Computing three-loop master integrals for the production of two off-shell vector bosons

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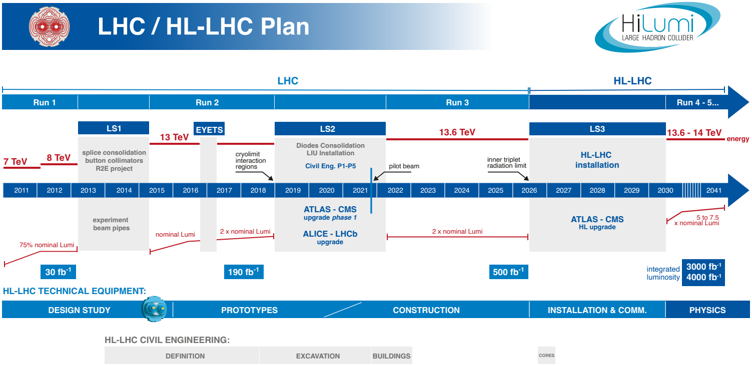
Introduction



Future 14 TeV HL-LHC runs + Clear signatures left by vector boson productions (leptonic decays)

⇓

Observables measured with an accuracy well below one percent

⇓

Precision achieved theoretically only if N3LO corrections included!!!

- QCD amplitudes expanded in α_s → 3-loop contributions to hard-scattering cross section @ N3LO

$$d\hat{\sigma}^{N3LO}_{ij\to f} \sim \int d\Phi^{(n-2)} \left(2\,\mathrm{Re}\left[\mathcal{M}^{(3)}_n (\mathcal{M}^{(0)}_n)^*\right]\right) + \dots$$

- $\mathcal{M}^{(3)}_n$ is a sum of three-loop Feynman Diagrams → Feynman Integrals ($d = 4 - 2\epsilon$)

$$\mathcal{M}^{(3)}_n = \sum_t C_t F_t \quad \text{with} \quad F_t = \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \mathrm{d}^d k_3 \prod_{i\in t} \frac{1}{D_i^{a_i}}\,, \quad D_i = (k_i + p_i)^2 - m_i^2$$

which satisfy integration-by-part relations [K. Chetyrkin, F. Tkachov, 1981; S. Laporta, 2004] → Master Integrals

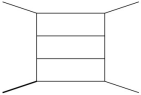
$$\int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \mathrm{d}^d k_3 \frac{\partial}{\partial k_l^\mu} \prod_{i\in t} \frac{1}{D_i^{a_i}} = 0 \quad \longrightarrow \quad \{\mathcal{J}_i\}$$

- $\mathcal{M}^{(3)}_n$ expressed as a sum of Master Integrals multiplied by process-dependent coefficients

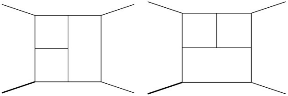
$$\boxed{\mathcal{M}^{(3)}_n = \sum_{i=1}^{N_{\mathrm{MIs}}} \tilde{C}_i \mathcal{J}_i}$$

Frontier in Three-Loop Feynman Integral Computations

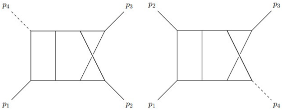
- Current frontier: Four-point families with one off-shell leg



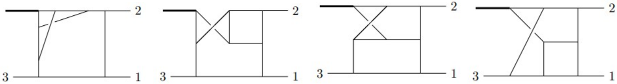
(a) Ladder-box
[S. Di Vita et al, 2014]



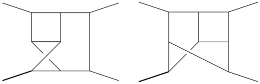
(b) Tennis-courts
[D.C., N. Syrrakos, 2021]



(c) Some non-planar Ladder-boxes
[J. Henn et al, 2023]

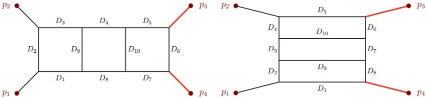


(d) Some non-planar Tennis-courts
[T. Gehrmann et al, 2024]

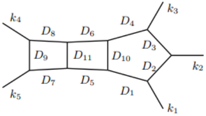


(e) More non-planar Tennis-courts
[J. Henn et al, 2024]

- Recently extended: Four-point with two off-shell legs (same mass) + Massless 5-point



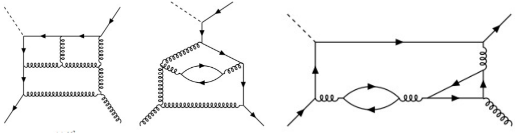
(f) Two Ladder-boxes
[M. Long, 2024]



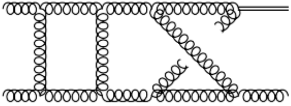
(g) Planar Penta-box-box
[Y. Liu et al, 2024]

Frontier in Three-Loop Amplitude Computations

- Amplitudes computed using some of the results of the previous slides

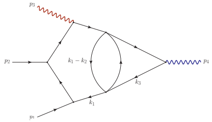


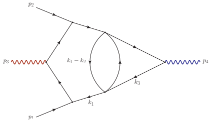
(h) Planar Vj production
[T. Gehrmann et al, 2023]

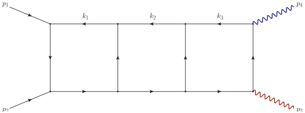


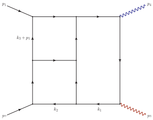
(i) Leading Color Hj production
[T. Gehrmann et al, 2024 & X.Chen et al, 2025]

Herein we extend the frontier to families with two external massive particles with unequal masses!!!









- These families contribute to leading color three-loop QCD amplitudes of processes like

$$q\bar{q}'/gg \to V_1 V_2 \to (l_1 \bar{l}_1')(l_2 \bar{l}_2') \quad \text{with} \quad V_1 V_2 = \gamma^*\gamma^*, W^+W^-, ZZ, W^\pm Z, W^\pm\gamma^*, Z\gamma^*$$

Notation and Kinematics

- Four-particle scattering + two different external masses → 4 independent invariants $\vec{s}$ (scales)

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 + p_3)^2, \quad m_3^2 = p_3^2, \quad m_4^2 = p_4^2, \quad p_1^2 = p_2^2 = 0$$

- Physical region in Mandelstams contains the root $R = \sqrt{m_3^4 + (m_4^2 - s_{12})^2 - 2m_3^2(m_4^2 + s_{12})}$

$$m_3^2, m_4^2 > 0, \quad s_{12} > (m_3 + m_4)^2, \quad \frac{m_3^2 + m_4^2 - s_{12} - R}{2} \leq s_{23} \leq \frac{m_3^2 + m_4^2 - s_{12} + R}{2}$$

- R can be rationalized using the parametrization $(R = m_3^2 x(1 - y))$ [J. Henn at al, 2014]

$$\frac{s_{12}}{m_3^2} = (1 + x)(1 + xy), \qquad \frac{s_{23}}{m_3^2} = -xz, \qquad \frac{m_4^2}{m_3^2} = x^2 y$$

- Physical region in (x, y, z) parametrization takes the form

$$\boxed{x > 0, \qquad 0 < y < 1, \qquad y < z < 1}$$

Integral Families Under Study

- Our three-loop four-point Feynman Integral families are of the form

$$F_{\alpha_1,\dots,\alpha_{15}} = \int \frac{d^dk_1 d^dk_2 d^dk_3}{(i\pi)^{3d/2}} \frac{e^{3\gamma_E\varepsilon}}{D_1^{\alpha_1}\cdots D_{15}^{\alpha_{15}}} \qquad \text{with} \qquad D_j = \left(\sum_{i=1}^{3} a_{ij}k_i + b_{ij}p_i\right)^2$$

where $a_{ij}, b_{ij} = 0, \pm 1$ and the last 5 propagators are auxiliary ones ($\alpha_i \leq 0$ for $i = 11, \dots, 15$).

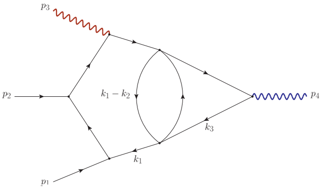
- All the planar FI families can be collected into two propagator super-families: F123 and F132.

- F123 is described by the following set of propagators

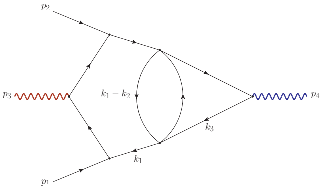
$$\begin{aligned}
&D_1 = k_1^2, \quad D_2 = (k_1+p_1)^2, \quad D_3 = (k_1+p_{12})^2, \quad D_4 = (k_1+p_{123})^2,\\
&D_5 = k_2^2, \quad D_6 = (k_2+p_1)^2, \quad D_7 = (k_2+p_{12})^2, \quad D_8 = (k_2+p_{123})^2,\\
&D_9 = k_3^2, \quad D_{10} = (k_3+p_1)^2, \quad D_{11} = (k_3+p_{12})^2, \quad D_{12} = (k_3+p_{123})^2,\\
&D_{13} = (k_1-k_2)^2, \quad D_{14} = (k_1-k_3)^2 \quad \text{and} \quad D_{15} = (k_2-k_3)^2
\end{aligned}$$

- F132 propagators can be obtained from the F123 ones via the transformation $p_2 \leftrightarrow p_3$.

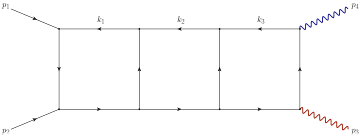
Four Planar Families



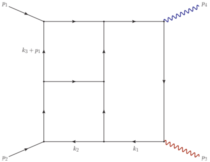
(a) RL1 (F123): 27 MIs



(b) RL2 (F132): 25 MIs



(c) PL1 (F123): 150 MIs



(d) PT4 (F123): 189 MIs

- Alphabet in the variables $\{x, y, z\}$ consists of the following 15 letters

$$\{x,\ y,\ z,\ 1+x,\ 1-y,\ 1-z,\ z-y,\ 1+y-z,\ 1+xy,\ 1+xz,\ z+xy,$$
$$1-z+y(1+x),\ 1+x(1+y-z),\ z-y(1-z-xz),\ z-x(y-z-yz)\}$$

- 50 new sectors with 132 genuinely new master integrals!

Differential Equations (DEs)

- Computation of MIs derivatives + Generation of IBPs → FFIntRed [T. Peraro, In-house package]

$$\int \left(\prod_{i=1}^{l} \frac{d^d k_i}{(2\pi)^d} \right) \frac{\partial}{\partial k_b^\mu} \left(u^\mu \frac{\bar{z}_1^{i_1} \cdots \bar{z}_{n_{ir}}^{i_{n_{ir}}}}{\prod_j D_j^{a_j}} \right) = 0 \qquad \text{with} \qquad u^\mu = a^i p_i^\mu + b_i k_i^\mu$$

- IBPs solved over finite fields → Reconstruct DEs [A. Kotikov, 1991] with FiniteFlow [T. Peraro, 2019]

$$\partial_\xi \vec{G} = B_\xi(\vec{s};\epsilon)\ \vec{G} \qquad \text{with} \qquad \xi \in \vec{s},$$

- What we reconstructed is not the above form but the following canonical one [J. Henn, 2013]

$$\partial_\xi \vec{I} = \epsilon\ \tilde{A}_\xi(\vec{s})\ \vec{I} \qquad \to \qquad \boxed{d\vec{I} = \epsilon\ dA(\vec{s})\ \vec{I}} \qquad \text{with} \qquad dA(\vec{s}) = \sum A_i\ d\log w_i$$

- Pure bases: bottom-up approach + sector-by-sector study in the maximal cut (MC)
  - Magnus exponential [M. Argeri, et al, 2014] & [T. Gehrmann et al, 2014]
  - Building-blocks [P. Wasser, 2018]
  - Candidates with integrands of $d\log$ form [P Wasser, 2018] & [J. Henn et al, 2020]

Pure Bases Construction: Magnus Exponential

- Choose appropriate candidates (if $N_p \leq 7 \rightarrow$ dot, else $\rightarrow$ ISPs)[1] that render MC DEs linear on ε

$$\partial_\xi \vec{G}^{\mathsf{MC}} = (H_{0,\xi} + \varepsilon H_{1,\xi})\vec{G}^{\mathsf{MC}}$$

- H_0 can be removed by rescaling MIs a matrix that satisfies the following DEs

$$\partial_\xi \tilde{T}^{\mathsf{MC}} = -\tilde{T}^{\mathsf{MC}} H_{0,\xi}$$

- New candidates defined as $\vec{I}^{\mathsf{MC}} = \tilde{T}^{\mathsf{MC}}\vec{G}^{\mathsf{MC}}$ acquire canonical DEs in MC

$$\partial_\xi \vec{I}^{\mathsf{MC}} = \varepsilon A_\xi^{\mathsf{MC}} \vec{I}^{\mathsf{MC}} \qquad \text{with} \qquad A_\xi^{\mathsf{MC}} = \tilde{T}^{\mathsf{MC}} H_{1,\xi} (\tilde{T}^{\mathsf{MC}})^{-1}$$

- Are $\vec{I}$ pure beyond MC? $\rightarrow$ relax cut conditions $\rightarrow$ may appear sub-sector entries linear on ε

$$\partial_\xi \vec{I} = \varepsilon A_\xi^{\mathsf{MC}} \vec{I}^{\mathsf{MC}} + (h_{0,\xi} + \varepsilon h_{1,\xi})\vec{I}^{\mathsf{LS}}$$

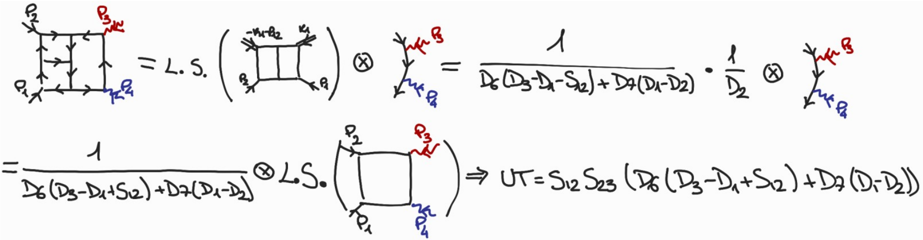
- To set them in canonical form rotate the lower sector ε^0 contributions by integrating out $h_{0,\xi}$

$$\vec{I} = \vec{I}^{\mathsf{MC}} + \tilde{T}^{\mathsf{LS}} \vec{I}^{\mathsf{LS}} \qquad \text{with} \qquad \partial_\xi \tilde{T}^{\mathsf{LS}} = -h_{0,\xi}$$

[1] Where with N_p we denote the number of propagators of the sector at hand

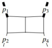
Pure Bases Construction: Building-Blocks

• Building Blocks: combine leading singularities (LS) of one/two-loop pure candidates as building blocks for creating a three-loop candidate with constant LS, in a graphical/heuristic approach



↓

$$\varepsilon^6 s_{12}s_{23}\,\big(G^{F123}_{0,0,1,1,1,1,-1,0,0,1,1,0,1,1,1} - G^{F123}_{0,0,1,1,1,0,0,0,0,1,1,0,1,1,1} - G^{F123}_{1,-1,1,1,1,1,-1,0,0,1,1,0,1,1,1} + G^{F123}_{1,0,0,1,1,0,0,0,0,1,1,0,1,1,1}] + s_{12}\,G^{F123}_{1,0,1,1,1,0,0,0,0,1,1,0,1,1,1}\big)$$



$$\epsilon^4 s\Big[(p_1^2 - p_3^2 + t)G_{1,1,1,0,1,1,1,0,0} + (p_3^2 - s)G_{1,1,1,1,1,1,1,-1,0}\Big]$$ [J. Henn et al, 2014]

Pure Bases Construction: FIs of $d\log$-Form

- DLogBasis: use spinor-helicity formalism to bring 4-dimensional integrand into $d\log$ form, e.g.

$$I_{box} = \frac{k^{-2}(k-p_1)^{-2}d^4k}{(k-p_{12})^2(k+p_4)^2} \longrightarrow (s_{12}s_{23})^{-1}d\log[f_1(k)] \times d\log[f_2(k)] \times d\log[f_3(k)] \times d\log[f_4(k)]$$

- Massive momenta $\rightarrow$ use a SDE parametrization [C. Papadopoulos, 2014] to re-express in terms of massless

$$\{p_1 \rightarrow xq_1,\ p_2 \rightarrow yq_2,\ p_3 \rightarrow q_{13} - xq_1,\ p_4 \rightarrow q_{24} - yq_2\} \qquad \text{with} \qquad q_1^2 = q_2^2 = q_3^2 = q_4^2 = 0$$

- $d\log$ form in d-dimensional integrand using Baikov representation [P. Baikov, 1996] in MC and beyond

$$G^{n\times cut}_{\alpha_1,\ldots,\alpha_N} \equiv \frac{e^{L\gamma_E\varepsilon}C_N^L}{\mathcal{G}^{(d-E-1)/2}}\left(\prod_{a=n+1}^{N}\int dD_a\right)\left(\prod_{c=1}^{n}\oint_{D_c=0} dD_c\right)\frac{(P_N^L)^{(d-L-E-1)/2}}{D_1^{\alpha_1}\ldots D_N^{\alpha_N}}$$

- For example, in MC for the top sector of PT4 after keeping the leading term on ε we have[2]

$$\frac{1}{D_2^2 D_7(D_7-s_{12})R_{D_2}} \xrightarrow{\times D_2^2} (s_{12}R)^{-1}d\log[f(D_2)] \times d\log[f'(D_2,D_7)]$$

$$\rightarrow s_{12}R\ G^{F123}_{1,-2,1,1,1,1,0,0,0,1,1,0,1,1,1}$$

[2] $R_{D_2} = \sqrt{D_2^2(m_3^4 + (m_4^2 - s_{12})^2 - 2m_3^2(m_4^2 + s_{12})) + s_{12}^2 s_{23}^2 + 2D_2 s_{12}((m_4^2 - s_{12})s_{23} + m_3^2(s_{23} - 2m_4^2))}$

Analytic Solution

- Solve the (x, y, z) DEs in terms of MPLs in the physical region (base-point $(x, y, z) = (0, 0, 0)$)

$$\mathcal{G}_{1\ldots n} \equiv \mathcal{G}(a_1, \ldots, a_n; X) = \int_0^X \frac{dt}{t - a_1} \mathcal{G}(a_2, \ldots, a_n; t) \quad \text{with} \quad \mathcal{G}(\vec{0}_n; X) = \frac{1}{n!} \log^n(X),$$

- Solution obtained up to order 6 in the ε expansion $\rightarrow$ MPLs only up to weight 6 appear

$$\begin{aligned}
\vec{I} = {} & \varepsilon^0 \left(\vec{b}_0 \right) \\
& + \varepsilon^1 \left(c_1^i \mathcal{G}_i \vec{b}_0 + \vec{b}_1 \right) \\
& + \varepsilon^2 \left(c_2^{ij} \mathcal{G}_{ij} \vec{b}_0 + c_2^i \mathcal{G}_i \vec{b}_1 + \vec{b}_2 \right) \\
& + \varepsilon^3 \left(c_3^{ijl} \mathcal{G}_{ijl} \vec{b}_0 + c_3^{ij} \mathcal{G}_{ij} \vec{b}_1 + c_3^i \mathcal{G}_i \vec{b}_2 + \vec{b}_3 \right) \\
& + \varepsilon^4 \left(c_4^{ijlk} \mathcal{G}_{ijlk} \vec{b}_0 + c_4^{ijl} \mathcal{G}_{ijl} \vec{b}_1 + c_4^{ij} \mathcal{G}_{ij} \vec{b}_2 + c_4^i \mathcal{G}_i \vec{b}_3 + \vec{b}_4 \right) \\
& + \varepsilon^5 \left(c_5^{ijlkn} \mathcal{G}_{ijlkn} \vec{b}_0 + c_5^{ijlk} \mathcal{G}_{ijlk} \vec{b}_1 + c_5^{ijl} \mathcal{G}_{ijl} \vec{b}_2 + c_5^{ij} \mathcal{G}_{ij} \vec{b}_3 + c_5^i \mathcal{G}_i \vec{b}_4 + \vec{b}_5 \right) \\
& + \varepsilon^6 \left(c_6^{ijlkno} \mathcal{G}_{ijlkno} \vec{b}_0 + c_6^{ijlkn} \mathcal{G}_{ijlkn} \vec{b}_1 + c_6^{ijlk} \mathcal{G}_{ijlk} \vec{b}_2 + c_6^{ijl} \mathcal{G}_{ijl} \vec{b}_3 + c_6^{ij} \mathcal{G}_{ij} \vec{b}_4 + c_6^i \mathcal{G}_i \vec{b}_5 + \vec{b}_6 \right)
\end{aligned}$$

where $c_\alpha^{i\ldots o}$ are constant matrices and $\vec{b}_\alpha$ boundary constants at order ε^α

Boundary conditions

Regularity Conditions

AMFlow + PSLQ

For computing boundaries we followed the procedure of [S. Badger et al, 2023]:

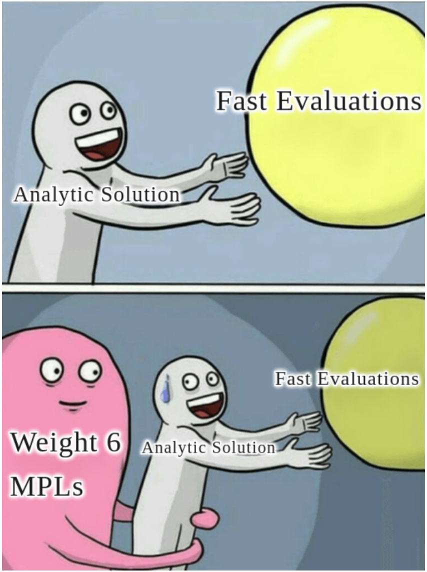
- Computed MIs in 70 digits precision using AMFLOW [X. Liu, Y. Ma, 2022] for the point $(x_0, y_0, z_0) = (3/2, 1/5, 1/2)$.
- Computed the MPLs of the analytic solution in 70 digits precision using DIFFEXP [M. Hidding, 2020] at the same point.
- The boundary constants can contain only the transcendental constants listed in the table below $\rightarrow$ performed PSLQ order by order.

Expansion	ε^0	ε^1	ε^2	ε^3	ε^4	ε^5	ε^6
Boundaries	1	$i\pi$	π^2	$i\pi^3, \zeta_3$	$\pi^4, i\pi\zeta_3$	$i\pi^5, \pi^2\zeta_3, \zeta_5$	$\pi^6, i\pi^3\zeta_3, \zeta_3^2, i\pi\zeta_5$

For example at order ε^3, we have

$$\vec{I}^{(3)}_{\text{AMFLOW}} - c_3^{ijl}\mathcal{G}_{ijl}^{\text{DIFFEXP}}\vec{b}_0 - c_3^{ij}\mathcal{G}_{ij}^{\text{DIFFEXP}}\vec{b}_1 - c_3^{i}\mathcal{G}_{i}^{\text{DIFFEXP}}\vec{b}_2 = i\pi^3\vec{C}_1 + \zeta_3\vec{C}_2 \equiv \vec{b}_3$$

Analytic Solution: Evaluation



Family	# MPLs	# W6
RL1	3619	1631
PL1	5416	2554
RT4	8290	3709
F123	8360	3739
RL2	10973	5219
ALL	19306	8951

Many MPLs of weight 6 ($\sim 1s$ per MPL)

↓

Much time spent for their evaluation

↓

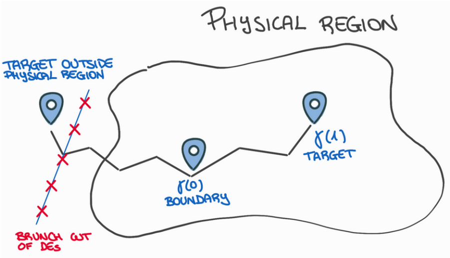
Analytic solution inefficient in current form!

↓

Generalized Power Series [F. Moriello, 2019]

Generalized Series Expansion Method

- Semi-numerical method applied for solving DEs



$$\gamma(t): \quad \gamma(0) = \vec{s}_b \quad \text{and} \quad \gamma(1) = \vec{s}_t \quad \text{with} \quad t \in [0,1]$$

- Divide path from $\vec{s}_b$ to $\vec{s}_t$ into segments $\to$ Expand and solve DEs therein!
- $\gamma(t)$ in the physical region $\to$ no physical singularity crossed $\to$ no analytic continuation!
- Boundary conditions computed using AMFlow [X. Liu, Y. Ma, 2022].
- DiffExp [M. Hidding, 2020], SeaSyde [T. Armadillo et al, 2022], AMFlowSolver [X. Liu, Y. Ma, 2022], LINE [R. Prisco et al, 2025].

Numerical results using DiffExp

Results obtained for 40 digits using a personal laptop (Apple M1 Pro, 8 cores, 16GB RAM)

	Timings per Point		Timings per Segment	
	$\{s_{12}, s_{23}, m_3^2, m_4^2\}$	$\{x, y, z\}$	$\{s_{12}, s_{23}, m_3^2, m_4^2\}$	$\{x, y, z\}$
RL1	175 s	55 s	8 s	9 s
RL2	169 s	64 s	9 s	11 s
PL1	2765 s	818 s	126 s	136 s
PT4	4987 s	1478 s	226 s	246 s

- DiffExp solution on $\{x, y, z\}$ faster than analytic one.
- $\{x, y, z\}$ DEs solved faster than $\{s_{12}, s_{23}, m_3^2, m_4^2\}$ ones.
- DiffExp timings scale with the size of the DEs.
- Analytic solution timings scale with the # of MPLs.
- E.g. for RL2: Analytic $\sim$ hours, DiffExp $\sim$ 1 min).

Conclusions

Results

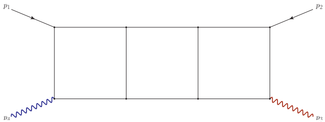
- Computed four integrals families relevant for di-boson production @ N3LO QCD.
- Constructed canonical DEs → solved analytically in terms of MPLs and semi-analytically using DiffExp.

What's next

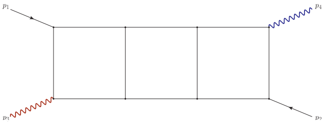
- Computation of remaining 5 planar families.
- Optimization of analytic solution → Basis of special functions.
- Usage of alternative tools (AMFlowSolver and LINE) for faster numerical evaluations.
- Computation of leading color di-boson amplitudes using our results.

Thank you very much for your attention!!!

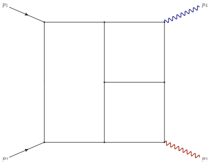
Backup Slides: Remaining Planar Families



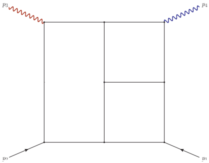
(a) PL2: 143 MIs



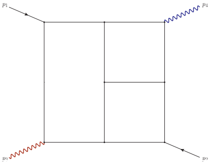
(b) PL3: 142 MIs



(c) PT1: 344 MIs

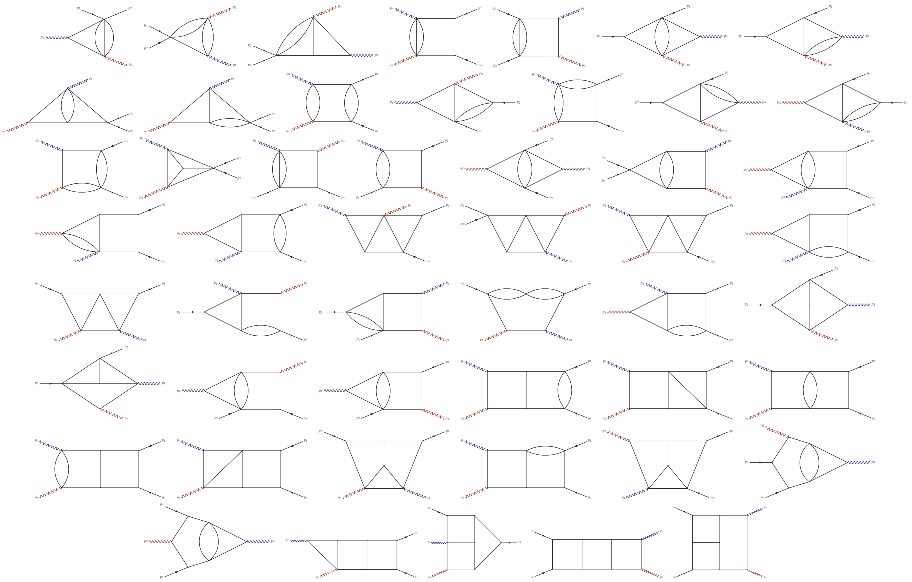


(d) PT2: 252 MIs



(e) PT3: 240 MIs

Backup Slides: 50 New Sectors



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