Two analytical formulas of heavy-ion fusion cross sections

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Various attempts have been made to describe the heavy-ion fusion excitation functions across a wide energy region. The Wong formula includes the effects of a quantum mechanical tunneling through the Coulomb barrier and can reproduce the excitation function at the range larger than 0.1 mb. The beauty of the Wong formula lies in its simple, analytical form with only three parameters, while other more sophisticated theoretical models calculate the fusion cross sections with numerical technology. Here We discuss two formulas, which keep the simple, analytical forms, and have significantly performance across the whole energy region.

1) By assuming one parameter of the Wong formula, the curvature of the potential barrier, as an energy-dependent one, namely $\hbar\omega \rightarrow \hbar\omega \exp[\lambda \frac{E-V_1}{V_1}]$, a modified-Wong formula $\sigma_M(E)$ has been developed:

$$\sigma_M(E) = \frac{R_1^2}{2E}\hbar\omega \exp\Big[\lambda\frac{E-V_1}{V_1}\Big] \ln\Big[1+\exp\Big(2\pi/(\hbar\omega\exp\Big[\lambda\frac{E-V_1}{V_1}\Big])(E-V_1)\Big)\Big]. \quad (1)$$

Here, λ is an additional adjustable parameter and $\hbar\omega$ becomes the curvature at the top of the barrier $E = V_1$. R_1 and V_1 are the radius and barrier height.

2) Starting from an exact equation: $\sigma(E) = \frac{1}{E}\int^{E}\Big(\int^{E'}\frac{d^2(E''\sigma(E''))}{dE''^2}dE''\Big)dE'$, where $\frac{d^2(E''\sigma(E''))}{dE''^2} = \pi R_2^2 D_{test}(E'')$ and $D_{test}(E'')$ is the well known fusion barrier distribution introduced by Rowley *et al.*, by assuming that $D_{test}(E'')$ is a Gaussian distribution, $\frac{1}{\sqrt{2\pi}W}exp\Big[-(\frac{E''-V_2}{\sqrt{2}W}\Big)^2\Big]$, another analytical $\sigma_G(E)$ formula is obtained:

$$\sigma_G(E) = \frac{\pi R_2^2 W}{\sqrt{2}E}\Big[\sqrt{\pi}Zerfc(Z) + exp(-Z^2)\Big], (2)$$

with $Z = (E - V_2)/\sqrt{2}W$. Here, R_2 and W are the centroid and the standard deviation of the Gaussian distribution.

Both $\sigma_M(E)$ and $\sigma_G(E)$ reproduce very well many light to heavy systems across the whole energy region including the hindrance behavior at extreme sub-barrier energies (see figures). If in the expression for $\sigma_G(E)$, D_{test} is taken as a multi-Gaussian distribution, then it gives a 'real' and analytical barrier distribution, without the ambiguities that appeared in the ones obtained previously by using the double-differentiation procedures proposed by Rowley *et al.*

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