Particle Avenues in the Dark Universe Arena (PADUA): Axions

Axion Dark Matter from Strings

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The Abdus Salam International Centre for Theoretical Physics

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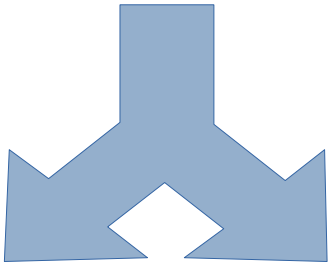
12 - 14 September 2022

based on: 1806.04677 & 2007.04990 with M. Gorghetto and E. Hardy

Non-Thermal Cold Axions



$v_{PQ} \gtrsim \max(H_I, T_{\rm max})$

pre-inflation scenario

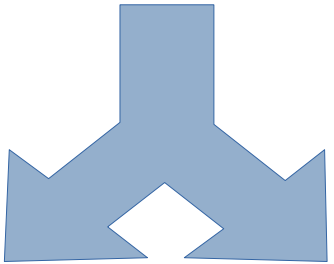
PQ-symmetric phase
not restored after inflation

$v_{PQ} \lesssim \max(H_I, T_{\rm max})$

post-inflation scenario

PQ-symmetric phase
restored after inflation

Non-Thermal Cold Axions



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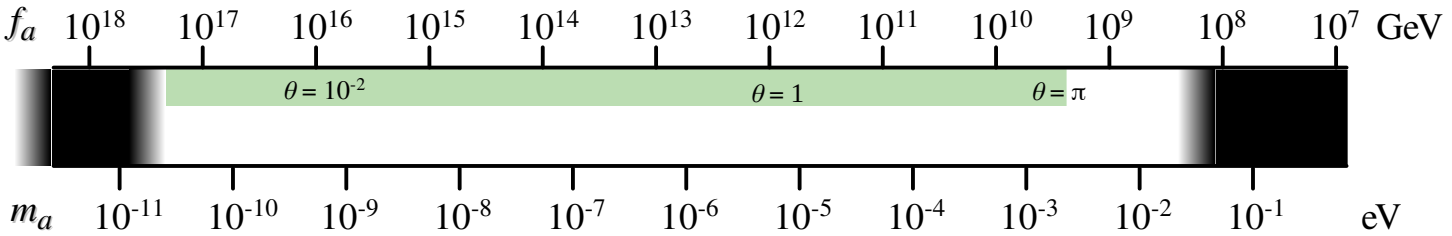
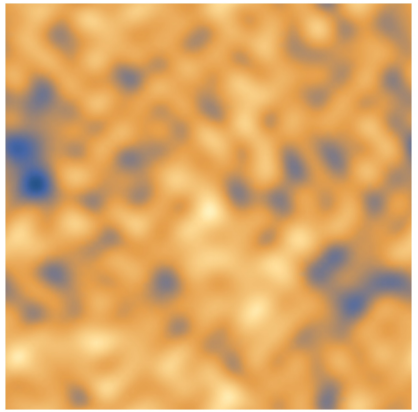


DM Axions from misalignment: Post Inflation scenario

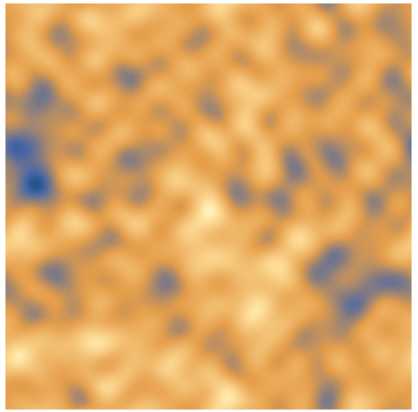
PQ-phase unbroken after Inflation $\max\{H_I, T_{\rm max}\} \gtrsim v_{\rm PQ}$

$$\Omega_a(m_a, ...) = \Omega_{\rm DM}$$

axion mass prediction

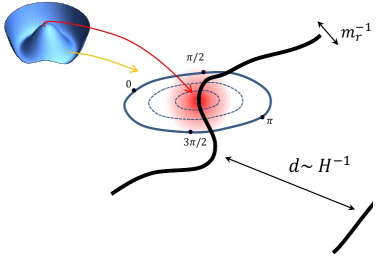


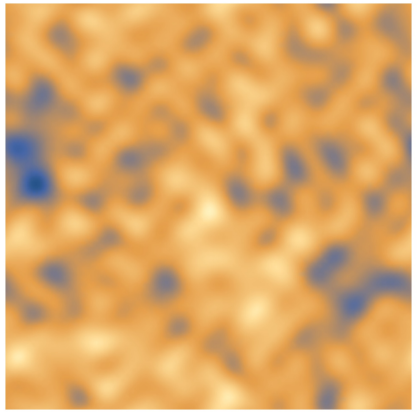
Complex non-linear evolution



Complex non-linear evolution

Strings





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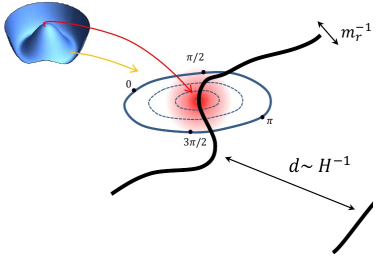
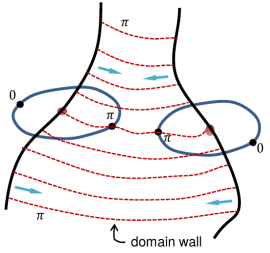
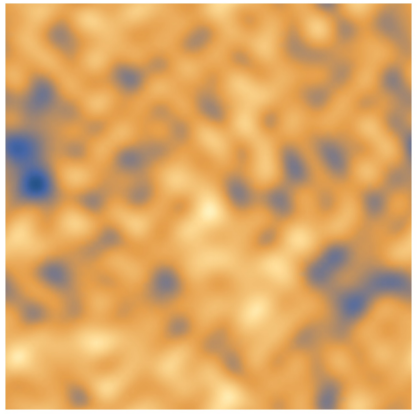


Domain Walls





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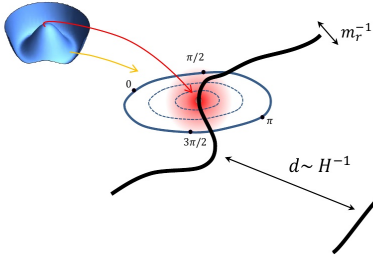

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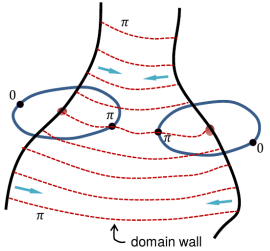
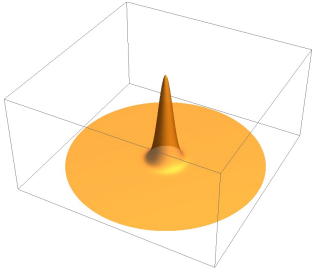
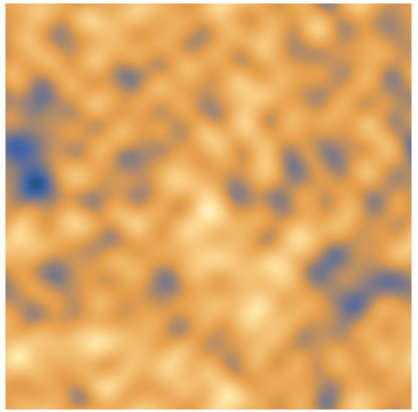


Oscillons





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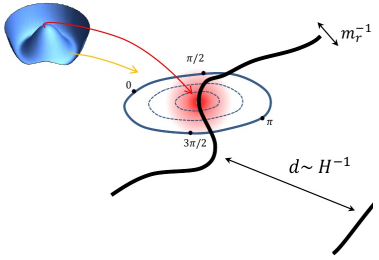

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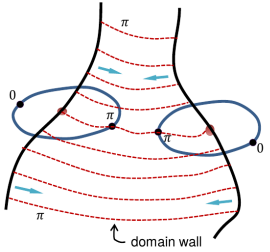

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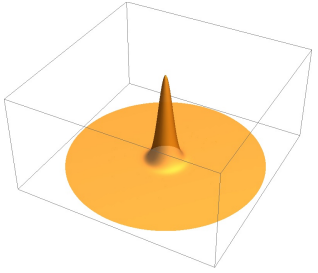
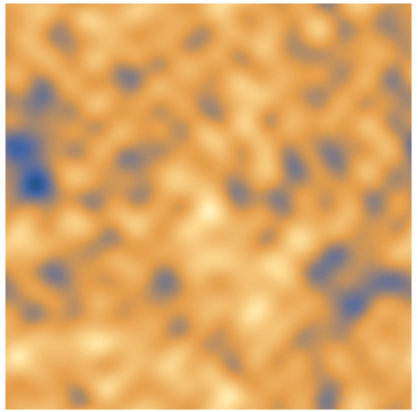


Mini-Clusters

from 1804.05857



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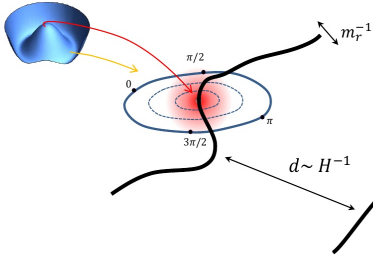

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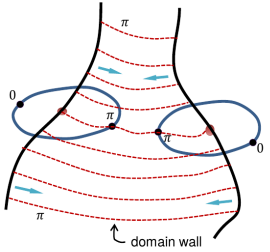

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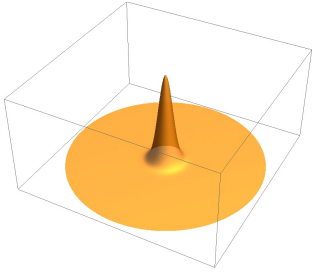
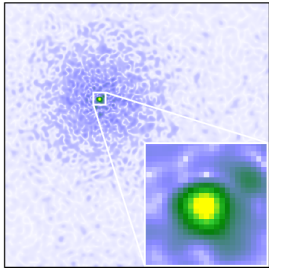


Mini-Clusters

Axion-Stars



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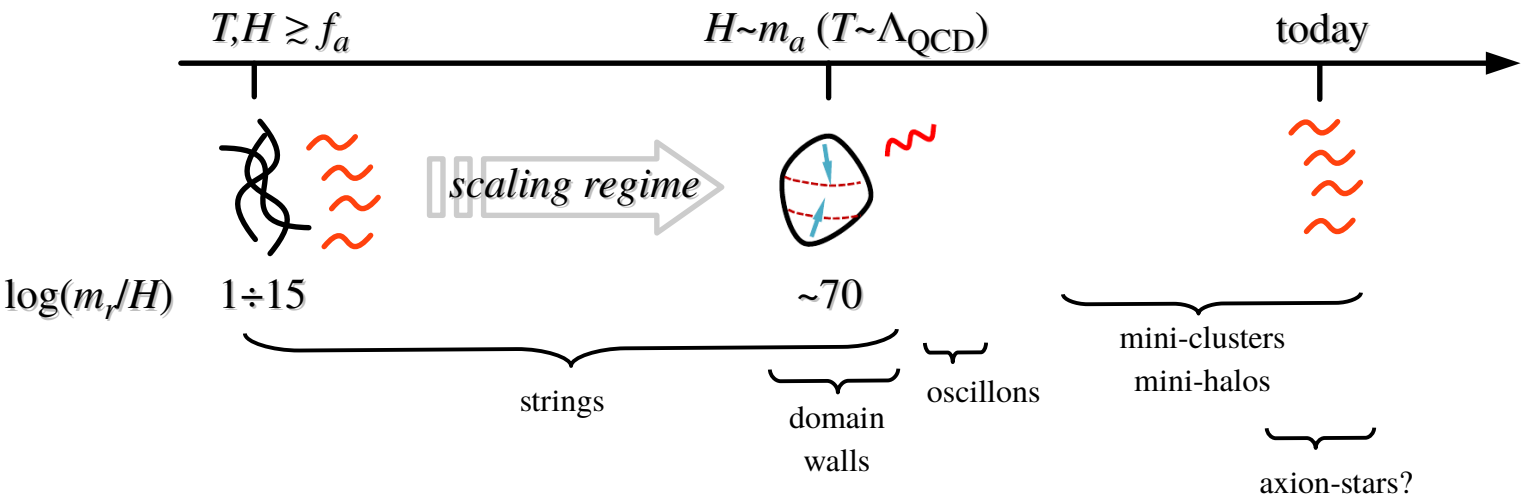
T,H ≳ f_a
H~m_a (T~Λ_QCD)
today
scaling regime
log(m_r/H)
1÷15
~70
strings
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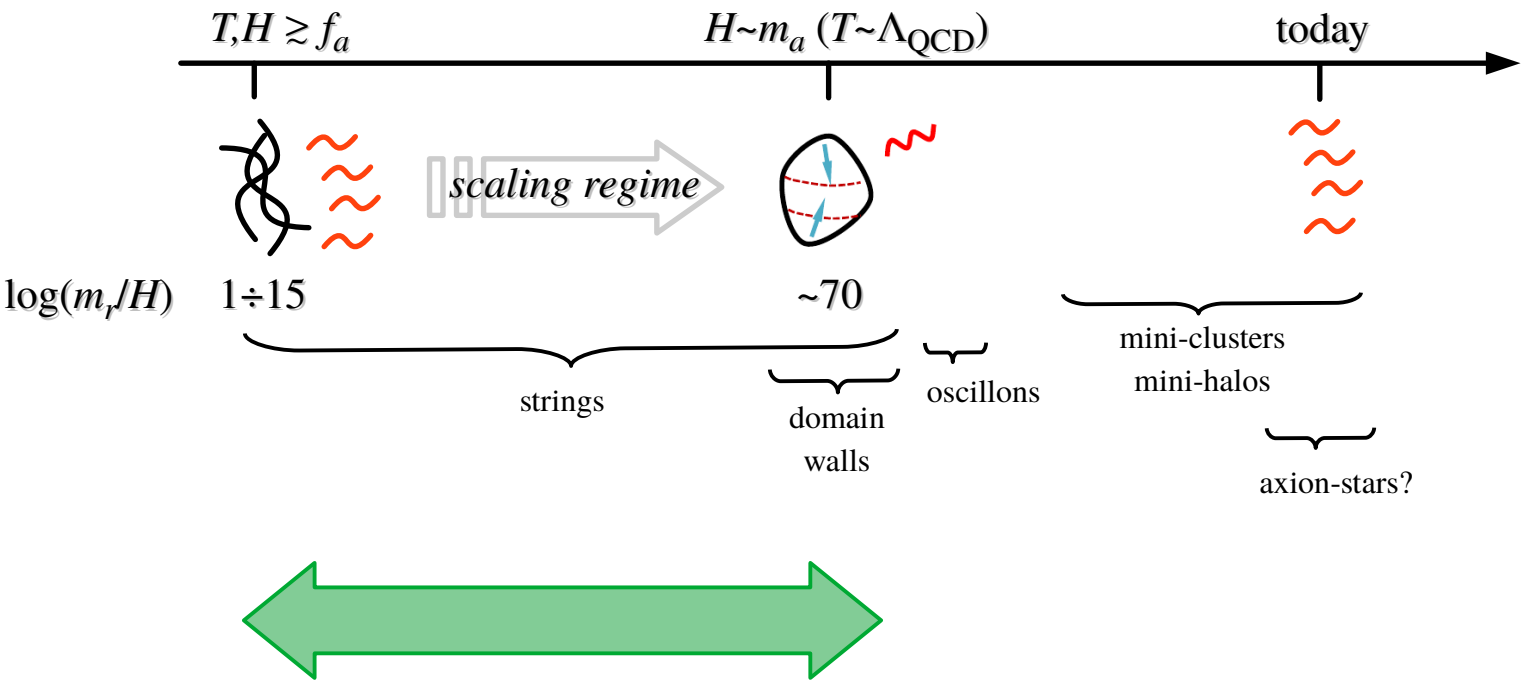
mini-halos
axion-stars?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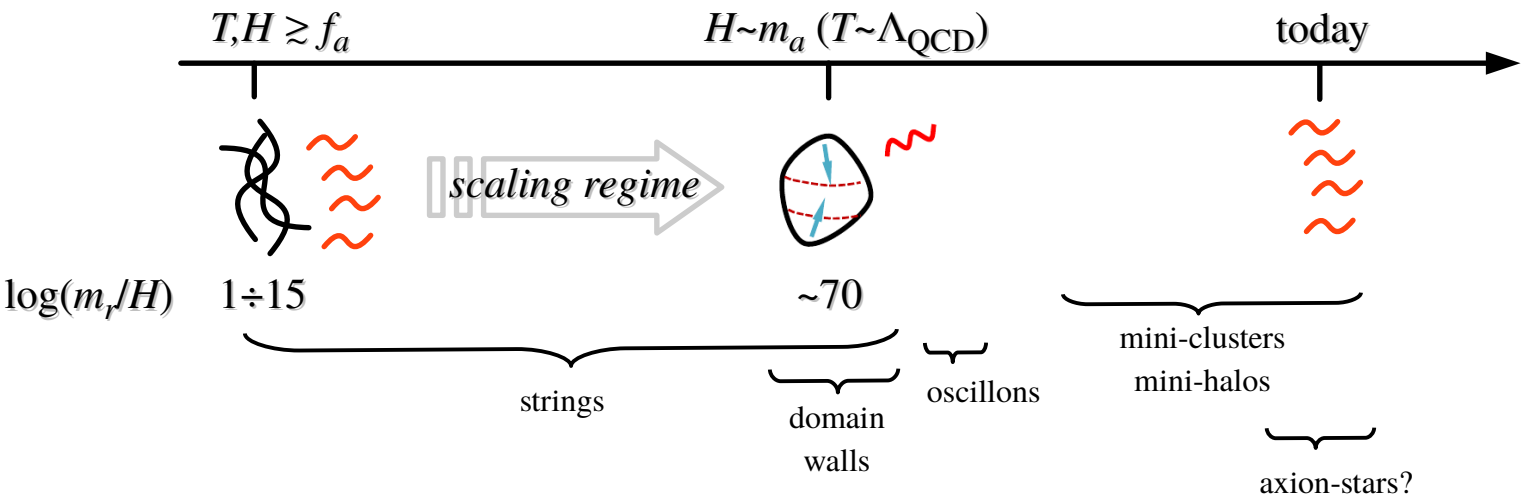
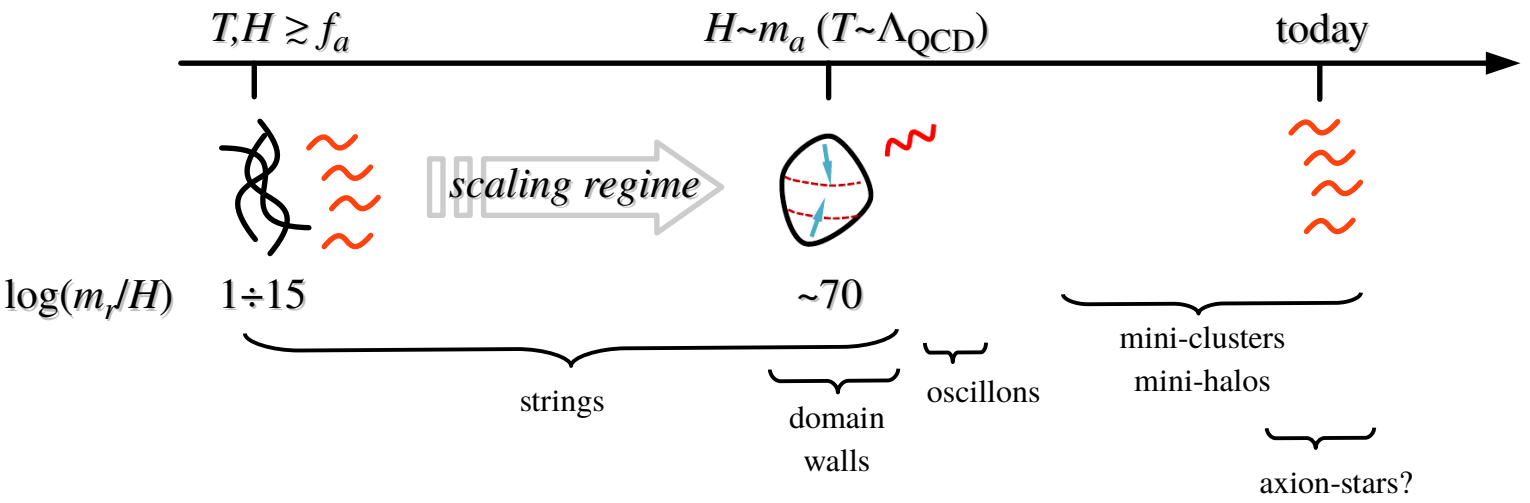


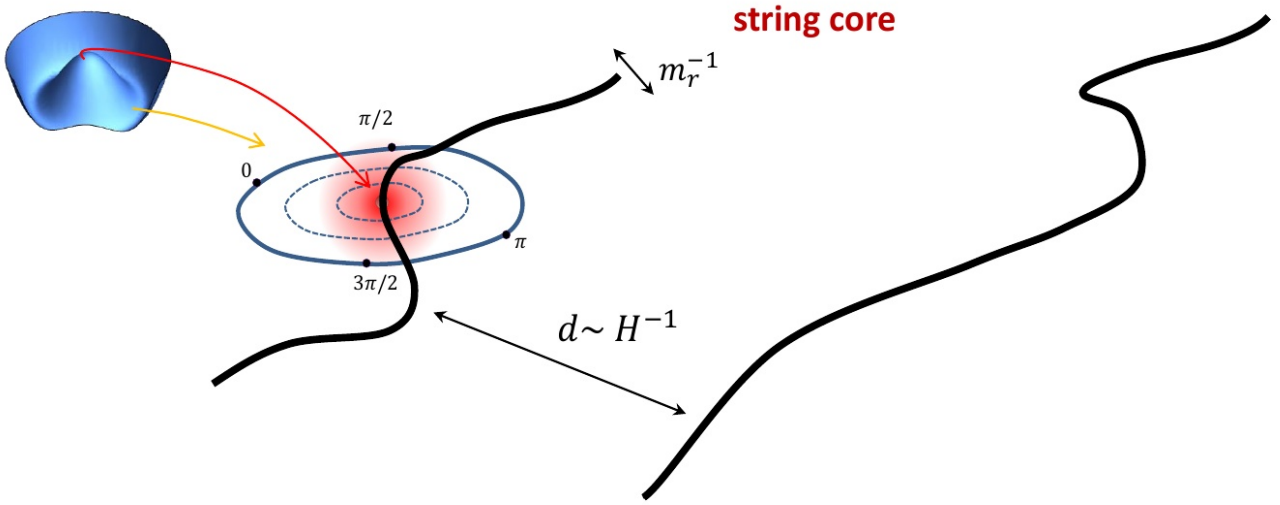
Non-Linear Evolution ⇒ ☹ ~~Analytic~~



Non-Linear Evolution → ☹ ~~Analytic~~

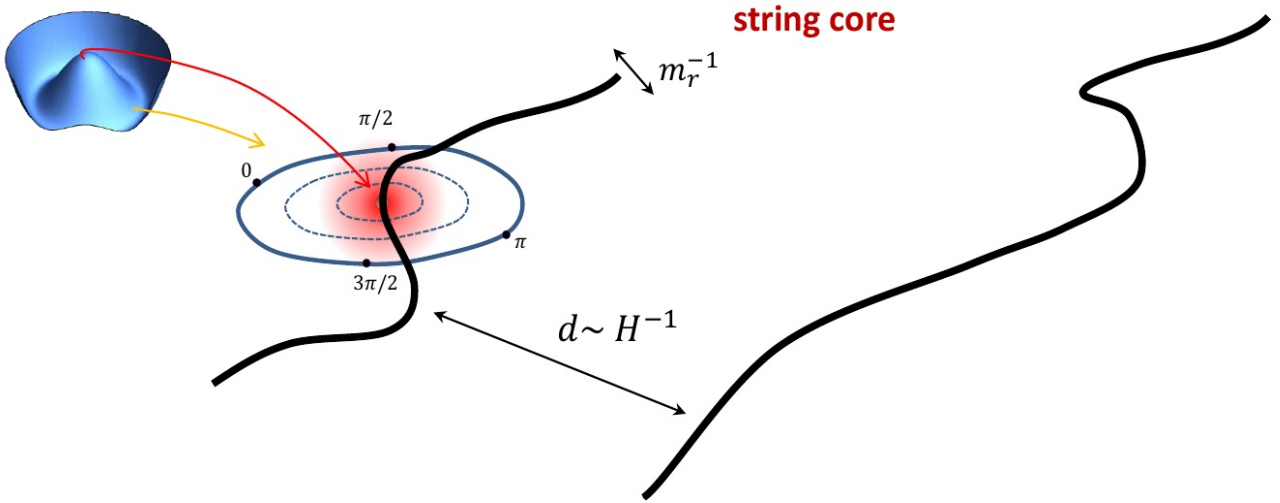
Multi-Scale Problem → ☹ ~~Numerical~~

Axionic Strings



string tension $$\mu = \frac{E}{L} = \pi v_{PQ}^2 \log(m_r \ell_{IR})$$

Axionic Strings



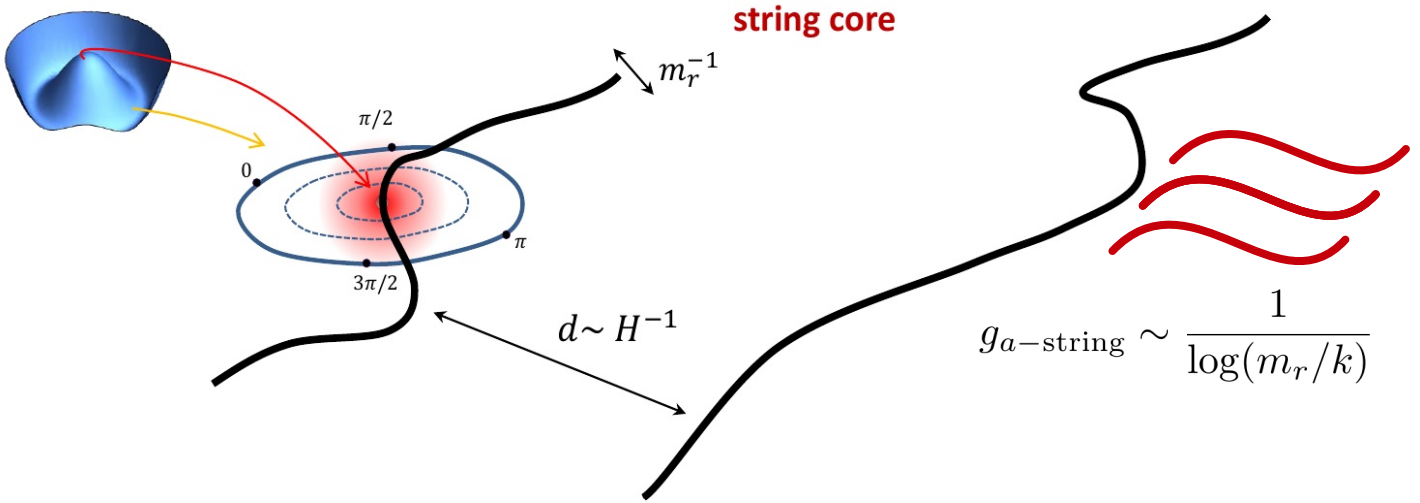
string tension

$$\mu = \frac{E}{L} = \pi v_{PQ}^2 \log(m_r \ell_{IR}) \simeq \pi v_{PQ}^2 \log\left(\frac{m_r}{H}\right)$$

radial core

axion field

Axionic Strings



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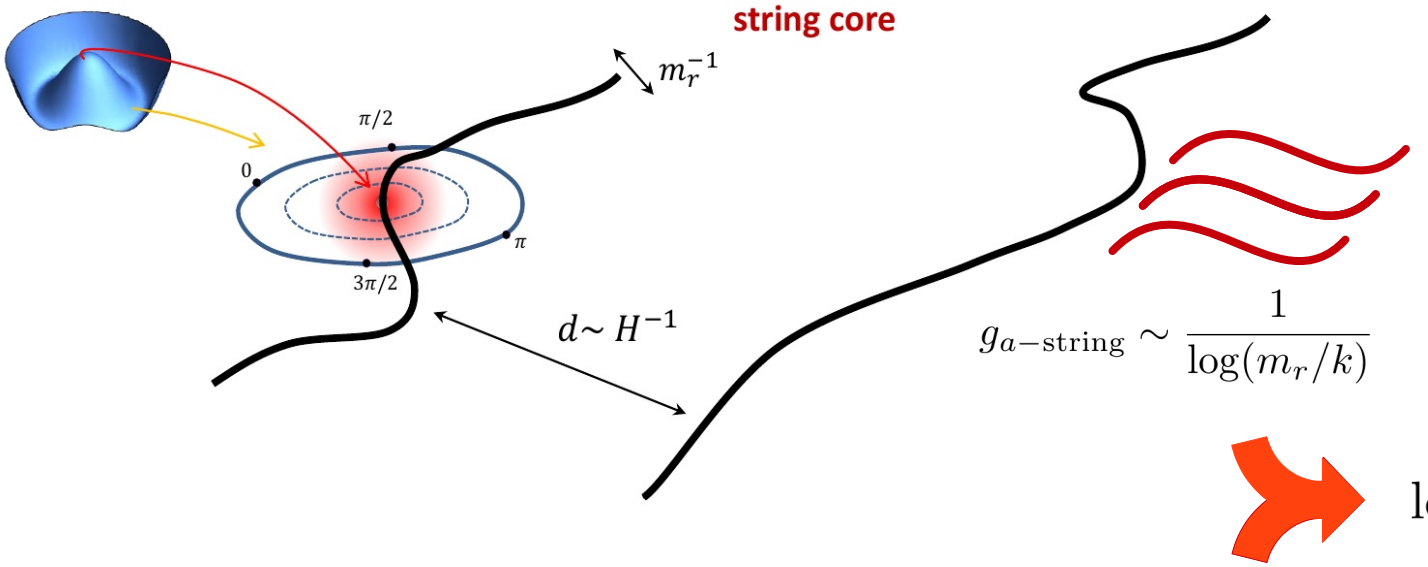

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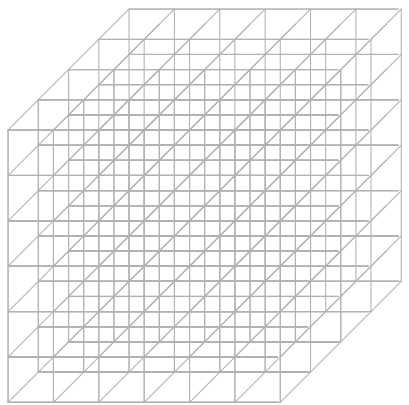
radial core

axion field

Numerical Simulation

$$V(\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$$

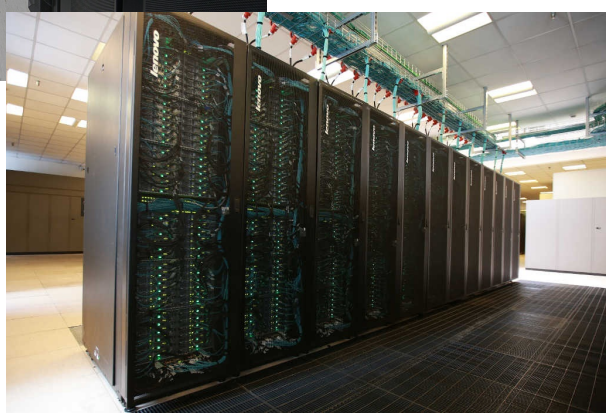
$$\Phi = \frac{f_a + r}{\sqrt{2}}\, e^{ia/f_a}$$



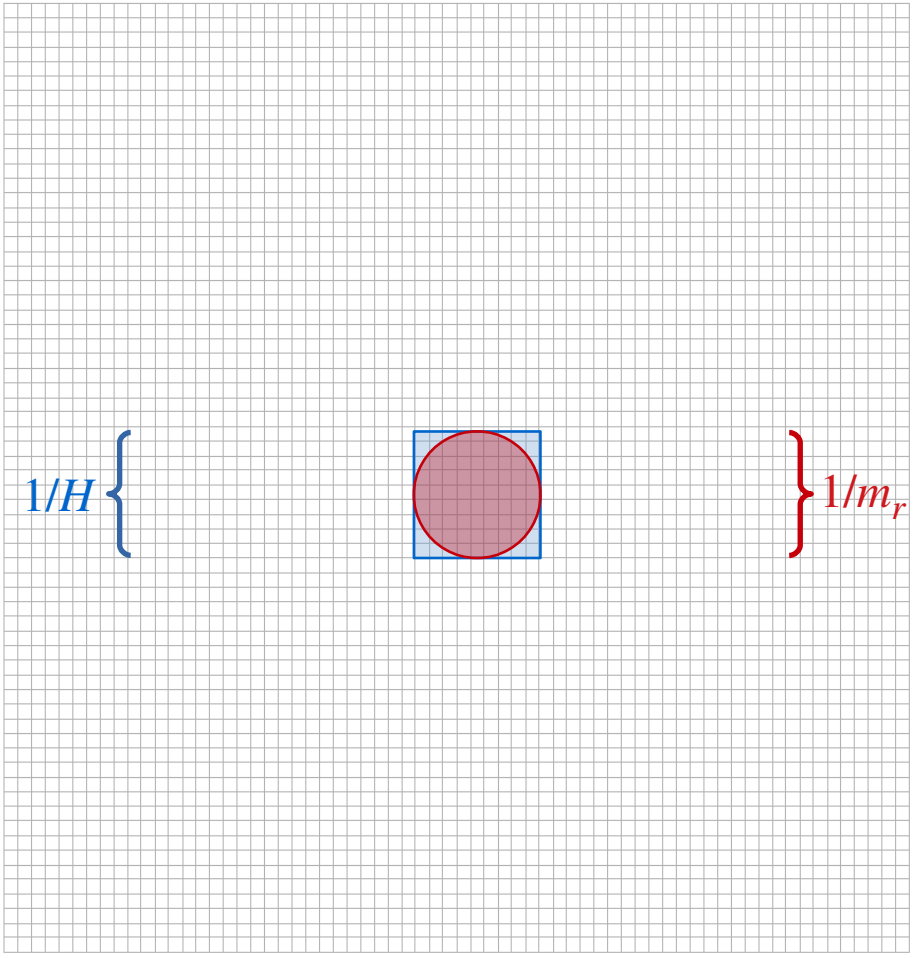
$$L \approx 4 \cdot 10^3\, a$$

ICTP/SISSA HPC

CINECA



The Bottle Neck



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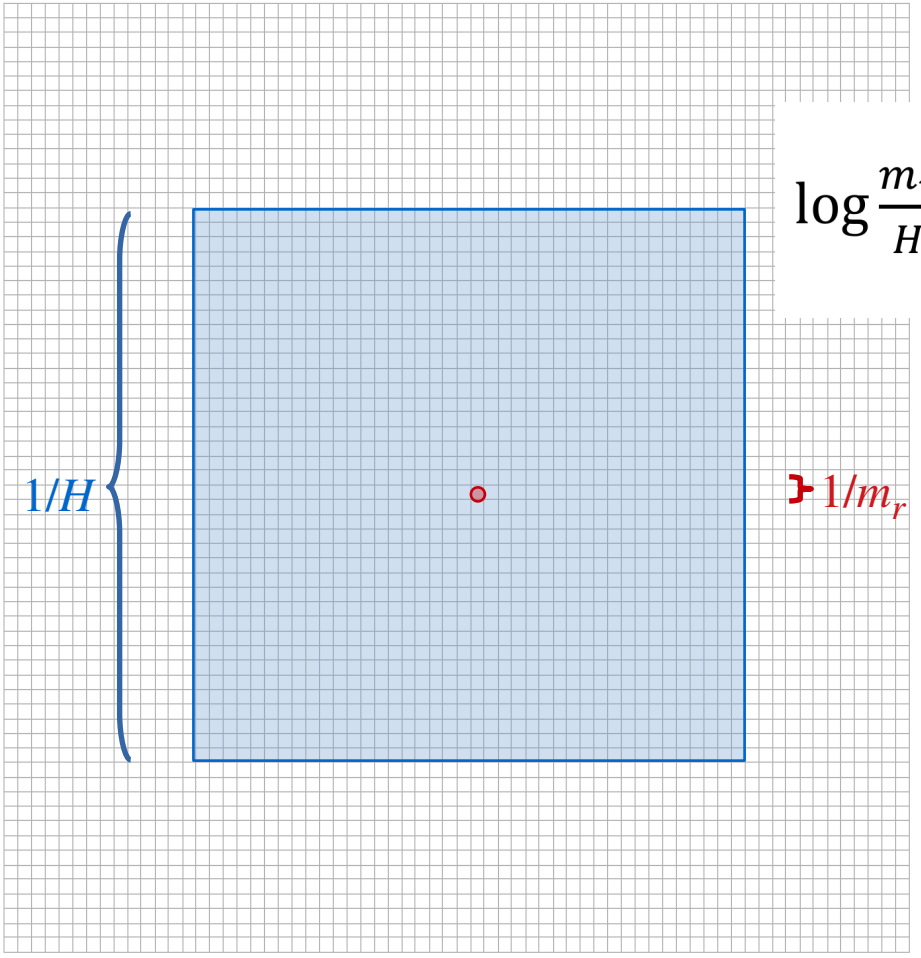
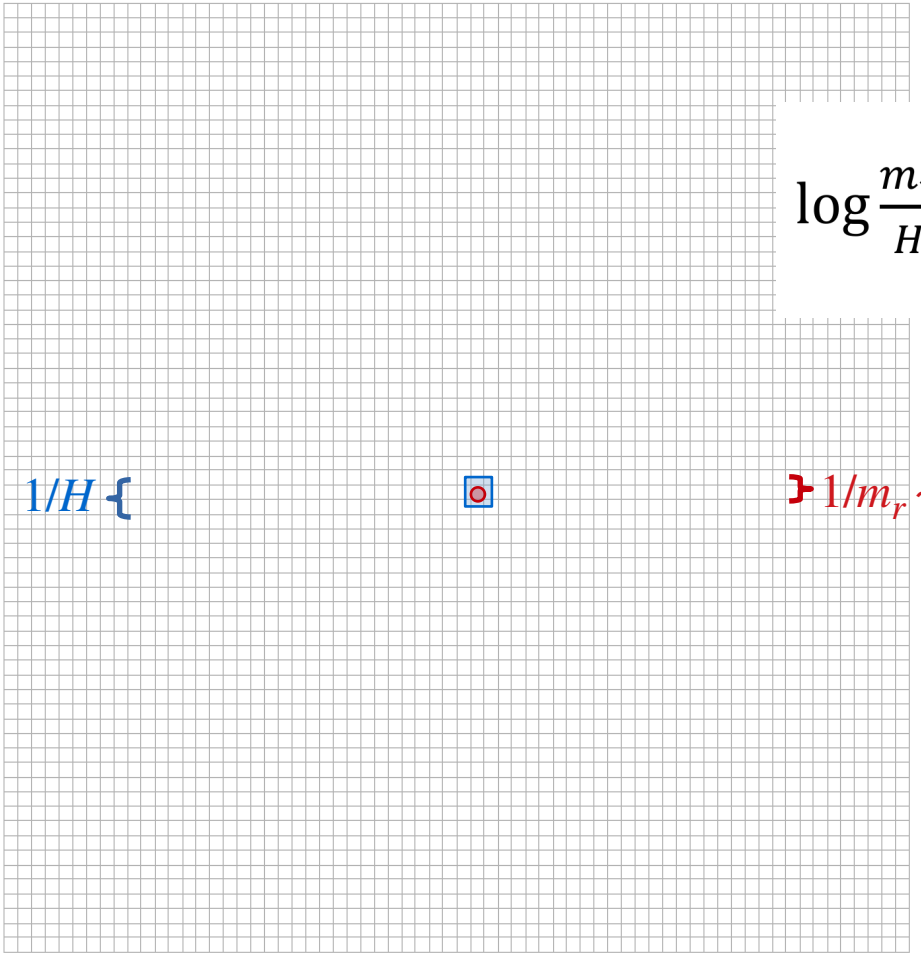


$$\log \frac{m_r}{H} \leq \log(\frac{\square}{\bigcirc}) \sim 8 \ll 70$$

The Bottle Neck



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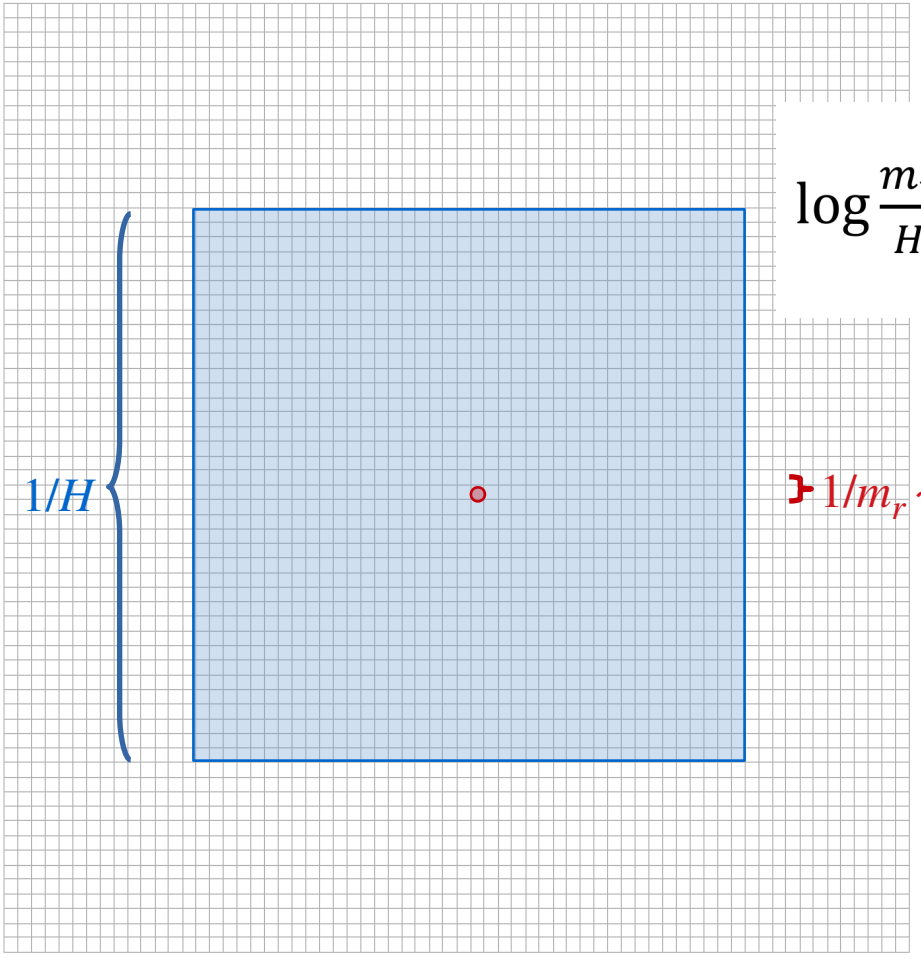

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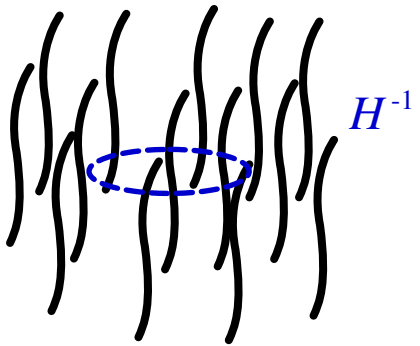
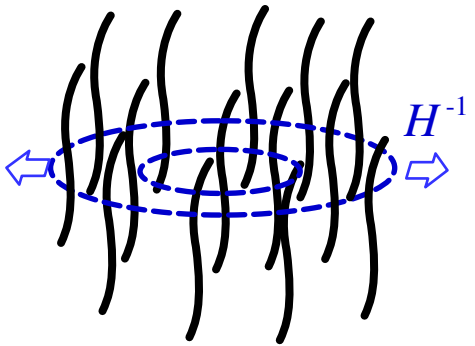
fat string trick

free strings



$$\xi = \frac{\#\text{ strings}}{\text{Hubble Volume}}$$

free strings



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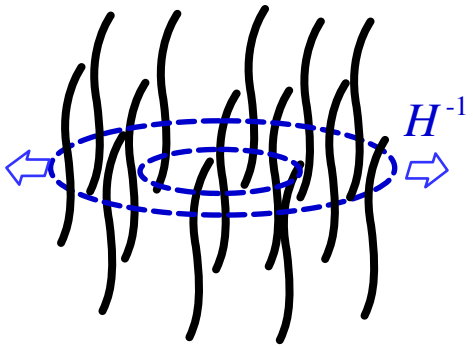
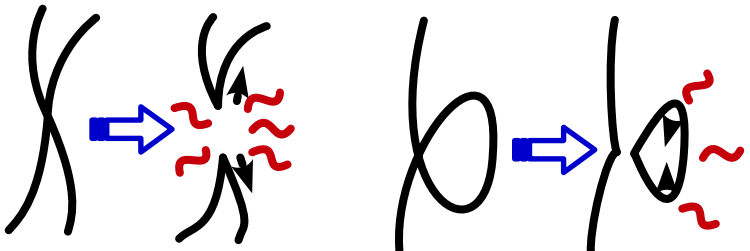


string recombination



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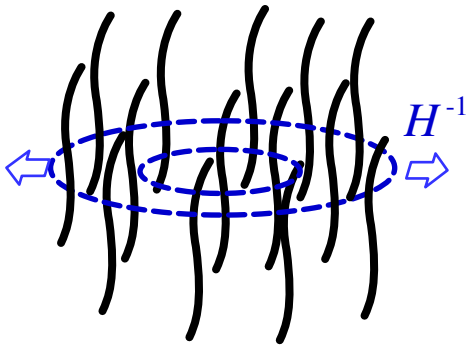

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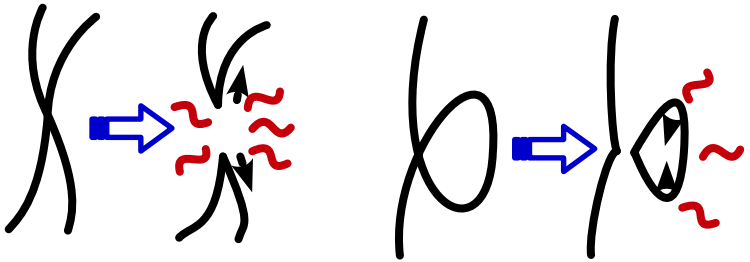

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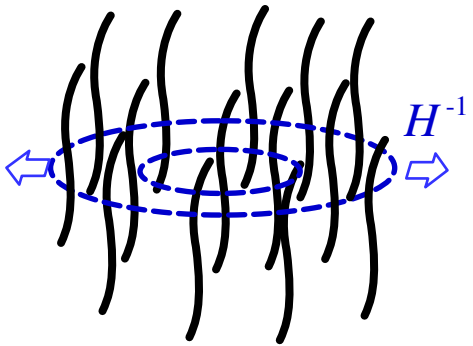
Self-Organized Criticality

free strings



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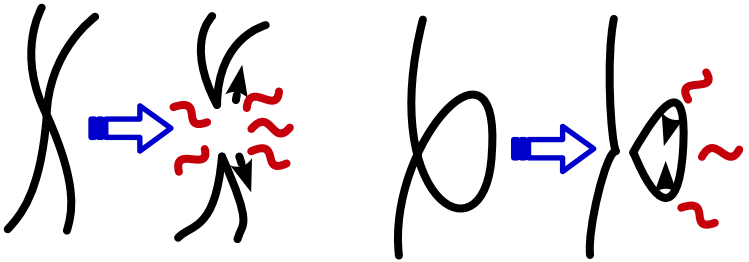


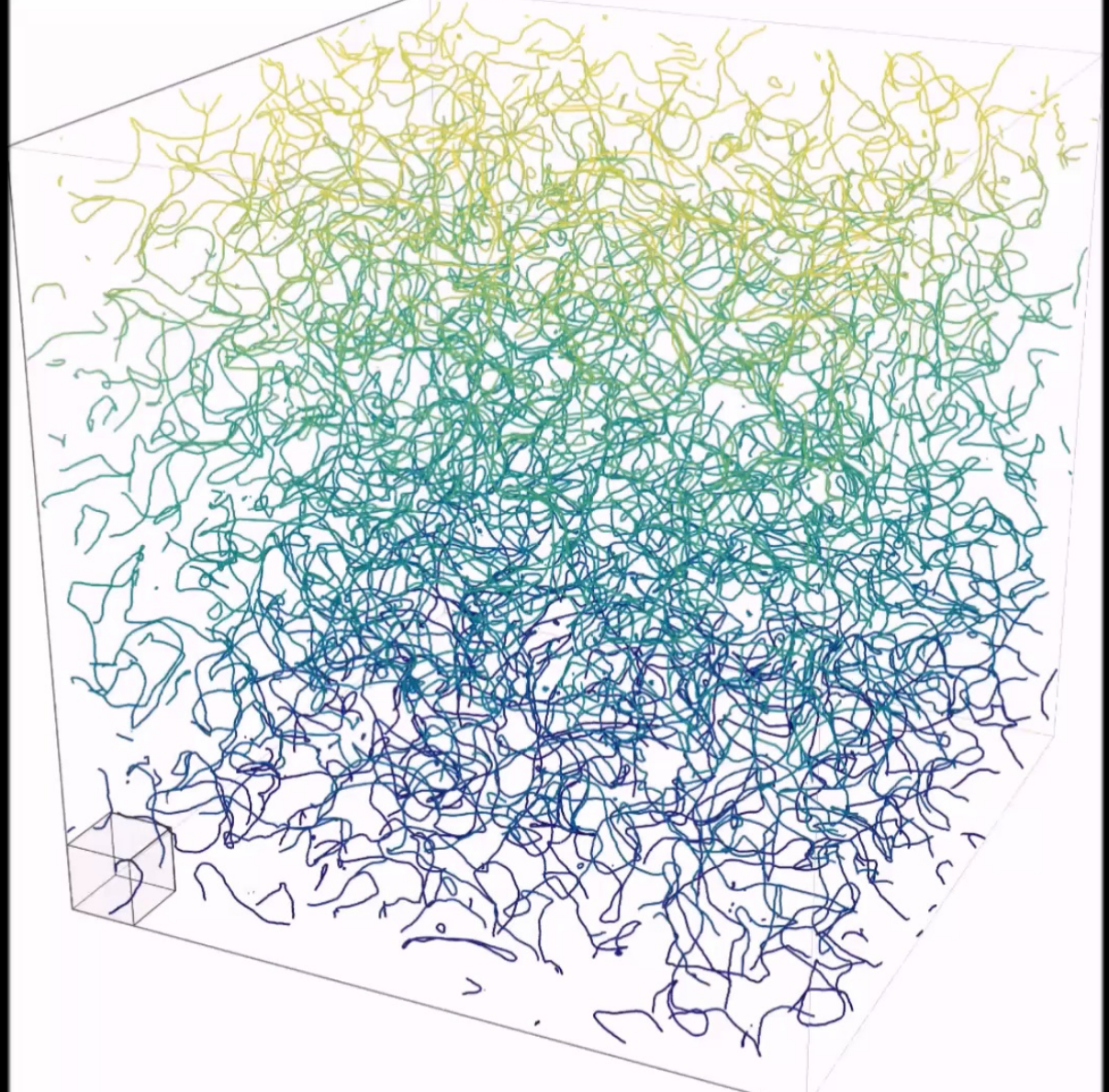
$$\xi = \frac{\#\ \text{strings}}{\text{Hubble Volume}}$$

ξ

Self-Organized Criticality

Scaling Solution $\Rightarrow$ $\xi \Rightarrow \xi_{\text{scaling}}$

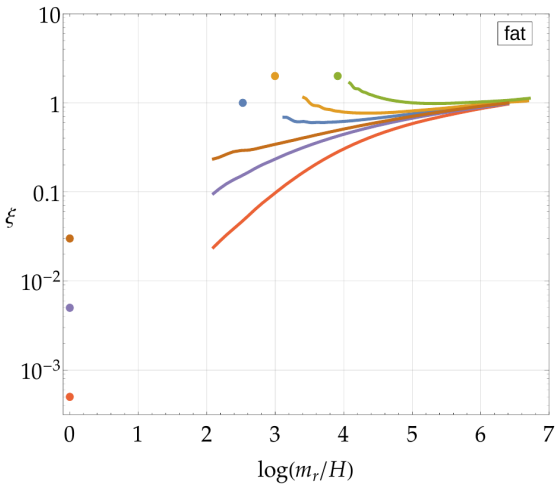
Albrecht, Allen, Battye, Bennett, Bouchet, Davis, Dabholkhar, Hagmann, Kibble, Martins, Quashnock, Shellard, Sikivie, Turok, Vilenkin, ...



String Density Evolution

$$\rho_s = \xi \frac{\mu}{t^2}$$

the attractor



Gorghetto, Hardy, GV '18

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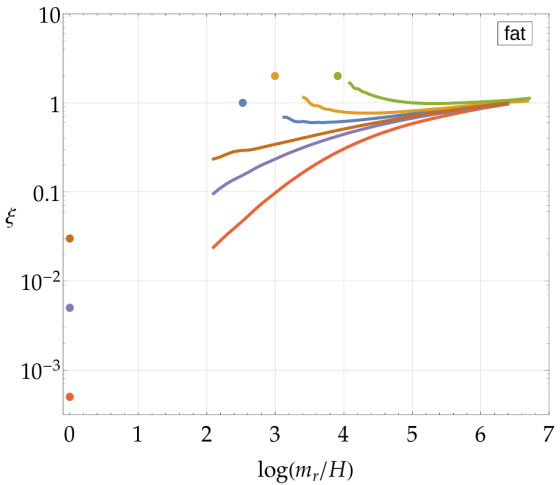
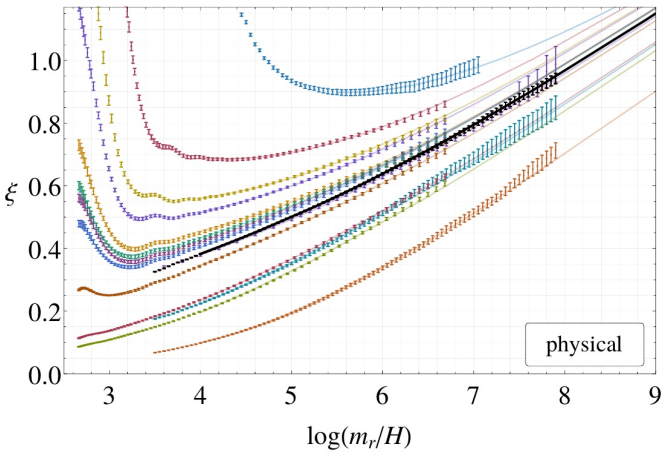


Gorghetto, Hardy, GV '18

scaling violations



Gorghetto, Hardy, GV '20

Also:
Fleury, Moore '15
Klaer, Moore '17, '19
Kawasaki et al. '18
Vaquero et al. '18
Buschmann et al. '19, '21

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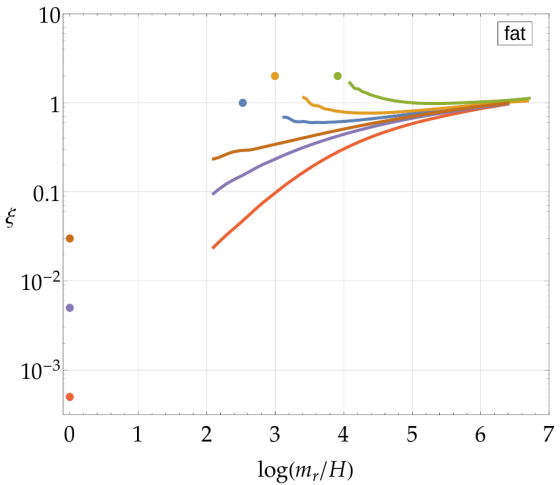

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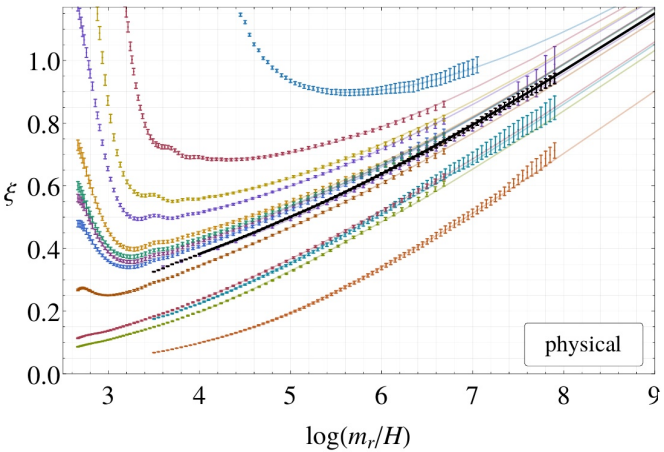

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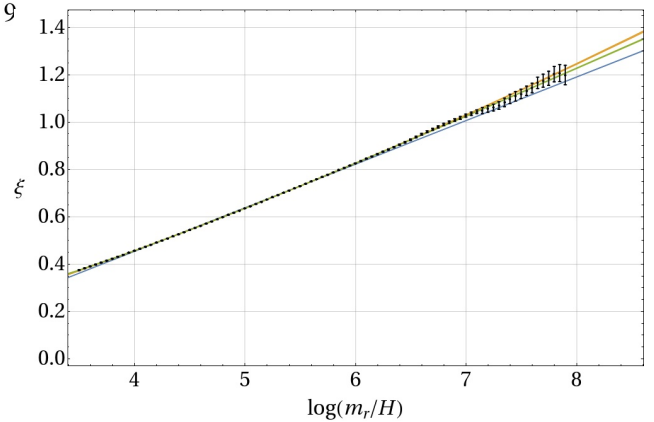
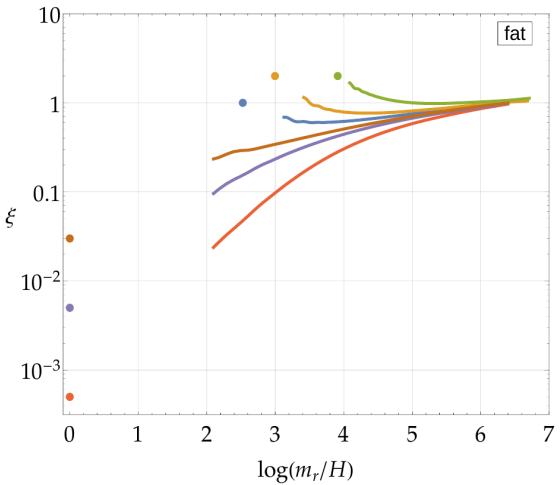


String Density Evolution

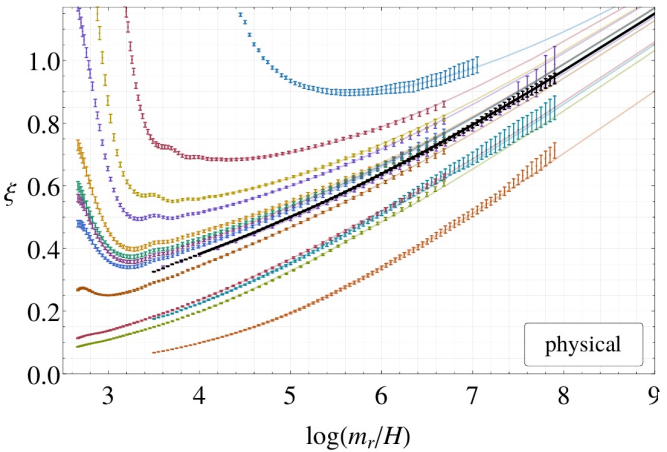
$$\rho_s = \xi \frac{\mu}{t^2}$$

the attractor



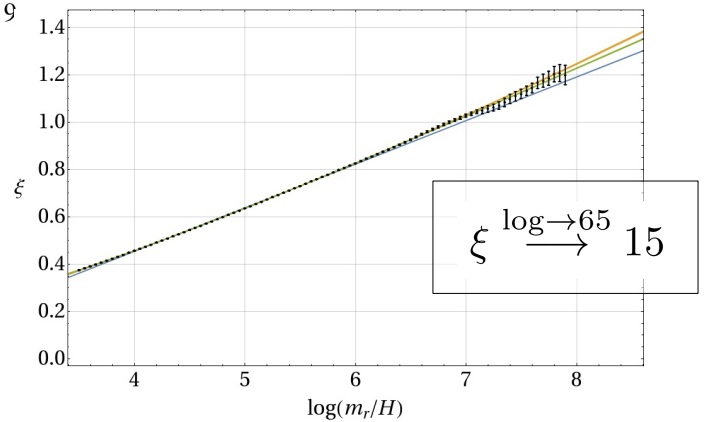
Gorghetto, Hardy, GV '18

scaling violations



Gorghetto, Hardy, GV '20

Also:
Fleury, Moore '15
Klaer, Moore '17, '19
Kawasaki et al. '18
Vaquero et al. '18
Buschmann et al. '19, '21



Axion Emission Energy

$$\rho_{free} \sim \rho_s(t_0)\frac{R_0^2}{R^2} = \rho_s(t_0)\frac{t_0}{t} \qquad \textit{vs} \qquad \rho_s = \xi\frac{\mu}{t^2}$$

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scaling requires the network to release of energy

$$\Delta\rho_a \sim \rho_s = \xi\frac{\mu}{t^2}$$

per Hubble time

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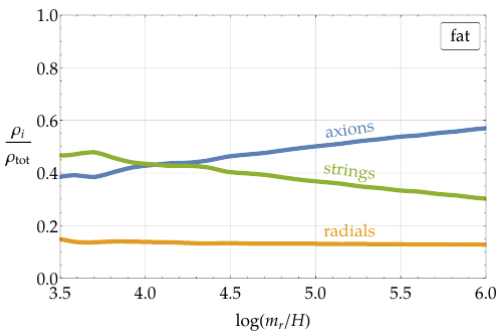

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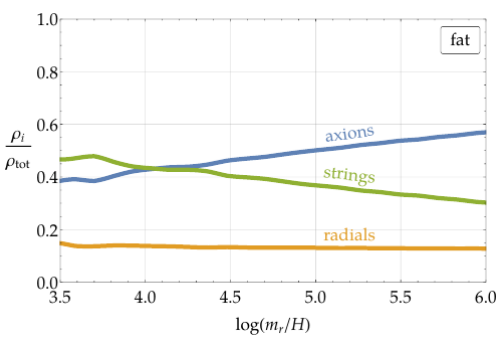
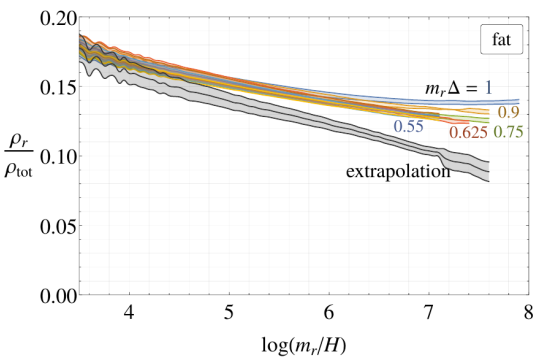


radial modes



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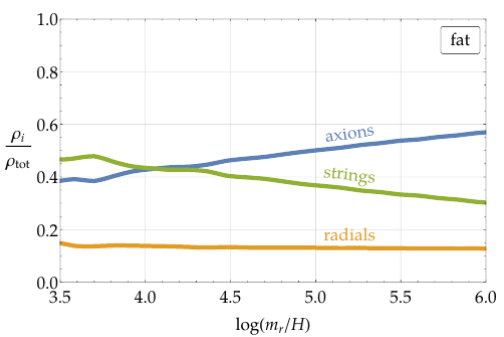
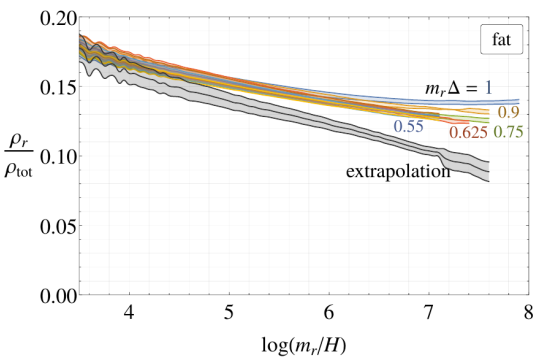


radial modes



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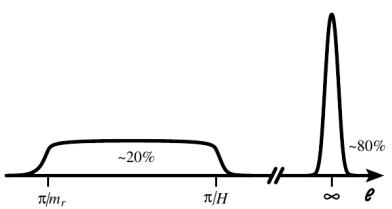

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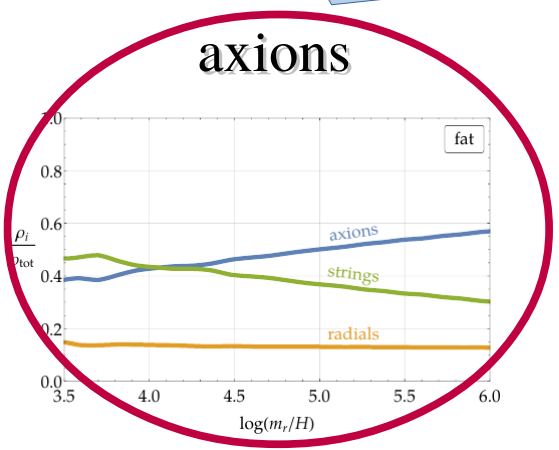
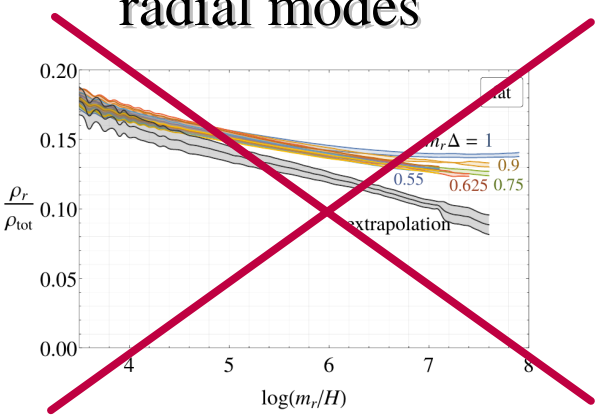


radial modes



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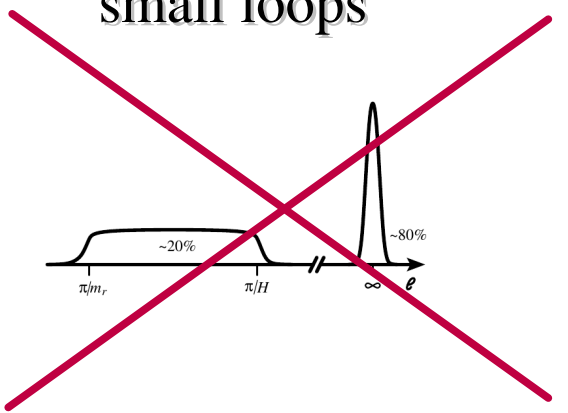

Axion Spectra VS Axion Number

$$n \sim \frac{\rho}{\langle k \rangle}$$

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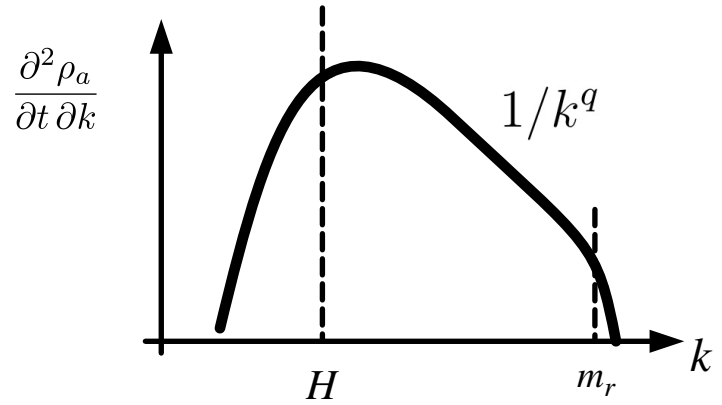

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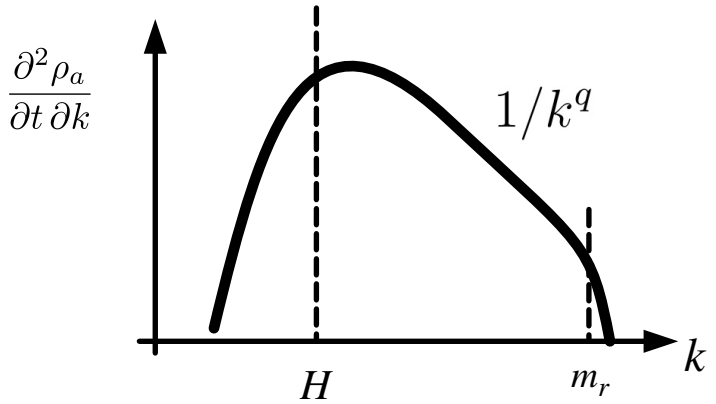

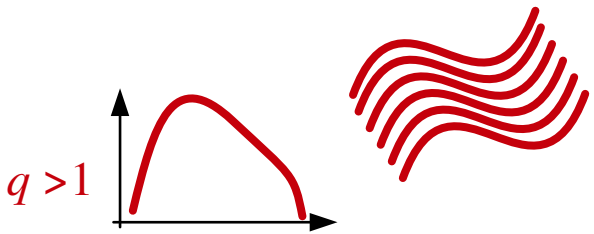

Davies, Shellard, ...

$$n \sim \frac{\rho}{H} \sim \xi \log f^2 H \sim \xi \log n^{mis}$$

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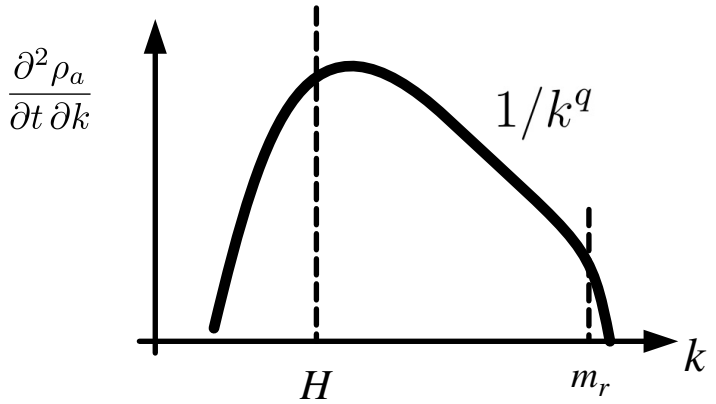

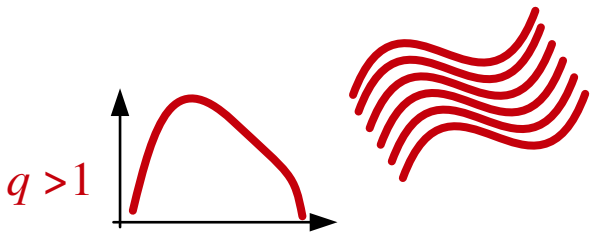

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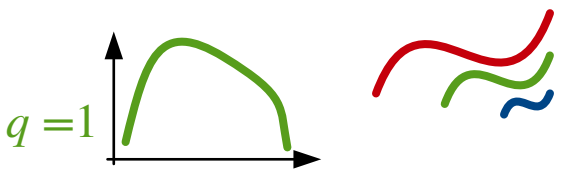

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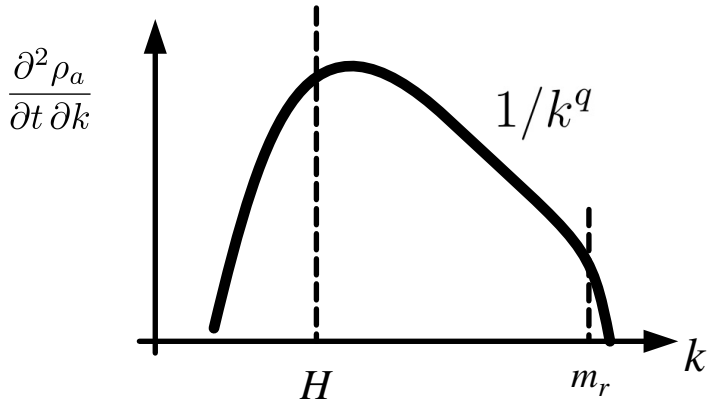

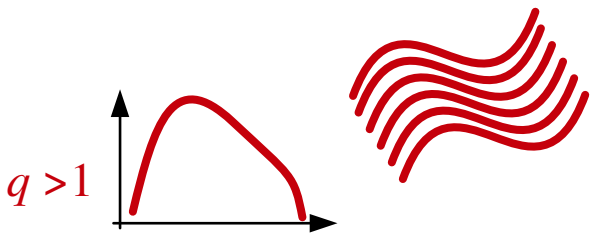

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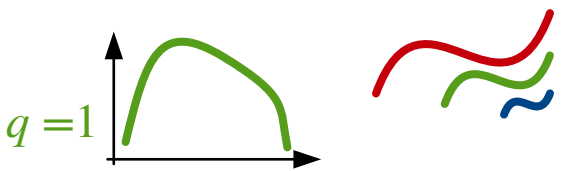

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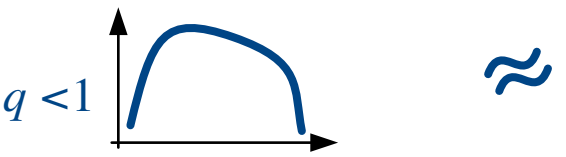
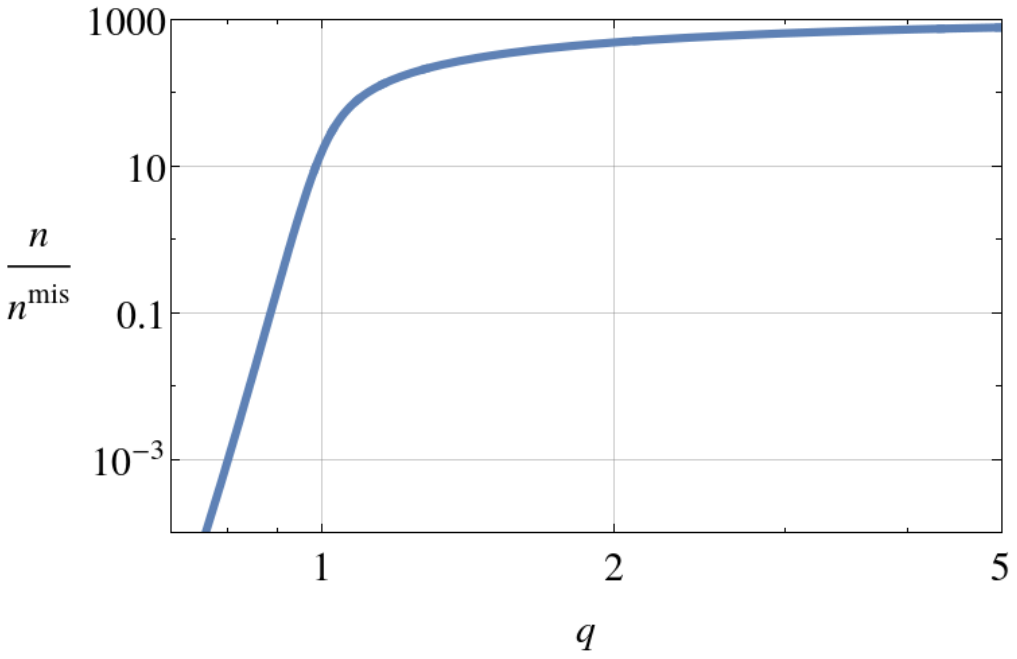


$$n \sim \frac{\rho}{H} \left(\frac{H}{m_r}\right)^{1-q} \sim n^{mis} \left(\frac{H}{m_r}\right)^{1-q}$$

Axion Spectra VS Axion Number



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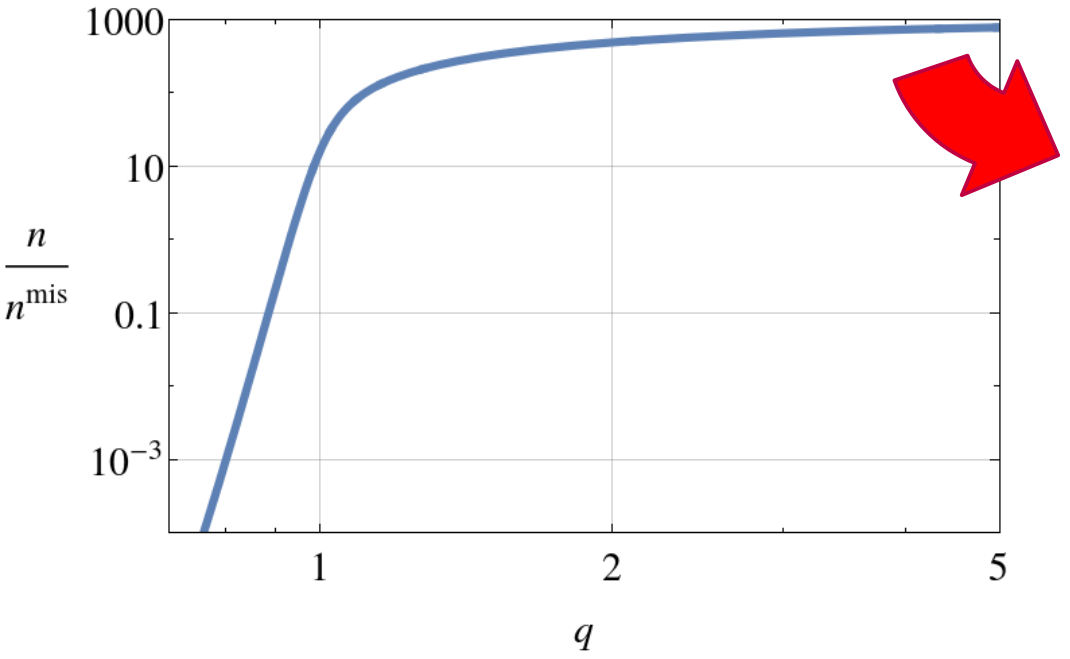
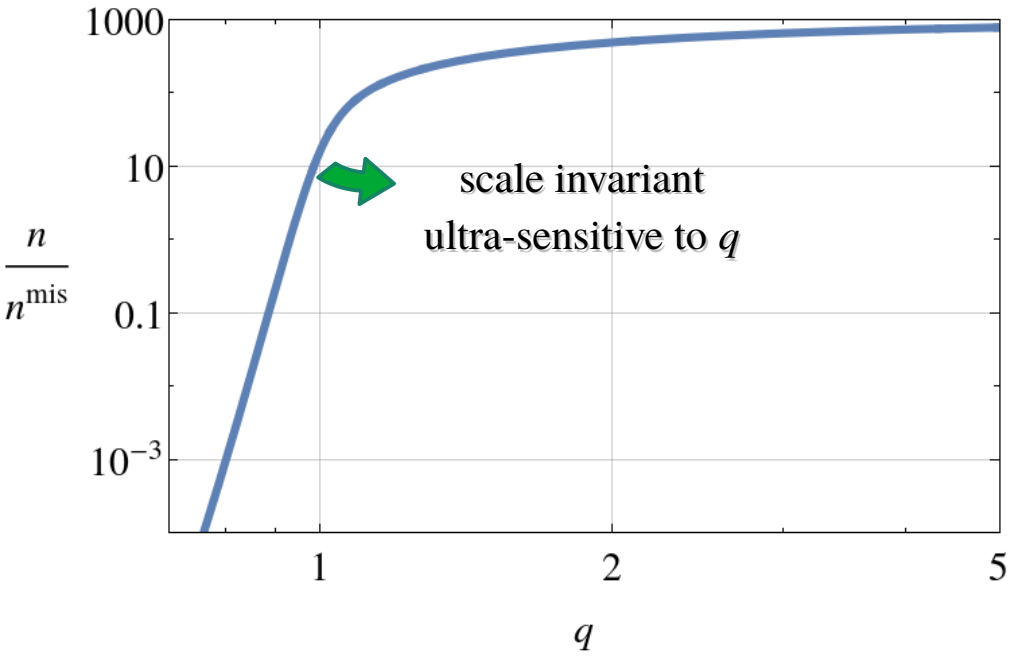


IR dominated
insensitive to q

Axion Spectra VS Axion Number



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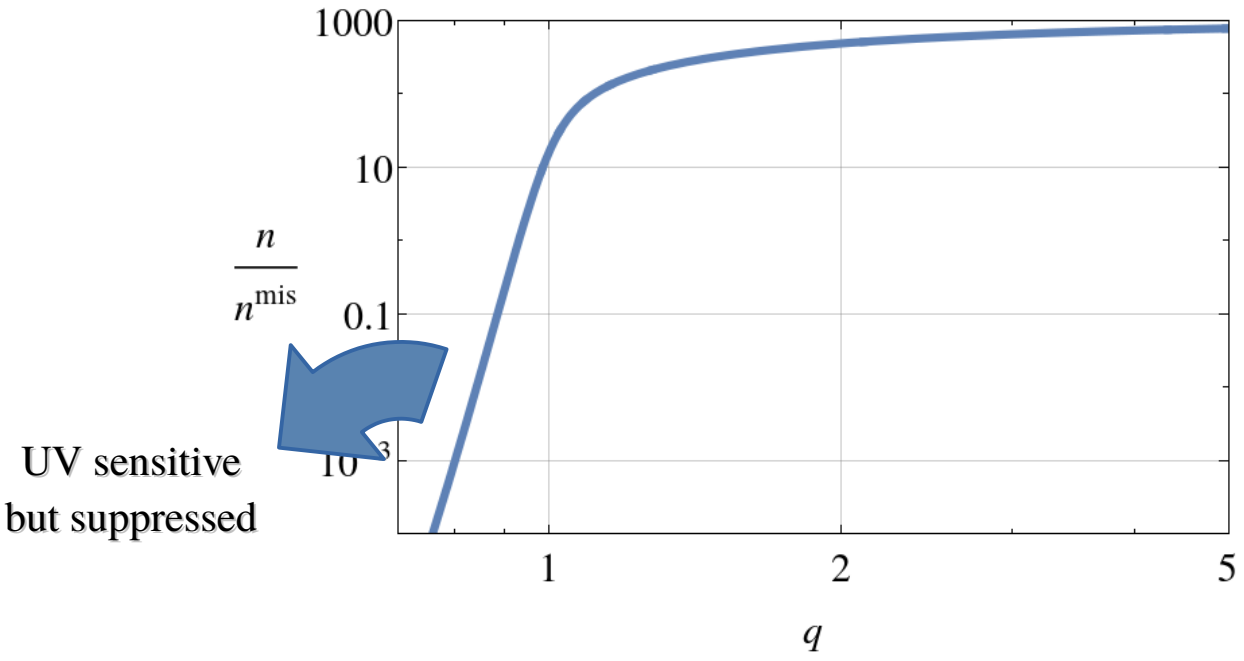

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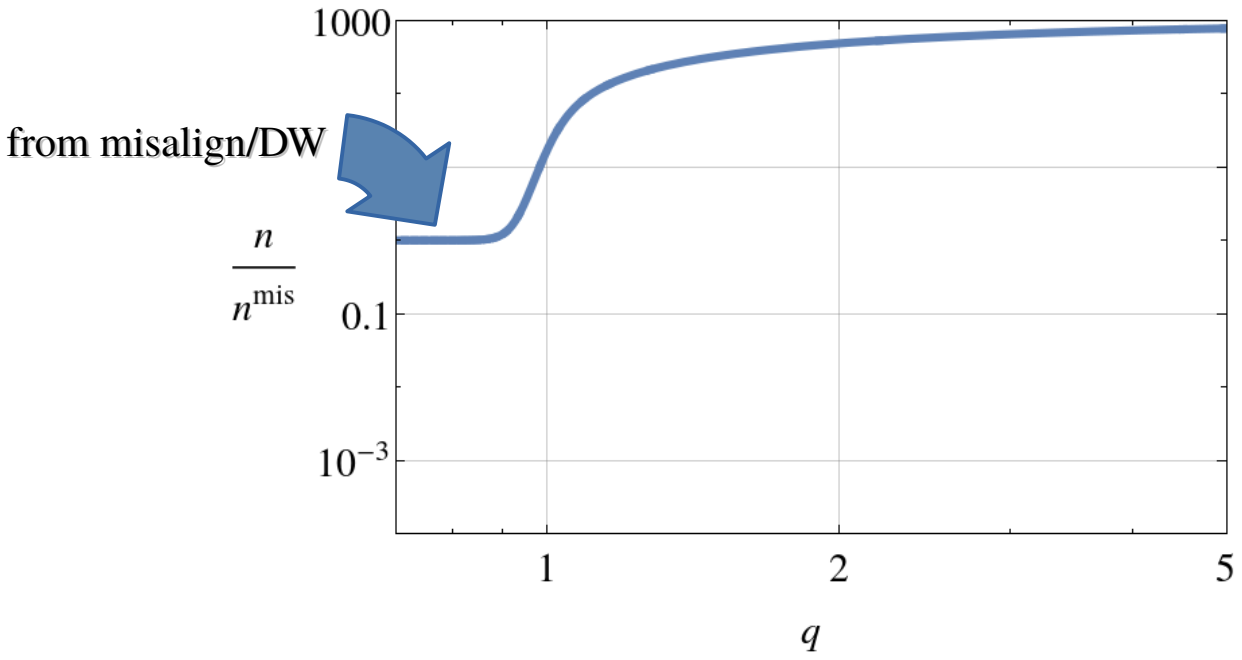
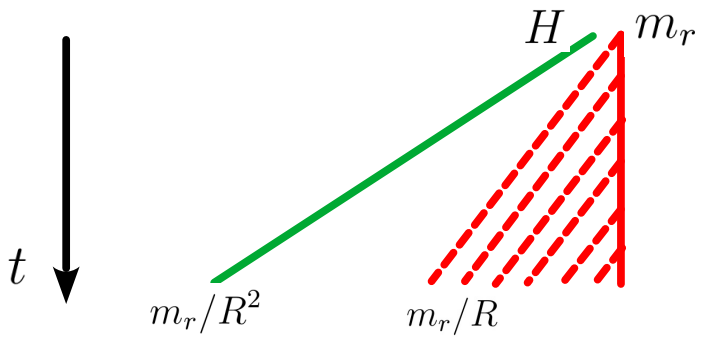
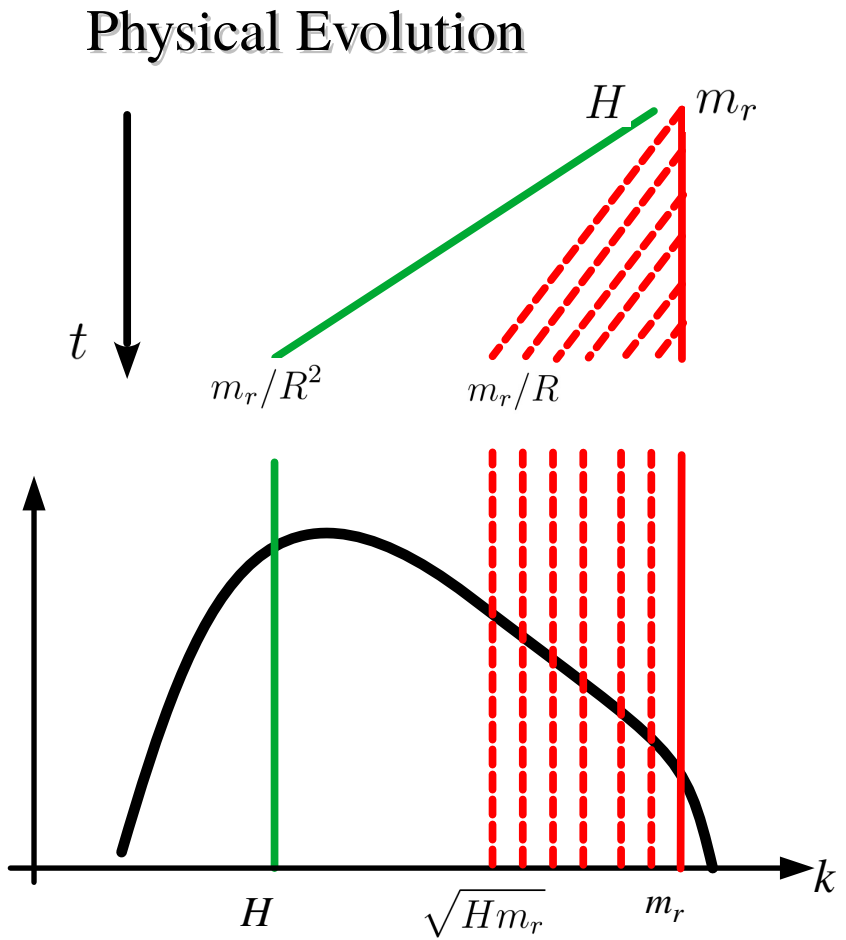


Physical Evolution

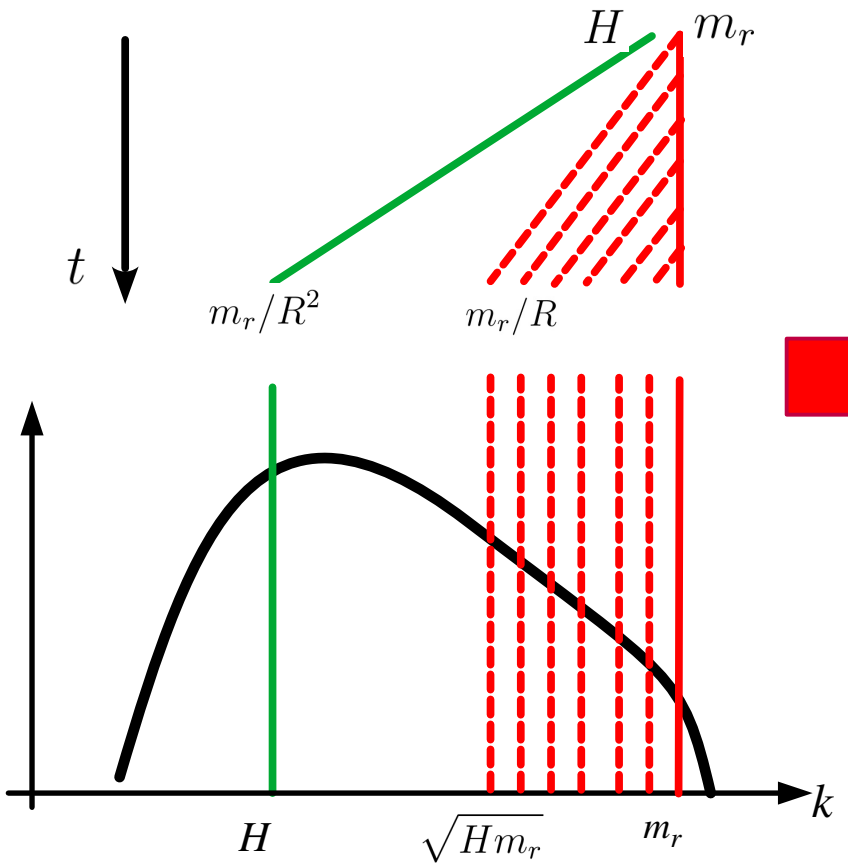
H m_r

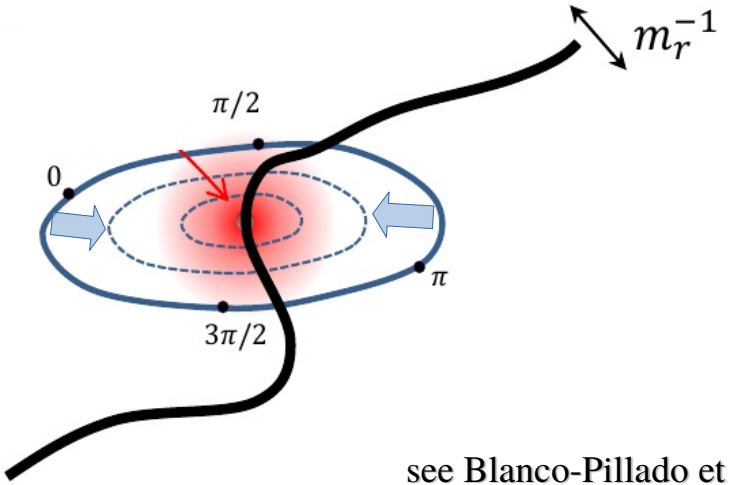
Physical Evolution




Physical Evolution
H
m_r
t
m_r/R^2
m_r/R
k
H
$\sqrt{Hm_r}$
m_r

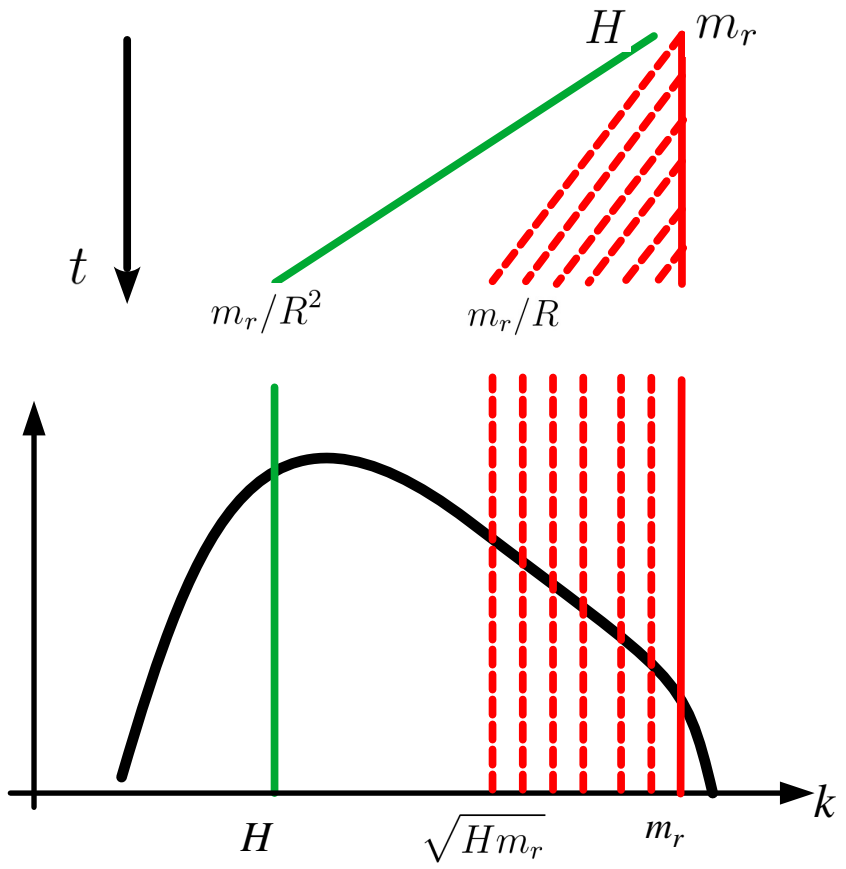
Physical Evolution





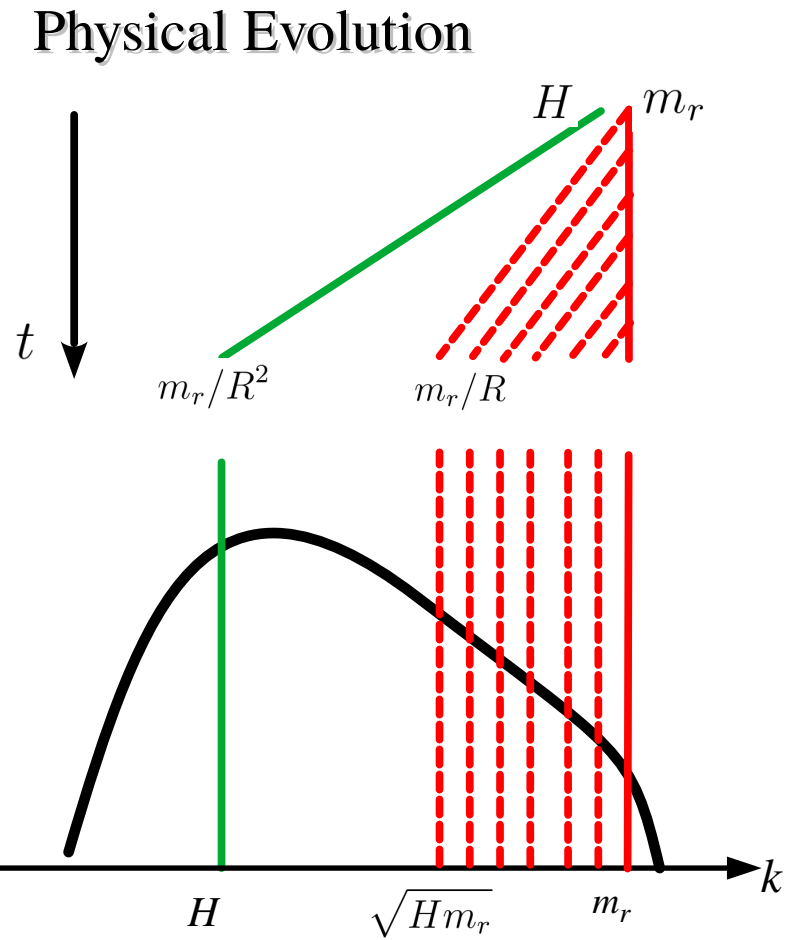
see Blanco-Pillado et al. '21
2107.02215

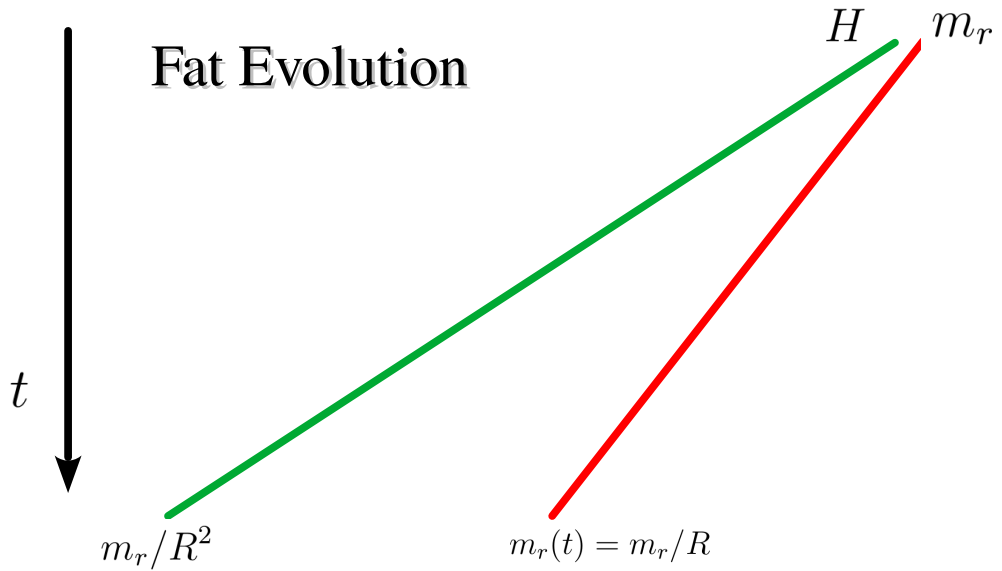
Physical Evolution

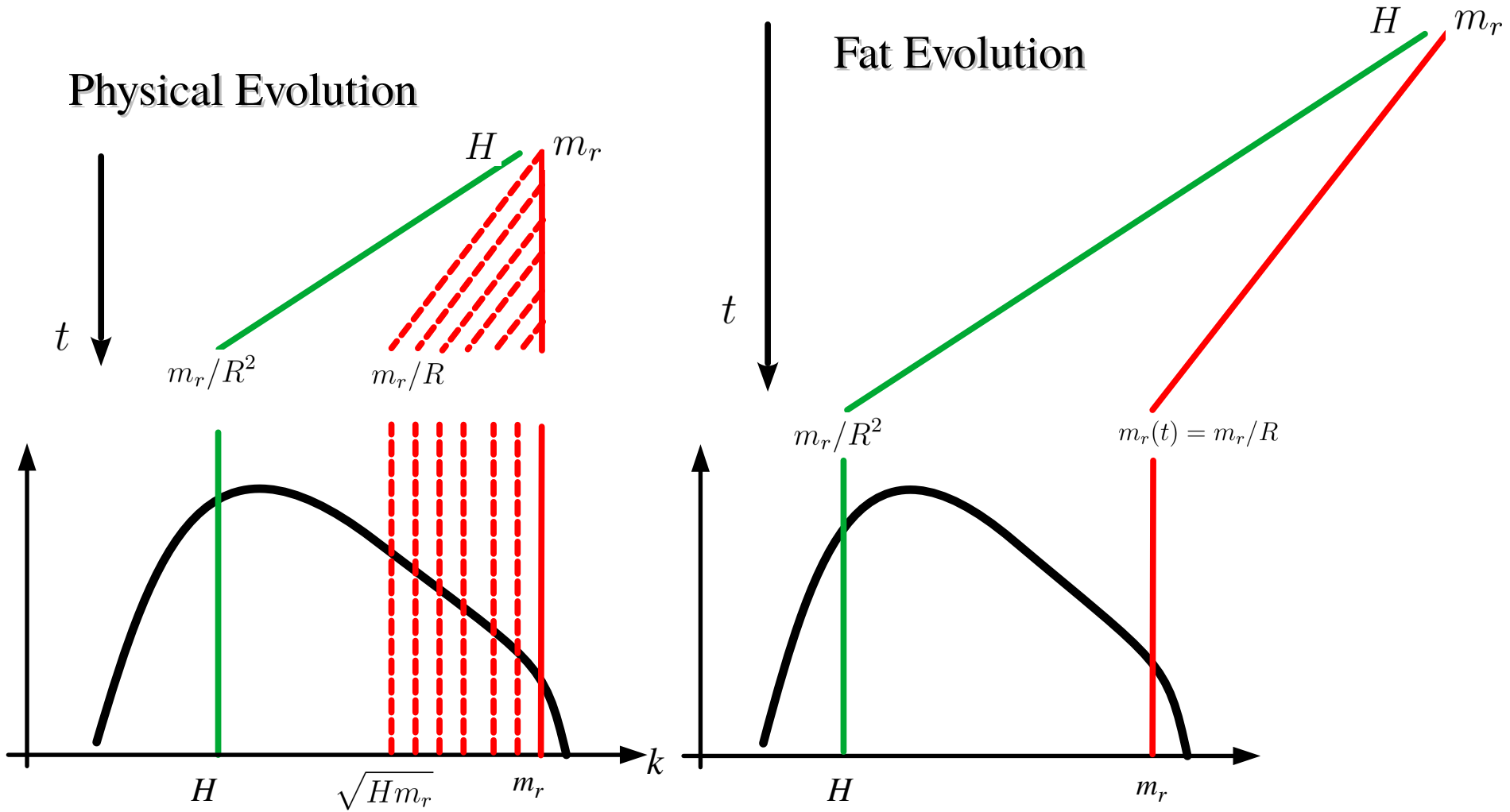


Fat Evolution

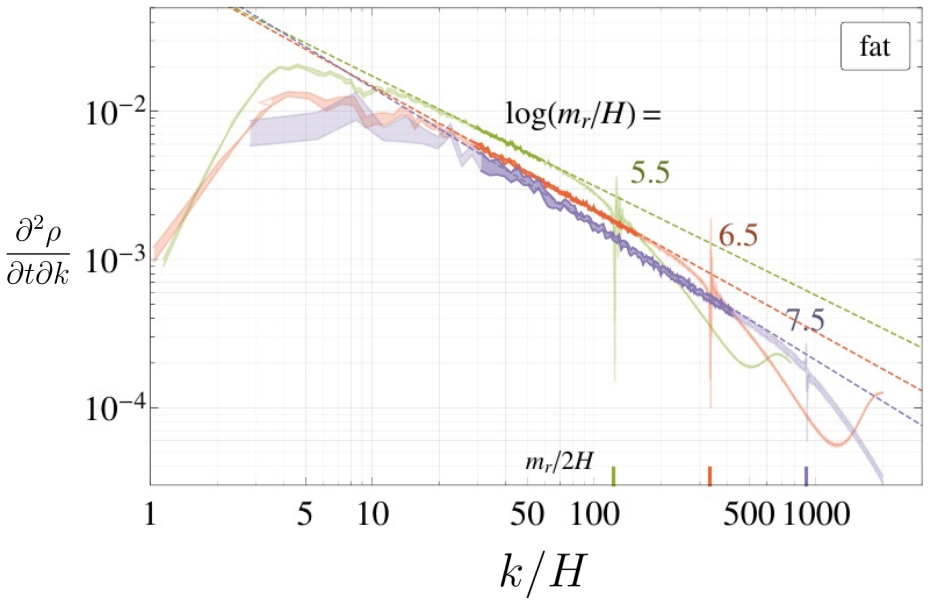
H m_r


Physical Evolution
H
m_r
t
m_r/R^2
m_r/R
H
$\sqrt{Hm_r}$
m_r
k


Fat Evolution
H
m_r
t
m_r/R^2
$m_r(t) = m_r/R$

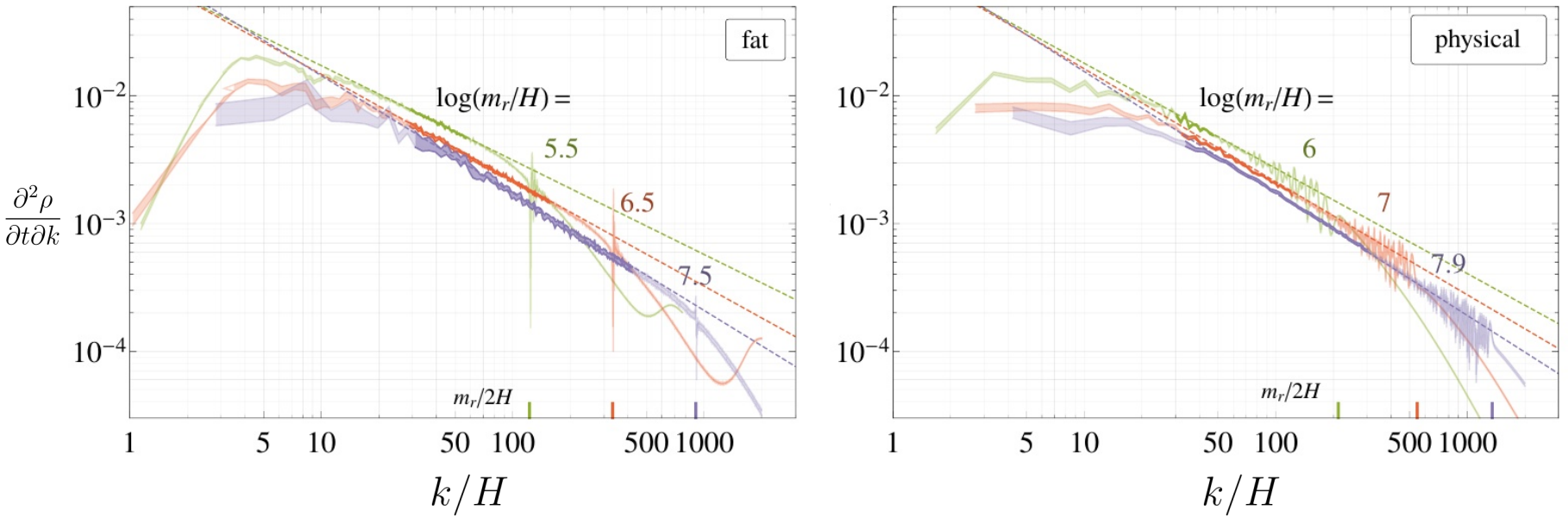

Physical Evolution
H
m_r
t
m_r/R^2
m_r/R
H
$\sqrt{Hm_r}$
m_r
k
Fat Evolution
H
m_r
t
m_r/R^2
$m_r(t) = m_r/R$
H
m_r

Axion Spectra

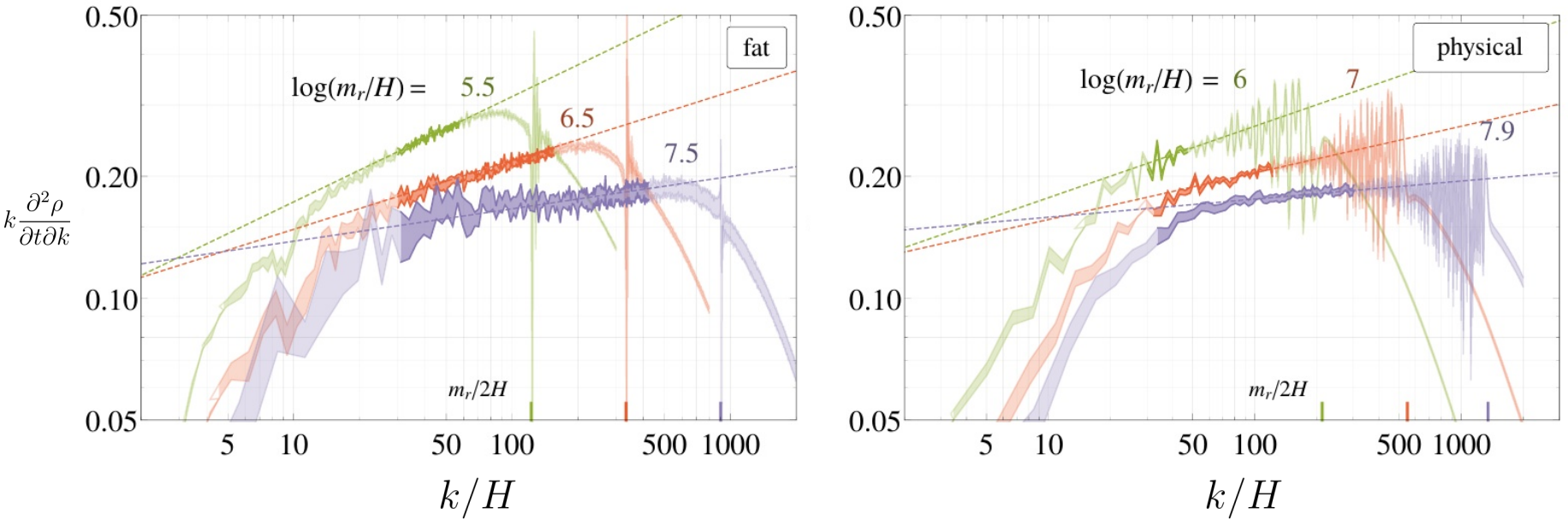


k/H

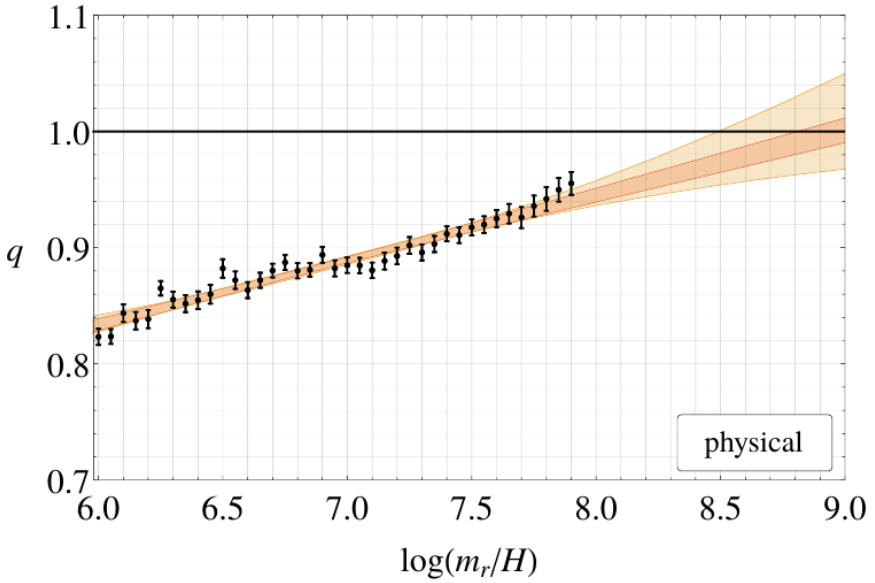
Axion Spectra

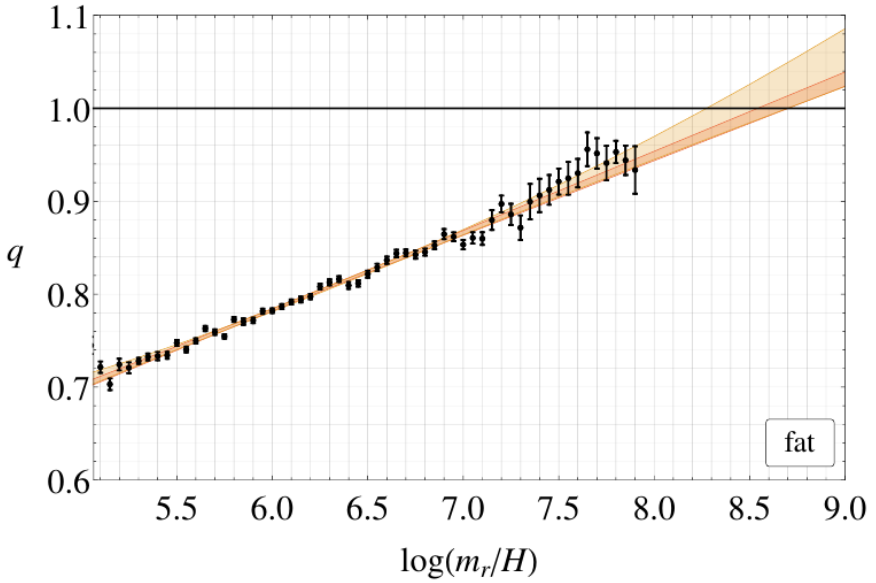


Axion Spectra



Axion Spectral Index Evolution





$$\rho_a \sim \frac{\xi\mu}{t^2} \sim 10^{3\div 4}\left[\frac{\xi\log}{10^3}\right] H^2 f_a^2$$

$$\gg \rho_a^{mis} \sim \theta_0^2 m_a^2 f_a^2 \qquad @\ m_a = H$$

Non linearities at the QCD transition

$$@\ m_a = H_\star \qquad \rho_a \sim \xi \log\ \rho^{(mis)}$$

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$$m_a \gtrsim 0.5 \text{ meV}\,, \qquad f_a \lesssim 10^{10} \text{ GeV} \qquad \text{KSVZ}$$

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$$m_a \gtrsim 3.5 \text{ meV}\,, \qquad f_a \lesssim 2 \cdot 10^{9} \text{ GeV} \qquad \text{DSFZ}$$

Future Scenarios

- **late $q < 1$** : ~~strings,~~ misalign/DW - dynamics is UV sensitive
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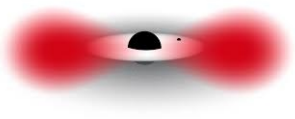
1 Wish

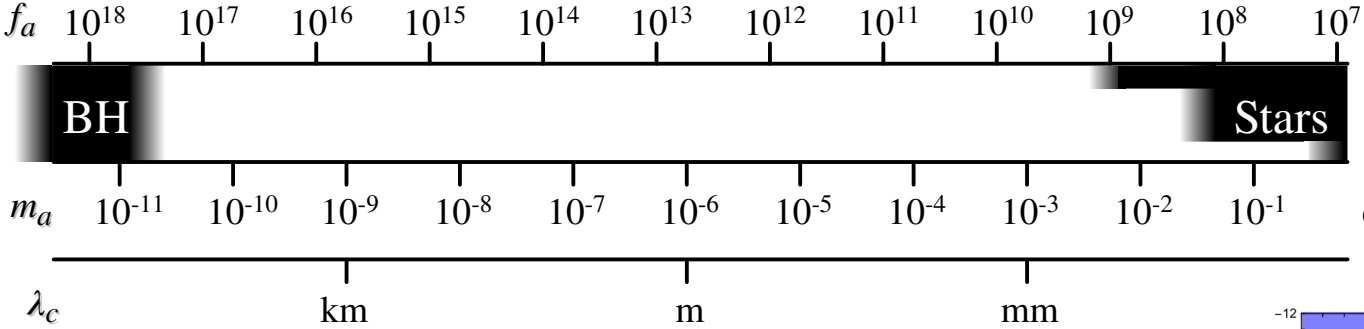
- Theoretical understanding of various behaviors

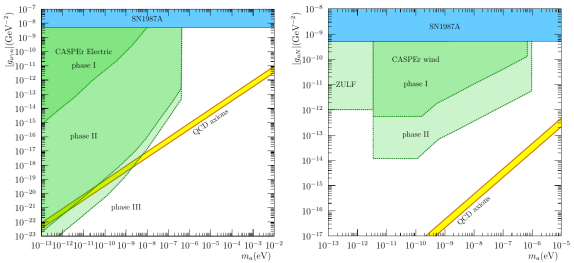
Thank You

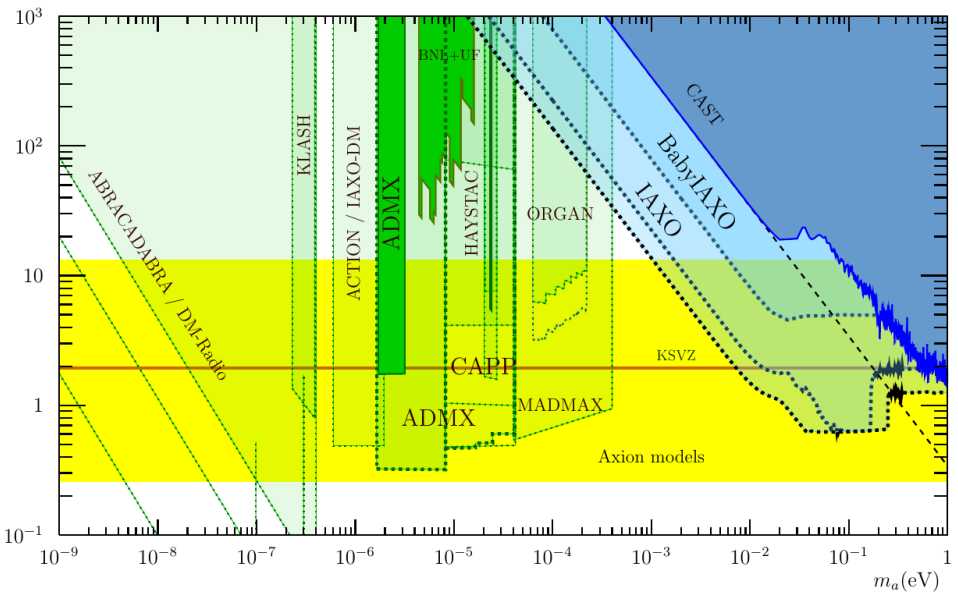
Axions Searches

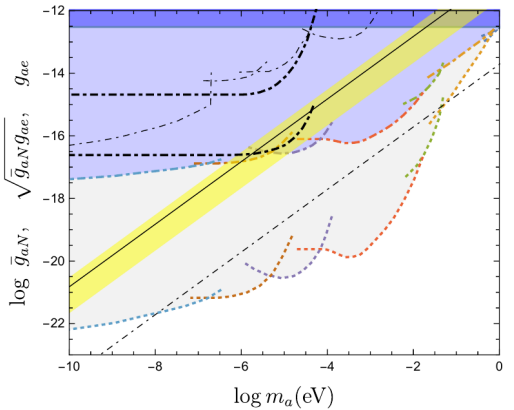


Arvanitaki et al. ‘09



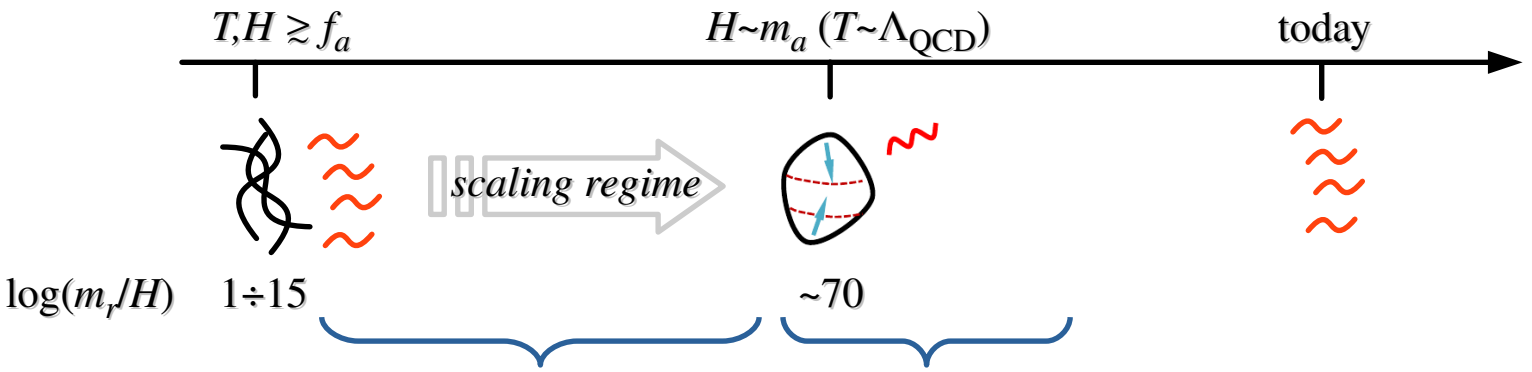






Irastorza, Redondo ‘18

Full Simulation Not Reliable
Smaller Goal: Lower Bound on DM

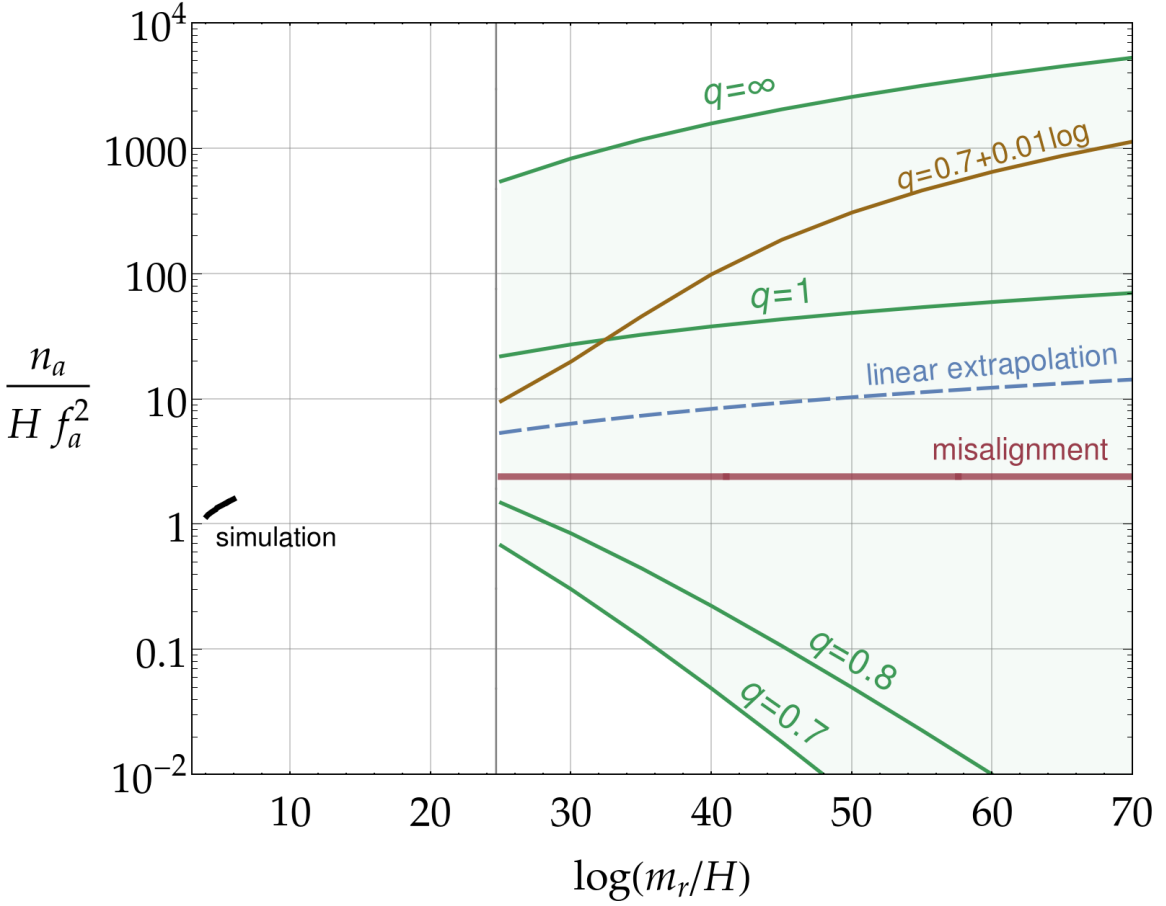


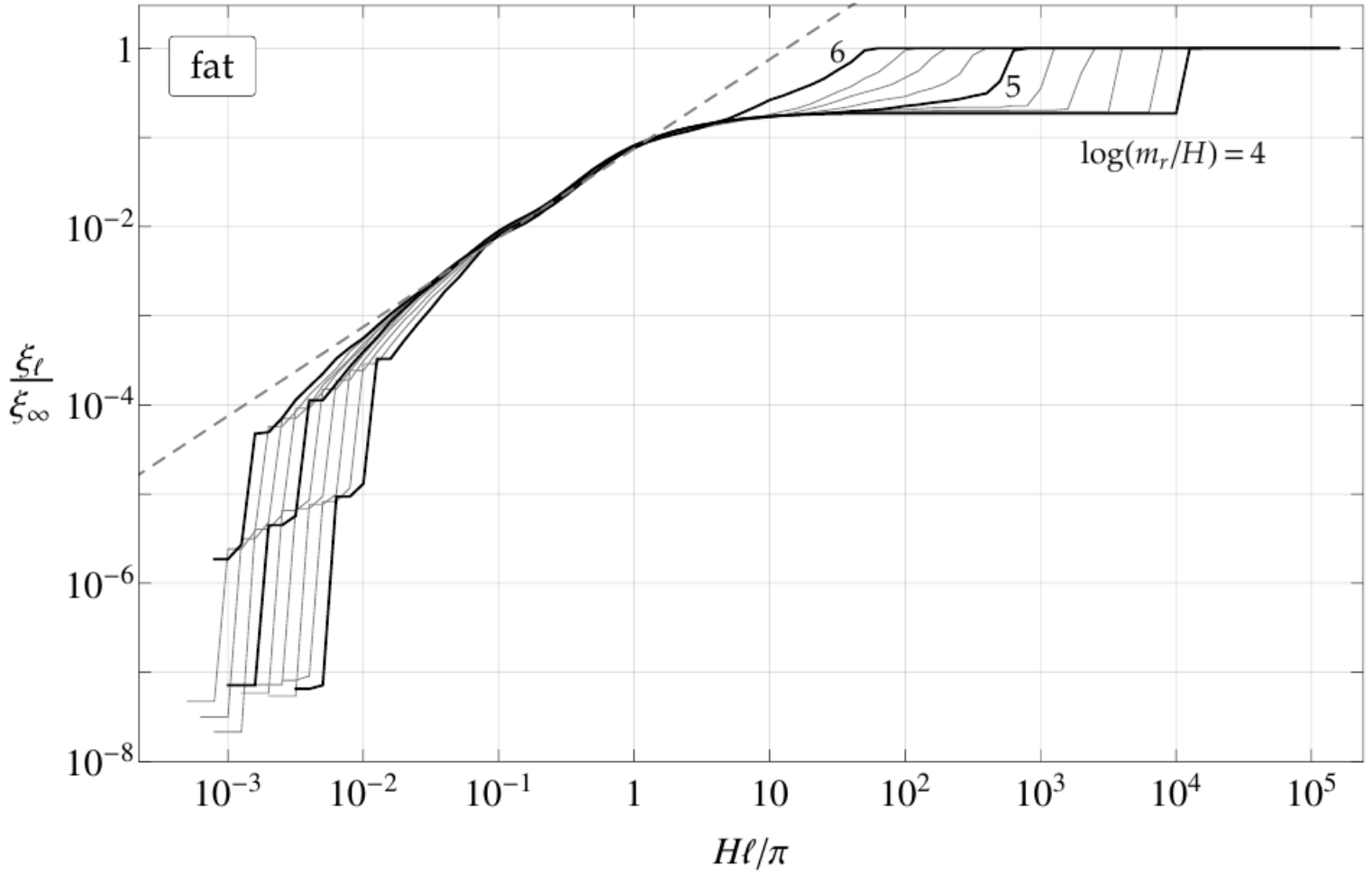
1) Energy in axions: $\frac{d\rho_a}{dt} \simeq \frac{\xi\mu}{t^3}$ **# of strings**

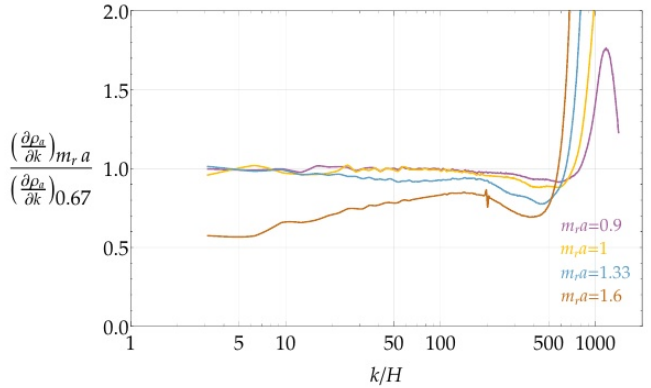
2) Number of axions: $n_a = \int \frac{dk}{\omega_k} \frac{\partial \rho_a}{\partial k}$ **spectrum**

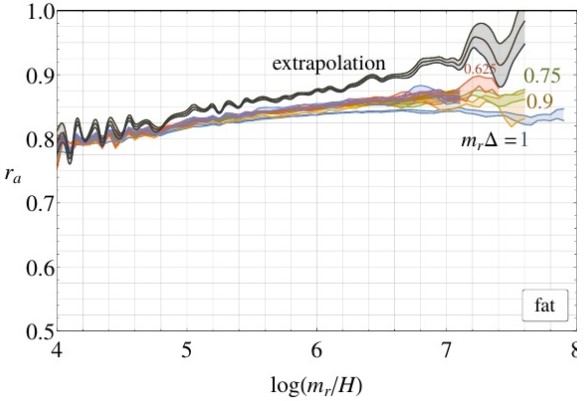
3) Number density transmission through non-linear regime

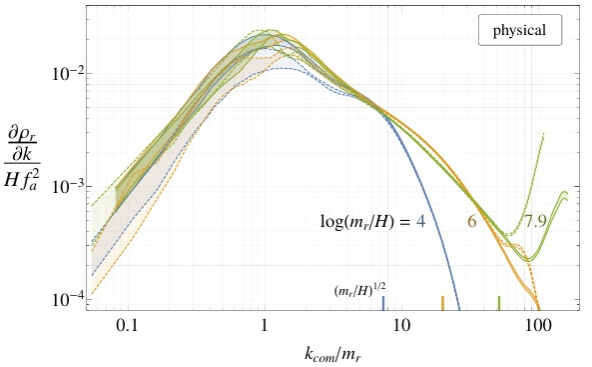
Axion Number Density → Extrapolation

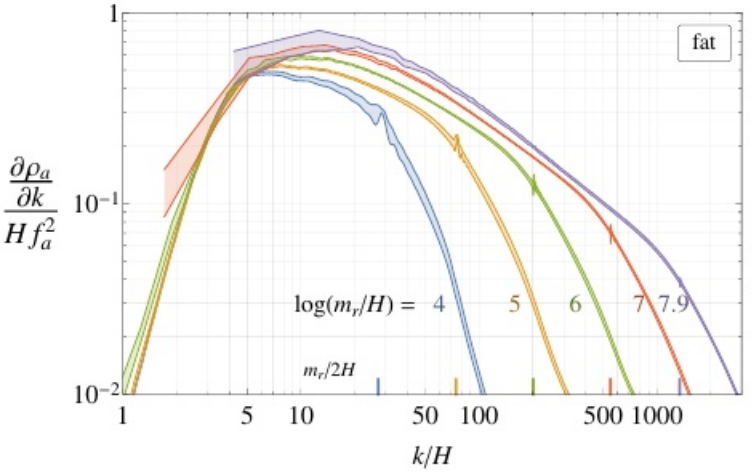


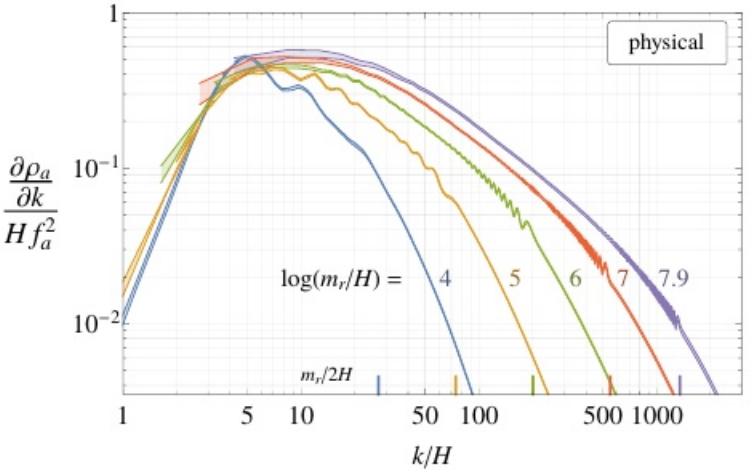

fat
6
5
$\log(m_r/H) = 4$
$\frac{\xi_\ell}{\xi_\infty}$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-3}
10^{-2}
10^{-1}
1
10
10^2
10^3
10^4
10^5
$H\ell/\pi$

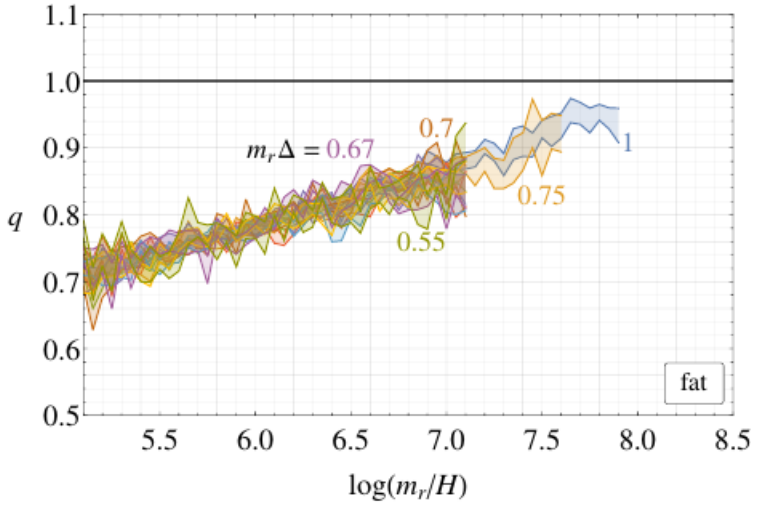

$\frac{\left(\frac{\partial \rho_a}{\partial k}\right)_{m_r a}}{\left(\frac{\partial \rho_a}{\partial k}\right)_{0.67}}$
$m_r a$=0.9
$m_r a$=1
$m_r a$=1.33
$m_r a$=1.6
k/H

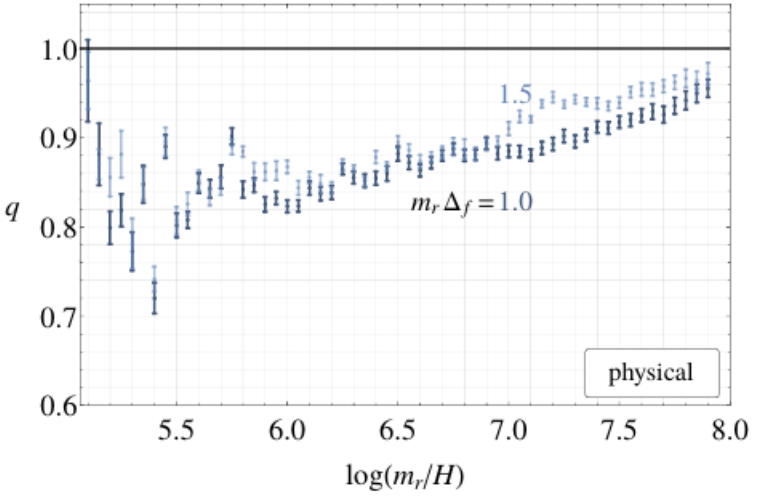

extrapolation
0.625
0.75
0.9
$m_r\Delta = 1$
fat
r_a
$\log(m_r/H)$

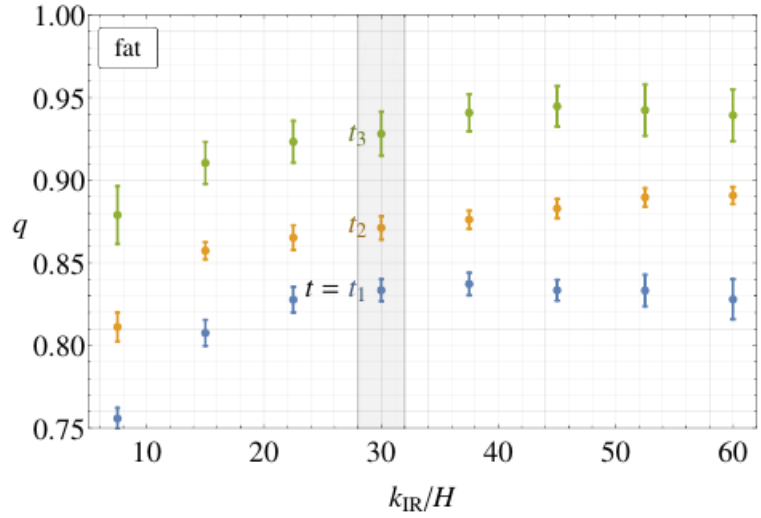

physical
$\frac{\partial \rho_r}{\partial k}$
$H f_a^2$
$\log(m_r/H) = 4$
6
7.9
$(m_r/H)^{1/2}$
k_{com}/m_r

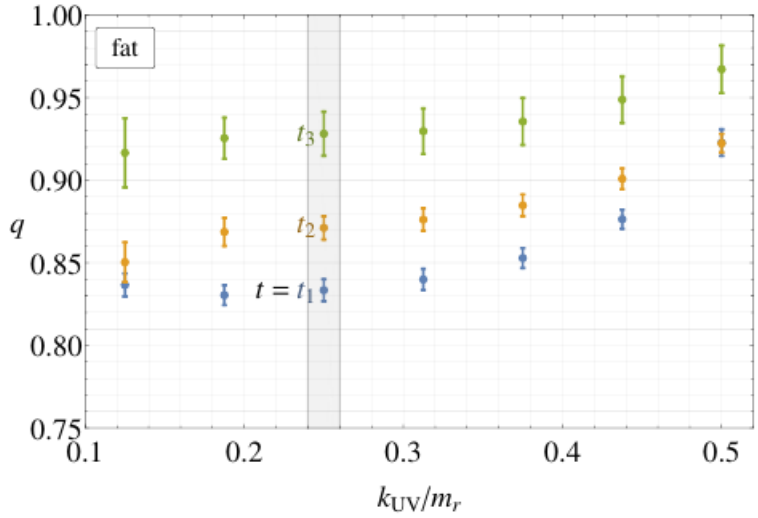

fat
$\frac{\partial \rho_a}{\partial k}$
$H f_a^2$
$\log(m_r/H) =$ 4 5 6 7 7.9
$m_r/2H$
k/H

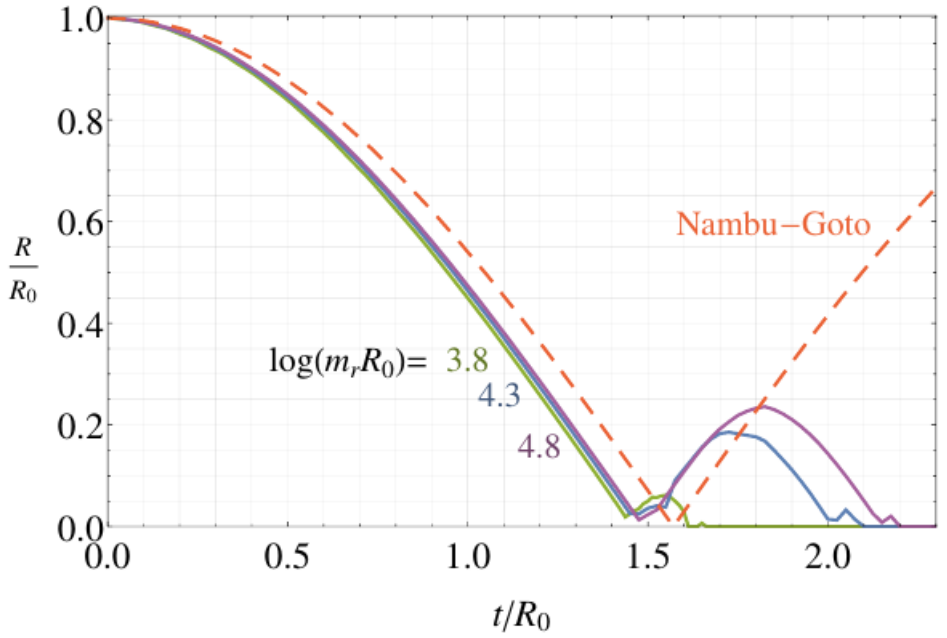

physical
$\frac{\partial \rho_a}{\partial k}$
$H f_a^2$
$\log(m_r/H) =$ 4 5 6 7 7.9
$m_r/2H$
k/H


1.1
1.0
0.9
0.8
0.7
0.6
0.5
q
$m_r\Delta = 0.67$
0.7
0.75
0.55
1
fat
5.5
6.0
6.5
7.0
7.5
8.0
8.5
$\log(m_r/H)$


1.0
0.9
0.8
0.7
0.6
q
1.5
$m_r\Delta_f = 1.0$
physical
5.5
6.0
6.5
7.0
7.5
8.0
$\log(m_r/H)$


1.00
0.95
0.90
0.85
0.80
0.75
q
fat
t_3
t_2
$t = t_1$
10
20
30
40
50
60
k_{IR}/H


1.00
0.95
0.90
0.85
0.80
0.75
q
fat
t_3
t_2
$t = t_1$
0.1
0.2
0.3
0.4
0.5
k_{UV}/m_r


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
$\frac{R}{R_0}$
t/R_0
Nambu−Goto
$\log(m_r R_0)=$ 3.8
4.3
4.8