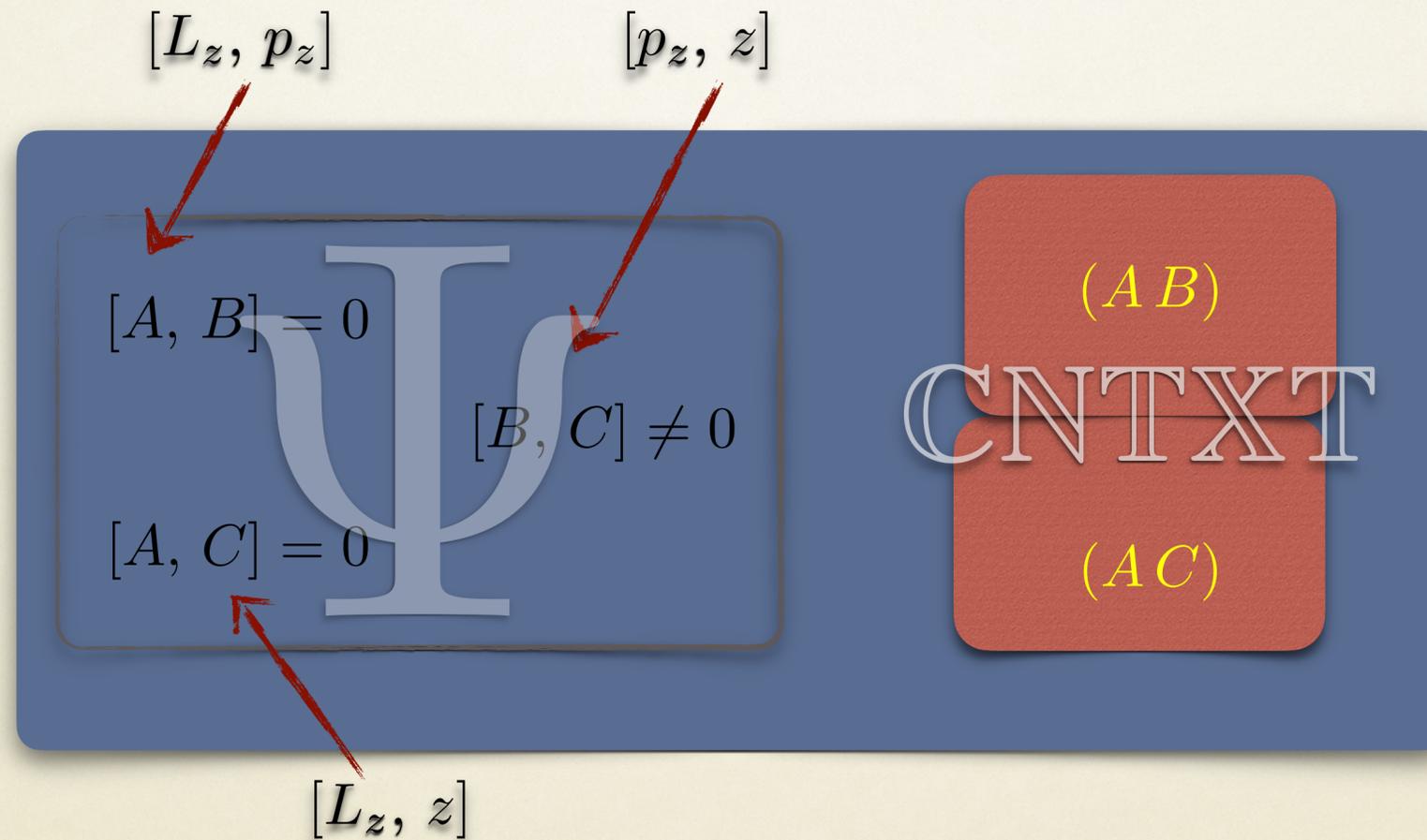


Io stímo piú il trovar un vero, benché di cosa leggiera,
che 'l dísputar lungamente delle massíme questioní
senza conseguír verità nissuna.

1

Quantum contextuality

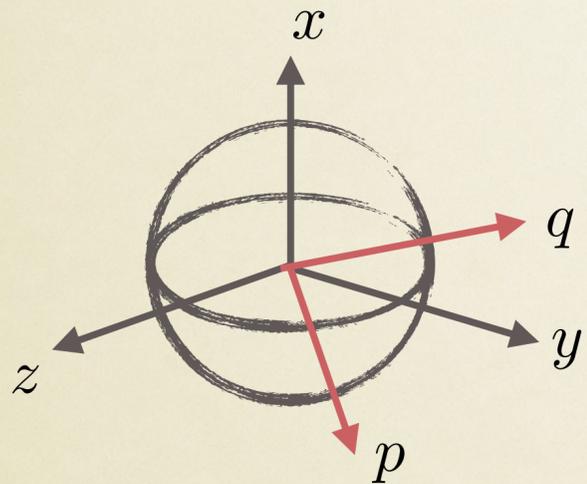


J. S. Bell, *On the Problem of Hidden Variables in Quantum Mechanics*, *Rev. Mod. Phys.* **38** (1966) 447–452.

S. Kochen and E. Specker, *The Problem of Hidden Variables in Quantum Mechanics*, *J. Math. Mech.* **1** (1968) 59–87.

Bell-Kochen-Specker theorem

Quantum contextuality



$$S^2 = S_x^2 + S_y^2 + S_z^2$$

$$S^2 = 2$$

commuting

$$[S_x^2, S_y^2] = [S_y^2, S_z^2] = [S_z^2, S_x^2] = 0$$

$$[S_x^2, S_p^2] = [S_p^2, S_q^2] = [S_q^2, S_x^2] = 0$$

$$[S_z^2, S_r^2] \neq 0$$

$$[S_y^2, S_p^2] \neq 0$$

the context

$$S_z^2 = 0 \quad S_y^2 = 1$$

$$S_p^2 = 1 \quad S_q^2 = 1$$



$$S_x^2 = 1$$

$$S_x^2 = 0$$

$$\langle \overset{-}{\mathcal{O}}_1 \overset{+}{\mathcal{O}}_2 \rangle + \langle \overset{+}{\mathcal{O}}_2 \overset{-}{\mathcal{O}}_3 \rangle + \langle \overset{-}{\mathcal{O}}_3 \overset{+}{\mathcal{O}}_4 \rangle + \langle \overset{+}{\mathcal{O}}_4 \overset{-}{\mathcal{O}}_5 \rangle + \langle \overset{-}{\mathcal{O}}_5 \overset{-}{\mathcal{O}}_1 \rangle \geq -3 .$$

$$\mathcal{O}_i = 1 - 2\Pi_i$$

$$\text{CNT}\chi\text{T}_N \equiv \sum_{i=1}^N \langle \Pi_i \rangle = \sum_{i=1}^N \text{Tr}(\rho \Pi_i) \leq c_N ,$$

$$c_5 = 2 \quad c_9 = 3$$

$$\text{CNT}\chi\text{T}' \equiv \langle (\sigma_3 \otimes \sigma_1)(\sigma_1 \otimes \sigma_3) \rangle - \langle (\sigma_3 \otimes \sigma_3)(\sigma_1 \otimes \sigma_1) \rangle \leq 0$$

experimental tests of contextuality

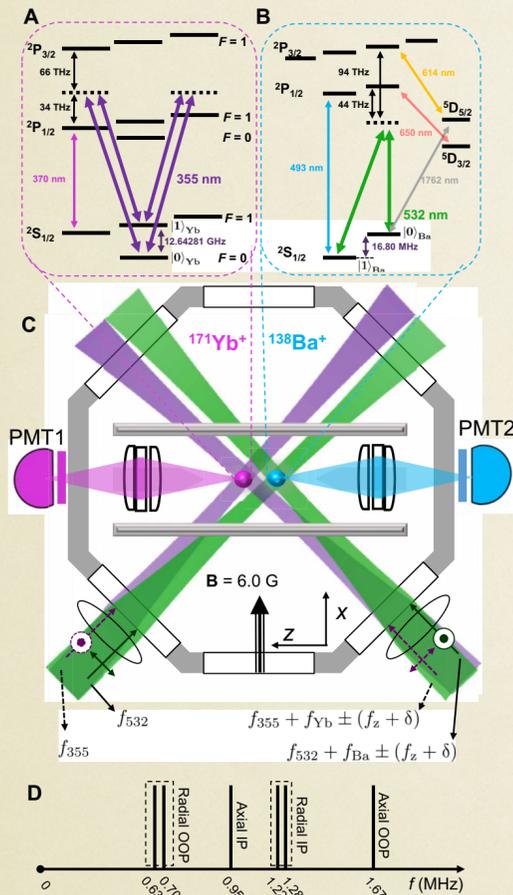


Fig. 2. Experimental setup. (A and B) are the energy level diagrams of $^{171}\text{Yb}^+$ and $^{138}\text{Ba}^+$ ion, respectively. Only relevant Raman transitions are shown here. (C) Ion trap in the octagon chamber and schematic diagram for Raman beams. Two different photomultiplier tubes (PMTs) with different spectral responses and filters are used to detect two ions fluorescence independently, which are located at the top and the bottom of the chamber in the actual experimental system. Solid and dashed arrows indicate the directions and the polarizations of 532- and 355-nm laser beams, respectively. In the figure, f_{Yb} and f_{Ba} are the qubit frequencies of $^{171}\text{Yb}^+$ and $^{138}\text{Ba}^+$, respectively; $f_z = 1.67$ MHz is the frequency of the axial out-of-phase (OOP) mode; and δ is the detuning of the laser from the OOP mode sideband, when δ is zero, then the Raman transition is directly red and blue sideband transitions. For the M-S gate, δ should match to the sideband Rabi frequency and determines the duration of the M-S interaction as $1/\delta$. $\delta = 22.0$ kHz here. (D) Frequencies of vibrational modes of a single $^{171}\text{Yb}^+$ and a single $^{138}\text{Ba}^+$ ions. Axial OOP mode is used for the Mølmer-Sørensen (M-S) interaction. IP, in-phase mode.

photons

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molecular
nuclear spin

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systems

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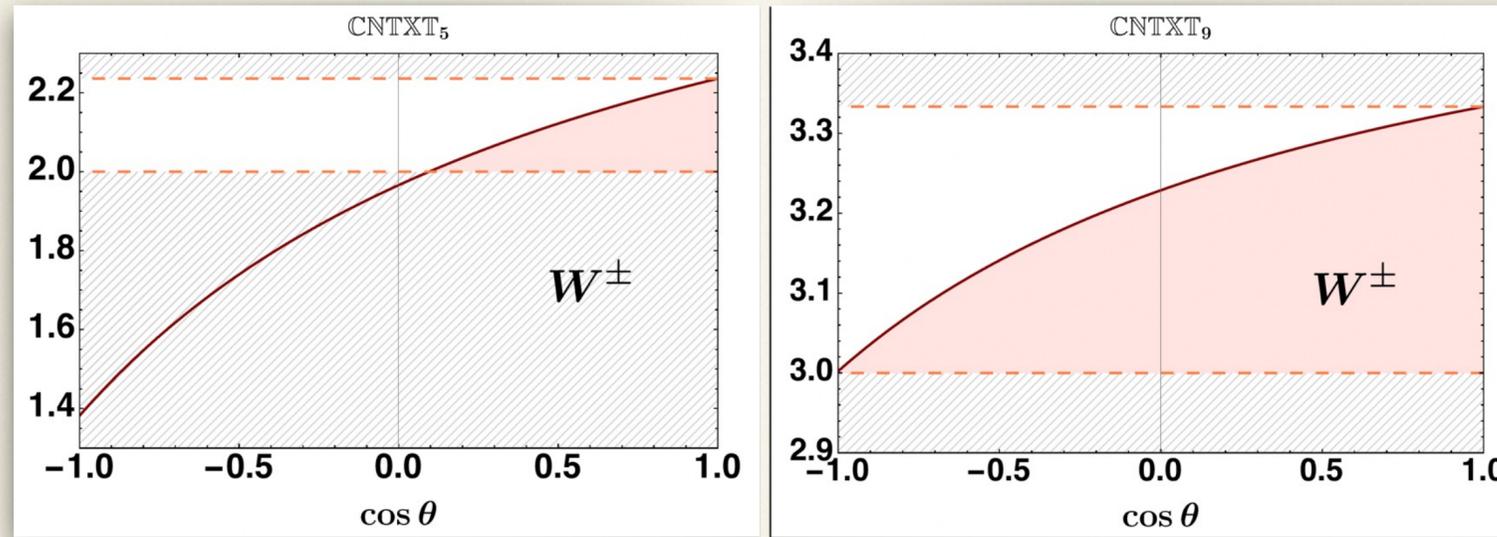
nuclear spins

S. B. van Dam, J. Cramer, T. H. Taminiou, R. Hanson, Multipartite entanglement generation and contextuality tests using nondestructive three-qubit parity measurements. *Phys. Rev. Lett.* **123**, 050401 (2019).

loopholes: sharpness, detection, compatibility

W^\pm

$t \rightarrow W^+ b$



M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum contextuality of spin-1 massive particles*, arXiv:2503.14587.

J. A. Aguilar-Saavedra and J. Bernabeu, *W polarisation beyond helicity fractions in top quark decays*, Nucl. Phys. B **840** (2010) 349–378, [arXiv:1005.5382].

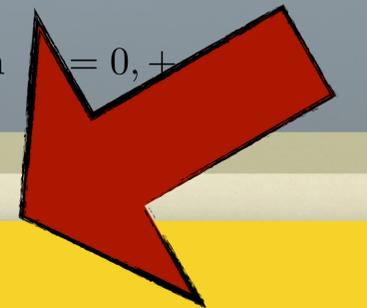
$$\rho_1 = \begin{pmatrix} \frac{1}{3} + \frac{s_3}{2} + \frac{t_0}{\sqrt{6}} & -\frac{s_1}{2} - \frac{t_1}{\sqrt{2}} & t_2 \\ -\frac{s_{-1}}{2} - \frac{t_{-1}}{\sqrt{2}} & \frac{1}{3} - \frac{2t_0}{\sqrt{6}} & -\frac{s_1}{2} + \frac{t_1}{\sqrt{2}} \\ t_{-2} & -\frac{s_{-1}}{2} + \frac{t_{-1}}{\sqrt{2}} & \frac{1}{3} - \frac{s_3}{2} + \frac{t_0}{\sqrt{6}} \end{pmatrix}.$$

ATLAS Collaboration, G. Aad et al., *Measurement of the polarisation of W bosons produced in top-quark decays using dilepton events at s=13 TeV with the ATLAS experiment*, Phys. Lett. B **843** (2023) 137829, [arXiv:2209.14903].

$$\rho_W \propto \begin{pmatrix} m_{++}(1 + \cos \theta) & 0 & 0 \\ 0 & m_{00}(1 + \cos \theta) & m_{0-} \sin \theta e^{-i\varphi} \\ 0 & m_{0-} \sin \theta e^{i\varphi} & m_{--}(1 - \cos \theta) \end{pmatrix}$$

$$\begin{aligned} \frac{m_{00}}{m_{00} + m_{--} + m_{++}} &= F_0 = 0.684 \pm 0.015, \\ \frac{m_{--}}{m_{00} + m_{--} + m_{++}} &= F_- = 0.318 \pm 0.009, \\ \frac{m_{++}}{m_{00} + m_{--} + m_{++}} &= F_+ = -0.002 \pm 0.014, \end{aligned}$$

$$F_i \equiv \frac{\Gamma_i(t \rightarrow Wb)}{\Gamma_T(t \rightarrow Wb)}, \quad \text{with } i = 0, +, -$$



$\text{CNTXT}_5 = 2.175 \pm 0.015$ and $\text{CNTXT}_9 = 3.311 \pm 0.012$,

$t \bar{t}$

$$p + p \rightarrow t + \bar{t}.$$

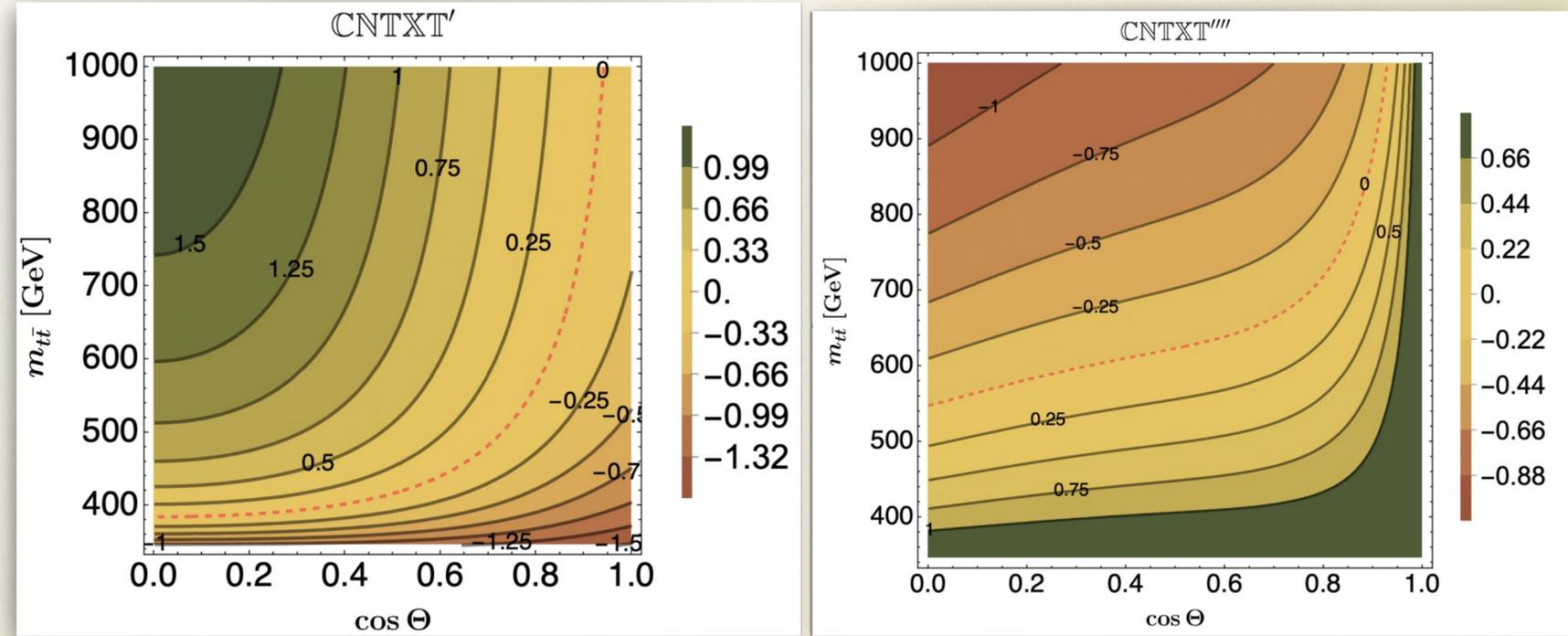
$$\rho_{1/2 \times 1/2} = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{i,j} C_{ij} (\sigma_i \otimes \sigma_j) \right],$$

$$\frac{d\sigma}{d\Omega dm_{t\bar{t}}} = \frac{\alpha_s^2 \beta_t}{64\pi^2 m_{t\bar{t}}^2} \left\{ L^{gg}(\tau) \tilde{A}^{gg}[m_{t\bar{t}}, \Theta] + L^{qq}(\tau) \tilde{A}^{qq}[m_{t\bar{t}}, \Theta] \right\}$$

$$C_{ij}[m_{t\bar{t}}, \Theta] = \frac{L^{gg}(\tau) \tilde{C}_{ij}^{gg}[m_{t\bar{t}}, \Theta] + L^{qq}(\tau) \tilde{C}_{ij}^{qq}[m_{t\bar{t}}, \Theta]}{L^{gg}(\tau) \tilde{A}^{gg}[m_{t\bar{t}}, \Theta] + L^{qq}(\tau) \tilde{A}^{qq}[m_{t\bar{t}}, \Theta]},$$

$$L^{gg}(\tau) = \frac{2\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{dz}{z} q_g(\tau z) q_g\left(\frac{\tau}{z}\right) \quad \text{and} \quad L^{qq}(\tau) = \sum_{q=u,d,s} \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{dz}{z} q_q(\tau z) q_{\bar{q}}\left(\frac{\tau}{z}\right),$$

CMS Collaboration, A. Hayrapetyan et al.,
Measurements of polarization and spin correlation
and observation of entanglement in top quark pairs
using lepton+jet events from proton-proton collisions
at $s=13$ TeV, Phys. Rev. D **110** (2024), no. 11
112016, [arXiv:2409.11067].



	threshold	boosted	central boosted
C_{nn}	0.540 ± 0.042	0.175 ± 0.028	0.661 ± 0.064
C_{rr}	0.269 ± 0.070	-0.202 ± 0.044	-0.678 ± 0.083
C_{kk}	0.427 ± 0.074	0.040 ± 0.050	-0.69 ± 0.12
C_{nr}	0.01 ± 0.08	-0.03 ± 0.05	0.05 ± 0.10
C_{nk}	0.07 ± 0.12	-0.05 ± 0.05	-0.12 ± 0.14
C_{rn}	0.07 ± 0.08	-0.04 ± 0.05	0.09 ± 0.10
C_{rk}	-0.01 ± 0.14	-0.05 ± 0.07	-0.01 ± 0.15
C_{kn}	0.00 ± 0.12	-0.02 ± 0.05	-0.06 ± 0.14
C_{kr}	0.05 ± 0.14	-0.06 ± 0.07	0.01 ± 0.15
B_n^+	0.015 ± 0.029	0.007 ± 0.017	0.004 ± 0.027
B_r^+	0.004 ± 0.034	0.006 ± 0.014	-0.026 ± 0.029
B_k^+	0.001 ± 0.022	0.000 ± 0.011	-0.015 ± 0.034
B_n^-	-0.012 ± 0.029	-0.003 ± 0.011	-0.004 ± 0.027
B_r^-	-0.063 ± 0.033	-0.013 ± 0.014	-0.010 ± 0.029
B_k^-	0.003 ± 0.22	0.026 ± 0.017	0.002 ± 0.034

$300 < m_{tt} < 400$ GeV

threshold

$$\text{CNTXT}'''' = 1.09 \pm 0.09,$$

$m_{tt} > 800$ GeV

boosted

$$\text{CNTXT}' = 0.40 \pm 0.08$$

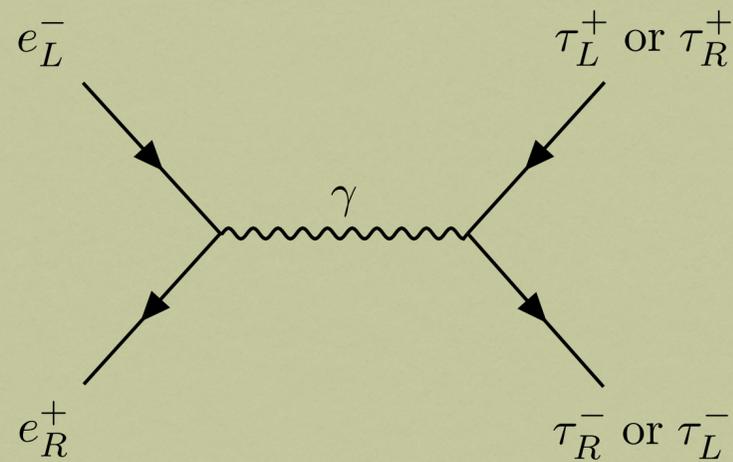
$m_{tt} > 800$ GeV and $|\cos \Theta| < 0.4$

central boosted

$$\text{CNTXT}' = 1.35 \pm 0.17$$

2

Entanglement



$$\zeta_1 |\tau_L^- \rangle |\tau_L^+ \rangle + \zeta_2 |\tau_R^- \rangle |\tau_L^+ \rangle + \zeta_3 |\tau_L^- \rangle |\tau_R^+ \rangle + \zeta_4 |\tau_R^- \rangle |\tau_R^+ \rangle$$

$$\left(\sum_i |\zeta_i|^2 = 1 \right)$$

$$\underbrace{(1 + \cos \Theta)}_{\zeta_2 = D_{1,1}^{(1)}(\Theta)} |\tau_R^- \rangle |\tau_L^+ \rangle + \underbrace{(1 - \cos \Theta)}_{\zeta_3 = D_{1,-1}^{(1)}(\Theta)} |\tau_L^- \rangle |\tau_R^+ \rangle$$

$$J = \pm 1 \quad J_z = \pm 1 \quad (\Theta = 0)$$

$$|\tau_R^- \rangle |\tau_L^+ \rangle$$

separable

$$J = \pm 1 \quad J_z = 0 \quad (\Theta = \pi/2)$$

$$\frac{1}{\sqrt{2}} \left(|\tau_R^- \rangle |\tau_L^+ \rangle + |\tau_L^- \rangle |\tau_R^+ \rangle \right)$$

entangled (Bell state)

Entanglement vs. classical correlations

Spin

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+\frac{1}{2}\rangle_1 |-\frac{1}{2}\rangle_2 - |-\frac{1}{2}\rangle_1 |+\frac{1}{2}\rangle_2 \right)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+\frac{1}{2}\rangle_1 |-\frac{1}{2}\rangle_2 - |-\frac{1}{2}\rangle_1 |+\frac{1}{2}\rangle_2 \right)$$



$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+\frac{1}{2}\rangle_1 |-\frac{1}{2}\rangle_2 - |-\frac{1}{2}\rangle_1 |+\frac{1}{2}\rangle_2 \right)$$

R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum entanglement*, *Rev. Mod. Phys.* **81** (2009) 865–942, [[quant-ph/0702225](https://arxiv.org/abs/quant-ph/0702225)].

Charge

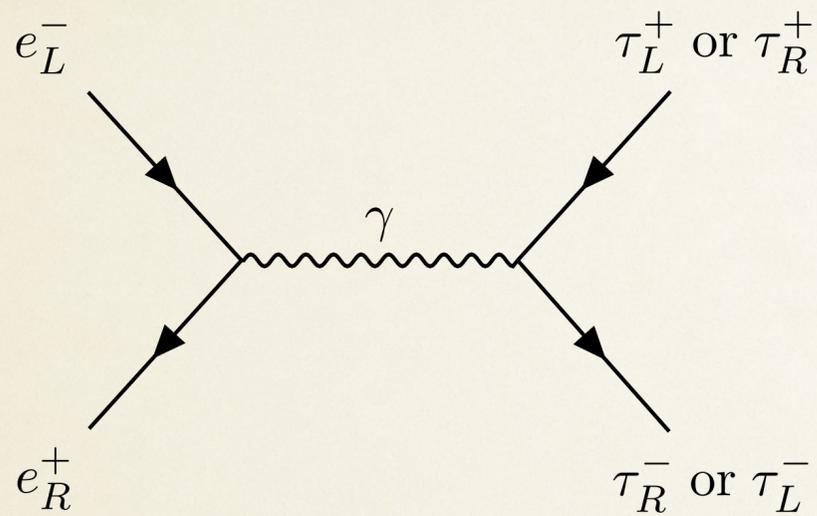


$$|\Psi\rangle = |+-\rangle \text{ or } |-+\rangle$$

$$|\Psi\rangle = |+-\rangle \text{ or } |-+\rangle$$



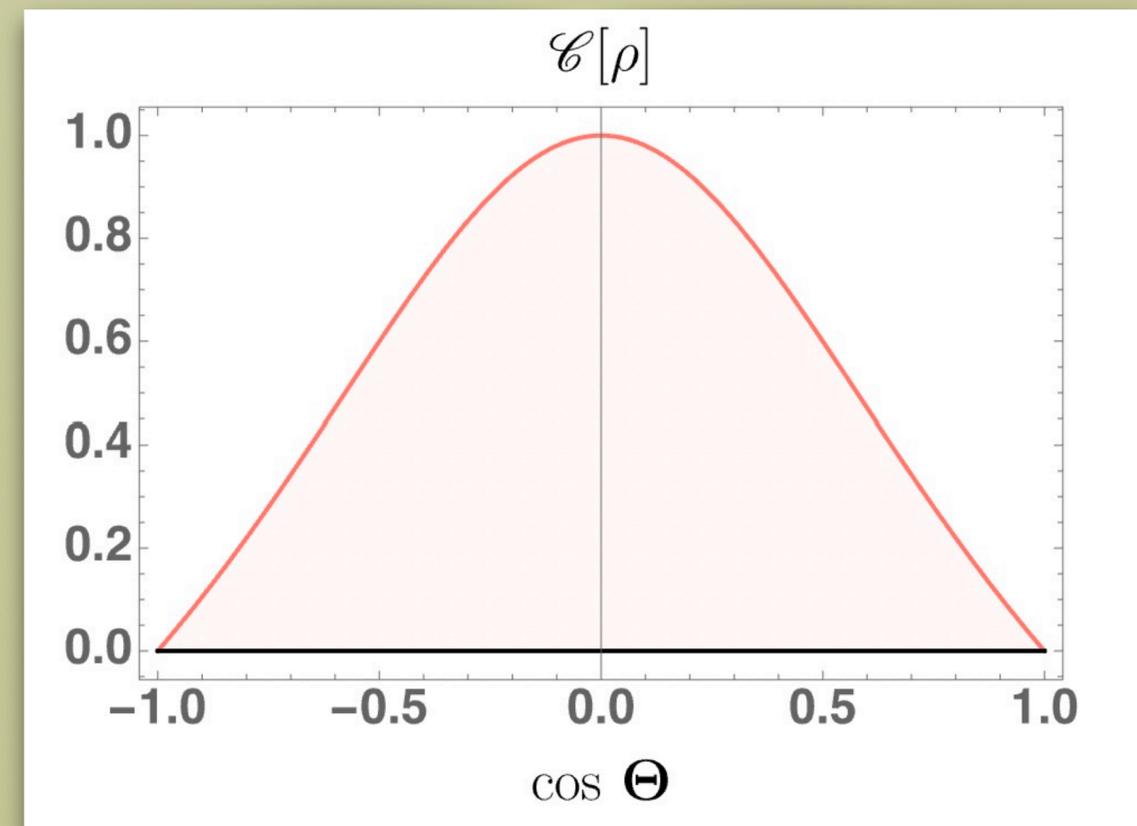
$$|\Psi\rangle = |+-\rangle \text{ or } |-+\rangle$$



$$(1 + \cos \Theta) |\tau_R^-\rangle |\tau_L^+\rangle + (1 - \cos \Theta) |\tau_L^-\rangle |\tau_R^+\rangle$$

Concurrence

$$C[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2 \Theta}{1 + \cos^2 \Theta}$$



Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Angular Correlation of Scattered Annihilation Radiation*

C. S. WU AND I. SHAKNOV

Papin Physics Laboratories, Columbia University, New York, New York
November 21, 1949

AS early as 1946, J. A. Wheeler¹ proposed an experiment to verify a prediction of pair theory, that the two quanta emitted in the annihilation of a positron-electron pair, with zero relative angular momentum, are polarized at right angles to each other. This suggestion involves coincidence measurements of the scattering of both the annihilation photons at various azimuths. The detailed theoretical investigations were reported by Pryce and Ward² and by Snyder, Pasternack, and Hornbostel.³ The predicted maximum asymmetry ratio of coincidence counts when the two counters are at right angles to each other to coincidence counts when the counters are co-planar is as large as 2.85 and occurs at a scattering angle of $\theta=82^\circ$. Bleuler and Bradt⁴ used two end-window G-M counters as detectors and observed an asymmetry ratio not inconsistent with the theory. Nevertheless, the margin of error associated with their results is so large that a detailed comparison between the theory and experiments is made rather difficult. In the meantime, Hanna⁵ performed similar experiments with more efficient counter arrangements and found the asymmetry ratio observed to be consistently smaller than those predicted. Therefore, it appeared to be highly desirable to reinvestigate this problem by using more efficient detectors and more favorable conditions.

The recently developed scintillation counter has been proved to be a reliable and highly efficient gamma-ray detector. With this improved efficiency, which is around ten times that of G-M counters, there will be an increase in the coincidence counting rate of one hundred times. In our experiments, two RCA 5819 photo-multiplier tubes and two anthracene crystals $1 \times 1 \times \frac{1}{4}$ in. were used. The efficiency for the annihilation radiation obtained with these anthracene crystals is seven to eight percent which compares favorably with the calculated value. The geometrical arrangement is schematically shown in Fig. 1.

The positron source Cu^{64} was activated by deuteron bombardment on a copper target in the Columbia cyclotron. The electroplating method was employed to separate Cu activity from other

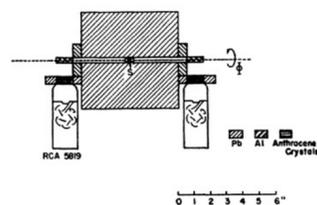


FIG. 1. Schematic diagram of experiment.

contaminations. The active Cu^{64} was packed in a small Al capsule of 8-mm diameter and 8-mm length. The annihilation radiation was collimated by a lead block $6 \times 6 \times 6$ in. with a $\frac{1}{4}$ -in. channel drilled through the center of the block, such that the spread of the beam was found to be less than 3° . The aluminum scatterers were $\frac{1}{2}$ in. in diameter and 1-in. long. They were designed to absorb about 40 percent of the annihilation radiation lengthwise and to limit the multiple scattering of the radiation scattered at 90° to less than 15 percent. The crystal of the counter subtends an angle of 43° at the point in the scatterer where 20 percent of the incident radiation has been absorbed—that is, at the absorption midpoint of the scatterer. The mean scattering angle is very close to 82° , the predicted maximum of anisotropy. Under these conditions, the scattered radiation taken as the counting difference detected by the scintillation counter with and without the scatterer in place is three times the over-all background.

In taking the coincidence measurements, one detector was kept fixed in position, and the second detector was oriented to four different positions with azimuth differences (ϕ) of 0° , 90° , 180° , and 270° between the detector axis. After that, the second detector was kept fixed and the first one rotated. The total period of measurement lasted about 30 continuous hours. On account of the high coincidence rate observed (the true coincidence rates for the perpendicular position at the beginning was of the order of four per minute), the statistical deviations are much improved as compared to the results from G-M counters. The asymmetry ratio from our best run is

$$\frac{\text{Coincidence counting rate } (\perp)}{\text{Coincidence counting rate } (\parallel)} = 2.04 \pm 0.08,$$

where ± 0.08 is the probable mean error. The calculated asymmetry ratio for our geometrical arrangement is 2.00. Therefore, the agreement is very satisfactory. Further work is being planned to extend the investigations to more ideal geometrical conditions.

We wish to express our appreciation to Professors J. R. Dunning, W. W. Havens, Jr., and L. J. Rainwater for their constant interest and encouragement. We also wish to thank the cyclotron group for preparing the Cu^{64} source and the U. S. AEC which aided materially in the performance of this research.

* Partially supported by the AEC.
¹ J. A. Wheeler, *Ann. New York Acad. Sci.* **48**, 219 (1946).
² M. H. L. Pryce and J. C. Ward, *Nature* **160**, 435 (1947).
³ Snyder, Pasternack, and Hornbostel, *Phys. Rev.* **63**, 440 (1948).
⁴ E. Bleuler and H. L. Bradt, *Phys. Rev.* **73**, 1398 (1948).
⁵ R. C. Hanna, *Nature* **162**, 332 (1948).

The Optical Detection of Radiofrequency Resonance

M. H. L. PRYCE

Clarendon Laboratory, Oxford, England
October 31, 1949

IN a recent paper under this title, Bitter¹ discusses the effect of a radiofrequency field on the optical Zeemann effect. He illustrates the question by treating an atomic system whose ground state is 2S , making optical transitions to a 2P state. The atoms are in a steady magnetic field H_0 , on which is superposed a rotating radiofrequency field H_1 , in the xy plane, of angular frequency ω . According to Bitter, when there is a resonance between ω and $\omega_0 = g\mu_0 H_0/h$, the precession frequency of the spin moment in the 2S ground state, certain observable changes happen to the Zeemann effect.

Such an effect certainly occurs, but Bitter's discussion is incorrect, and it is very doubtful if the effect could be observed in practice. Bitter calculates the frequencies of the Zeemann lines by means of a *mean energy* of the ground level in the presence of the radiofrequency field. This is a fallacious argument. The fre-

1949

Bell inequality violation

probabilities

$$\mathcal{P}(\uparrow_{\hat{n}_i}; -)$$

spin of one tau-lepton up
in the direction n_j

$$\mathcal{P}(\uparrow_{\hat{n}_i}; \downarrow_{\hat{n}_j})$$

spin of one tau-lepton up in the direction n_i
other tau-lepton spin down in the direction n_j

Bell inequality violation

stochastic variables

$$\mathcal{P}(\uparrow \hat{n}_1; -) = \int d\lambda \eta(\lambda) \underline{p_\lambda(\uparrow \hat{n}_1; -)}$$

$$\int d\lambda \eta(\lambda) = 1$$

$$\mathcal{P}(\uparrow \hat{n}_1; \uparrow \hat{n}_2) = \int d\lambda \eta(\lambda) \underline{p_\lambda(\uparrow \hat{n}_1; \uparrow \hat{n}_2)}$$

$$p_\lambda(\uparrow \hat{n}; \downarrow \hat{m}) = p_\lambda(\uparrow \hat{n}; -) p_\lambda(-; \downarrow \hat{m})$$

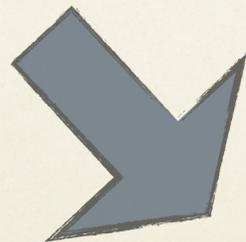
Bell locality assumption

probability independence

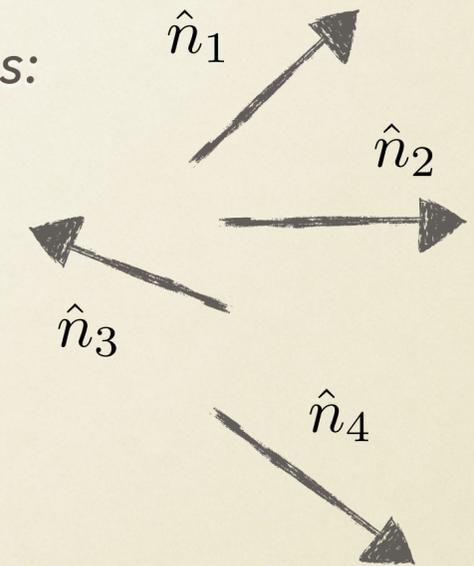
Bell inequality violation

any four non-negative numbers

$$x_1x_2 - x_1x_4 + x_3x_2 + x_3x_4 \leq x_3 + x_2$$



four directions:



$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_4}) \leq \mathcal{P}(\uparrow_{\hat{n}_3}; -) + \mathcal{P}(-; \uparrow_{\hat{n}_2})$$



$$\underline{\Psi = |\tau_R^- \rangle |\tau_L^+ \rangle} \quad \longrightarrow \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i}; \uparrow_{\hat{n}_j}) = \frac{1}{4} \langle \Psi | (1_{2 \times 2} + \hat{n}_i \cdot \vec{\sigma}) \otimes (1_{2 \times 2} + \hat{n}_j \cdot \vec{\sigma}) | \Psi \rangle = \frac{1}{4} (1 - \hat{n}_i^z + \hat{n}_j^z - \hat{n}_i^z \hat{n}_j^z)$$

$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{-1}{\sqrt{2}}(\hat{z} + \hat{x}), \quad \hat{n}_3 = -\hat{x}, \quad \hat{n}_4 = \frac{1}{\sqrt{2}}(\hat{z} - \hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_4}) = \frac{1}{2}$$

$$\leq \mathcal{P}(\uparrow_{\hat{n}_3}; -) + \mathcal{P}(-; \uparrow_{\hat{n}_2}) = 1 - \frac{\sqrt{2}}{4}$$





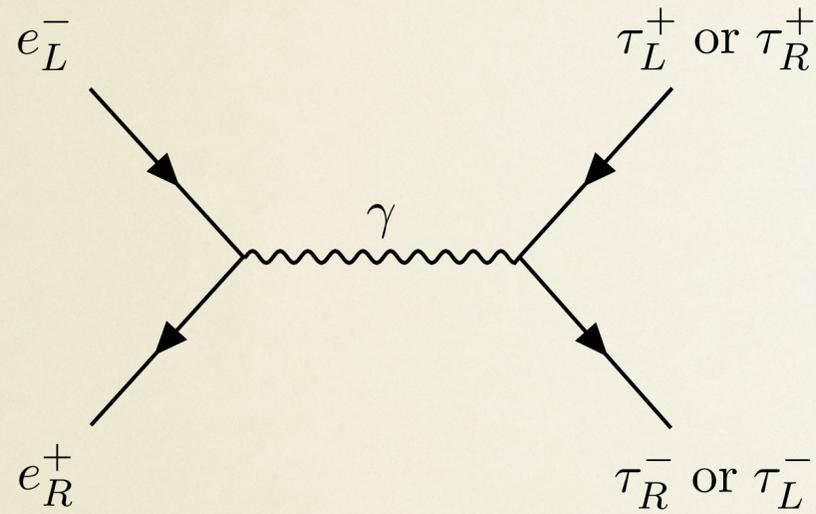
$$\underline{\Psi = \frac{1}{\sqrt{2}} \left(|\tau_R^- \rangle |\tau_L^+ \rangle + |\tau_L^- \rangle |\tau_R^+ \rangle \right)} \quad \longrightarrow \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathcal{P}(\uparrow_{\hat{n}_i}; \uparrow_{\hat{n}_j}) = \frac{1}{4} \langle \Psi | (1_{2 \times 2} + \hat{n}_i \cdot \vec{\sigma}) \otimes (1_{2 \times 2} + \hat{n}_j \cdot \vec{\sigma}) | \Psi \rangle = \frac{1}{4} (1 + \hat{n}_i^x \hat{n}_j^x + \hat{n}_i^y \hat{n}_j^y - \hat{n}_i^z \hat{n}_j^z)$$

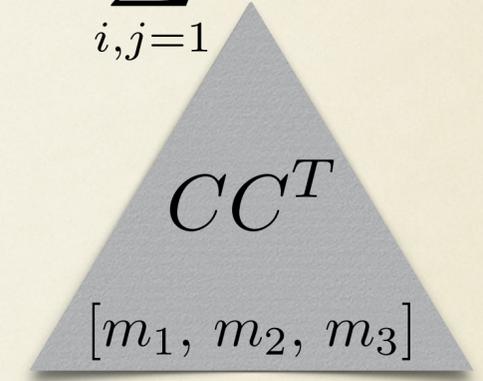
$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{-1}{\sqrt{2}}(\hat{z} + \hat{x}), \quad \hat{n}_3 = -\hat{x}, \quad \hat{n}_4 = \frac{1}{\sqrt{2}}(\hat{z} - \hat{x})$$

$$\mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_2}) - \mathcal{P}(\uparrow_{\hat{n}_1}; \uparrow_{\hat{n}_4}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_2}) + \mathcal{P}(\uparrow_{\hat{n}_3}; \uparrow_{\hat{n}_4}) = \frac{1}{2} + \frac{\sqrt{2}}{2}$$

$$\not\leq \mathcal{P}(\uparrow_{\hat{n}_3}; -) + \mathcal{P}(-; \uparrow_{\hat{n}_2}) = 1$$



$$\rho = \frac{1}{4} \left[1_2 \otimes 1_2 + \sum_{i=1}^3 B_i^+ (\sigma_i \otimes 1_2) + \sum_{i=1}^3 B_i^- (1_2 \otimes \sigma_i) + \sum_{i,j=1}^3 C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

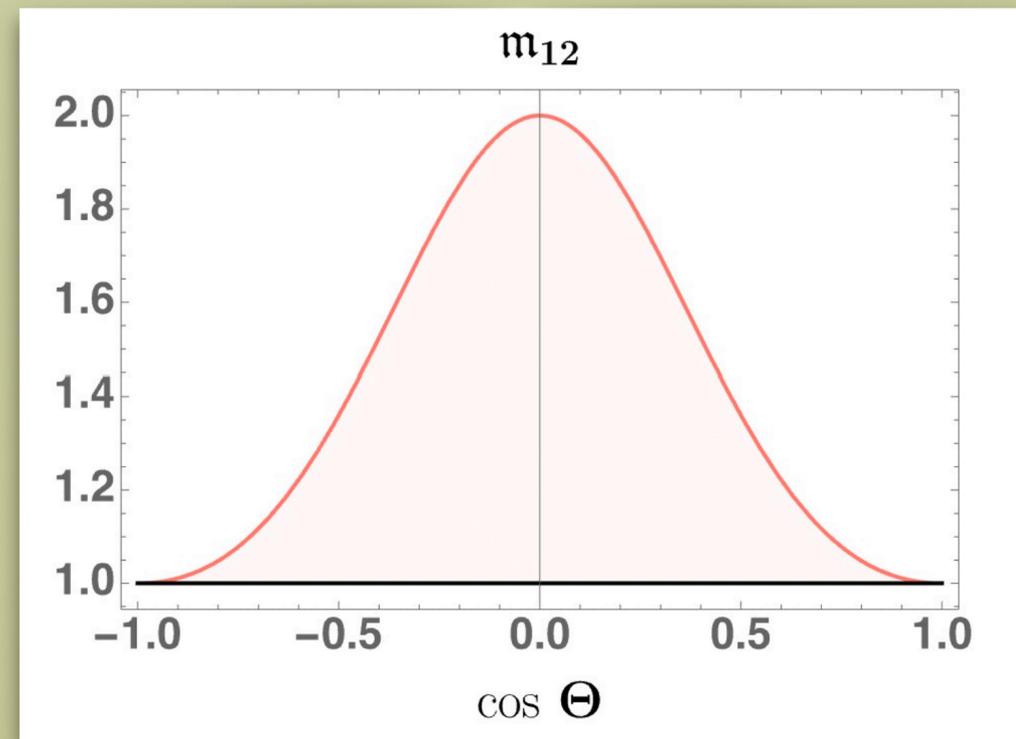


$$(1 + \cos \Theta) |\tau_R^-\rangle |\tau_L^+\rangle + (1 - \cos \Theta) |\tau_L^-\rangle |\tau_R^+\rangle$$

Non-locality witness
Horodecki condition

$$\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4 \Theta}{(1 + \cos^2 \Theta)^2}$$



> 2 more nonlocal than QM!

B. S. Cirelson, *Quantum Generalizations of Bell's inequality*, *Lett. Math. Phys.* **4** (1980) 93–100.

Low-energy tests with photons and solid-state devices

A. Aspect, J. Dalibard and G. Rogers, Phys. Rev. Lett. 49 (1982) 5039

J.F. Clauser, M.A. Horne, Phys. Rev. D 10 (1974) 526

J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. 23 (1969) 880

G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. 81 (1998) 5039

W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Phys. Rev. Lett. 81 (1998) 3563

M. Ansmann et al, Nature 461 (2009) 504

VOLUME 81, NUMBER 17 PHYSICAL REVIEW LETTERS 26 OCTOBER 1998

Violation of Bell Inequalities by Photons More Than 10 km Apart

W. Tittel,* J. Brendel, H. Zbinden, and N. Gisin

Group of Applied Physics, University of Geneva, 20, Rue de l'Ecole de Médecine, CH-1211 Geneva 4, Switzerland
(Received 10 June 1998)

A Franson-type test of Bell inequalities by photons 10.9 km apart is presented. Energy-time entangled photon pairs are measured using two-channel analyzers, leading to a violation of the inequalities by 16 standard deviations without subtracting accidental coincidences. Subtracting them, a two-photon interference visibility of 95.5% is observed, demonstrating that distances up to 10 km have no significant effect on entanglement. This sets quantum cryptography with photon pairs as a practical competitor to the schemes based on weak pulses. [S0031-9007(98)07478-X]

Article | [Open access](#) | [Published: 10 May 2023](#)

Loophole-free Bell inequality violation with superconducting circuits

[Simon Storz](#) ✉, [Josua Schär](#), [Anatoly Kulikov](#), [Paul Magnard](#), [Philipp Kurpiers](#), [Janis Lütolf](#), [Theo Walter](#), [Adrian Copetudo](#), [Kevin Reuer](#), [Abdulkadir Akin](#), [Jean-Claude Besse](#), [Mihai Gabureac](#), [Graham J. Norris](#), [Andrés Rosario](#), [Ferran Martin](#), [José Martínez](#), [Waldimar Amaya](#), [Morgan W. Mitchell](#), [Carlos Abellán](#), [Jean-Daniel Bancal](#), [Nicolas Sangouard](#), [Baptiste Royer](#), [Alexandre Blais](#) & [Andreas Wallraff](#) ✉

Nature 617, 265–270 (2023) | [Cite this article](#)

VOLUME 47, NUMBER 7

PHYSICAL REVIEW LETTERS

17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

...near polarization correlation of the photons emitted from a high-efficiency source provided an excellent test of local realistic theories. Our results, in excellent agreement with quantum theory, strongly violate the generalized Bell inequalities. No significant violation of local realistic theories is observed for analyzer separations of up to 6.5 m.

VOLUME 81

7 DECEMBER 1998

NUMBER 23

PHYSICAL REVIEW LETTERS

Violation of Bell's Inequality under Strict Einstein Locality Conditions

Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
(Received 6 August 1998)

We observe strong violation of Bell's inequality in an Einstein-Podolsky-Rosen-type experiment with independent observers. Our experiment definitely implements the condition of locality, a central assumption in the derivation of Bell's inequality. The spacelike separation of the observations is achieved by ultrafast and random setting of the

PRL 115, 250402 (2015)

Selected for a Viewpoint in PHYSICS
PHYSICAL REVIEW LETTERS

week ending
18 DECEMBER 2015

Strong Loophole-Free Test of Local Realism*

Lynden K. Shalm,^{1,3} Evan Meyer-Scott,² Bradley G. Christensen,³ Peter Bierhorst,¹ Michael A. Wayne,^{3,4} Martin J. Stevens,¹ Thomas Gerrits,¹ Scott Glancy,¹ Deny R. Hamel,⁵ Michael S. Allman,¹ Kevin J. Coakley,¹ Shellee D. Dyer,¹ Carson Hodge,¹ Adriana E. Lita,¹ Varun B. Verma,¹ Camilla LAMBROCCO,¹ Edward Tortorici,¹ Alan L. Migdall,^{4,6} Yanbao Zhang,⁷ Daniel R. Kumar,³ William H. Farr,⁷ Francesco Marsili,⁷ Matthew D. Shaw,⁷ Jeffrey A. Stern,⁸ Carlos Abellán,⁸ Waldimar Amaya,⁸ Valerio Pruneri,^{8,9} Thomas Jennewein,^{2,10} Morgan W. Mitchell,^{8,9} Paul G. Kwiat,³ Joshua C. Bienfang,^{4,6} Richard P. Mirin,¹ Emanuel Knill,¹ and Sae Woo Nam^{1,2}

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³Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

⁴National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA

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⁶Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA

⁷Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109, USA

⁸ICFO-Institut de Ciències Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain

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¹⁰Quantum Information Science Program, Canadian Institute for Advanced Research, Toronto, Ontario, Canada
(Received 10 November 2015; published 16 December 2015)

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7} . We therefore reject the hypothesis that local realism governs our experiment.

1976

Quantum mechanics and hidden variables: A test of Bell's inequality by the measurement of the spin correlation in low-energy proton-proton scattering

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(Received 11 August 1975; revised manuscript received 26 July 1976)

The inequality of Bell has been tested by the measurement of the spin correlation in proton-proton scattering. Measurements were made at $E_p = 13.2$ and 13.7 MeV using carbon analyzers of 18.6 and 29 mg/cm², respectively, accumulating a total of 10^4 coincidences. The experimental analyzing power, geometric correlation coefficients, and energy spectra are compared to the result of a Monte Carlo simulation of the apparatus. The results are in good agreement with quantum mechanics and in disagreement with the inequality of Bell if the same additional assumptions are made. The conditions for comparing the results of the experiments to the inequality of Bell are discussed.

I. INTRODUCTION

Since the beginning of quantum mechanics (QM) a number of physicists who contributed the most to the development of this theory had serious doubts about its logical foundations. Most of the problems were illustrated by a number of paradoxes, such as those of Einstein, Podolsky, and Rosen¹ and Schrödinger (namely, the cat paradox).² These discussions never died down and even today there is no theory of measurement which satisfies everybody.

One attempt to overcome these difficulties was to suppose that there are some supplementary variables outside the scope of QM ("hidden variables") which determine the result of the individual measurement. A theorem derived by J. von Neumann was taken for a long time as proof that such interpretations are impossible. But Bohm³ in 1952 developed a model of the hidden-variables theory which was in complete agreement with the predictions of QM, and Bell⁴ showed in 1965 why the theorem of von Neumann was not valid as applied to physical systems. Bell showed, too, that all hidden-variables models which give complete agreement with QM must have an undesirable feature. They do not obey the principle of locality as stated by Einstein⁵: "If S_1 and S_2 are two systems that have interacted in the past but are now arbitrarily distant, the real, factual situation of system S_1 does not depend on what is done with system S_2 , which is spatially separated from the former."

Developments⁶⁻⁸ of the argumentation of Bell led for the first time in a more than 30-year-old discussion to the possibility of a critical experimental test which could distinguish among the different interpretations. The consequences of such experimental verifications have more profound implications than just eliminating special models which

interpret the measuring process. They will test the validity of a general conception of the foundations of microphysics: the principle of locality or, as written more precisely in Ref. 7, the validity of objective local theories.

II. BELL'S INEQUALITY

The first derivation of the inequality, which later led to an experimental test, was given by Bell.⁴ It has been generalized by Clauser *et al.*^{6,7} In the meantime various ways of demonstration have been derived and can be found in Ref. 9 together with a description of the actual state of hidden-variables theories. We will follow here a demonstration given by Bell.⁸

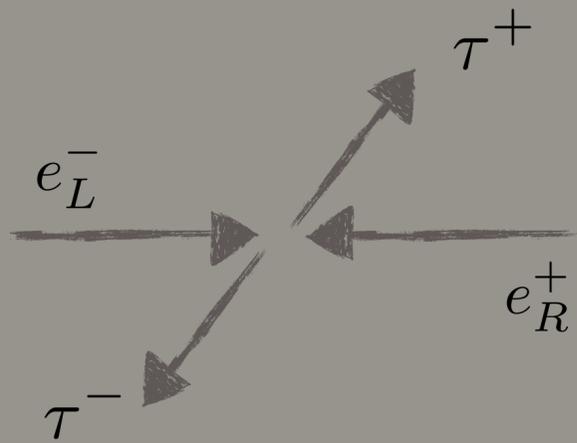
To be definite we take the example of two spin- $\frac{1}{2}$ particles which have been coupled in the past in a singlet state and which are now widely separated. The principle of locality as formulated by Einstein means that each of these particles has some properties, which we will denote by λ (λ can be a whole set of variables) which do *not* depend on what is happening to the other particle. The result of the measurement is determined by these properties λ . We denote by A , B the result of the measurement in the direction \vec{a} and \vec{b} of the sign of the spin of the two particles respectively. For a realistic apparatus and/or if the dependence on λ is not strictly deterministic, but only stochastic, one will have

$$|A(\lambda, \vec{a})| \leq 1 \text{ and } |B(\lambda, \vec{b})| \leq 1.$$

The correlation function $P(\vec{a}, \vec{b})$ is defined to be the mean value of the product AB and thus

$$P(\vec{a}, \vec{b}) = \int A(\lambda, \vec{a})B(\lambda, \vec{b})\rho(\lambda)d\lambda,$$

where $\rho(\lambda)$ denotes the frequency of the properties λ with the normalization condition $\int \rho(\lambda)d\lambda = 1$. Thus,

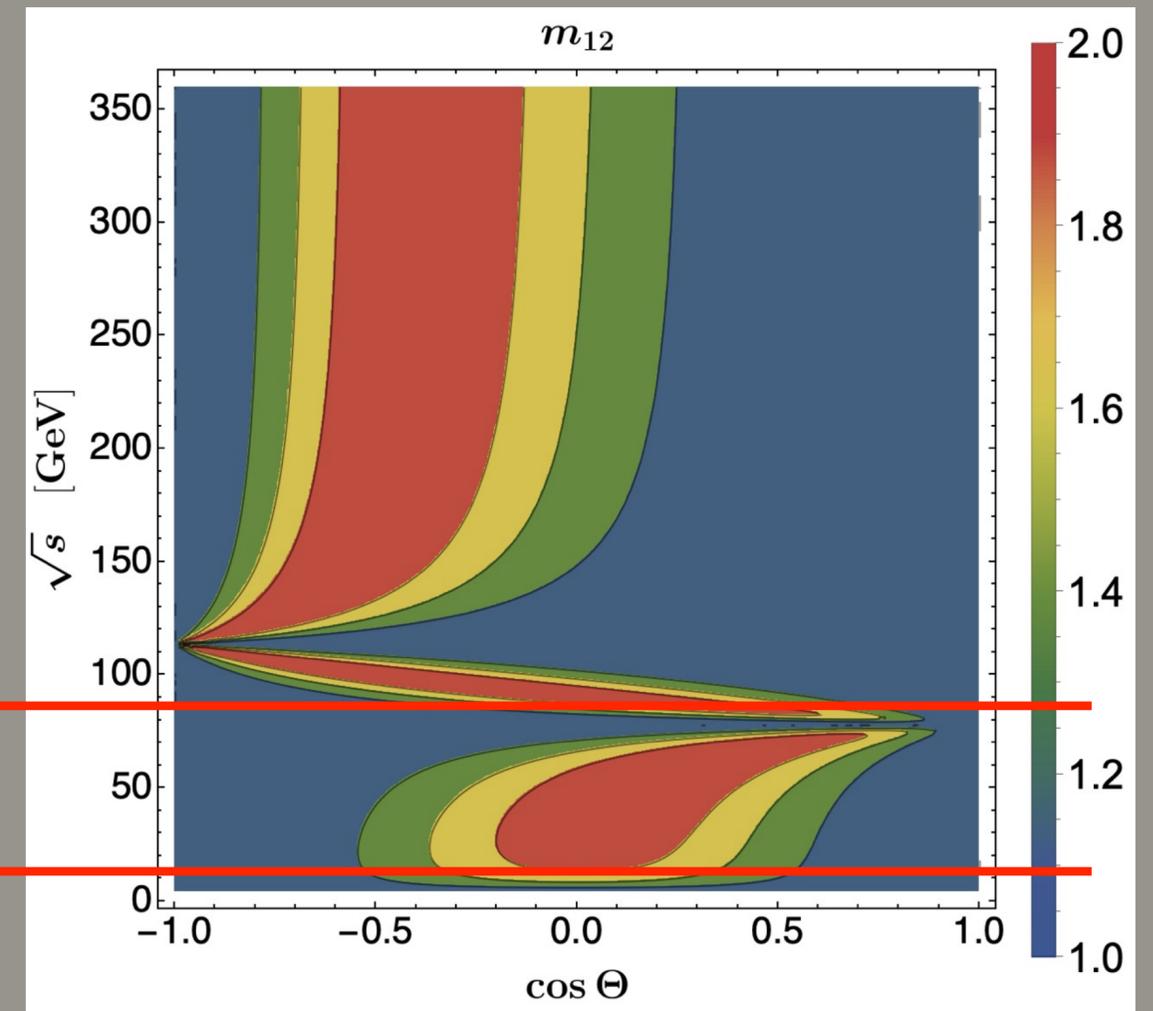
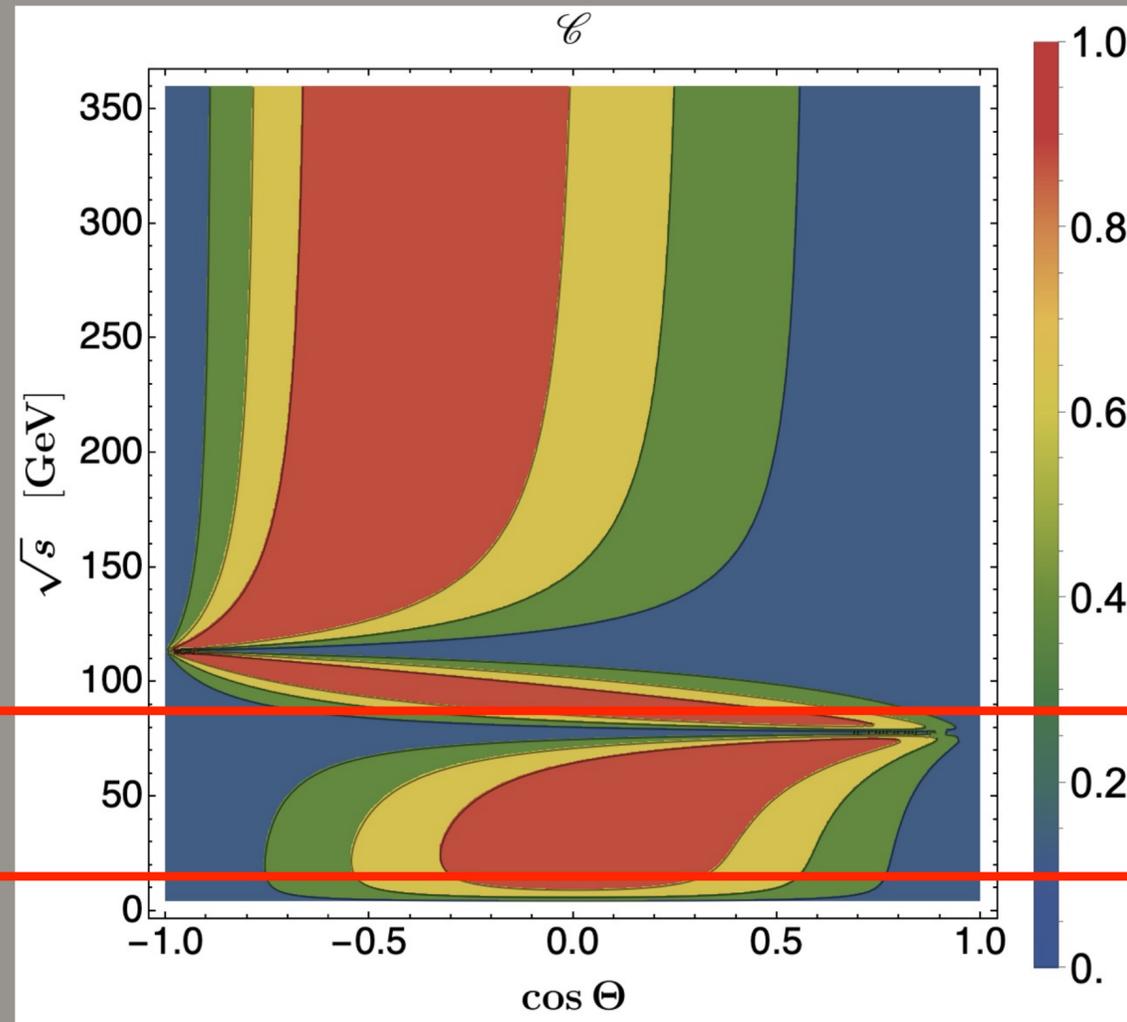


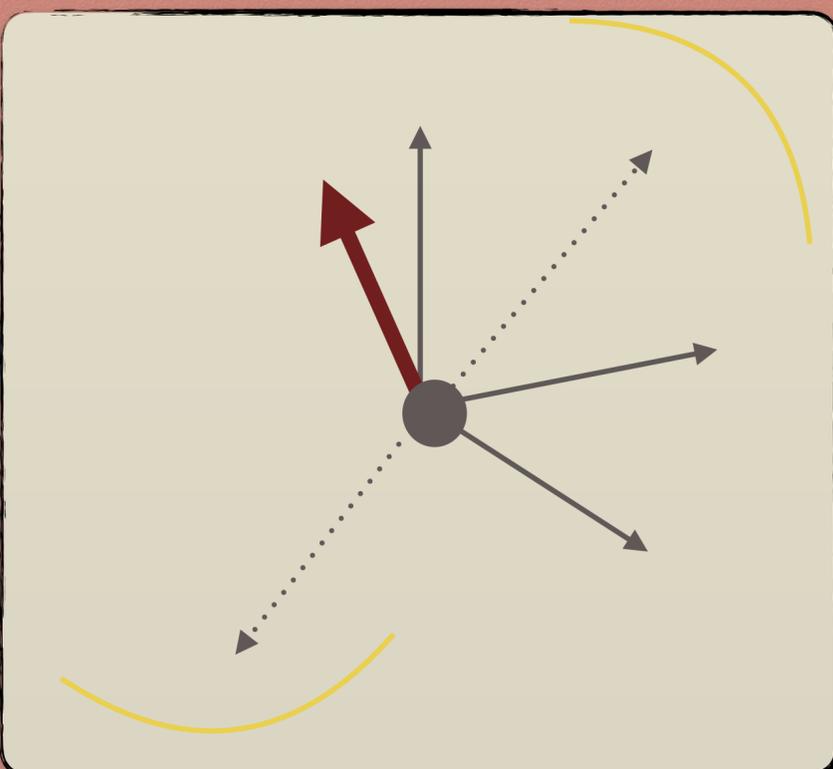
Concurrence

Horodecki condition

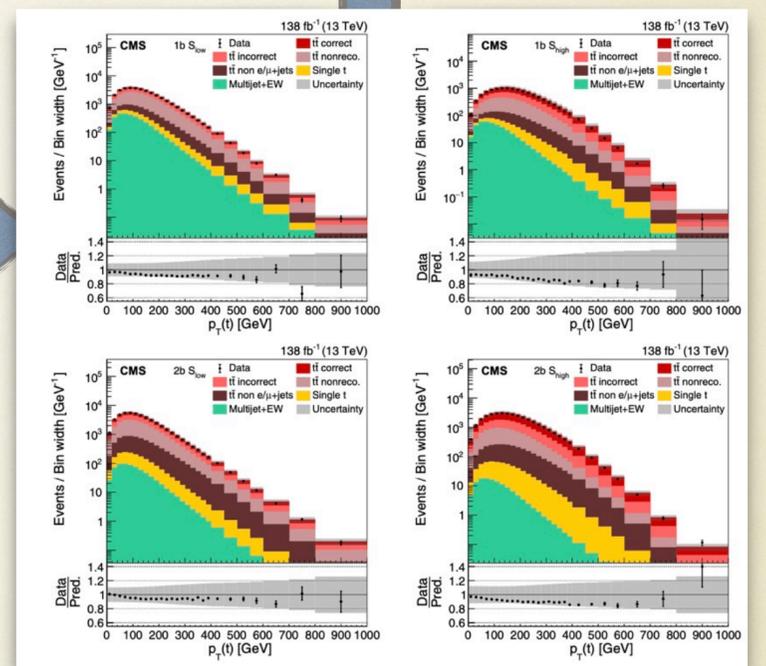
FCC-ee

Belle II





$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^\pm} = \frac{1}{2} \left(1 + \langle s_i \rangle \alpha_i \cos \theta_i \right)$$

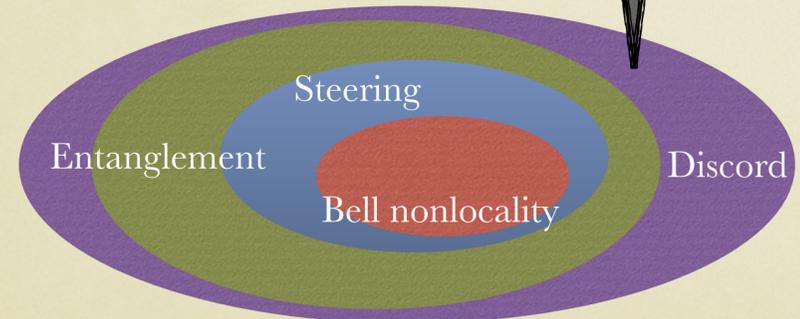
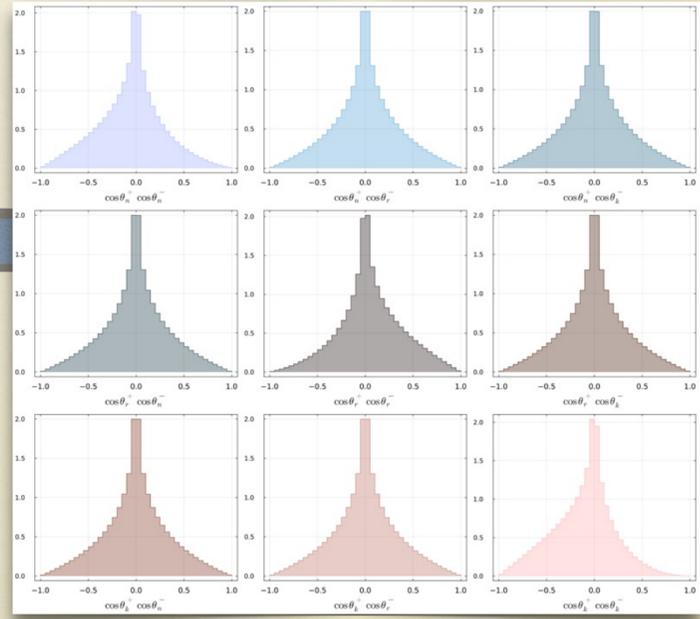


Contextuality
 Concurrence
 Bell nonlocality
 Trace distance
 Magic
 Steering

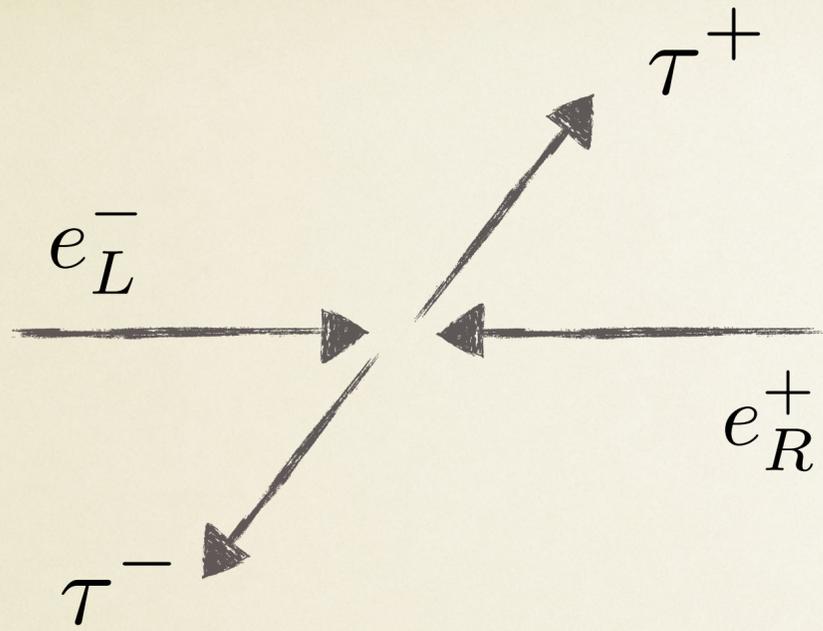
Quantum state tomography

$$\rho_{\lambda_1 \lambda_2, \lambda'_1 \lambda'_2} \propto w_{\lambda_1 \lambda_2} w_{\lambda'_1 \lambda'_2}^* \sum_k D_{k, \lambda_1 - \lambda_2}^{(J)*}(0, \Theta, 0) D_{k, \lambda'_1 - \lambda'_2}^{(J)}(0, \Theta, 0)$$

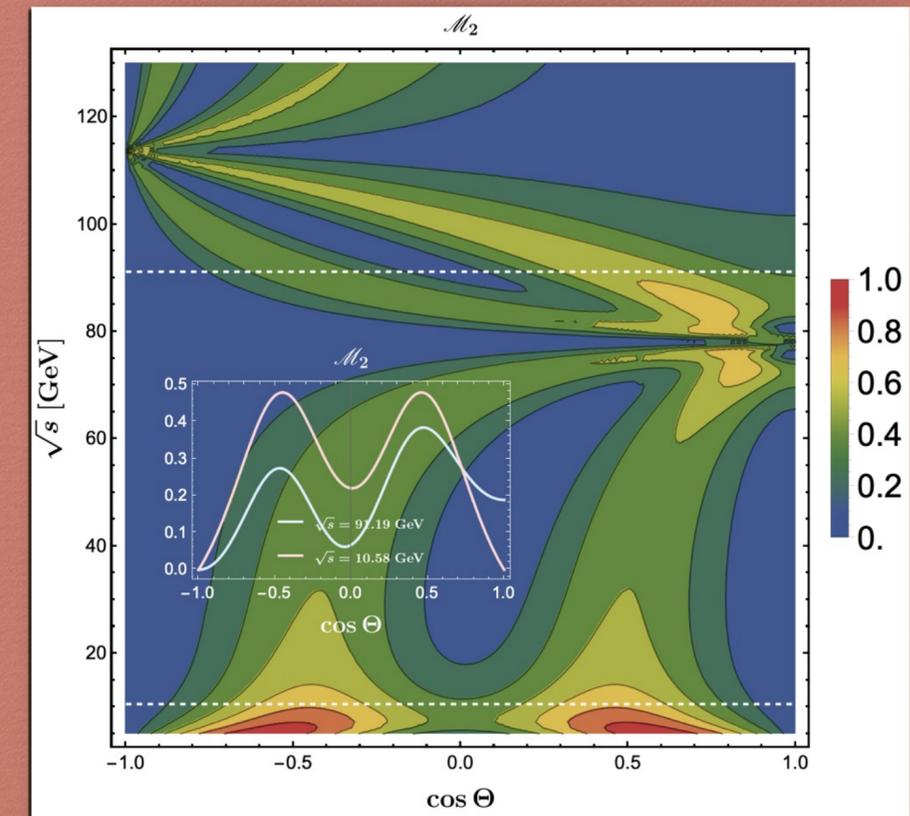
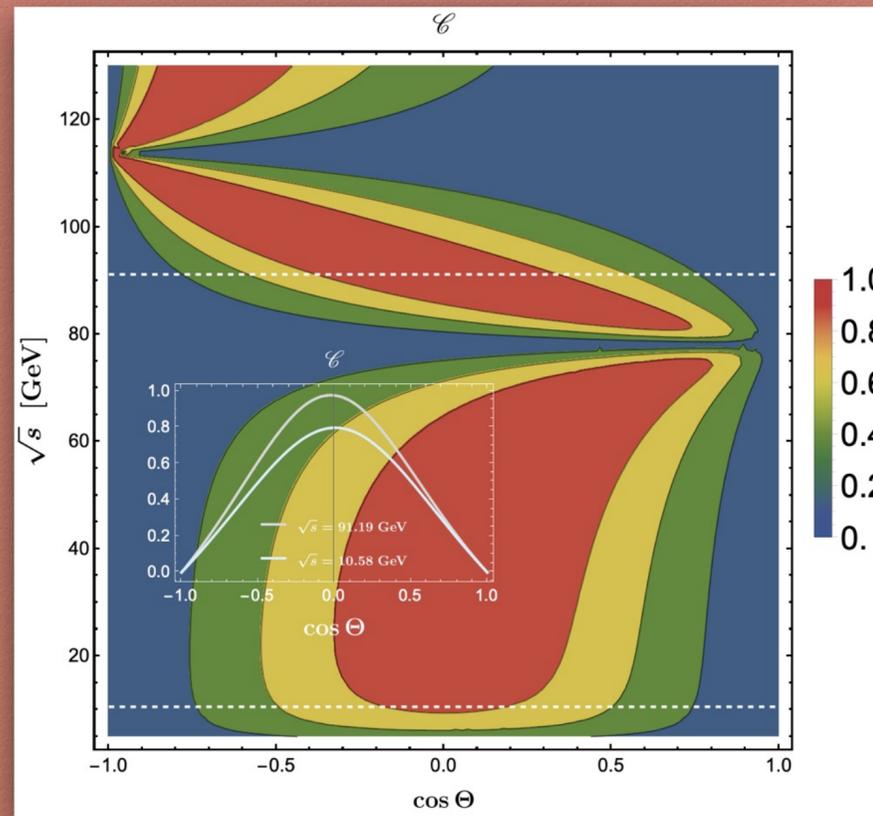
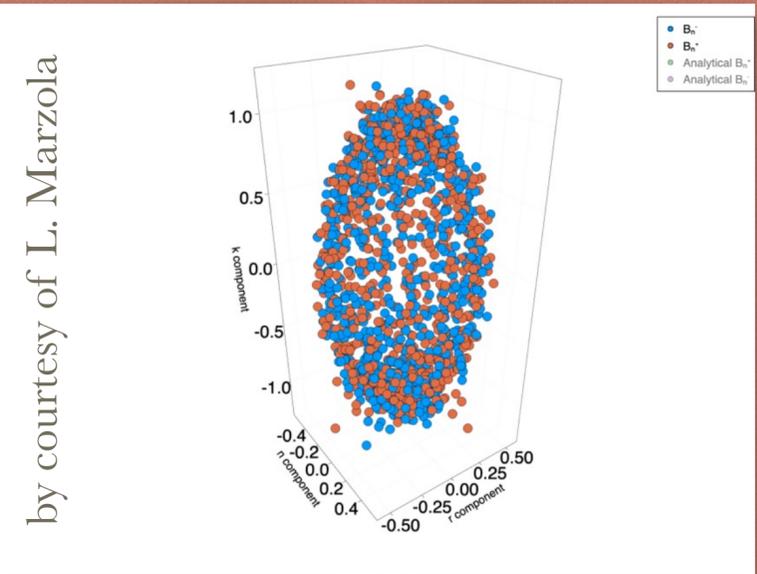
$C_{ij} \quad B_i^\pm$



$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^+ d \cos \theta_j^-} = \frac{1}{2} \left(1 + \alpha_i B_i^+ \cos \theta_i^+ + \alpha_i B_i^- \cos \theta_i^- + C_{ij} \alpha_i \alpha_j \cos \theta_i^+ \cos \theta_j^- \right)$$



A bit of steering



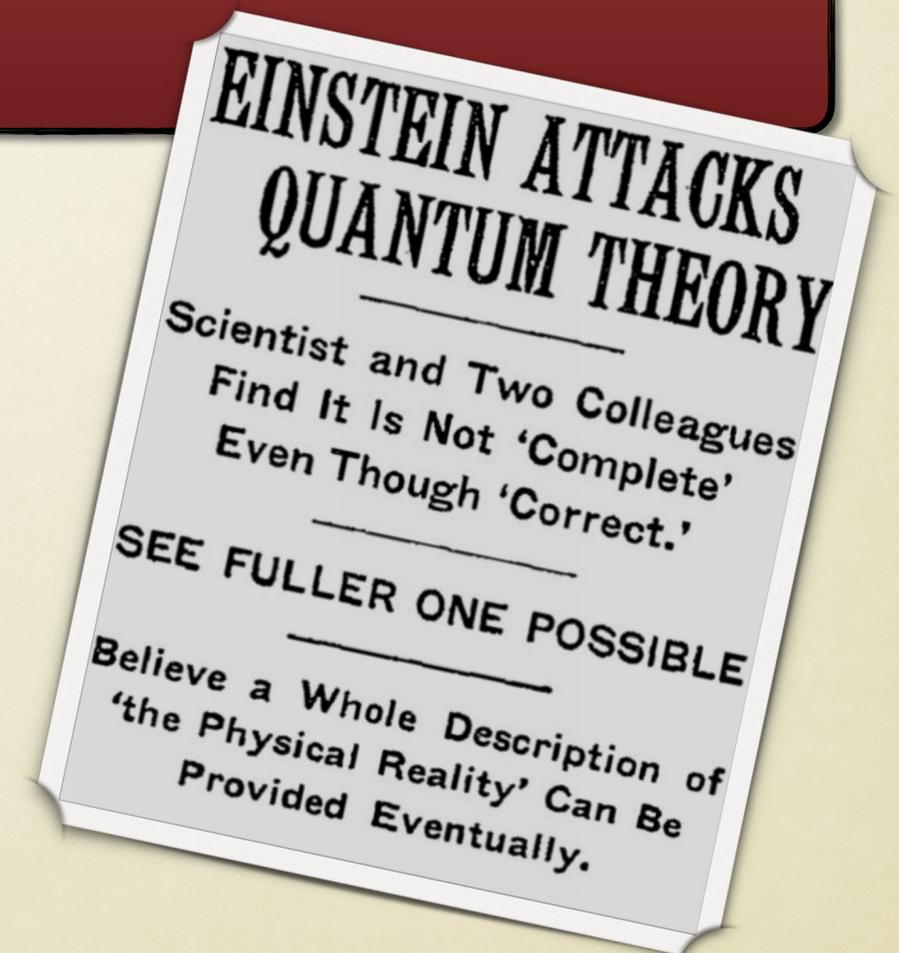
A bit of magic

Figure 5: Concurrence and magic profiles over the entire kinematic space spanned by \sqrt{s} and $\cos \Theta$ for the process $e^+e^- \rightarrow \tau^+\tau^-$. The dashed white horizontal lines mark the center of mass energies of Belle (lower line) and LEP3 (upper line). The insets show the dependence on the scattering angle at these two energies.

INTERMISSION

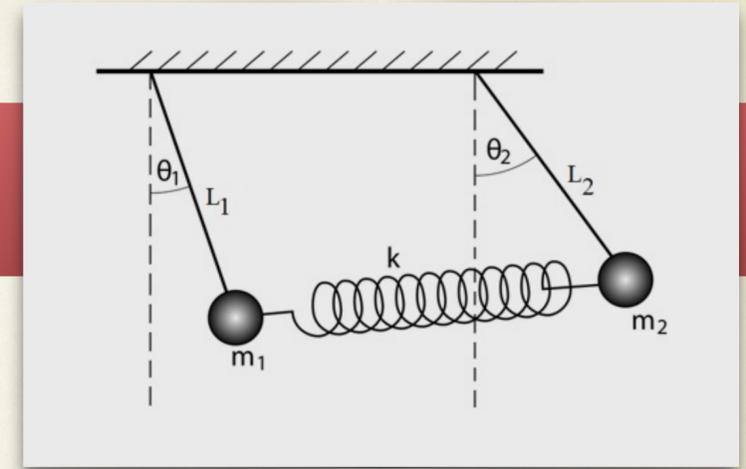
Where have we already seen
entanglement or Bell inequality violation
at high energies?

New York Times headline
May 4th, 1935



1

Flavor space



$K^0 \bar{K}^0$ and $B^0 \bar{B}^0$
oscillations

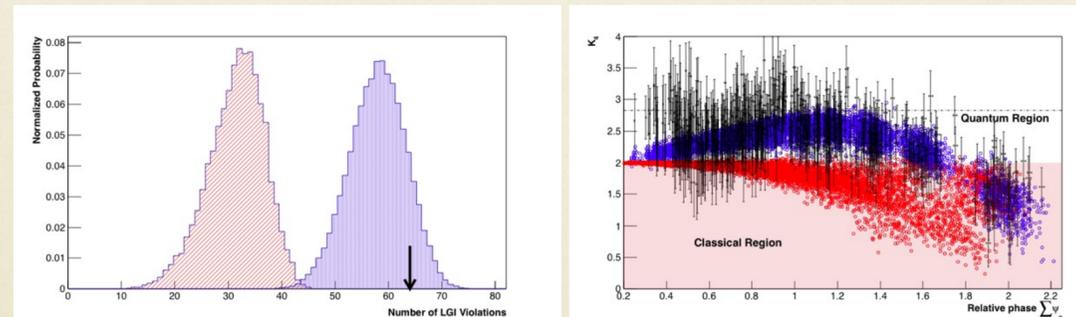


F Benatti and R Floreanini, *Phys. Rev. D* 57 (1998) R1332, *Eur. Phys. J. C* 13 (2000) 267

A Go, Belle Collaboration, *Phys. Phys. Lett.* 99 (2007) 131802

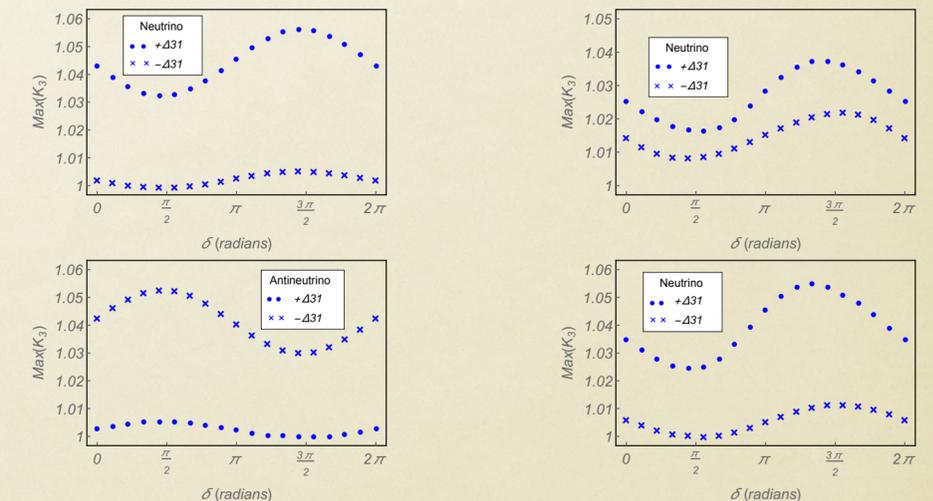
Leggett-Garg inequality
violation

neutrino
oscillations



Minos (6σ)

JA Formaggio, DI Kaiser, MM Murskyj and TE Weiss,
Phys. Rev. Lett. 117 (2016) 050402



Dune

T2K/No ν a

J Naikoo et al, *Phys. Rev. D* 99 (2019) 095001

2

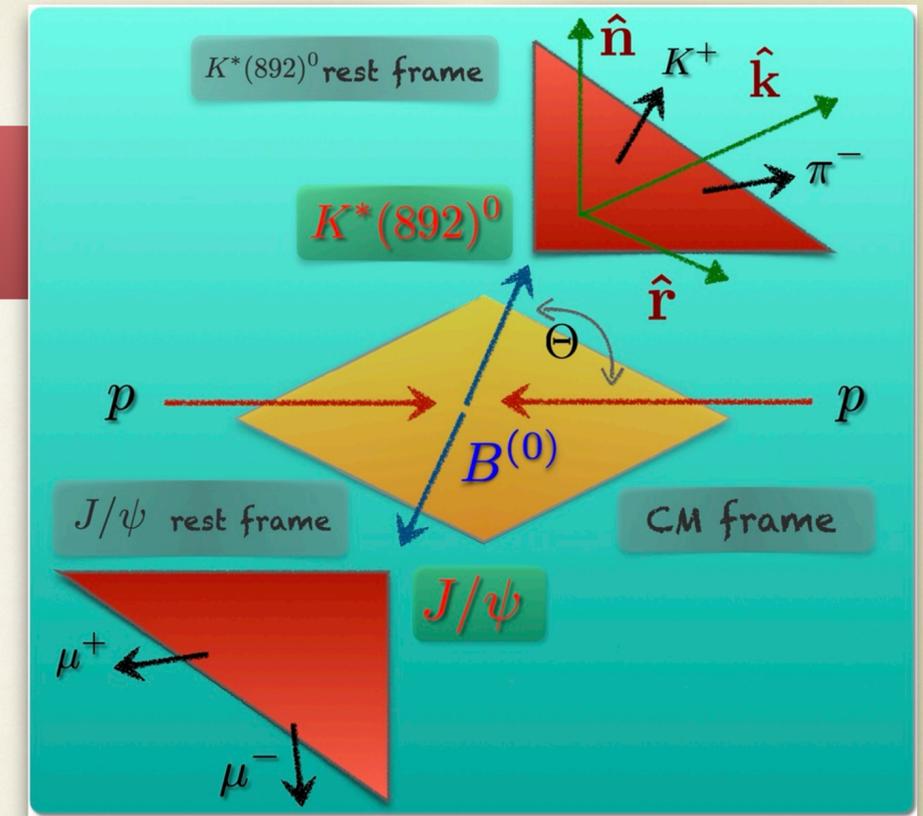
B-meson decays

	\mathcal{E}	\mathcal{I}_3
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015
• $B^0 \rightarrow \phi K^*(892)^0$ [22]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$
• $B^0 \rightarrow \rho K^*(892)^0$ [23]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$
• $B_s \rightarrow \phi \phi$ [24]	0.734 ± 0.037	2.525 ± 0.064
• $B_s \rightarrow J/\psi \phi$ [25]	0.731 ± 0.032	2.462 ± 0.080

entanglement



Bell inequality



8.2σ

Parameter	Result			
$ A_0 ^2$	$0.384 \pm 0.007 \pm 0.003$			
$ A_\perp ^2$	$0.310 \pm 0.006 \pm 0.003$			
δ_\parallel [rad]	$2.463 \pm 0.029 \pm 0.009$			
δ_\perp [rad]	$2.769 \pm 0.105 \pm 0.011$			

	$ A_0 ^2$	$ A_\perp ^2$	δ_\parallel	δ_\perp
$ A_0 ^2$	1	-0.342	-0.007	0.064
$ A_\perp ^2$		1	0.140	0.088
δ_\parallel			1	0.179
δ_\perp				1

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023) [arXiv:2304.06198 [hep-ex]].

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Bell inequality is violated in $B^0 \rightarrow J/\psi K^{*0}(892)$ decays*, Phys. Rev. D **109** (2024), no. 3 L031104.

E. Gabrielli and L. Marzola, arXiv:2406.17772 (2024)

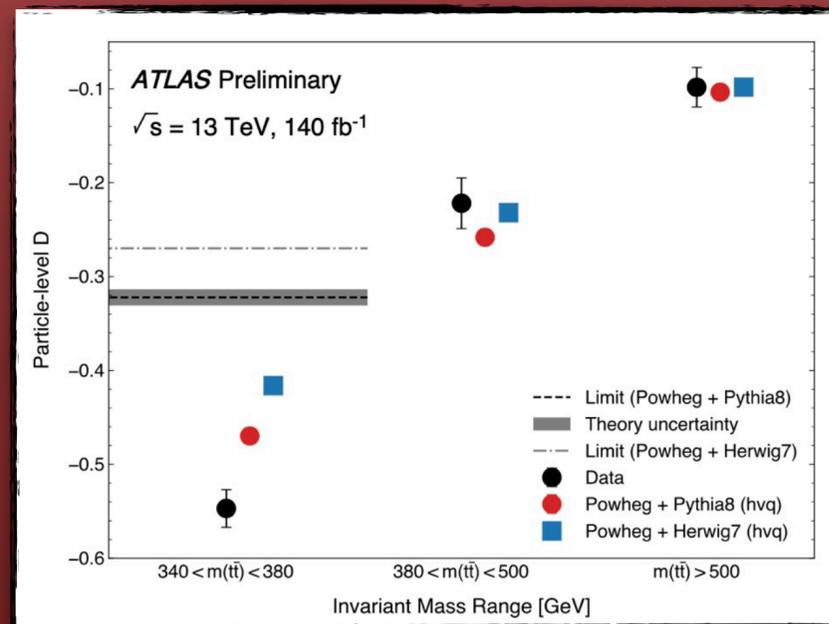
3

Pairs of top quarks

Y. Afik and J.R.M. de Nova, *Eur. Phys. J. Plus* **136** (2021) 907

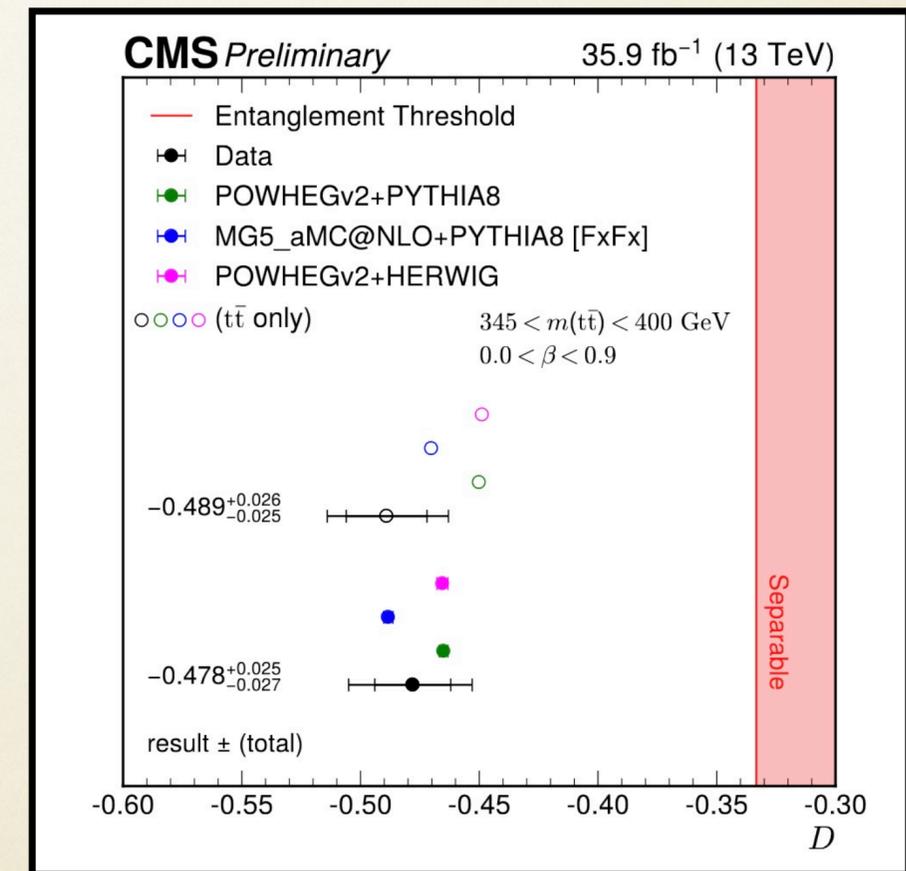
$$pp \rightarrow t + \bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \phi} = \frac{1}{2} (1 - D \cos \phi)$$



$$D = -0.547 \pm 0.002 [\text{stat}] \pm 0.021 [\text{syst}]$$

ATLAS Collaboration, *Nature* 633 (2024) 542



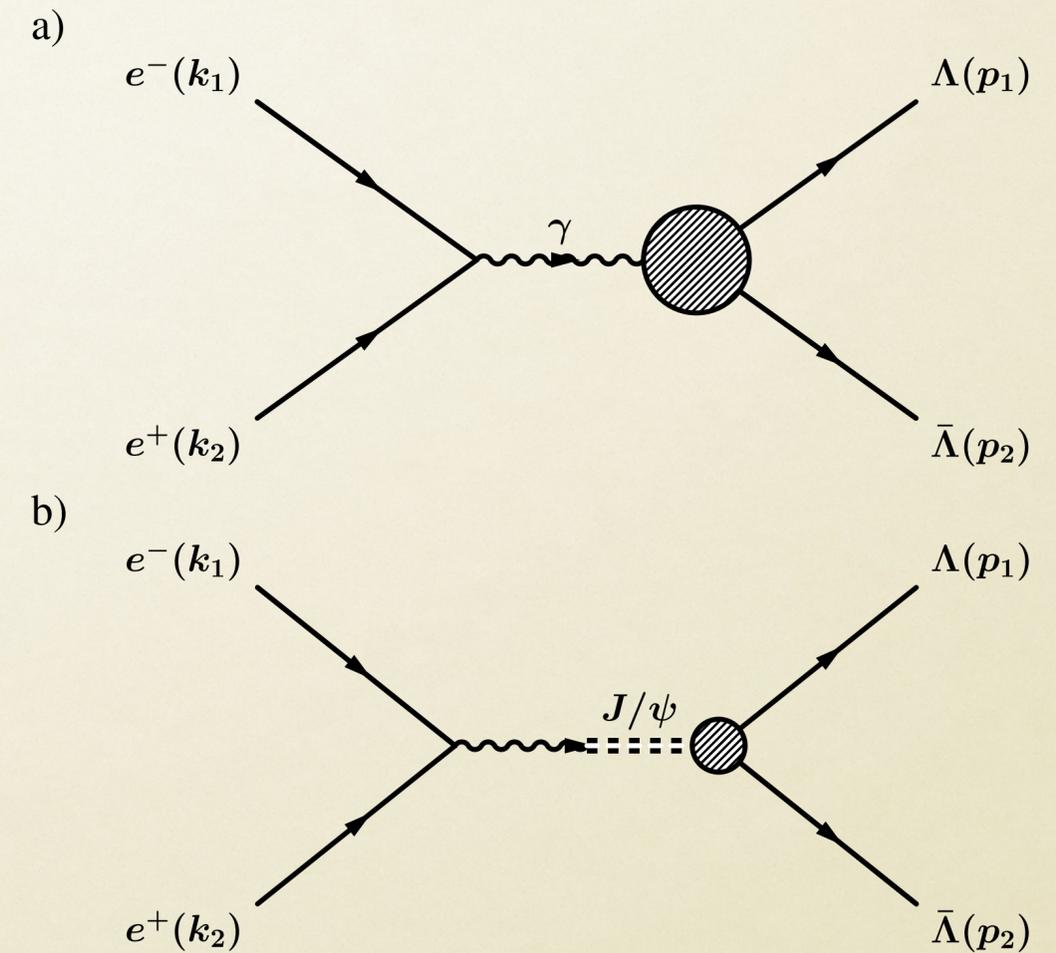
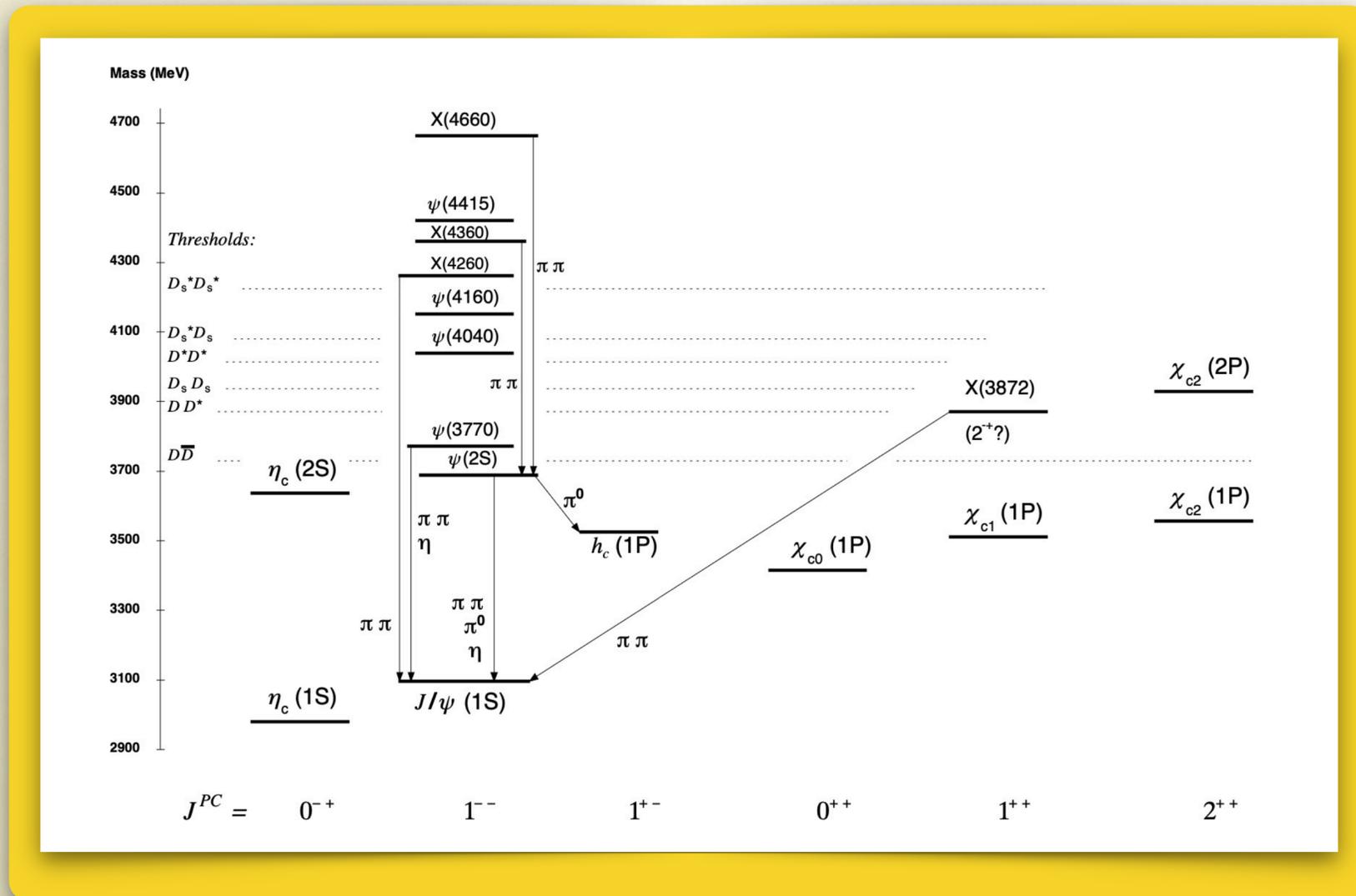
$$D = -0.478^{+0.025}_{-0.027}$$



CMS Collaboration, [arXiv:2406.03976](https://arxiv.org/abs/2406.03976) (2024)
 CMS Collaboration, [arXiv:2409.11067](https://arxiv.org/abs/2409.11067): (2024)

4

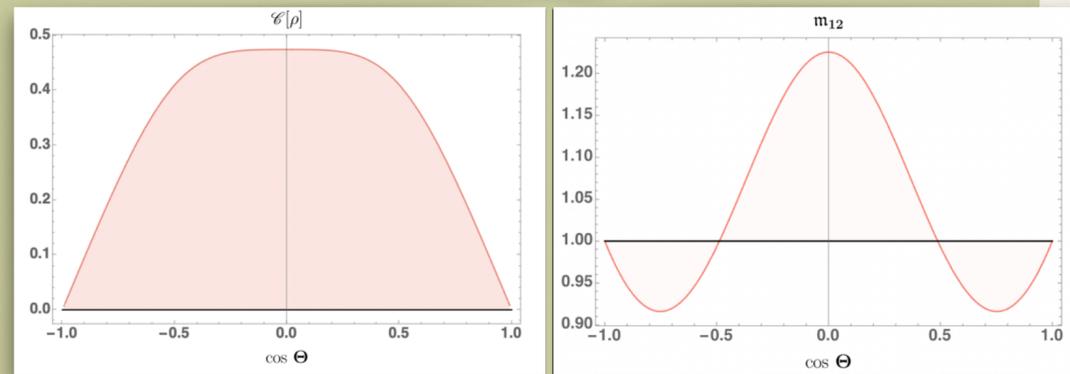
Charmonium



Charmonium spin-1 states

$$J/\psi \rightarrow \Lambda + \bar{\Lambda} \quad \text{and} \quad \psi(3686) \rightarrow \Lambda + \bar{\Lambda}$$

$$\begin{aligned} |\psi_{\uparrow}\rangle &\propto w_{\frac{1}{2}\frac{1}{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle \\ |\psi_{\downarrow}\rangle &\propto w_{-\frac{1}{2}-\frac{1}{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle \\ |\psi_0\rangle &\propto w_{\frac{1}{2}-\frac{1}{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle + w_{-\frac{1}{2}\frac{1}{2}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle, \end{aligned}$$



$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}} \quad \text{and} \quad \Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}}.$$

BESIII Collaboration, M. Ablikim et al.,
Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ - $\bar{\Lambda}$ Pairs, *Phys. Rev. Lett.* **129** (2022), no. 13 131801,
[\[arXiv:2204.11058\]](https://arxiv.org/abs/2204.11058).

Concurrence

$$\mathcal{C} = 0.475 \pm 0.0039 \quad (122\sigma)$$

Horodecki condition

$$m_{12} = 1.225 \pm 0.004 \quad (56\sigma)$$

Bell inequality violation

decay	m_{12}	significance
$J/\psi \rightarrow \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \rightarrow \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \rightarrow \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \rightarrow \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \rightarrow \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \rightarrow \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \rightarrow \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \rightarrow \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \rightarrow \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \rightarrow \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

ongoing work

$$pp \rightarrow t\bar{t}$$

LHC, data already available

Analysis under way



$$pp \rightarrow H \rightarrow ZZ^*$$

LHC, data already available

Analysis under way



$$e^+e^- \rightarrow \tau^+\tau^-$$

Belle II, data already available

Analysis under way



local vs. nonlocal entanglement

$$\rho_{1/2 \times 1/2} = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{i,j} C_{ij} (\sigma_i \otimes \sigma_j) \right],$$

	threshold	boosted	central boosted
C_{nn}	0.540 ± 0.042	0.175 ± 0.028	0.661 ± 0.064
C_{rr}	0.269 ± 0.070	-0.202 ± 0.044	-0.678 ± 0.083
C_{kk}	0.427 ± 0.074	0.040 ± 0.050	-0.69 ± 0.12
C_{nr}	0.01 ± 0.08	-0.03 ± 0.05	0.05 ± 0.10
C_{nk}	0.07 ± 0.12	-0.05 ± 0.05	-0.12 ± 0.14
C_{rn}	0.07 ± 0.08	-0.04 ± 0.05	0.09 ± 0.10
C_{rk}	-0.01 ± 0.14	-0.05 ± 0.07	-0.01 ± 0.15
C_{kn}	0.00 ± 0.12	-0.02 ± 0.05	-0.06 ± 0.14
C_{kr}	0.05 ± 0.14	-0.06 ± 0.07	0.01 ± 0.15
B_n^+	0.015 ± 0.029	0.007 ± 0.017	0.004 ± 0.027
B_r^+	0.004 ± 0.034	0.006 ± 0.014	-0.026 ± 0.029
B_k^+	0.001 ± 0.022	0.000 ± 0.011	-0.015 ± 0.034
B_n^-	-0.012 ± 0.029	-0.003 ± 0.011	-0.004 ± 0.027
B_r^-	-0.063 ± 0.033	-0.013 ± 0.014	-0.010 ± 0.029
B_k^-	0.003 ± 0.22	0.026 ± 0.017	0.002 ± 0.034

CMS Collaboration, A. Hayrapetyan et al.,
Measurements of polarization and spin correlation
and observation of entanglement in top quark pairs
using lepton+jets events from proton-proton collisions
at $s=13$ TeV, Phys. Rev. D **110** (2024), no. 11
112016, [arXiv:2409.11067].

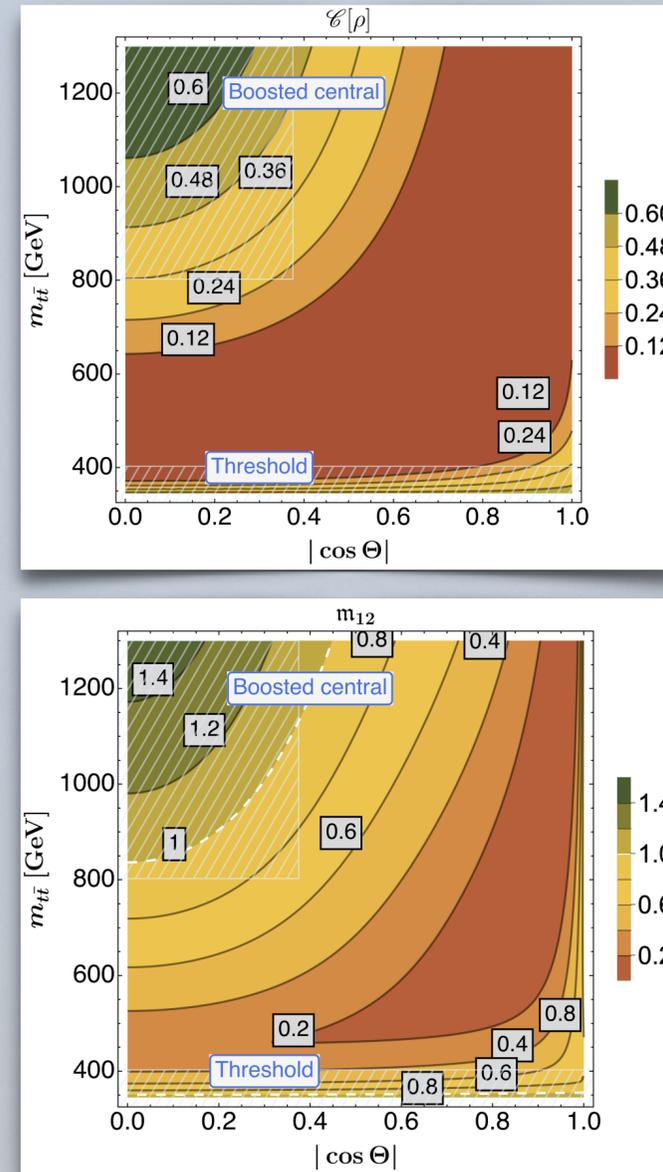


FIG. 2. The behavior of the concurrence, $\mathcal{C}[\rho]$, and of the Horodecki parameter $m_{12}[C]$ over the considered kinematic space. The white, dashed line marks the $m_{12}[C] = 1$ contour, above which the condition for Bell nonlocality is satisfied. The hatched areas denote two of the bins used by the CMS collaboration in their data analysis [25].

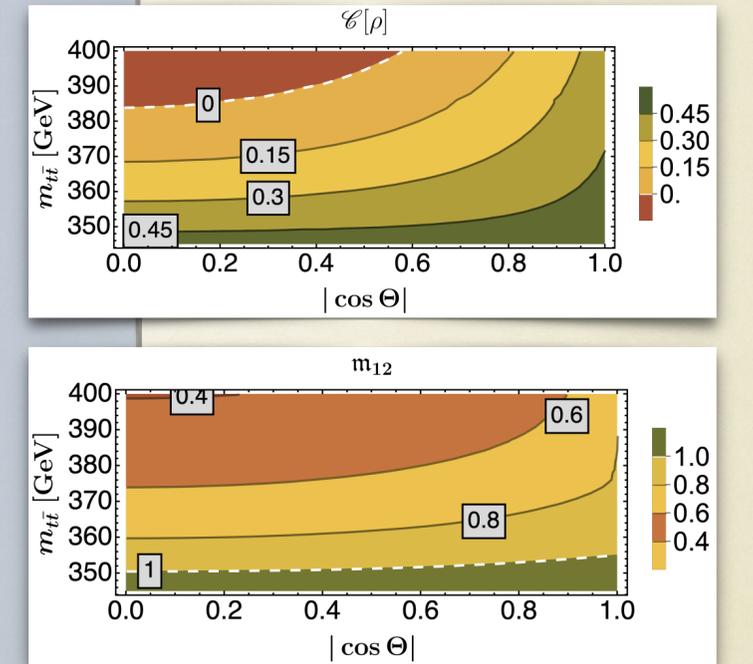


FIG. 3. Enlarged view of concurrence $\mathcal{C}[\rho]$ and $m_{12}[C]$ parameter in the threshold bin defined by the invariant mass $340 < m_{t\bar{t}} < 400$. The white, dashed lines mark the Horodecki condition for Bell nonlocality, in the lower panel, and a vanishing concurrence in the upper one.

	Threshold	Boosted central
$\mathcal{C}[\rho]$	0.133 ± 0.055	0.52 ± 0.06
$\mathcal{D}(\tilde{\mathcal{D}})$	-0.382 ± 0.030	(0.662 ± 0.052)
$m_{12}[C]$	0.548 ± 0.084	1.05 ± 0.13

TABLE I. Concurrence and Horodecki parameter as computed from the CMS data pertaining to the two bin: **Threshold** ($300 < m_{tt} < 400$ GeV) and **Boosted central** ($m_{tt} > 800$ GeV, $|\cos \Theta| < 0.4$). The value of the coefficient \mathcal{D} and $\tilde{\mathcal{D}}$ are those quoted in [25].

entanglement vs. discord

	threshold	boosted	central boosted
C_{nn}	0.540 ± 0.042	0.175 ± 0.028	0.661 ± 0.064
C_{rr}	0.269 ± 0.070	-0.202 ± 0.044	-0.678 ± 0.083
C_{kk}	0.427 ± 0.074	0.040 ± 0.050	-0.69 ± 0.12
C_{nr}	0.01 ± 0.08	-0.03 ± 0.05	0.05 ± 0.10
C_{nk}	0.07 ± 0.12	-0.05 ± 0.05	-0.12 ± 0.14
C_{rn}	0.07 ± 0.08	-0.04 ± 0.05	0.09 ± 0.10
C_{rk}	-0.01 ± 0.14	-0.05 ± 0.07	-0.01 ± 0.15
C_{kn}	0.00 ± 0.12	-0.02 ± 0.05	-0.06 ± 0.14
C_{kr}	0.05 ± 0.14	-0.06 ± 0.07	0.01 ± 0.15
B_n^+	0.015 ± 0.029	0.007 ± 0.017	0.004 ± 0.027
B_r^+	0.004 ± 0.034	0.006 ± 0.014	-0.026 ± 0.029
B_k^+	0.001 ± 0.022	0.000 ± 0.011	-0.015 ± 0.034
B_n^-	-0.012 ± 0.029	-0.003 ± 0.011	-0.004 ± 0.027
B_r^-	-0.063 ± 0.033	-0.013 ± 0.014	-0.010 ± 0.029
B_k^-	0.003 ± 0.22	0.026 ± 0.017	0.002 ± 0.034

CMS Collaboration, A. Hayrapetyan et al.,
Measurements of polarization and spin correlation
and observation of entanglement in top quark pairs
using lepton+jets events from proton-proton collisions
at $s=13$ TeV, Phys. Rev. D **110** (2024), no. 11
112016, [arXiv:2409.11067].

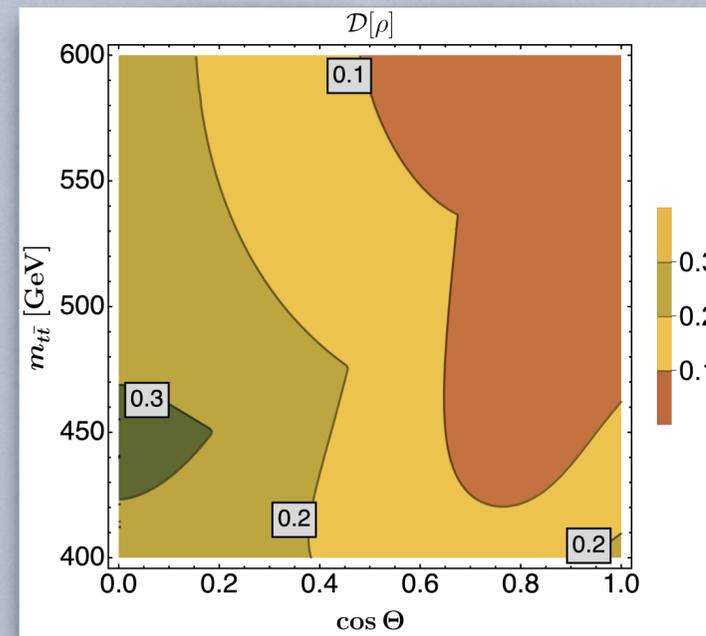
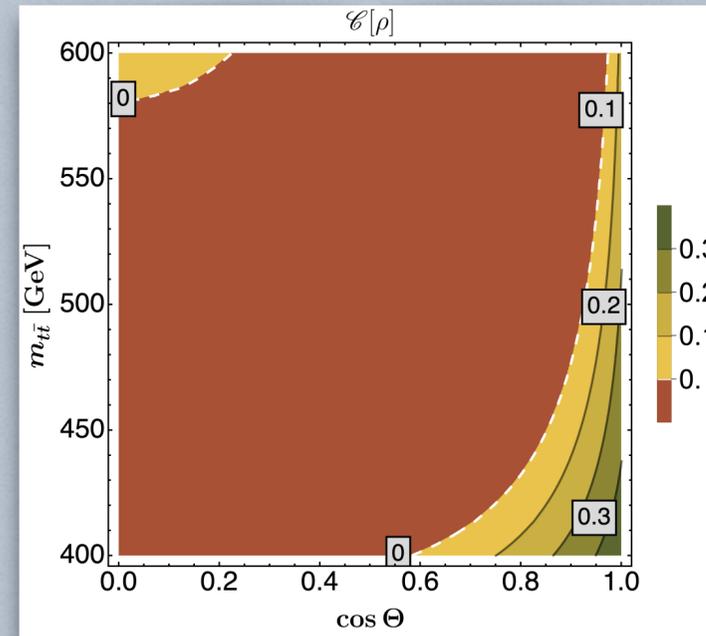


FIG. 4. Concurrence $\mathcal{E}[\rho]$ and discord $\mathcal{D}[\rho]$ in the intermediate bin $400 < m_{t\bar{t}} < 600$ GeV. The reddish region between the white dashed-lines, in the concurrence plot on top, contains separable states for which the concurrence (and the negativity) vanish. On the other hand, the plot below shows non-vanishing values of the discord in the same kinematic region.

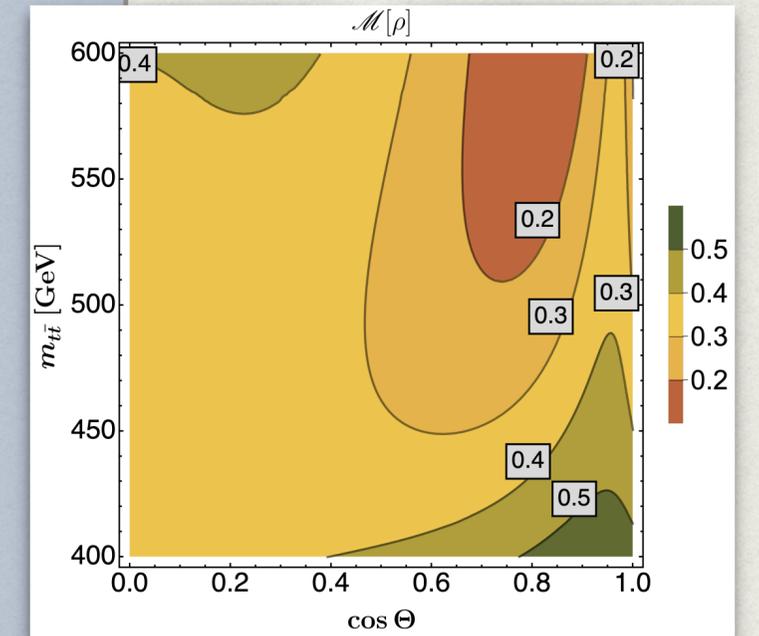


FIG. 5. Magic $\mathcal{M}[\rho]$ of the top-quark pairs produced at the LHC in the intermediate region $400 < m_{t\bar{t}} < 600$ GeV.

	Intermediate
$\mathcal{E}[\rho]$	0.0 ± 0.02
$\mathcal{N}[\rho]$	0.0 ± 0.02
$\mathcal{D}[\rho]$	0.097 ± 0.013
$\mathcal{M}[\rho]$	0.292 ± 0.019

TABLE II. Concurrence, negativity, discord and magic as computed from the CMS data pertaining to the Intermediate bin ($400 < m_{t\bar{t}} < 600$ GeV).

IF THERE IS TIME LEFT

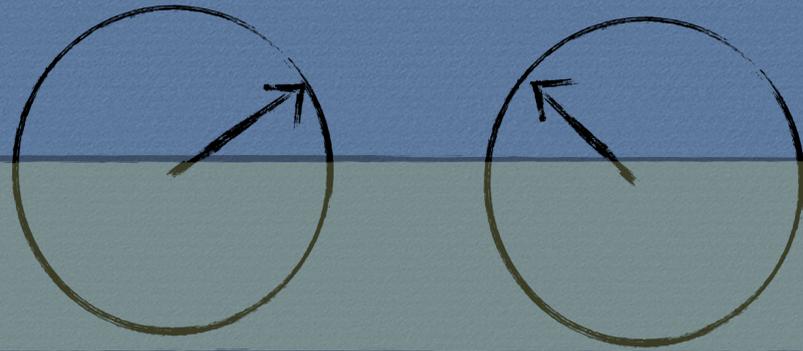
Testing new physics with QI observables

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- [15] M. Fabbrichesi, R. Floreanini, and E. Gabrielli, *Constraining new physics in entangled two-qubit systems: top-quark, tau-lepton and photon pairs*, *Eur. Phys. J. C* **83** (2023), no. 2 162, [[arXiv:2208.11723](#)].
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- [19] A. Bernal, P. Caban, and J. Rembieliński, *Entanglement and Bell inequalities violation in $H \rightarrow ZZ$ with anomalous coupling*, *Eur. Phys. J. C* **83** (2023), no. 11 1050, [[arXiv:2307.13496](#)].
- [20] R. Aoude, E. Madge, F. Maltoni, and L. Mantani, *Probing new physics through entanglement in diboson production*, *JHEP* **12** (2023) 017, [[arXiv:2307.09675](#)].
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- [22] M. Fabbrichesi and L. Marzola, *Quantum tomography with τ leptons at the FCC-ee: Entanglement, Bell inequality violation, $\sin\theta W$, and anomalous couplings*, *Phys. Rev. D* **110** (2024), no. 7 076004, [[arXiv:2405.09201](#)].

Testing new physics with QI observables

Trace distance

$$\mathcal{D}^T(\rho, \varsigma) = \frac{1}{2} \text{Tr} \sqrt{(\rho - \varsigma)^\dagger (\rho - \varsigma)}$$



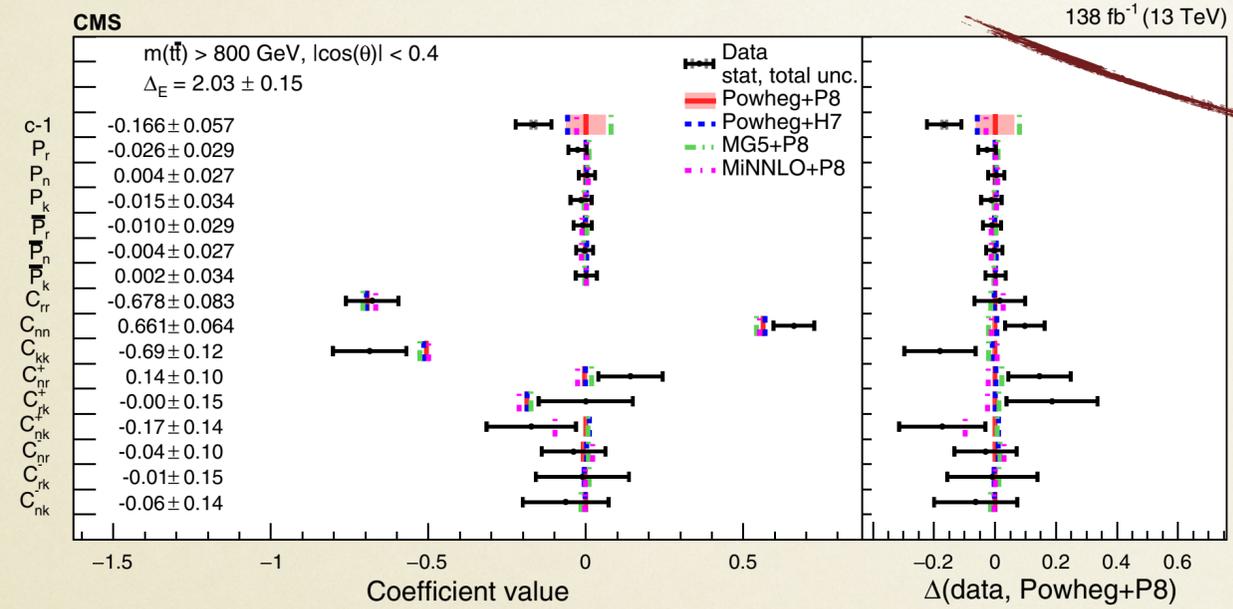
$$\rho = \frac{1}{2} [\mathbb{1} + \vec{r} \cdot \vec{\sigma}], \quad \varsigma = \frac{1}{2} [\mathbb{1} + \vec{s} \cdot \vec{\sigma}],$$

$$\mathcal{D}^T(\rho, \varsigma) = \frac{\|\vec{r} - \vec{s}\|}{2},$$

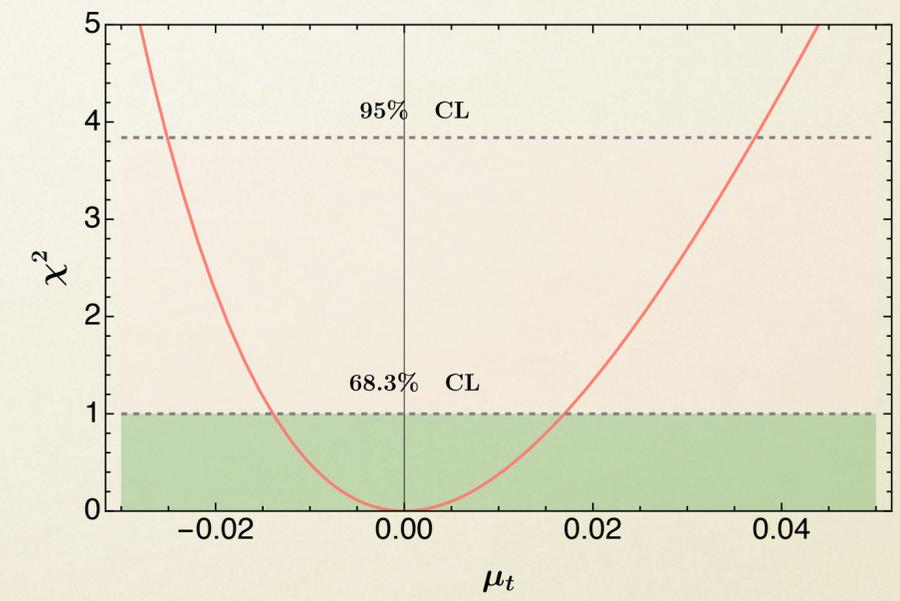
M. Fabbrichesi, M. Low, and L. Marzola, The trace distance between density matrices, a nifty tool in new-physics searches, *Phys. Rev. D* **112**, 013003 (2025).

CMS Collaboration, A. Hayrapetyan et al.,
Measurements of polarization and spin correlation and observation of entanglement in top quark pairs using lepton+jets events from proton-proton collisions at $\sqrt{s} = 13$ TeV,
 arXiv:2409.11067.

$$\mathcal{L}_{\text{dipole}} = \frac{c_{tG}}{\Lambda^2} (\mathcal{O}_{tG} + \mathcal{O}_{tG}^\dagger) \quad \text{with} \quad \mathcal{O}_{tG} = g_s (\bar{Q}_L \sigma^{\mu\nu} T^a t_R) \tilde{H} G_{\mu\nu}^a$$



$$\mu_t = -\frac{\sqrt{2} m_t v}{\Lambda^2} c_{tG}$$

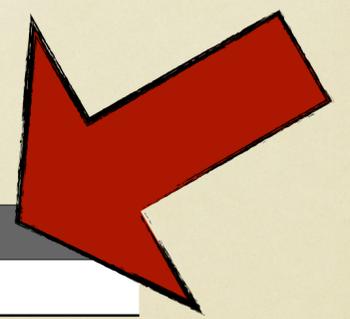


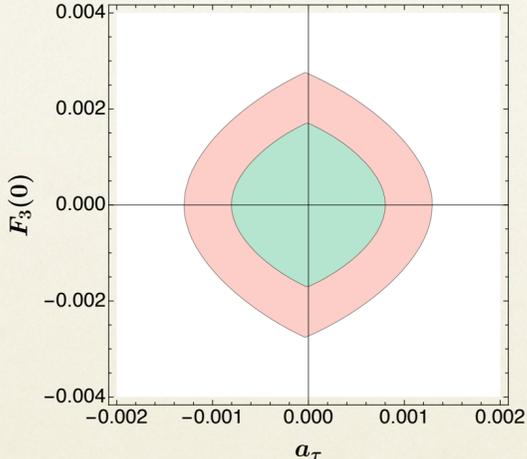
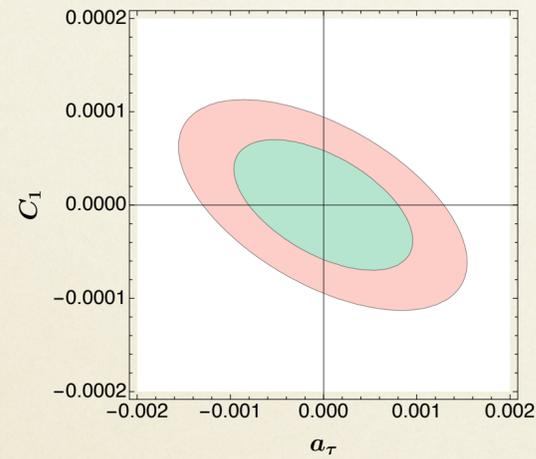
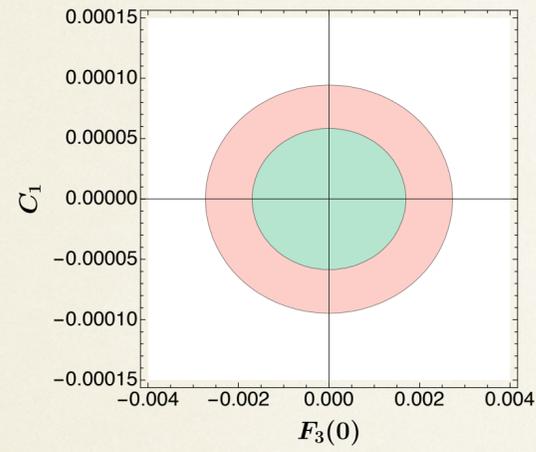
$$\left(\frac{\mathcal{D}^T[\rho_{\text{NP}}(\mu_t), \rho_{\text{SM}}]}{\sigma_{\mathcal{D}^T}} \right)^2 \leq (1.00) 3.84.$$

Figure 1: χ^2 test for the chromomagnetic dipole moment of the top quark μ_t obtained through the trace distance.

LHC

Benchmark	This work
$-0.046 \leq \mu_t \leq 0.040$	$-0.025 \leq \mu_t \leq 0.037$

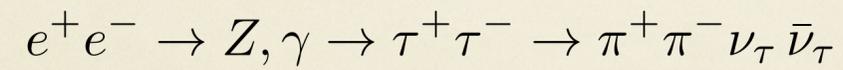




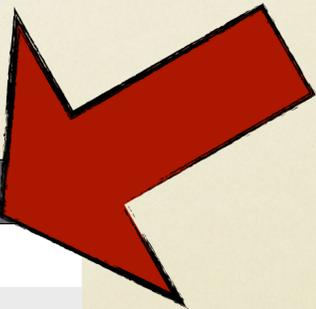
$$\chi^2(\lambda) = \left(\frac{\mathcal{D}^T[\rho_{\text{NP}}(\lambda), \rho_{\text{SM}}]}{\sigma_{\mathcal{D}^T}} \right)^2 + \left(\frac{\sigma_{\text{NP}}(\lambda) - \sigma_{\text{SM}}}{\sigma_{\sigma}} \right)^2 \leq (1.00) 3.84,$$

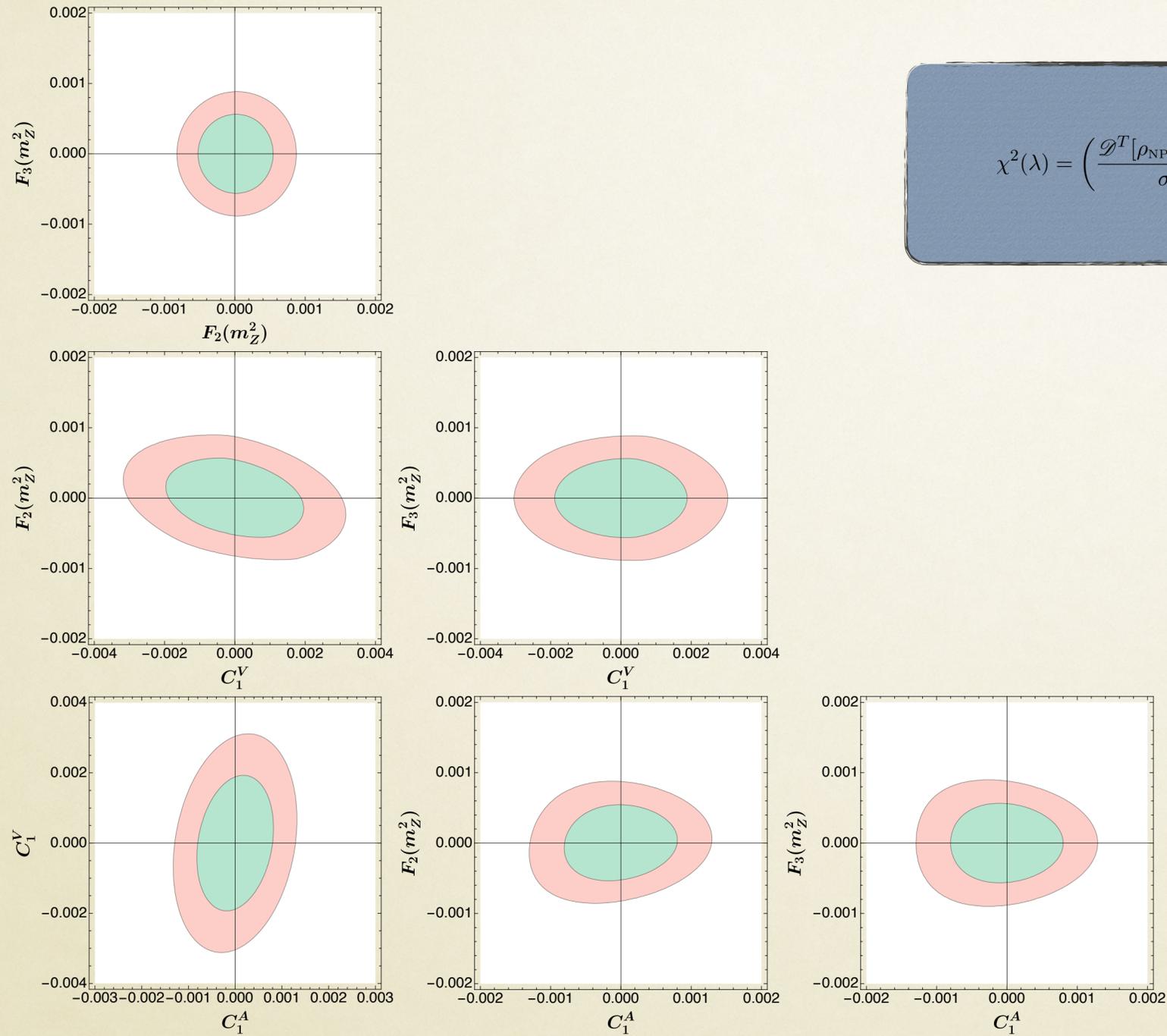
Belle II

Figure 2: Limits on the electromagnetic anomalous couplings of the τ lepton at Belle obtained with the test in Eq. (4.9).



PDG (2022)	This work
$-1.9 \times 10^{-17} \leq d_{\tau} \leq 6.1 \times 10^{-18} \text{ e cm}$	$ d_{\tau} \leq 1.5 \times 10^{-17} \text{ e cm}$
$-5.2 \times 10^{-2} \leq a_{\tau} \leq 1.3 \times 10^{-2}$	$ a_{\tau} \leq 1.0 \times 10^{-3}$
$ C_1 \leq 1.0 \times 10^{-5}$	$ C_1 \leq 1.0 \times 10^{-4}$





$$\chi^2(\lambda) = \left(\frac{\mathcal{D}^T[\rho_{\text{NP}}(\lambda), \rho_{\text{SM}}]}{\sigma_{\mathcal{D}^T}} \right)^2 + \left(\frac{\sigma_{\text{NP}}(\lambda) - \sigma_{\text{SM}}}{\sigma_\sigma} \right)^2 \leq (1.00) 3.84,$$

$$e^+e^- \rightarrow Z, \gamma \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^- \nu_\tau \bar{\nu}_\tau$$

Benchmark	This work
$-0.002 \leq F_2(m_Z^2) \leq 0.003$	$ F_2(m_Z^2) \leq 0.001$
$ F_3(m_Z^2) \leq 0.001$	$ F_3(m_Z^2) \leq 0.001$
$-0.009 \leq C_1^V \leq 0.010$	$ C_1^V \leq 0.003$
$ C_1^A \leq 0.001$	$ C_1^A \leq 0.001$

FCC-ee

Figure 4: Limits on the four form factors entering the coupling of the τ lepton to the Z boson obtained from the $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$ process at the Z peak via quantum tomography. The shown joint confidence intervals use the test in Eq. (5.6).

LEP3

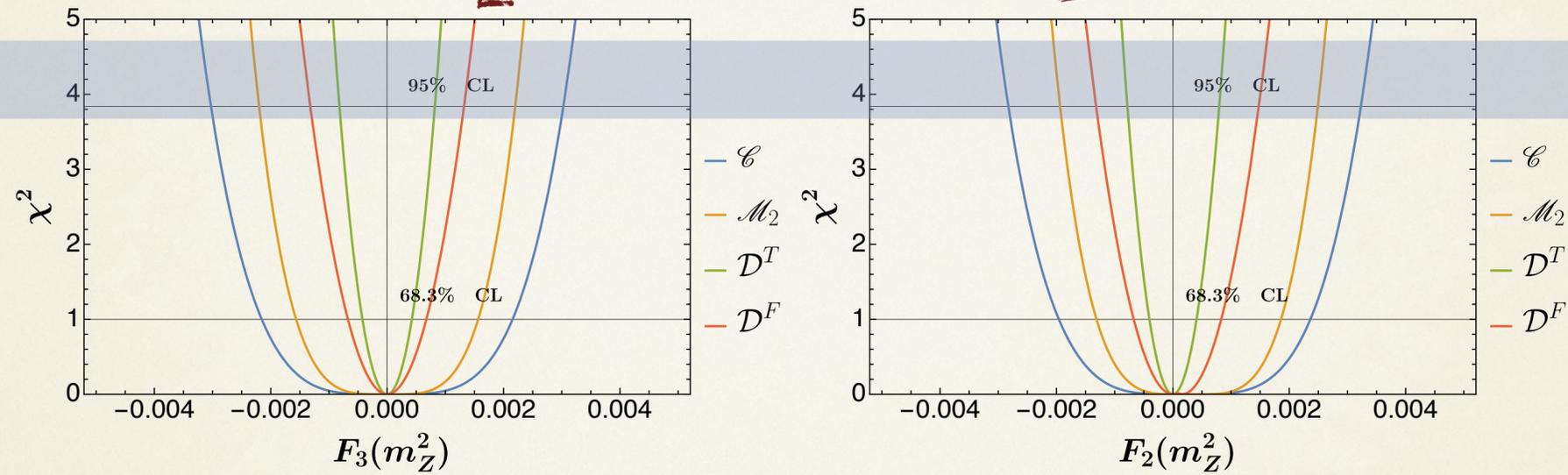


Figure 8: Comparison of χ^2 tests obtained for \mathcal{C} , \mathcal{M}_2 , \mathcal{D}^T , and \mathcal{D}^F as we vary individually the F_2 and F_3 form factors of the τ lepton in the setup of the LEP3 experiment.

Belle

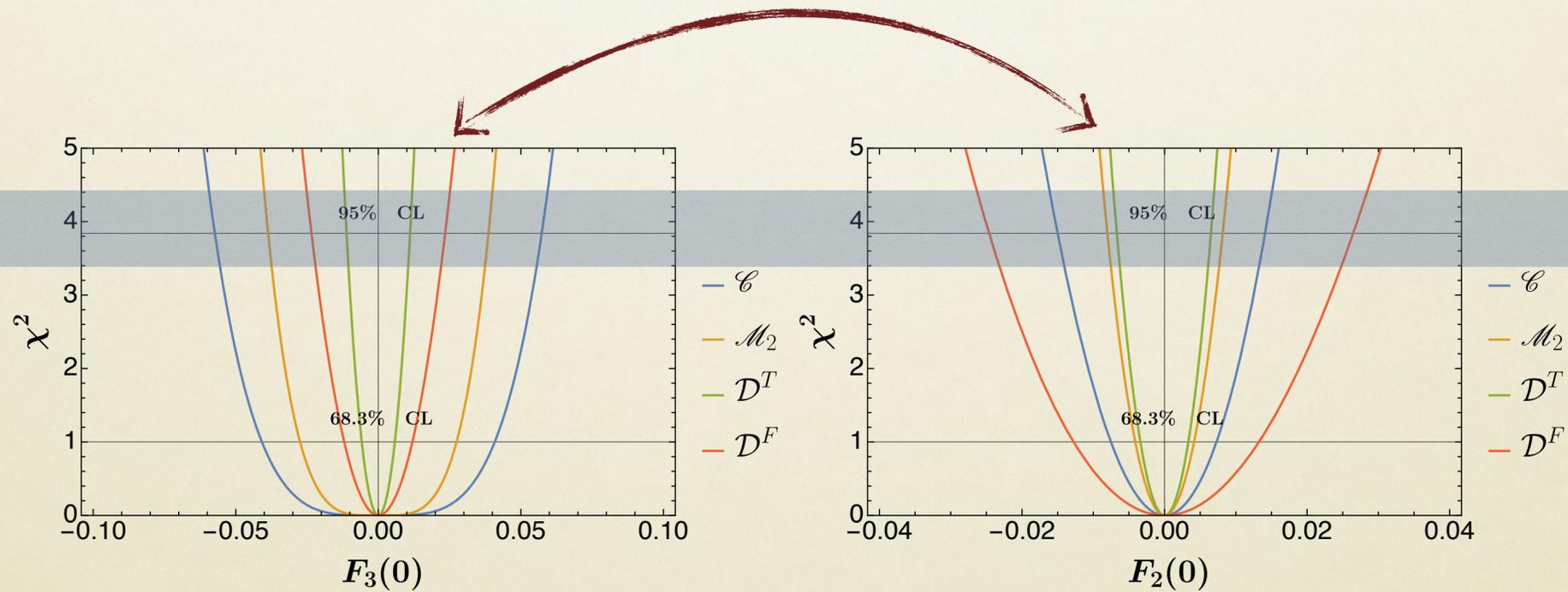


Figure 7: Comparison of χ^2 tests obtained for \mathcal{C} , \mathcal{M}_2 , \mathcal{D}^T , and \mathcal{D}^F as we vary individually the F_2 and F_3 form factors of the τ lepton in the setup of the Belle experiment.

$$\mathcal{F}(\rho, \varsigma) = \text{Tr} \sqrt{\sqrt{\rho} \varsigma \sqrt{\rho}}, \quad \mathcal{D}^F = \sqrt{1 - \mathcal{F}^2}.$$

IN CASE SOMEONE ASKS

Z

$$H \rightarrow ZZ^*$$

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Quantum contextuality of spin-1 massive particles*, arXiv:2503.14587.

M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, *Bell inequalities and quantum entanglement in weak gauge boson production at the LHC and future colliders*, *Eur. Phys. J. C* **83** (2023), no. 9 823, [arXiv:2302.00683].

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

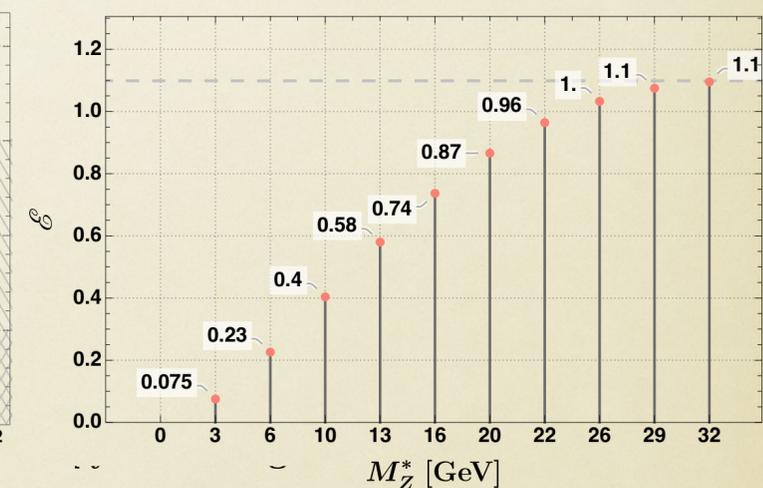
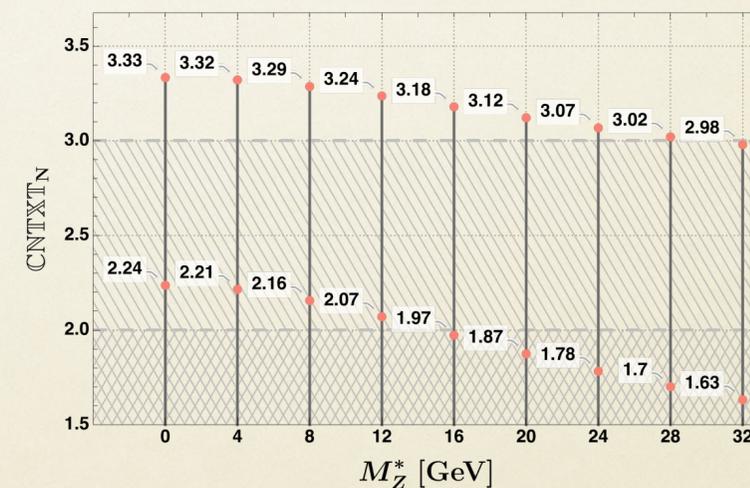
$$h_{16} = \frac{fM_V^2(-m_H^2 + (1+f^2)M_V^2)}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4},$$

$$h_{33} = \frac{1}{4} \frac{(m_H^2 - (1+f^2)M_V^2)^2}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4},$$

$$h_{44} = \frac{2f^2M_V^4}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4},$$

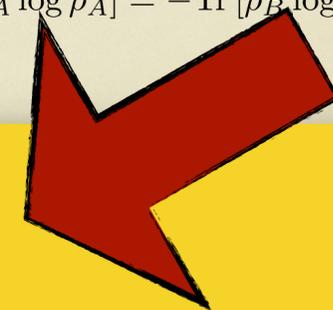
$$|\overline{\mathcal{M}}_H|^2 = \frac{g^2\xi_V^2}{4f^2M_V^2} [m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4].$$

$$\rho_{W,Z} = \text{Tr}_{W,Z} \rho_H = 2 \begin{pmatrix} h_{44} & 0 & 0 \\ 0 & 2h_{33} & 0 \\ 0 & 0 & h_{44} \end{pmatrix}.$$



$$\mathcal{E}[\rho] = -\text{Tr}[\rho_A \log \rho_A] = -\text{Tr}[\rho_B \log \rho_B],$$

$$\text{CNTXT}_5 = 2.1553 \pm 0.0006 \quad \text{and} \quad \text{CNTXT}_9 = 3.2860 \pm 0.0003,$$



$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle + \langle \mathcal{O}_2 \mathcal{O}_3 \rangle + \langle \mathcal{O}_3 \mathcal{O}_4 \rangle + \langle \mathcal{O}_4 \mathcal{O}_5 \rangle + \langle \mathcal{O}_5 \mathcal{O}_1 \rangle \geq -3 .$$

J. Ahrens, E. Amselem, A. Cabello, and M. Bourennane, *Two fundamental experimental tests of nonclassicality with qutrits*, arXiv:1301.2887.

$$|v_j\rangle = \left(\cos \phi, \sin \phi \cos \frac{4\pi j}{5}, \sin \phi \sin \frac{4\pi j}{5} \right)^T \quad \text{with} \quad \cos^2 \phi = \frac{\cos \pi/5}{1 + \cos \pi/5},$$

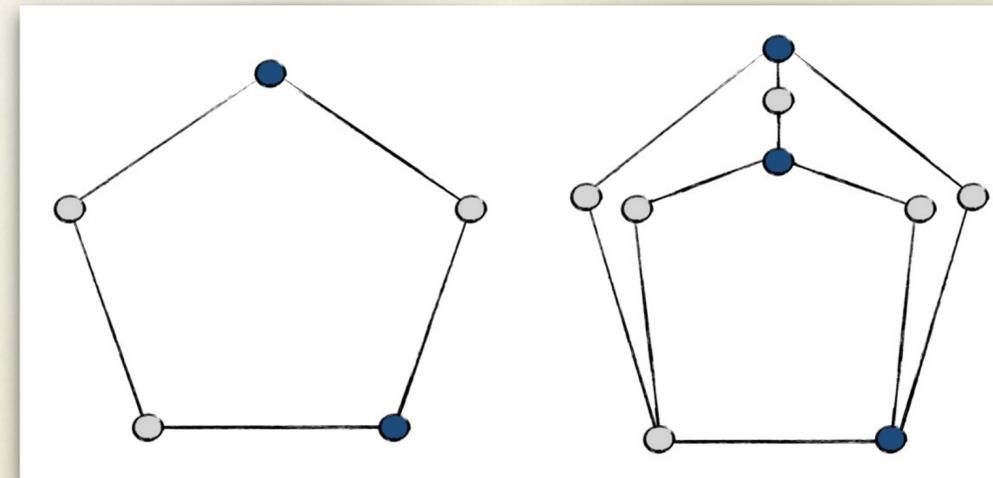
$$j = 1, 2, 3, 4 \text{ and } 5.$$

P. Kurzyński and D. Kaszlikowski, *Contextuality of almost all qutrit states can be revealed with nine observables*, *Phys. Rev. A* **86** (Oct, 2012) 042125.

$$\begin{aligned} |v_1\rangle &= (1, 0, 0)^T, & |v_2\rangle &= (0, 1, 0)^T, & |v_3\rangle &= (0, 0, 1)^T, \\ |v_4\rangle &= 1/\sqrt{2}(0, 1, -1)^T, & |v_5\rangle &= 1/\sqrt{3}(1, 0, -\sqrt{2})^T, & |v_6\rangle &= 1/\sqrt{3}(1, \sqrt{2}, 0)^T, \\ |v_7\rangle &= 1/2(\sqrt{2}, 1, 1)^T, & |v_8\rangle &= 1/2(\sqrt{2}, -1, -1)^T, & |v_9\rangle &= 1/2(\sqrt{2}, -1, 1)^T. \end{aligned}$$

$$\mathcal{O}_i = 1 - 2\Pi_i$$

$$\Pi_i \equiv |v_i\rangle\langle v_i|,$$



$$\text{CNTXT}_N \equiv \sum_{i=1}^N \langle \Pi_i \rangle = \sum_{i=1}^N \text{Tr}(\rho \Pi_i) \leq c_N,$$

$$c_5 = 2 \quad c_9 = 3 \quad c_{13} = 4$$

$$\langle \mathcal{O}_{11} \mathcal{O}_{12} \mathcal{O}_{13} \rangle + \langle \mathcal{O}_{21} \mathcal{O}_{22} \mathcal{O}_{23} \rangle + \langle \mathcal{O}_{31} \mathcal{O}_{32} \mathcal{O}_{33} \rangle + \langle \mathcal{O}_{11} \mathcal{O}_{21} \mathcal{O}_{31} \rangle + \langle \mathcal{O}_{12} \mathcal{O}_{22} \mathcal{O}_{32} \rangle - \langle \mathcal{O}_{13} \mathcal{O}_{23} \mathcal{O}_{33} \rangle \leq 4$$

$$\mathcal{O}_{11} = \sigma_3 \otimes \mathbb{1}$$

$$\mathcal{O}_{12} = \mathbb{1} \otimes \sigma_3$$

$$\mathcal{O}_{13} = \sigma_3 \otimes \sigma_3$$

$$\mathcal{O}_{21} = \mathbb{1} \otimes \sigma_1$$

$$\mathcal{O}_{22} = \sigma_1 \otimes \mathbb{1}$$

$$\mathcal{O}_{23} = \sigma_1 \otimes \sigma_1$$

$$\mathcal{O}_{31} = \sigma_3 \otimes \sigma_1$$

$$\mathcal{O}_{32} = \sigma_1 \otimes \sigma_3$$

$$\mathcal{O}_{33} = \sigma_2 \otimes \sigma_2 .$$

$$\mathcal{O}_{33} = \mathbb{1} \otimes \mathbb{1}$$

$$\text{CNT}\cancel{\text{X}}\text{T}' \equiv \langle (\sigma_3 \otimes \sigma_1)(\sigma_1 \otimes \sigma_3) \rangle - \langle (\sigma_3 \otimes \sigma_3)(\sigma_1 \otimes \sigma_1) \rangle \leq 0$$

$$(U \otimes V)^\dagger \cdot \left[(\sigma_3 \otimes \sigma_1)(\sigma_1 \otimes \sigma_3) - (\sigma_3 \otimes \sigma_3)(\sigma_1 \otimes \sigma_1) \right] \cdot (U \otimes V) .$$

$$\text{CNT}\cancel{\text{X}}\text{T}'' \equiv \langle (\sigma_1 \otimes \sigma_2)(\sigma_2 \otimes \sigma_1) \rangle - \langle (\sigma_1 \otimes \sigma_1)(\sigma_2 \otimes \sigma_2) \rangle \leq 0$$

$$\text{CNT}\cancel{\text{X}}\text{T}''' \equiv \langle (\sigma_2 \otimes \sigma_3)(\sigma_3 \otimes \sigma_2) \rangle - \langle (\sigma_2 \otimes \sigma_2)(\sigma_3 \otimes \sigma_3) \rangle \leq 0 .$$

N. D. Mermin, *Simple unified form for the major no-hidden-variables theorems*, Phys. Rev. Lett. **65** (1990), no. 27 3373.

A. Peres, *Incompatible results of quantum measurements*, Phys. Lett. A **151** (1990) 107–108.

A. Peres, *Two simple proofs of the Kochen-Specker theorem*, J. Phys. A **24** (1991), no. 4 L175.

C. Budroni, A. Cabello, O. Gühne, M. Kleinmann, and J.-r. Larsson, *Kochen-Specker contextuality*, Rev. Mod. Phys. **94** (2022), no. 4 045007, [[arXiv:2102.13036](https://arxiv.org/abs/2102.13036)].

Y. Nambu, *Proposed Experimental Test for Proving Quantum Contextuality with Non-entangled Photons*, [arXiv:0805.3398](https://arxiv.org/abs/0805.3398).

the toolbox

Qubits

$$\rho = \frac{1}{4} \left[\mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_{i=1}^3 B_i^+ (\sigma_i \otimes \mathbb{1}_2) + \sum_{i=1}^3 B_i^- (\mathbb{1}_2 \otimes \sigma_i) + \sum_{i,j=1}^3 C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

$$R = \rho (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$$

Concurrence $\mathcal{C}[\rho] = \max(0, r_1 - r_2 - r_3 - r_4)$

$$CC^T \quad [m_1, m_2, m_3]$$

Horodecki condition $\mathbf{m}_{12} \equiv m_1 + m_2 > 1$

Qutrits

$$\rho = \frac{1}{9} \left[\mathbb{1}_3 \otimes \mathbb{1}_3 + \sum_{a=1}^8 f_a [T^a \otimes \mathbb{1}_3] + \sum_{a=1}^8 g_a [\mathbb{1}_3 \otimes T^a] + \sum_{a,b=1}^8 h_{ab} [T^a \otimes T^b] \right]$$

$$\mathcal{C}_2 = 2 \max \left[-\frac{2}{9} - 12 \sum_a f_a^2 + 6 \sum_a g_a^2 + 4 \sum_{ab} h_{ab}^2; \right. \\ \left. -\frac{2}{9} - 12 \sum_a g_a^2 + 6 \sum_a f_a^2 + 4 \sum_{ab} h_{ab}^2, 0 \right]$$

Entropy $\mathcal{E}[\rho] \equiv -\text{Tr}[\rho_A \ln \rho_A] = -\text{Tr}[\rho_B \ln \rho_B]$

Negativity $\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2}$

Bell operator $\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}_3]$



A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Quantum entanglement and Bell inequality violation at colliders, *Prog. Part. Nucl. Phys.* **139**, 104134 (2024).

$$e^+e^- \rightarrow Z, \gamma \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$$

$$p_{\tau^+}^\mu + p_{\tau^-}^\mu = p_{e^+e^-}^\mu$$

$$\begin{aligned} (p_{\tau^+} - p_{\pi^+})^2 = m_\nu^2 = 0 & \quad \text{and} \quad (p_{\tau^-} - p_{\pi^-})^2 = m_\nu^2 = 0 \\ p_{\tau^+}^2 = m_\tau^2 & \quad \text{and} \quad p_{\tau^-}^2 = m_\tau^2. \end{aligned}$$

$$\mathbf{d}_{min} = \mathbf{d} + \frac{[(\mathbf{d} \cdot \mathbf{n}_+)(\mathbf{n}_- \cdot \mathbf{n}_+) - \mathbf{d} \cdot \mathbf{n}_-] \mathbf{n}_- + [(\mathbf{d} \cdot \mathbf{n}_-)(\mathbf{n}_- \cdot \mathbf{n}_+) - \mathbf{d} \cdot \mathbf{n}_+] \mathbf{n}_+}{1 - (\mathbf{n}_- \cdot \mathbf{n}_+)^2}.$$

statistical errors

FCC Collaboration, A. Abada et al., *FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2*, *Eur. Phys. J. ST* **228** (2019), no. 2 261–623.

We model the detector resolution with the following uncertainties:

$$\frac{\sigma_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{\text{GeV}} \quad \text{and} \quad \sigma_{\theta, \phi} = 0.1 \times 10^{-3} \text{ rad}$$

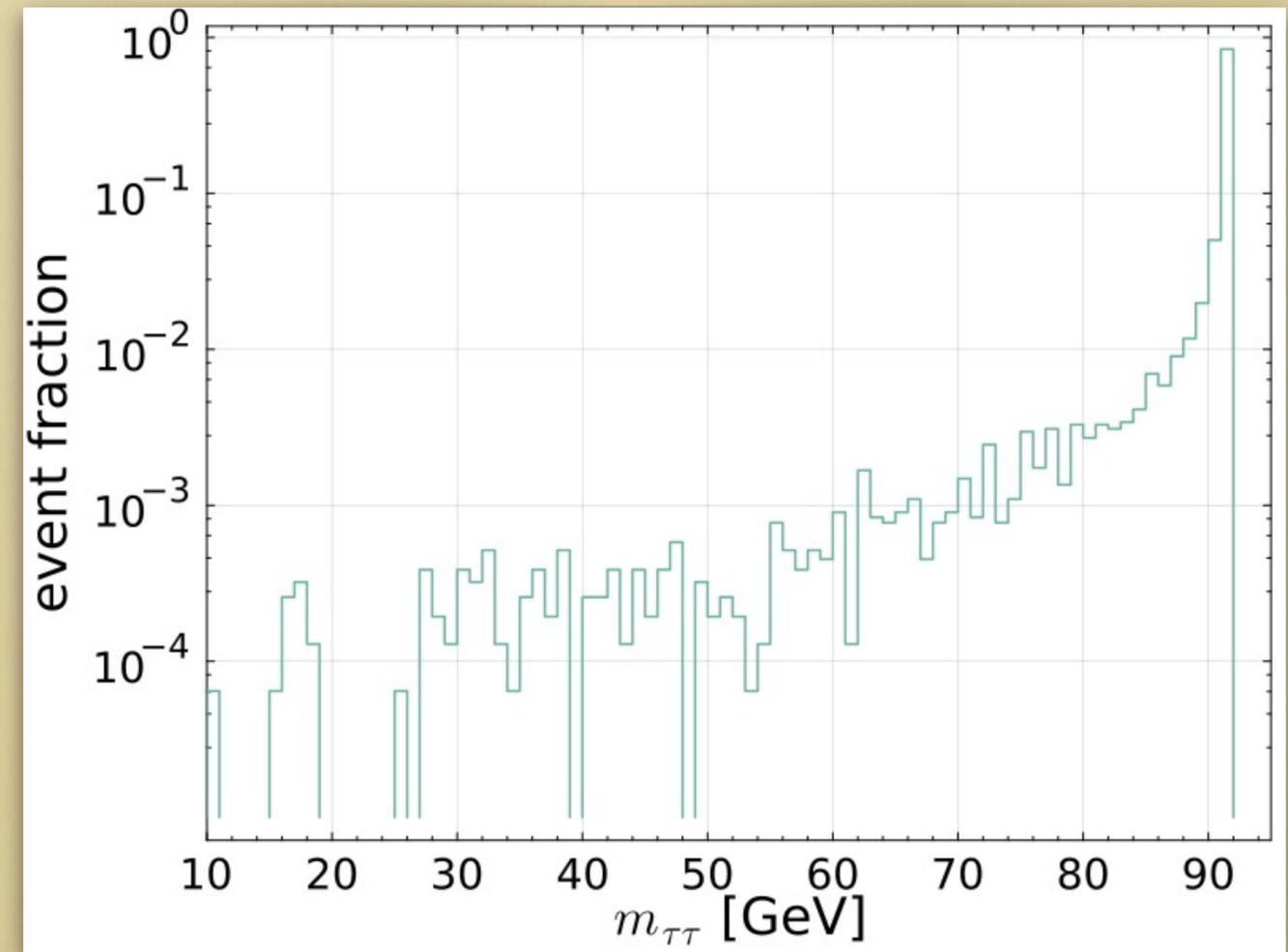
for the tracks proper and

$$\sigma_b = 3 \mu\text{m} \oplus \frac{15 \mu\text{m}}{\sin^{2/3} \Theta} \frac{\text{GeV}}{p_T}$$

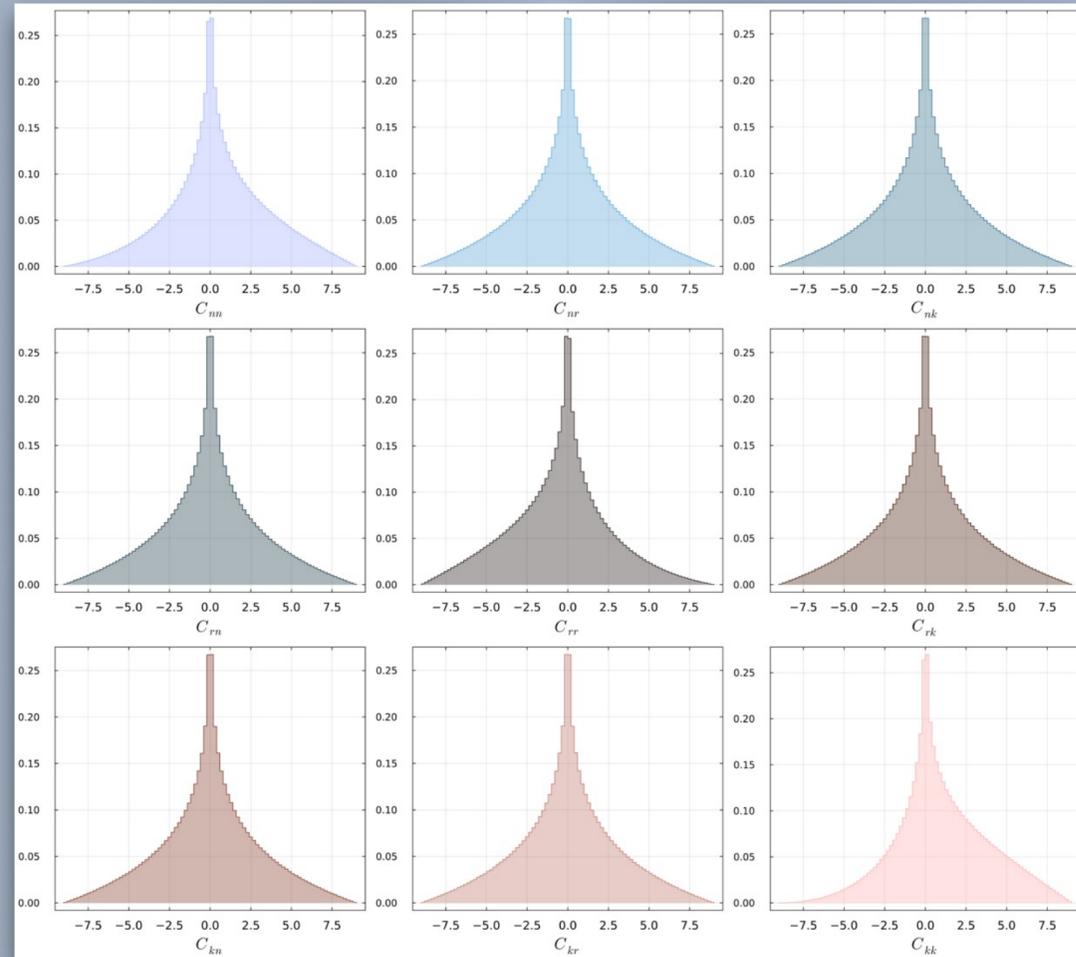
more statistical errors

C. Bierlich et al., *A comprehensive guide to the physics and usage of PYTHIA 8.3*, *SciPost Phys. Codeb.* **2022** (2022) 8, [[arXiv:2203.11601](https://arxiv.org/abs/2203.11601)].

plus systematic errors



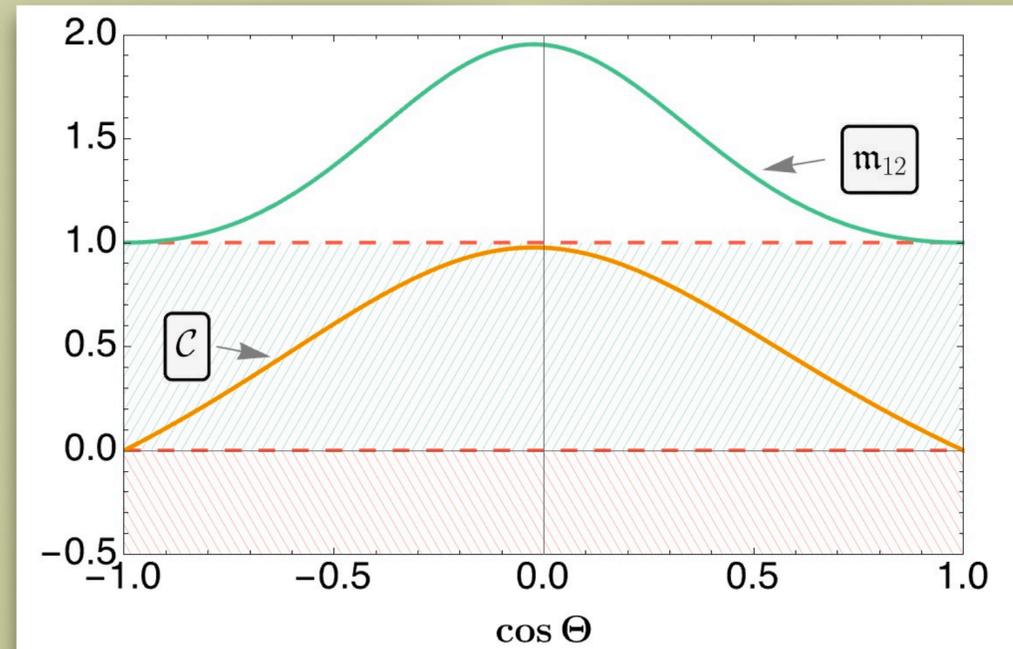
$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^+ d \cos \theta_j^-} = \frac{1}{4} \left(1 + C_{ij} \cos \theta_i^+ \cos \theta_j^- \right)$$



J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079, [[arXiv:1405.0301](https://arxiv.org/abs/1405.0301)].

K. Hagiwara, T. Li, K. Mawatari, and J. Nakamura, *TauDecay: a library to simulate polarized tau decays via FeynRules and MadGraph5*, *Eur. Phys. J. C* **73** (2013) 2489, [[arXiv:1212.6247](https://arxiv.org/abs/1212.6247)].

$$\rho = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{i,j} C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

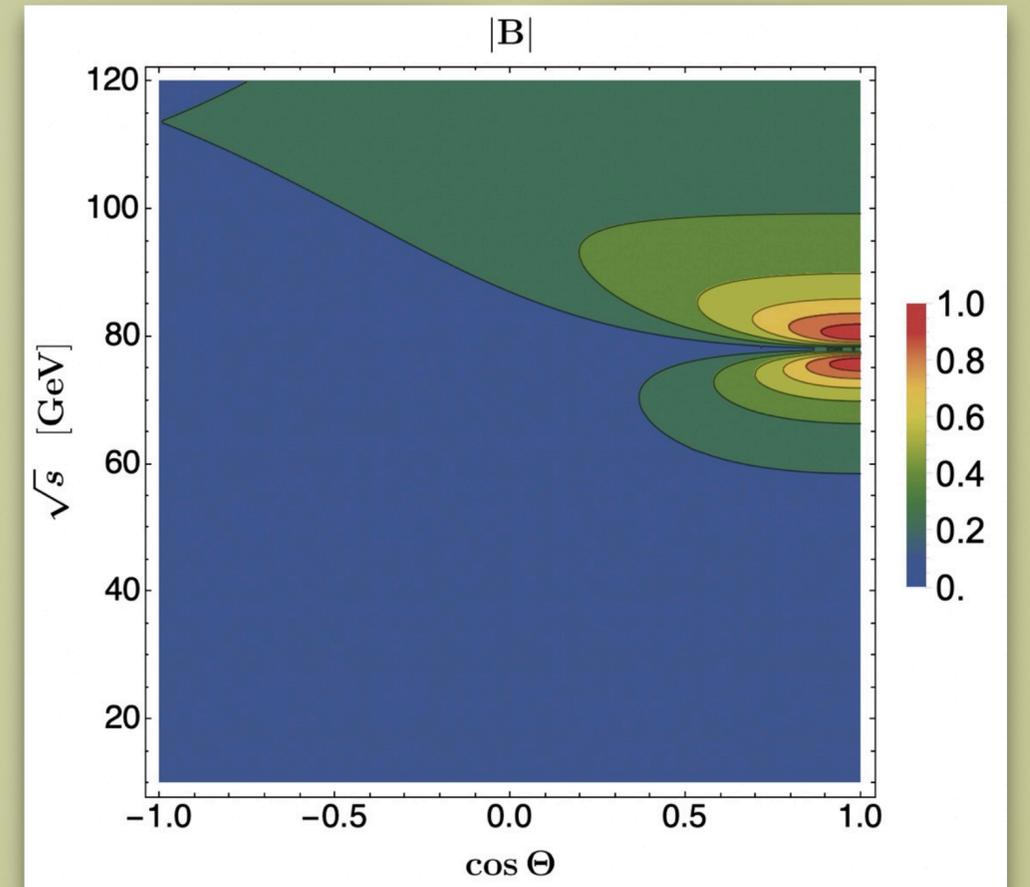
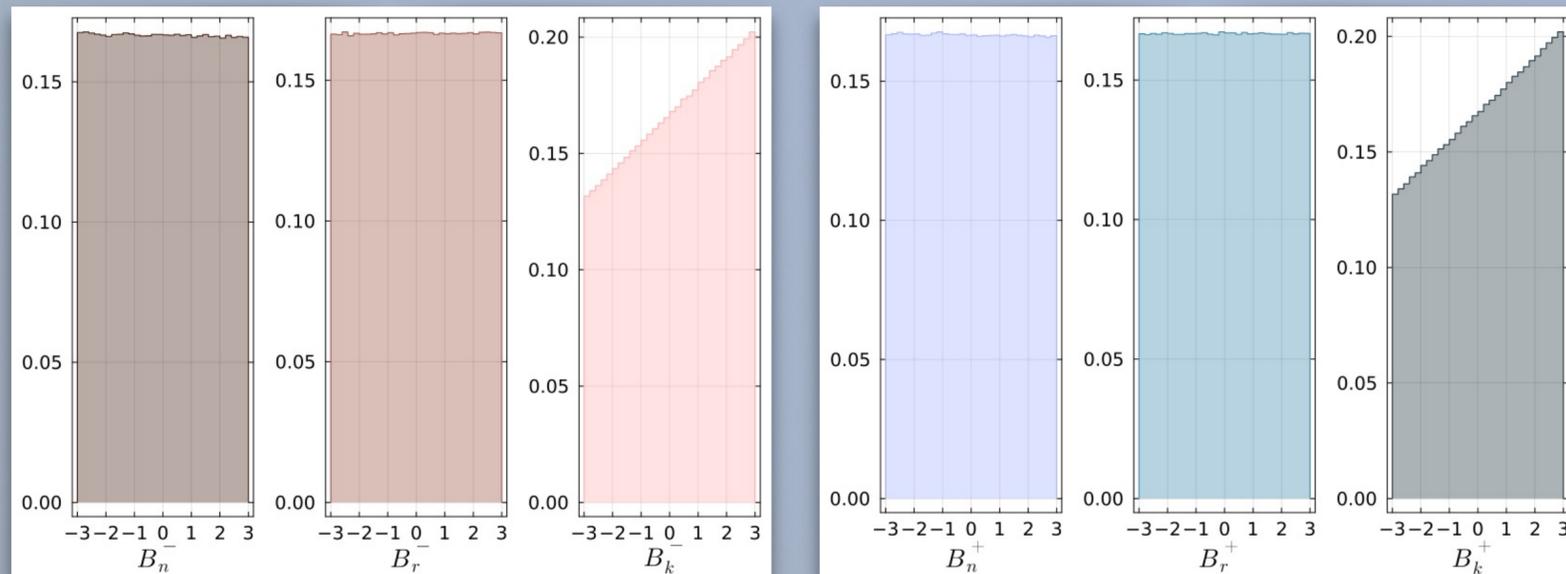


$$\mathcal{C} = 0.4805 \pm 0.0063|_{\text{stat}} \pm 0.0012|_{\text{syst}},$$

$$m_{12} = 1.239 \pm 0.017|_{\text{stat}} \pm 0.008|_{\text{syst}},$$

MF and L. Marzola, [arXiv:2405.09201](https://arxiv.org/abs/2405.09201)

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^\pm} = \frac{1}{2} (1 \mp B_i^\pm \cos \theta_i^\pm)$$



$$\langle P \rangle_\tau = \frac{1}{2} (B_k^+ + B_k^-) = 0.2203 \pm 0.0044|_{\text{stat}} \pm 0.0008|_{\text{syst}},$$

$$P_\tau(\cos \Theta) = \frac{\mathcal{A}_\tau (1 + \cos^2 \Theta) + 2 \cos \Theta \mathcal{A}_e}{1 + \cos^2 \Theta + 2 \cos \Theta \mathcal{A}_\tau \mathcal{A}_e},$$

$$\mathcal{A} = \mathcal{A}_e = \mathcal{A}_\tau = \frac{2(1 - 4 \sin^2 \theta_W)}{1 + (1 - 4 \sin^2 \theta_W)^2},$$

$$\sin^2 \theta_W = 0.2223 \pm 0.0006|_{\text{stat}} \pm 0.0001|_{\text{syst}},$$



Top-quark pairs

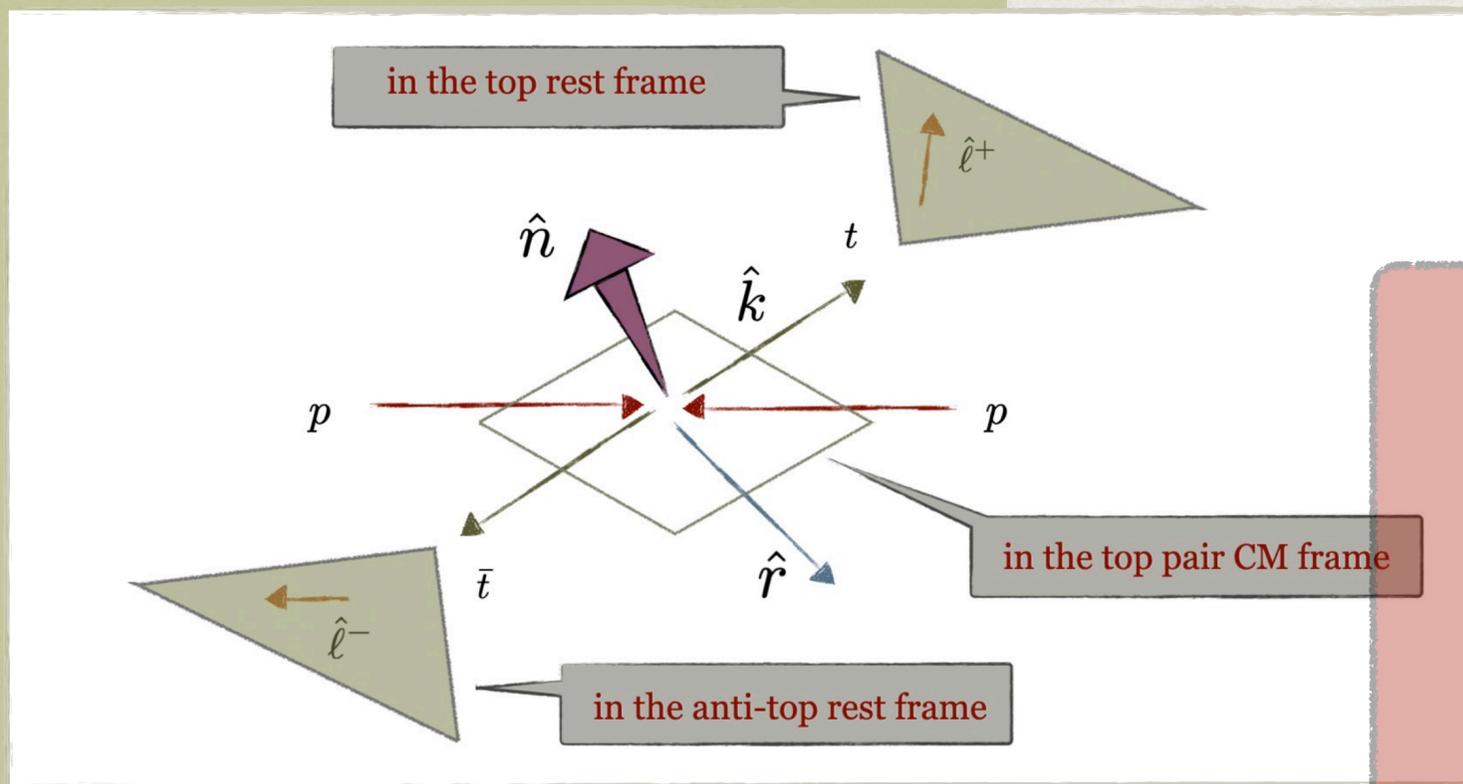
$$pp \rightarrow t + \bar{t} \rightarrow \ell^{\pm} \ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$



Event generation

MadGraph5 (NNPDF23)

DELPHES (fast simulation
ATLAS detector)



exactly two opposite sign lepton of different flavor

at least 2 anti-k_t jets with R=0.4

at least 1 b-tagged jet

$p_T > 25 \text{ GeV}$ $|\eta| < 2.5$ jets

$p_T > 20 \text{ GeV}$ $|\eta| < 2.47$ leptons

neutrino weighting technique

Implementing at the LHC

W. Bernreuther, D. Heisler, and Z. G. Si, J. High Energy Phys. 12 (2015) 026.

Y. Afik and J. R. M. de Nova, Eur. Phys. J. Plus 136, 907 (2021).

$$pp \rightarrow t + \bar{t} \rightarrow \ell^\pm \ell^\mp + \text{jets} + E_T^{\text{miss}}$$

$$\xi_{ab} = \cos \theta_+^a \cos \theta_-^b$$

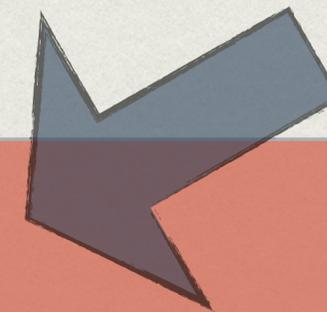
3 x 3 matrix

$$C_{ab}[\sigma(m_{t\bar{t}}, \cos \Theta)] = -9 \frac{1}{\sigma} \int d\xi_{ab} \frac{d\sigma}{d\xi_{ab}} \xi_{ab}$$

diagonalization for each value of invariant mass and scattering angle

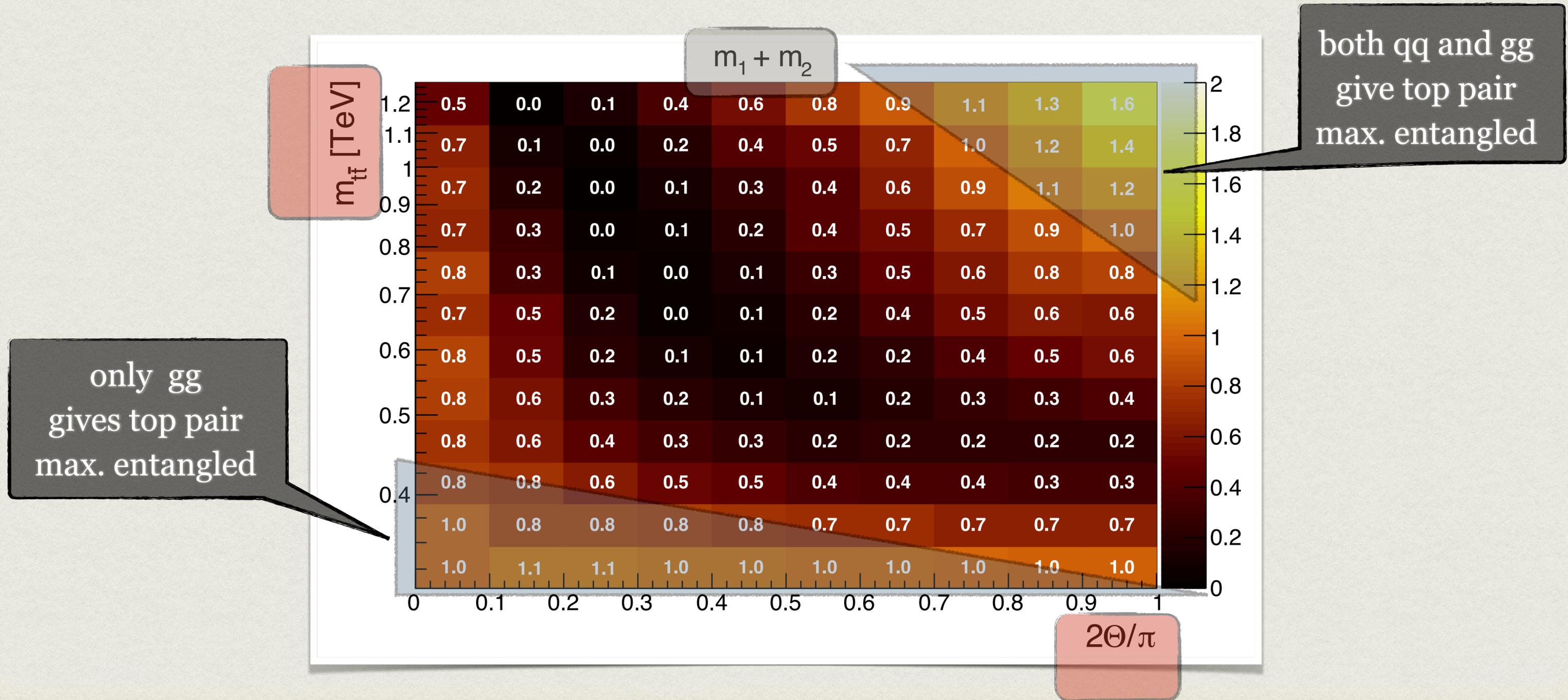
	label	\hat{a}	\hat{b}
transverse	n	$\text{sign}(y_p) \hat{n}_p$	$-\text{sign}(y_p) \hat{n}_p$
r axis	r	$\text{sign}(y_p) \hat{r}_p$	$-\text{sign}(y_p) \hat{r}_p$
helicity	k	\hat{k}	$-\hat{k}$

$$m_1 + m_2 > 1$$



MC analysis

M. Fabbriches, R. Floreanini, and G. Panizzo,
*Testing Bell Inequalities at the LHC with
Top-Quark Pairs, Phys. Rev. Lett.* **127** (2021),
no. 16 161801, [[arXiv:2102.11883](https://arxiv.org/abs/2102.11883)].





Results

bins

$$\frac{2\Theta}{\pi} \gtrsim 0.7 \quad m_{t\bar{t}} \gtrsim 0.9 \text{ TeV}$$

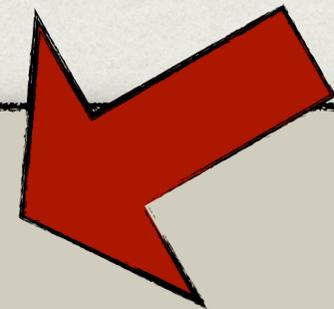
1.1	1.3	1.6
1.0	1.2	1.4
0.9	1.1	1.2

null hypothesis: $m_1 + m_2 \leq 1$

Hypothesis test

$$\chi^2 = \sum_i \frac{(1 - m_1^i - m_2^i)^2}{s_i^2}$$

violation: **98% CL** w/ Run II data (139 fb⁻¹)
99.99% CL with Run III



systematic uncertainties (e.g. from unfolding) not included