Inflationary magnetogenesis with massive vector fields and Schwinger effect

Astrophysics and Space Science in Marche III: Big Bang, Big stars, Big Computers

Mattia Dubbini

Università di Camerino, Via Madonna delle Carceri, Camerino, 62032, Italy
Dipartimento di Fisica "Ettore Pancini", Università degli Studi di Napoli “Federico II”, Via Cinthia, Edificio 6, 80126, Napoli, Italy

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Abstract

We investigate inflationary magnetogenesis considering massive electromagnetic fields, including scalar particle production via the Schwinger effect and the backreaction induced on the background electric energy density.

- Without Schwinger production:
  - complete quantum approach,
  - calculation of longitudinal, electric and magnetic energy densities (with backreaction),
  - plausible masses do not alter the dynamics,
  - the electric contribution remains dominant (background),
  - calculation of the magnetic field.
- Schwinger effect (complex scalar fields):
  - separation between electric background (affected by Schwinger) and other contributions (unaffected),
  - semiclassical approach,
  - the electric background is reduced by the particle production.

The massless framework

Breaking the conformal invariance of the Maxwell electromagnetism through the coupling $-\frac{f^2(\phi)}{4}F^{\mu\nu}F_{\mu\nu}$ with the inflaton field:

- $f^2(\phi) = 1 \to$ restoration of the Maxwell theory;
- dynamical $f^2(\phi) \to$ inflationary magnetogenesis scenario: negative friction-like term within the EM equation of motion (amplification).

The inflationary dynamics is governed by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 2\frac{\partial(\ln f)}{\partial\phi}\rho_{\mathrm{E}}. \tag{1}$$

- As long as the source is negligible, we can take $f(\phi) \equiv f(\phi(\eta)) \equiv f(\eta) = \left(\frac{a}{a_e}\right)^{\alpha}$, with $\alpha < -2$.
- When the source becomes relevant, the backreaction regime is established, and it can be analytically treated considering $\alpha = -2$, yielding almost constant electric fields.

Two different approaches have been followed in scientific literature.

1. Quantization of the electromagnetic field, computation of the electric and magnetic energy densities from the electromagnetic energy-momentum tensor. In a homogeneous and isotropic universe, we have $\rho_{\mathrm{E}} \gg \rho_{\mathrm{B}}$.

   J. Martin, J. Yokoyama. In: JCAP 01 (2008) 025. DOI: 10.1088/1475-7516/2008/01/025. e-Print: 0711.4307.

2. Separation between electric background and magnetic quantum fluctuations: classical time-evolution equation for the electric energy density and calculation of the magnetic component as inhomogeneous quantum fluctuations of the electromagnetic field. The Schwinger effect is included adding a source term proportional to $E \cdot j$ in the time-evolution equation for the electric energy density.

   O.O. Sobol, E.V. Gorbar, M. Kamarpour, S.I. Vilchinskii. In: Phys.Rev.D 98 (2018) 6, 063534. DOI: 10.1103/PhysRevD.98.063534. e-Print: 1807.09851.

Inflationary magnetogenesis within the Maxwell–Proca electromagnetism

Previous works dealt with inflationary magnetogenesis within massive electromagnetism

- considering dynamical mass-couplings between inflaton and EM field,
- without including the Schwinger effect in the model.

A.V. Lysenko, O.O. Sobol, S.I. Vilchinskii. In: Phys.Rev.D 113 (2026) 6, 063525. DOI: 10.1103/lpph-hrdr. e-Print: 2509.24963.

Differently, we consider

- the well-known Proca theory of massive electromagnetism,
- masses compatible with the experimental constraints,
- the impact of the Schwinger production in the electric background.

We aim to investigate

- the impact of the mass term in the transverse-mode dynamics,
- to which extent the longitudinal modes affect the electric background.

We take $A_\mu = (A_0, -\mathbf{A})$, and conventionally we define

$$(\mathbf{E})_i = \frac{F_{0i}}{a} \implies \mathbf{E} = -\frac{\dot{\mathbf{A}} + \nabla A_0}{a}, \qquad (2)$$
$$(\mathbf{B})_i = -\frac{1}{2a^2}\epsilon_{ijk}F_{jk} \implies \mathbf{B} = \frac{\nabla \wedge \mathbf{A}}{a^2}.$$

We exploit the Helmholtz decomposition of the vector potential, $\mathbf{A} = \mathbf{A}_\perp + \mathbf{A}_\parallel$. The dynamics of the electromagnetic field is governed by 4 equations:

$$\nabla \dot{\mathbf{A}}_\parallel = -\left(\nabla^2 - \frac{a^2 m^2}{f^2}\right) A_0, \qquad (3a)$$

$$\ddot{\mathbf{A}}_\perp - \frac{\nabla^2 \mathbf{A}_\perp}{a^2} + \left(H + 2\frac{\dot{f}}{f}\right)\dot{\mathbf{A}}_\perp + \frac{m^2}{f^2}\mathbf{A}_\perp = 0, \qquad (3b)$$

$$\ddot{\mathbf{A}}_\parallel + \left(H + 2\frac{\dot{f}}{f}\right)\dot{\mathbf{A}}_\parallel + \frac{m^2}{f^2}\mathbf{A}_\parallel = -\nabla\left[\dot{A}_0 + \left(H + 2\frac{\dot{f}}{f}\right)A_0\right], \qquad (3c)$$

$$\dot{A}_0 + 3HA_0 = -\frac{\nabla \mathbf{A}_\parallel}{a^2}. \qquad (3d)$$

The longitudinal modes

Longitudinal modes quantization:

$$\hat{A}_0(t,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}} fm} \left[\frac{i}{a} \hat{b}_{\mathbf{k},\mathrm{L}} \mathcal{D}_L(t,k) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right], \tag{4a}$$

$$\hat{\mathbf{A}}_{\parallel}(t,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}} fm} \left[\frac{\mathbf{k}}{k} \hat{b}_{\mathbf{k},\mathrm{L}} \mathcal{A}_L(t,k) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]. \tag{4b}$$

In Fourier space we obtain

$$\begin{cases} \mathcal{A}_\mathrm{L}' + \frac{\alpha}{\eta}\mathcal{A}_\mathrm{L} = k\left(1 + \frac{a^2m^2}{k^2f^2}\right)\mathcal{D}_\mathrm{L} \\ \mathcal{D}_\mathrm{L}' + \frac{\alpha-2}{\eta}\mathcal{D}_\mathrm{L} = -k\mathcal{A}_\mathrm{L} \end{cases} . \tag{5}$$

The mass term is negligible as long as experimental bounds are respected, yielding simple solutions:

$$\mathcal{A}_\mathrm{L}(\eta) = \frac{f}{a}\sqrt{\frac{k}{2}}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta}, \quad \mathcal{D}_\mathrm{L}(\eta) = -i\frac{f}{a}\sqrt{\frac{k}{2}}e^{-ik\eta}. \tag{6}$$

The transverse modes

Transverse modes quantization:

$$\hat{A}_{\perp j}(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}} f} \sum_{\lambda=1}^{2} \left[\epsilon_j^{(\lambda)}(k) \hat{b}_{\mathbf{k},\lambda} \mathcal{A}_\lambda(t, k) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]. \tag{7}$$

In Fourier space we obtain

$$\mathcal{A}_\lambda'' + k^2 \left(1 + \frac{a^2 m^2}{k^2 f^2} - \frac{\alpha(\alpha+1)}{k^2 \eta^2} \right) \mathcal{A}_\lambda = 0, \tag{8}$$

where the mass term is proved to be negligible once again, yielding

$$\mathcal{A}_\lambda(\eta) = \sqrt{-\eta} \left[c_1 J_{\alpha+\frac{1}{2}}(-k\eta) + c_2 Y_{\alpha+\frac{1}{2}}(-k\eta) \right] \xrightarrow{\text{BD}} e^{\frac{i\pi}{2}(1+\alpha)} \sqrt{-\eta} \frac{\sqrt{\pi}}{2} H^{(1)}_{\alpha+\frac{1}{2}}(-k\eta). \tag{9}$$

Notice that the solution depends on α, thus matching with– and without–backreaction solutions is necessary to study the entire dynamics.

The electromagnetic fields

Once known the mode-solutions, we can find the electromagnetic fields:

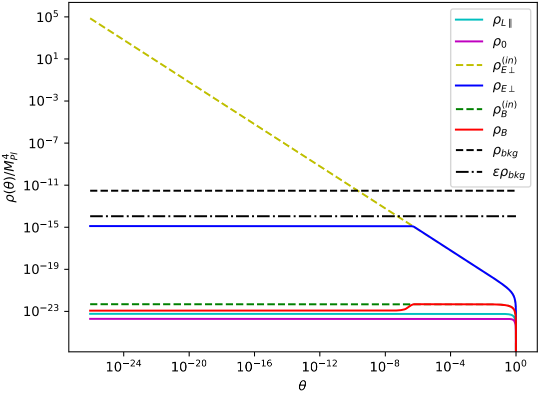
$$\hat{\mathbf{E}}_{\parallel} = -\frac{1}{a^2}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}} fm}\left[\frac{\mathbf{k}}{k}\hat{b}_{\mathbf{k},\mathrm{L}}\left(\mathcal{A}'_{\mathrm{L}} + \frac{\alpha}{\eta}\mathcal{A}_{\mathrm{L}} - k\mathcal{D}_{\mathrm{L}}\right)e^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{h.c.}\right], \tag{10a}$$

$$(\hat{\mathbf{E}}_{\perp})_j = -\frac{1}{a^2 f}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\sum_{\lambda=1}^{2}\left[\epsilon_j^{(\lambda)}(k)\hat{b}_{\mathbf{k},\lambda}\left(\mathcal{A}'_{\lambda} + \frac{\alpha}{\eta}\mathcal{A}_{\lambda}\right)e^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{h.c.}\right], \tag{10b}$$

$$(\hat{\mathbf{B}})_m = \frac{1}{a^2 f}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\sum_{\lambda=1}^{2}\left[i\epsilon_{mij}k_i\epsilon_j^{(\lambda)}(k)\hat{b}_{\mathbf{k},\lambda}\mathcal{A}_{\lambda}e^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{h.c.}\right]. \tag{10c}$$

The electromagnetic energy density

Finally, we compute the various components of the electromagnetic energy density, with respect to $\theta \equiv -k_{\rm i}\eta$. We show only the dominant terms in the plot below.



$$\rho_{\rm L\parallel} \equiv \frac{m^2}{2a^2}\left\langle \mathbf{A}_\parallel^2 \right\rangle , \qquad (11a)$$

$$\rho_0 \equiv \frac{m^2}{2}\left\langle A_0^2 \right\rangle , \qquad (11b)$$

$$\rho_{\rm E\perp} \equiv \frac{f^2}{2}\left\langle \mathbf{E}_\perp^2 \right\rangle , \qquad (11c)$$

$$\rho_{\rm B} \equiv \frac{f^2}{2}\left\langle \mathbf{B}^2 \right\rangle . \qquad (11d)$$

The magnetic field at our time

- We compute the value of the magnetic energy density at the end of inflation: $\frac{\rho_{\rm B}(\theta_{\rm min})}{M_{\rm Pl}^4} \simeq 1.18 \times 10^{-23}$.
- We compute the magnetic field at the end of the inflation:

$$B_e \equiv B(\theta_{\rm min}) = \sqrt{\frac{2\rho_B(\theta_{\rm min})}{f^2(\theta_{\rm min})}} \simeq 4.86 \times 10^{-12} M_{\rm Pl}^2. \qquad (12)$$

- The magnetic field at our time, say B_0, can be computed from B_e through the magnetic flux conservation law, implying that $B \propto a^{-2}$:

$$B_0 \equiv \left(\frac{a_e}{a_0}\right)^2 B_e \simeq 2.67 \times 10^{-17}\ \mathrm{G} \in [10^{-18}, 10^{-9}]\ \mathrm{G}. \qquad (13)$$

This value is not significantly affected by the presence of the photon mass as long as the latter remains within the experimental bounds.

The Schwinger effect

The Schwinger effect is commonly invoked within inflationary magnetogenesis as the most natural way to suppress the strong electric fields produced during the inflation, explaining why nowadays we observe no electric fields but only magnetic ones.

We consider the scalar QED Lagrangian for the complex scalar fields:

$$\begin{aligned}\mathcal{L}_{\mathrm{Schw}} &= \sqrt{-g}\,(D_\mu\chi)\,(D^\mu\chi)^* = \\ &= \sqrt{-g}\Big[\,(\partial_\mu\chi)\,(\partial^\mu\chi^*) + e^2 A^\mu A_\mu \chi\chi^* + ieA_\mu\,(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi)\Big].\end{aligned} \tag{14}$$

The scalar-field dynamics is governed by

$$\ddot{\chi} - \frac{\nabla^2\chi}{a^2} + 3H\dot{\chi} + 2ieA^\mu\partial_\mu\chi - e^2 A^\mu A_\mu \chi = 0. \tag{15}$$

The theory is now interacting, and the fully quantum approach would become challenging.

However, by virtue of the previous results, we can deal with the electric background and the perturbative components separately.

Electric (background)

- The Schwinger current affects the background.
- The problem is interacting.
- Semiclassical approach: classical electromagnetic field and quantum scalar field.
- Consistency check for $e = 0$ with the other (fully quantum) approach.

Magnetic and longitudinal

- Unaltered by the Schwinger particle production.
- The problem is free.
- Fully quantum approach: the electromagnetic field is quantized.
- By definition, nothing changes from the case $e = 0$.

The resolution of the coupled dynamics

In presence of the scalar field, the equations governing the electromagnetic-field dynamics turn to

$$\left(m^2 + 2e^2 \langle \chi\chi^* \rangle\right) A_0 = -e \langle J_0 \rangle \,, \tag{16a}$$

$$\ddot{A}_i + \left(H + 2\frac{\dot{f}}{f}\right)\dot{A}_i + \left(\frac{m^2 + 2e^2 \langle \chi\chi^* \rangle}{f^2}\right) A_i = -\frac{e}{f^2} \langle J_i \rangle \,. \tag{16b}$$

Even though the Gauss' law, at first glance, does not imply $A_0 = 0$, it can be proved that this is actually the unique physically-reasonable result.

About the scalar-field modes, instead, we get

$$\xi''_{\mathbf{k}} + \left[(k_i + eA_i)^2 - \frac{2}{\eta^2}\right]\xi_{\mathbf{k}} = 0, \tag{17}$$

where the modes refer to

$$\hat{\chi}(t, \mathbf{x}) = \frac{1}{a}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \left[\hat{c}_{\mathbf{k}}\xi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{d}^{\dagger}_{\mathbf{k}}\xi^*_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]. \tag{18}$$

In terms of $\mathcal{A}_i = fA_i$ and with respect to conformal time, specializing the vector potential along the z-axis we get

$$\begin{cases} \xi_{\mathbf{k}}'' + \left[k^2 + 2k_z e_{\text{eff}} \mathcal{A}_i + e_{\text{eff}}^2 \mathcal{A}_i^2 - \frac{2}{\eta^2}\right] \xi_{\mathbf{k}} = 0 \\ \mathcal{A}_i'' + \left[2e_{\text{eff}}^2 \langle \xi \xi^* \rangle - \frac{\alpha(\alpha+1)}{\eta^2}\right] \mathcal{A}_i = -e_{\text{eff}} \langle \mathcal{J}_i \rangle \end{cases}, \tag{19}$$

where the mass-term can be once again neglected – within the experimental bounds – and the new VEVs are a^2 times the old ones.

Writing the VEVs explicitly, we obtain

$$\begin{cases} \xi_{\mathbf{k}}'' + \left[k^2 + 2kxe_{\text{eff}} \mathcal{A}_i + e_{\text{eff}}^2 \mathcal{A}_i^2 - \frac{2}{\eta^2}\right] \xi_{\mathbf{k}} = 0 \\ \mathcal{A}_i'' + \left[\frac{e_{\text{eff}}^2}{2\pi^2} \int_{-1}^{1} dx \int_{k_{\text{i}}}^{k_H} k^2 |\xi_{\mathbf{k}}|^2 dk - \frac{\alpha(\alpha+1)}{\eta^2}\right] \mathcal{A}_i = -\frac{e_{\text{eff}}}{2\pi^2} \int_{-1}^{1} dx \int_{k_{\text{i}}}^{k_H} k^3 x |\xi_{\mathbf{k}}|^2 dk \end{cases}. \tag{20}$$

Numerical resolution of the problem

Solving the coupled system numerically requires considering

- rescaled dimensionless mode-solutions and momentum variables,
- the evolution with respect to $N \equiv -\ln(\theta)$,
- logarithmic and time-dependent momentum variables.

Even adopting these common precautions, the system is still numerically rather unstable.

- The scalar modes $\xi_{\mathbf{k}}$ are almost constant in x, and thus the integrand in the r.h.s. of the EM equation is almost odd.
- The integration in the symmetric domain $[-1, 1]$, at leading order, should give exactly zero, and the biggest non-null term should be given by a very small first-order correction.
- When the integration step is not small enough in order to correctly detect the cancellation of the leading-order contributions, a numerical instability appears.

We address this problem writing

$$\tilde{\xi}_{\mathbf{k}} = \tilde{\xi}_{\mathbf{k}}^{(0)} + \delta\tilde{\xi}_{\mathbf{k}}, \tag{21}$$

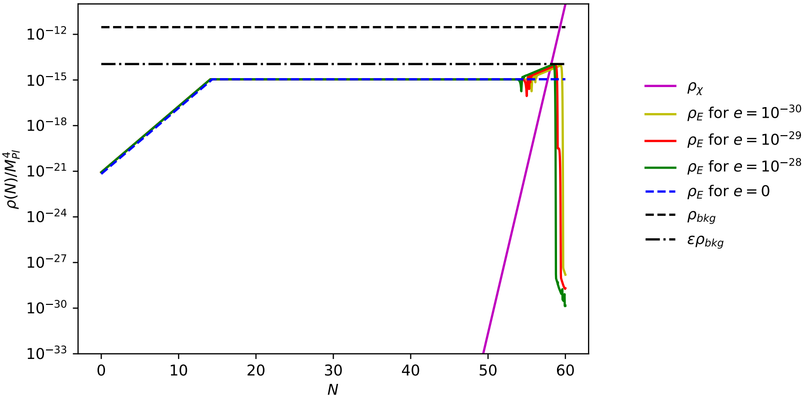
where all the dependence on x is contained in the small perturbation $\delta\tilde{\xi}_{\mathbf{k}}$. Finally we obtain

$$\begin{cases} \frac{d^2\tilde{\xi}_{\mathbf{k}}^{(0)}}{dN^2} + \frac{d\tilde{\xi}_{\mathbf{k}}^{(0)}}{dN} + \left[e^{-2y} + \delta^2\tilde{\mathcal{A}}_i^2 - 2\right]\tilde{\xi}_{\mathbf{k}}^{(0)} = 0 \\ \frac{d^2\delta\tilde{\xi}_{\mathbf{k}}}{dN^2} + \frac{d\delta\tilde{\xi}_{\mathbf{k}}}{dN} + \left[e^{-2y} + \delta^2\tilde{\mathcal{A}}_i^2 - 2\right]\delta\tilde{\xi}_{\mathbf{k}} = -2e^{-y}x\delta\tilde{\mathcal{A}}_i\tilde{\xi}_{\mathbf{k}}^{(0)} \\ \dfrac{d^2\tilde{\mathcal{A}}_i}{dN^2} + \dfrac{d\tilde{\mathcal{A}}_i}{dN} + \left[\dfrac{\sigma^2}{2}\displaystyle\int_{-1}^{1}dx\int_0^N e^{-3y}|\tilde{\xi}_{\mathbf{k}}^{(0)}|^2 dy - \alpha(\alpha+1)\right]\tilde{\mathcal{A}}_i = \\ \qquad = -\zeta\displaystyle\int_{-1}^{1}dx\int_0^N e^{-4y}x\mathrm{Re}\left\{\tilde{\xi}_{\mathbf{k}}^{(0)*}\delta\tilde{\xi}_{\mathbf{k}}\right\}dy \end{cases} . \tag{22}$$

We extract from the coupled system the following quantities:

$$\frac{\rho_{\mathrm{E}}(N)}{M_{\mathrm{Pl}}^4} = \frac{1}{2}\left(\frac{H_I}{M_{\mathrm{Pl}}}\right)^4 \left(\frac{d\tilde{\mathcal{A}}_i}{dN} - \alpha\tilde{\mathcal{A}}_i\right)^2 e^{-2N}, \tag{23a}$$

$$\frac{\rho_{\chi}(N)}{M_{\mathrm{Pl}}^4} = \frac{1}{2\pi^2}\left(\frac{k_{\mathrm{i}}}{M_{\mathrm{Pl}}}\right)^2 \left(\frac{H_I}{M_{\mathrm{Pl}}}\right)^2 e^{3N} \int_0^N e^{-3y} \left(\left|\frac{d\tilde{\xi}_{\mathbf{k}}^{(0)}}{dN}\right|^2 + e^{-2y}\left|\tilde{\xi}_{\mathbf{k}}^{(0)}\right|^2\right) dy. \tag{23b}$$



Conclusions

The main results of this work are summarized below.

- A small photon's mass compatible with the experimental constraints does not alter the transverse-mode dynamics. Moreover, it gives rise to negligible longitudinal contributions to the EM energy density.
- The magnetic-field value at our time is not affected by the photon's mass.
- The electric fields dominate the energy density, constituting the background and can be computed classically.
- The Schwinger effect can be invoked in order to reduce the electric fields. The interacting dynamics can be solved numerically if considering a classical electric background. This is licit as long as $\rho \sim \rho_{\mathrm{E}}$.

Future works

- Making the whole theory gauge-invariant, studying the implications in terms of magnetic and electric field generation. In particular, gauge invariance could be restored through the Stueckelberg mechanism, adding a new real scalar field whose physical interpretation should be discussed.
- Investigating fermion particle production via the Schwinger effect.
- Exploring innovative methods to deal with the Schwinger production, without necessarily separating background evolution from perturbative contributions.
- Unifying magnetogenesis and baryogenesis frameworks in a unique more general scenario, describing both electromagnetic fields and baryon asymmetry generation.