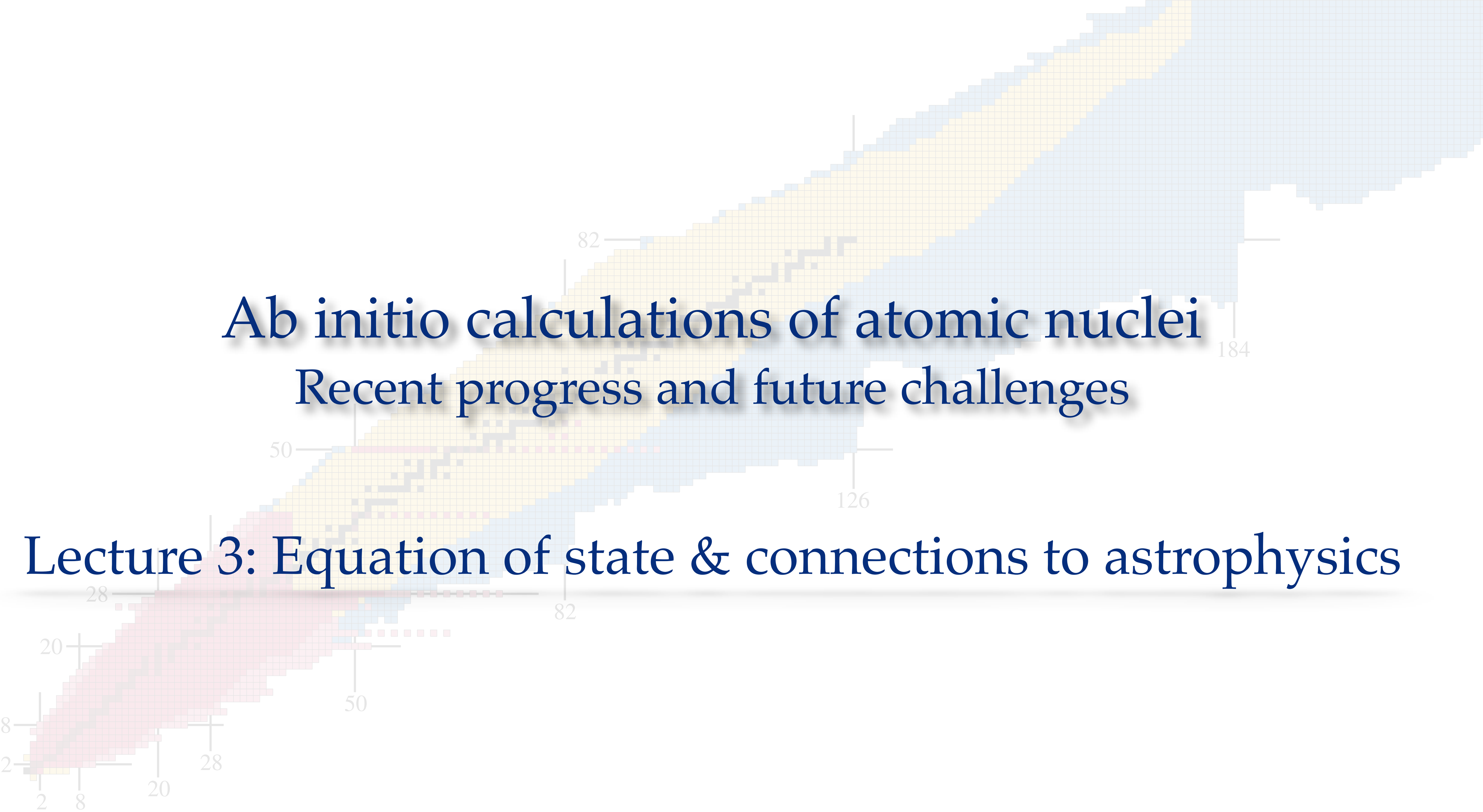

Ab initio calculations of atomic nuclei

Recent progress and future challenges

Lecture 3: Equation of state & connections to astrophysics

Università di Padova
7-10 June 2021

Vittorio Somà
CEA Saclay

Contents

1. Inter-nucleon forces

- Brief introduction to the nuclear many-body problem
- Properties and modelling of nuclear forces
- The modern view: chiral effective field theory

2. Ab initio techniques for the nuclear many-body problem

- Configuration-interaction approaches
- Techniques to mitigate the “curse of dimensionality” (SRG, NO2B, IT)
- Mean field and correlations
- Expansion methods for closed-shell nuclei
- Symmetry breaking
- Expansion methods for open-shell nuclei
- State of the art and open problems

3. Equation of state of nuclear matter & connections to astrophysics

- Neutron stars & Tolman-Oppenheimer-Volkoff equations
- Equation of state of neutron-star matter
- Astrophysical constraints on the nuclear EoS

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- **Neutron stars & Tolman-Oppenheimer-Volkoff equations**
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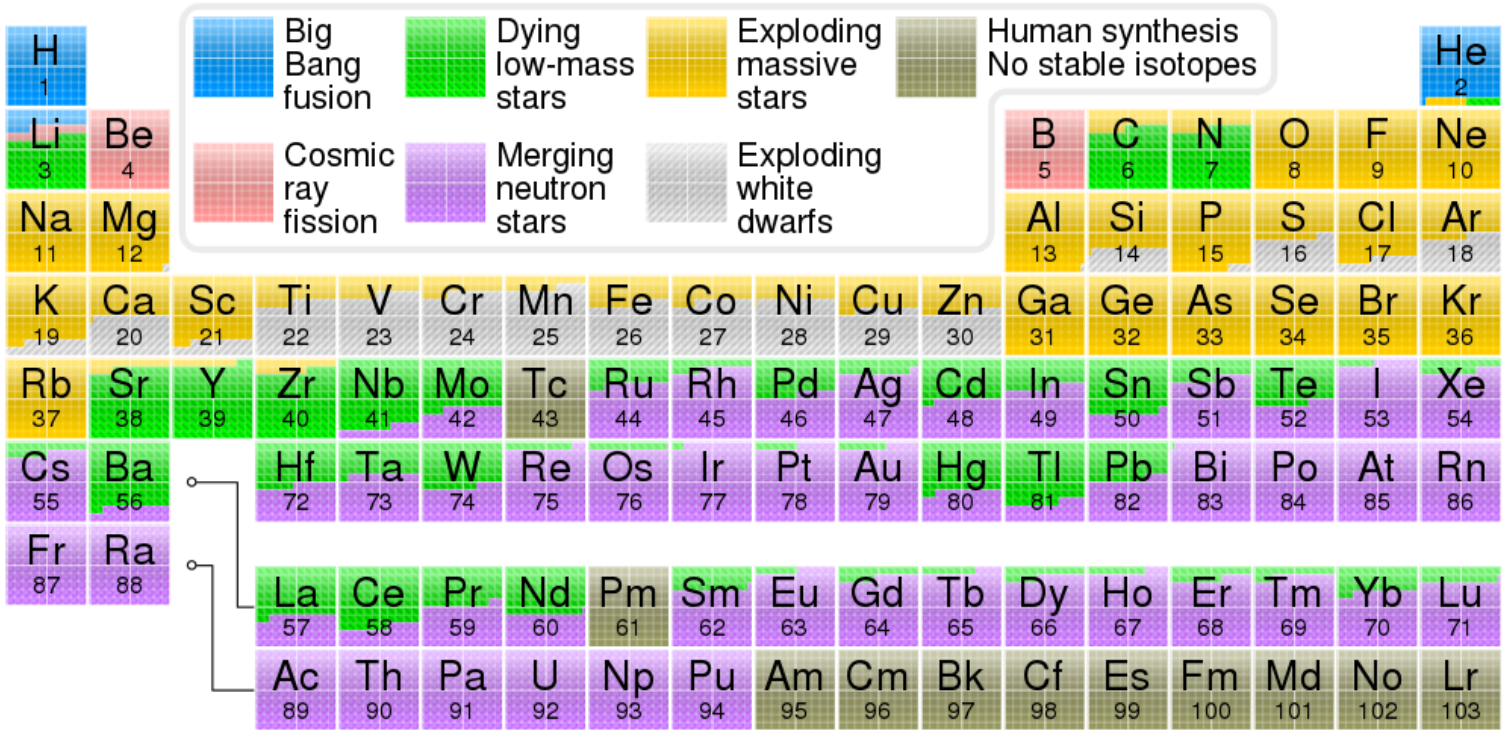
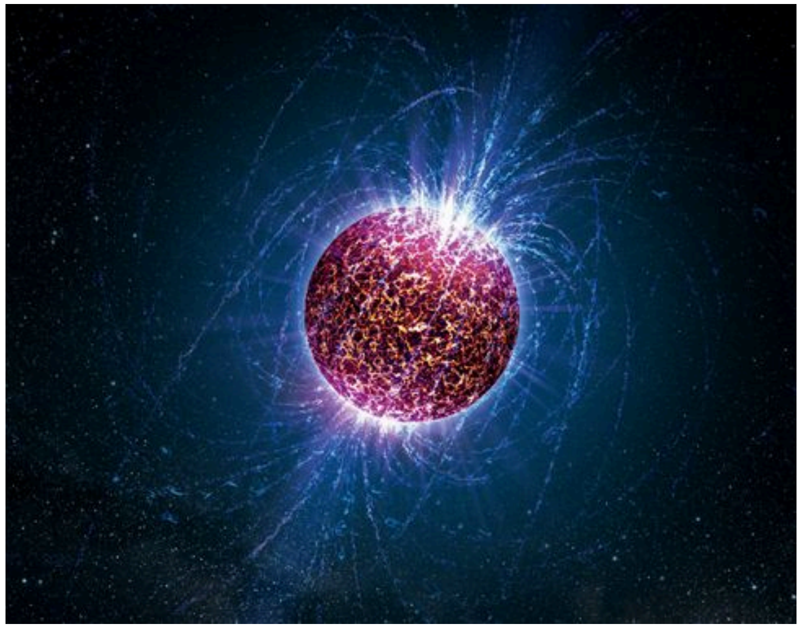
Connections to astrophysics

- **Nucleosynthesis**
  - Big-bang
  - Cosmic rays
  - Stellar burning
  - Supernovae explosions
  - Neutron stars mergers



- **Neutron stars**
  - Birth (supernovae explosions)
  - Life (NS properties: mass, radius, cooling, …)
  - Death (neutron star mergers)



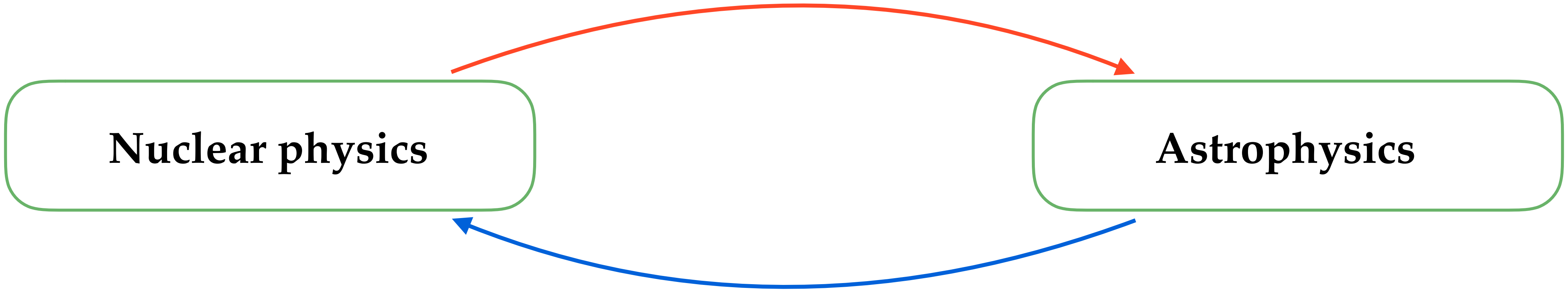
Inputs and constraints

○ Properties of **finite nuclei** (BE, spectra, decay rates, scattering cross sections, …)

→ Many nuclei of interest are currently out of reach for ab initio calculations

○ EoS of **nuclear matter** (P or E vs. density for different proton fractions and temperatures)

→ Ab initio calculations of nuclear matter available



needed for accurate modelling

Nuclear physics

Astrophysics

provides experimental constraints

⦿ Constraints on nuclear properties under **extreme conditions** (unattainable on Earth)

○ Very neutron-rich nuclei

○ Nuclear matter at high densities (and low temperatures)

→ Complementary to what accessed in high-energy experiments (low density- high T)

What are neutron stars?

⦿ **Neutron stars are different things for different people** [*Credits*: I. Vidaña]

○ For **astronomers**, they are tiny stars visible as radio pulsars or sources of X- or γ-rays

○ For **particle physicists**, they are neutrino sources and probably the only place in the universe where deconfined quark matter may be abundant

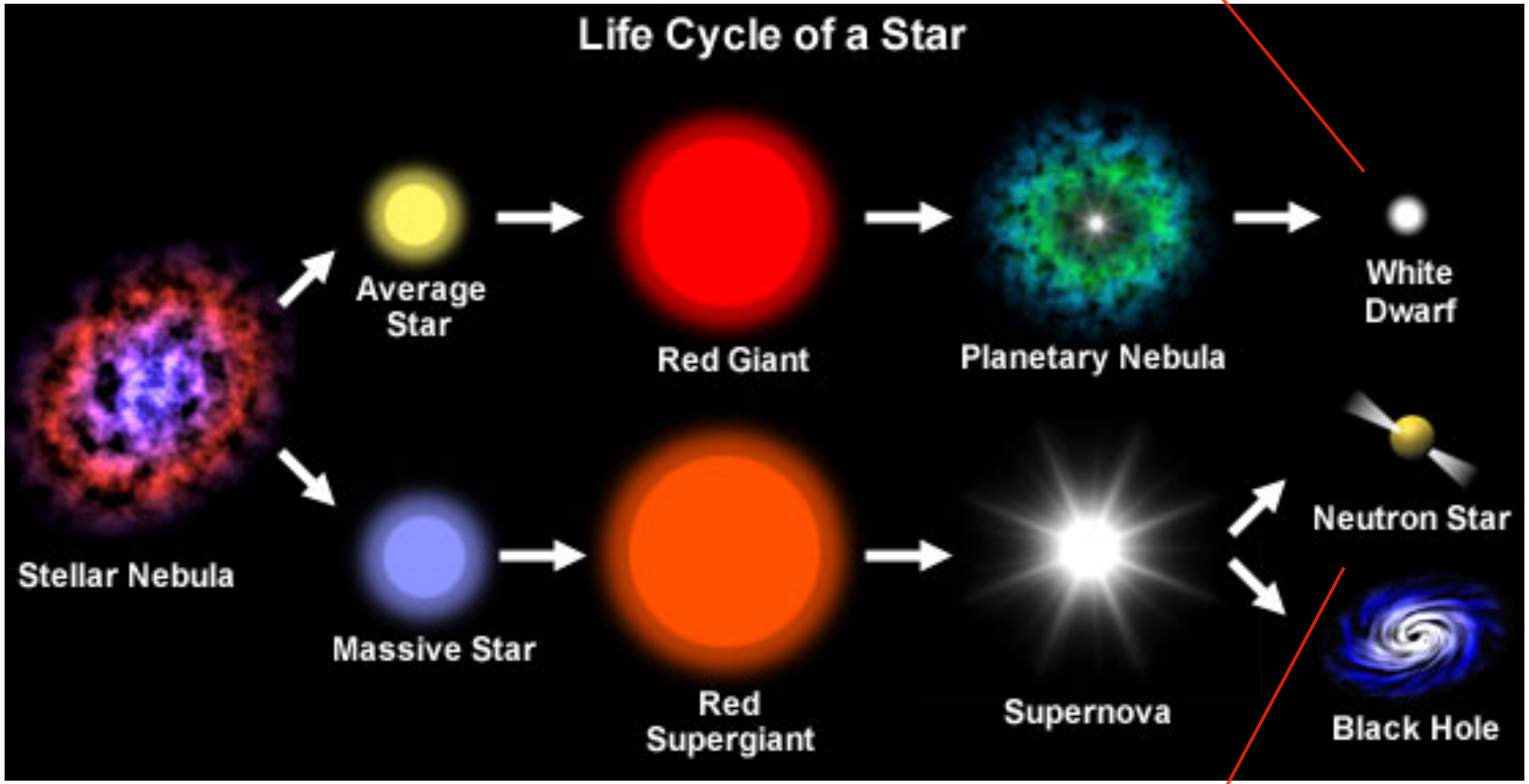
○ For **cosmologists**, they are almost black holes

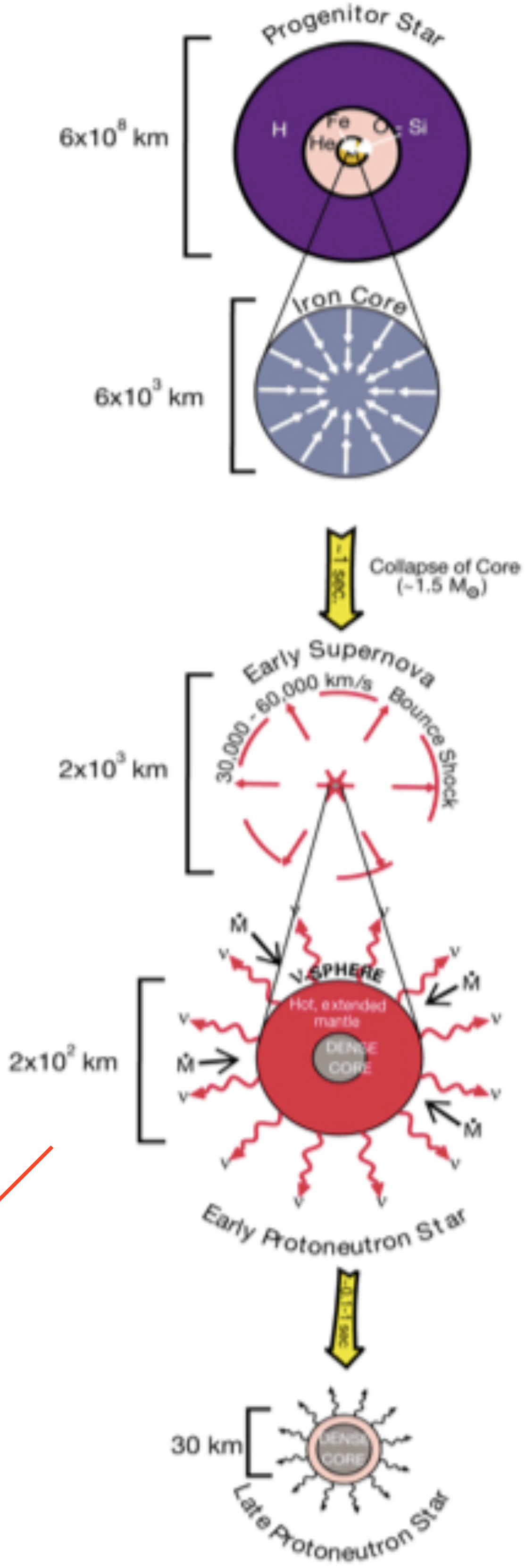
○ For **nuclear physicists**, they are the largest (neutron-rich) nuclei in the universe ($A = 10^{56}$-10^{57}; $M \sim 1\text{-}2\ M_\odot$; $R \sim 10$ km)

⦿ **However everybody agrees** on the fact that they are a type of stellar compact remnant originating from the gravitational collapse of a massive star (8 - $25\ M_\odot$) during a supernova event

Birth of a neutron star

White dwarf: stabilised by degeneracy pressure of electrons





Neutron star: stabilised by degeneracy pressure of neutrons

Modelling of **supernova explosions** involves nuclear physics ingredients (EoS across a large range of densities, proton fraction and temperatures)

→ Complex multi-scale modelling, hinders **feedback on nuclear physics**

From a minimal model…

⦿ **Minimal constituents**

- **Neutrons** (to generate the needed degeneracy pressure stabilising the star)
- **Protons** & **electrons** (neutrons do decay → β-equilibrium needed)

⦿ **Three simple conditions must be satisfied**

- **Charge neutrality** $n_{\mathrm{p}} = n_{\mathrm{e}}$

  → Even a small charge unbalance would make the star blow up

- **Beta equilibrium** $\mu_{\mathrm{n}} = \mu_{\mathrm{p}} + \mu_{\mathrm{e}}$

  → Neutrinos fly out of the star shortly after they are produced

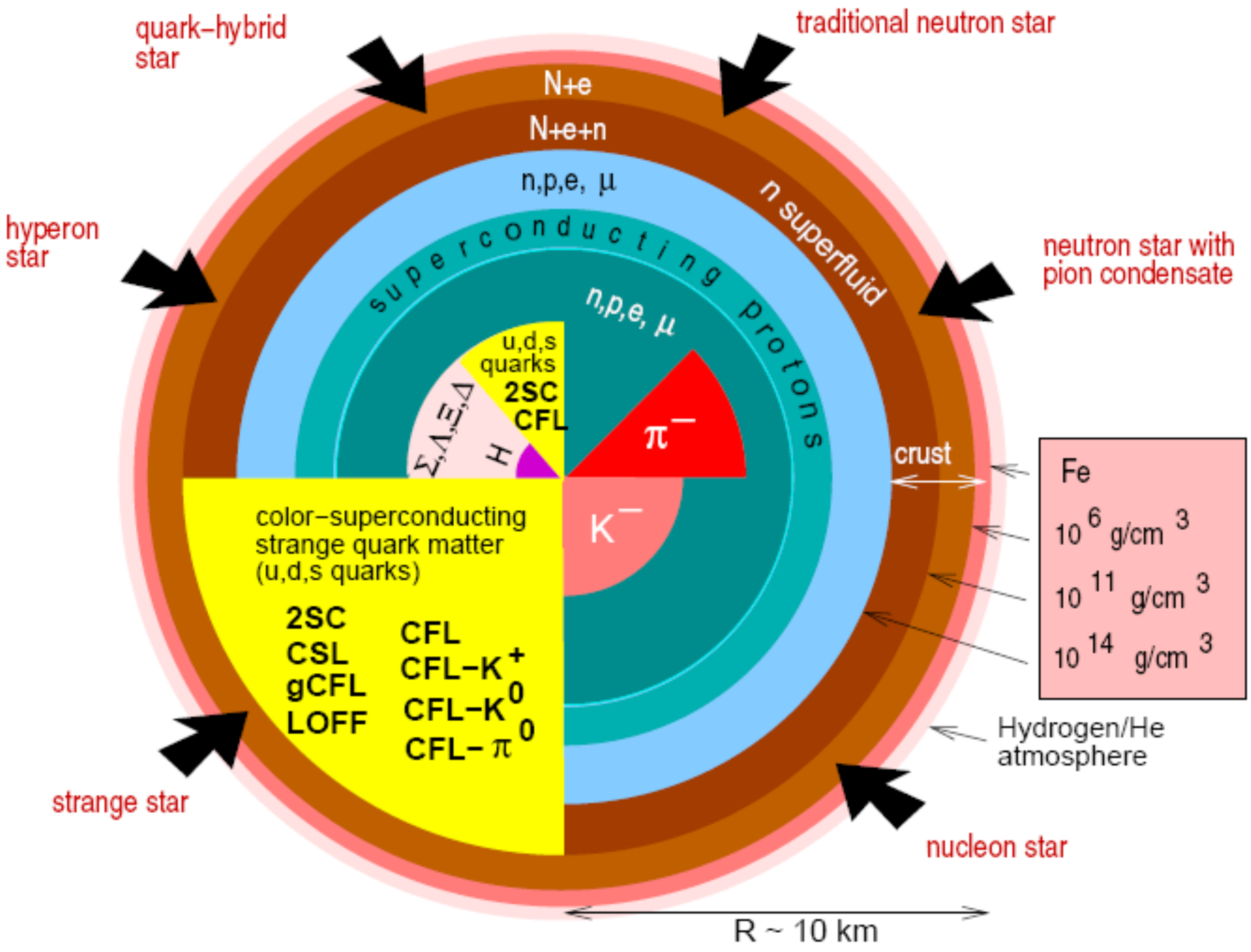
- **Conservation of total baryon number** $n_{\mathrm{B}} = n_{\mathrm{p}} + n_{\mathrm{n}}$

Charge-neutral β-equilibrium matter

→ **Proton fraction**

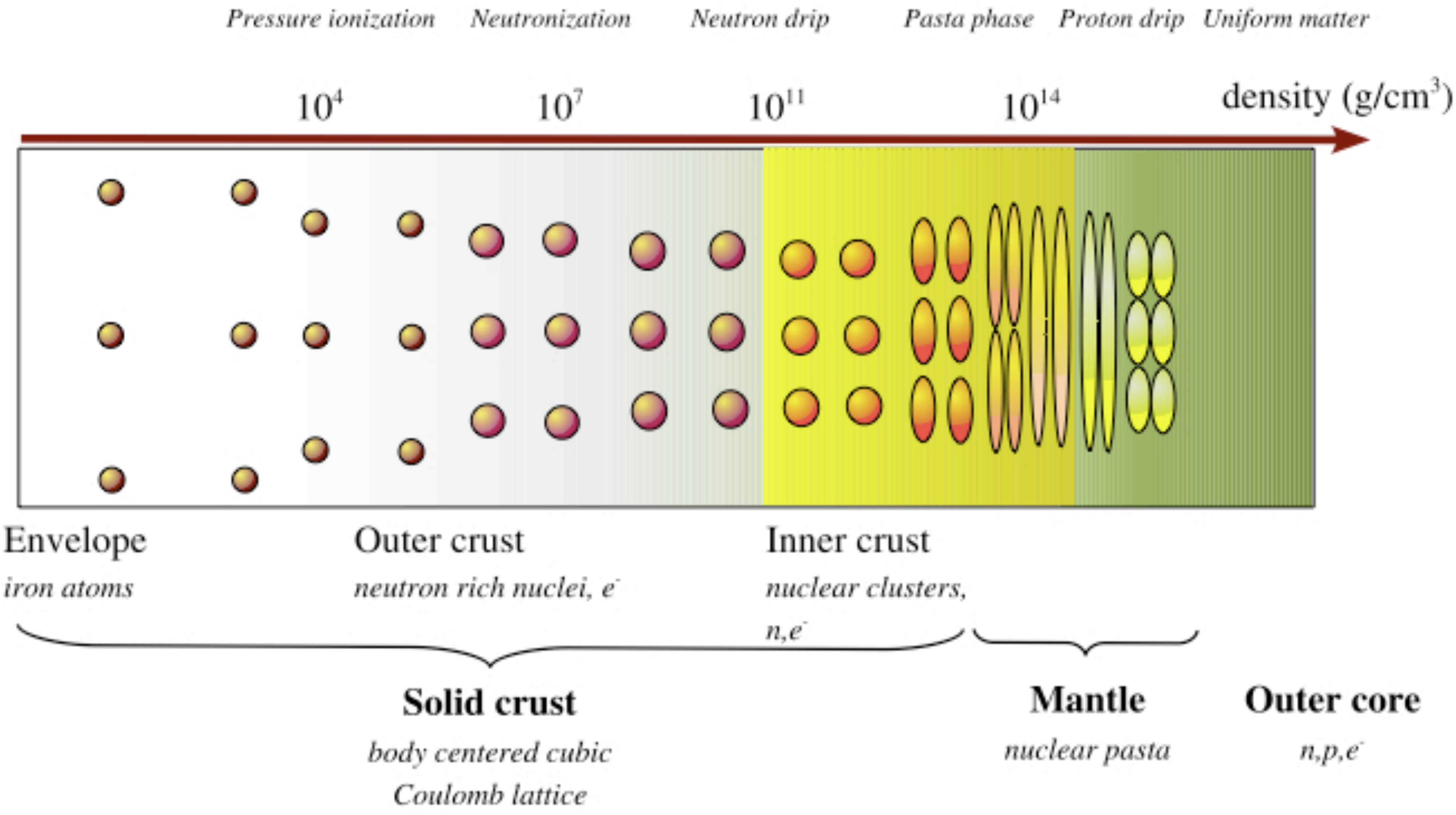
- Simple estimate (non-interacting nucleons) 3%
- More refined calculations 11%

…to, possibly, a very rich structure



[*Figure*: F. Weber]

Neutron star crust



[*Figure:* …]

TOV equations

- **Basic description of neutron-star structure**
  - Hydrostatic equilibrium equations

  \+ General relativity corrections necessary given the compactness of these objects

Tolman-Oppenheimer-Volkoff (TOV) equations

pressure

$$\frac{dP}{dr} = -G\frac{m(r)\epsilon(r)}{r^2}\left(1+\frac{4\pi r^3 P}{m(r)c^2}\right)\left(1+\frac{P}{\epsilon(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi\epsilon(r)r^2$$

mass

energy density

Boundary conditions

$r{=}0 \rightarrow \quad P = P_c \text{ and } m = 0$

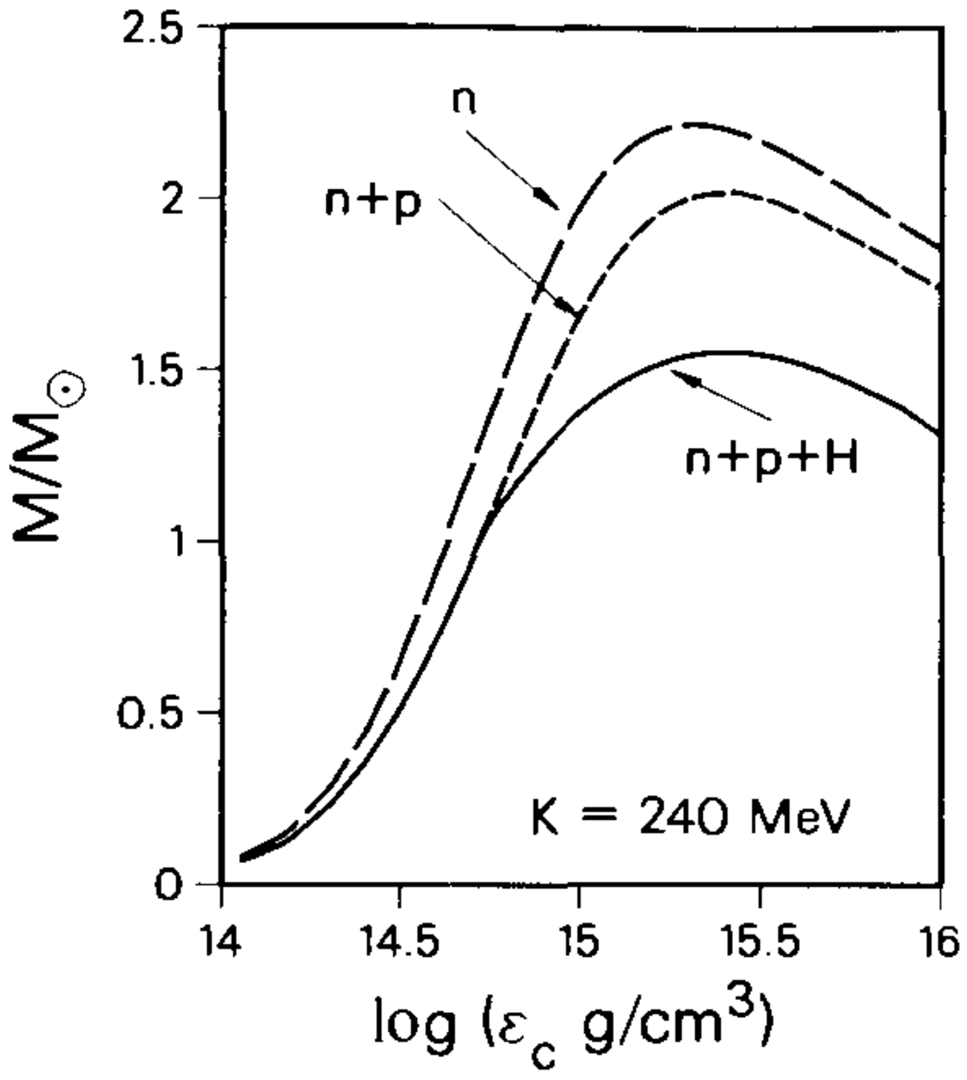
$r{=}R \rightarrow \quad P = 0 \text{ and } m(R) = M$

Relation between pressure and energy density required to close the system of equations

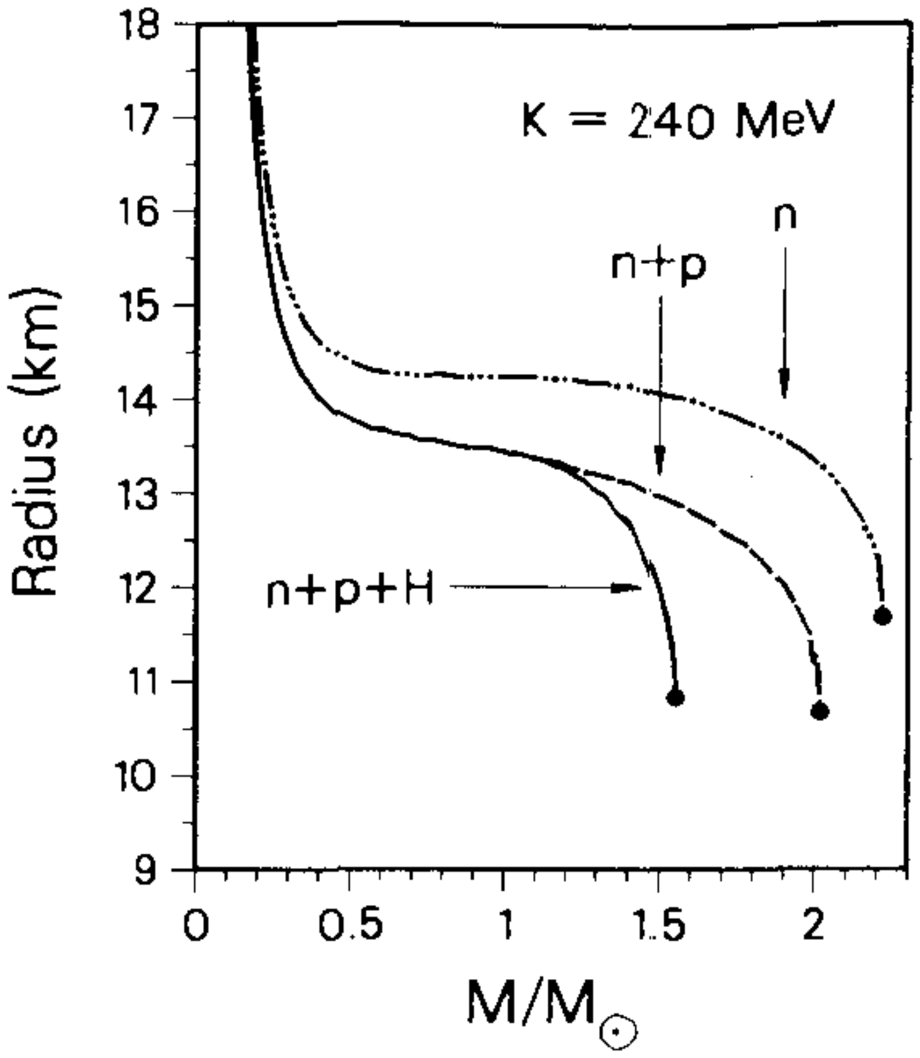
$P = P(\epsilon)$ → **Equation of state (EOS)**

Mass-radius relation

- Solution determines **mass M** and **radius R** of the compact object
- **Central pressure** acts as a parameter



Mass vs. central energy density



Mass vs. radius

[Glendenning, *Compact stars,* 2000]

Equation of state of neutron-star matter

⦿ **In the minimal model, equation of state of neutron-proton-electron matter needed**

○ Electrons produce negligible pressure → proton-neutron ("neutron-star") matter

○ Neutron stars have temperatures T ~ 10^7-10^8 K; $E_{\rm th} = 8$ keV ≪ $E_{\rm F}$ → T = 0 good approximation

⦿ **Equation of state of neutron-star matter**

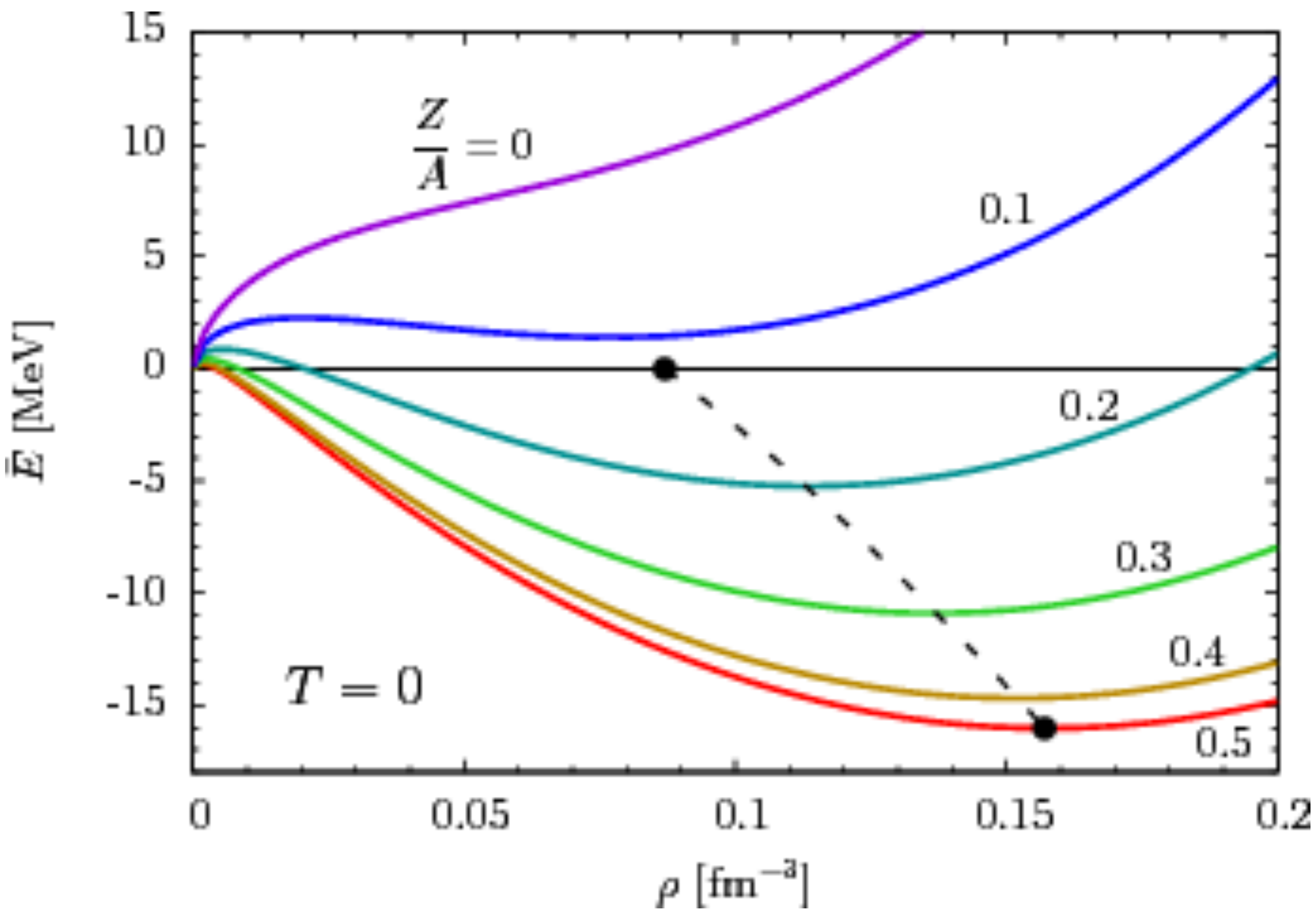
○ Baryon density $n = \frac{A}{V}$

○ Energy density $\epsilon = \frac{E}{V} = n\,\frac{E}{A}$

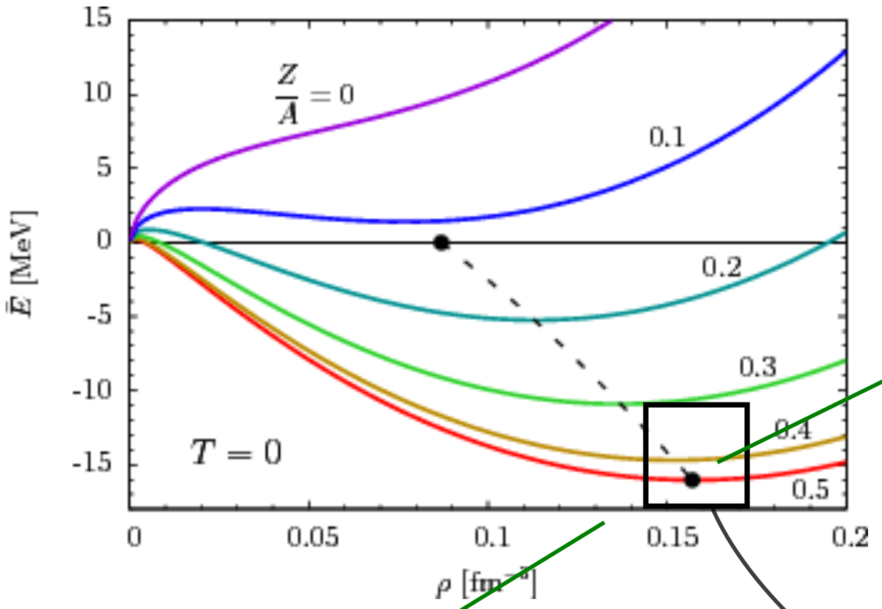
○ Pressure $P = n^2 \frac{\partial\, E/A}{\partial\, n}$

Equation of state $\frac{E}{A}(n, x)$

with proton fraction $x \equiv n_{\rm p}/n$



Equation of state of neutron-star matter



1. Saturation density

2. Saturation energy

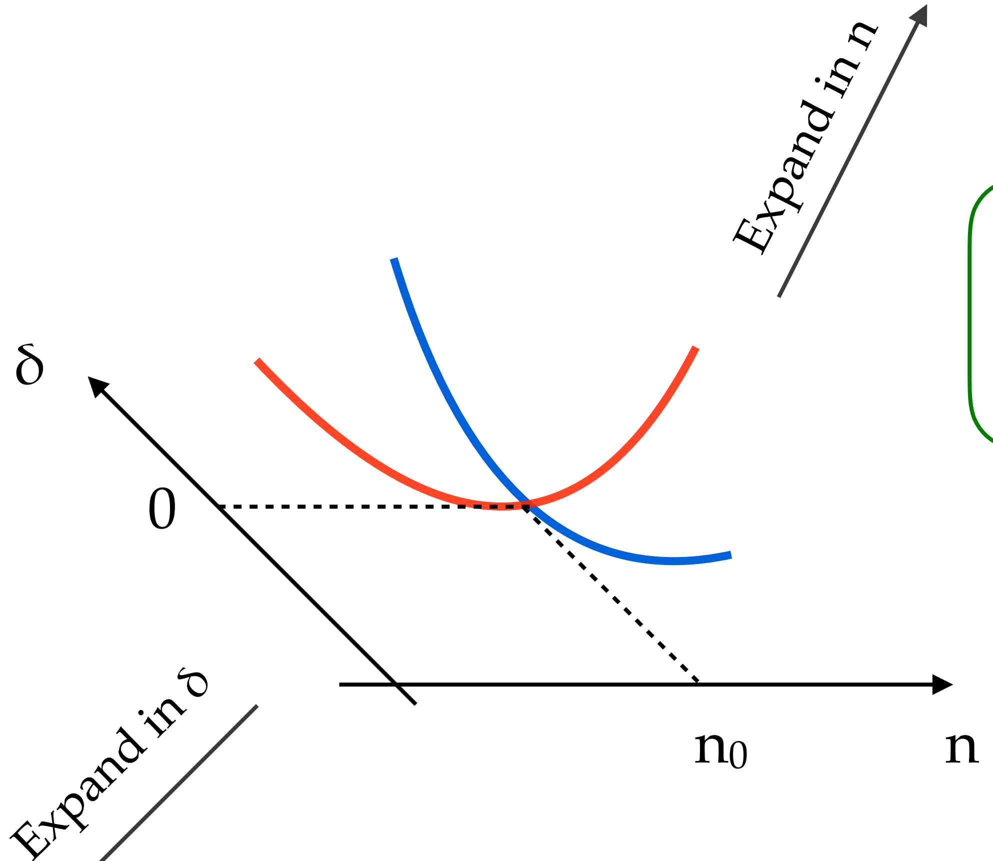
$$E_0 = E(n_0, 0)/A$$

3. Incompressibility

$$E(n, \delta = 0)/A \approx E_0 + (K/2)\,\eta^2$$

$$\eta = (n - n_0)/(3n_0)$$

$$\delta = (n_n - n_p)/n$$



5. Symm. energy slope

4. Symmetry energy

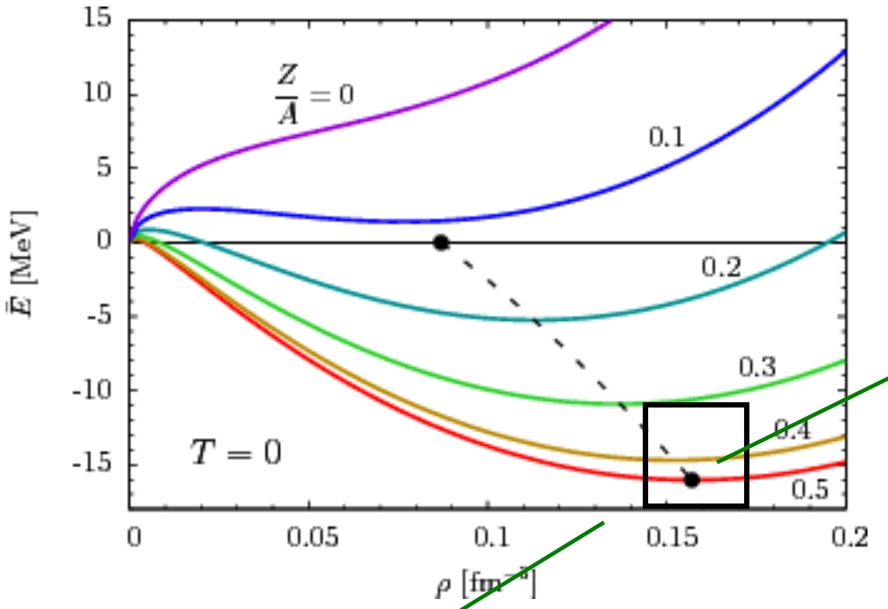
$$E_{\rm sym}(n) \approx S_v + L\,\eta$$

$$E(n,\delta)/A = E(n,0)/A + E_{\rm sym}(n)\,\delta^2$$

Pure neutron matter

$$E(n,1)/A \approx (E_0 + S_v) + L\,\eta$$

Equation of state of neutron-star matter



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$$E_0 \;=\; E(n_0, 0)/A$$

3. Incompressibility

$$E(n, \delta \;=\; 0)/A \;\approx\; E_0 + (K/2)\,\eta^2$$

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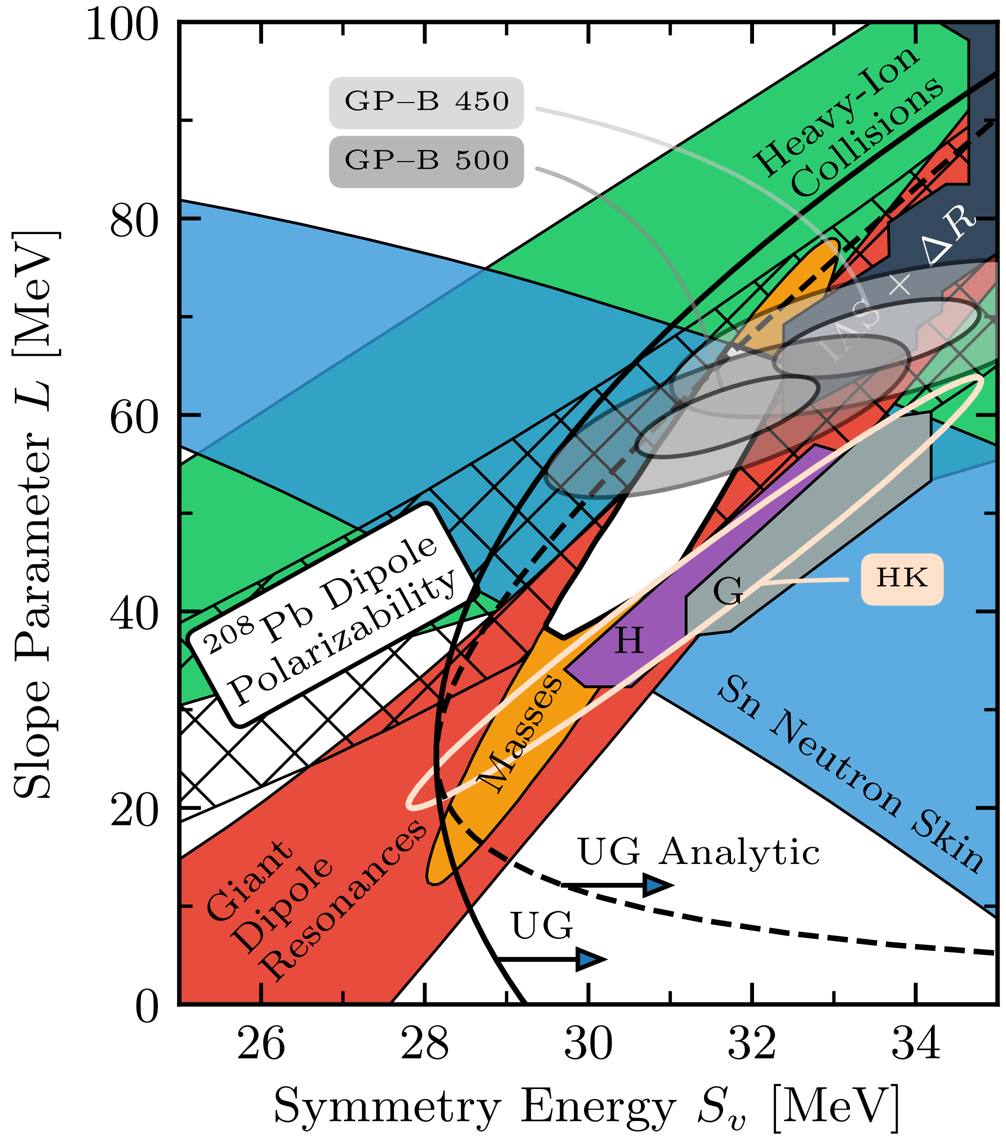


[Drischler *et al.* 2021]

Simulating nuclear matter

- Modelling of nuclear matter
  - **Homogeneous system** of nucleons interacting via strong interactions (Coulomb switched off)
  - Thermodynamic limit ($A \to \infty$, $\mathcal{V} \to \infty$, n=$A/\mathcal{V}$ constant)
  - Pure neutron matter is simpler (weaker 3N forces, tensor components, …)
  - Basis: plane waves well suited (translational invariance)
  - Calculations easier than for finite nuclei (no surface, symmetry breaking often negligible, …)

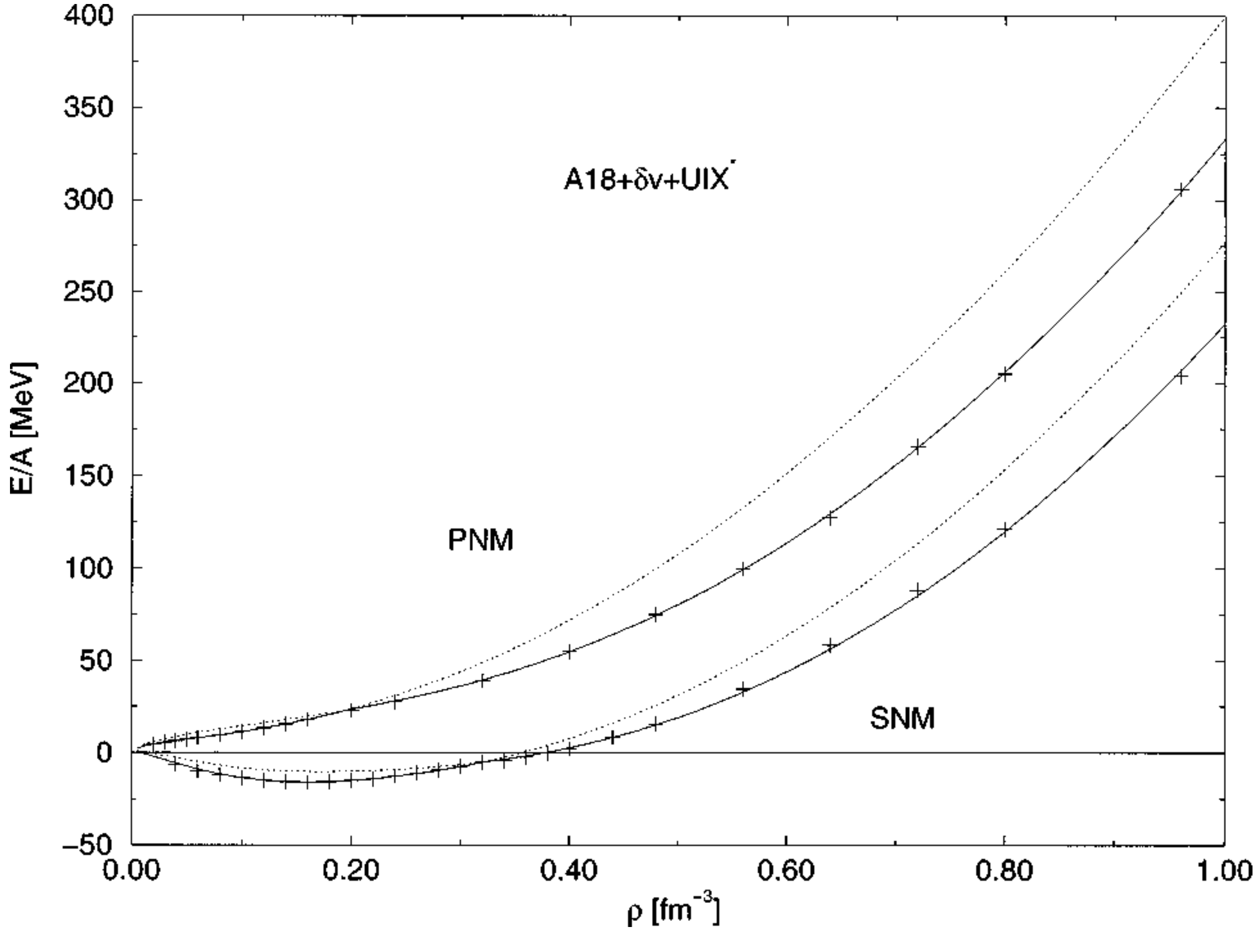
- Nuclear matter as a **theoretical laboratory** to test interactions & many-body methods

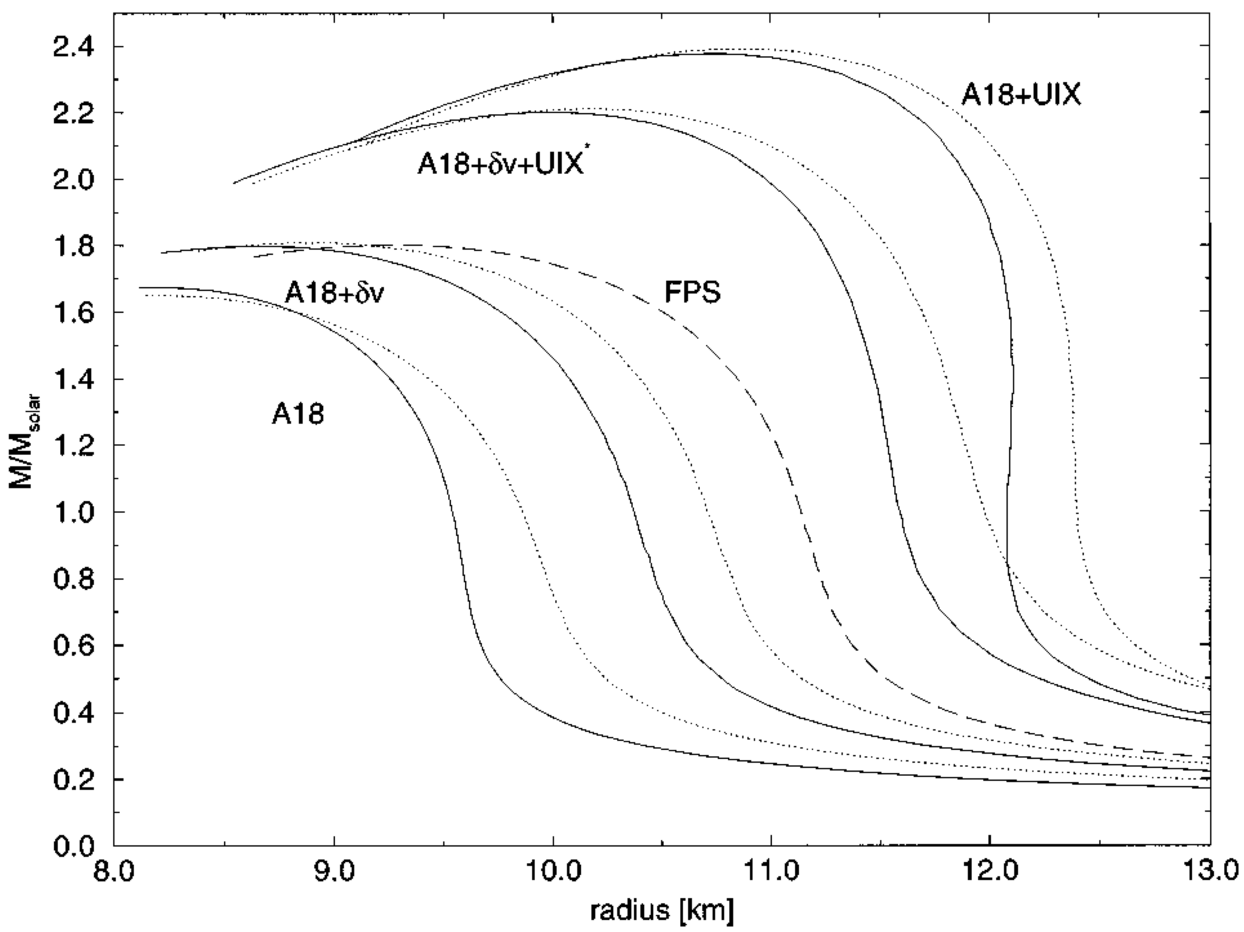
- Long history of ab initio nuclear matter calculations
  - **Interactions:** schematic interactions, OBE potentials, forces from chiral EFT, SRG evolution, …
  - **Many-body approaches:** exact, correlation-expansion (both non-perturbative and perturbative)

Frequent benchmarks between different calculations

First generation of ab initio results

- **Akmal-Pandharipande-Ravenhall (APR) equation of state** [Akmal *et al.* 1998]
  - “Milestone” calculation with AV18 + three-body forces (+ relativistic correction)





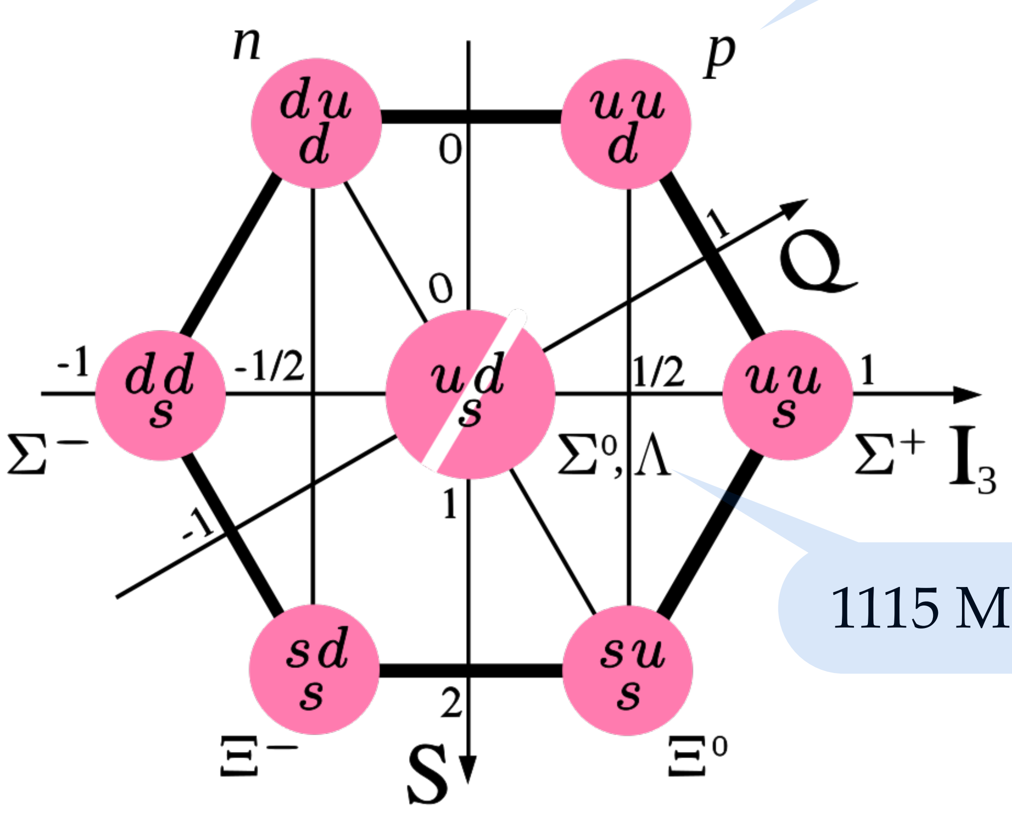
- Not many constraining data on the mass-radius relation at the time
- Is a **description in terms of only nucleons** realistic at such high densities?

Hyperons?

Quarks?

Hyperons

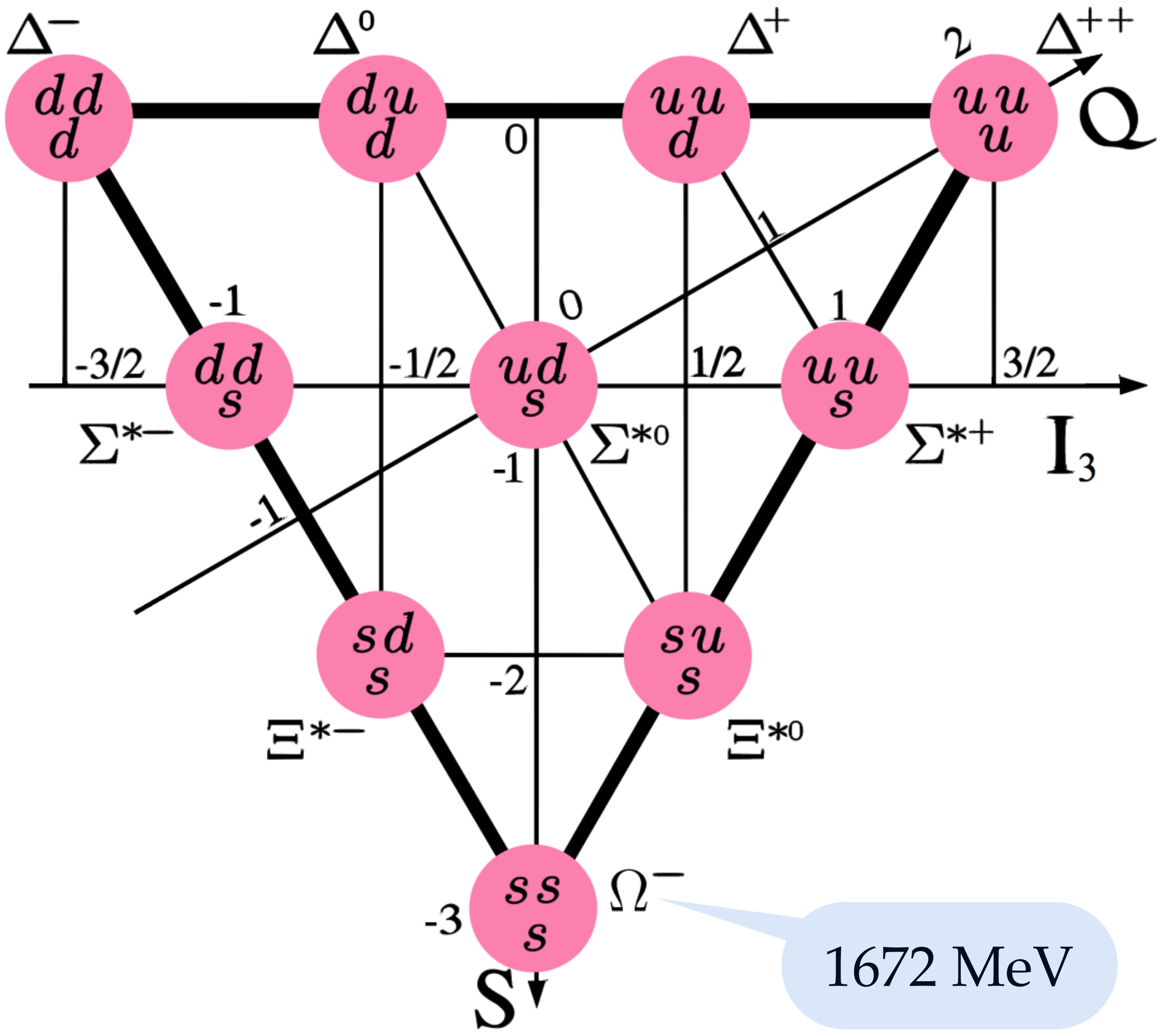
***usd* baryon octet**



938 MeV

1115 MeV

***usd* baryon decuplet**



1672 MeV

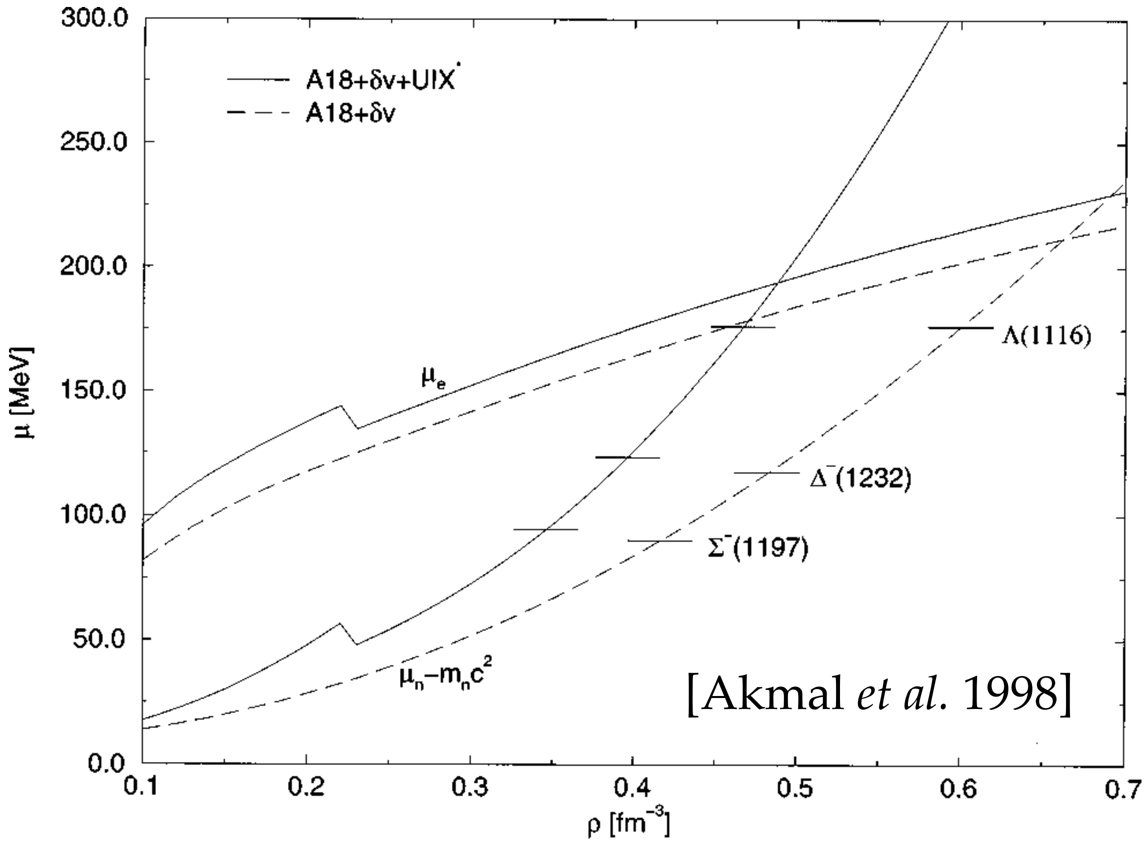
- **Hyperon:** a baryon containing one, two or three **s** quarks in addition to **u** and **d** quarks
  - Hyperons decay weakly (strangeness conserved by strong and EM interactions)
  - New quantum number: strangeness

Hyperon puzzle

- **Hyperon production**
  - Becomes energetically favourable at n ~ 2-3 n_0
  - β-equilibrium equations get modified

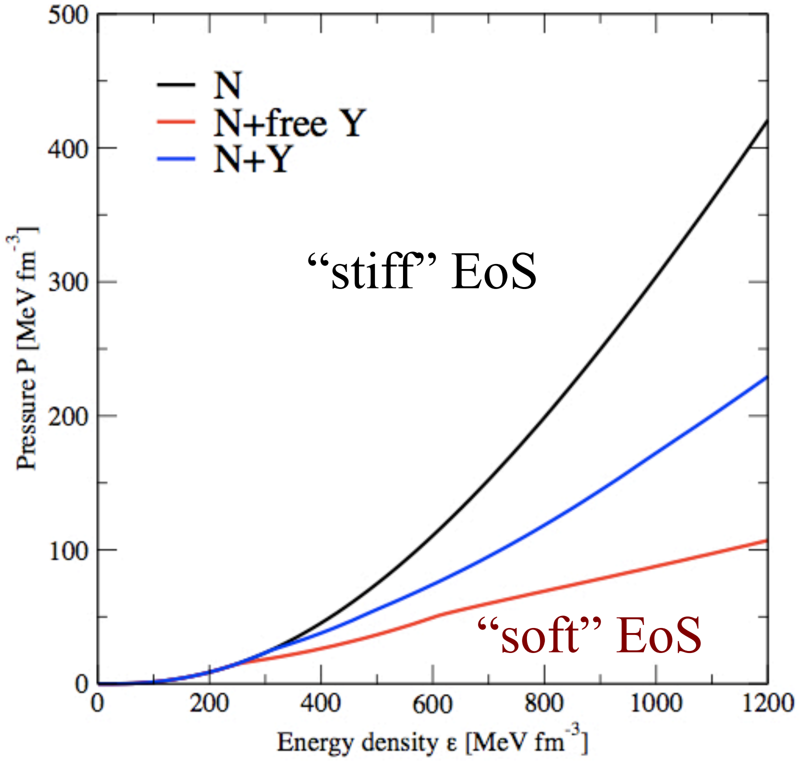
$$n + n \rightarrow n + \Lambda$$
$$p + e^- \rightarrow \Lambda + \nu_{e^-}$$
$$n + n \rightarrow p + \Sigma^-$$
$$n + e^- \rightarrow \Sigma^- + \nu_{e^-}$$

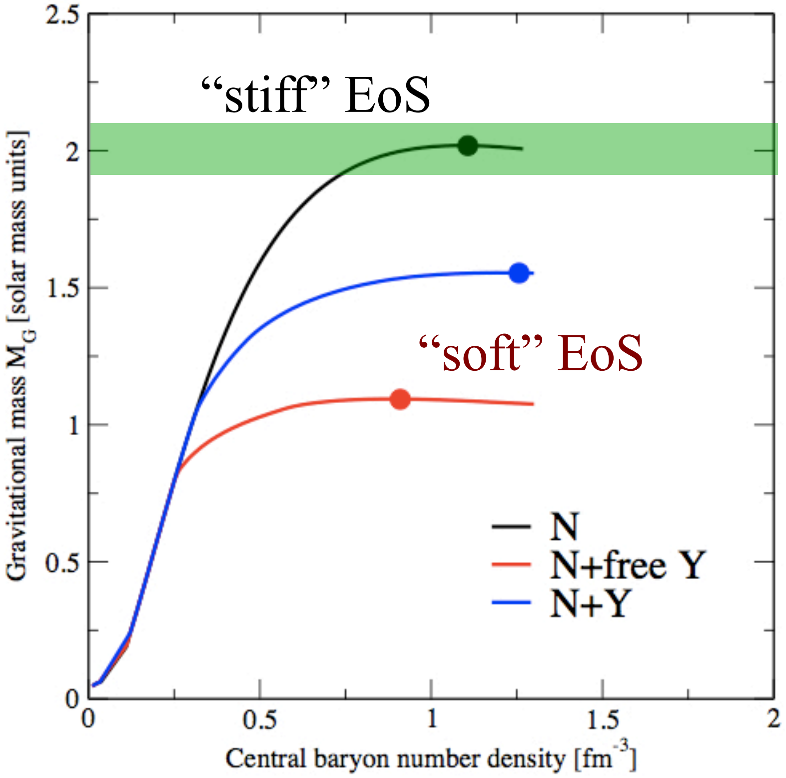
$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-} - \mu_{\nu_{e^-}}$$
$$\mu_\Lambda = \mu_n$$



[Akmal *et al.* 1998]

- New degrees of freedom
- Softening of the EoS
- Lower maximum mass
- In 2010, **2M⊙ star observed!**

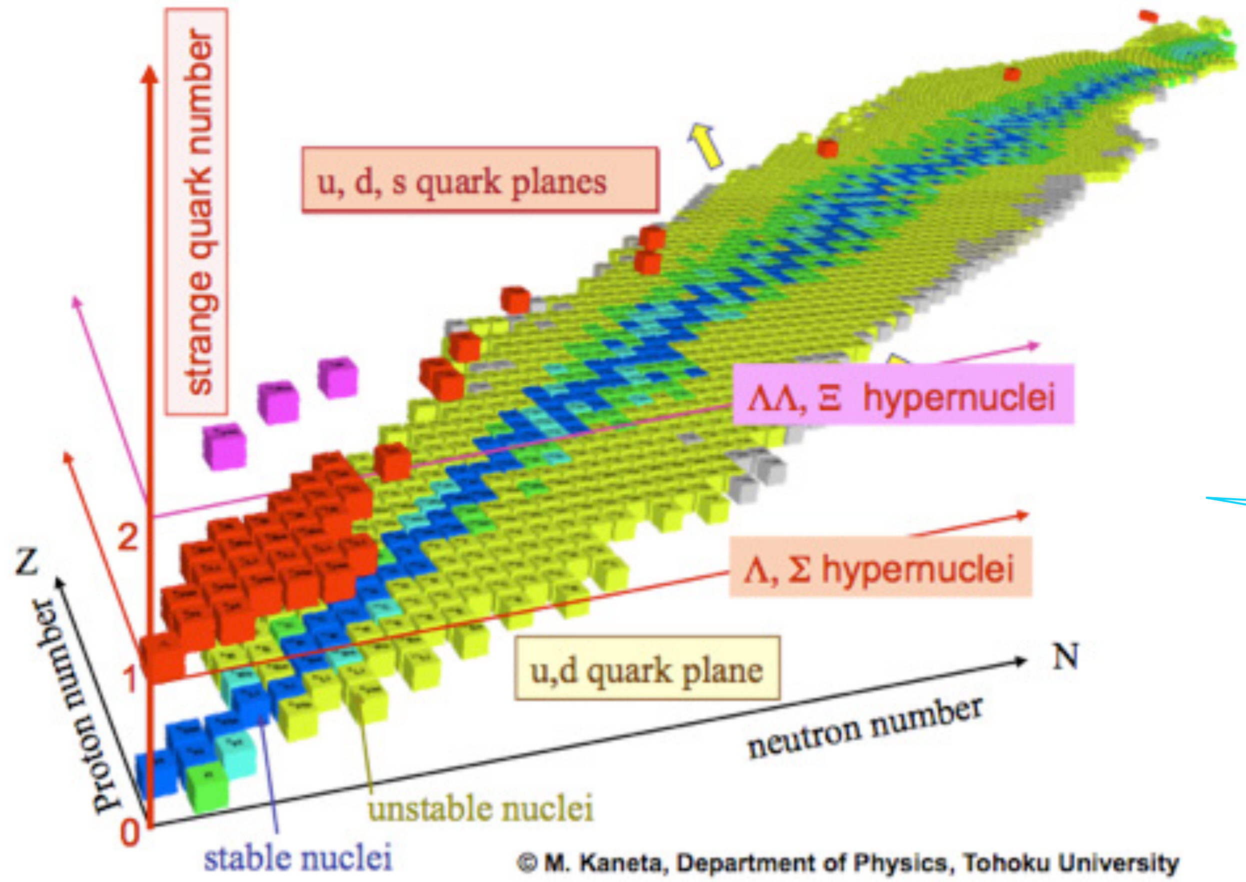




Hypernuclei & hyperon interactions

- **Hyperon-nucleon scattering**
  - Difficult experiments, scarce data

- **Hypernuclei:** nuclei with at least one hyperon in addition to protons and neutrons



© M. Kaneta, Department of Physics, Tohoku University

About 40 hypernuclei produced
+ various experiments planned

Idea: deduce YN interaction from reverse-engineering NCSM results

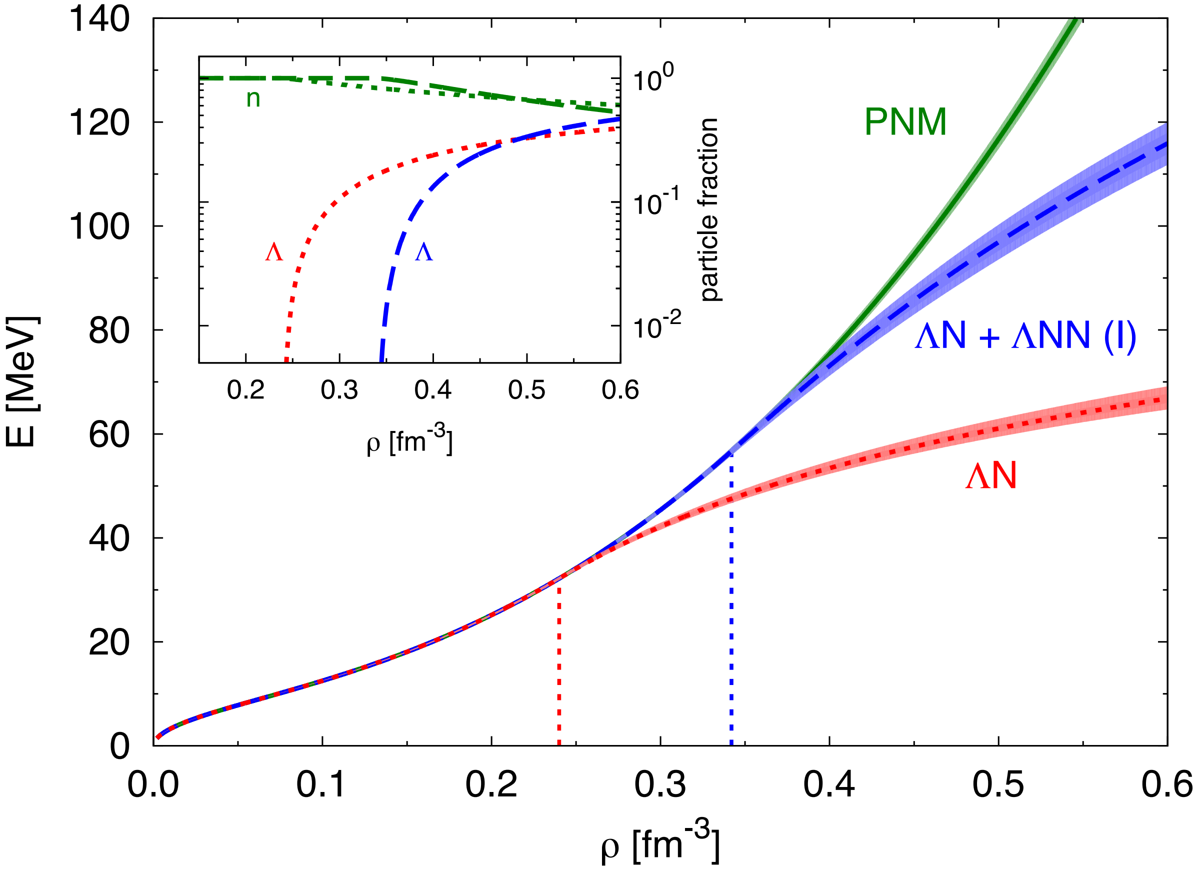
- **Relativistic pp collisions**
  - Study of correlations between strange baryonic products

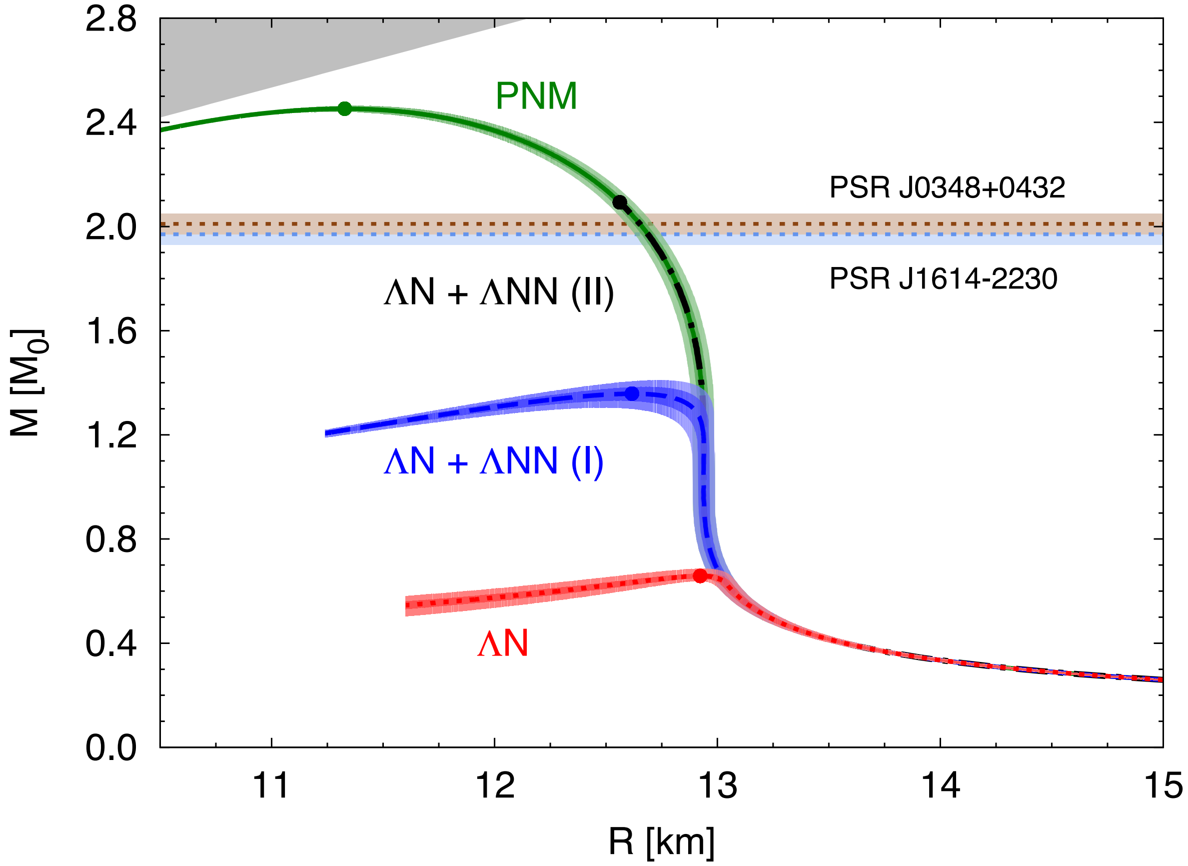
➪ Possible game changer: **interactions from lattice QCD**

Hyperons in neutron stars

- **Modern calculations leave the problem open**
  - Strong sensitivity on (poorly known) YNN interactions

[Lonardoni *et al*. 2015]





- Possible solutions:
  - Poor knowledge of YN, YY, YN, YYN, YYY interactions
  - Critical density not reached
  - Transition to quark matter in the neutron star interior
  - …. ?

Quark stars

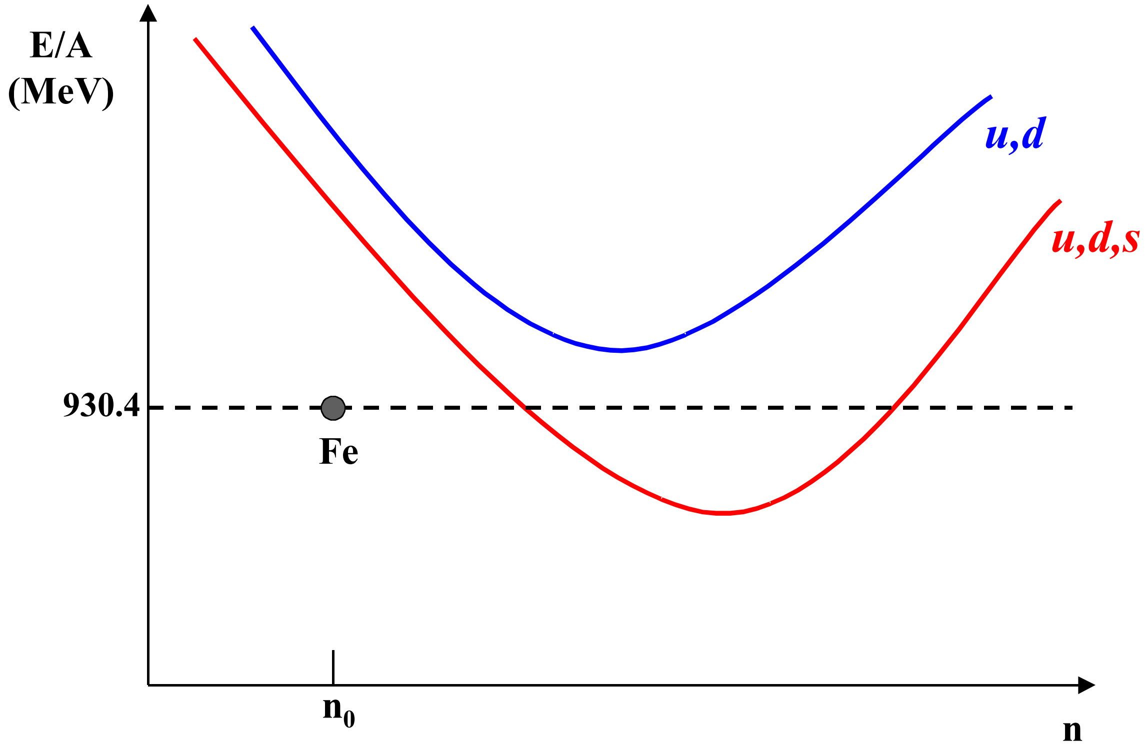
- **Hybrid stars → outer hadronic layers, deconfined quarks in the interior**
  - Also leads to a softening of the equation of state

- **Stars made purely of quarks → "Strange stars"**

Strange quark matter hypothesis

[Bodmer 1971, Terazawa 1979, Witten 1984]

3-flavour u,s,d quark matter could be the **true ground state** of strongly interacting matter



Why nuclei don't decay into droplets of strange quark matter?

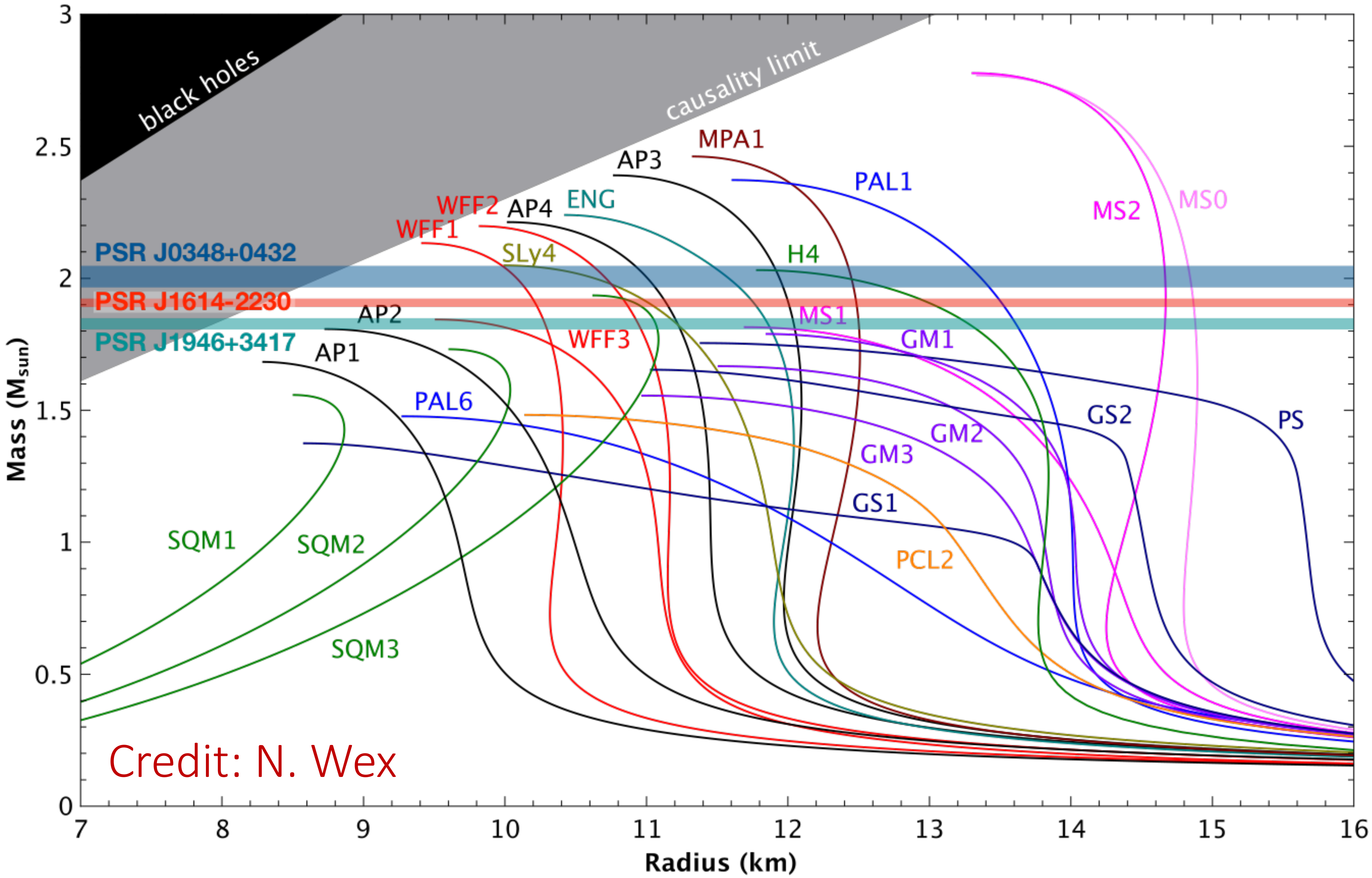
Probability for direct decay very small

$$^{56}Fe \rightarrow {}^{56}(SQM)$$

Sequential decay energetically forbidden

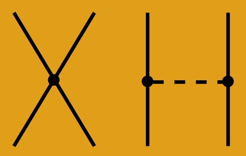
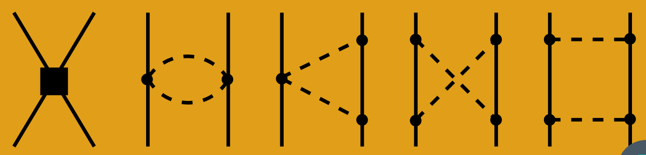
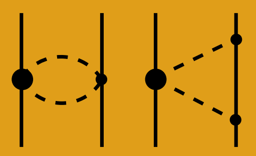
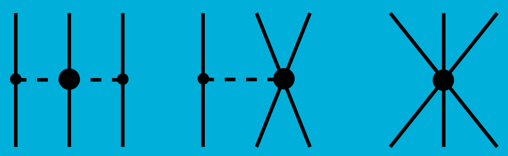
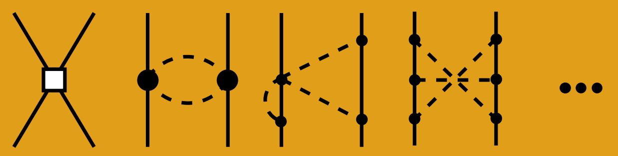
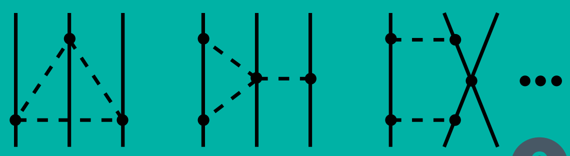
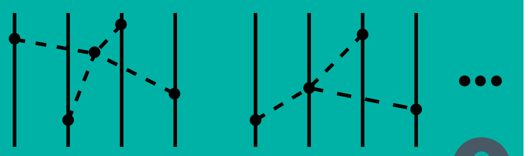
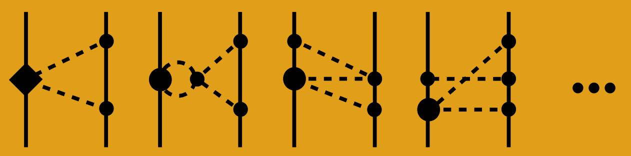
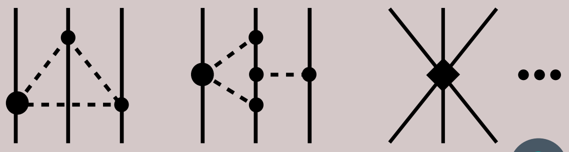
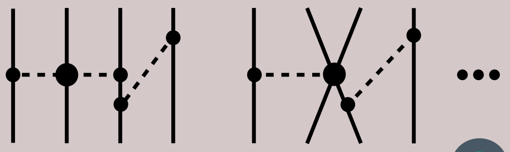
$$^{56}Fe \rightarrow X^{56}_{\Lambda} \rightarrow Y^{56}_{\Lambda\Lambda} \rightarrow \ldots \rightarrow {}^{56}(SQM)$$

Proliferation of EoS



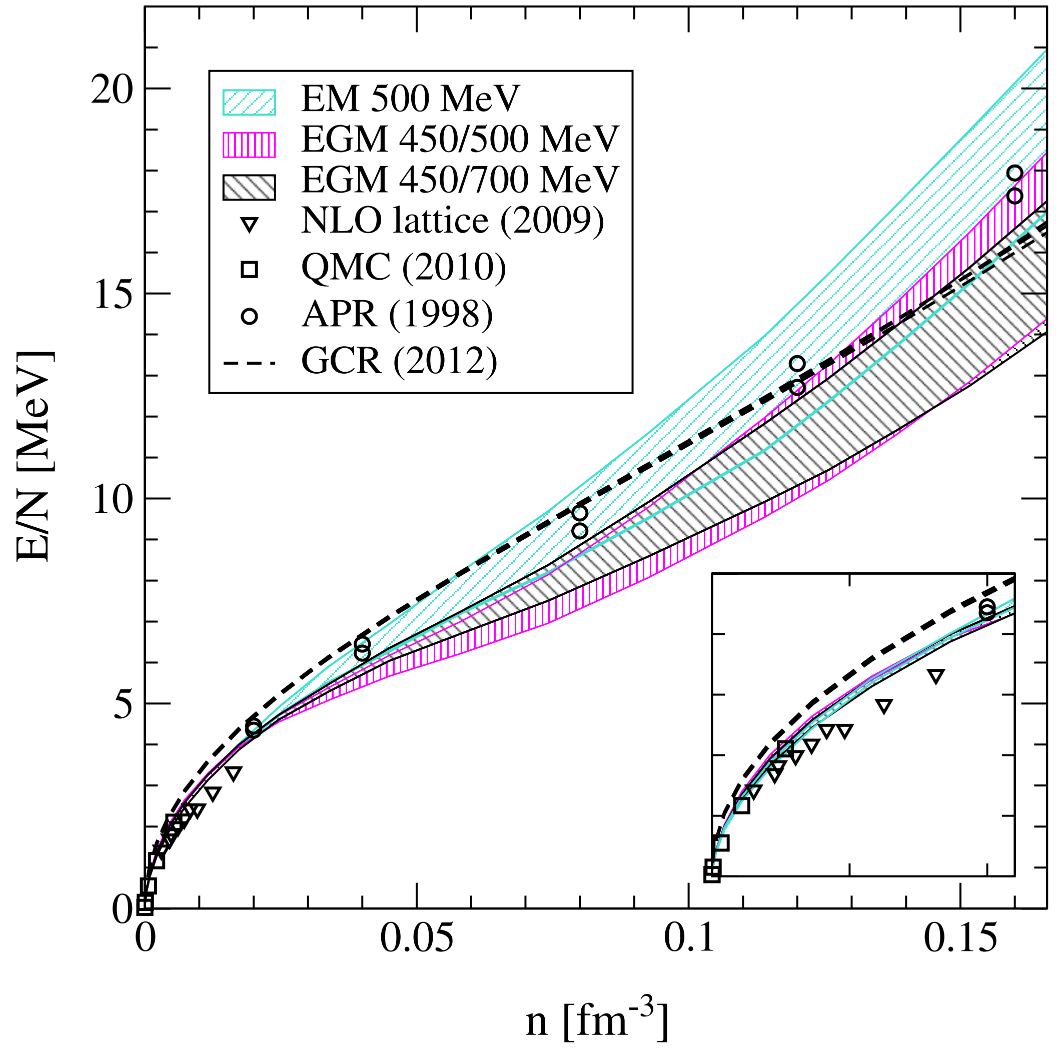
Credit: N. Wex

Chiral EFT

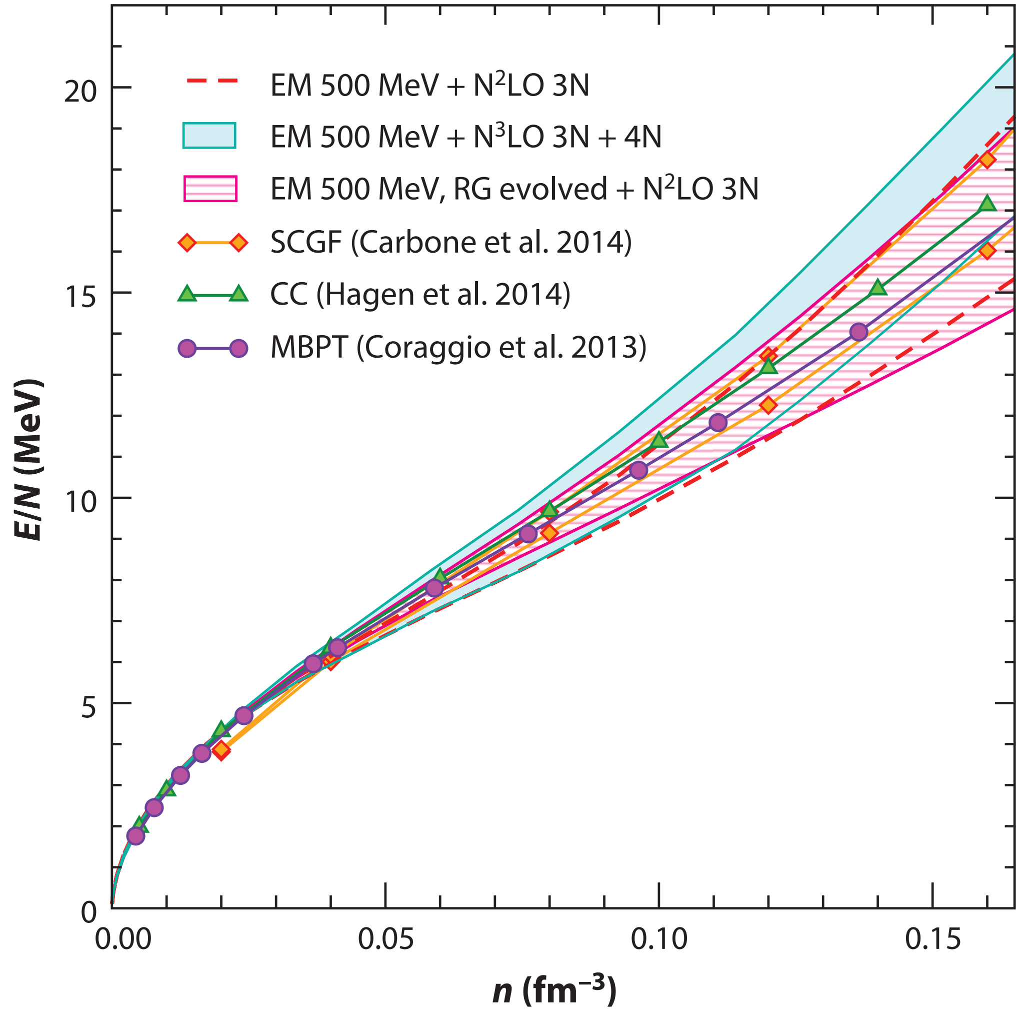
	NN forces	3N forces	4N forces
LO (Q^0)	 2	—	—
NLO (Q^2)	 7	—	—
N^{2}LO (Q^3)	 0	 2	—
N^{3}LO (Q^4)	 12	 0	 0
N^{4}LO (Q^5)	 0	 ?	 ?

➪ No new 3N and 4N LECs in neutron matter until N^{4}LO

Equation of state of neutron matter



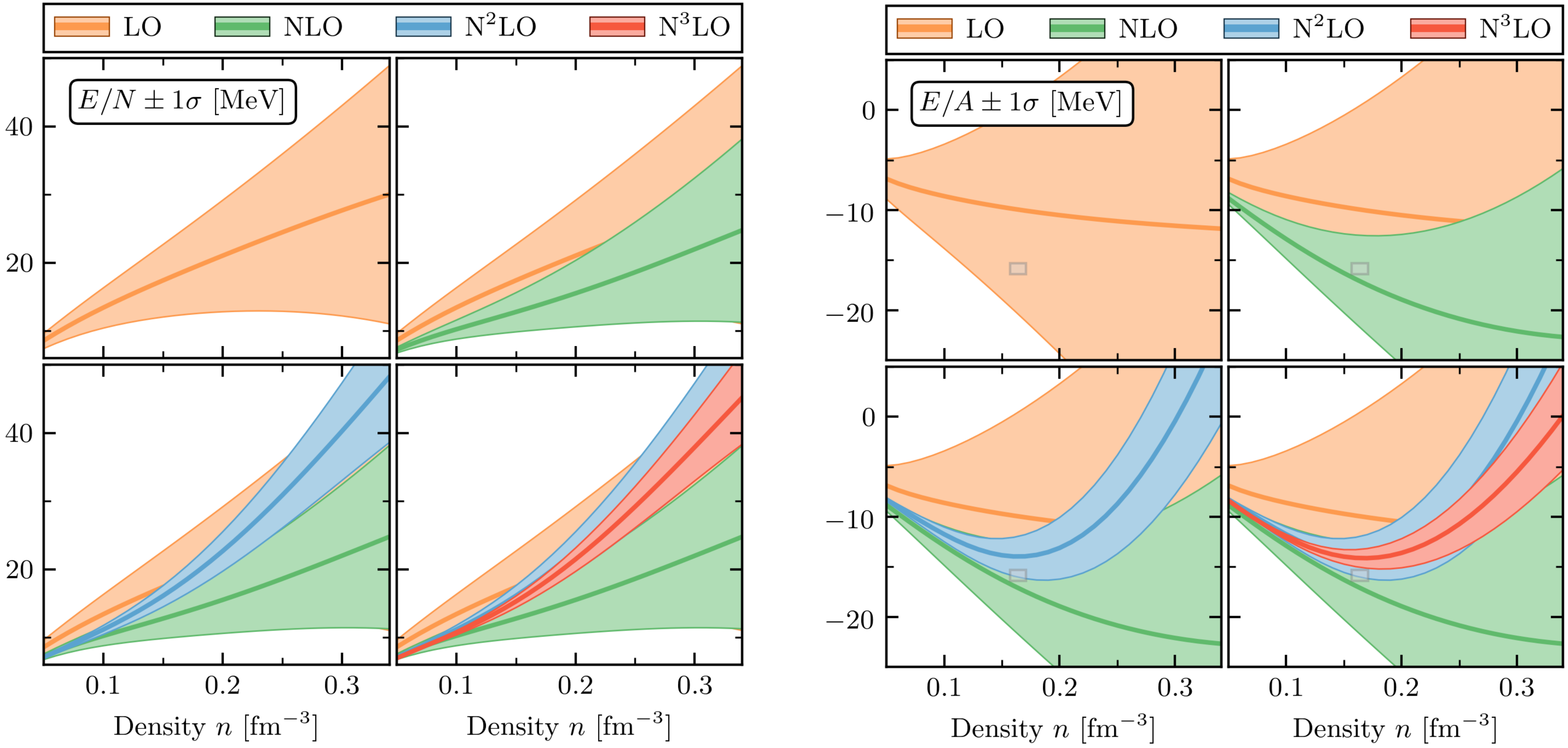
[Tews *et al.* 2013]



[Hebeler *et al.* 2015]

- Benchmarks between several calculations (perturbative & non-perturbative)
- Low-density part constrained by exact results
- Uncertainties grow with density → breakdown of chiral expansion

Equation of state of neutron & symmetric matter

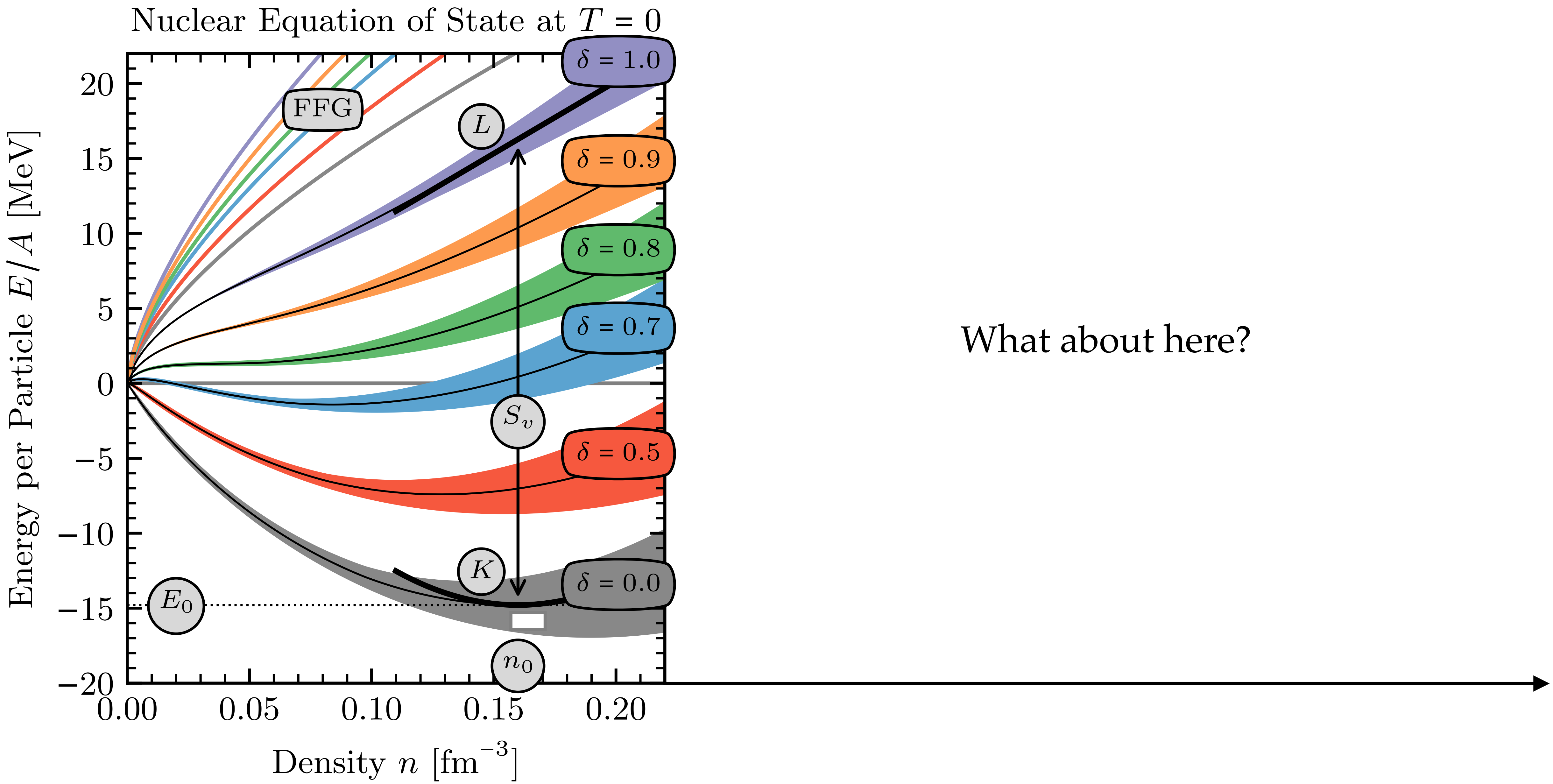


[Drischler *et al.* 2021]

- ⊙ Order-by-order convergence of chiral interactions
- ⊙ Going beyond N^{3}LO currently problematic

Limited applicability of chiral EoS

- Asymmetric matter can be computed as well…



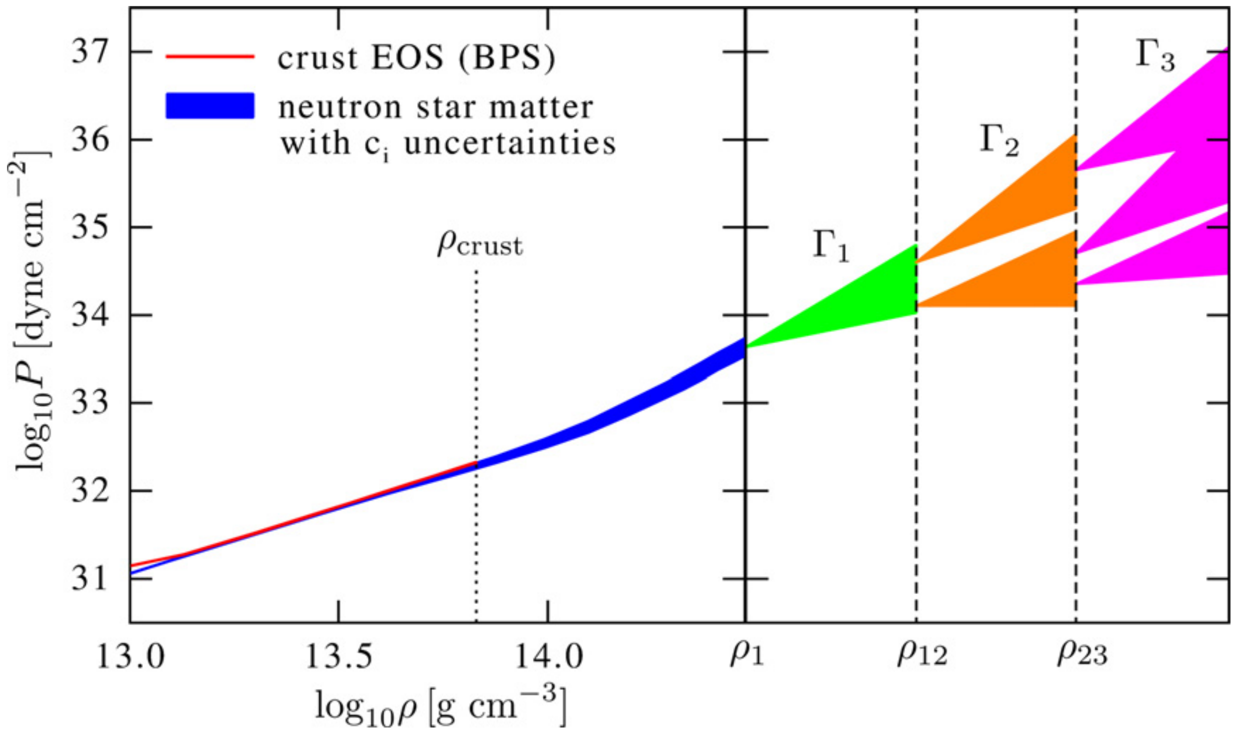
What about here?

[Drischler *et al.* 2021]

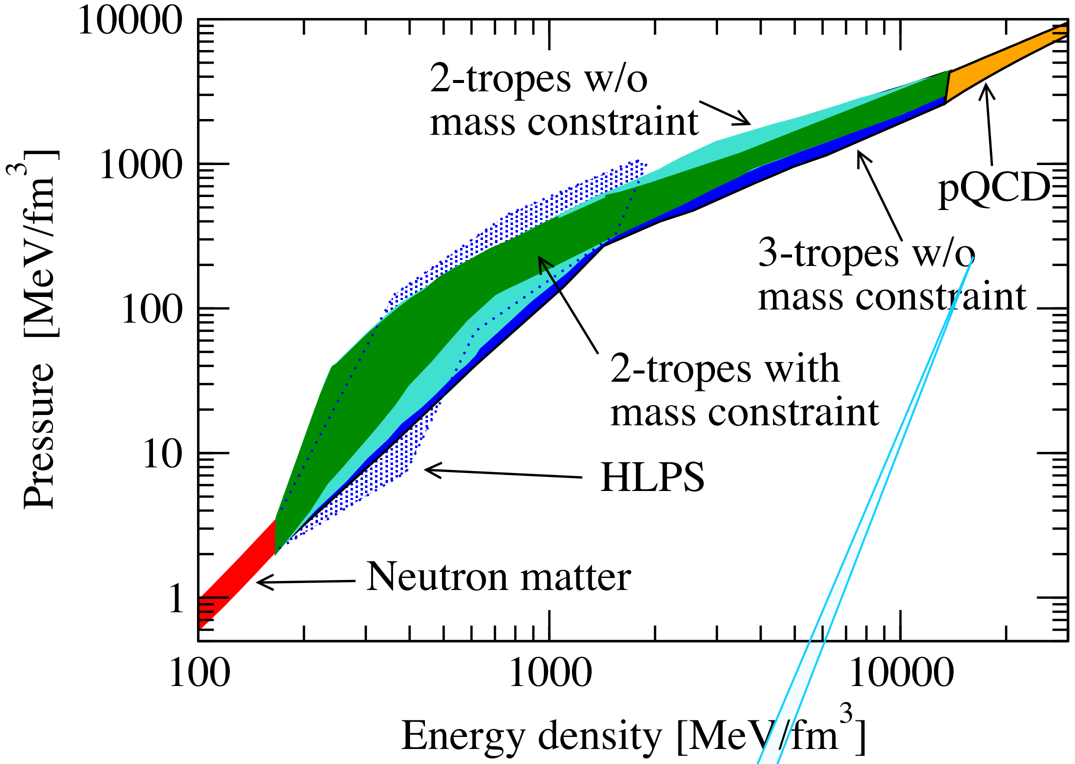
High-density extrapolations

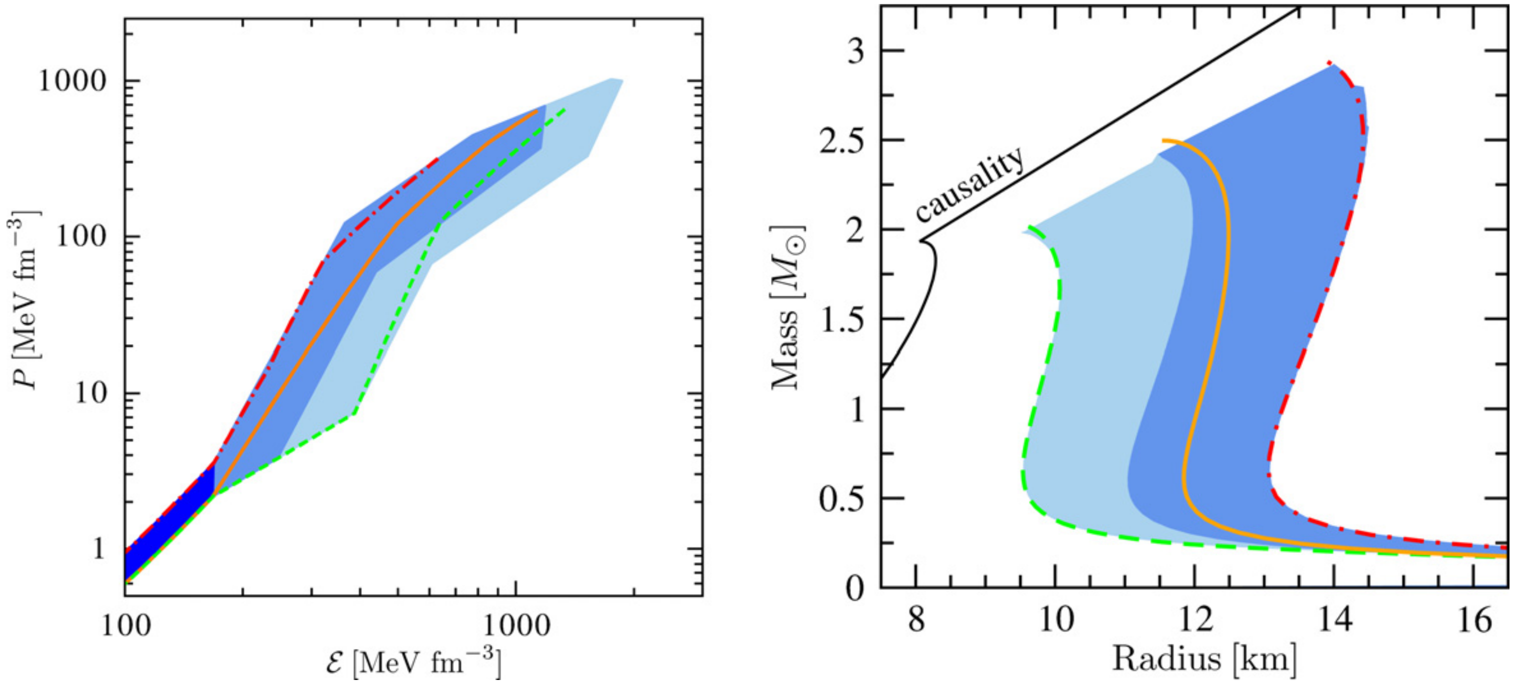
- Agnostic approach: series of piecewise polytropes

[Hebeler *et al.* 2010]



[Kurkela *et al.* 2014]



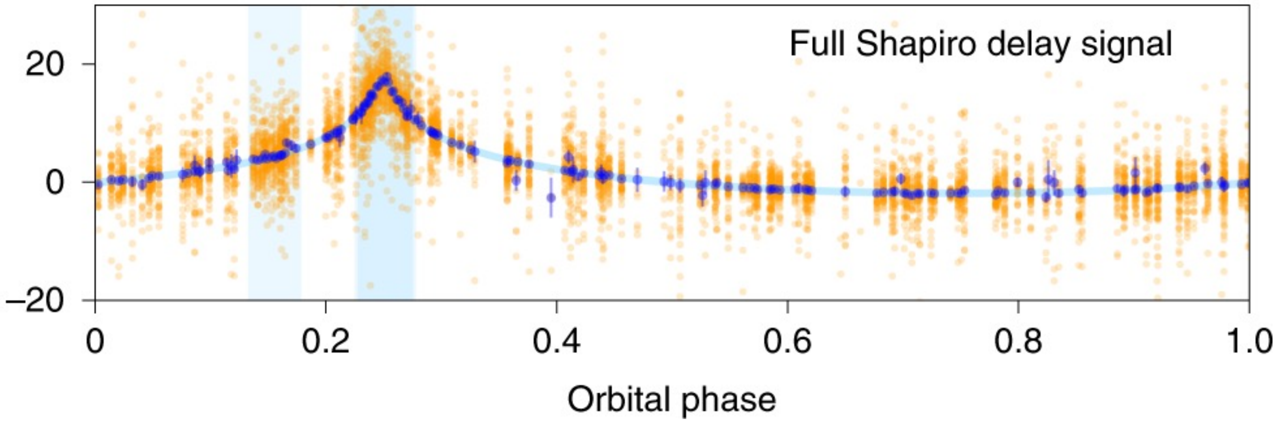


One can further constrain the very high density region

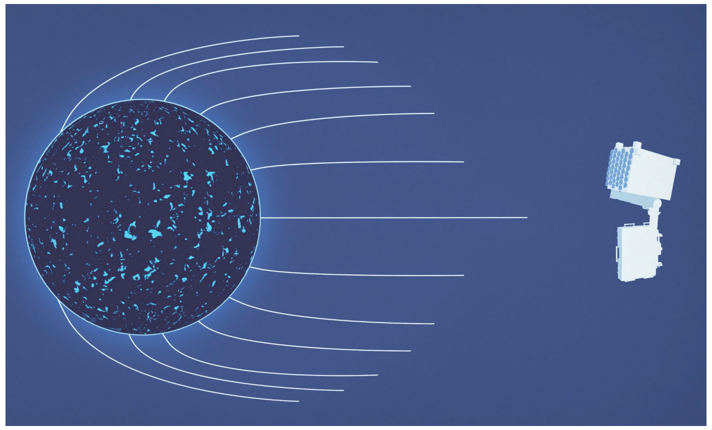
→ Perturbative QCD can be applied

Astrophysical constraints

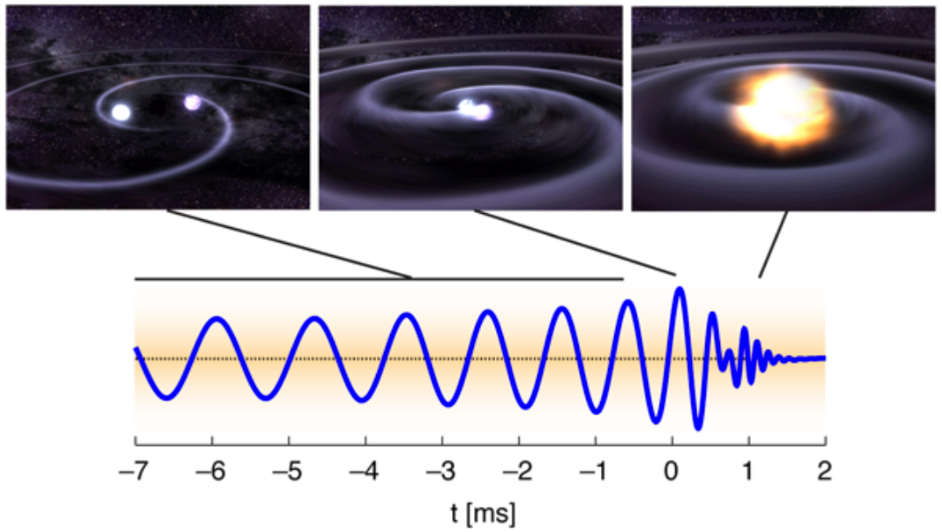
- **Measurements of NS masses**
  - Shapiro delay technique
  - Measurements of M > 2 M$_\odot$



- **Measurements of NS radii**
  - NICER experiment → rotation of hot spots
  - Measurements of R ~ 12-13 km

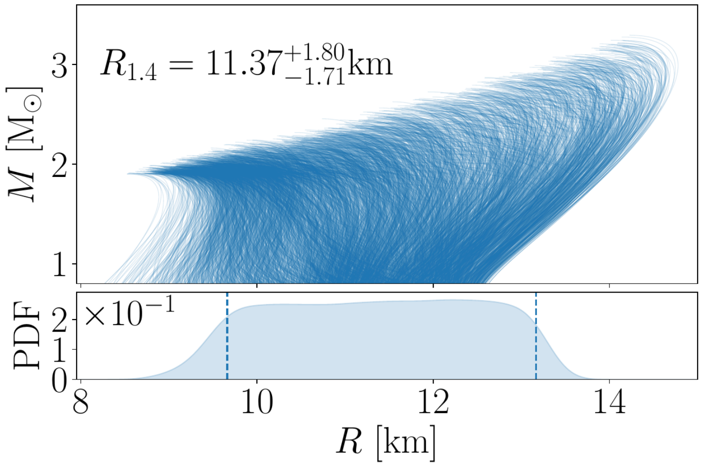


- **Gravitational waves / multi messenger observations**
  - GW signal → tidal deformability (→ radius)
  - Oscillation frequency of merger remnant → radius
  - Fate of merger remnant → maximum mass

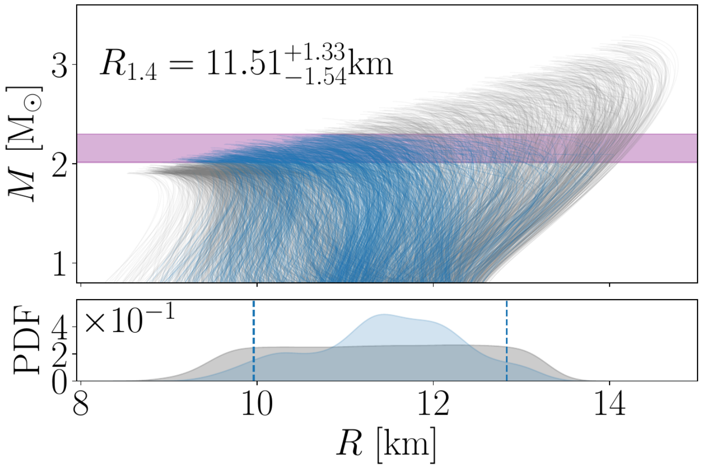


Multi-step analysis

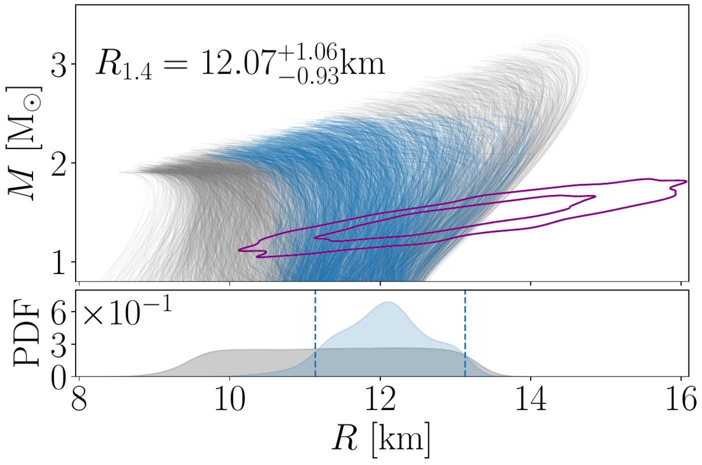
(A) Chiral effective field theory:
EOS derived with the chiral EFT framework



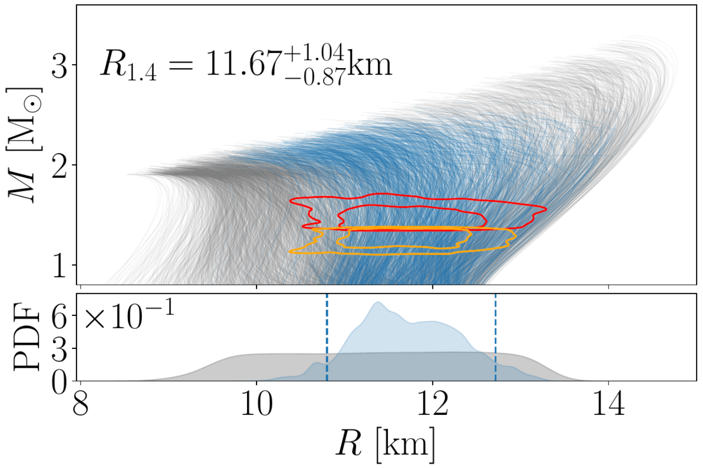
(B) Maximum Mass Constraints:
PSR J0740+6620/ PSR J0348+4032/ PSR J1614-2230 and GW170817/AT2017gfo remnant classification



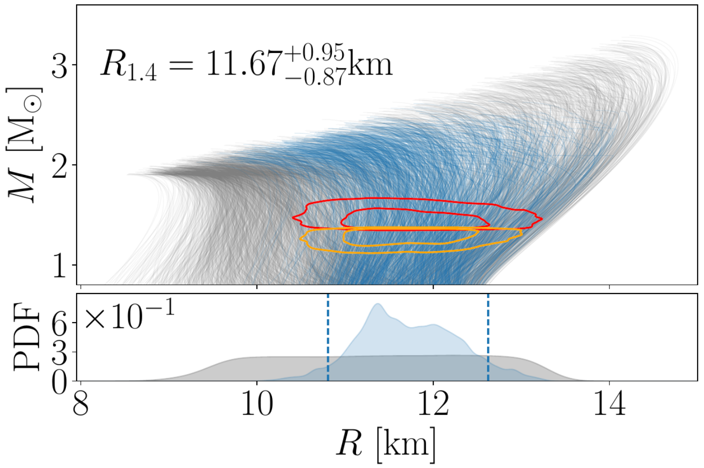
(C) NICER:
PSR J0030+0451



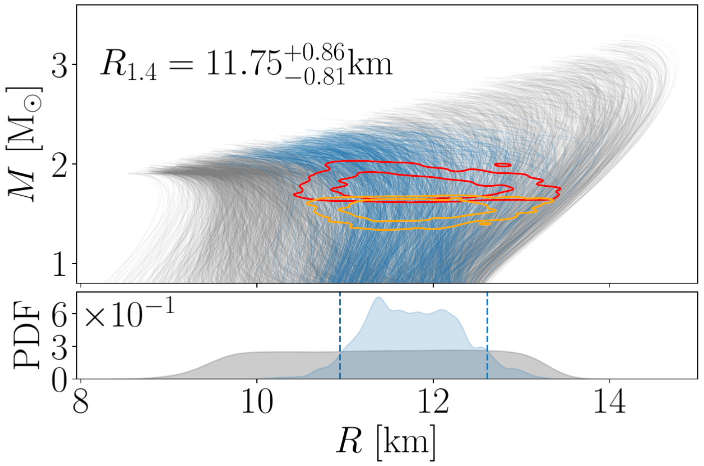
(D) GW170817:
reanalysis with IMRPhenomPv2_NRTidalv2



(E) AT2017gfo:
analysis of the observated lightcurves



(F) GW190425:
reanalysis with IMRPhenomPv2_NRTidalv2



[Dietrich *et al.* 2020]

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