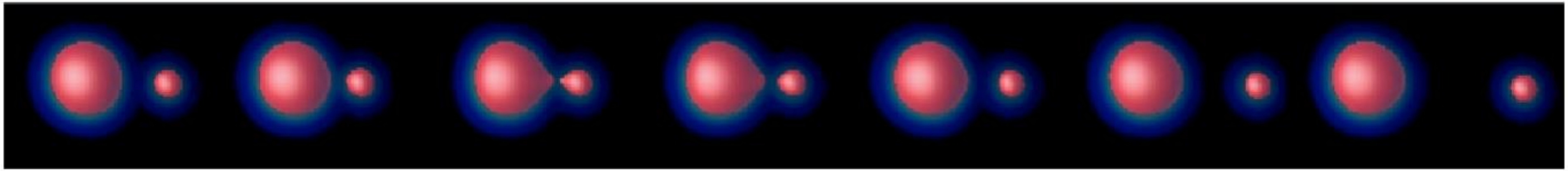
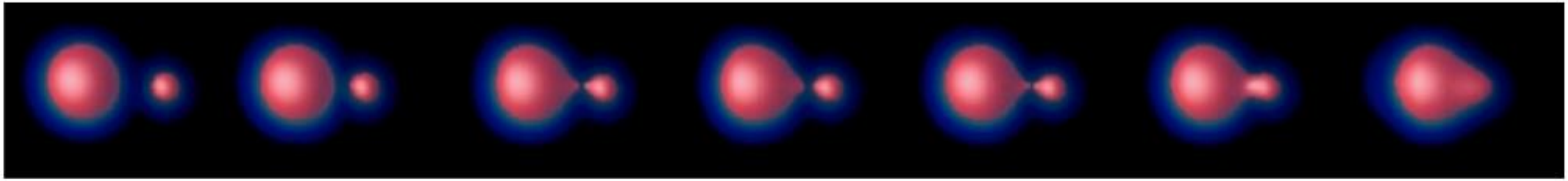


Pauli energy and dynamical contributions to nucleus-nucleus interaction

Cédric Simenel

Australian National University

Collaborators: K. Godbey (TAMU,FRIB), A. S. Umar (Vanderbilt University)



Goal

- **Fusion phenomenology**
- **Quantitative predictions (no free parameter)**
- **From deep sub-barrier to above barrier**
- **Isolate contributions to nucleus-nucleus potentials**
    - Pauli repulsion
    - Dynamics (shape polarisation, transfer…)

Outline

- **Microscopic approach to nucleus-nucleus potential**
  FHF, DCFHF, DC-TDHF
- **Application to $^{16}O+^{208}Pb$**
- **Dynamical isovector contribution to the potential**
- **Pauli energy distribution**

Microscopic approach

Hartree-Fock (HF)

$$\delta \langle \Phi | \hat{H} | \Phi \rangle = 0$$

Independent nucleons
= Slater determinant (=> Pauli)

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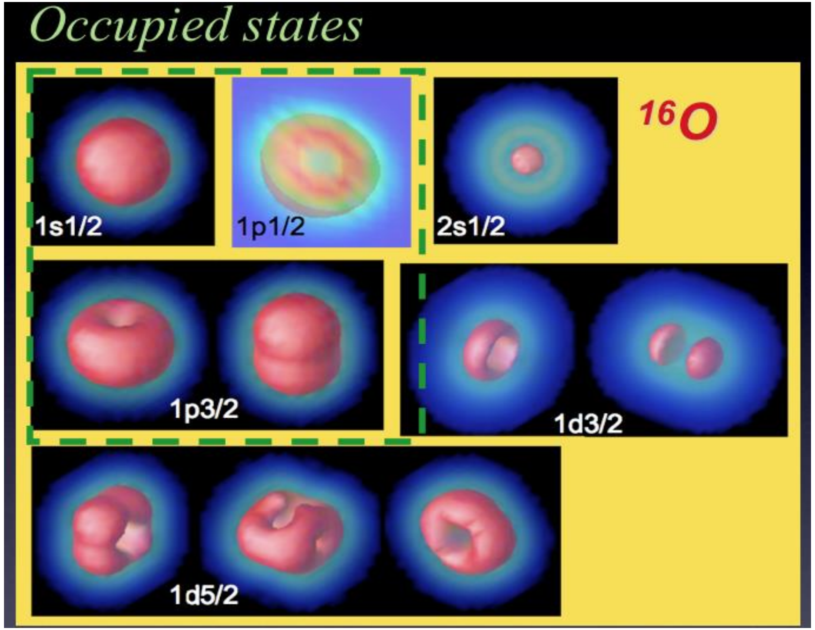

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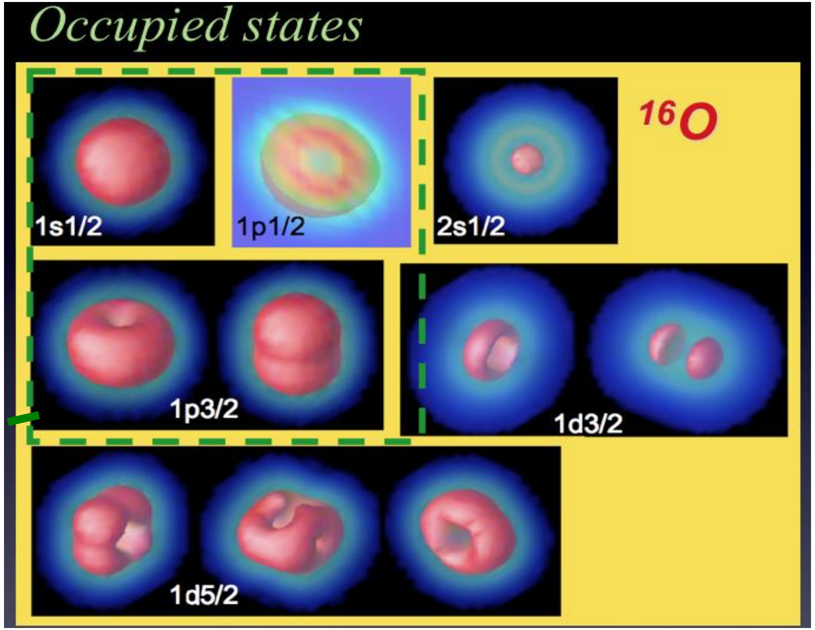
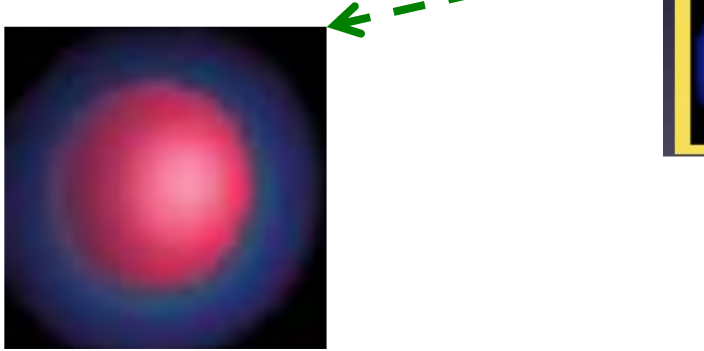




$\rho(\mathbf{r})$

$$E[\rho] = \int d^3r\, \mathcal{H}(\mathbf{r})$$

Skyrme SLy4d functional

Microscopic approach

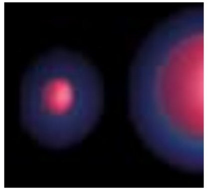
Frozen Hartree-Fock (FHF)

Brueckner *et al.*, PR **173**, 944 (1968)

$$\delta \langle \Phi | \hat{H} | \Phi \rangle = 0$$

$$V_{FHF}(R) = E[\rho_1 + \rho_2] - E[\rho_1] - E[\rho_2]$$

(No Pauli)



$\rho_1(\mathbf{r}) + \rho_2(\mathbf{r} - \mathbf{R})$

Microscopic approach

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Density-Constrained Frozen Hartree-Fock (DCFHF)

Simenel *et al.*, PRC **95**, 031601 (2017)

$$\delta \langle \Phi | \left[\hat{H} - \int d\mathbf{r}\, \lambda(\mathbf{r}) \rho(\mathbf{r}) \right] | \Phi \rangle = 0$$

Microscopic approach

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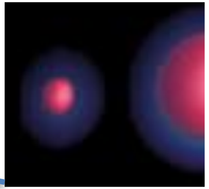

$\rho_1(\mathbf{r}) + \rho_2(\mathbf{r} - \mathbf{R})$

$$V_{DCFHF}(R) = \langle\Phi|\hat{H}|\Phi\rangle - E[\rho_1] - E[\rho_2]$$

Microscopic approach

Frozen Hartree-Fock (FHF)

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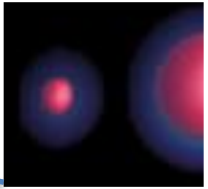


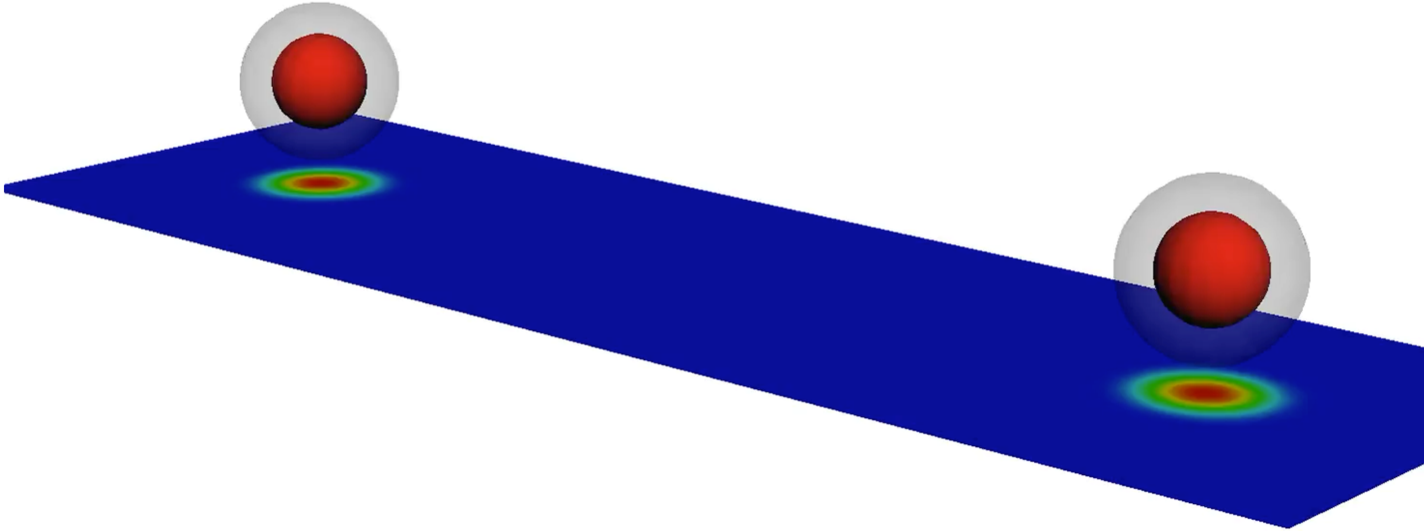
$\rho_1(\mathbf{r}) + \rho_2(\mathbf{r}-\mathbf{R})$

$$V_{DCFHF}(R) = \langle\Phi|\hat{H}|\Phi\rangle - E[\rho_1] - E[\rho_2]$$

Dynamics

Time-Dependent HF

$$\delta \langle \Phi | \left[\hat{H} - i\partial_t \right] | \Phi \rangle = 0$$



^{12}C+^{12}C at E~V_B

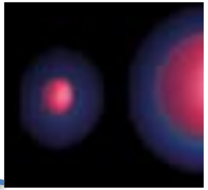
Courtesy of K. Godbey

Microscopic approach

Density-Constrained Time-Dependent Hartree-Fock (DC-TDHF)

Simenel *et al.*, PRC **95**, 031601 (2017)

$$\delta \langle \Phi | \left[\hat{H} - \int d\mathbf{r}\, \lambda(\mathbf{r}) \rho(\mathbf{r}) \right] | \Phi \rangle = 0$$



$$\rho_{TDHF}(\mathbf{r}, t)$$

$$V_{DCTDHF}[R(t)] = \langle \Phi | \hat{H} | \Phi \rangle - E[\rho_1] - E[\rho_2]$$

$$\delta \langle \Phi | \left[\hat{H} - i\partial_t \right] | \Phi \rangle = 0$$

Microscopic approach

Frozen Hartree-Fock (FHF)

Static, no Pauli

Density-Constrained Frozen Hartree-Fock (DCFHF)

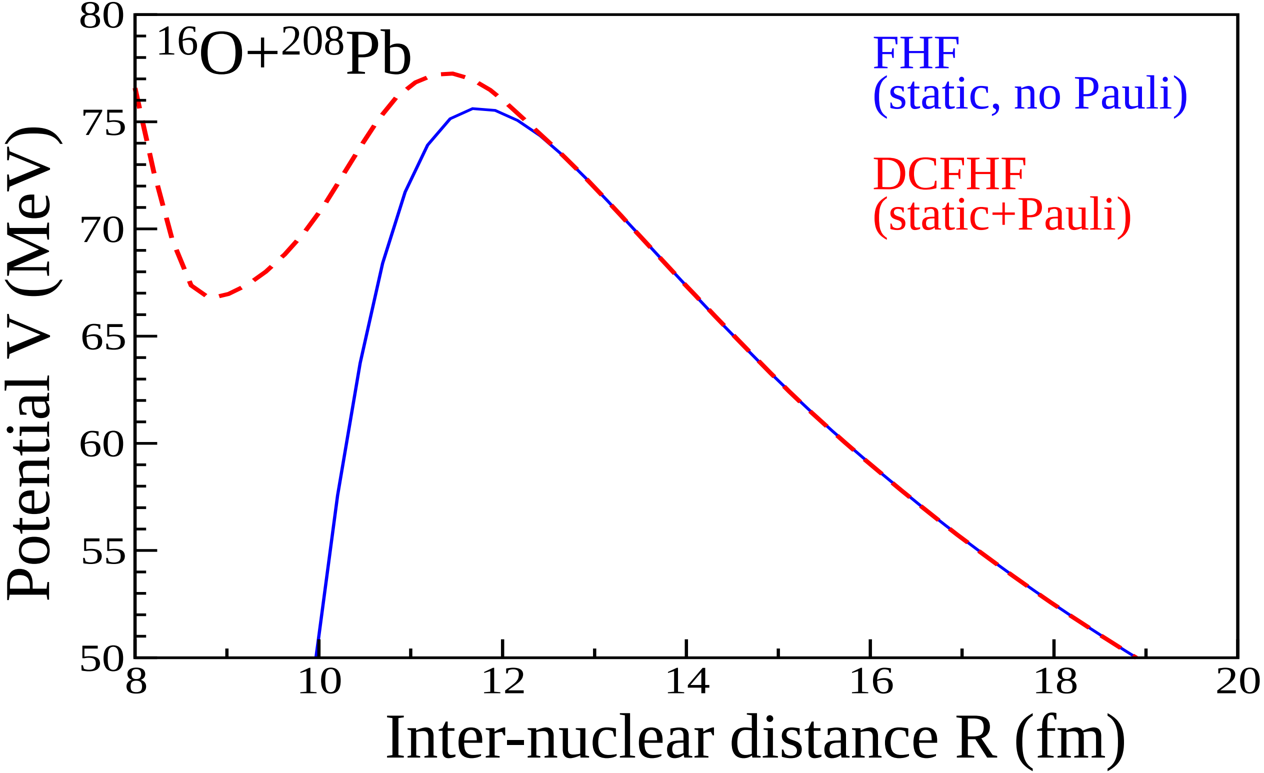
Static + Pauli

Density-Constrained Time-Dependent Hartree-Fock (DC-TDHF)

Dynamic + Pauli

Nucleus-nucleus potential

Pauli repulsion from DCFHF



Nucleus-nucleus potential

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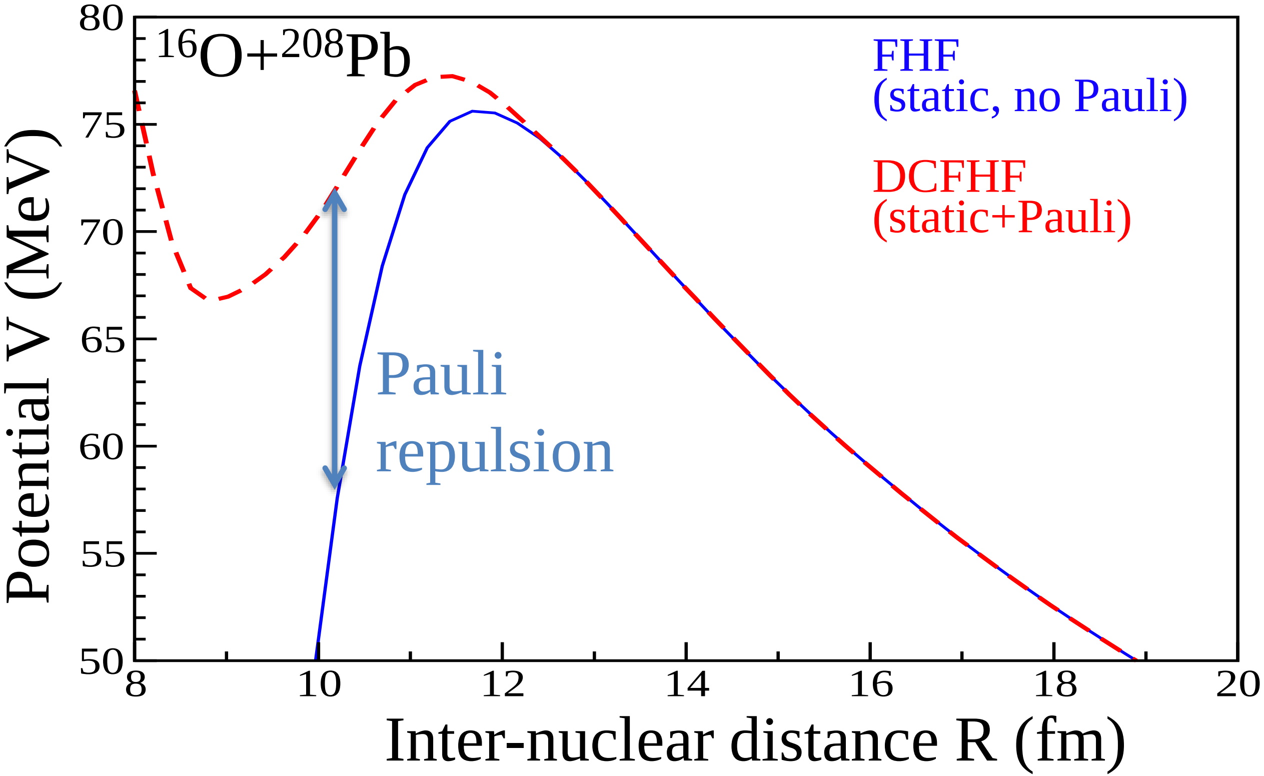

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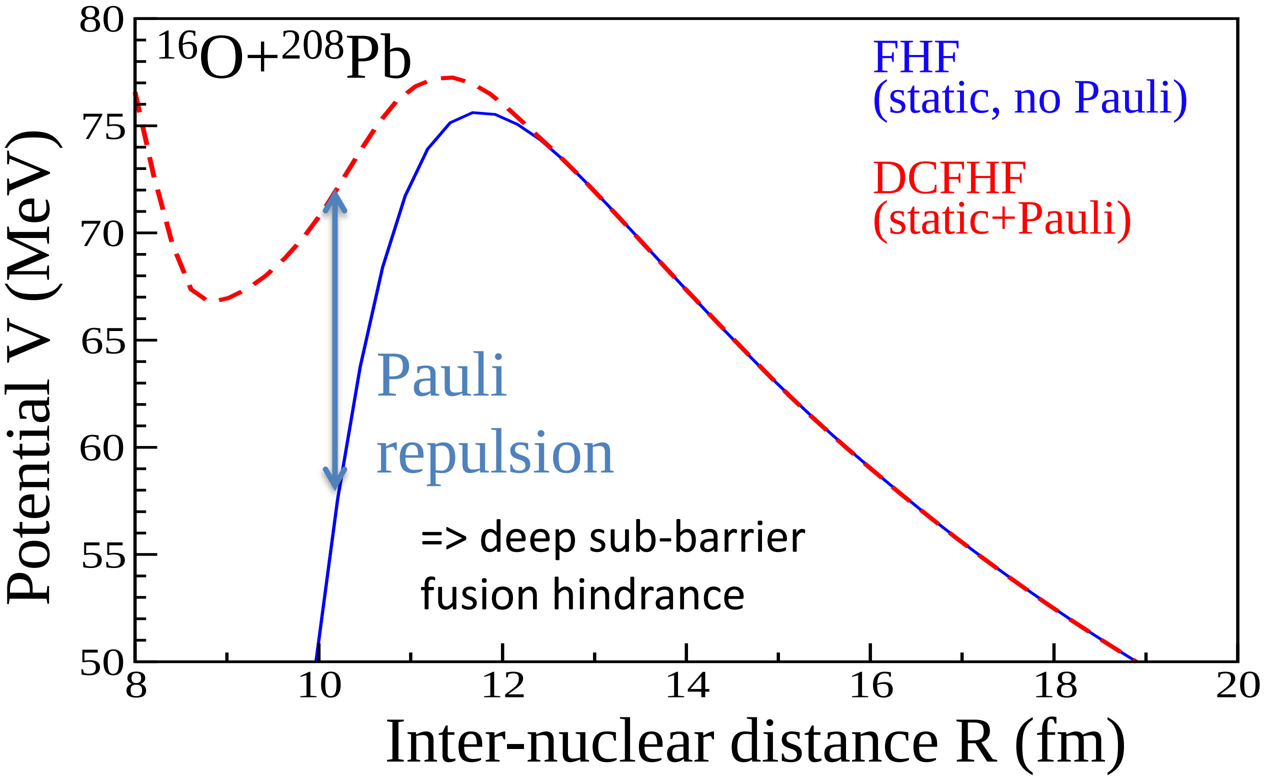
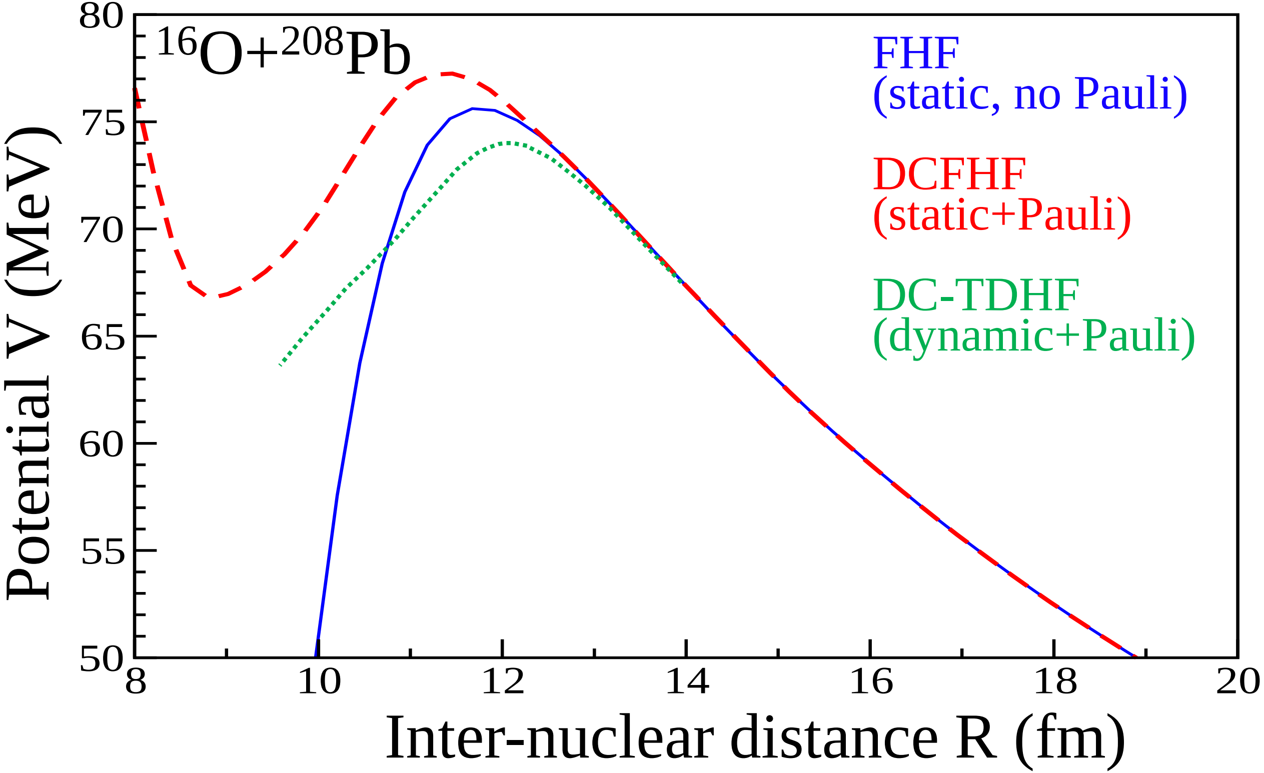


=> deep sub-barrier fusion hindrance

Nucleus-nucleus potential

Pauli repulsion from DCFHF



Nucleus-nucleus potential

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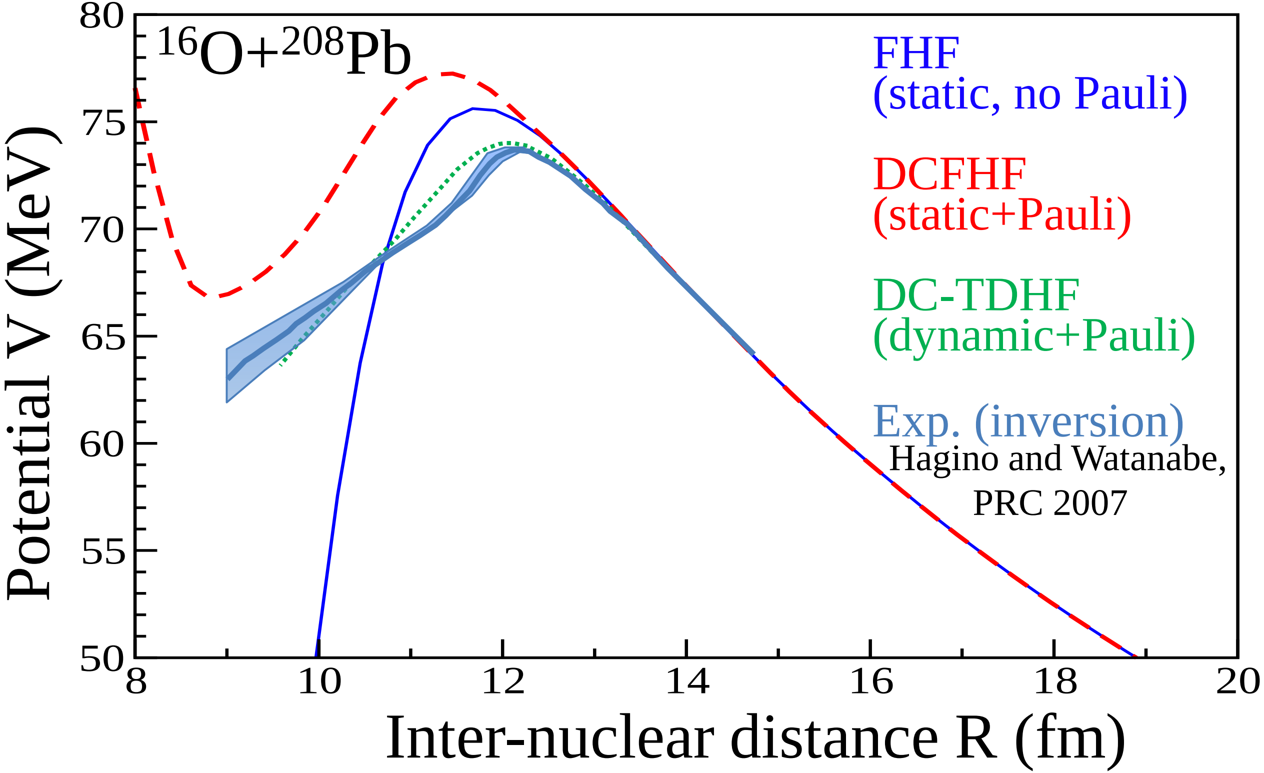
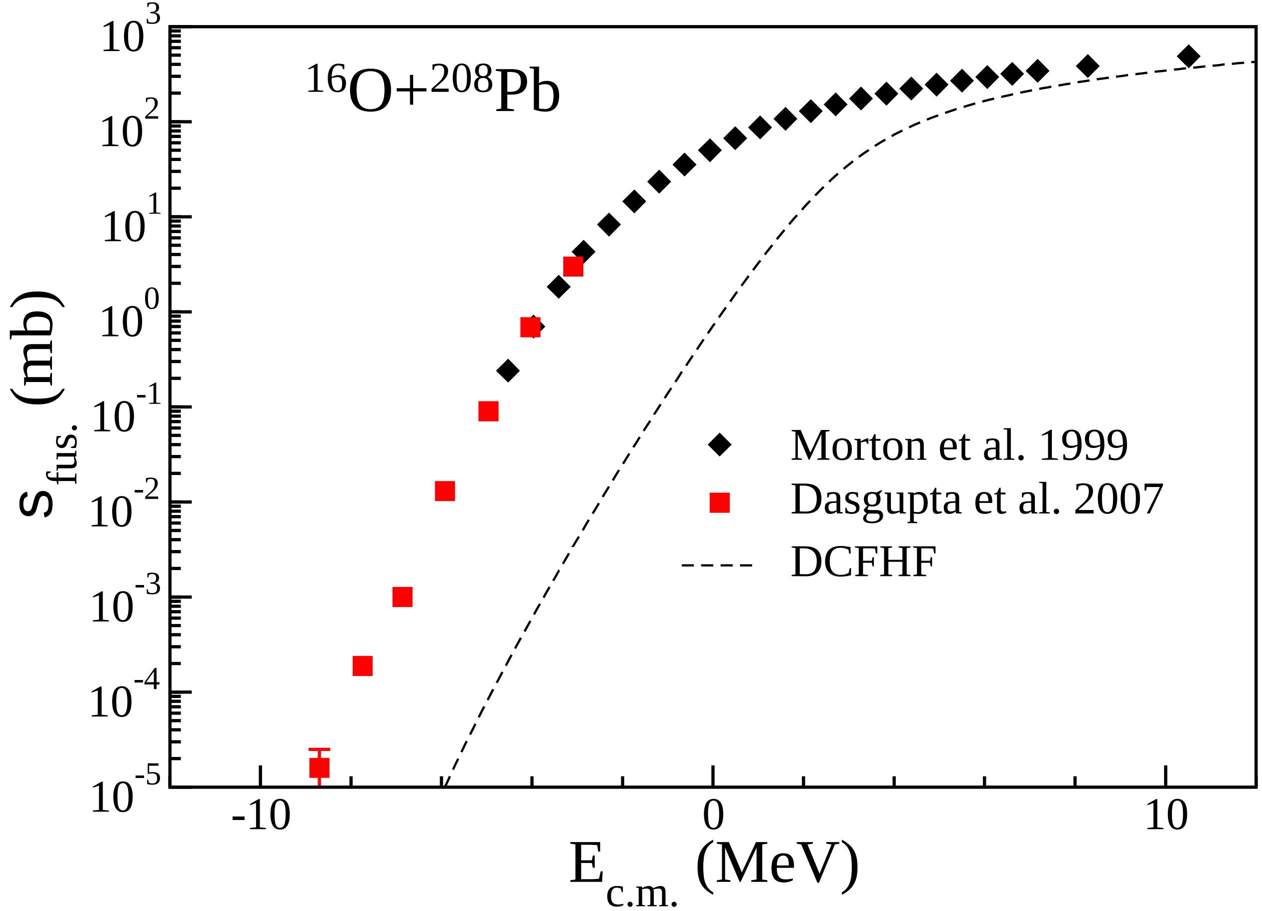
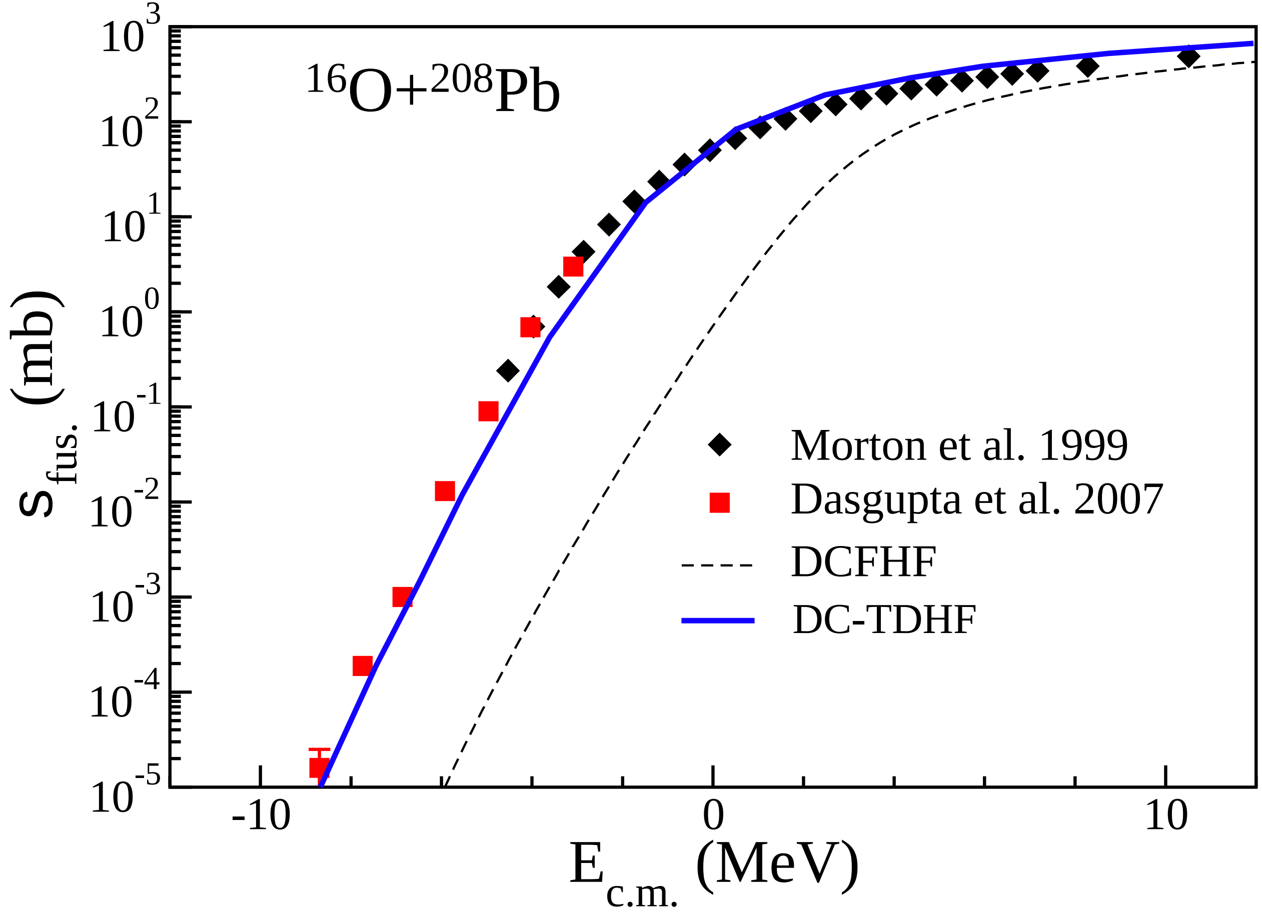


Deep sub-barrier fusion



Deep sub-barrier fusion



Dynamics

Isovector (transfer) dynamics with DCTDHF

Godbey, Umar, Simenel, PRC (R) 2017

$$H\ (\mathrm{r}) = \frac{\hbar^2}{2m} t_0 + H_0(\mathrm{r}) + H_1(\mathrm{r}) + H_C(\mathrm{r})$$

Dynamics

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$$H(\mathrm{r}) = \frac{\hbar^2}{2m} t_0 + H_0(\mathrm{r}) + H_1(\mathrm{r}) + H_C(\mathrm{r})$$

isoscalar isovector

Dynamics

Isovector (transfer) dynamics with DCTDHF

Godbey, Umar, Simenel, PRC (R) 2017

$$H(\mathrm{r}) = \frac{\hbar^2}{2m} t_0 + \underset{\substack{\uparrow \\ \text{isoscalar}}}{H_0(\mathrm{r})} + \underset{\substack{\uparrow \\ \text{isovector}}}{H_1(\mathrm{r})} + H_C(\mathrm{r})$$

$$\Rightarrow \quad V(R) = v_0(R) + \underset{\substack{\uparrow}}{v_1(R)} + V_C(R)$$

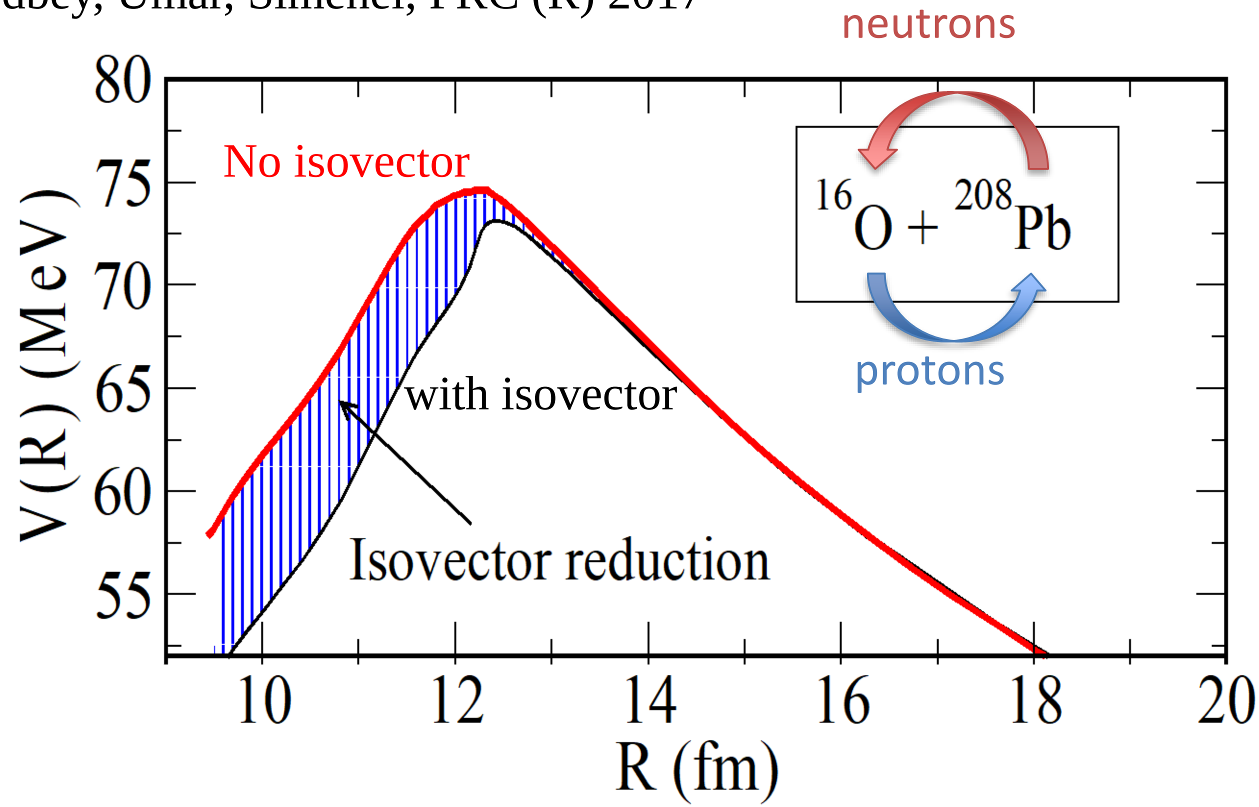
= 0 in FHF

=> purely dynamical (polarisation and transfer)

Dynamics

Isovector (transfer) dynamics with DCTDHF

Godbey, Umar, Simenel, PRC (R) 2017



Pauli energy distribution

Nucleon localisation function

Probability of finding 2 nucleons

$$P_{qs}(\mathbf{r},\mathbf{r}') = \rho_q(\mathbf{r}s,\mathbf{r}s)\rho_q(\mathbf{r}'s,\mathbf{r}'s) - |\rho_q(\mathbf{r}s,\mathbf{r}'s)|^2$$

Short range behaviour (**r~r**’) => localisation measure

$$D_{qs_\mu} = \tau_{qs_\mu} - \frac{1}{4}\frac{|\boldsymbol{\nabla}\rho_{qs_\mu}|^2}{\rho_{qs_\mu}} - \frac{|\mathbf{j}_{qs_\mu}|^2}{\rho_{qs_\mu}}$$

Pauli energy distribution

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Pauli kinetic energy

$$E^{\mathrm{P}}_{qs} = \frac{\hbar^2}{2m}\int d^3r\, D_{qs}(\mathbf{r})$$

Pauli (static) repulsion

$$\Delta E^{\mathrm{P(F)}}_{q\mu}(R) = \frac{\hbar^2}{2m}\sum_{s_\mu}\int d^3r\,\left[D^{\mathrm{DCFHF}}_{qs_\mu}(\mathbf{r},R) - D^{\mathrm{FHF}}_{qs_\mu}(\mathbf{r},R)\right]$$

Dynamic

$$\Delta E^{\mathrm{P(D)}}_{q\mu}(R) = \frac{\hbar^2}{2m}\sum_{s_\mu}\int d^3r\,\left[D^{\mathrm{DC-TDHF}}_{qs_\mu}(\mathbf{r},R) - D^{\mathrm{DCFHF}}_{qs_\mu}(\mathbf{r},R)\right]$$

Pauli energy distribution

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Probability of finding 2 nucleons

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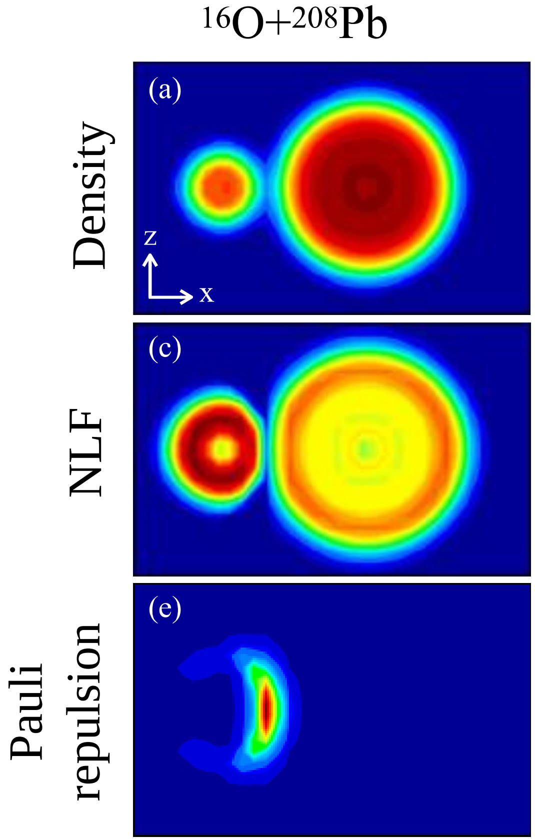
Dynamic $\Delta E^{\mathrm{P(D)}}_{q\mu}(R)=\frac{\hbar^2}{2m}\sum_{s_\mu}\int d^3r\,\,[D^{\mathrm{DC-TDHF}}_{qs_\mu}(\mathbf{r},R)-D^{\mathrm{DCFHF}}_{qs_\mu}(\mathbf{r},R)]$



Umar, Simenel, Godbey, PRC 2021

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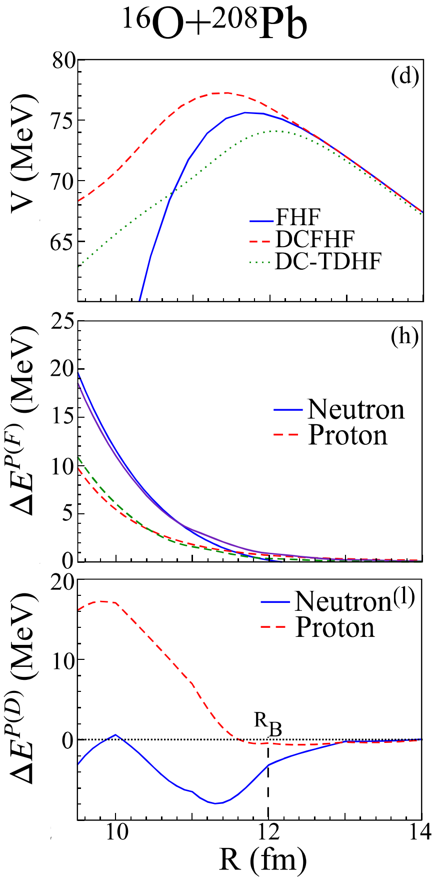
^{16}O+^{208}Pb

Umar, Simenel, Godbey, PRC 2021

Conclusions

- Microscopic predictions (no free parameters)
- FHF, DCFHF, and DC-TDFHF to isolate Pauli repulsion and dynamics
- Applications to ^{16}O+^{208}Pb
- Pauli repulsion inside the fusion barrier => Deep sub-barrier fusion hindrance
- Isovector dynamics (transfer)
- NLF => Pauli energy
- Pauli repulsion in the neck
- Different dynamical effects for protons and neutrons