

# A self-consistent wave description of axion miniclusters and their survival in the galaxy

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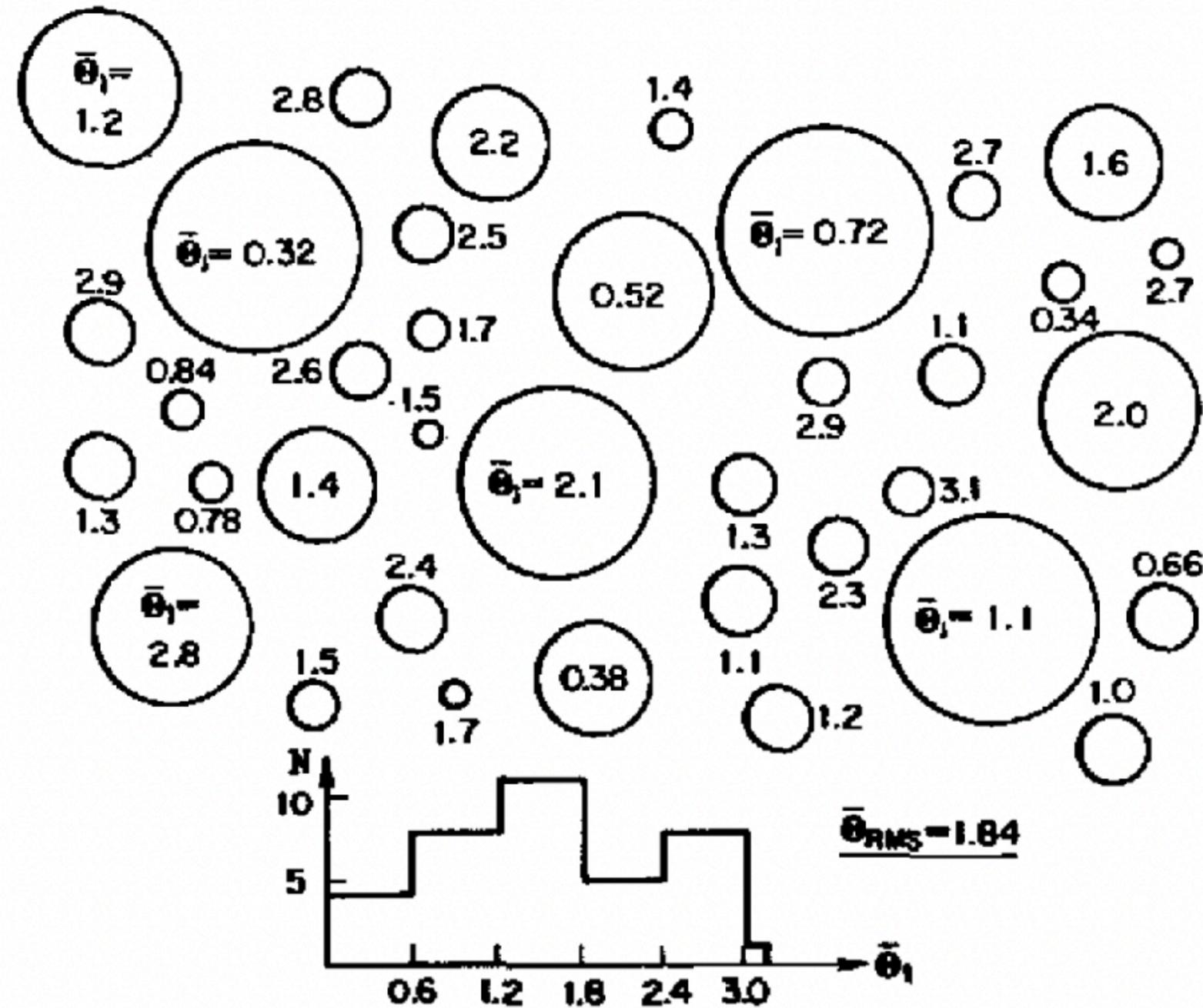
with V. Dandoy, T. Schwetz (KIT)

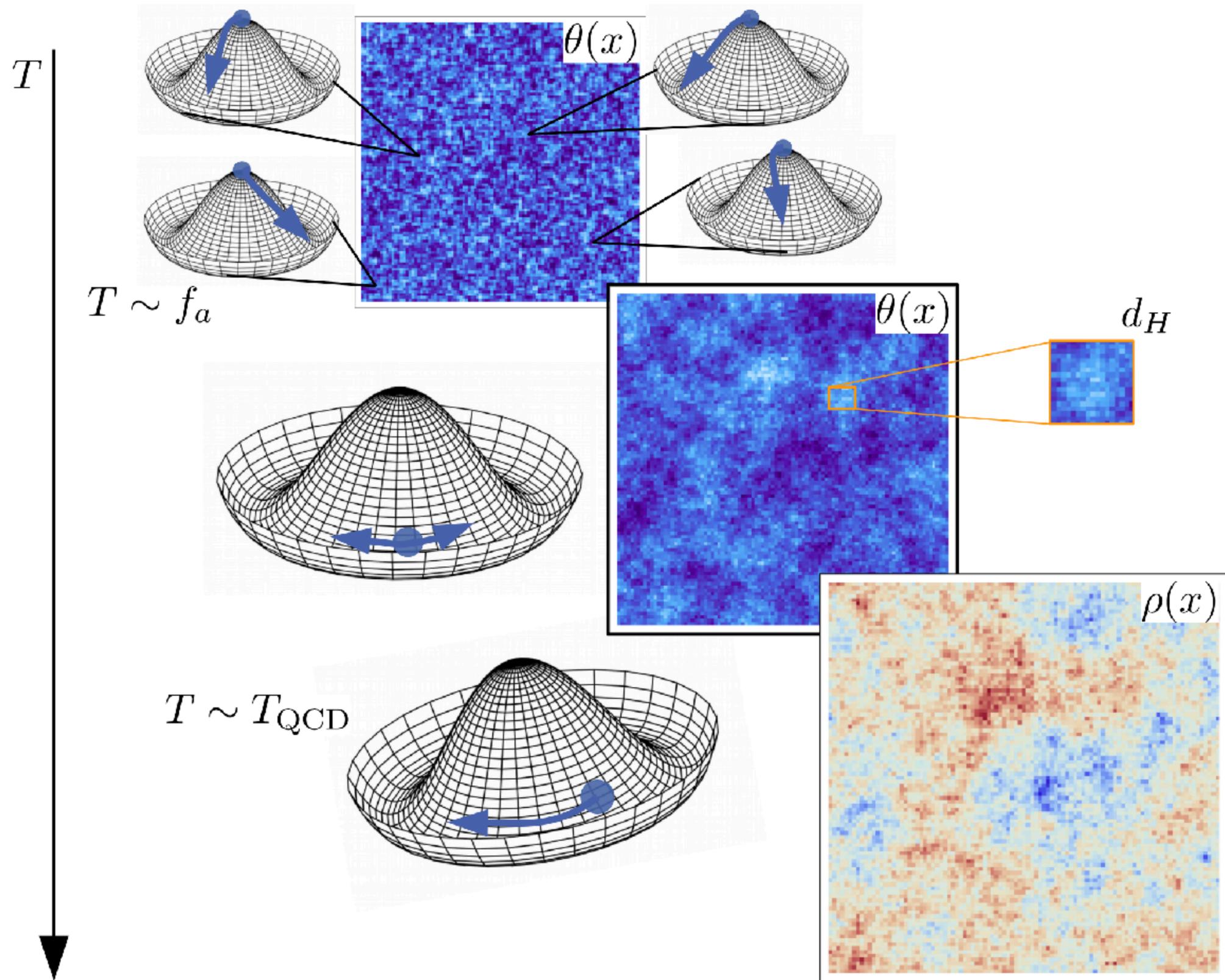
Based on 2206.04619  
(to appear in JCAP)

Padova, 12.09.2022



# Axion miniclusters



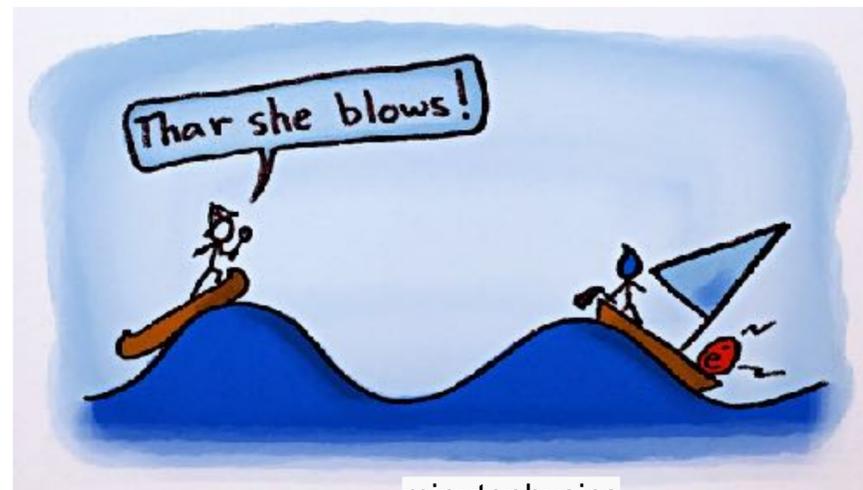


# Occupation numbers

- (Very) rough estimates for a QCD axion with  $m_a = 10^{-5}$  eV
- Axion mass is turned on at  $m_a(t_1) \simeq H(t_1)$
- Typical radius and mass of miniclusters are determined by the size of the horizon at  $t_1$

$$R \sim 10^8 \text{ km} \quad M \sim 10^{-12} M_\odot$$

- Typical phase space occupancy  $\mathcal{N} \sim 10^{51}$
- We describe the minicluster as an incoherent superposition of classical axion waves

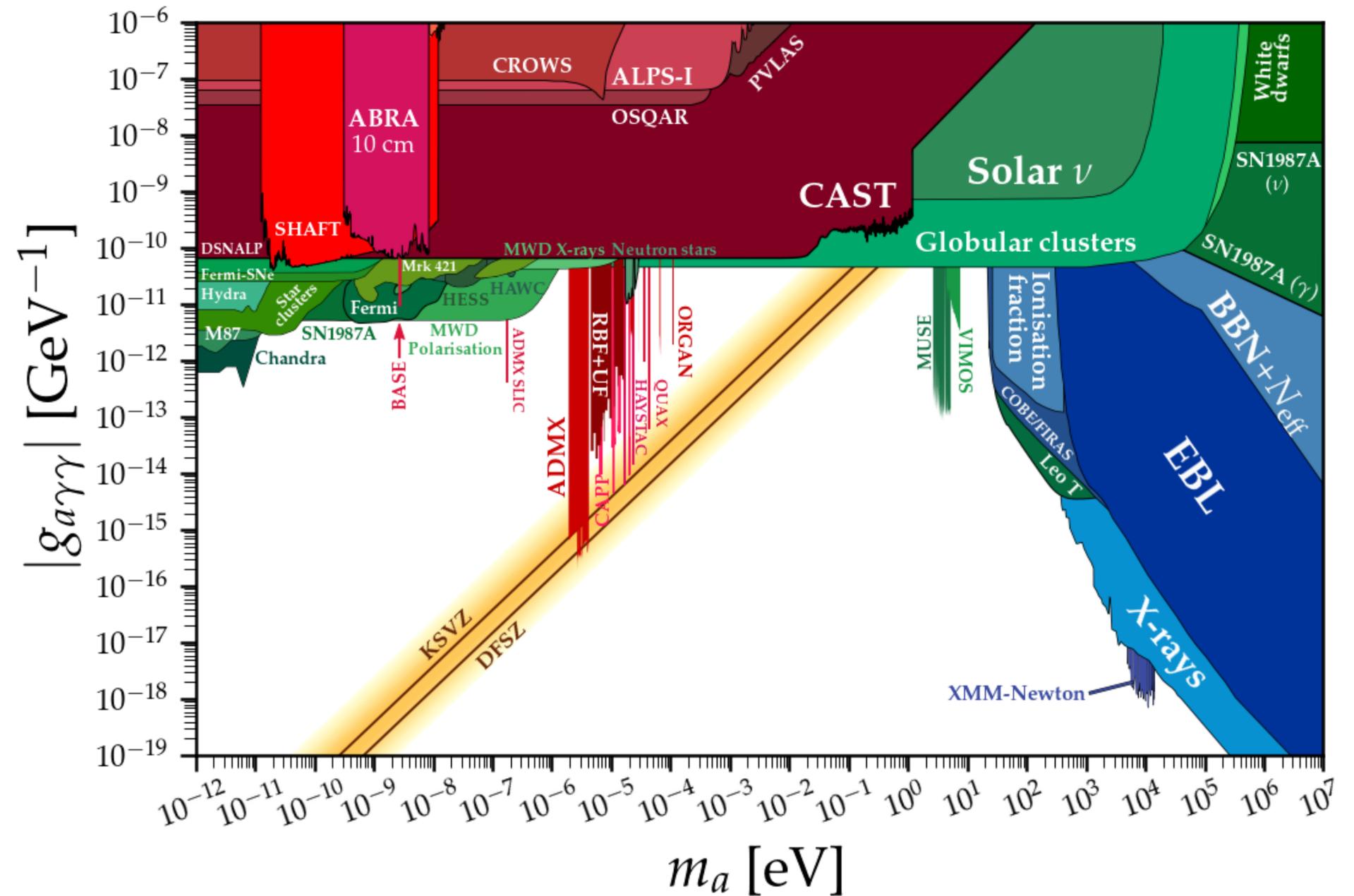


# Miniclusters and dark matter detection

- If all the local dark matter is in miniclusters, the rate of encounters with Earth is

$$R_{enc} \sim 10^{-4} \text{ yr}^{-1}$$

- Problem for haloscope type experiments!
- Astrophysical signatures

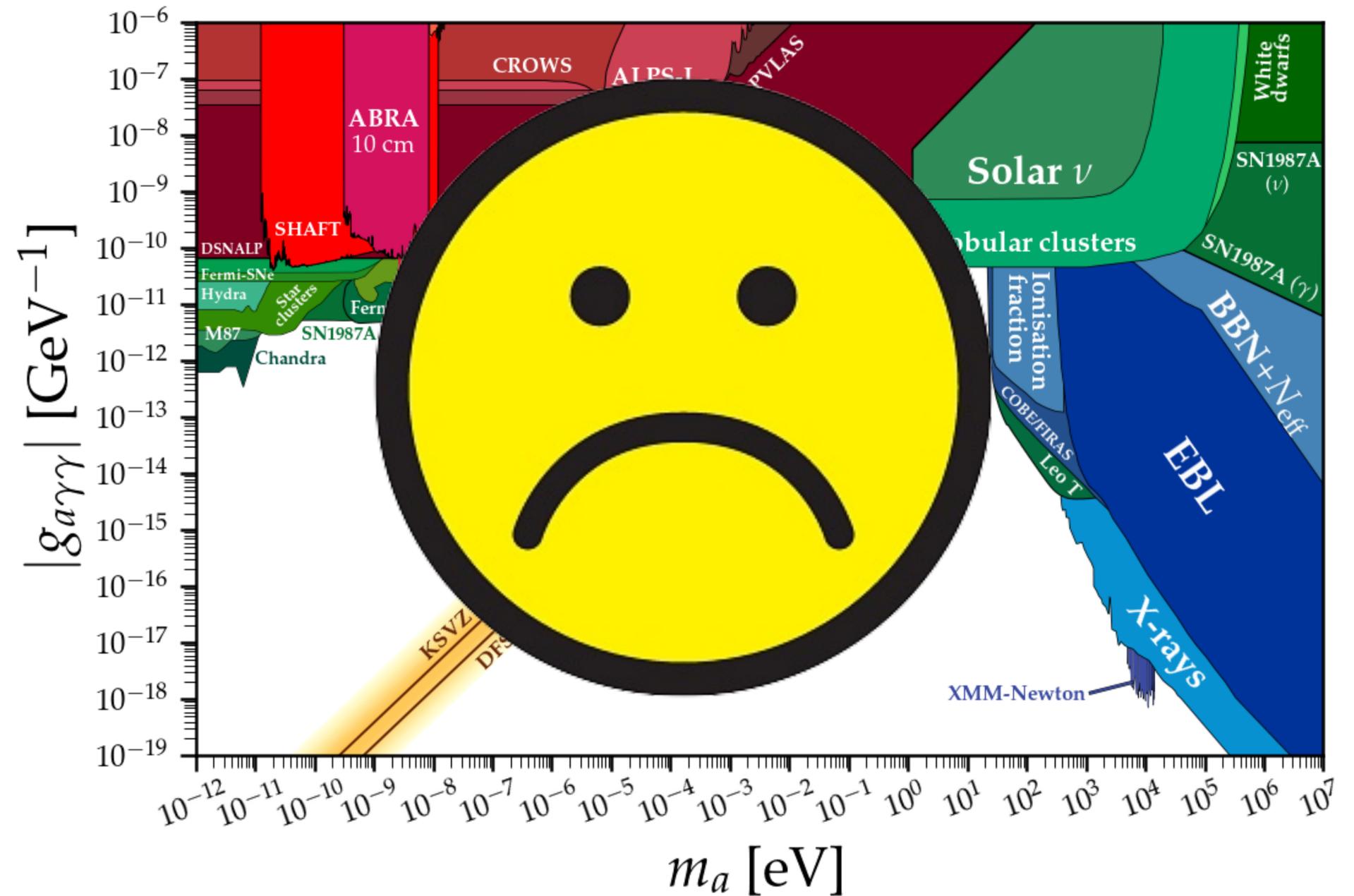


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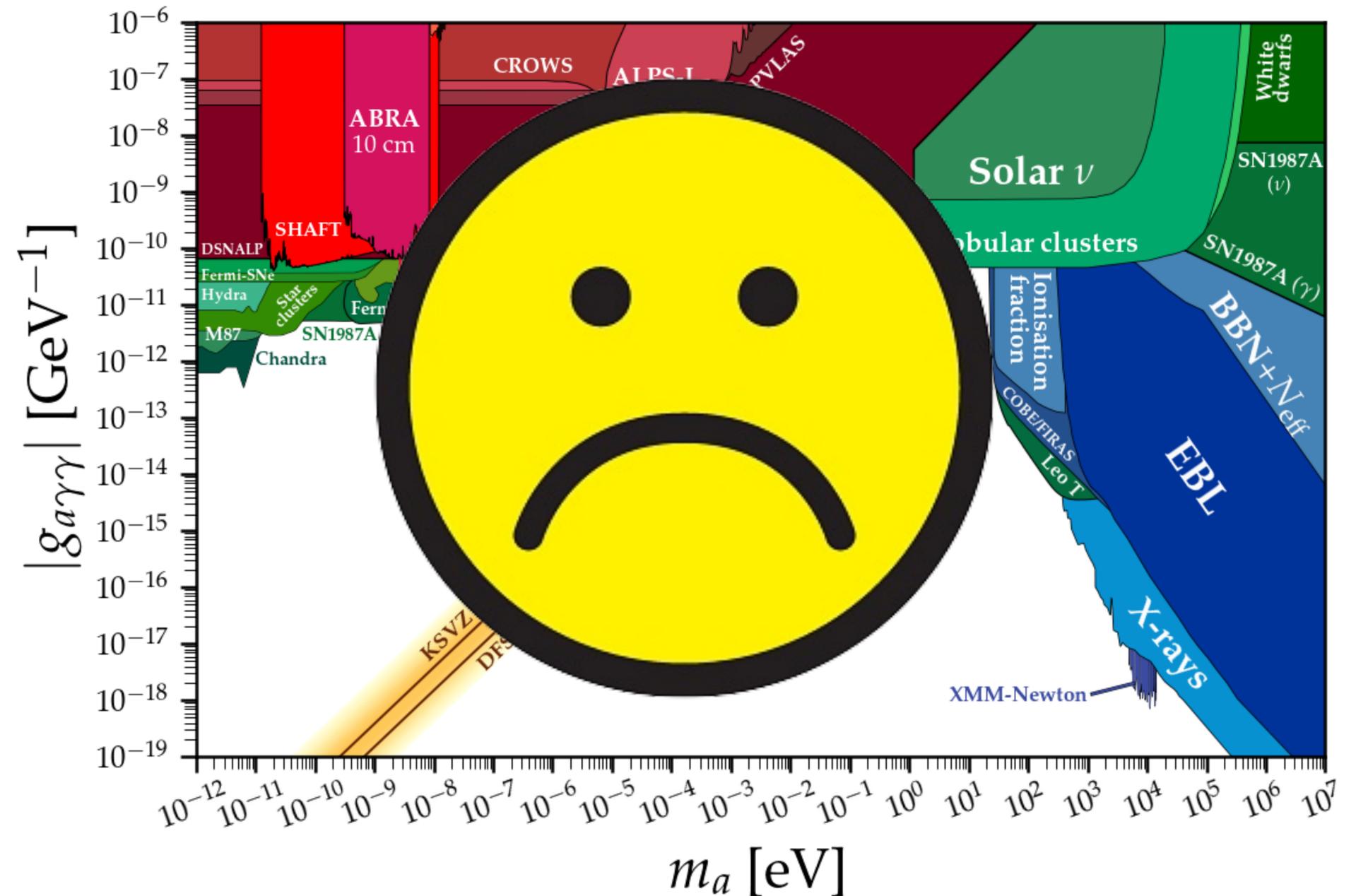


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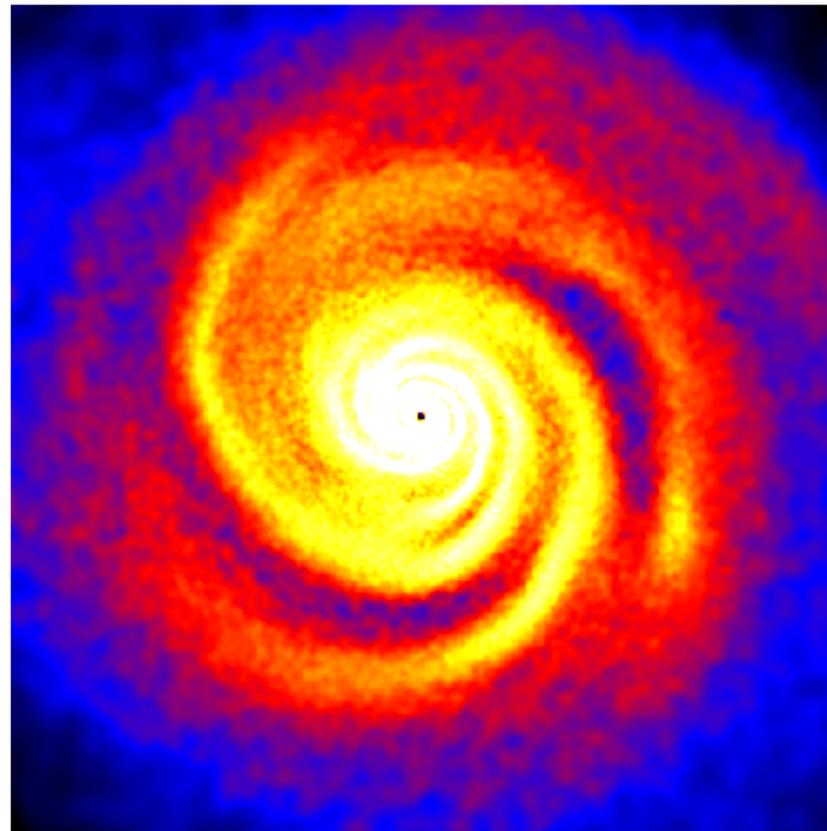
- Problem for haloscope type experiments!
- Astrophysical signatures
- We need to understand if miniclusters survive until today
- Tidal disruption in the galaxy effective?



# Other work on tidal disruption of miniclusters

- P. Tinyakov, I. Tkachev, and K. Zioutas, JCAP 1601 (2016), no. 01 035, [1512.02884]
- V. I. Dokuchaev, Y. N. Eroshenko, and I. I. Tkachev, J. Exp. Theor. Phys. 125 (2017), no. 3 434–442, [1710.09586]
- B. J. Kavanagh, T. D. P. Edwards, L. Visinelli, and C. Weniger, Phys. Rev. D 104 (2021), no. 6 063038, [2011.05377]
- X. Shen, H. Xiao, P. F. Hopkins, K. M. Zurek, [2207.11276]

# Construction of a self-gravitating system



# WKB solution to Schrödinger equation

- Eigenfunctions

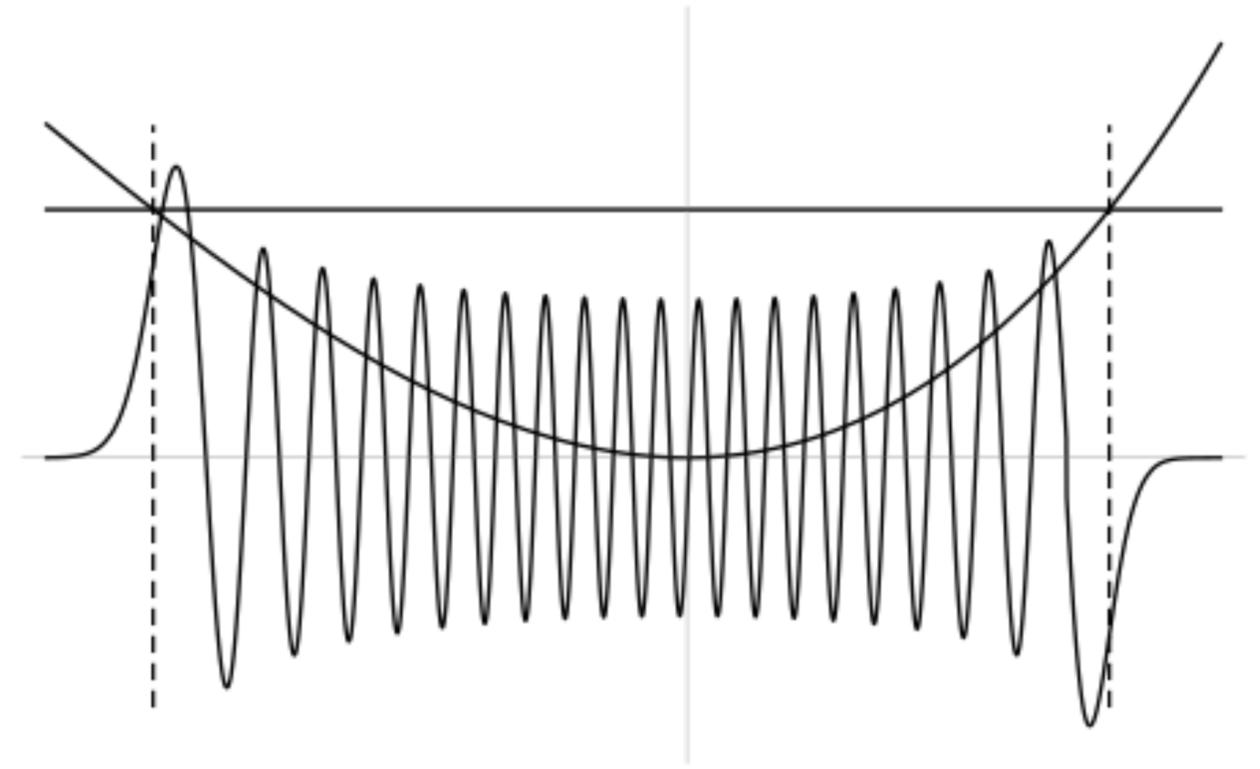
$$\psi_{nlm}(\vec{r}) = R_{nl}(r)Y_{lm}(\theta, \varphi)$$

- $R_{nl}(r)$  is WKB solution to the radial equation with effective potential

$$V_l(r) = \frac{l^2}{2m_a r^2} + m_a \phi(r)$$

- Validity of WKB

$$\frac{\lambda}{2\pi} = \frac{1}{\sqrt{2m_a(E_n - V_l(r))}} \ll R$$



Typical angular momentum

$$l \sim m_a \sqrt{GM R}$$

must be large

# WKB solution to Schrödinger equation

$$\psi(\vec{r}, t) = \sum_{nlm} C_{nlm} R_{nl}(r) Y_{lm}(\theta, \varphi) e^{-iE_n t}$$

- Coefficients with random phases

$$C_{nlm} = \sqrt{(2\pi)^3 f(E_n) dn/dE_n dE_n} e^{i\phi_{nlm}}$$

- Energy density

$$\begin{aligned} \rho(r) &= m_a \langle |\psi(\vec{r}, t)|^2 \rangle \\ &= 4\pi m_a^2 \int_{m_a \phi(r)}^0 dE f(E) \sqrt{2m_a(E - m_a \phi(r))} \end{aligned}$$

# Construction of a self-gravitating system

- Choose a  $\phi(r)$ ,  $\rho(r)$  pair that satisfies Poisson equation

- Find  $\rho(\phi)$

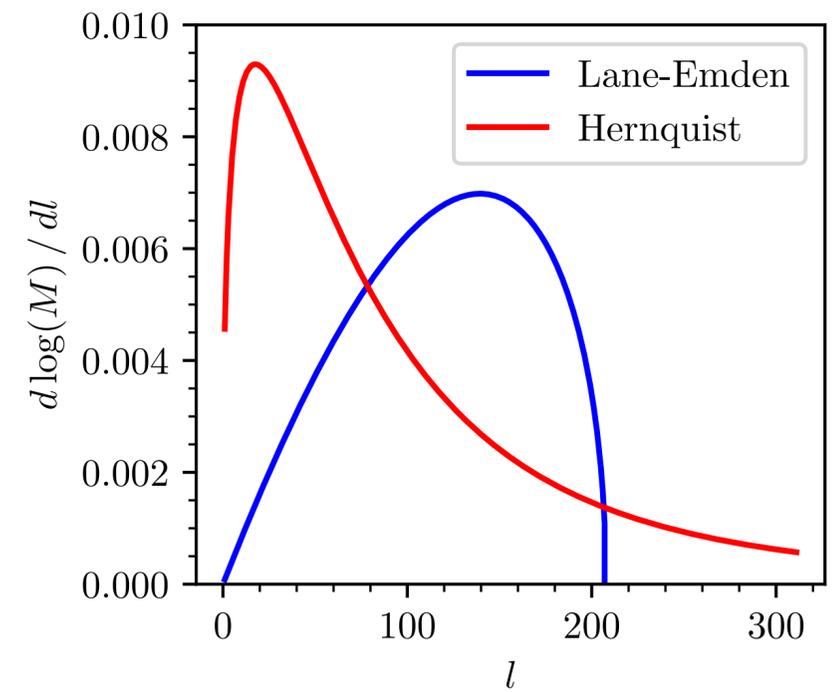
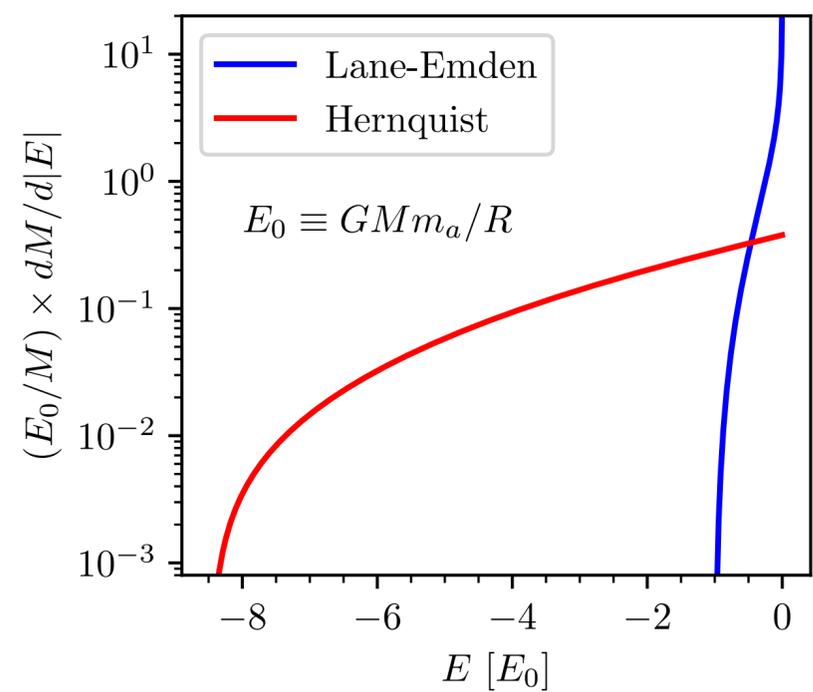
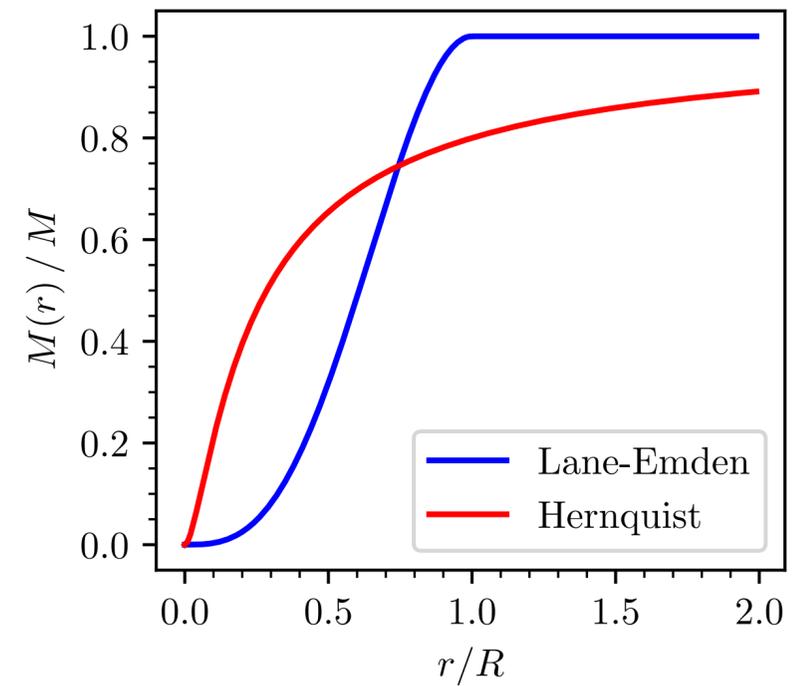
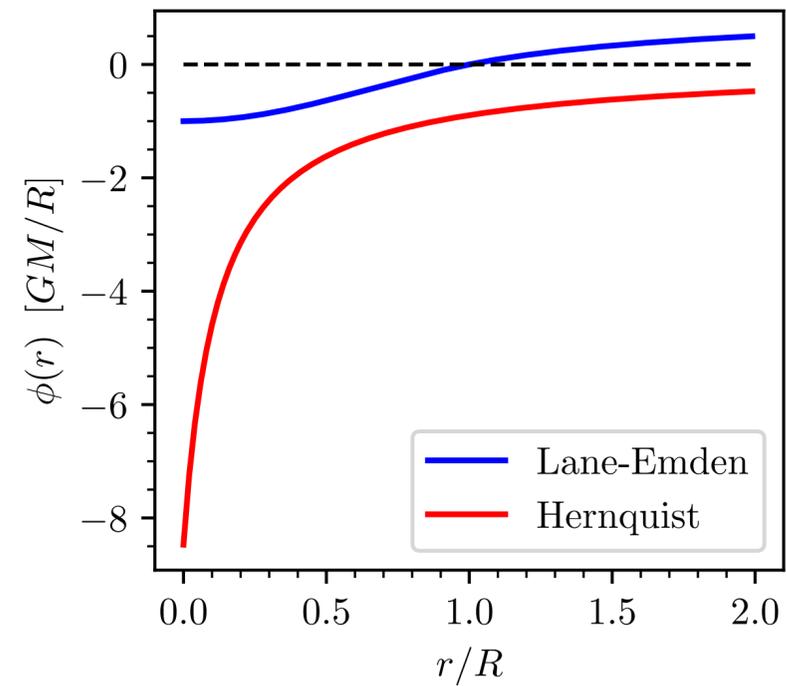
- Use Eddington's formula to obtain  $f(E)$

$$f(E) = \frac{1}{2\pi^2 m_a^2} \frac{d}{dE} \int_{E/m_a}^0 d\phi \frac{d\rho/d\phi}{\sqrt{2m_a(m_a\phi - E)}}$$

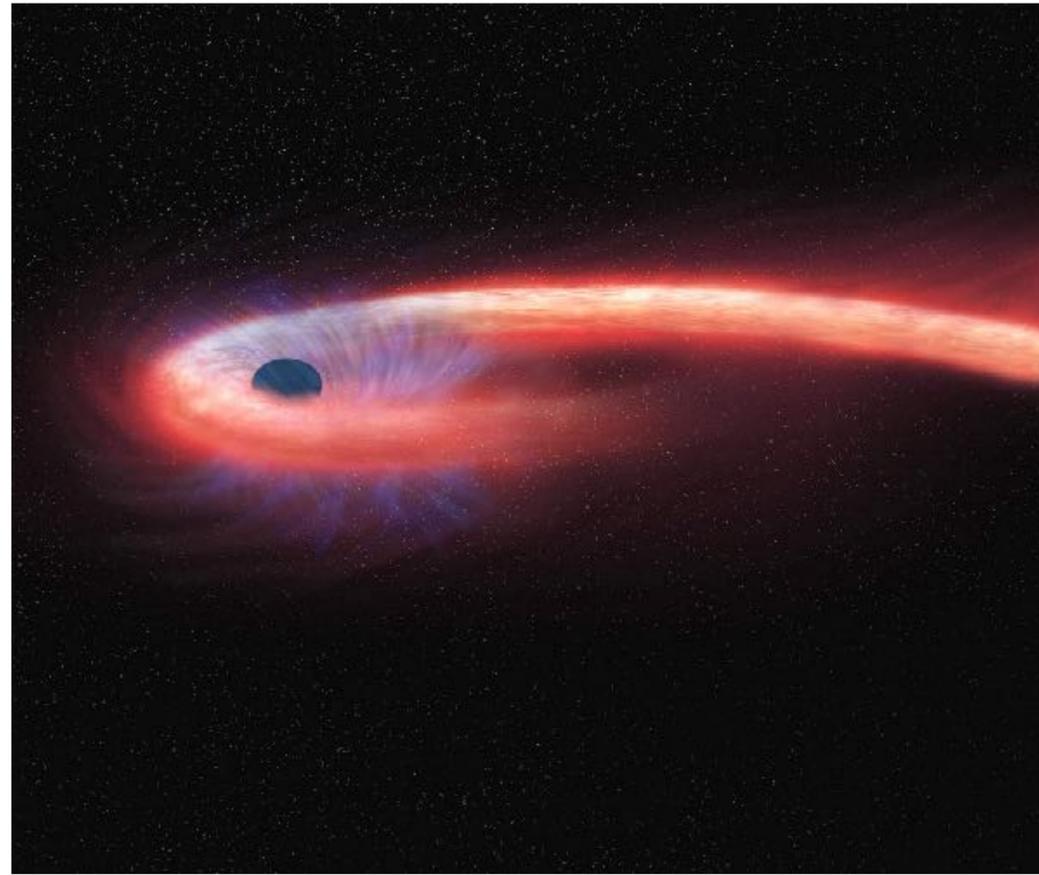
- Construct WKB eigenfunctions
- Construct coefficients
- Resulting average clump is in virial equilibrium



# Two density profiles



# Tidal perturbation of the clump

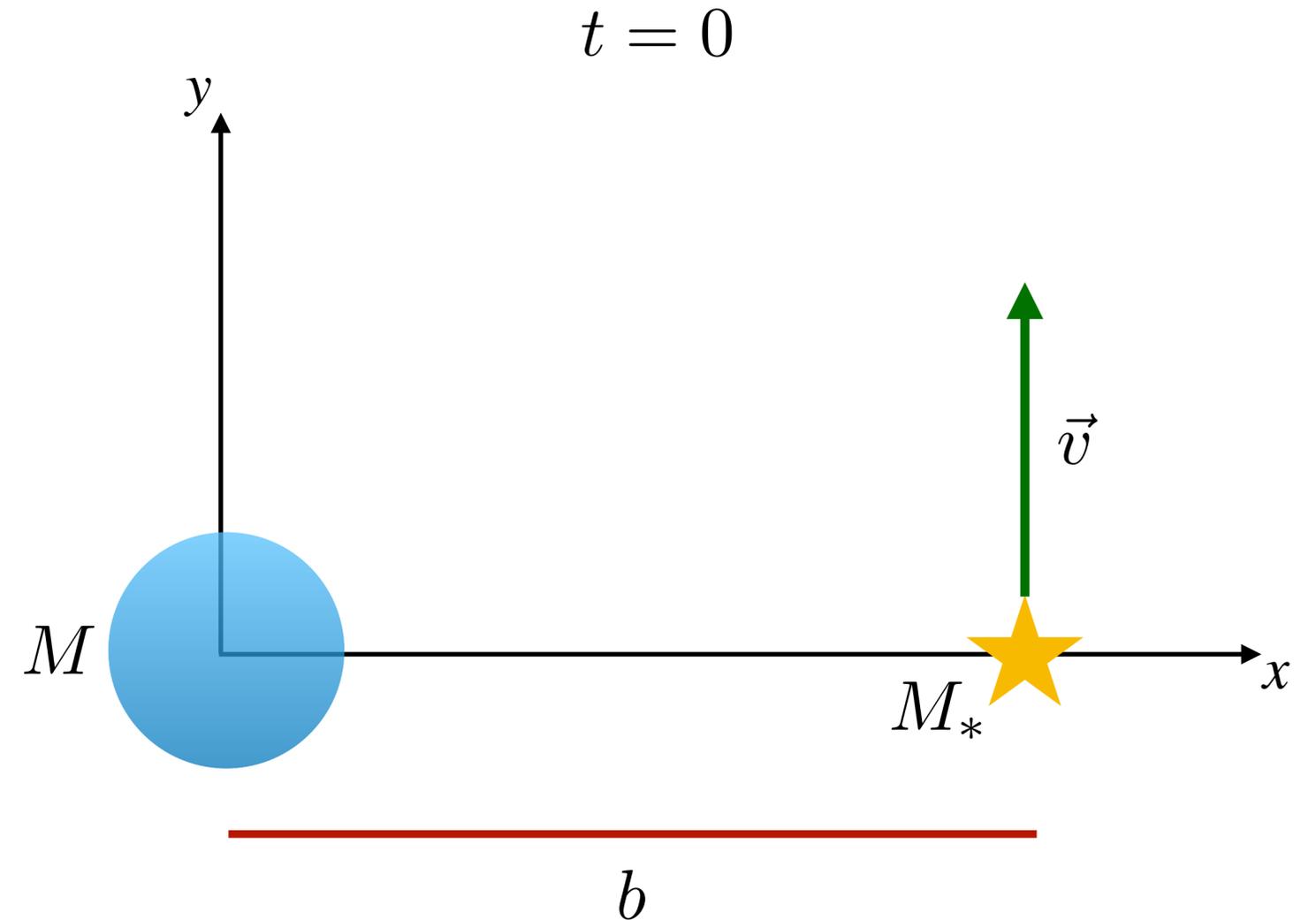


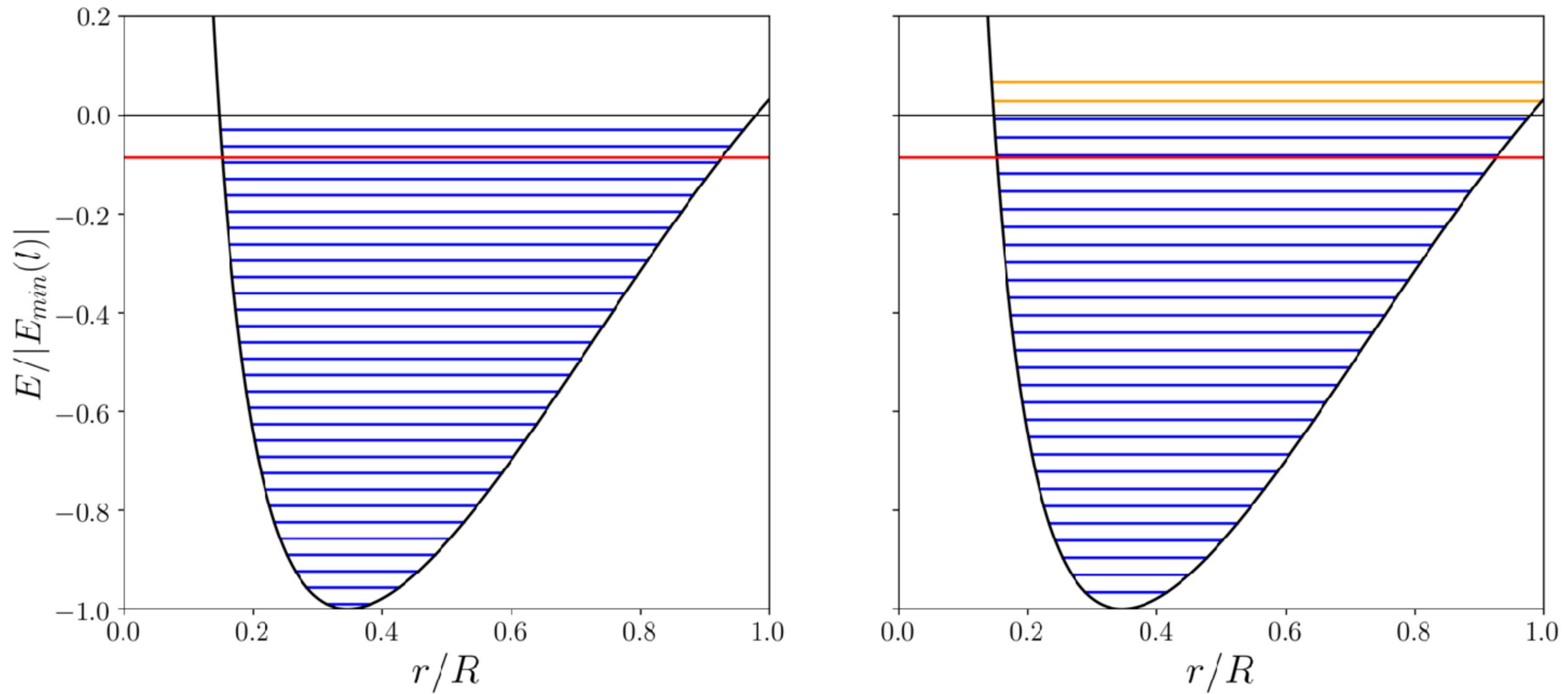
# A star passing by

$$H_1(\vec{r}, t) = -\frac{GM_* m_a r^2}{(b^2 + v^2 t^2)^{3/2}} P_2(\cos \gamma(t))$$

- Perturb the coefficients  $C_{nlm}$  up to second order
- Impulse approximation

$$\frac{b}{v} \ll R \sqrt{\frac{R}{GM}}$$





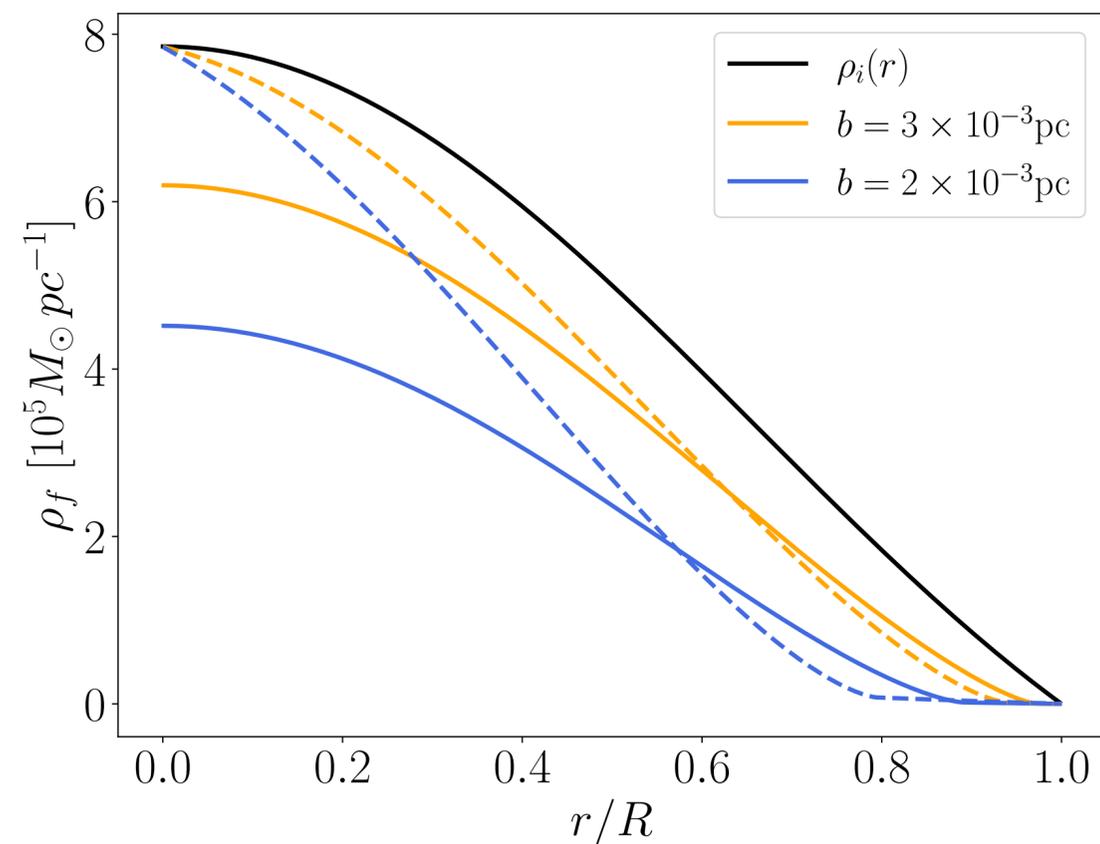
- Energy shift of a level due to the impact of the star

$$\delta E(E, l) = \left( \frac{2GM_*}{b^2 v} \right)^2 \frac{m_a}{4} \langle nl | r^2 | nl \rangle$$

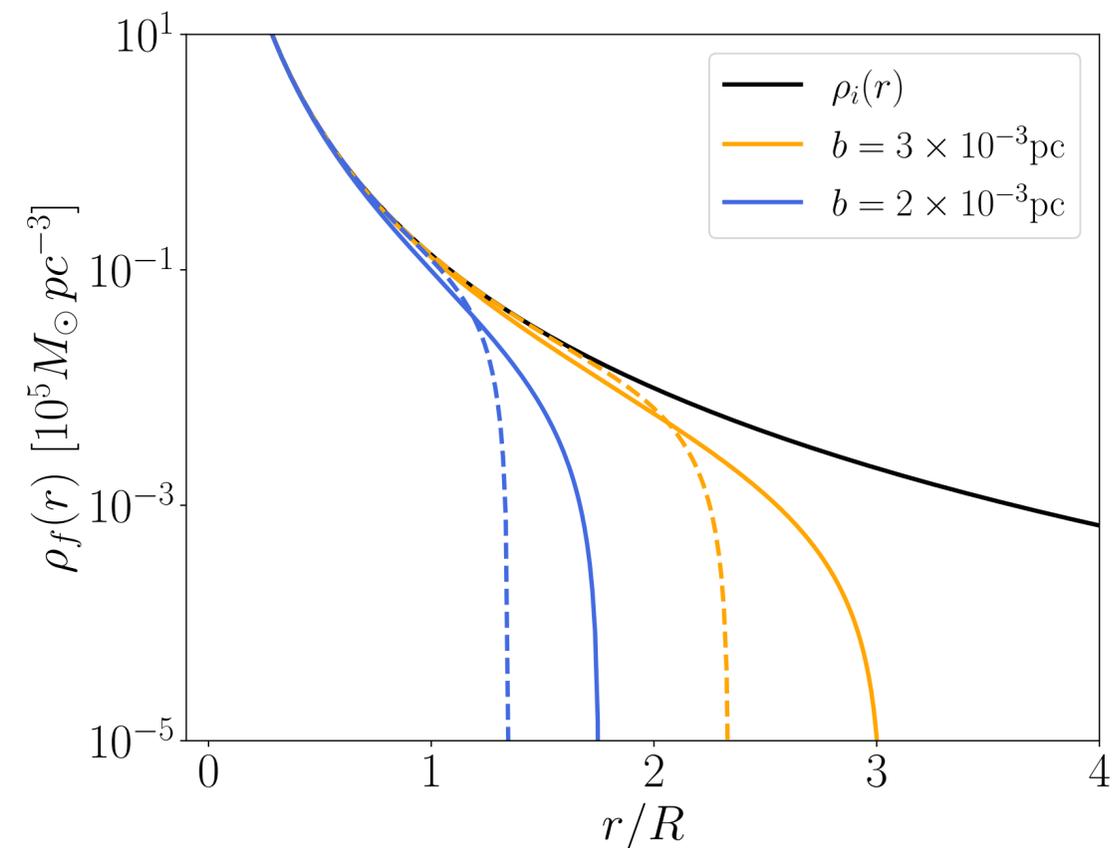
- All level with energy above a critical value become unbound

# Density profile after encounter

Lane-Emden

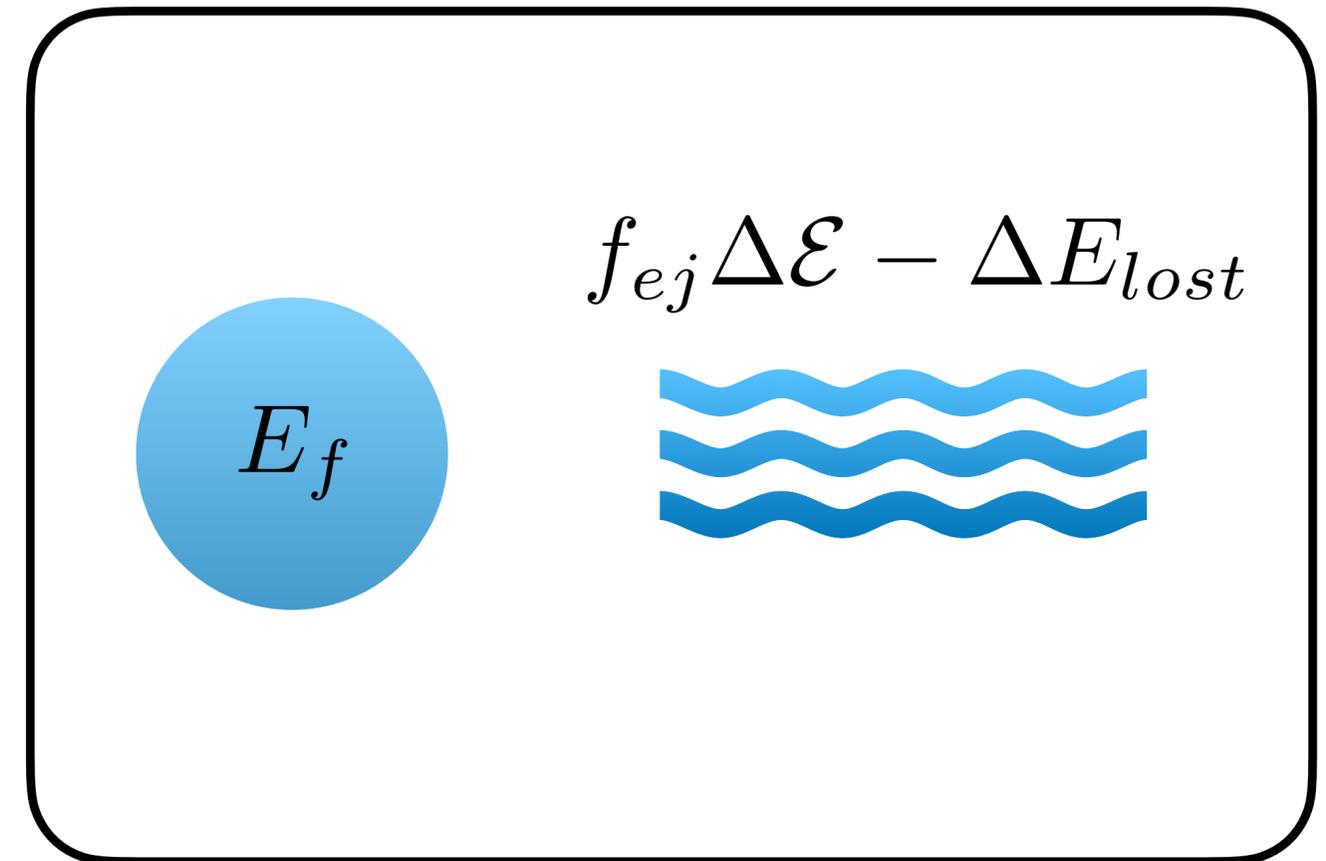
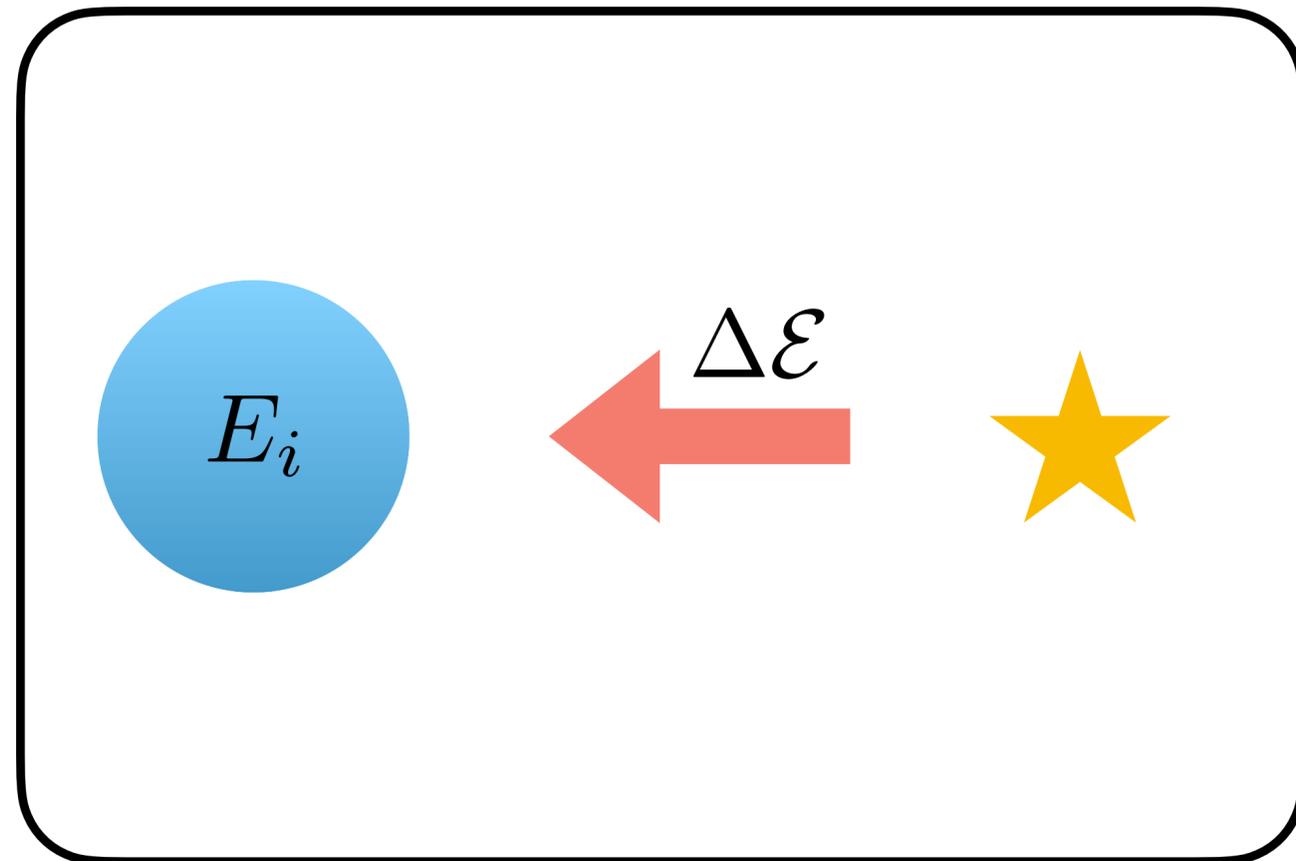


Hernquist



$$M = 10^{-12} M_\odot, \quad R = 10^{-6} \text{pc}, \quad M_* = M_\odot, \quad v = 10^{-4}$$

# Energy balance



$$E_f = E_i + \underbrace{\Delta E_{lost} + (1 - f_{ej})\Delta\mathcal{E}}_{\Delta E}$$

$$\Delta E_{lost} = \Delta E_K + \Delta E_B$$

# Re-virialization and new radius

- Assume cluster re-virializes to the same profile keeping it's total energy fixed
- Relaxation time is smaller than times between star encounters for our typical cluster
- Virial equilibrium

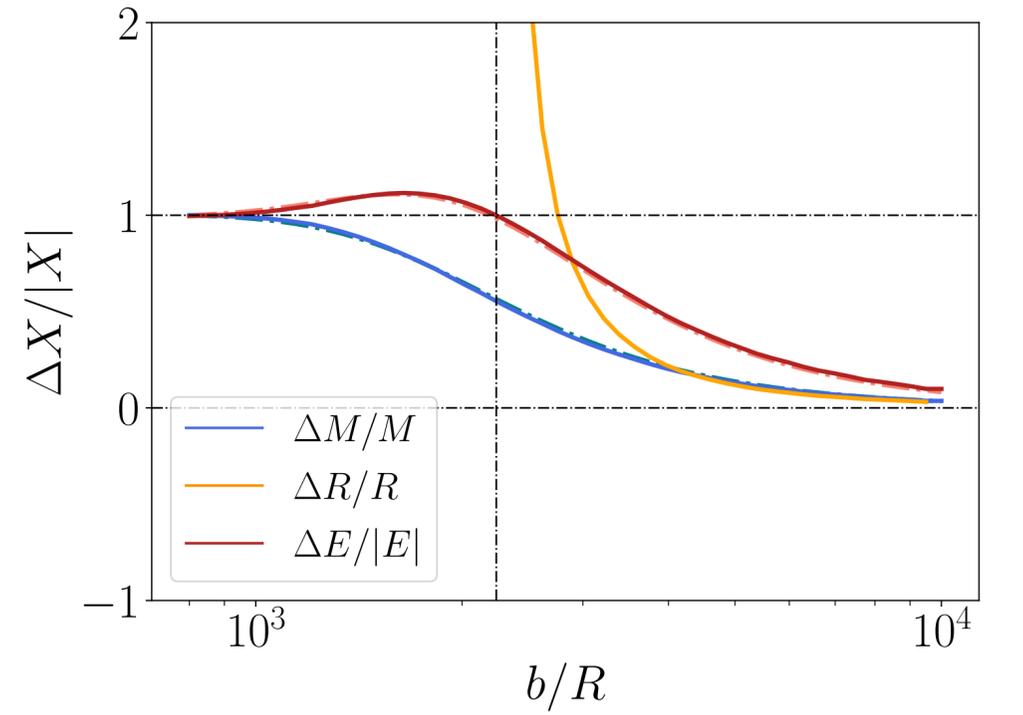
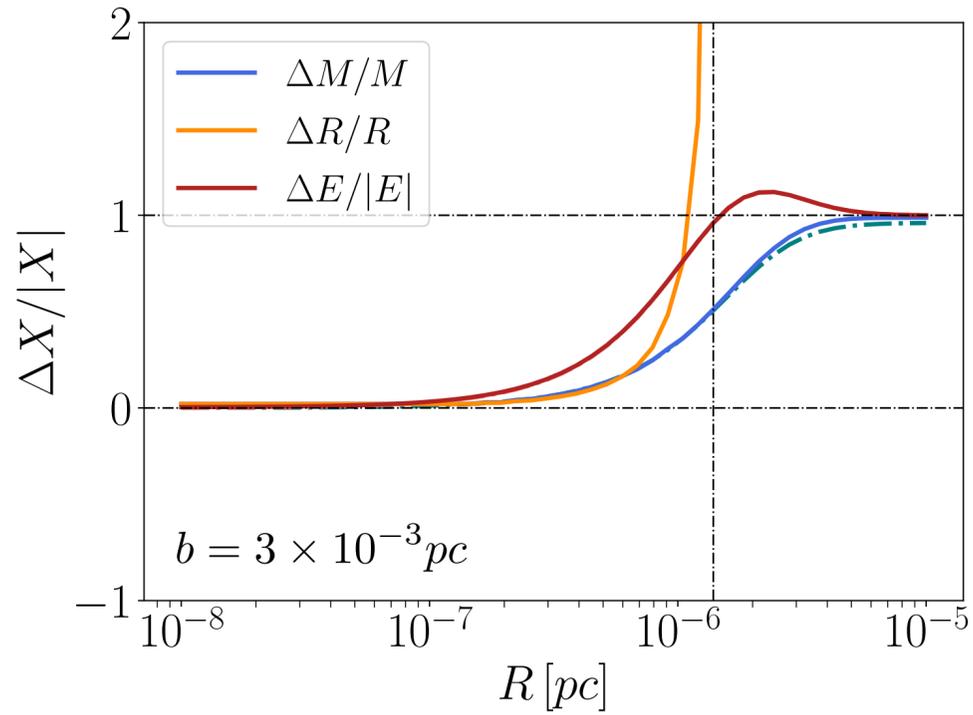
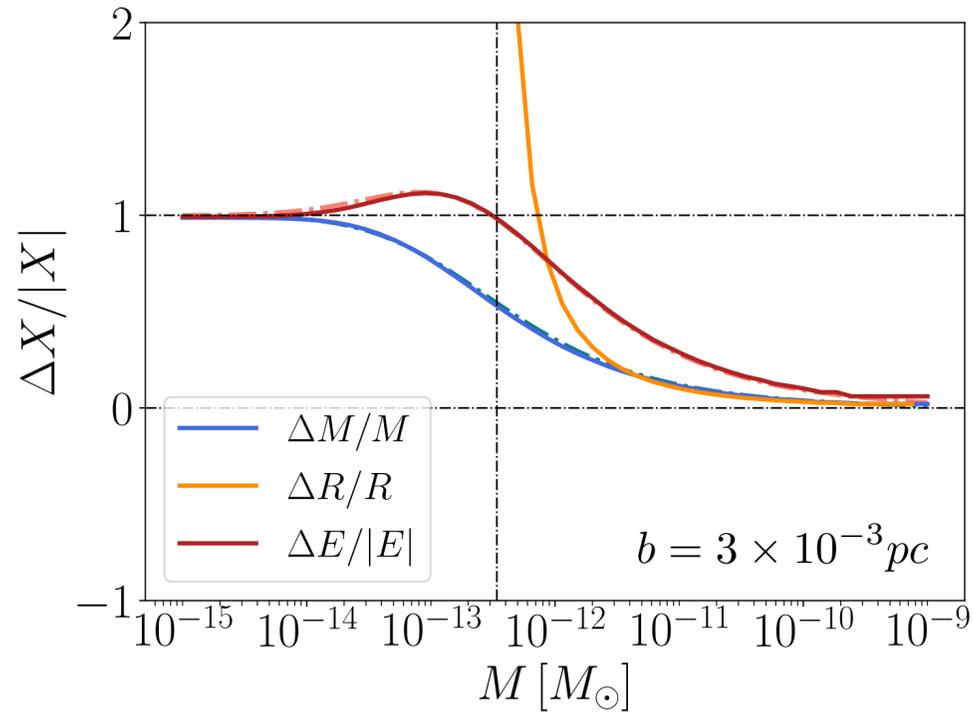
$$E_{tot} = \frac{E_B}{2}$$

- New radius

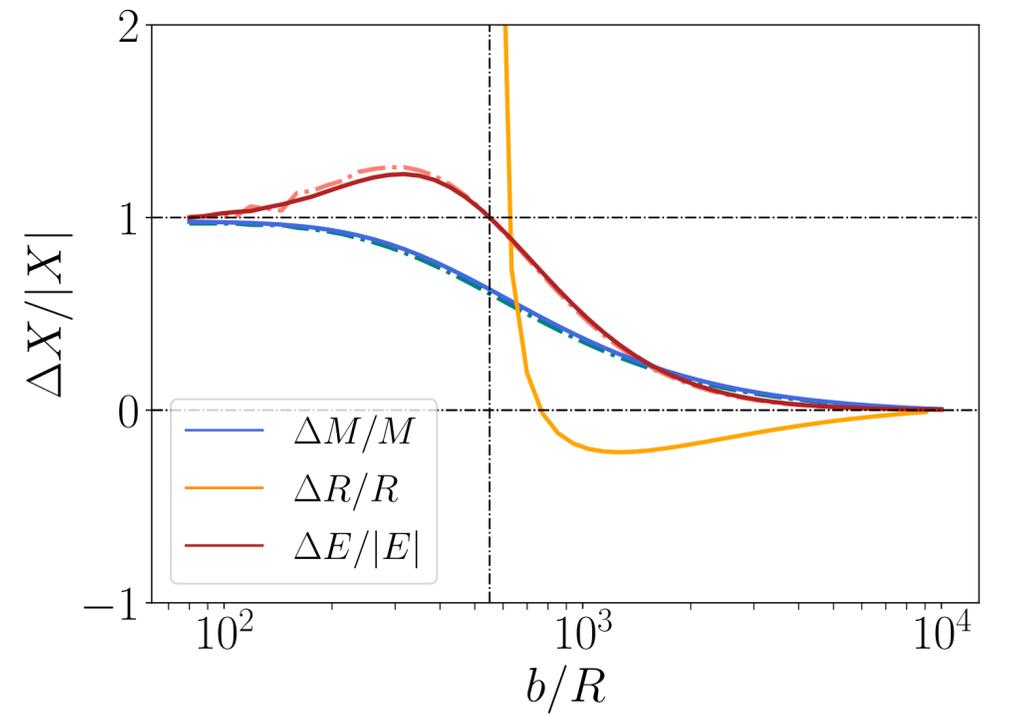
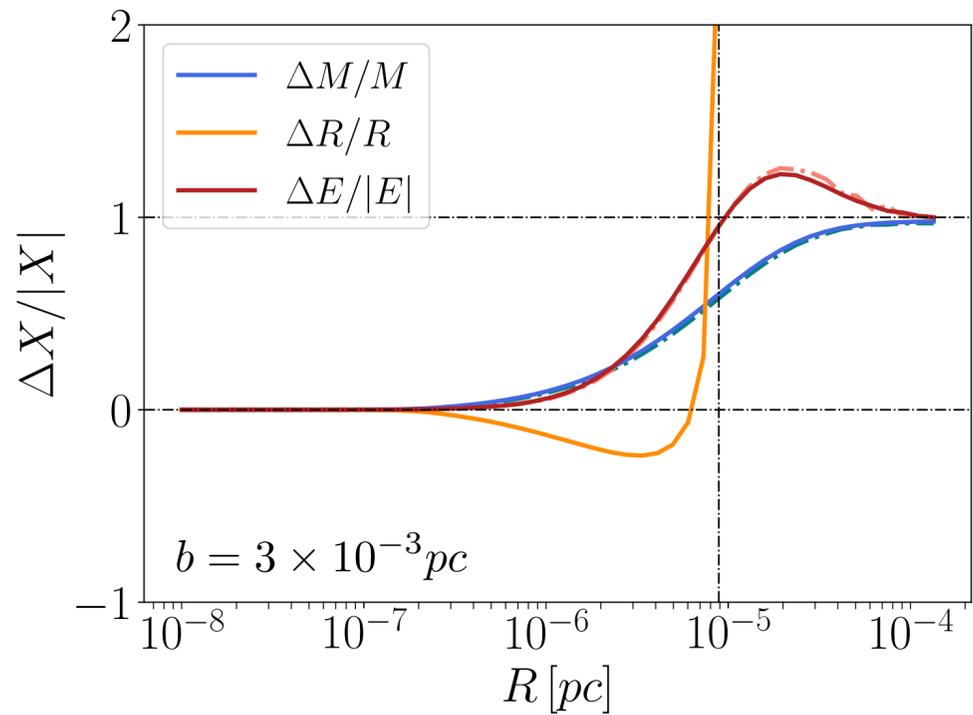
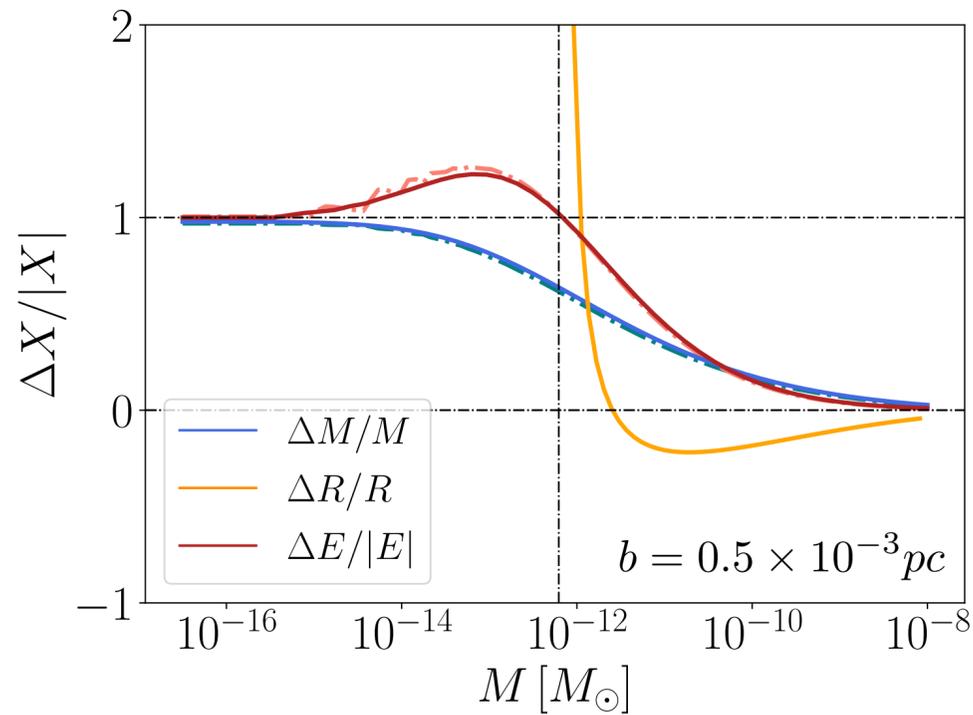
$$E_f = -\alpha \frac{G(M - \Delta M)^2}{R_f} \Rightarrow R_f = R_i \left( 1 - 2 \frac{\Delta M}{M} + \frac{\Delta E}{|E_i|} \right)$$



### Lane-Emden



### Hernquist



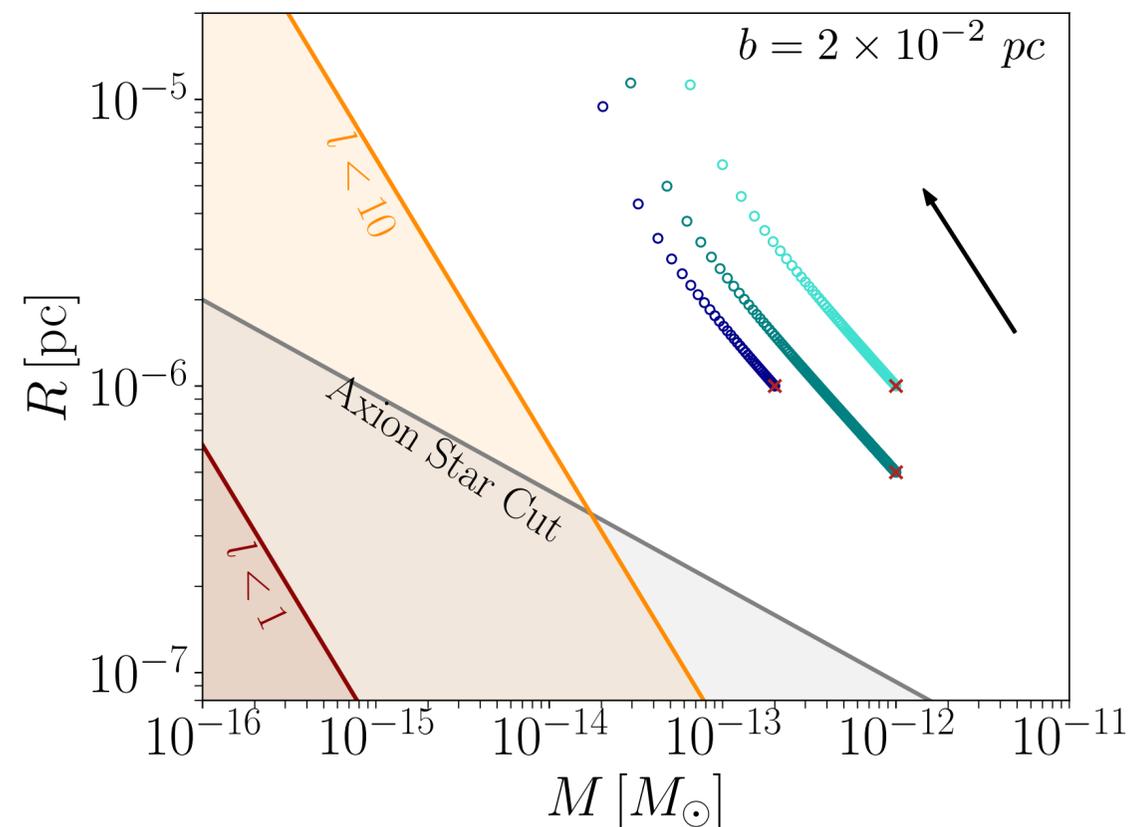
$$M = 10^{-12} M_\odot, \quad R = 10^{-6} pc, \quad M_* = M_\odot, \quad v = 10^{-4}$$

# Critical density and repeated perturbations

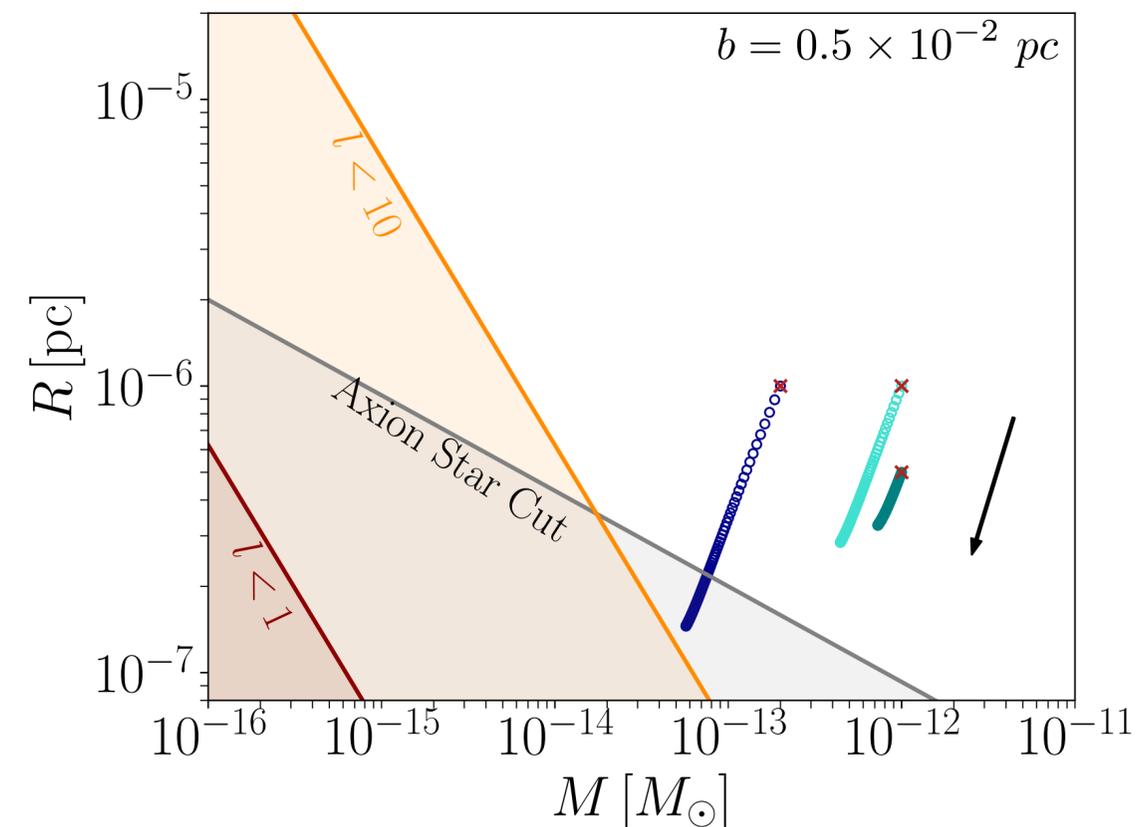
- Critical density, below which the cluster gets destroyed

$$\bar{\rho}_{\text{crit}} \approx \left( \frac{M_*}{1 M_\odot} \right)^2 \left( \frac{10^{-3} \text{ pc}}{b} \right)^4 \left( \frac{10^{-4}}{v} \right)^2 \times \begin{cases} 0.7 \times 10^{-11} M_\odot (10^{-6} \text{ pc})^{-3} & \text{(LE)} \\ 2.1 \times 10^{-14} M_\odot (10^{-6} \text{ pc})^{-3} & \text{(H)} \end{cases}$$

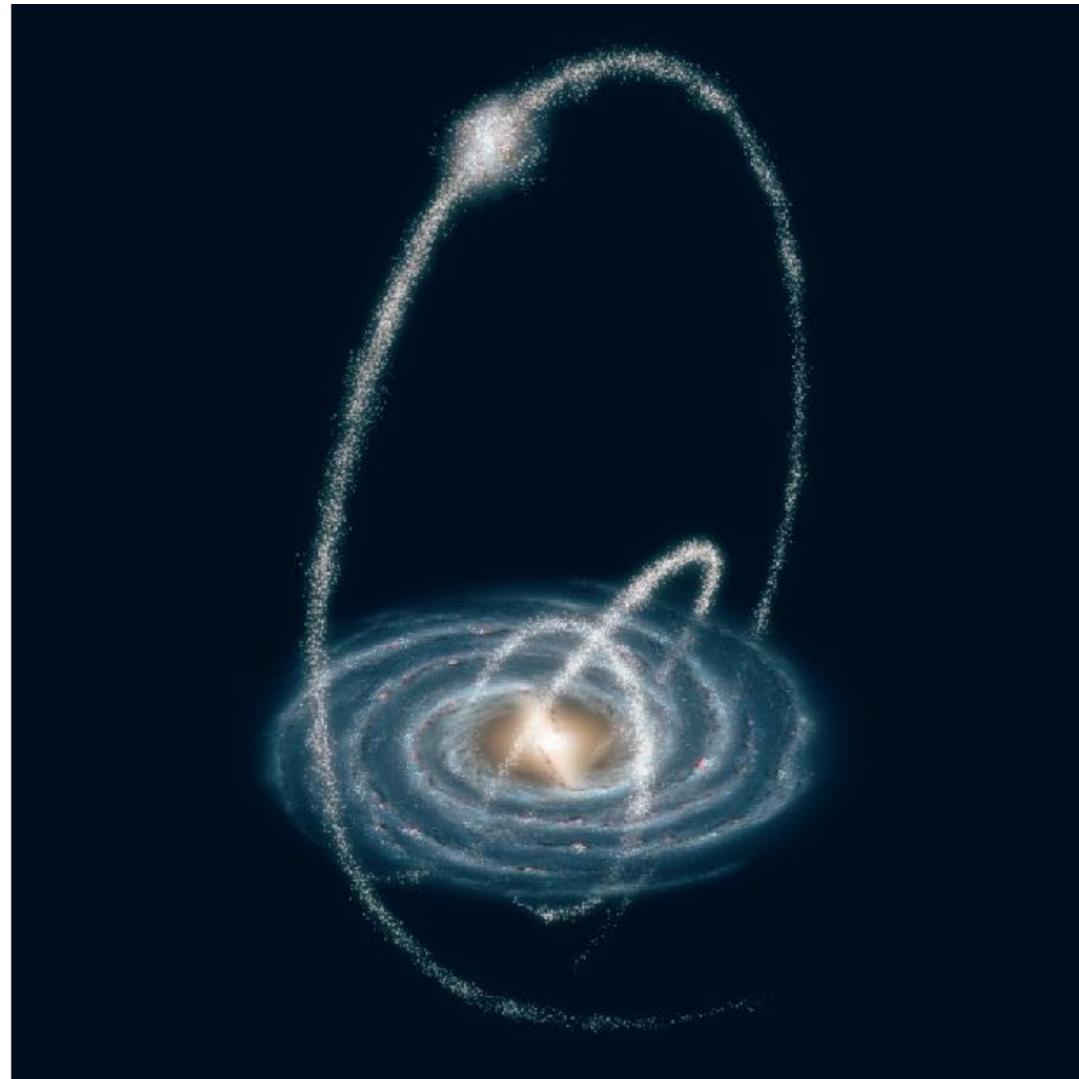
Lane-Emden



Hernquist



# Survival in the galaxy



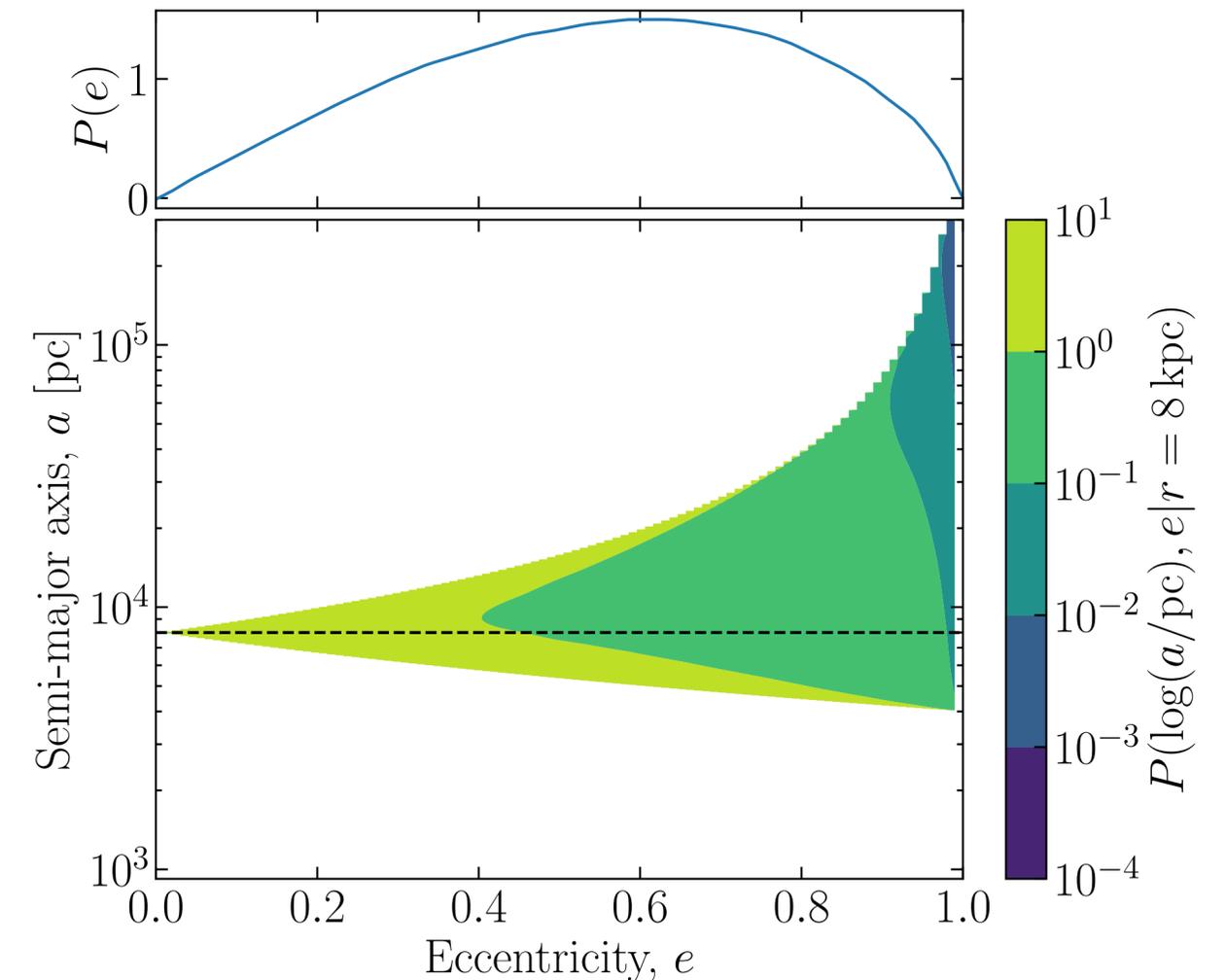
# A halo made of miniclusters

- Fix  $M$ ,  $R$  and the galactic radius  $r$
- Probability that a minicluster at  $r$  is on an orbit with given semi-major axis and eccentricity

$$P(a, e|r)$$

such that the galaxy has an NFW density profile

- Create sample  $N_{\text{MC}}^{(i)}(M, R, r)$
- The orbit fixes the average number of star encounters as a minicluster travels in the galaxy
- For each encounter sample  $P(b)$ ,  $P(v)$

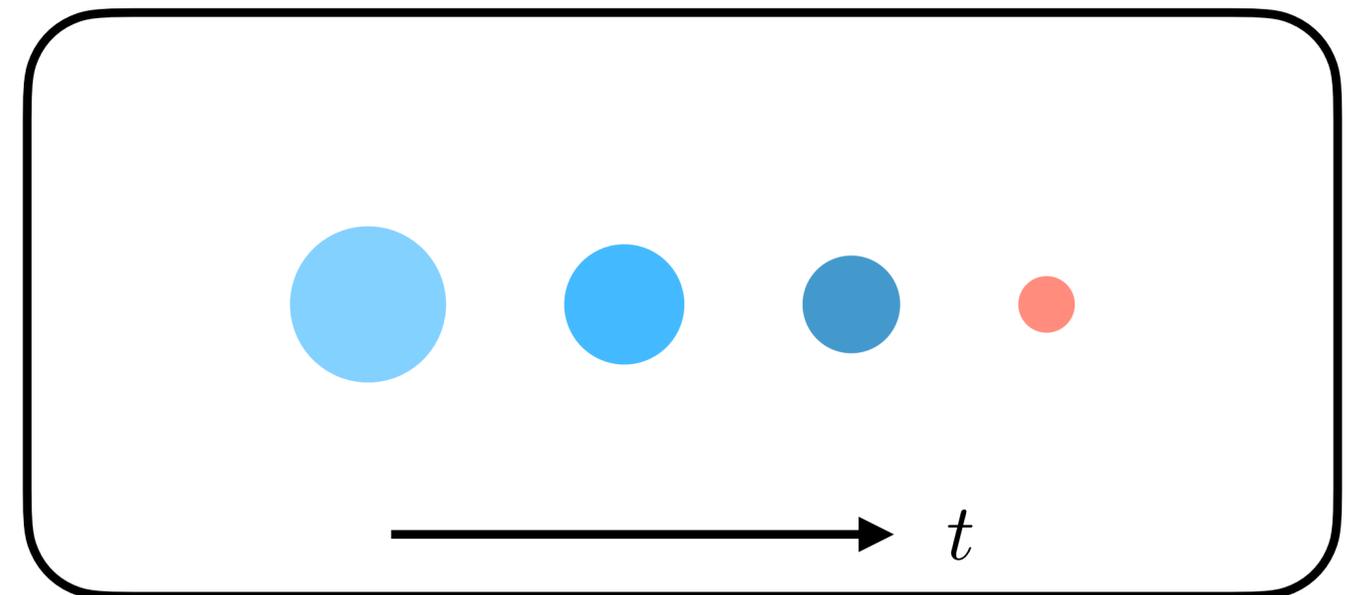
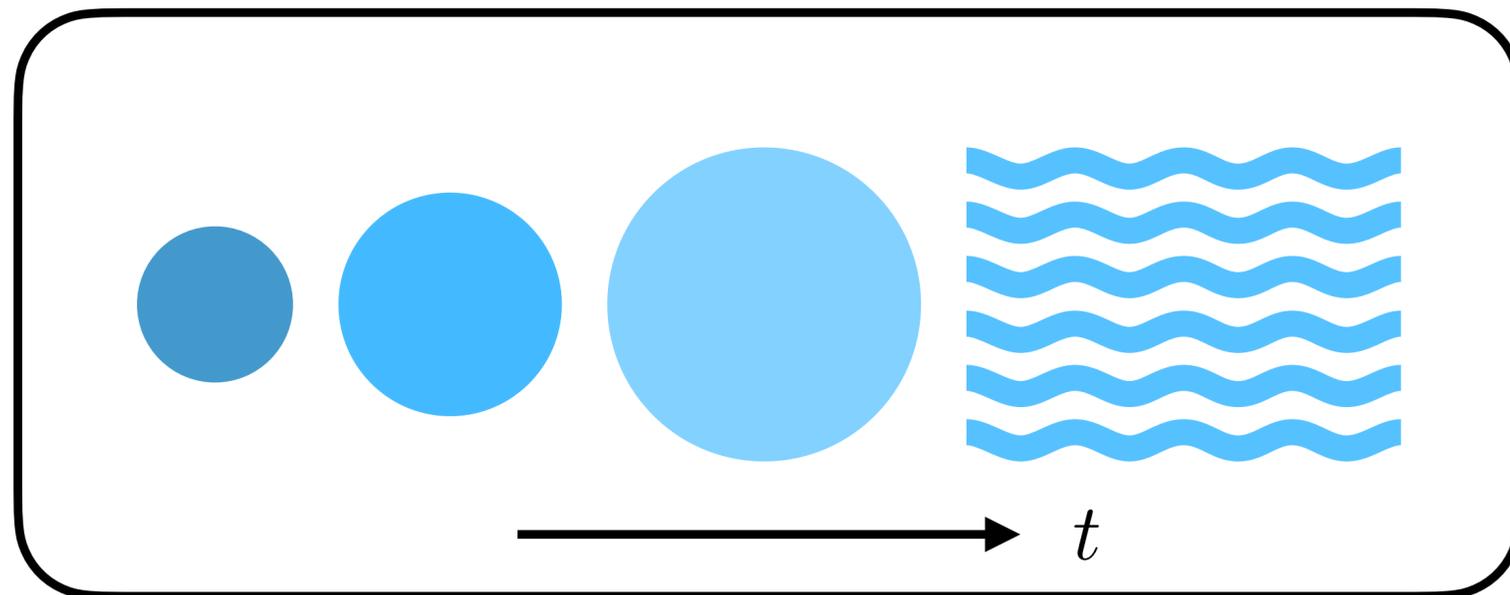


Kavanagh et al.  
Phys. Rev. D 104, 063038 (2021)

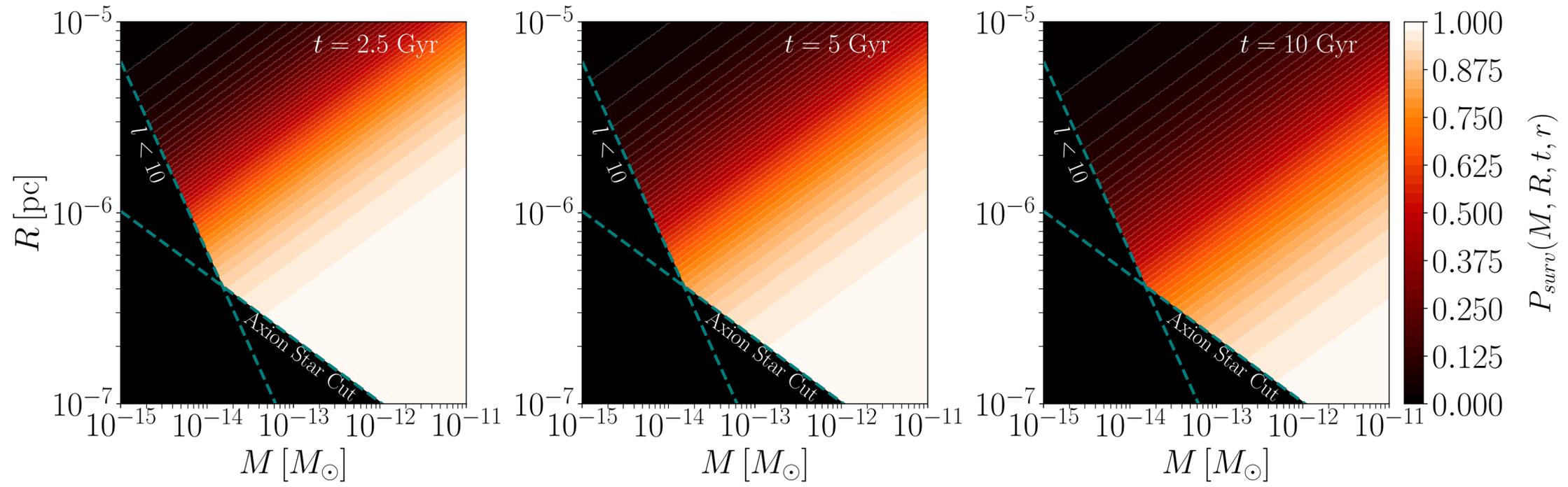
# Survival probability

$$P_{\text{surv}}(M, R, t, r) = \frac{N_{\text{MC}}^{(f)}(M, R, r, t)}{N_{\text{MC}}^{(i)}(M, R, r)}$$

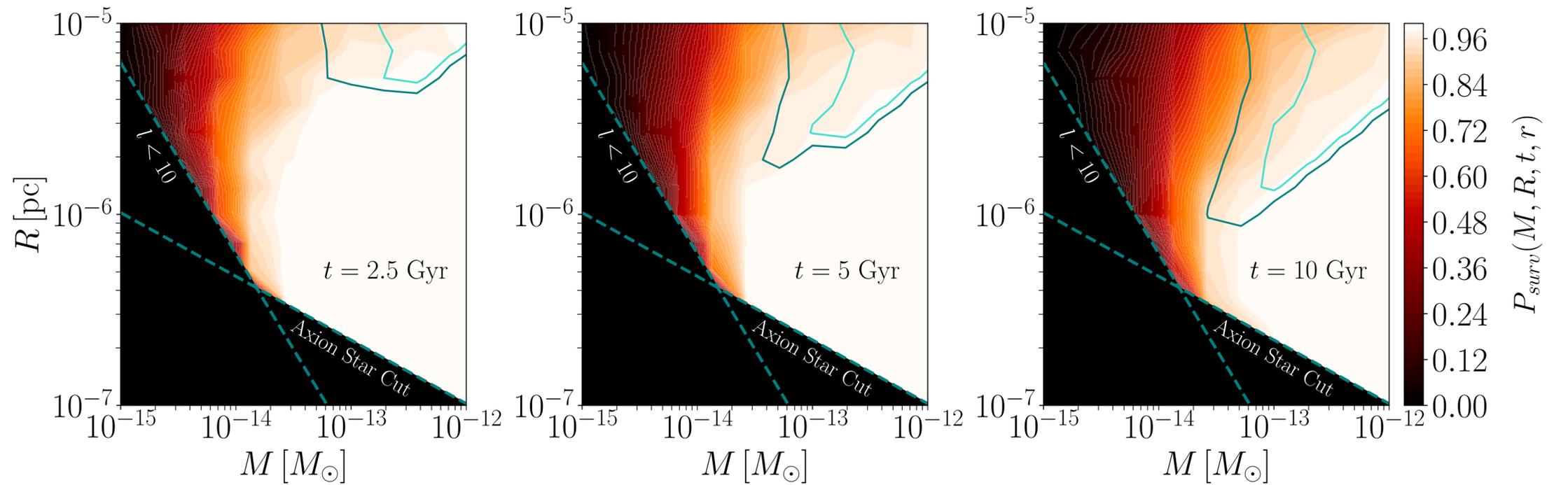
- 2 ways for a minicluster to be destroyed
  1. Become less and less tightly bound until completely unbound
  2. Become more and more tightly bound until turning into an axion star



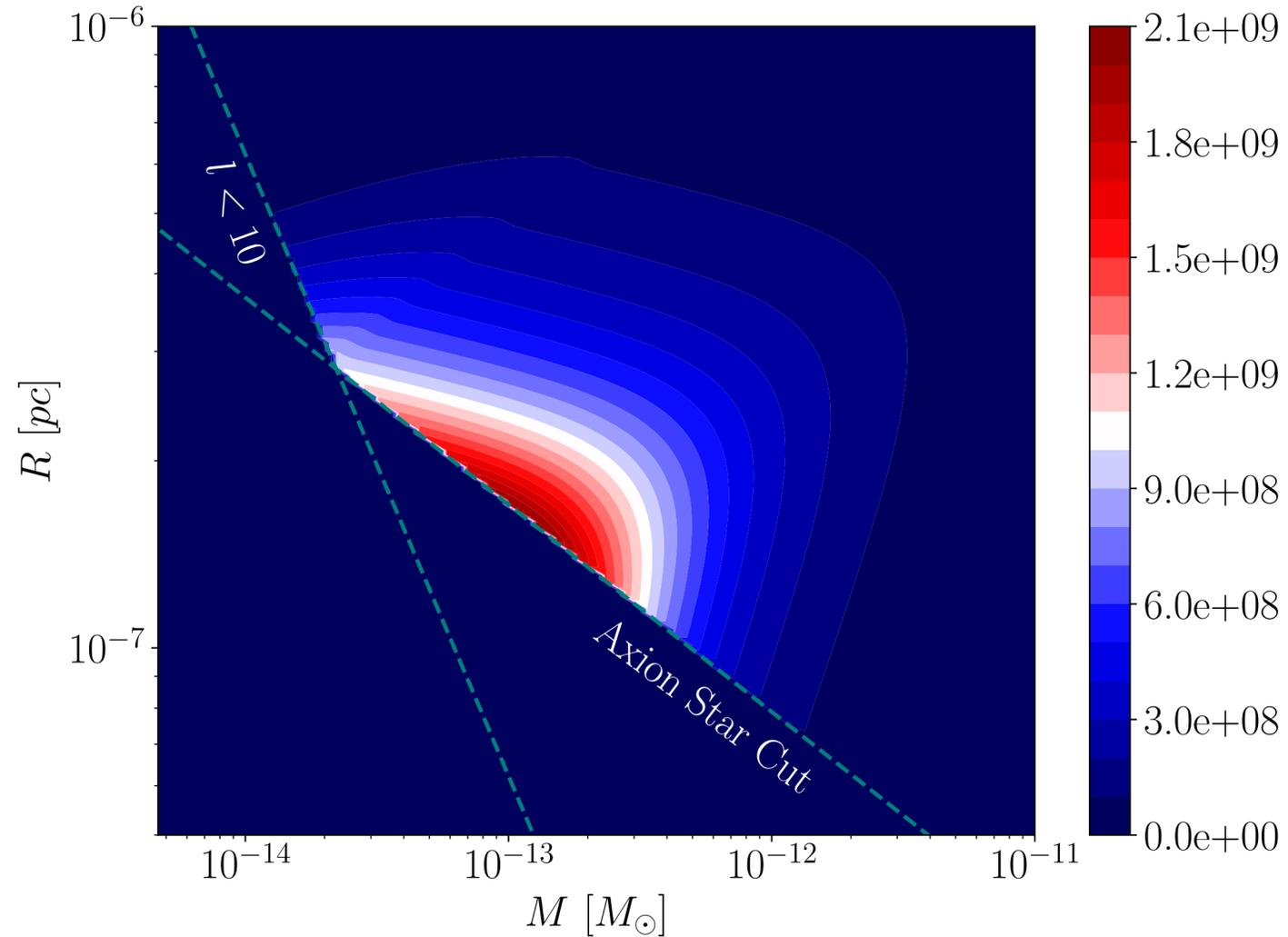
## Lane-Emden



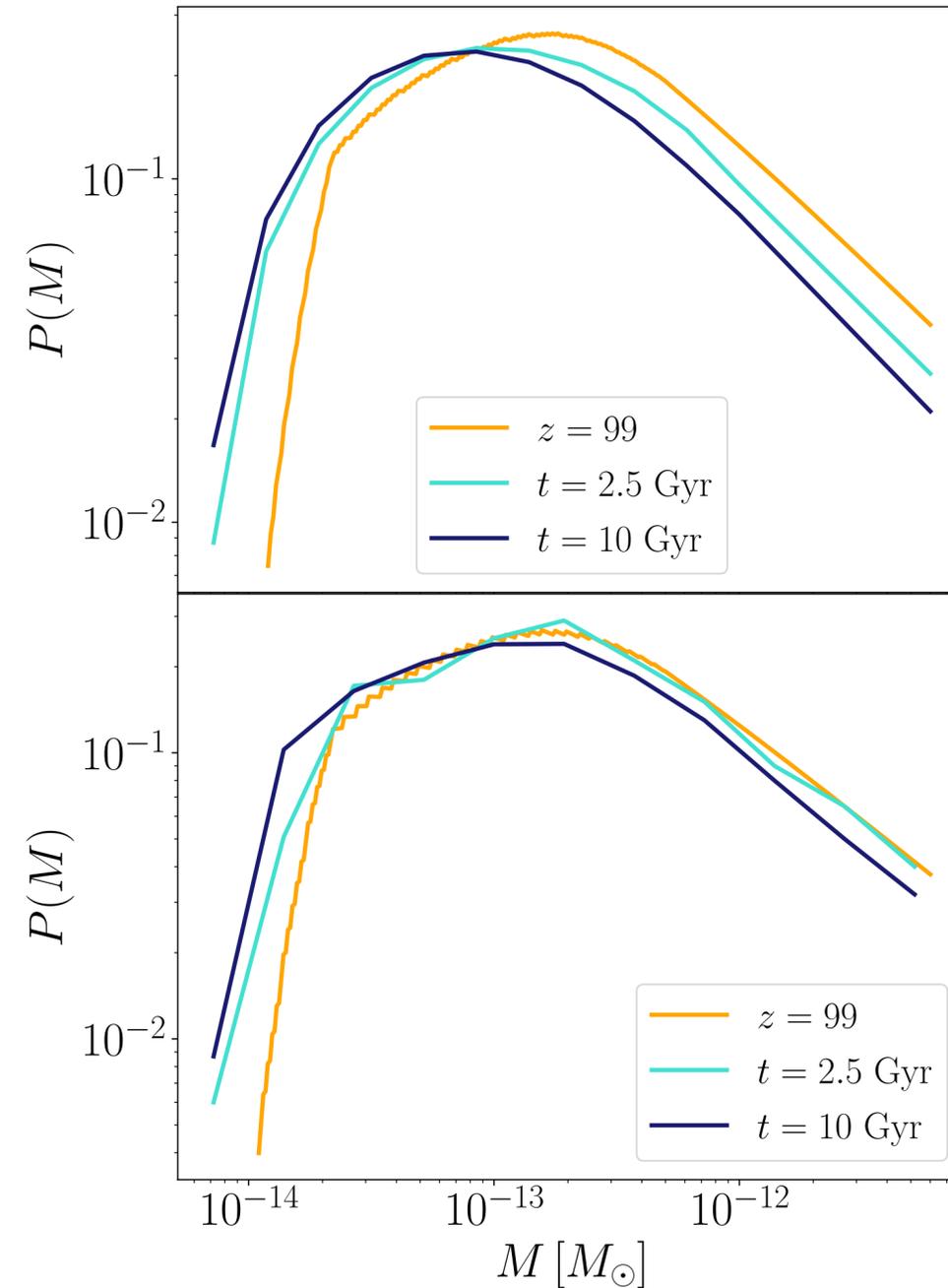
## Hernquist



# Distribution of radii and masses



$P_{z=99}(R, M)$



- Kavanagh et al., PRD 104, 063038 (2021)
- Eggemeier et al., PRL 125, 041301 (2020)
- Fairbairn et al., PRL 119, 021101 (2017)
- Buschmann et al., PRL 124, 161103 (2020)

# Results

	$P_{\text{surv}}$		
	$t = 2.5 \text{ Gyr}$	$t = 5 \text{ Gyr}$	$t = 10 \text{ Gyr}$
Lane-Emden	0.94	0.90	0.82
Hernquist	0.99	0.98	0.94

**Kavanagh et al., Phys. Rev. D 104  
(Particle phase-space density)**

Power law profile

$$P_{\text{surv}} = 0.99$$

NFW profile

$$P_{\text{surv}} = 0.46$$

Run for  $t = 13.5 \text{ Gyr}$

**Shen et al., 2207.11276 (N-Body)**

NFW profile

$$P_{\text{surv}} = 0.87$$

Different initial distribution of miniclusters

No relaxation between encounters

# Conclusions

- Construction of self-consistent clump of axion waves in the WKB limit (see also Yavetz et al., PRD 105 (2022) 2, 023512)
- Calculation of survival against tidal stripping from encounters with stars
- Limitations and future work
  - Modelling of relaxation after encounter
  - Effect of axion star
  - Evolution of the Milky Way
  - Initial distribution of minicluster masses and radii

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