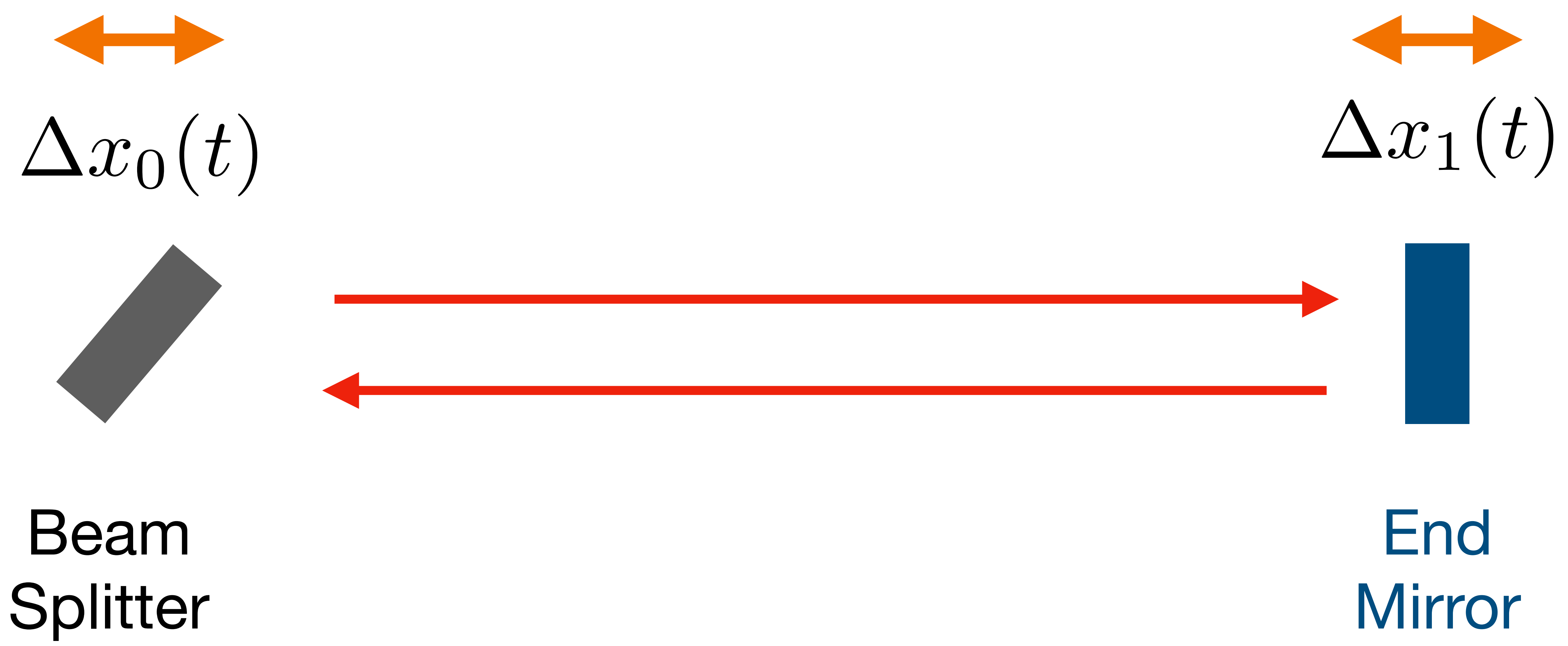
laser hits Mirror at **( t - L )**

laser returns to BS at **( t )**

Distance traveled by laser

$$2L = [x_1(t-L) - x_0(t)] + [x_1(t-L) - x_0(t-2L)]$$

*Quasiparticle will perturb the position of test mass and beam splitter*




## Perturbed distance traveled by laser

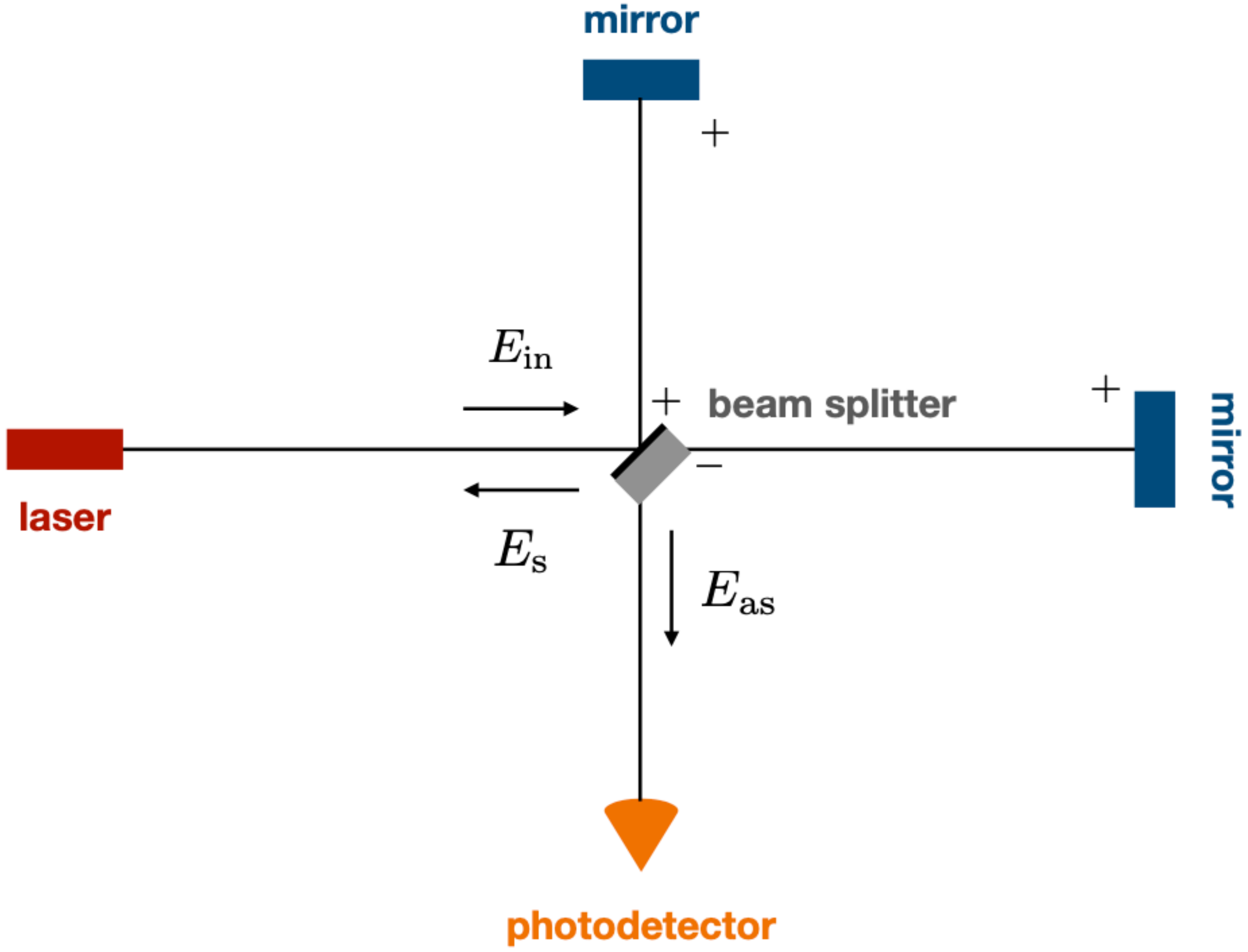
$$2\Delta L = [\Delta x_1(t-L) - \Delta x_0(t)] + [\Delta x_1(t-L) - \Delta x_0(t-2L)]$$

$$\widetilde{\Delta L}(f) = e^{2\pi i f L}[\widetilde{\Delta x_1}(f) - \cos(2\pi f L)\widetilde{\Delta x_0}(f)]$$

*response of the system to QP*

$$\widetilde{\Delta L}(f) \sim \frac{1}{(2\pi f)^2}\int \frac{d^3k}{(2\pi)^3}[e^{i\vec{k}\cdot\vec{x}_1} - \cos(2\pi f L)]a(f,\vec{k})$$

*acceleration by QP*




$$\bar{a} = \frac{G m_{\rm eff}}{\lambda^2}$$

$$\tau = \frac{1}{m\sigma^2}$$

*Combining the effects along the two arms one finds*

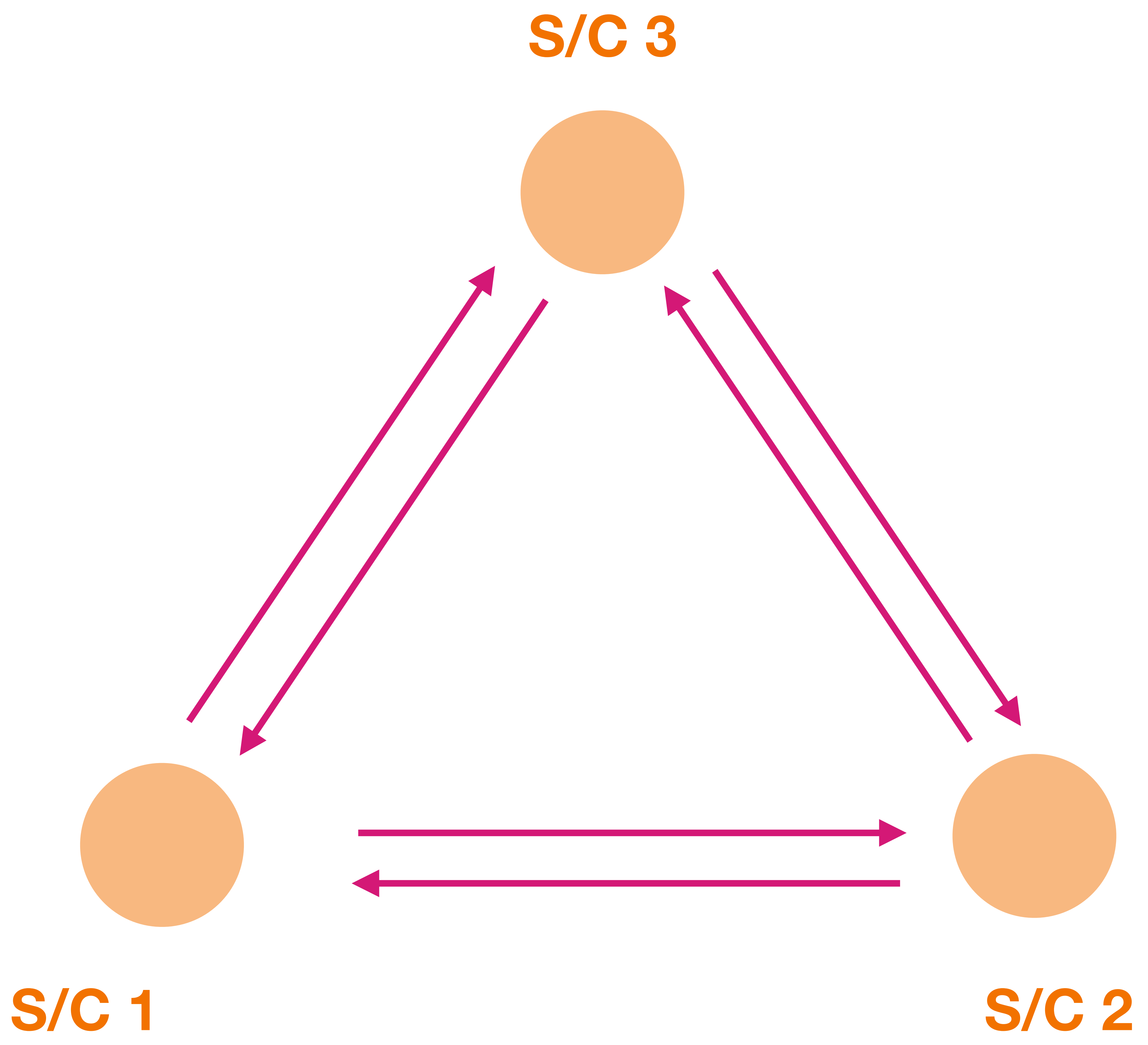
*response of Mich. Interferometer*
*B ~ $(L/\lambda)^2$ if L << $\lambda$ (tidal limit)*

$$S_{\Delta L/L}(f) = \frac{\bar{a}^2 \tau}{(2\pi f)^4 L^2}[\sigma^4 A_{\rm Mich}(f) + B_{\rm Mich}(f)]$$

*rms fluctuation of a single test mass $(\Delta L/L)^2_{rms}$ over $\Delta t \sim 1/f$*

$$\bar{a} = \frac{G m_{\rm eff}}{\lambda^2}$$

$$\tau = \frac{1}{m\sigma^2}$$



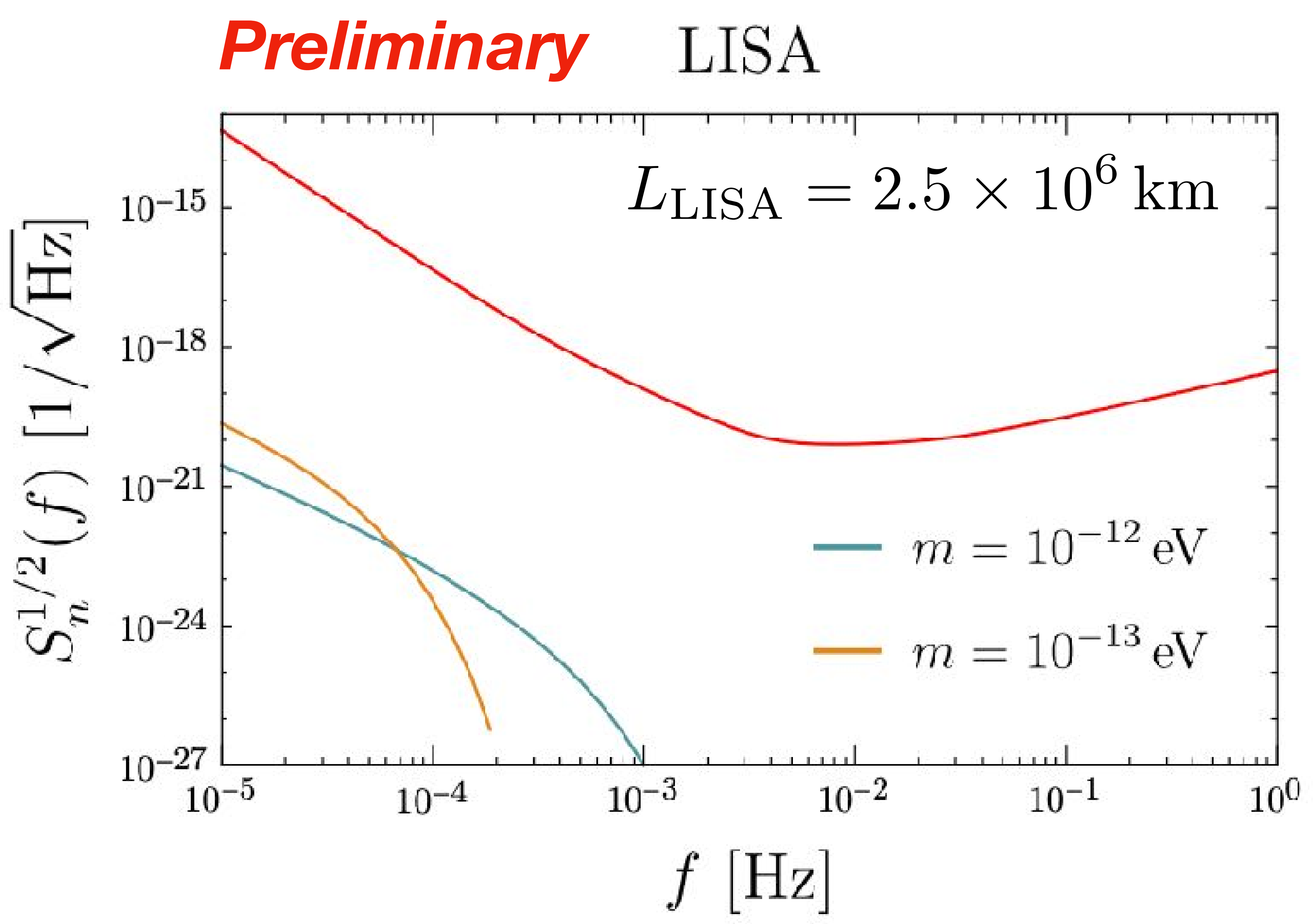

*one can find a similar expression for other types of interferometer*

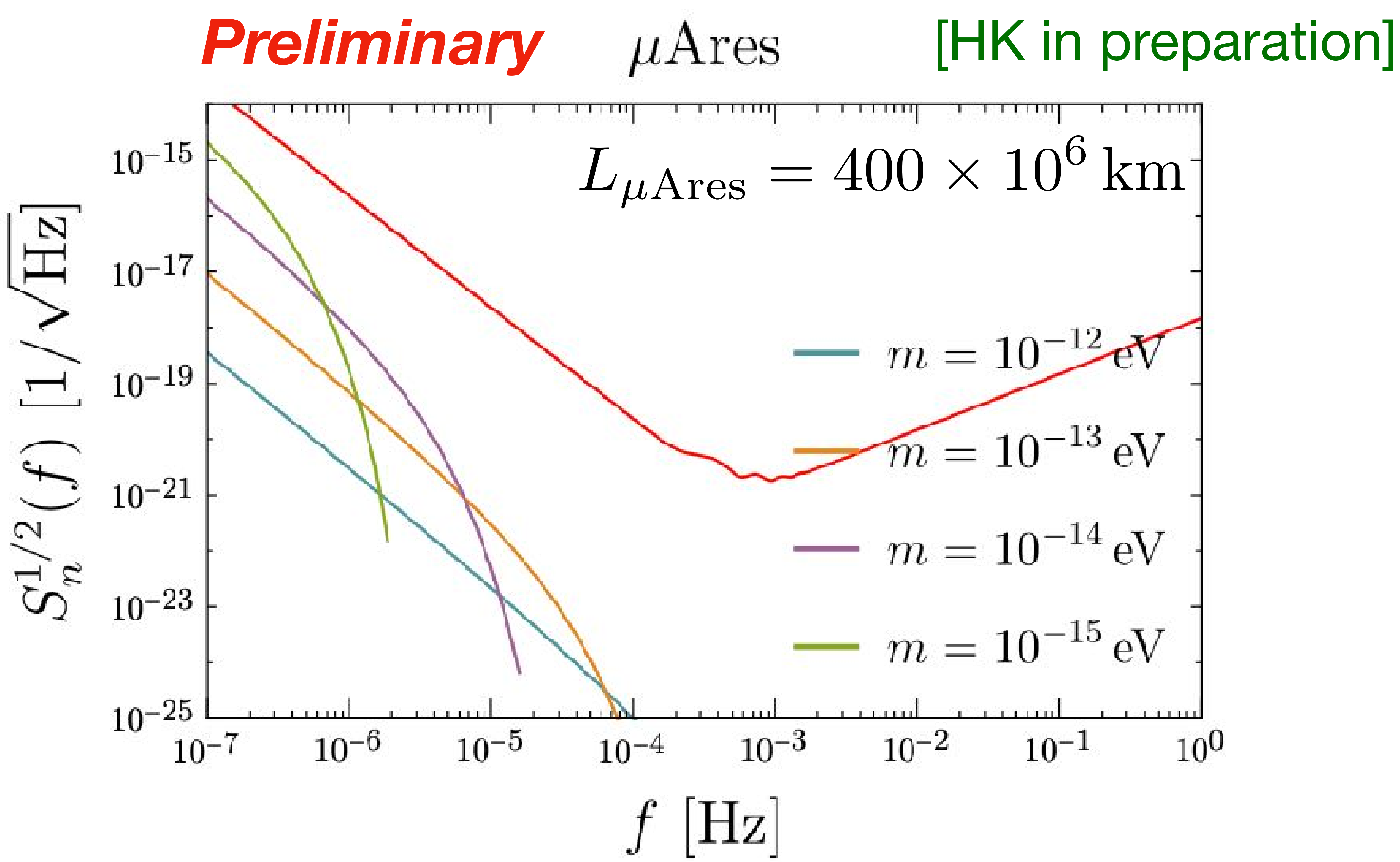
$$S_{\Delta L/L}(f) = \frac{\bar{a}^2\tau}{(2\pi f)^4 L^2}[\sigma^4 A_{\rm TDI}(f) + B_{\rm TDI}(f)]$$

*rms fluctuation over Δt~1/f*

*response of TDI Interferometer*
*B ~ (L/λ)² if L << λ (tidal limit)*

*when ULDM signal is translated into strain power spectrum*




*Even for space-borne interferometers, ULDM-induced noise are subdominant*

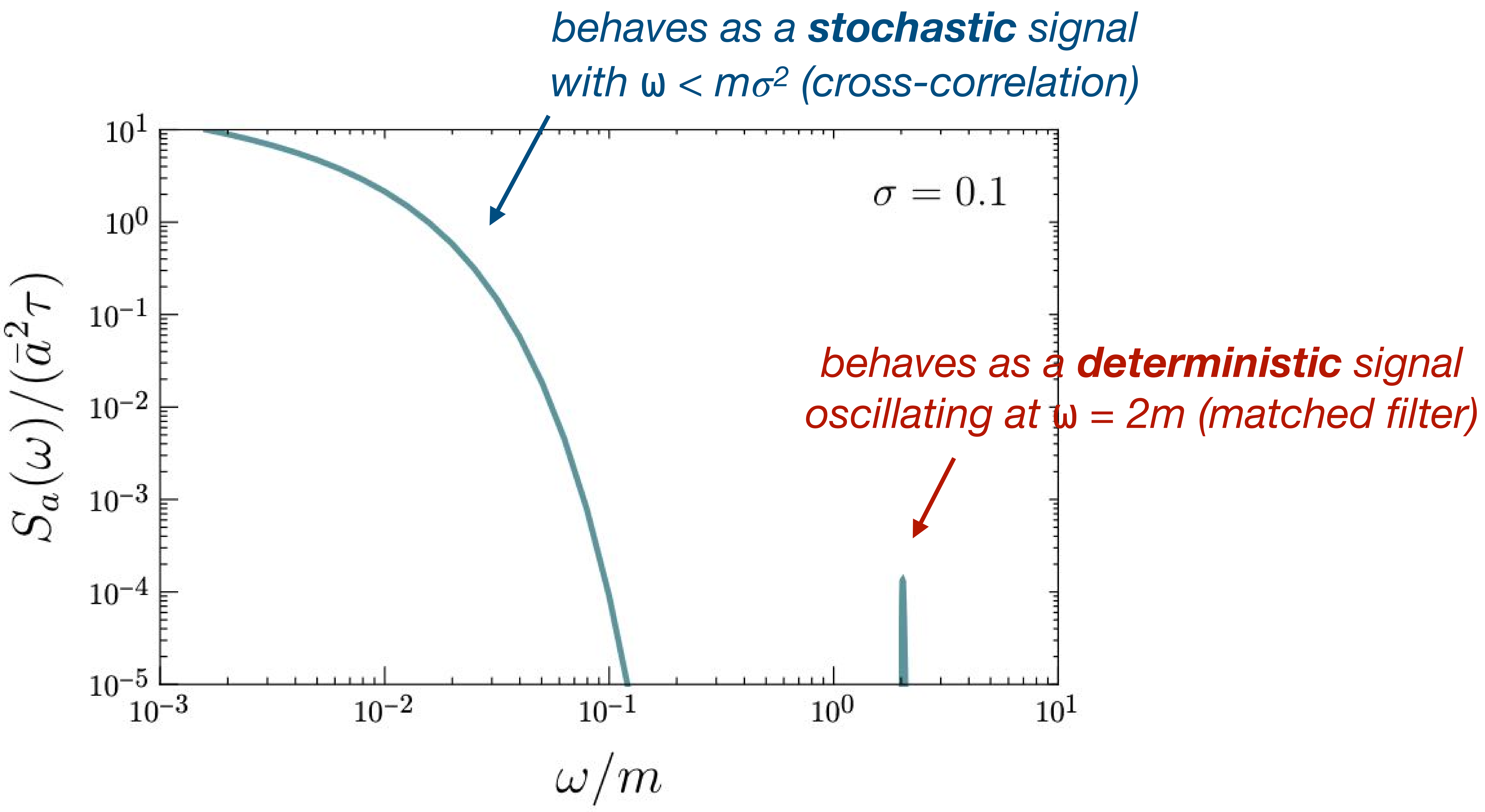
Two questions:

*1. Do we have to worry about* ***ULDM-induced noise***
*in the current and future gravitational waves?*

*for current and future GW intereferometers*
*gravitational interaction of ULDM leaves subdominant noise*

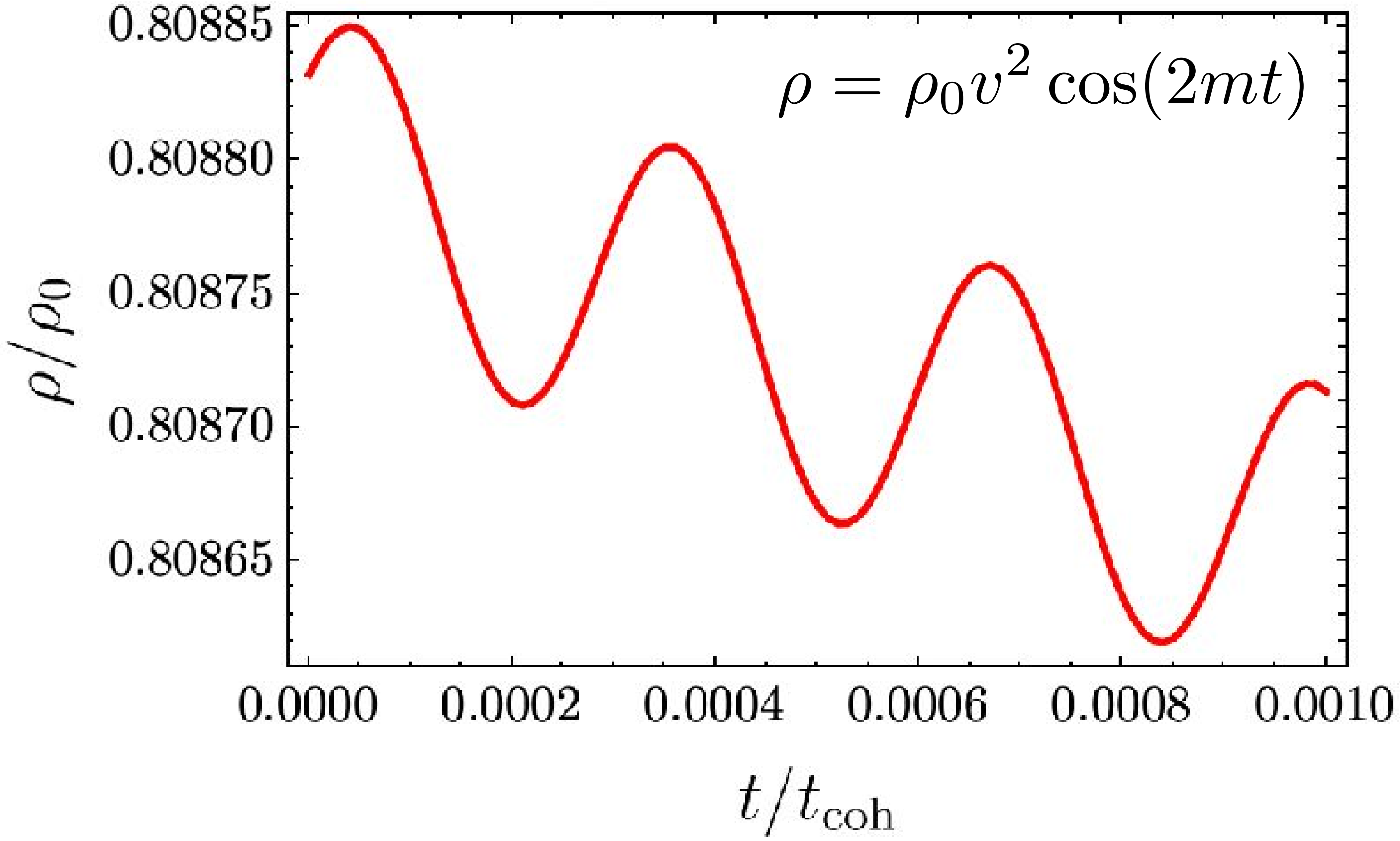
2. Can current and future GW interferometers
***probe ultralight dark matter gravitationally?***

*more specifically*
*can we constrain dark matter density in the solar system*
*through gravitational interaction with GW interferometers?*


behaves as a **stochastic** signal
with ω < mσ² (cross-correlation)
behaves as a **deterministic** signal
oscillating at ω = 2m (matched filter)
$\sigma = 0.1$
$S_a(\omega)/(\bar{a}^2\tau)$
$\omega/m$
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$

## *the deterministic signal*

*we have seen coherently oscillating mode in $\rho$*




*the position of the test mass will also behave in a similar way*

$$\Delta L/L \propto \cos(2mt)$$

*As we know the shape of the signal*
*we can 'filter' the detector output such that it coherently picks up the signal*
*by choosing the optimal filter K(t)*

$$\int dt\, d(t) K(t)$$

$$d(t) = s(t) + n(t)$$

$$K(t) \propto \cos(2mt)$$

*the signal is coherently added up*
*while the noise is added incoherently*

$$\frac{S}{N} = \left[ T \int_{-\infty}^{\infty} df \, \frac{S_s(f)}{S_n(f)} \right]^{1/2}$$

*coherent addition of signal*

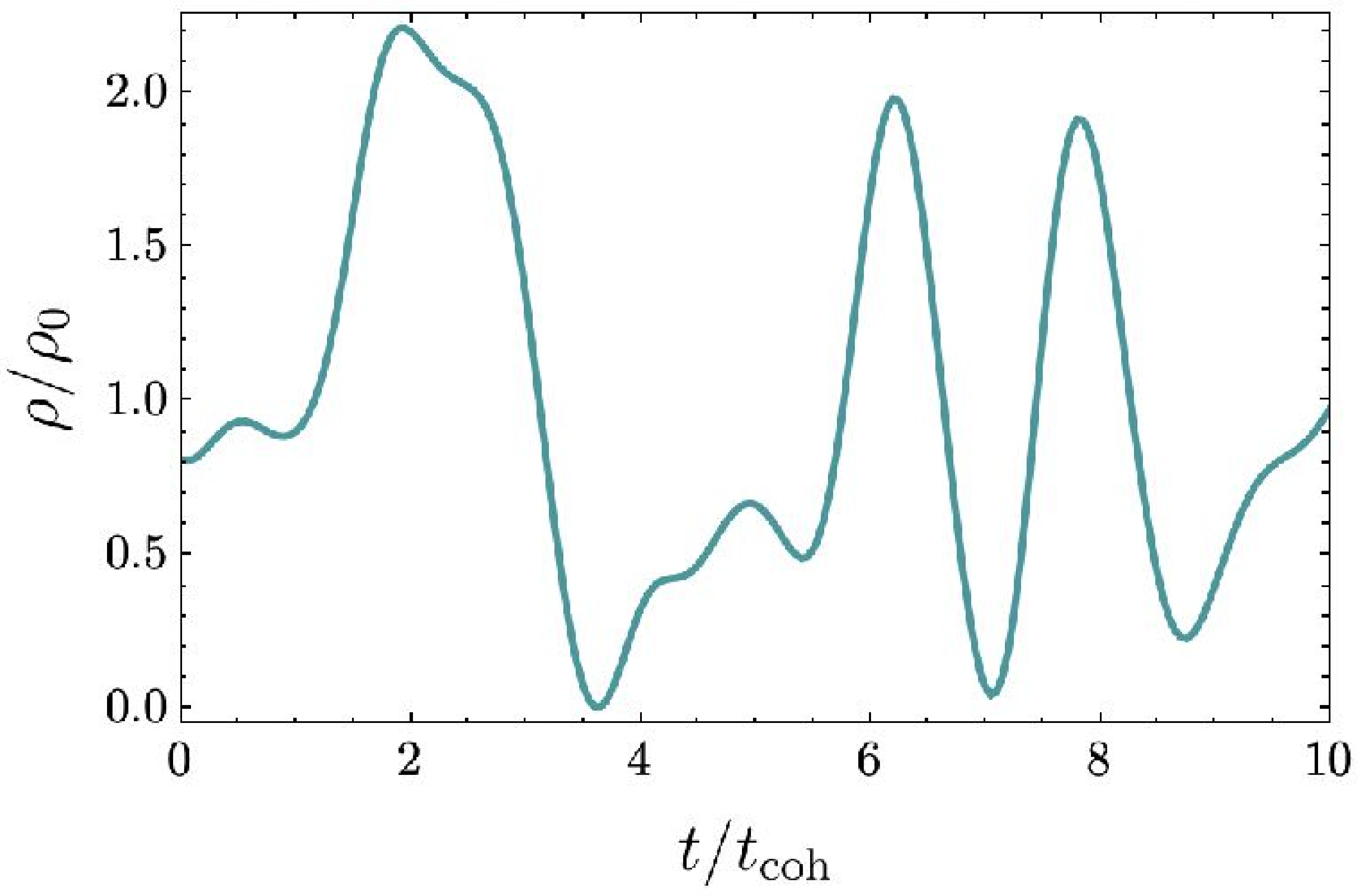
*$(\Delta L/L)_{rms}$ over $\Delta t_m \sim 1/m$ due to ULDM*

*$(\Delta L/L)_{rms}$ due to detector noise*

$$\sim \frac{[\bar{a}\sigma^2/(2\pi f_m)^2 L]}{[S_n(f_m)/\tau]^{1/2}} \left[ T \int df\, A_{\rm Mich} \right]^{1/2}$$

*the stochastic signal*

*we have seen random changes in $\rho$ over coherent time scale $t_{coh}$*

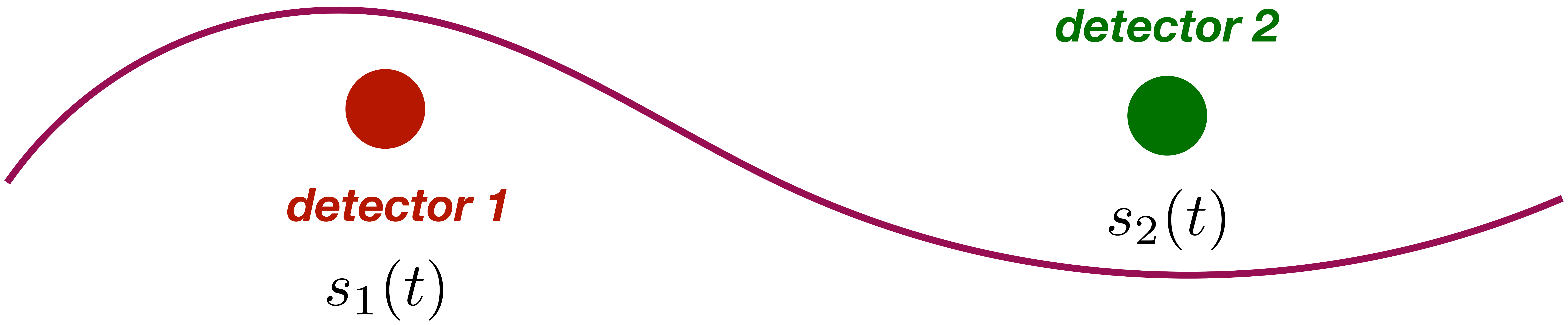



*the form of the signal is unknown, and hence, matched filter cannot be used*

## *the stochastic signal*

*if we have more than two detectors*
*we can cross-correlate the signals*

$$Y = \int dt \int dt' s_1(t) s_2(t') Q(t-t')$$




*the noise is expected to be uncorrelated*
*the correlated signal can be picked up by choosing an optimal filter Q(t)*

*the stochastic signal*

*if we have more than two detectors*
*we can cross-correlate the signals*

$$\frac{S}{N} = \left[ T \int_{-\infty}^{\infty} df \, \frac{|S_{12}(f)|^2}{S_n^2(f)} \right]^{1/2}$$

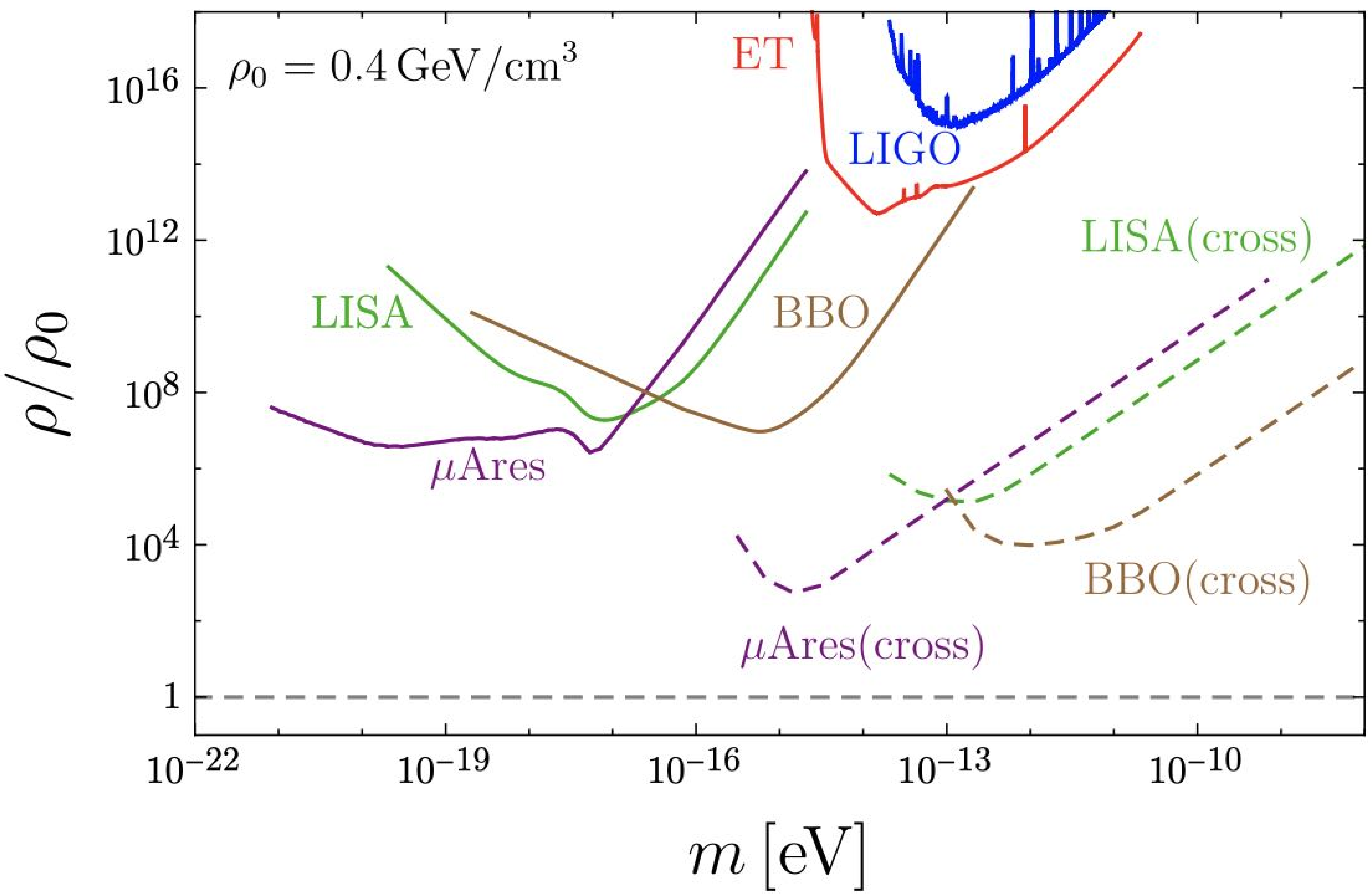
*coherent addition of signal*

*$(\Delta L/L)^2_{rms}$ over $\Delta t \sim t_{coh}$ due to ULDM*

$$\sim \frac{\bar{a}^2}{(2\pi f_0)^4 L^2} \left[ T \int df \, \frac{(f_0/f)^8 |B_{\rm cross}|^2}{f_0^2 S^2_{\Delta L/L}(f)} \right]^{1/2}$$

*$(\Delta L/L)^4_{rms}$ due to detector noise*

*the noise is expected to be uncorrelated*
*the correlated signal can be picked up by choosing an optimal filter Q(t)*




[HK in preparation]

*A few remarks on local dark matter density*

$$\rho_0 = 0.4\,\mathrm{GeV/cm^3}$$

*is a measured value over the volume $V > [O(10^2)\ \mathrm{pc}]^3$*

see reviews e.g. [Read (14)]; [de Salas, Widmark (20)]

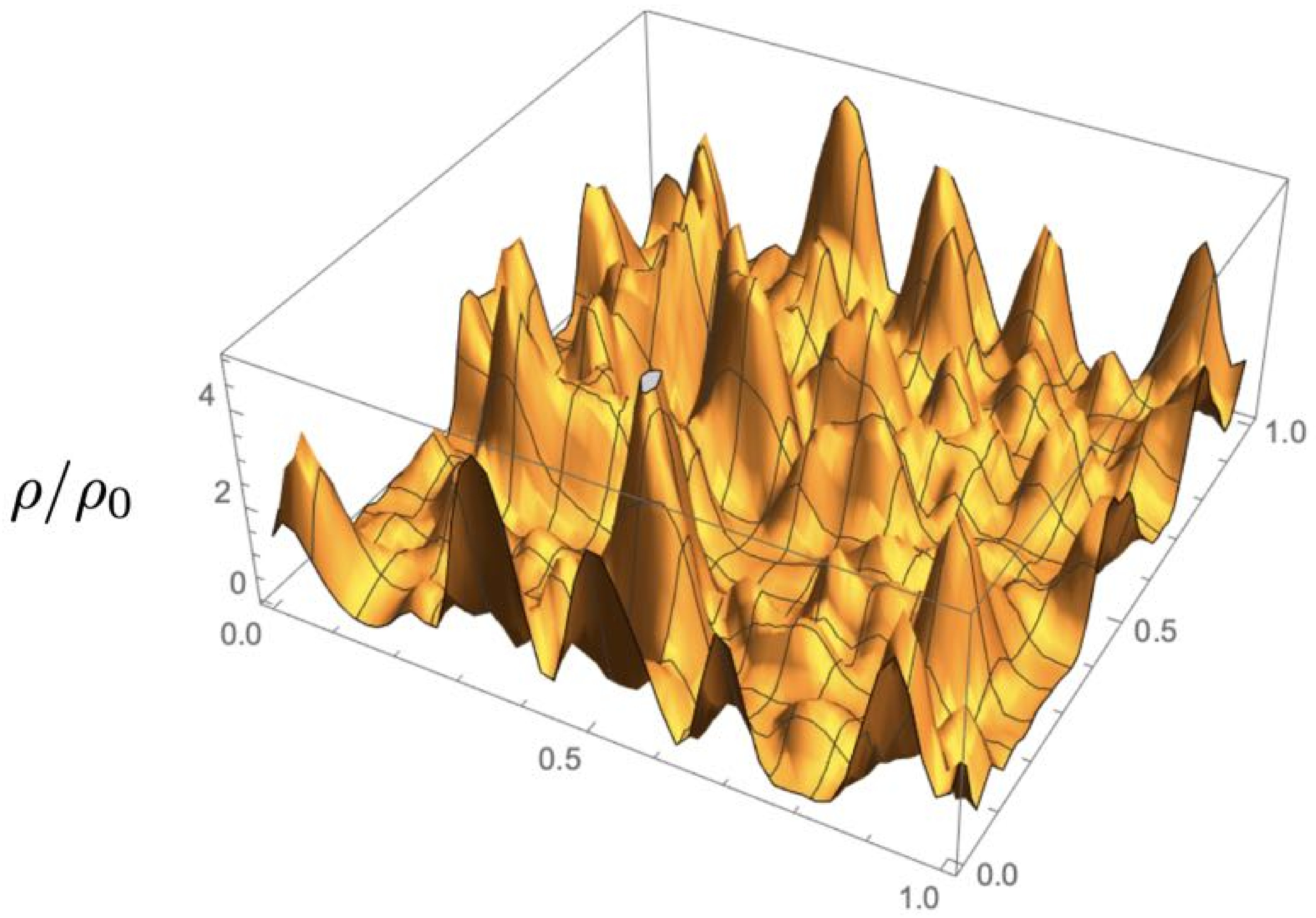
*currently no direct measurement of dark matter density in the solar system but only constraints exist*

$$\rho/\rho_0 \lesssim 10^4$$

From solar system ephemerides [Pitjev, Pitjeva (13)]

$$\rho/\rho_0 \lesssim 10^{11}$$

From geodetic satellite and LLR [Adler (08)] *in addition we have seen that there could be easily O(1) density fluctuation in the wave DM halo*

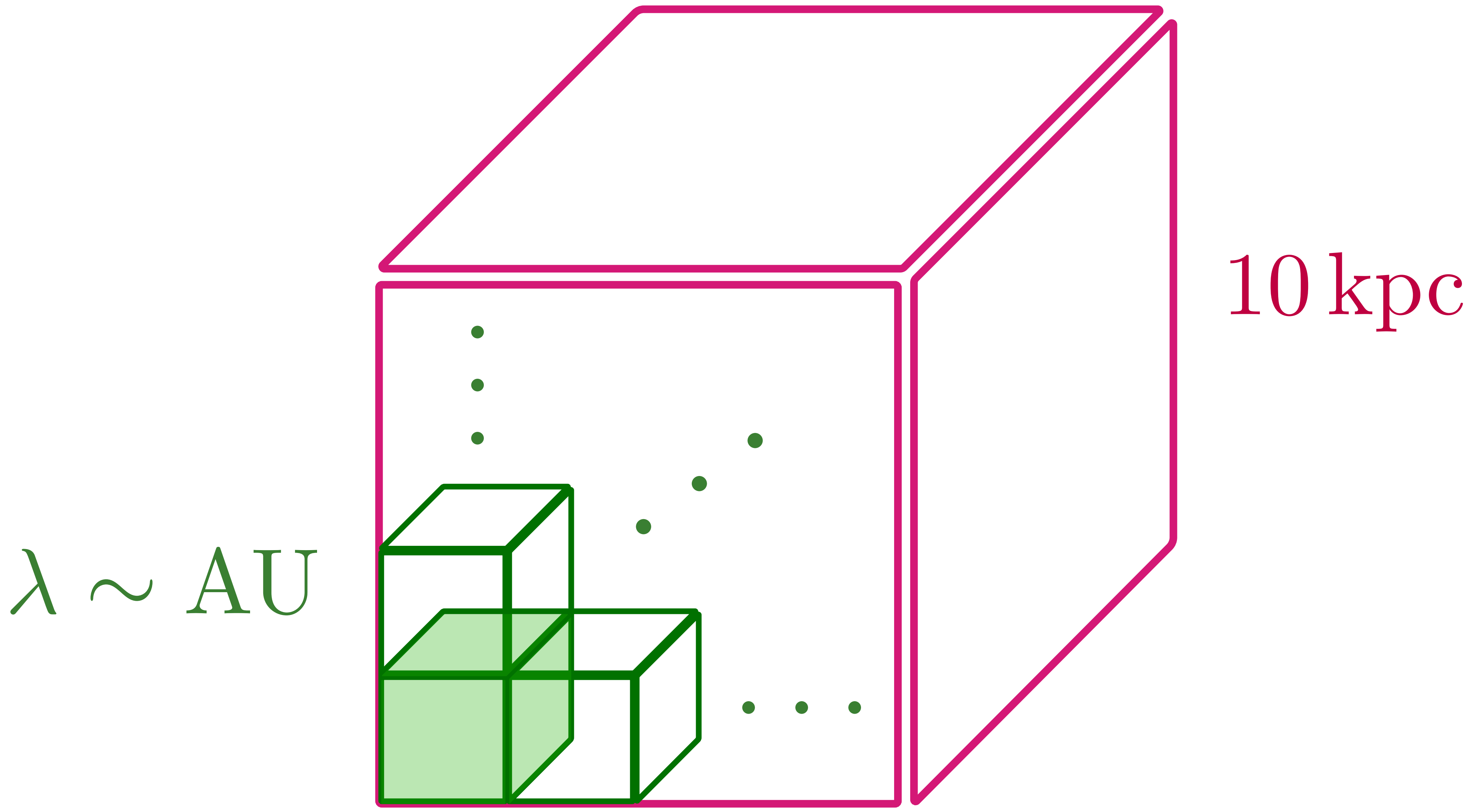



$$p(\rho)d\rho = \frac{1}{\rho_0}\exp\left[-\frac{\rho}{\rho_0}\right]d\rho$$

$$P(\rho > \rho_c) = e^{-\rho_c/\rho_0}$$

the probability of finding $\rho/\rho_0 > 5$ is small (~ 1%)
but it happens quite often because there are many of such patches

consider e.g. $m \sim 10^{-15}$ eV where the wavelength is ~AU scale
in the volume of $V = (10\ \text{kpc})^3$ there are $10^{28}$ AU-sized patches




statistically speaking
there will be ~ 100 patches
in this volume with $\rho > 60\ \rho_0$!

$$N_{\text{patches}} = (10\,\text{kpc}/\text{AU})^3 \simeq 10^{28}$$

$$P(\rho > 60\rho_0) = e^{-60} \simeq 10^{-26}$$