QUANTUM ADVANTAGES

IN FUNDAMENTAL PHYSICS

OUTLINE

PREPARING YOUR DETECTOR IN AN APPROPRIATE QUANTUM STATE CAN IMPROVE YOUR SENSITIVITY BY ORDERS OF MAGNITUDE AND EVEN GENERATE QUALITATIVELY NEW PHENOMENA

(INSPIRATIONAL BUT IMPOSSIBLE)

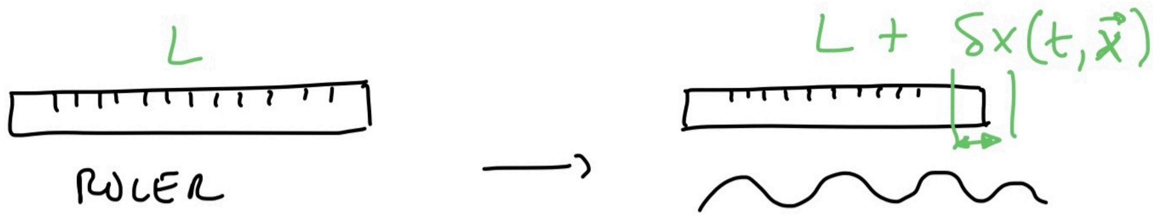
→ STOCHASTIC GWs DETECTION

→ COHERENT "COLLIDERS"

(MENTION $h \sim \frac{1}{\sqrt{N}}$, $h \sim \frac{1}{N}$)

STOCHASTIC GWs

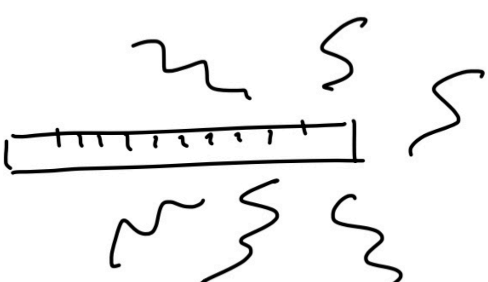
"STANDARD" GW



$$\eta_{\mu\nu} \qquad \longrightarrow \qquad \eta_{\mu\nu} + h_{\mu\nu}(t,\vec{x})$$

$$\delta x(t,\vec{x}) \simeq h(t,\vec{x})\, L$$

STOCHASTIC GW



$$h_{\mu\nu}(t,\vec{x}) = \sum_i h_{\mu\nu}(t,\vec{x}\,|\,\theta_i)$$

EX.

$$h^i_{\mu\nu} \propto \cos\left((\omega_g + \delta\omega_i)t + \phi_i\right)$$

$$\theta_i = (\delta\omega_i, \phi_i)$$

USEFUL TO THINK OF θ AS "NUISANCE" PARAMETERS COMING FROM SOME DISTRIBUTION IN FREQUENCY SPACE, I.E.

$$\tilde{h}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{-i\omega t}\, h(t)$$

$$h(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{i\omega t}\, \tilde{h}(\omega)$$

FOR SIMPLICITY AND BECAUSE OF LINEARITY

$$h_{\mu\nu}(t, \vec{x} \,|\, \vec{\theta}) = h(t \,|\, \vec{\theta})\, f_{\mu\nu}(\vec{x})$$

$$\langle \tilde{h}(\omega \,|\, \vec{\theta}) \rangle_{\theta} \equiv \underbrace{\langle \tilde{h}(\omega \,|\, \vec{\theta}) \rangle = 0}_{\text{PRIMORDIAL STOCHASTIC GW}}$$

$$\boxed{\langle \tilde{h}(\omega \,|\, \vec{\theta})\, \tilde{h}^{*}(\omega' \,|\, \vec{\theta}) \rangle \equiv \delta(\omega - \omega')\, S_{hh}(\omega)}$$

SIGNAL: GAUSSIAN, ZERO MEAN, (ISOTROPIC) (UNPOLARIZED), STATIONARY ⊛

$\Downarrow$

PHASE TRANSITIONS
INFLATION
...

$\langle h(t) h^*(t') \rangle \propto$
$\propto \delta(t-t')$

WHAT YOU MEASURE

$$\delta x(t,\vec{x}) = L \cdot h(t,\vec{x}|\vec{\vartheta}) + N(t,\vec{x}) =$$

$$= L \cdot h(t|\vec{\vartheta}) + N(t)$$

ISOTROPIC

$$\langle N(\omega) N^*(\omega') \rangle = S_{NN}(\omega)\,\delta(\omega-\omega')$$

$$\langle N(\omega) \rangle = 0$$

IT'S ALSO A RANDOM VARIABLE WITH THE SAME PROPERTIES OF THE SIGNAL ⊛

EXPERIMENTAL FACT

WE ARE GOING TO LOOK FOR THESE SIGNALS AT $\omega \gg$ kHz BECAUSE:

1) NO ASTRO BKGS (*)

2) $\omega_S \simeq 100 \text{ MHz} \left(\frac{T}{10^{15} \text{ GeV}}\right)$

$P(\text{GeV}) \simeq 0.3\, B(T) \cdot R(m)$

LET'S SAY $R \sim R_\oplus \simeq 6 \cdot 10^3$ km

FOR A COLLIDER AT THIS ENERGY YOU NEED

THEN $B \simeq 10^{12}$ T

IS THIS EVEN POSSIBLE?

$E_{\text{MAGNETS}} \simeq$

$\simeq B^2 \cdot R \cdot A^2 / \mu_0 \simeq \ldots$ J

(*) why?

OBJECT OF SIZE R, $\omega_{max} = \frac{c}{R} = \frac{1}{R}$ ($c=1$)

$R_{MIN} = R_{BH}$

$R_{BH}^{MIN} \simeq G_N M_\odot$ (CHANDRASEKAR BOUND)

$\Downarrow$

$\omega_{MAX}^{ASTRO} \simeq \frac{1}{G_N M_\odot} \simeq$ kHz

"THE HARDEST MEASUREMENT IN PHYSICS"

$$\Omega_g(\omega) \equiv \frac{\omega^3}{24\pi H_0^2} S_{hh}(\omega) \qquad \text{(DEFINITION)}$$

$$\boxed{\int \frac{d\omega}{\omega} \Omega_g(\omega) \lesssim 10^{-6} \quad \text{BBN}}$$

$$H_0 = \text{HUBBLE TODAY} \simeq 70 \frac{\text{Km}}{\text{Mpc}} \frac{1}{s} \simeq 10^{-18} \frac{1}{s}$$

BUT EXPERIMENTS MEASURE STRAIN h

$$S_{hh}(\omega) \simeq \frac{h^2}{\Delta\omega} \sim \frac{h^2}{\omega_g}$$

$$\int \frac{d\omega}{\omega} \Omega_g \sim \frac{\omega_g^2 h^2}{24\pi H_0^2} \lesssim 10^{-6} \Rightarrow h \lesssim 10^{-26} \left(\frac{\text{MHz}}{\omega_g}\right)$$

CAN WE DO IT?

LET'S CHECK OUR BEST "CLASSICAL" DETECTOR = INTERFEROMETERS

1) WHY DO WE MEASURE TEST MASS POSITIONS?

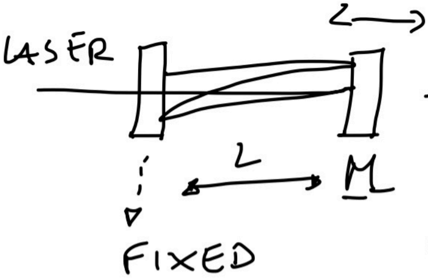
$\hookrightarrow$ BECAUSE OF THE EQUIVALENCE PRINCIPLE!

$$M_I \ddot{x} \simeq F_{ext} + M_G \ddot{h} L$$

$$M_I = M_G = M \Rightarrow M \to \infty$$

$$\frac{F_{ext}}{M} \to 0 \quad \text{(NO NOISE)}$$

2) TOY MODEL



$\dashrightarrow$ FABRY-PÉROT CAVITY

FREE HAMILTONIAN

$$H_0 = \underbrace{\hbar \omega a^\dagger a}_{\text{LASER}} + \underbrace{\hbar \omega_m d^\dagger d}_{\text{MIRROR}}$$

$H_0 \to$ CONVENIENCE

$U H_0 U^\dagger - \hbar \omega_L a^\dagger a$

$U = e^{i \omega_L a^\dagger a t}$

$$\omega = \omega(L) \simeq \omega(L_0) + \delta x \cdot g_0' \equiv \omega_0 + \delta x \cdot g_0'$$

$$\delta x = \frac{d + d^\dagger}{\sqrt{2}} x_0 \,, \quad g_0' \equiv \left. \frac{\partial \omega}{\partial x} \right|_{x = L_0} \quad g_0 \equiv g_0' x_0$$

$$H = \hbar \Delta a^\dagger a + \hbar \omega_m d^\dagger d + \hbar g_0 a^\dagger a \frac{(d^\dagger + d)}{\sqrt{2}}$$

LASER: $H_0 \to H_0 + \hbar E (a + a^\dagger)$

THIS IS THE SAME AS GIVING A VEV TO a, a^+

$$a \to a + \alpha$$

$$b \to b + \beta$$

(ALSO THE CAVITY LENGTH CHANGES)

$$\alpha = -\frac{E}{\omega_0 + 2\beta g_0}$$

$$\Rightarrow H_0 = \hbar \Delta a^+ a + \hbar \omega_m d^+ d + \hbar g \left(a^+ + a\right)\frac{\left(d^+ + d\right)}{\sqrt{2}} + \dots$$

$g \equiv g_0 \alpha \gg g_0$ (SO WE NEGLECT THE TERM $g_0 a^+ a \left(d^+ + d\right)$

$$X \equiv \frac{a + a^\dagger}{\sqrt{2}} \qquad Y \equiv \frac{i\,(a - a^\dagger)}{\sqrt{2}}$$

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m \omega_m^2 x^2 + \Delta \left(X^2 + Y^2\right) + \sqrt{2}\, g\, x\, X \qquad \boxed{\hbar = 1}$$

DISSIPATION

$$H_R = \sum_e \int d\omega \Big[\omega\, b_e^\dagger(\omega)\, b_e(\omega) + \left(i g_e\, b_e^\dagger(\omega)\, a + h.c. \right) \Big]$$

THE CAVITY HAS "HOLES"
↓
ℓ SPANS THEM

MINIMALLY ONE FOR THE LASER AND ONE TO READ IT OUT

$b_e^\dagger, b_e$ ARE THE OPERATORS FOR

PHOTONS OUTSIDE THE CAVITY,

THE ONES WE MEASURE!

NOW E.O.M.S

$$\dot{O} = i\left[H, O\right]$$

AS USUAL, IT'S USEFUL TO DEFINE

$$a_{IN}^{e} \equiv -\frac{1}{\sqrt{2\pi}}\int d\omega\, e^{-i\omega(t-t_0)}\, b_e(\omega, t_0)$$

$$a_{OUT}^{e} \equiv -\frac{1}{\sqrt{2\pi}}\int d\omega\, e^{-i\omega(t-t_1)}\, b_e(\omega, t_1)$$

$$t_1 > t, \quad t_0 < t$$

THEN $\kappa_e = 2\pi g_e^2 \quad \kappa \equiv \sum_e \kappa_e$

$$\dot{X} = \Delta Y - \frac{\kappa}{2} X + \sum_e \sqrt{\kappa_e}\, X_e^{IN,OUT}$$

$$\dot{Y} = -\Delta X - \frac{\kappa}{2} Y + \sum_e \sqrt{\kappa_e}\, Y_e^{IN,OUT}$$

$$\dot{x} = \frac{P}{M}$$

QUALITATIVE DISCUSSION OF THESE TWO TERMS

$$\dot{P} = -M\omega_m^2 x - \gamma_m P - g X + F_{ext}$$

SOLVE: $(\Delta = 0)$

ONLY MEASUREMENT

ASSUMING NO LOSSES $\kappa = \kappa_m$

1 $$\underbrace{S^{OUT}_{Y_m Y_m}}_{\text{what you measure}} = \underbrace{S^{IN}_{Y_m Y_m}}_{\text{what you put in the cavity}} + \frac{4\kappa^2 g^4 \, S^{IN}_{X_m X_m}}{M^2\left(\frac{\kappa^2}{4} + \omega^2\right)^2 \left(\omega^2 \gamma_m^2 + (\omega^2 - \omega_m^2)^2\right)}$$

NO CLASSICAL NOISE

$F_{ext} = 0$

ADD WAVE:

$$\dot{P} = F_G + \dots$$

$$F_G = \int d^3x\, \rho(\vec{x})(\vec{x})_i U_j^m(\vec{x})\, \partial_t^2 h_{ij}^{TT}(t, \vec{x})$$

SOLVE:

2 $$S_{hh}(\omega) = \frac{(\kappa^2 + 4\omega^2)(\omega^2 \gamma_m^2 + (\omega^2 - \omega_m^2)^2)}{4 g^2 L^2 \omega^4 |I|^2 \kappa m} S_{yy}^{OUT}(\omega)$$

1 + 2 $$= S_{hh} \sim \underbrace{g^2 S_{XX}^{IN}}_{\text{BACKACTION}} + \underbrace{\frac{1}{g^2} S_{yy}^{IN}}_{\text{SHOT NOISE}}$$

$$g^2 \sim P_{LASER}$$

FROM HERE FIND g_* THAT MINIMIZES S_{hh}

$$\Rightarrow \boxed{S_{hh}^{MIN} = \frac{2\hbar}{M L^2 \omega^2}}$$

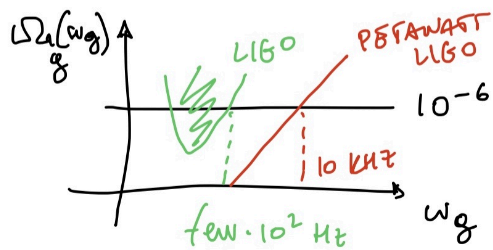
SQL OF INTERFEROMETERS (OR ANY POSITION MEASUREMENT) MODULO $O(1)$ FACTORS

WHAT DOES IT MEAN IN PRACTICE?

LET'S BE EVEN MORE OPTIMISTIC, TAKE $M \to \infty$

THEN $S_{hh} \sim P_{LASER}^{-1} \cdot S^{IN}_{y_m y_m}$

$$S^{IN}_{y_m y_m} = 1/2 \text{ (VACUUM)}$$



$$h \sim \frac{1}{P_{LASER}^{1/2}} \sim \frac{1}{N_\gamma^{1/2}}$$

CAN WE DO BETTER? YES!

$S^{IN}_{y_m y_m} \to 0$ HEISENBERG LIMIT

STILL NEED ONE PHOTON WITHIN t_{INT}

SO IT WOULD SEEM THAT THERE IS NO GAIN, FALSE!

OUR TOY MODEL SOLUTION WAS WRONG

WE DID THE SHIFT $Q \to Q + \alpha$ ONLY IN THE CAVITY, Q_{IN} IS NOT IN VACUUM, I.E. b_e CONTAINS THE LASER

WE NEGLECTED

$$S_{y_m^{IN} F_G} \neq 0$$

$$S_{hh}(\omega) = \frac{(\kappa^2+4\omega^2)\left(\omega^2\gamma_m^2 + (\omega^2-\omega_m^2)\right)}{4 g^2 L^2 \omega_g^4 |I|^2 \kappa_m \cdot \Delta} S^{OUT}_{y_m^2 y_m^2}(\omega)$$

$$\Delta = \left(\frac{\kappa - 2\kappa_m}{\kappa}\right)^2 \cdot S^{IN}_{y_m y_m}(0)$$

LASER

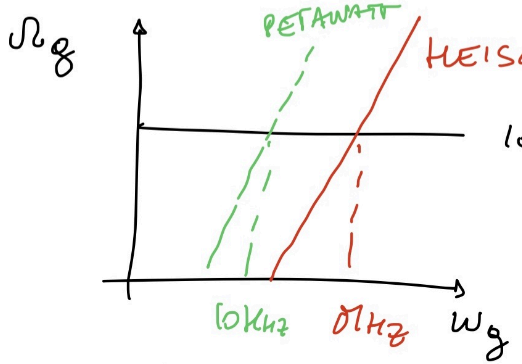
$$\boxed{S^{OUT}_{y_m^2 y_m^2} \sim N_\gamma} \qquad g^2 \sim N_\gamma$$

$$S_{y_m y_m} \sim N_\gamma \qquad h \sim \frac{1}{\sqrt{N_\gamma}}$$

SPECIAL QUANTUM STATE

$$\boxed{S^{OUT}_{y_m^2 y_m^2} \to 0}$$

BUT STILL NEED ONE γ FROM SIGNAL $\to h \sim 1/N_\gamma$



ALSO THIS IS A BAD TOY MODEL BTW (MENTION CAVES STORY)

1. THE LASER

CONSIDER ONE E.M. MODE

$$\omega, \; a, \; a^\dagger \qquad [a, a^\dagger] = 1$$

$$N_\gamma = a^\dagger a \qquad \text{VACUUM: } a|0\rangle = 0$$

COHERENT STATE (LASER): $D(\alpha)|0\rangle \equiv$

$$\equiv |\alpha\rangle \;, \quad D(\alpha) \equiv e^{\alpha a^\dagger - \alpha^* a}$$

USEFUL PROPERTY $D^\dagger(\alpha)\, a\, D(\alpha) =$

$$= a + \alpha$$

$$D^\dagger(\alpha)\, a^\dagger D(\alpha) = a^\dagger + \alpha^*$$

$$\langle N_\gamma \rangle = \langle \alpha | N_\gamma | \alpha \rangle = |\alpha|^2$$

$$\langle \Delta N \rangle = \sqrt{\langle N_\gamma^2 \rangle - \langle N_\gamma \rangle^2} = |\alpha|$$

EX. CHECK USING $[a, a^\dagger] = 1$

COHERENT "COLLIDER"

THIS IS MOSTLY FOR FUN

BUT ...

<u>LHC</u>

$\sqrt{s} = 13\ \text{TeV}$

$\} \; N_p \simeq 10^{11}$

$$M_{MAX} \lesssim \sqrt{s}$$

CAN WE DO BETTER?

IN PRINCIPLE WE HAVE $N_p \sqrt{s} \simeq 10^{12}$ TeV AT OUR DISPOSAL.

<u>TOY EXAMPLE</u> TWO SCALAR FIELDS

$$V = \frac{m_\phi^2}{2}\phi^2 + \frac{m_\chi^2}{2}\chi^2 + \frac{g^2}{2}\phi^2\chi^2$$

PREPARE ϕ IN A COHERENT STATE (LIKE A LASER BUT FOR THE QUANTUM FIELD)

$$\hat{\phi}|\phi_*\rangle = \phi_*|\phi\rangle$$ WITH ϕ_* A REAL NUMBER $\lambda_{dB} = \frac{1}{m_\phi v_\phi}$

IF $n_\phi = m_\phi \phi_*^2$ $\quad n_\phi \lambda_{dB}^3 \gg 1$ CLASSICAL FIELD $\phi(t) = \Phi \cos(m_\phi t)$

THEN (E.O.M. FROM THE POTENTIAL)

$$\frac{d^2}{dz^2}\chi_k(z) + \left[A_k + 2q\cos(2z)\right]\chi_k(z) = 0$$

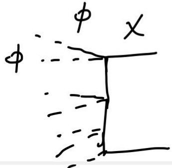
MATHIEU EQUATION — VERY WELL KNOWN

$$z \equiv mt \qquad A_k \equiv \frac{k^2 + m_k^2}{m_\phi^2} \qquad q \equiv \frac{g^2\,\bar{\Phi}}{4 m_\phi^2}$$

CAN PRODUCE AN EXPONENTIALLY LARGE NUMBER OF χ'S

UP TO $m_\chi^2 \lesssim g\,\bar{\Phi}\, m_\phi \gg$

$$\gg m_\phi^2/4$$



E.O.M.S ARE RESUMMING ALL THESE DIAGRAMS

THIS IS THE SAME PRINCIPLE AS THE LASER. INDEED WHEN $E_0 \gtrsim m_e^2$ YOU EXPECT e^+e^- PAIRS TO BE PRODUCED (SCHWINGER EFFECT) EVEN IF SINGLE PHOTONS HAVE ENERGIES

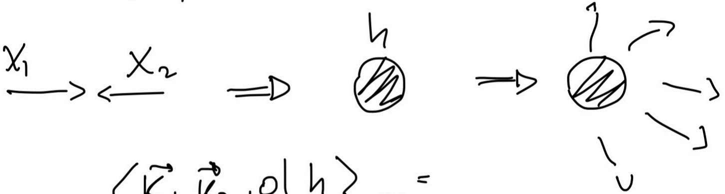
$$\omega_L \simeq \mathrm{THz} \simeq 10^{-4}\,\mathrm{eV} \ll m_e$$

COLLECTIVE EFFECT FROM

$$N_\gamma \simeq \frac{m_e}{10^{-4}\,\mathrm{eV}} \simeq 10^{10} \text{ PHOTONS}$$

UNFORTUNATELY THE LASER IS NOT GOOD (DISCHARGES INTO

ELECTRONS), SO? $\Rightarrow$ HIGGS BOSON



$$ {}_{IN}\langle \vec{K}_1, \vec{K}_2, 0 | h \rangle_{OUT} = $$

$$ = \frac{\int_{h(t\to-\infty)=0}^{h(t\to+\infty)=h} \mathcal{D}X\, \mathcal{D}h\; e^{-S(X,h)}\, X(\vec{k}_1)\, X(\vec{k}_2)}{\int \mathcal{D}X\, \mathcal{D}h\; e^{-S(X,h)}} \overset{\text{COLEMAN CALLAN '77}}{=} A e^{-B} $$

$B \sim M^4 \cdot V \cdot \underline{\Delta t}$

$\Delta t \downarrow$ LIFETIME OF CONDENSATE

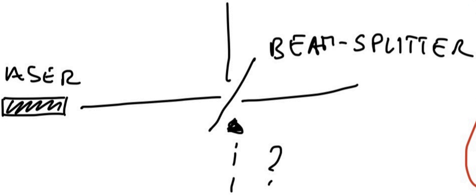
FOR

$M \simeq M_{GUT}$

$V \simeq 1\,m^3$

$\Delta t \simeq 1/m_h \Rightarrow B = \simeq 10^{87}$

2. THE UNUSED PORT



NORMALLY JUST $|0\rangle$, BUT IS IT THE BEST CHOICE?