Mixed QCD-EW Corrections To Drell-Yan Processes

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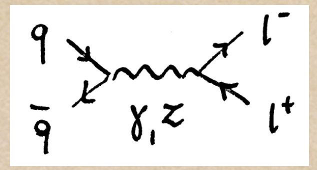
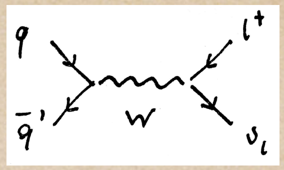
Plan of the talk

- Brief Introduction: why Drell-Yan?
- Theoretical Predictions in the literature
- The calculation of Mixed QCDxEW corrections
- Pheno Relevance of the mixed corrections

Introduction





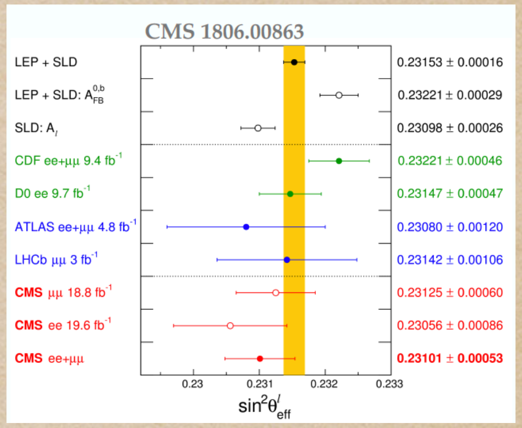
- NC and CC Drell-Yan processes, $pp(\bar{p}) \to l^- l^+$ and $pp(\bar{p}) \to l\nu$ are of fundamental importance for an accurate check of the SM at hadron colliders. Sizable cross sect and high sensitivity to the properties of the gauge bosons!

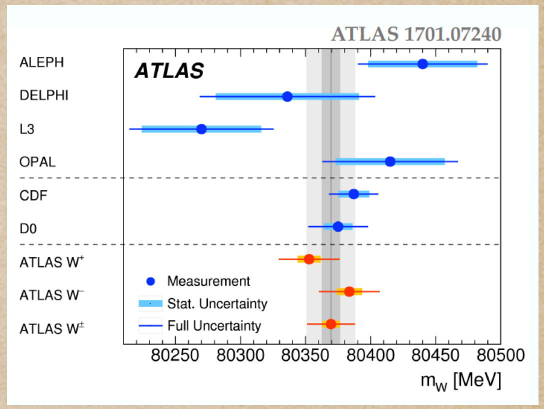
- NC DY is important for the detector calibration, accurate determination of input SM parameters, as the measurement of $\sin^2\theta_{eff}^{lep}$ that starts to compete with LEP (permille level at Tevatron and LHC)

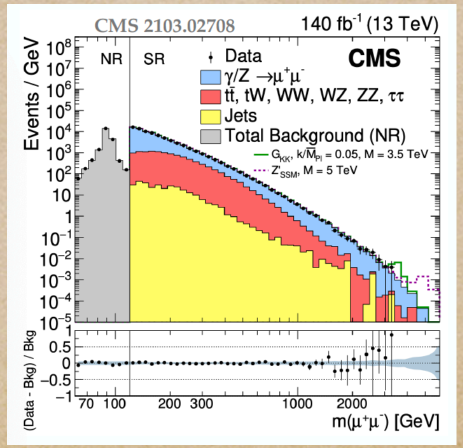
- CC DY production is important for the determination of the W mass (via transverse mass and lepton transverse momentum distributions using electron-neutrino and muon-neutrino final states), that was measured at Tevatron with accuracy of 16 MeV (arXiv:1204.0042) and at LHC with similar accuracy. Global fit: $\delta m_W \sim 8$ MeV. Requires accurate theoretical control on the distributions

- NC and CC Drell-Yan processes constitute the ``SM background'' for processes of new physics as Z' or W' production. Tail of kinematic distributions (high transverse momentum, high invariant mass distributions).

- Measurement of the di-lepton invariant mass distribution expected at O(1%) at $m_{l^+l^-} \sim 1$ TeV requires analogous control of SM predictions



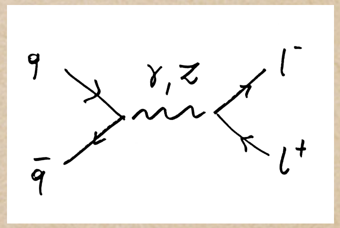




mass window [GeV]	stat. unc. $140 fb^{-1}$	stat. unc. $3 ab^{-1}$
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

Drell-Yan TH predictions in the Literature

- DY was one of the first hadronic processes for which perturbative corrections were calculated



- Dominant Perturbative Contributions come from QCD:
  - ✦ Total cross section at NLO and NNLO

    G. Altarelli, R. K. Ellis, G. Martinelli, Nucl.Phys.B 157 (1979) 461

    R. Hamberg, W. van Neerven and T. Matsuura, Nucl.Phys.B 359 (1991) 343
  - ✦ Differential cross section at NNLO

    C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. Lett. 91 (2003) 182002

    C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. D 69 (2004) 094008

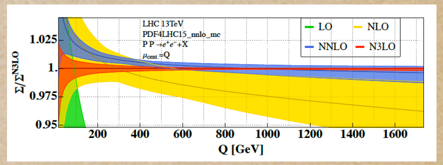
    K. Melnikov and F. Petriello, Phys. Rev. D 74 (2006) 114017

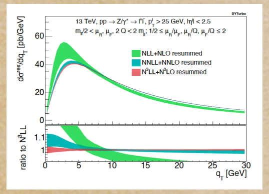
    S. Catani, L. Cieri, G, Ferrera, D. De Florian and M. Grazzini, Phys. Rev. Lett. 103 (2009) 082001
  - ✦ Total cross section at NNNLO

    C. Duhr, F. Dulat and B. Mistlberger, Phys. Rev. Lett. 125 (2020) 172001

    C. Duhr, F. Dulat and B. Mistlberger, JHEP 11 (2020) 143

    C. Duhr, B. Mistlberger, JHEP 03 (2022) 116





  - ✦ Lept-pair less inclusive at NNNLO, fiducial cross section and rapidity distribution

    S. Camarda, L. Cieri and G. Ferrera, Phys.Rev.D 104 (2021) 11, L111503

    X. Chen, T. Gehrmann, N. Glover, A. Huss, T. Yang and H. X. Zhu, Phys.Rev.Lett. 128 (2022) 5, 052001
  - ✦ Resummation …. 1987 …. up to NNNLL

    S. Moch, A. Vogt, 2005; V. Ravindran, 2006; S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini, 2014; …

Drell-Yan TH predictions in the Literature

- DY was one of the first hadronic processes for which perturbative corrections were calculated

- NLO EW Contributions

$$\alpha_S^2 \sim \alpha$$

  - CC Drell-Yan

    S. Dittmaier and M. Kraemer, Phys. Rev. D 65 (2002) 073007
    U. Baur and D. Wackeroth, Phys. Rev. D 70 (2004) 073015
    V. Zykunov, Phys. Atom. Nucl. 69 (2006) 1522
    A. Arbuzov at al., Eur. Phys. J. C 46 (2006) 407
    C. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 12 (2006) 016

  - NC Drell-Yan

    U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, Phys. Rev. D 65 (2002) 033007
    V. Zykunov, Phys. Rev. D 75 (2007) 073019
    C. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 10 (2007) 109
    Arbuzov at al., Eur. Phys. J. C 54 (2008) 451
    S. Dittmaier and M. Huber, JHEP 01 (2009) 060

- Differential NLO corrections both QCD and EW + PS

  MCFM, HORACE, POWHEG, MADGRAPH, MC@NLO

Drell-Yan TH predictions in the Literature

$$\alpha_S^3 \sim \alpha_S \alpha$$

- Mixed QCD-EW NNLO Corrections
  - On-Shell Z/W production
    - Mixed QCD-QED corrections to the inclusive prod of an on-shell Z
      D. de Florian, M. Der and I. Fabre, Phys. Rev. D 98 (2018) 094008
    - Fully differential mixed QCD-QED corrections to the prod of an on-shell Z
      M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, JHEP 01 (2019) 043
    - Mixed QCD-EW corrections to the inclusive prod of an on-shell Z (analytic)
      R.B., F. Buccioni, N. Rana and A. Vicini, Phys. Rev. Lett. 125 (2020) 232004
    - Fully differential mixed QCD-EW corr. inclusive prod of on-shell Z and W
      F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, Phys. Lett. B 811 (2020) 135969
      A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov and R. Roentsch, PRD 103 (2021) 013008
  - Beyond On-Shell production
    - Dominant QCD-EW corrections in resonant region, neutral and charged DY
      S. Dittmaier, A. Huss and C. Schwinn, Nucl. Phys. B 885 (2014) 318
      S. Dittmaier, A. Huss and C. Schwinn, Nucl. Phys. B 904 (2016) 216
    - Mixed QCD-QED corrections to neutrino pair production
      L. Cieri, D. de Florian, M. Der and J. Mazzitelli, JHEP 09 (2020) 155
    - Mixed QCD-EW corrections to (helicity) amplitudes for lepton pair prod
      M. Heller, A. Von Manteuffel, R. M. Schabinger and H. Spiesberger, JHEP 05 (2021) 216
      T. Armadillo, R.B., S. Devoto, N. Rana, A. Vicini, JHEP 05 (2022) 072
  - Mixed QCD-EW corrections to $h_1 + h_2 \to l\bar{\nu} + X$ and $h_1 + h_2 \to l^+l^- + X$
    L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini and F. Tramontano, Phys.Rev.D 103 (2021) 114012
    R.B., L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini, Phys.Rev.Lett. 128 (2022) 1, 012002
    F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M. Heller, A. Von Manteuffel, K. Melnikov, et al. , 2203.11237

Theoretical framework: Perturbative QCD

At LHC hadronic collisions $h_1 + h_2 \to l^+ l^- + X$

we rely on Factorization Theorem

PDFs: Universal Part
Evolution with Fact scale predicted by the theory

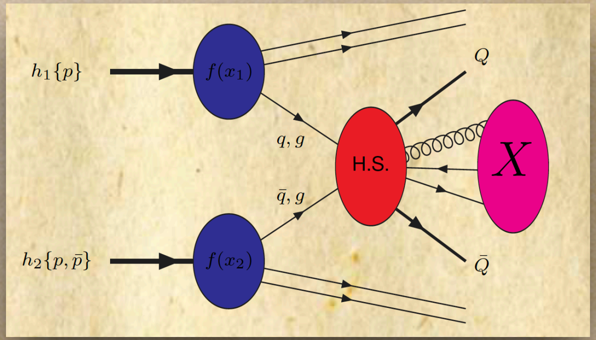
Partonic CS: Process-dep Part
Calculation in PT Theory

$$\sigma_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \, f_{i,h_1}(x_1,\mu_F) \, f_{j,h_2}(x_2,\mu_F) \, \hat{\sigma}_{ij}(\hat{s}, m^2, \alpha_S(\mu_R), \mu_F, \mu_R)$$

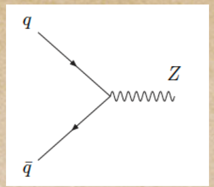
NNLO
S. Moch, J. Vermaseren, A. Vogt, Nucl. Phys. B688 (2004) 101
A. Vogt, S. Moch, J. Vermaseren, Nucl. Phys. B691 (2004) 129

NNNLO
S. Moch, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, Phys. Lett. B782 (2018) 627
J. Davis, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, Nucl. Phys. B915 (2017) 335
S. Moch, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt, JHEP 10 (2017) 041

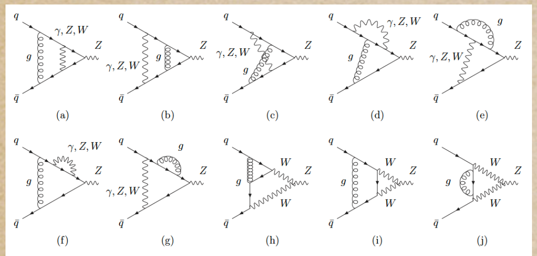
NNLO mixed
D. de Florian, G. F. R. Sborlini and G. Rodrigo, Eur. Phys. J. C 76 (2016) 282



Resonant contribution: Z on-shell



- Although this is not a realistic setup, it is interesting for many reasons:
  - It constitutes a benchmark calculation important for the total cross section normalisation
  - It allows a detailed understanding of the dependence on different input parameter schemes
  - It allows a detailed study of the impact of the choice of different PDFs sets
- The calculation was carried out integrating the contribution of the real radiation all over the phase space, through Reverse Unitarity

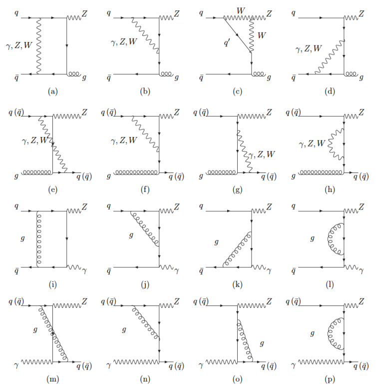


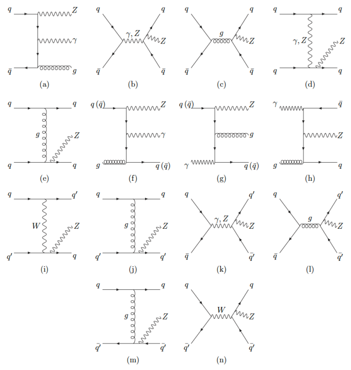
← VV

C. Anastasiou, K. Melnikov, Nucl. Phys. B 646 (2002) 220

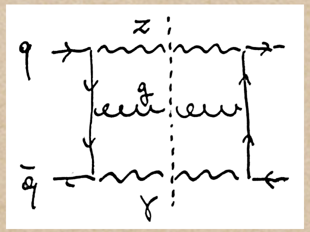
$$\delta(p^2 - m^2) \to \frac{1}{2\pi i}\left(\frac{1}{p^2 - m^2 + i0^+} - \frac{1}{p^2 - m^2 - i0^+}\right)$$

RV





RR



- The calculation proceeds via reduction to MIs and differential equations for their evaluation

R.B., F. Buccioni, N. Rana and A. Vicini, Phys. Rev. Lett. 125 (2020) 232004

R.B., F. Buccioni, N. Rana and A. Vicini, JHEP 02 (2022) 095

Resonant contribution: Z on-shell

We studied the impact of the corrections for Tevatron and LHC at different c. m. energies, with different PDFs sets and different input parameter schemes

- Absolute Size of the corrections at LHC (13 TeV): comparing the complete NNLO with the only-QCD NNLO we see a decrease of -0.57%

- Stabilisation of the result w.r.t. the scales variation

  (7-point variation of Ren and Fact scales around mZ)

	G_μ-scheme	$\alpha(0)$-scheme
B_3	$55469^{+0.65\%}_{-1.01\%}$	$55340^{+0.68\%}_{-1.13\%}$

- Different input param schemes: stabilisation of the scheme dependence thanks to mixed QCDxEW corrections

$$B_1 = \sigma_{LO} + \sigma_{10} + \sigma_{20}\,,$$
$$B_2 = \sigma_{LO} + \sigma_{10} + \sigma_{01} + \sigma_{20}\,,$$
$$B_3 = \sigma_{LO} + \sigma_{10} + \sigma_{01} + \sigma_{11} + \sigma_{20}$$

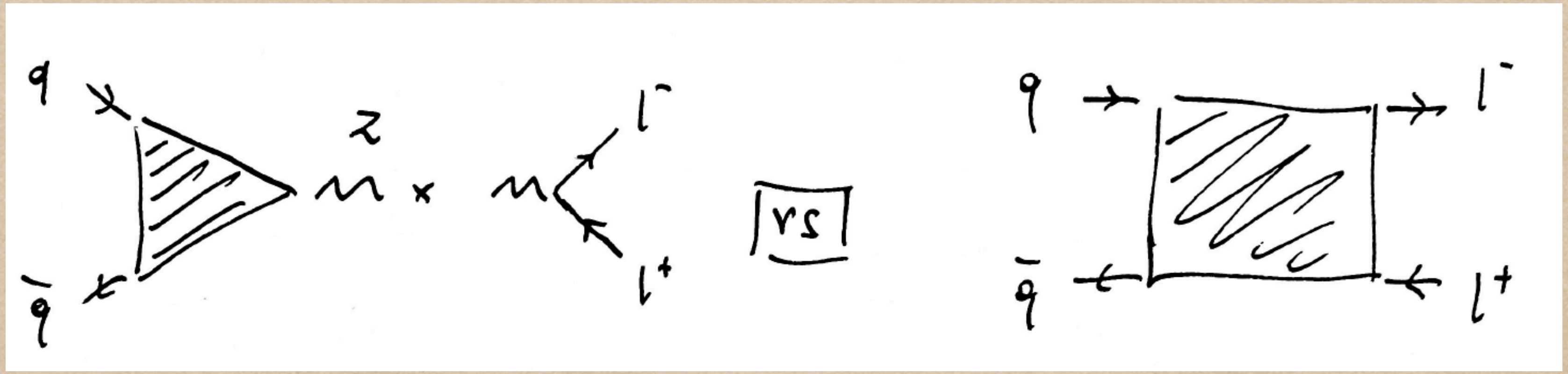
order	G_μ	$\alpha(0)$	$\delta_{G_\mu-\alpha(0)}$ (%)
A_1	55787	53884	3.53
B_1	55651	53753	3.53
B_2	55501	55015	0.88
B_3^γ	55516	55029	0.88
B_3	55469	55340	0.23

- We convoluted the partonic cross section with different PFD sets NNPDF31, MMHT2015, CT18: we register a spread that ranges between 1% and 2% dep on the energy. Compatible with PDF errors

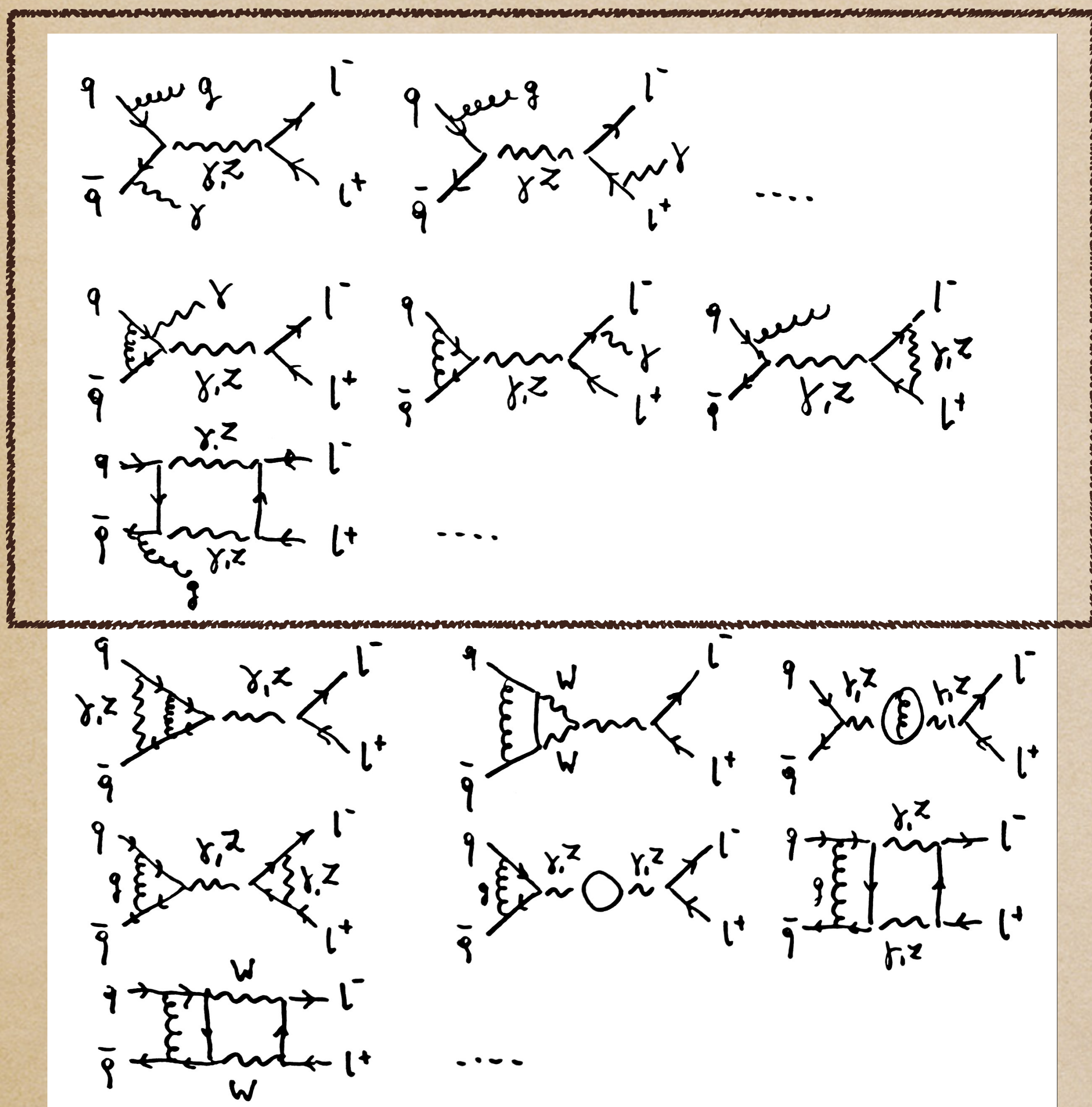
R.B., F. Buccioni, N. Rana and A. Vicini, Phys. Rev. Lett. 125 (2020) 232004

R.B., F. Buccioni, N. Rana and A. Vicini, JHEP 02 (2022) 095

Let us now move to a more realistic description that involves non factorable corrections



EXACT CALCULATION: Partonic Cross Section



2 -> 4 RR

2 -> 3 RV

2 -> 2 VV

Computed with RECOLA and OPENLOOPS

- The three sets are separately divergent: for a differential cross section we need IR counterterms in a subtraction scheme: we used Qt subtr scheme

S. Catani and M. Grazzini, Phys. Rev. Lett. *98* (2007) 222002

The qT subtraction formalism was recently generalised to include emission from a massive final state, at first for top-antitop prod. and then extended to the production of a lepton-neutrino pair.

S. Catani, S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli and H. Sargsyan, Phys. Rev. D 99 (2019) 051501
L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini and F. Tramontano, Phys.Rev.D 103 (2021) 114012

need no collinear div in the final-state —> massive leptons

- However: Collinear divergences cancel in the amplitude when we add s-t and s-u box contributions (we explicitly checked the cancellation of the highest pole). Remaining collinear divergences come from factorized one-loop diagrams —> easy to compute
- Therefore, we can set the leptonic mass to zero in the most complicated diagrams (the ones with many scales). Basically we set up the following computation:
  - QCDxQED corrections ⟶ massive leptons
  - One/Two boson exchange ⟶ massless leptons

Two-Loop Amplitude

The UV-renormalised amplitude has the following IR structure

$$
\begin{aligned}
|\mathcal{M}^{(1,1)}\rangle =& \frac{1}{\epsilon^4}\left\{\frac{1}{4}e_q^2 C_F\,|\mathcal{M}^{(0)}\rangle\right\} \\
&+\frac{1}{\epsilon^3}\left\{\frac{1}{2}C_F\left[\left(\frac{3}{2}+i\pi\right)e_q^2-\Gamma_t^{(0,1)}\right]|\mathcal{M}^{(0)}\rangle\right\} \\
&-\frac{1}{2\epsilon^2}\left\{e_q^2|\mathcal{M}_{fin}^{(1,0)}\rangle+C_F|\mathcal{M}_{fin}^{(0,1)}\rangle+C_F\left[\left(\frac{7}{12}\pi^2-\frac{9}{8}-\frac{3}{2}i\pi\right)e_q^2+\left(\frac{3}{2}+i\pi\right)\Gamma_t^{(0,1)}\right]|\mathcal{M}^{(0)}\rangle\right\} \\
&-\frac{1}{2\epsilon}\left\{\left[\left(\frac{3}{2}+i\pi\right)e_q^2-2\Gamma_t^{(0,1)}\right]|\mathcal{M}_{fin}^{(1,0)}\rangle+C_F\left(\frac{3}{2}+i\pi\right)|\mathcal{M}_{fin}^{(0,1)}\rangle\right. \\
&\left.+\frac{C_F}{8}\left[\left(\frac{3}{2}-\pi^2+24\zeta_3+\frac{2}{3}i\pi^3\right)e_q^2-\frac{3}{2}\pi^2\Gamma_t^{(0,1)}\right]|\mathcal{M}^{(0)}\rangle\right\} \\
&+|\mathcal{M}_{fin}^{(1,1)}\rangle
\end{aligned}
$$

$$
\Gamma_t^{(0,1)} = -\frac{1}{4}\left\{(e_{l^-}^2+e_{l^+}^2)(1-i\pi)+\sum_{i=1,2;\,j=3,4}e_ie_j\,\ln\left(\frac{(2p_i\cdot p_j)^2}{Q^2m_l^2}\right)+2e_{l^-}e_{l^+}\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right)-i\pi\left(\frac{1}{v}+1\right)\right]\right\}
$$

$$
v=\sqrt{1-\left(\frac{2m_l^2}{Q^2-2m_l^2}\right)^2}
$$

S.Catani, M. Grazzini and A. Torre, Nucl. Phys. B890 (2014) 518

L. Buonocore, M. Grazzini and F. Tramontano, Eur. Phys. J. C 80 (2020) 254

Computation of the Amplitude

... goes through the "usual" steps:

- Generation of the Feynman diagrams: QGRAF and FeynArts

  P. Nogueira, J. Comput. Phys. 105 (1993) 279

  T. Hahn, Comput.Phys.Commun. 140 (2001) 418-431

- We computed directly the Interference with the tree-level: FORM

  B. Ruijl, T. Ueda and J. A. M. Vermaseren, 1707.06453

- We used an anti commuting γ_5

  D. Kreimer, Phys. Lett. B 237 (1990) 59

- We renormalise in the G_μ scheme, keeping complex boson masses and accordingly complex s_W

  S.Dittmaier, A. Huss and C. Schwinn, Nucl. Phys. B 885 (2014) 318

- The Dim-Regularized scalar integrals were reduced to the MIs using IBP identities implemented in KIRA, Reduze2, LiteRed

  P. Maierhoefer, J. Usovitsch and P. Uwer, Comp. Phys. Commun. 230 (2018) 99

  J. Klappert, F, Lange, P. Maierhoefer and J. Usovitsch, Comput.Phys.Commun. 266 (2021) 108024

  A. Von Manteuffel and C. Studerus, 1201.4330

  R. N. Lee, 1212.2685

Treatment of γ_5 in Dim Regularisation

- γ_5 is a genuine 4-dim object. In 4 dimensions we have

  ✔ $\{\gamma_\mu, \gamma_5\} = 0$ ✔ $tr(\gamma_5 \gamma_{\mu_1} ... \gamma_{\mu_4}) = 4i\epsilon_{\mu_1 ... \mu_4}$ ✔ cyclicity

- When we move to $D \neq 4$ dimensions we cannot preserve anticommutation and cyclicity at the same time. We have two choices

  - We get rid of anticommutativity but we keep cyclicity ✘ $\{\gamma_\mu, \gamma_5\} = 0$ ✔ cyclicity

    Breaks gauge invariance that has to be restored with appropriate counterterms (finite renormalization)

    G. 't Hooft, M. Veltman, Nucl. Phys. B 44 (1972) 189

    P. Breitenlohner, D. Maison, Commun. Math. Phys. 52 (1977)11, 39, 55

  - We get rid of cyclicity but we keep anticommutativity ✔ $\{\gamma_\mu, \gamma_5\} = 0$ ✘ cyclicity

    Ward identities are automatically preserved but γ_5-odd traces have a reading point

    D. Kreimer, Phys. Lett. B 237 (1990) 59

- In the case of NC Drell-Yan it was proven that the two prescriptions give different amplitudes but equal IR-subtracted finite corrections!

  M. Heller, A. Von Manteuffel, R. M. Schabinger and H. Spiesberger, JHEP 05 (2021) 216

Treatment of γ_5 in Dim Regularisation

- However, In the case of NC Drell-Yan there are no anomalous diagrams i.e. any choice is basically ok
- We choose a naive anti commuting γ_5
  - We move all the γ_5 at the end of the Dirac trace using anticommutativity
  - We use $\gamma_5^2 = 1$ and we get either traces with one γ_5 or without γ_5
  - We replace the last γ_5 in the trace with $\gamma_5 = \frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$
- The contraction of two epsilon-tensors (one from the leptonic current and the other from the quark current) is done using

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4} = -\begin{vmatrix} \delta^{\nu_1}_{\mu_1} & \delta^{\nu_2}_{\mu_1} & \delta^{\nu_3}_{\mu_1} & \delta^{\nu_4}_{\mu_1} \\ \delta^{\nu_1}_{\mu_2} & \delta^{\nu_2}_{\mu_2} & \delta^{\nu_3}_{\mu_2} & \delta^{\nu_4}_{\mu_2} \\ \delta^{\nu_1}_{\mu_3} & \delta^{\nu_2}_{\mu_3} & \delta^{\nu_3}_{\mu_3} & \delta^{\nu_4}_{\mu_3} \\ \delta^{\nu_1}_{\mu_4} & \delta^{\nu_2}_{\mu_4} & \delta^{\nu_3}_{\mu_4} & \delta^{\nu_4}_{\mu_4} \end{vmatrix}$$

- As a non trivial check of the consistency of the treatment we observe the cancellation of the IR poles

UV renormalization

Three parameters: (G_μ, μ_W, μ_Z) or (α, μ_W, μ_Z)

- Feynman diagrams are computed in the Background Field Gauge that ensures the validity of Ward identities for the initial state vertex

  → UV-Finiteness of two-loop vertices with quark WF renormalisation

- We use the Complex Mass Scheme: mass counter-terms are defined at the complex pole of the propagator

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2\,,\ \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2\,,\ e_0 = e + \delta e \qquad s_{W0}^2 = 1 - \frac{\mu_{W0}^2}{\mu_{Z0}^2} \qquad \frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

- From the muon decay amplitude we derive the relation $\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2\mu_W^2 s_W^2}(1+\Delta r)$

- In the (G_μ, μ_W, μ_Z) scheme the couplings of Z and γ to fermions are

$$\frac{g_0}{c_{W0}} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}(1+\delta g_Z^{G_\mu}) \qquad g_0 s_{W0} = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s_W^2}(1+\delta g_A^{G_\mu})$$

- Finally, the renormalisation of the gauge boson propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2q^2\delta g_A \qquad \Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\frac{\delta s_W^2}{s_W c_W}$$

$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2(q^2-\mu_Z^2)\delta g_Z \qquad \Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\frac{\delta s_W^2}{s_W c_W}$$

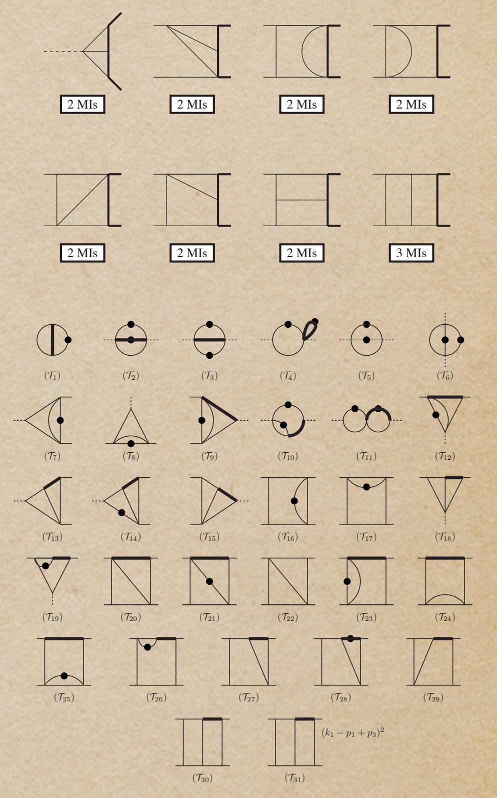
Master Integrals

- We basically divide the computation in two subsets

  - Massive final state MIs

    R.B., A. Ferroglia, T. Gehrmann, D. Maitre and C. Studerus, JHEP 07 (2008) 129
    R.B., A. Ferroglia, T. Gehrmann and C. Studerus, JHEP 08 (2009) 067
    P. Mastrolia, M. Passera, A. Primo and U. Schubert, JHEP 11 (2017) 198

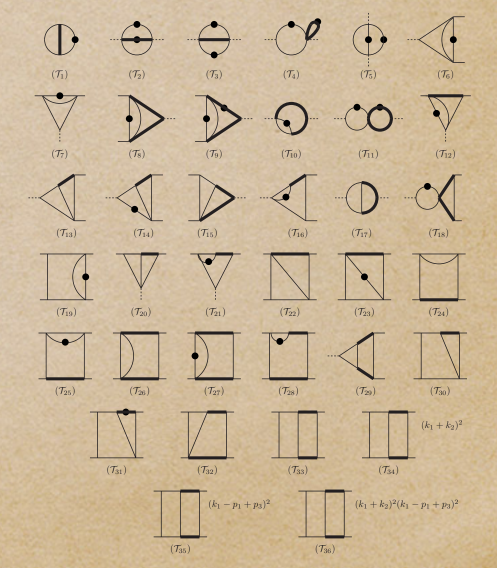
For the massive part of the calculation, all the MIs could be expressed in terms of GPLs.



  - Massless final state MIs

    U. Aglietti, R.B., Nucl.Phys.B 668 (2003) 3
    U.Aglietti, R.B., Nucl.Phys.B 698 (2004) 277
    R.B., S. Di Vita, P. Mastrolia and U. Schubert, JHEP 09 (2016) 091
    M. Heller, A. Von Manteuffel and R. M. Schabinger, Phys. Rev. D 102 (2020) 016025
    S. M. Hasan and U. Schubert, JHEP 11 (2020) 107

We rely on JHEP 09 (2016) 091 — All the MIs (with the exception of 5 two-mass BOXES) could be expressed in terms of GPLs. The 5 boxes still have a numeric int.



Differential Equations

- The MIs were computed using the Differential Equations Method

$$\frac{\partial}{\partial x_i} f(x,\epsilon) = A_{x_i}(x,\epsilon)\, f(x,\epsilon)$$

V. Kotikov, Phys. Lett. B 254 (1991) 158
Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B 412 (1994) 751
E. Remiddi, Nuovo Cim. A 110 (1997) 1435
T. Gehrmann and E. Remiddi, Nucl. Phys. B 580 (2000) 485

- Massless final state MIs were evaluated analytically using the canonical form for the system

$$df(x,\epsilon) = \epsilon\, dA(x)\, f(x,\epsilon)$$

J. M. Henn, Phys. Rev. Lett. 110(2013) 251601
M. Argeri, S. Di Vita, P. Mastrolia, E. Mirabella, J, Schlenk, U. Schubert and L. Tancredi, JHEP 03 (2014) 082

- However, the alphabet contains squared roots that could not be linearised simultaneously. Not all of the masters in GPLs —> remaining numeric integration

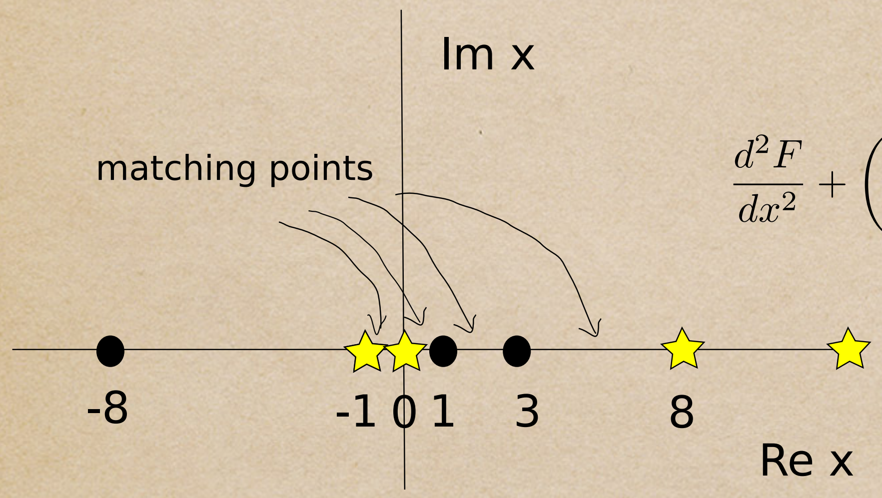
S. Caron-Huot and J. M. Henn, JHEP 06 (2014) 114

✤ The numeric evaluation of the remaining integration is not optimal, in particular for the analytic continuation …… What to do?

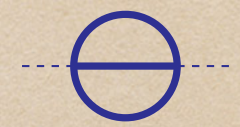
Differential Equations: Semi-analytic evaluation

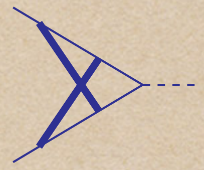
In some cases it is difficult to find closed-form solutions for the differential equations
What can be done is a solution of the relative differential equation in series expansion

S.Pozzorini and E. Remiddi, Comp. Phys. Comm. 175 (2006) 381



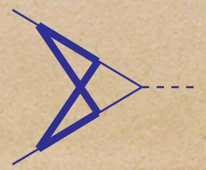
$$\frac{d^2F}{dx^2} + \left(\frac{3}{x} + \frac{1}{x+1} + \frac{1}{x-8}\right)\frac{dF}{dx} + \left[\frac{1}{x^2} + \frac{9}{8x} - \frac{4}{3(x+1)} + \frac{5}{24(x-8)}\right]F = \Omega(x)$$





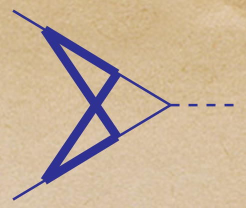
$$x = -\frac{s}{m^2}$$

- The differential equation and the solution are expanded in series around the singular points Every series depends on two arbitrary constants. Imposing the matching we express all of them in terms of the two constants
- Imposing initial conditions we fix the two constants. One can construct a numerical routine that evaluates F(x) for every value of x with arbitrary precision !!
- The convergence can be improved adding series expansions in intermediate regular points



U. Aglietti, R.B., L. Grassi and E. Remiddi, Nucl. Phys. B 789 (2008) 45
R. N. Lee, A. V. Smirnov and V. A. Smirnov, JHEP 03 (2018) 008
R.B., G. Degrassi, P. P. Giardino and R. Groeber, Comp. Phys. Comm. 241 (2019) 122

Example: elliptic vertex for ttbar production



$$\begin{cases} \dfrac{dM_9}{dx} = -\dfrac{2}{x}M_9 + \dfrac{4m^2}{x}M_{10} \\ \dfrac{dM_{10}}{dx} = -\dfrac{1}{16m^2}\left(\dfrac{1}{x} - \dfrac{1}{x-16}\right)M_9 - \left(\dfrac{1}{x} + \dfrac{1}{x-16}\right)M_{10} + \Omega_s(x) \end{cases} \qquad x = -\frac{s}{m^2}$$

$$\frac{d^2M_9}{dx^2} + \left(\frac{4}{x} + \frac{1}{x-16}\right)\frac{dM_9}{dx} + \left(\frac{9}{4x^2} - \frac{7}{64x} + \frac{7}{64(x-16)}\right)M_9 = \Omega(x)$$

Solution in $x = 0$

$$M_9^{(0)} = x^\alpha \sum_{n=0}^{\infty} a_n x^n \qquad \left(\alpha + \tfrac{3}{2}\right)^2 = 0$$

- Homogeneous solutions $M_9^{(0)}(x) = \frac{1}{\sqrt{x}}\sum_{n=-1}^{\infty} a_n x^n + \frac{\log x}{\sqrt{x}}\sum_{n=-1}^{\infty} b_n x^n \qquad a_0 = \frac{1}{64}a_{-1} + \frac{1}{32}b_{-1}\,, \quad b_0 = \frac{1}{64}b_{-1}$

  two independent solutions with $\begin{cases} a_{-1} = 1\,,\ b_{-1} = 0 \\ a_{-1} = 0\,,\ b_{-1} = 1 \end{cases}$

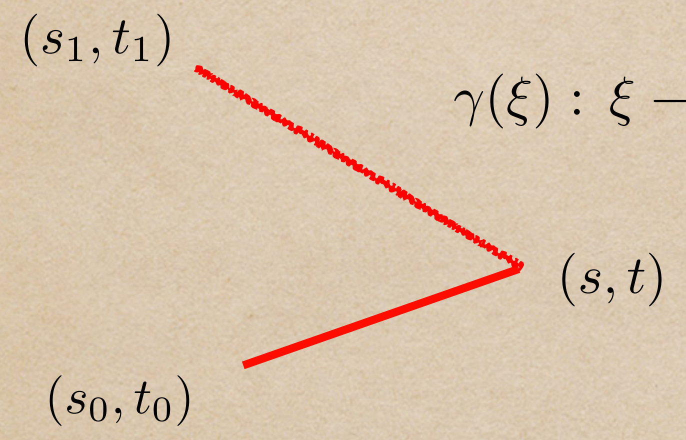
- Particular solutions $\Omega(x) = \sum_{n=-2}^{\infty} k_n x^n + \log x \sum_{n=-2}^{\infty} r_n x^n \qquad \tilde{M}_9(x) = \sum_{n=-1}^{\infty} p_n x^n + \log x \sum_{n=-1}^{\infty} q_n x^n$
- Matching with initial condition in $x = 0$ (only log) $M_9(x) = \sum_{n=0}^{\infty} p_n x^n + \log x \sum_{n=0}^{\infty} q_n x^n$
- The same in $x = 16$ + matching with the series in $x = 0$ … and so on
- Analitic continuation

L. Tancredi, A. Von Manteuffel, JHEP 06 (2017) 127

R.B., G. Degrassi, P. P. Giardino and R. Groeber, Comp. Phys. Comm. 241 (2019) 122

Differential Equations: Semi-analytic evaluation

- The examples above are one-dimensional, but the approach can be generalised to more dimesions and used for a general system of differential equations for the MIs
- The differential equations in s and t are combined and a one-dim diff eq is recovered and solved along a contour connecting two fixed points in the s-t plane



$$\gamma(\xi) : \xi \to \{s(\xi), t(\xi)\}$$

$$\frac{d}{d\xi} f(\xi, \epsilon) = A(\xi, \epsilon)\, f(\xi, \epsilon)$$

$$f(\xi, \epsilon) = \sum_{i=0}^{N} f^{(i)}(\xi)\epsilon^i$$

$$f^{(i)}(\xi) = \sum_{j=0}^{\infty} c^{(i,j)}(\xi - \xi_0)^j$$

$$f^{(i)}(\xi) = \sum_{j_1 \in S_1} \sum_{j_2=0}^{\infty} \sum_{j_3=0}^{\infty} c^{(i,j_1,j_2,j_3)}(\xi - \xi_0)^{w_{j_1}+j_2} \log^{j_3}(\xi - \xi_0)$$

- Analytical continuation is done expanding in the singular point and matching the series using Feynman prescription for the invariants
- The method is quite efficient and enables to compute fast a point in the phase space with arbitrary precision

F. Moriello, JHEP 01 (2020) 150

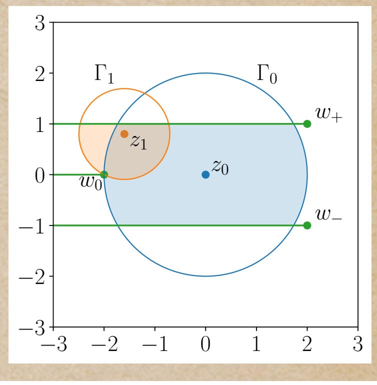
- Recently this method was implemented in a Mathematica code: DiffExp

M. Hidding, Comput.Phys.Commun. 269 (2021) 108125

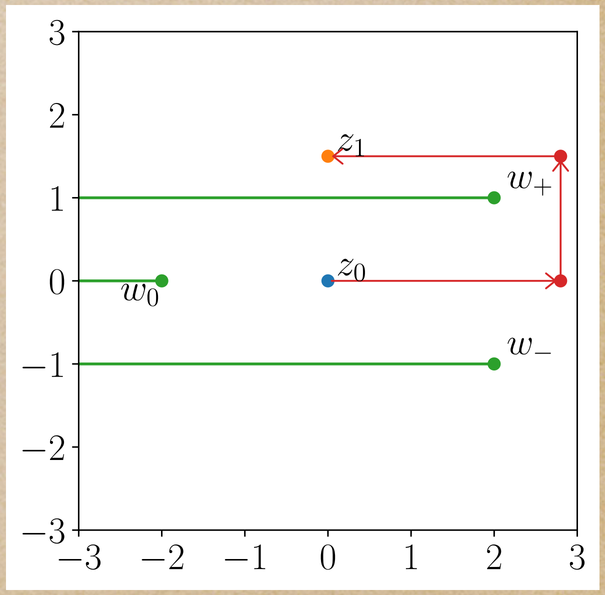
SeaSyde (Series Expansion Approach for Systems of Differential Equations)

- However DiffExp is designed for real masses. EW radiative corrections involve unstable particles Z, W, H, … : Complex Masses
- We developed an independent package, SeaSyde, to deal with series expansions solutions in the complex plane

$$z = -\frac{s}{m_V^2} \quad q = -\frac{t}{m_V^2} \qquad m_V^2 = M_V^2 - iM_V\Gamma_V$$



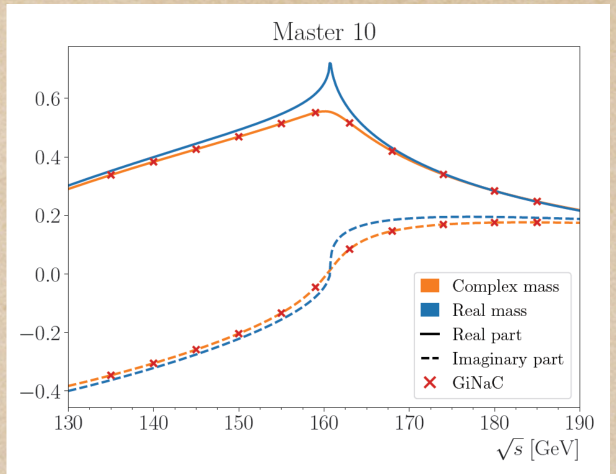
- Now we have cuts in the complex plane: we choose them to be parallel to the real axis, from the branching point to $-\infty$
- The path to avoid the cut proceeds via segments parallel to the real and to the imaginary axis in every complex variable, z and q
- We solve the equation in z, at fixed q (cuts in the complex z-plane) then the eq. in q at fixed z

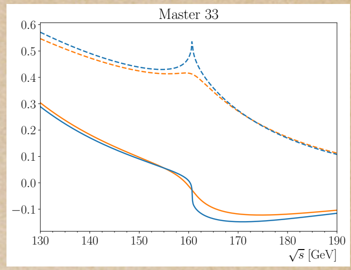


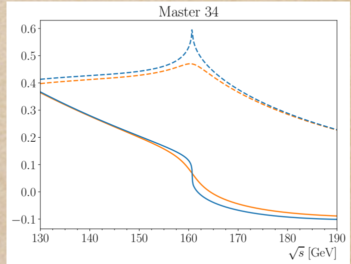
T. Armadillo, R.B., S. Devoto, N. Rana, A. Vicini, 2205.03345

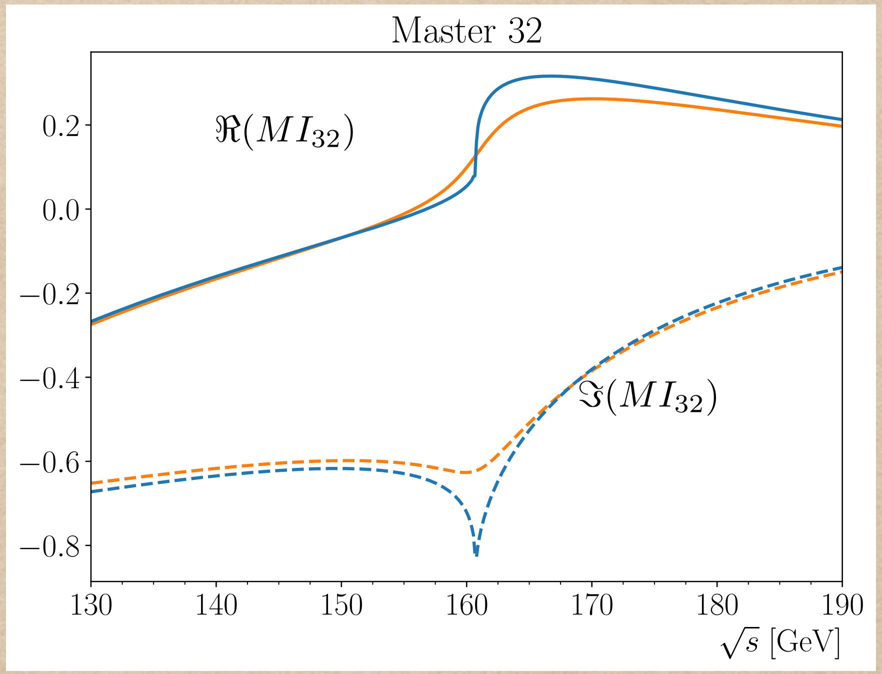
Effect of the complex mass

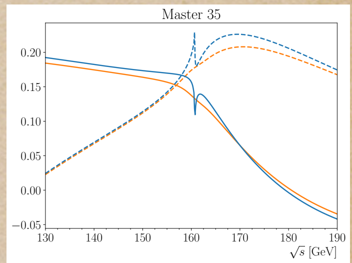
- The complex mass smoothens the behaviour near threshold
- The MIs can be computed with an arbitrary number of significant digits
- A point in the phase space is evaluated in O(10min)
- Starting from the initial conditions $s = t = -M_V^2$ every other point becomes the starting point for a subsequent step in the series expansion

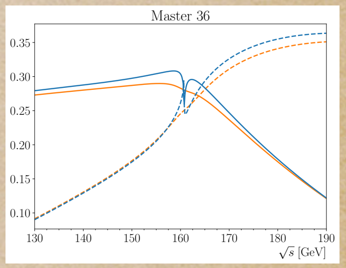












$\cos\theta = 0$

- ALL the results were checked against pySecDec, FIESTA (or numeric evaluation of analytical expressions done with GiNaC)

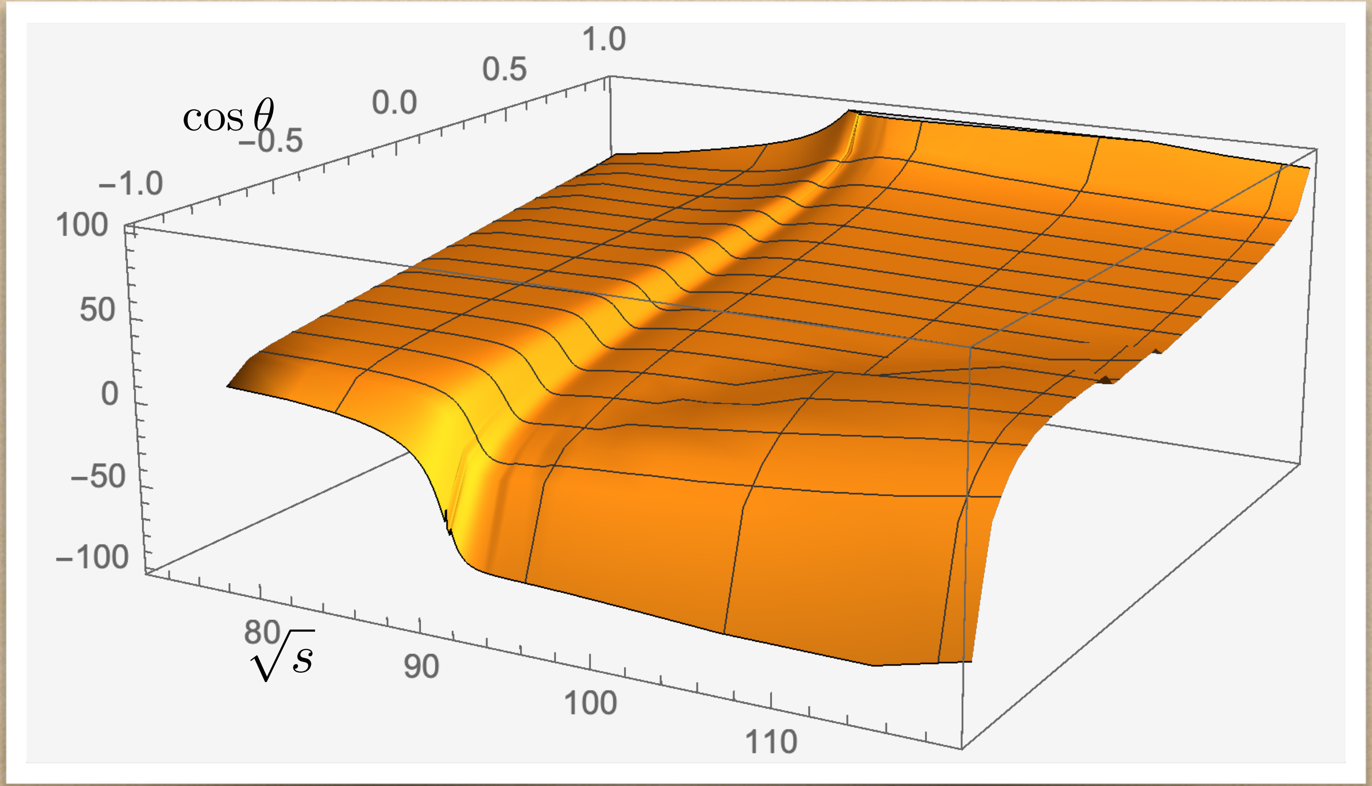
S. Borowka, G. Heinrich, S. Jahn, S. P. Jones, M. Kerner, J. Schlenk and T. Zirke, Comp. Phys. Commun. 222 (2018) 313

A. V. Smirnov, Comp. Phys. Commun. 204 (2016) 189

J. Vollinga and S. Weinzierl, Comput.Phys.Commun. 167 (2005) 177

The Computation of the IR subtracted amplitude

- We used a grid in $\sqrt{s}$ and $\cos\theta$ of 3250 points from 50 GeV to 3 TeV with smaller intervals around the Z (W) peak
- The entire grid is evaluated in O(3h) on a 32 cores machine
- The grid is interpolated with cubic splines to use it in the MC integrator
- The mass of the final leptons is kept wherever needed. The log of the lepton mass is subtracted from the grid and added back analytically



$$\frac{2\Re\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1,1)}_{fin}\rangle}{\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle}$$

In units $\frac{\alpha_S}{2\pi}\frac{\alpha}{2\pi}$

$$t = -\frac{s}{2}(1-\cos\theta)$$

Fiducial Cross Section: muon pair production

Setup: LHC at 14 TeV

PDFs: NNPDF31_nnlo_as_0118_luxqed

cuts: $p_{T,\mu^\pm} > 25\,\mathrm{GeV}\,, \quad |y_{\mu^\pm}| < 2.5\,, \quad m_{\mu\mu} > 50\,\mathrm{GeV}$

Massive muons (no photon lepton recombination)

Fixed scales: $\mu_R = \mu_F = m_Z$

σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)
$\sigma^{(m,n)}/\sigma_{\mathrm{LO}}$		+4.2 %	−4.3 %	~ 0 %	+0.5 %

- Large cancellation between qq and qg channels at NLO and NNLO in QCD
- Accidental cancellation between NLO QCD and NLO EW
  Importance of Mixed corrections
- Mixed QCD-EW corrections are bigger than NNLO QCD. Dominated by the qg channel

Differential distributions

Exact results are compared with

- Finite part of the 2-loop in Pole Approximation
- Factorized Approximation for QCD and EW

Setup: LHC at 14 TeV

PDFs: NNPDF31_nnlo_as_0118_luxqed

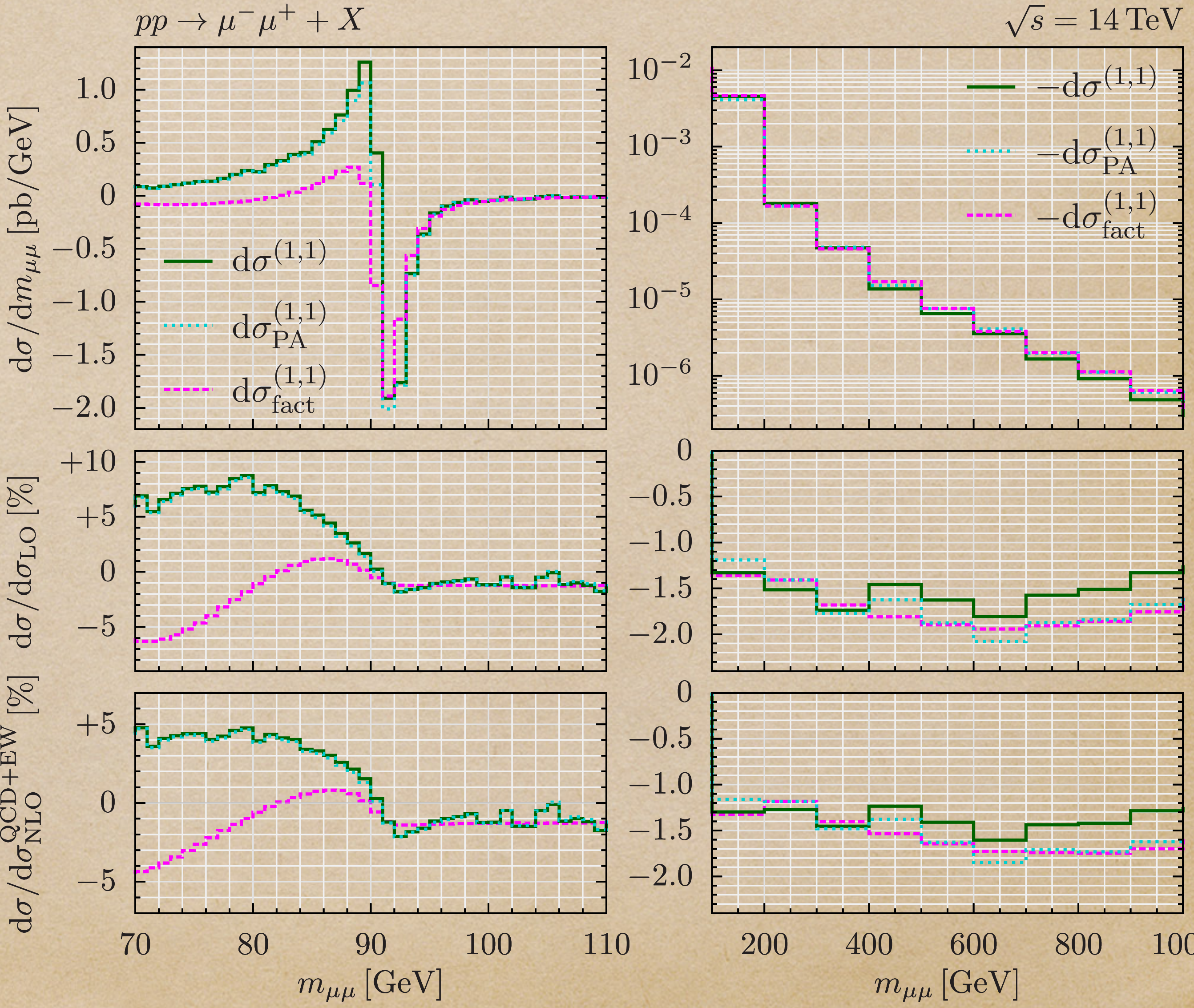
cuts: $p_{T,\mu^\pm} > 25\,\text{GeV}$, $|y_{\mu^\pm}| < 2.5$, $m_{\mu\mu} > 50\,\text{GeV}$

Massive muons (no photon lepton recombination)

Fixed scales: $\mu_R = \mu_F = m_Z$

The inv. mass distribution

- Below the peak sizeable effect of the corrections. Breakdown of the factorized hypothesis.
- The Pole Approx. provides an excellent description in the resonance region
- In the large inv. mass region the Mixed corrections are negative O(-1.5%). Deviations at O(0.5%) w.r.t. both PA and factored hypothesis



R.B., L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini, Phys.Rev.Lett. 128 (2022) 1, 012002

Differential distributions

Setup: LHC at 14 TeV

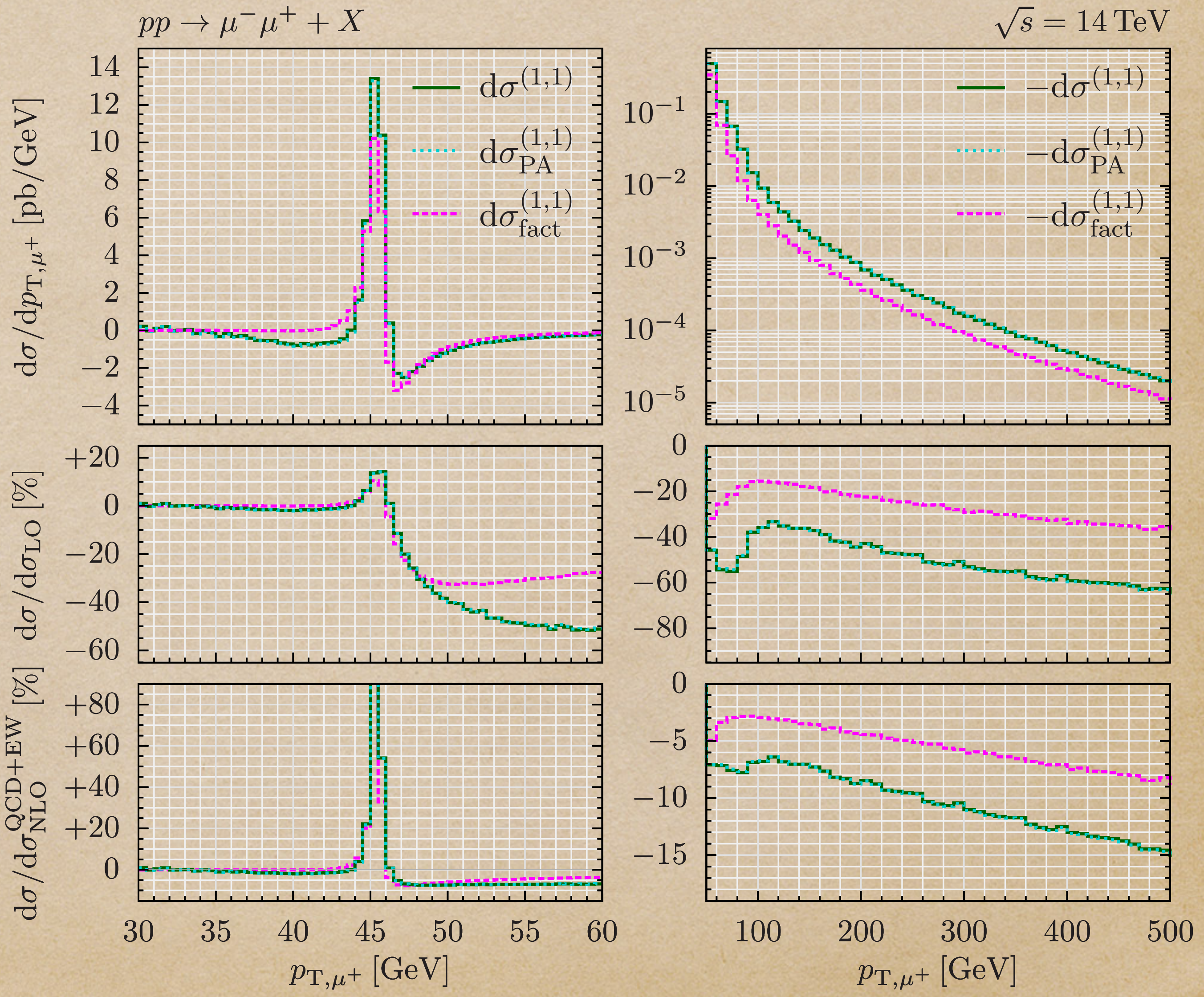
PDFs: NNPDF31_nnlo_as_0118_luxqed

cuts: $p_{T,\mu^\pm} > 25\,\mathrm{GeV}\,,\qquad |y_{\mu^\pm}| < 2.5\,,\quad m_{\mu\mu} > 50\,\mathrm{GeV}$

Massive muons (no photon lepton recombination)

Fixed scales: $\mu_R = \mu_F = m_Z$

The pt distribution



- In the peak region both PA and fact. reproduce quite well the exact result
- Beyond the peak the Mixed corrections are large and negative O(-60%) of the LO at 500 GeV
- The factored hypothesis overshoots the complete result

R.B., L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini, Phys.Rev.Lett. 128 (2022) 1, 012002

Conclusions

- We presented the calculation of the mixed QCD-EW corrections to the inclusive production of a lepton pair in proton proton collisions
- We developed SeaSyde for the calculation of the two-loop virtual amplitude using the differential equations method with series expansions. SeaSyde can deal with arbitrary complex masses
- We combined the virtual corrections with the real radiation. IR subtraction was performed in the Qt scheme. The mixed corrections are available in the MATRIX framework
- We find that the mixed corrections to the fiducial cross section amount to +0.5% with respect to the LO (the other corrections up to NNLO QCD basically vanish, amounting to -0.1% of the LO)
- For the anti-muon pt distribution we find that the Pole Approximation is in good agreement with the exact result all over the kinematic range
- For the di-muon invariant mass distribution we find that the Pole approximation is in good agreement with the exact result with small differences in the peak region . However in the high invariant mass region the PA undershoots the exact result by about 30%!
- OUTLOOKS: a more extended pheno study is ongoing; CCDY ...