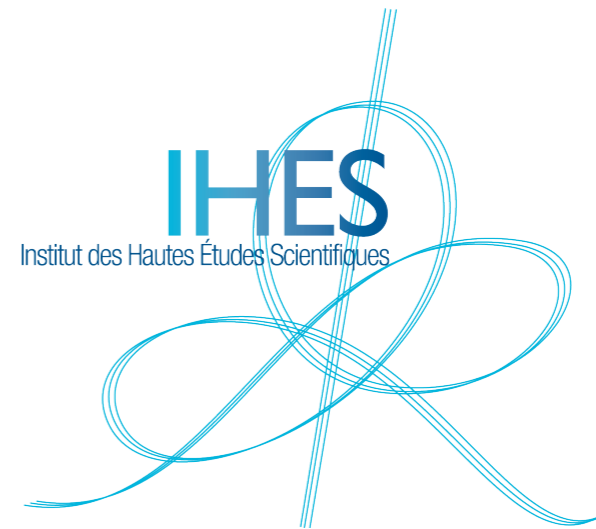


Naturalness of vanishing black-hole tides

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Based on *2510.20694* + *work in progress*

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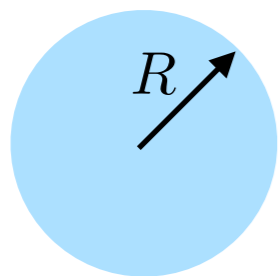
Motivations

- Tidal response and *Love numbers*.

Deformability of a body under external tidal fields. Gravitational analog of polarizability.

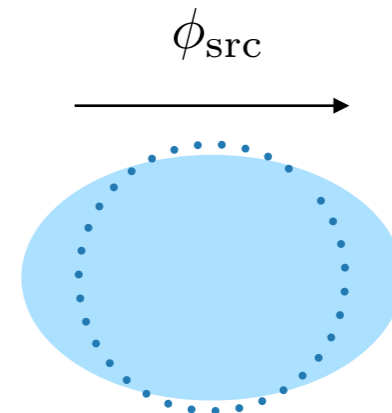
- Consider Newtonian gravity to begin with:

$$\nabla^2 \phi = 0 \quad (\text{exterior})$$



+

$$\phi_{\text{src}} = \sum_{\ell} a_{\ell} r^{\ell}$$



$$\phi_{\text{tail}} = \sum_{\ell} a_{\ell} r^{\ell} \left(1 + k_{\ell} \left(\frac{R}{r} \right)^{2\ell+1} \right)$$

$$k_{\ell} \sim \mathcal{O}(1) \quad \text{Love number}$$

Motivations

- *A naturalness puzzle* in classical physics.

Do black holes acquire induced multipoles under external tidal fields ?

- Yes if the tidal fields are time-dependent.

- No if the tidal fields are static, but only in $D=4$



Known for linear perturbations +
some nonlinear ones

Fang, Lovelace ; Damour, Nagar ; Bennington, Poisson ; Kol, Smolkin ; Le Tiec, Casals; Chia ;

Hui, Joyce, Penco, Santoni, Solomon; Riva, Vernizzi, Savic + [...]

- Valid only for black holes (cfr. neutron stars).

- This is puzzling! Seems to violate naturalness.

The problem becomes sharp in the worldline EFT formulation.

Goldberger, Rothstein

Motivations

- *Why* does this happen?

Some great work has been done to address this puzzle in BH perturbation theory:

- First contributions: NRG decomposition, isotropic coordinates and \mathbb{Z}_2

Kol and Smolkin *JHEP* 02 (2012) 010

- Linear perturbations : near zone symmetries and ladder symmetries

Dubovsky, Ivanov, Charalambous + Hui, Joyce, Penco, Santoni, Solomon

- Nonlinear axisymmetric perturbations (subclass of “electric” tides)

Combaluzier-Szteinszneider, L. Hui, L. Santoni, A. R. Solomon, S. S. C. Wong + Kehagias, Riotto

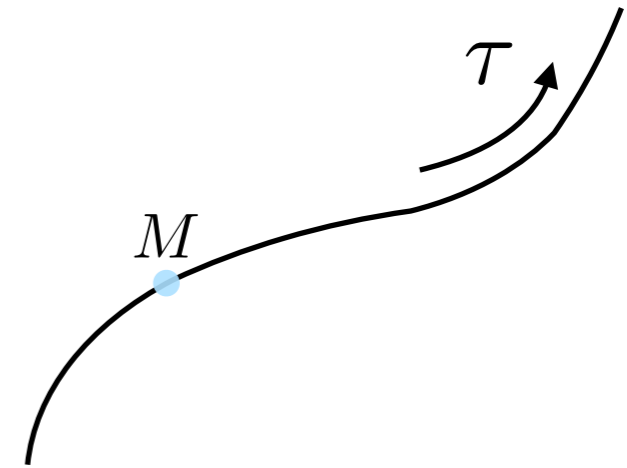
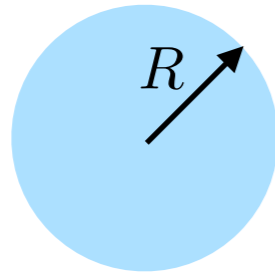
- Still many questions.

- Is the property valid to full nonlinear order, including for “magnetic” tides?

- Why is there no running from the EFT point of view?

- How is the symmetry realized in the worldline formalism?

The Worldline EFT (non-spinning objects)



- Massive point particle

$$S_{pp} = -M \int d\tau \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$

$$\dot{x}^\mu = \frac{dx^\mu}{d\tau}$$

- Bulk action: gravity in 4 dimension

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$$

The Worldline EFT (non-spinning objects)

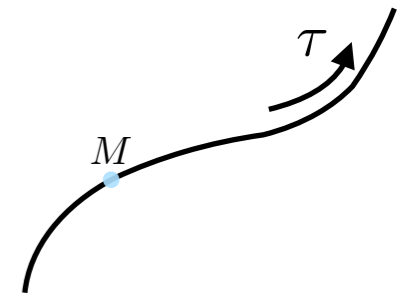
- Ricci-flat background: non-minimal couplings defined in terms of Weyl tensor

$$E_{\mu\nu} = C_{\mu\rho\nu\sigma} \dot{x}^\rho \dot{x}^\sigma$$

electric

$$B_{\mu\nu} = \tilde{C}_{\mu\rho\nu\sigma} \dot{x}^\rho \dot{x}^\sigma$$

magnetic



$$\dot{x}^\mu = \delta_0^\mu$$

rest frame

$$\left. \begin{aligned} (\partial E)_\ell &\equiv \partial_{\langle i_1 \dots \partial_{i_{\ell-2}} E_{i_{\ell-1} i_\ell} \rangle} \\ (\partial B)_\ell &\equiv \partial_{\langle i_1 \dots \partial_{i_{\ell-2}} B_{i_{\ell-1} i_\ell} \rangle} \end{aligned} \right] \text{angular momentum } \ell \geq 2$$

$\langle \dots \rangle$ symmetric and traceless combination

- Finite size terms: intrinsic multipoles

$$S_{\text{mult}} = \int d\tau (Q^{ij} E_{ij} + \dots) \quad \longrightarrow \quad \text{these are absent for black holes}$$

The Worldline EFT (non-spinning objects)

- Finite size terms: tidal effects



Static response

$$S_{\text{Love}}^{(1)} = \int d\tau \sum_{\ell} \left[\lambda_{\ell}^{(E)} (\partial E)_{\ell}^2 + \lambda_{\ell}^{(B)} (\partial B)_{\ell}^2 \right] \quad \longrightarrow \quad \text{linear Love numbers}$$

$$S_{\text{Love}}^{(n)} = \int d\tau \sum_{\vec{\ell}} \left[\lambda_{\vec{\ell},n}^{(E)} \left((\partial E)_{\ell_1} \dots (\partial E)_{\ell_{n+1}} \right) + \right. \\ \left. \lambda_{\vec{\ell},n}^{(B)} \left((\partial B)_{\ell_1} \dots (\partial B)_{\ell_{n+1}} \right) + \text{mixed} \right] \quad \longrightarrow \quad \text{nonlinear Love numbers}$$

Dynamic response

$$S_{\text{diss.}} = \int d\tau \sum_{\ell} \left[\lambda_{\ell,\omega}^{(E)} (\partial \dot{E}_{-})_{\ell} (\partial E_{+})_{\ell} + \dots \right] \quad \longrightarrow \quad \text{dissipation numbers}$$

$$S_{\text{dynamic}} = \int d\tau \sum_{\ell} \left[\lambda_{\ell,\omega^2}^{(E)} (\partial \dot{E})_{\ell} (\partial \dot{E})_{\ell} + \dots \right] \quad \longrightarrow \quad \text{dynamic Love numbers}$$

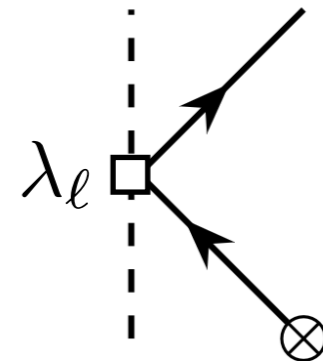
Matching and running effects in worldline EFT

- Matching UV theory and EFT

Full theory: response theory for BH perturbations

$$\mathcal{D}^2 \phi_\ell(r) = 0$$

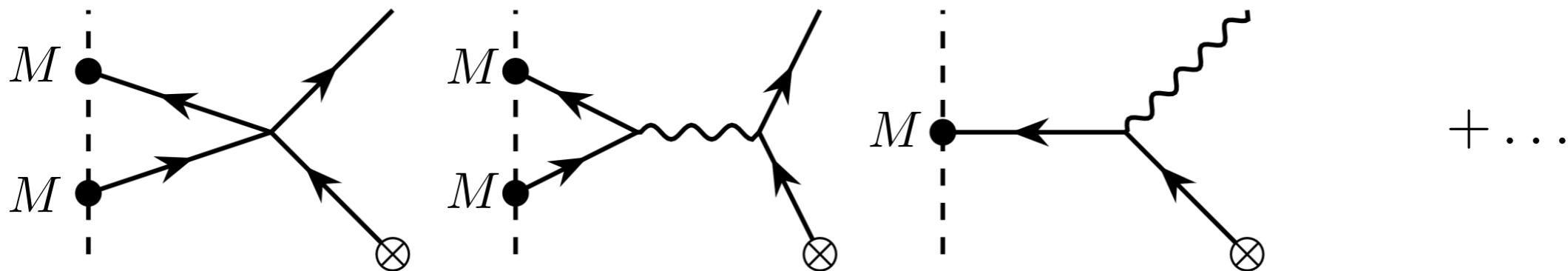
↔
matching



Kol, Smolkin ; Hui, Joyce, Penco, Santoni, Solomon; [...]

- Nonlinearities in the worldline EFT

→ Expect mixing and running!



see e.g. Ivanov, Zhou for probe scalar

Our proposal

- There is an accidental symmetry in the static sector of GR (at classical level)
- Some of its aspects are unique to $D=4$
- Broken by worldline coupling to the mass, but gives useful selection rules for BHs response
 - Implies radiative stability of Love numbers: no running (spurion analysis)
 - Analytic contributions are controlled: vanishing of Love numbers for BH in $D=4$
- Valid for both E, B at full nonlinear order

Time-independent metrics: Kaluza-Klein-like reduction

$$ds^2 = -e^{2\phi}(dt - A_i dx^i)^2 + e^{-2\phi} \gamma_{ij} dx^i dx^j$$

$$S_{\text{EH}} = -\frac{1}{16\pi G} \int d^4x \sqrt{\gamma} \left(-R^{(3)}[\gamma] + 2(\partial_i \phi)^2 - \frac{1}{4} e^{4\phi} F_{ij}^2 \right) + \mathcal{O}(\partial_t^2)$$

↓
3D

NB: this includes *stationary* metrics! Important to probe magnetic tides.

An accidental symmetry in the time-independent sector of GR

In $D = 4$ we can dualize the “graviphoton” A_i to a pseudo scalar “axion” $a(x)$:

$$\frac{1}{2} \varepsilon_i^{jk} F_{jk} = e^{-4\phi} \partial_i a$$

$$2(\partial_i \phi)^2 - \frac{1}{4} e^{4\phi} F_{ij}^2 \quad \longrightarrow \quad 2(\partial_i \phi)^2 + \frac{1}{2} e^{-4\phi} (\partial_i a)^2$$

Introduce a complex scalar field: $z = a + ie^{2\phi}$ ($\text{Im } z > 0$)

$$S_{\text{EH}} = -\frac{1}{16\pi G} \int d^4x \sqrt{\gamma} \left(-R^{(3)}[\gamma] + \frac{1}{2} \frac{\partial_i z \partial^i \bar{z}}{(\text{Im } z)^2} \right) + \mathcal{O}(\partial_t^2)$$

$\text{SL}(2, \mathbb{R})/\text{SO}(2)$ Nonlinear sigma model! (Ehlers group)

accidental

Recognized as solution generating symmetry in work by Buchdahl, Ernst, Ehlers, Geroch

SL(2,ℝ) symmetry

Symmetry action: $z \rightarrow \frac{az + b}{cz + d} \quad ad - bc = 1$

Generated by: $S : z \rightarrow -\frac{1}{z}, \quad T : z \rightarrow z + b$

In addition there is parity: $P : z \rightarrow -\bar{z}$

We can use a conformal map from the upper-half plane to the unit disk

$$w = \frac{z - i}{z + i} = \phi - i\frac{a}{2} + \dots \quad S : w \rightarrow -w$$

$$S_{\text{EH}} = -\frac{1}{16\pi G} \int dt d^3x \sqrt{\gamma} \left(-R^{(3)}[\gamma] + 2\frac{\partial_i w \partial^i \bar{w}}{(1 - \bar{w}w)^2} \right) + O(\partial_t^2)$$

This $SL(2, \mathbb{R})$ was mainly known as a generating symmetry (at the level of e.o.m.).

We see that it is an ordinary accidental symmetry of the time-independent sector

Spurionic symmetry in the “UV theory”

Nonlinear realized symmetry

Minkowski vacuum:

$$\mathrm{SL}(2, \mathbb{R})/\mathrm{SO}(2)$$

$$z = i \quad \leftrightarrow \quad w = 0$$

$$\mathrm{SO}(2): \quad w \rightarrow e^{i\theta} w$$

Schwarzschild background in isotropic coordinates

$$ds^2 = - \left(\frac{1 - X}{1 + X} \right)^2 dt^2 + (1 + X)^4 \delta_{ij} dx^i dx^j$$

$$X \equiv \frac{GM}{2r}$$

$$w_{\mathrm{Schw}} = -\frac{2X}{1 + X^2}, \quad \gamma_{ij} = (1 - X^2)^2 \delta_{ij}$$

The symmetry is broken.

We can restore S by assigning spurionic transformation rule

$$S : \begin{aligned} w &\rightarrow -w \\ M &\rightarrow -M \end{aligned}$$

The solution corresponds to that computed diagrammatically in the worldline EFT (series in GM)

Worldline coupling and spurionic symmetry

Worldline minimal coupling:

$$S_{pp} = -M \int d\tau e^\phi \supset \underbrace{-M \int d\tau \phi + \dots}$$

only term relevant for classical response computation

We can rewrite it in terms of our field variable w

$$S_{pp} = -M \int d\tau \operatorname{Re} w + \dots$$

$w \rightarrow -w$ is broken but we can assign spurionic transformation rule to M

$$S : w \rightarrow -w, \quad M \rightarrow -M$$

We can also rewrite the Weyl tensor as $E_{ij} - iB_{ij} = \partial_i \partial_j w + (\partial^2 \gamma)_{ij} + \dots$

Selection rules for static Love numbers

We can phrase all our computations in terms of w .

$$X \equiv \frac{GM}{2r}$$

$$w_{\text{tail}} \sim w_{\text{src}} \left(1 + \dots + k_\ell X^{2\ell+1} + \dots \right) \quad \longrightarrow \quad \text{linear Love numbers}$$

We use the symmetry

$$S : w \rightarrow -w, \quad M \rightarrow -M$$

$$w_{\text{src}} \left(1 + \dots + k_\ell X^{2\ell+1} + \dots \right) \rightarrow -w_{\text{src}} \left(1 + \dots - k_\ell X^{2\ell+1} + \dots \right)$$

This spurion analysis implies that LN are not renormalized to all orders in the worldline EFT.

Naturalness

In $D=4$ the LN response is analytic in X (perturbative in GM), therefore static Love Numbers also have to be zero.

Vanishing

Selection rules for static Love numbers

We can easily generalize the analysis to the nonlinear response.

$$S_{\text{Love}}^{(n)} = \int d\tau \sum_{\vec{\ell}} \left[\lambda_{\vec{\ell},n}^{(E)} \left((\partial E)_{\ell_1} \dots (\partial E)_{\ell_{n+1}} \right) + \lambda_{\vec{\ell},n}^{(B)} \left((\partial B)_{\ell_1} \dots (\partial B)_{\ell_{n+1}} \right) + \text{mixed} \right]$$

$$\vec{\ell} \equiv (\ell_1, \dots, \ell_{n+1})$$

$$|\vec{\ell}| = \sum_{i=0}^{n+1} \ell_i$$

Indices are contracted with γ_{ij} \longrightarrow $|\vec{\ell}|$ even

$$w_{\text{tail}} \sim \left(\prod_{\alpha=1}^n w_{\alpha} \right) \left(\dots + k_{\vec{\ell},n}^w X^{|\vec{\ell}|+1} + \dots \right) \quad \begin{array}{l} n \text{ odd} \\ \longrightarrow \\ k_{\vec{\ell},n}^w = 0 \end{array}$$

$$\gamma_{\text{tail}} \sim \left(\prod_{\alpha=1}^n w_{\alpha} \right) \left(\dots + k_{\vec{\ell},n}^{\gamma} X^{|\vec{\ell}|+1} + \dots \right) \quad \begin{array}{l} n \text{ even} \\ \longrightarrow \\ k_{\vec{\ell},n}^{\gamma} = 0 \end{array}$$

Static LN are not renormalized to all orders in the worldline EFT and vanish in D=4.

Accidental symmetry violations

- The symmetry is broken by time-dependent terms (with even number of derivatives).

Dynamic Love Numbers are not zero, but leading

$$S_{\text{dis.}} = \int d\tau \sum_{\ell} \left[\lambda_{\ell, \omega}^{(E)} (\partial \dot{E}_-)_\ell (\partial E_+)_\ell + \dots \right]$$

Dissipative Tides (linear and nonlinear) are not renormalized.

-
- Neutron stars and other compact objects can have non-vanishing LN.

In the worldline: there are non-minimal couplings (and new dimensionful scales beyond M)

Universal non-linearities from the leading point-particle coupling (“loops”),

do not induce any running.

-
- The symmetry is broken by higher curvature terms, e.g. R^3 , cosmological constant Λ , quantum corrections, couplings to generic matter fields (e.g. fermions)

Extensions

- In the higher dimensional case: we cannot dualize the vector to a scalar.

There is still a \mathbb{Z}_2 symmetry but restricted only to electric and tensor-type operators.

Power counting is different because $X = \frac{1}{4} \left(\frac{R_s}{r} \right)^{D-3} \propto GM$

$$g_{\text{tail}} \sim g_{\text{src}} \left(1 + \dots + k_\ell X^{2\hat{\ell}+1} + \dots \right) \quad \hat{\ell} = \frac{\ell}{D-3}$$

$$\hat{\ell} \in \mathbb{N}$$

Vanishing

$$\frac{2\hat{\ell} + 1}{2} \in \mathbb{N}$$

Running

$$2\hat{\ell} \notin \mathbb{N}$$

Non-renormalization

$$\phi \rightarrow -\phi, \quad M \rightarrow -M$$

Kol and Smolkin

- We can also extend the analysis to the case of a shift-symmetric scalar field coupled to gravity, in Schwarzschild background.

Relation to no-hair theorem + vanishing static LN for nonlinear even tides.

Selection rules for shift-symmetric scalars

Consider the action of a shift-symmetric scalar with minimal coupling

$$S_\psi = - \int d^4x \sqrt{\gamma} e^{-2\phi} P\left(e^{2\phi} \gamma^{ij} \partial_i \psi \partial_j \psi\right)$$

$\psi \rightarrow -\psi$ is an exact symmetry.

$w \rightarrow -w$, $M \rightarrow -M$ is a symmetry for a free scalar, or for the leading quadratic terms.

$$S_{\psi, \text{Love}}^{(n)} = \int d\tau \sum_{\vec{\ell}} \lambda_{\vec{\ell}, n}^{(\psi)} \left((\partial\psi)_{\ell_1} \dots (\partial\psi)_{\ell_{n+1}} \right)$$

$$\vec{\ell} \equiv (\ell_1, \dots, \ell_{n+1})$$

$$|\vec{\ell}| = \sum_{i=0}^{n+1} \ell_i$$

$$\psi_{\text{tail}} \sim \left(\prod_{\alpha=1}^n \psi_\alpha \right) \left(c_n + \dots + k_{\vec{\ell}, n}^{(\psi)} X^{|\vec{\ell}|+1} + \dots \right)$$

The response vanishes for any even n , because of $\psi \rightarrow -\psi$ symmetry. → Includes no-hair

The LN vanish for $n = 1$ for interacting scalar, because of $w \rightarrow -w$ symmetry.

Conclusions

- We presented an accidental symmetry of the static sector of GR
 - We explained how the symmetry is broken in a black hole background but can be restored assigning spurionic transformation rules to M
 - The selection rules imply non-renormalization (naturalness) and vanishing
 - The results are fully nonlinear and apply to both electric and magnetic tides
 - This symmetry is consistent with all previous results in $D=4$ and higher dim, and with all non-vanishing results
-
- The construction can be extended to EM charged black holes + also NUT charge
 - Extension to spinning BHs ?