Gluing via Intersection Theory

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based on 2411.07330 with G. Crisanti, B. Eden, M. Gottwald and P. Mastrolia and ongoing work

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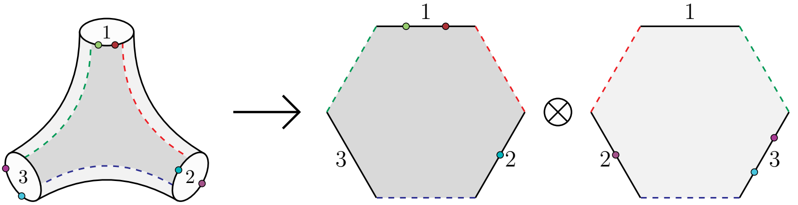

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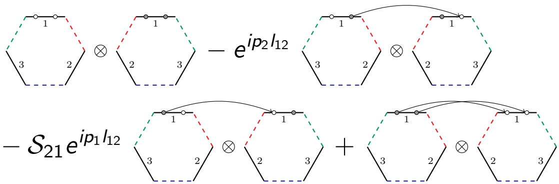

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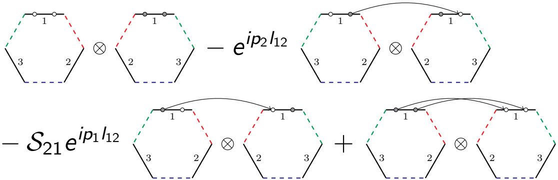

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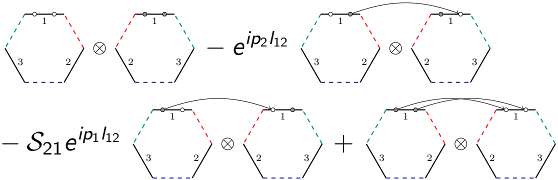

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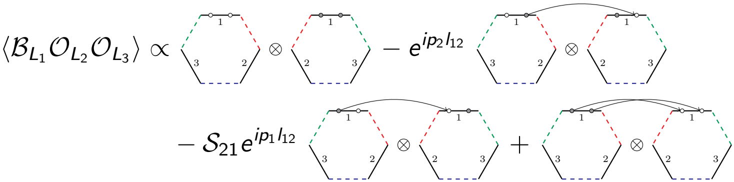

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  - higher loop orders $\to$ gluing corrections

Gluing Corrections

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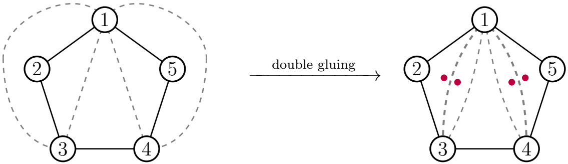

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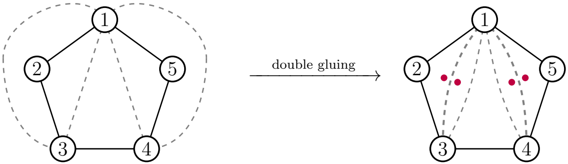

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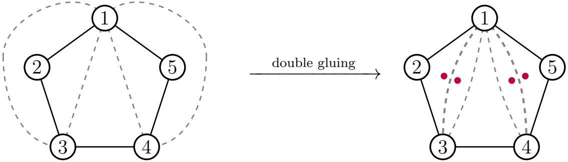

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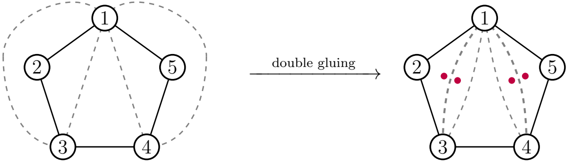

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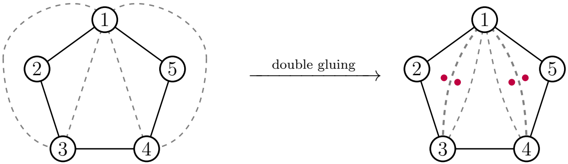

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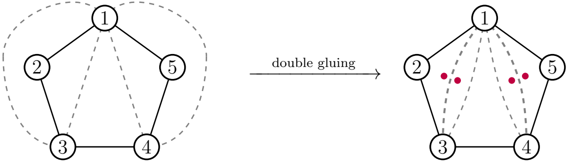

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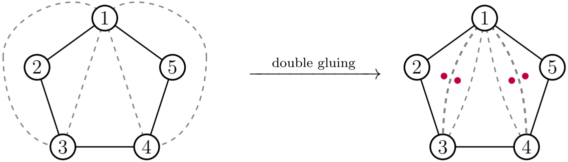

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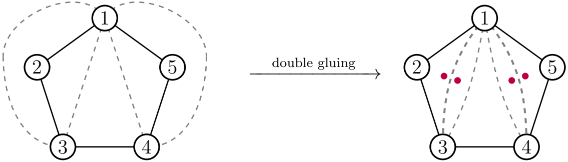

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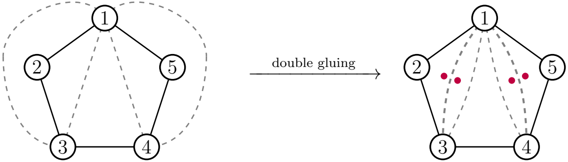

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  - double-gluing is more complicated [Fleury, Komatsu '18]

The Double-Gluing Process

- for simplicity, we only consider the so-called $\mathcal{X}$-element of the S-matrix by restricting to $\phi_1^1\phi_2^0$-states: [De Leeuw, Eden, Le Plat, Meier, Sfondrini '19]

$$I_{\mathcal{X}} = \sum_{K,\,L=1}^{\infty} \sum_{k,\,l=0}^{K-1,L-1} \int \frac{du\,dv}{4\pi} \frac{K\,L\,g^2(v^+-u^-)\,\Sigma^{KL}(u,v)\,\hat{\mathcal{X}}_k^{k,l}(u,v)\,W(u,v)}{(u^-)^2(v^+)^2u^+v^-(u^+-v^-)}$$

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- picking residua, we can decompose this as follows

$$I_{\mathcal{X}} = (S_1 + S_2) + S_W + S_{mes} + S_{mat}$$

$$S_1 = \sum a^{\sigma_{lm}} b^{\sigma_{jkm}} y^{\sigma_{Lm}} z^{\sigma_{jK}} \frac{\sigma_{kKm}}{j\sigma_{jkKm}} * \frac{\Gamma_{1km}\Gamma_{1lm}\Gamma_{Km}\Gamma_{Lm}\Gamma_{jKLm}\Gamma_{1jklLM}}{\Gamma_{1k}\Gamma_{1l}\Gamma_{K}\Gamma_{L}\Gamma_{1m}^2\Gamma_{1lLM}\Gamma_{1jKm}\Gamma_{1jkLM}},$$

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- a, b, y, z are defined by cross-ratios of the planar kinematics

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- the polynomials q_i are in general quadratic in all variables

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- we can also decompose derivatives of masters: $\partial_x \langle e_i | = \langle \nabla_{\sigma_x} e_i | = \Omega_{ij} \langle e_j |$

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$$\langle \varphi_L | \varphi_R \rangle = \sum_{p \in \mathcal{P}_\omega} \mathrm{Res}_{z=p}(\psi_p \, \varphi_R)$$

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- putting everything together in the end, we can confirm the expected identity $S_2 = S_1(a \leftrightarrow b,\ y \leftrightarrow z)$

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Thank You!