Quantum Simulation: overview and perspective

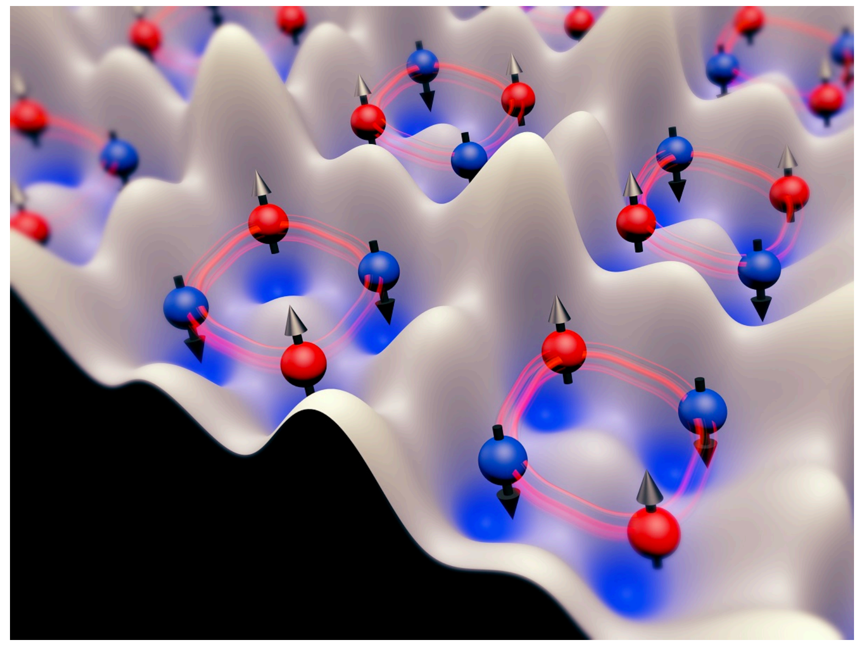
Marco Di Liberto

Dipartimento di Fisica e Astronomia "Galileo Galilei"
Quantum Technologies and Research Center (QTech)

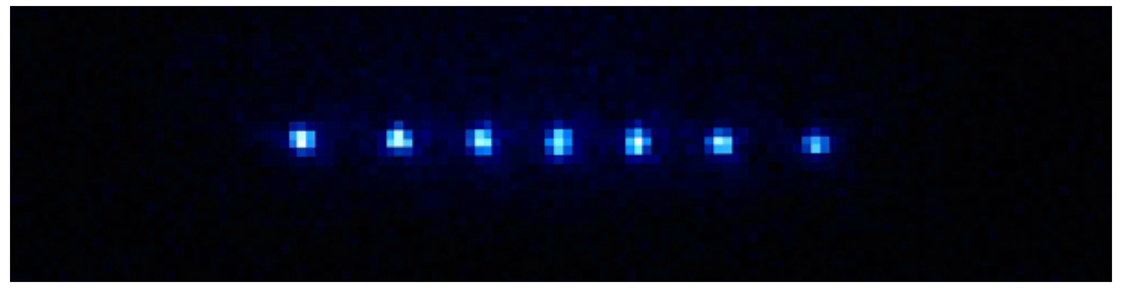
Università degli Studi di Padova

Quantum platforms

Ultracold atoms

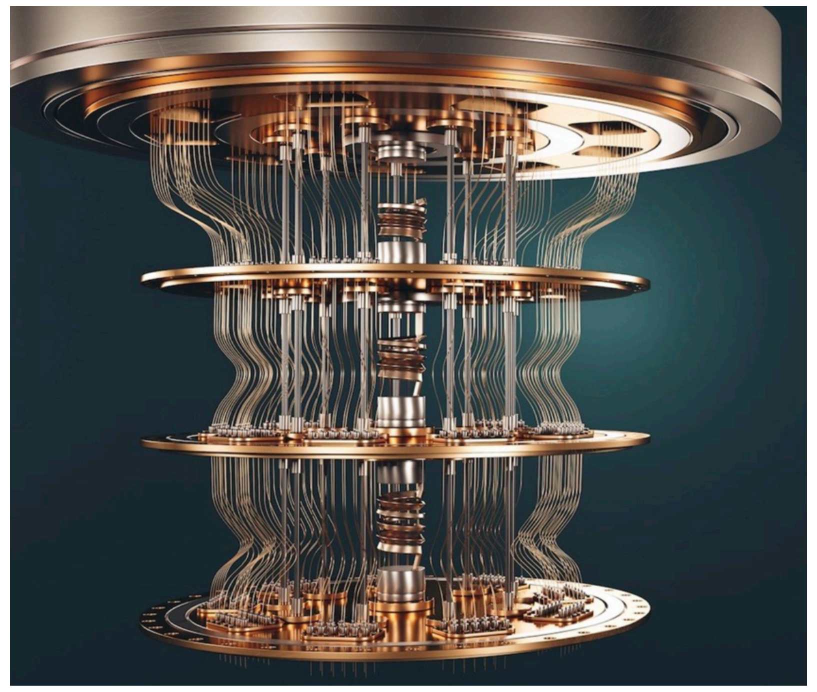


Trapped ions

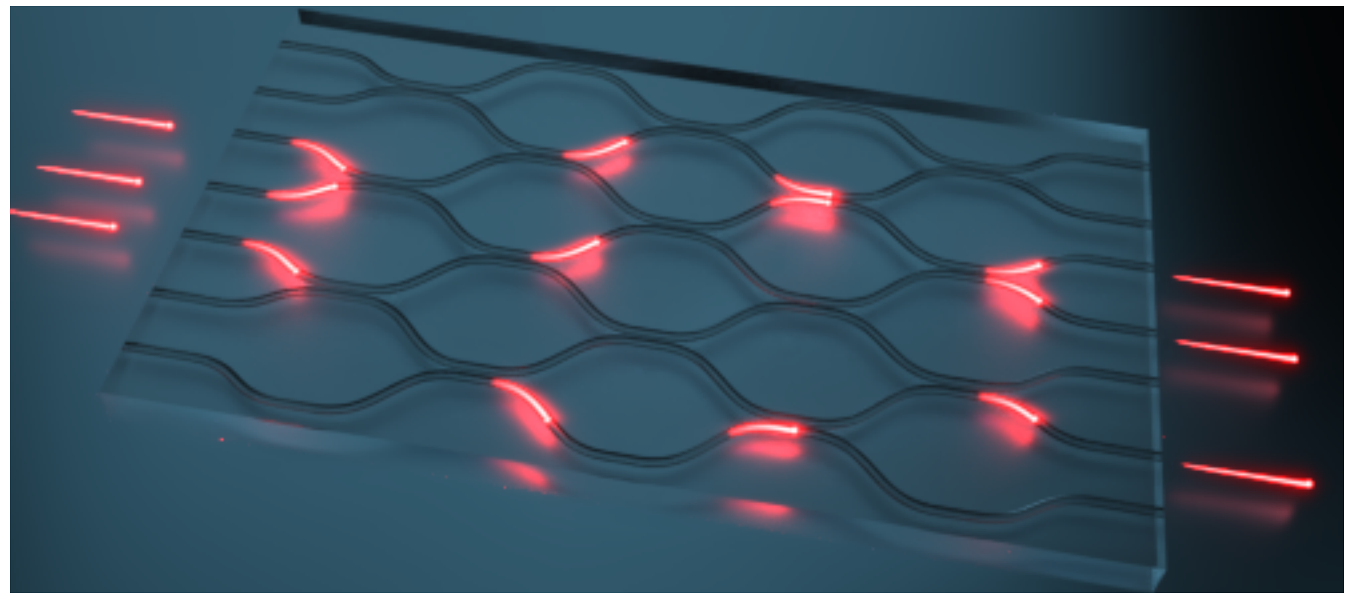


$$\text{“}i\hbar\partial_t\psi = H\psi\text{”}$$

Superconducting circuits

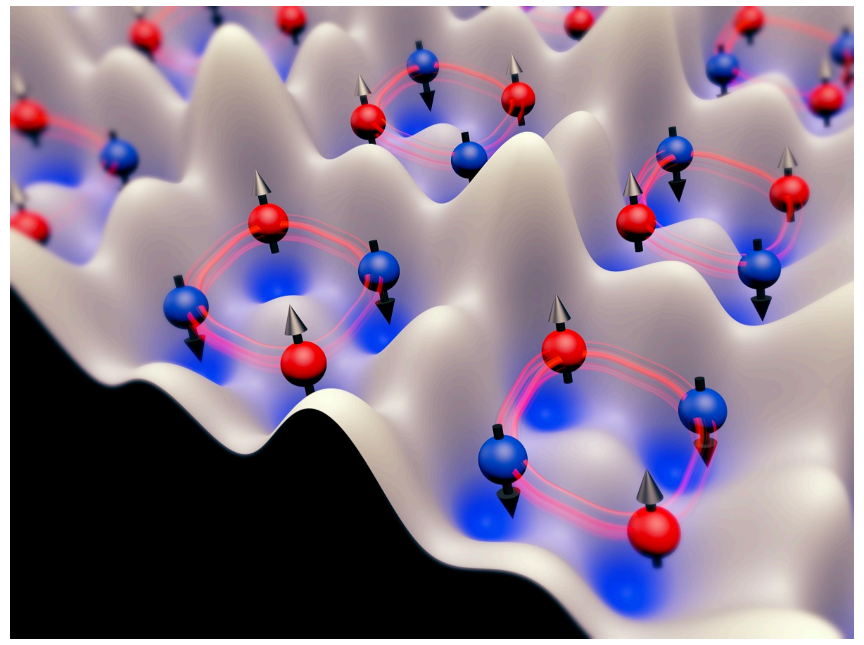


Photonics boson sampling



To do what?

Simulating many-body quantum systems



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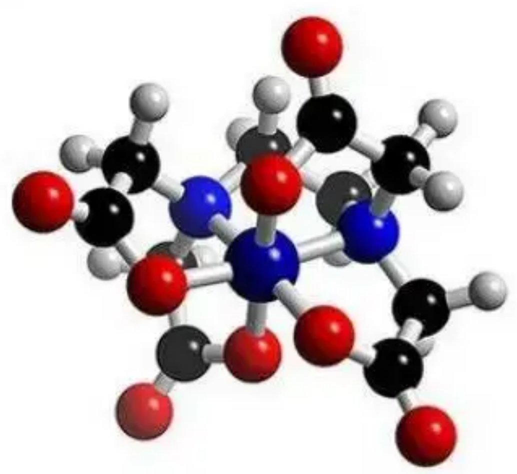
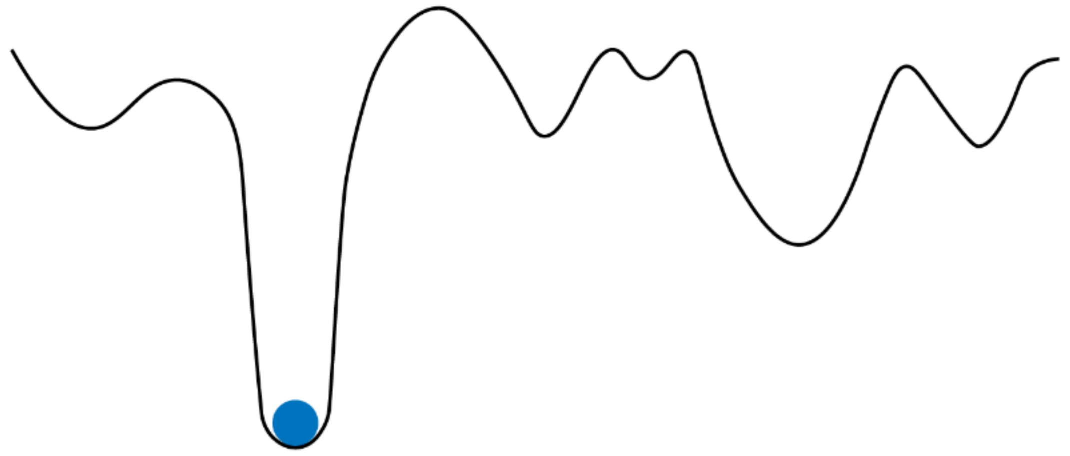
Design/model materials

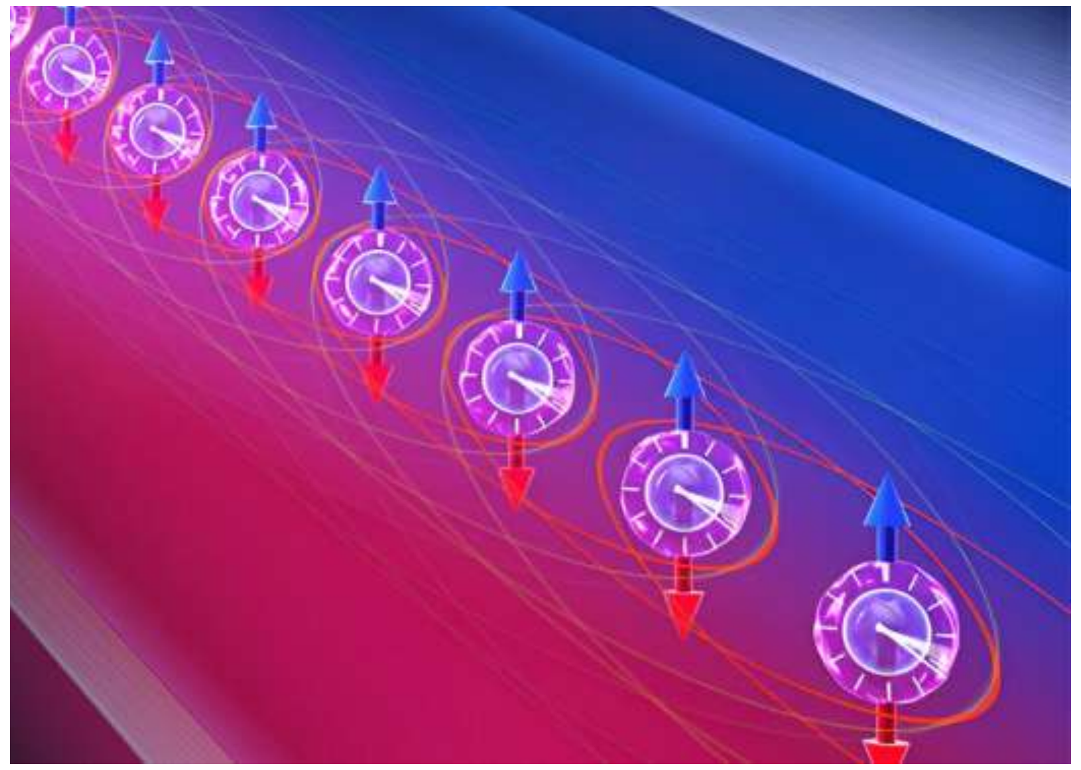


Cryptography 1994

Optimisation/search problems



Quantum metrology and sensing



Quantum advantage?

nature — Google

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

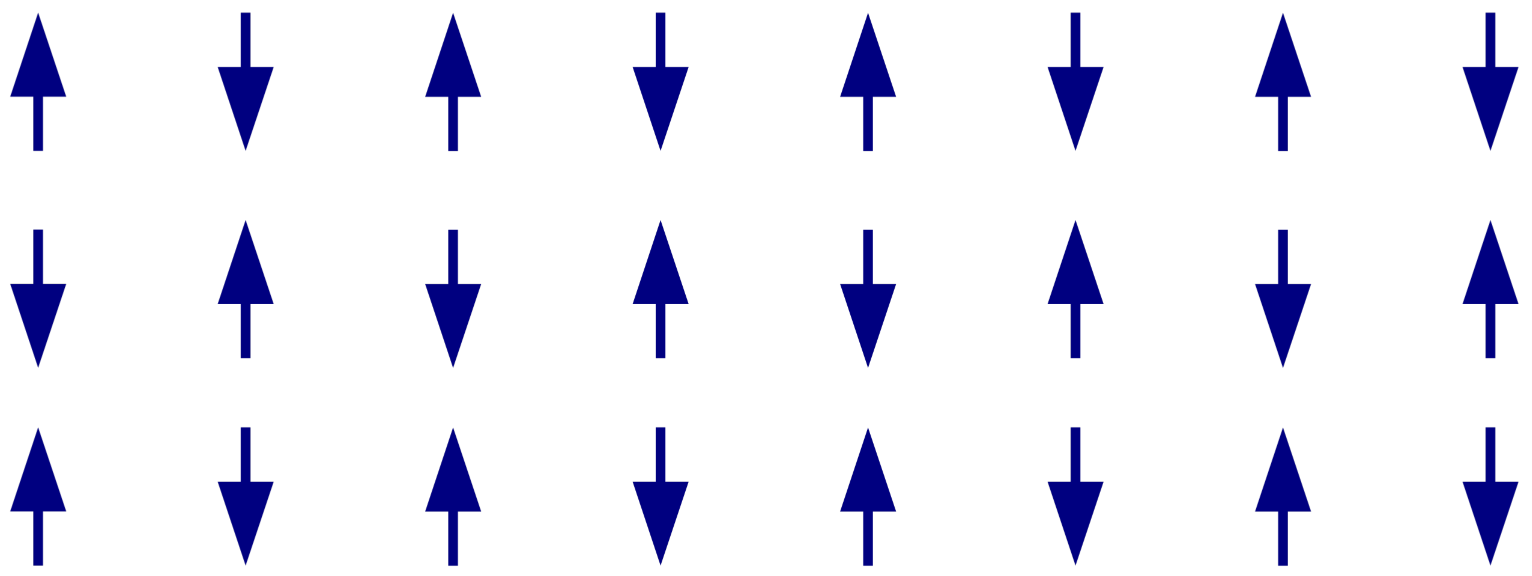
nature — IBM

Article | Open access | Published: 14 June 2023

Evidence for the utility of quantum computing before fault tolerance

Quantum complexity

Quantum Spin model



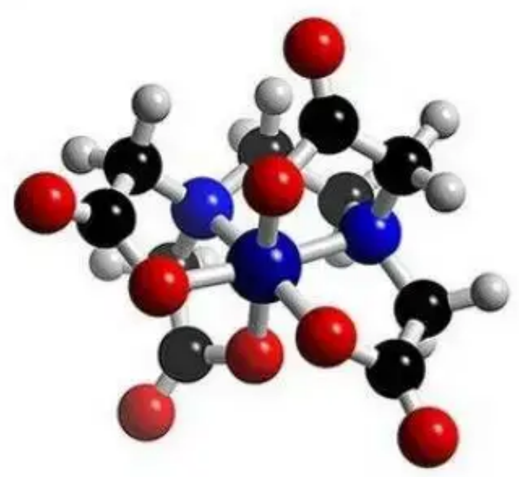
$$\hat{H} = \sum_{\langle i,j \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z + h \sum_i \hat{\sigma}_i^x$$

Many-body wavefunction

$$|\Psi\rangle = \sum_{\{\vec{\sigma}\}} C(\sigma_1, \ldots, \sigma_N) \, |\sigma_1, \ldots, \sigma_N\rangle$$

2^N numbers ⟷ 2^N quantum states

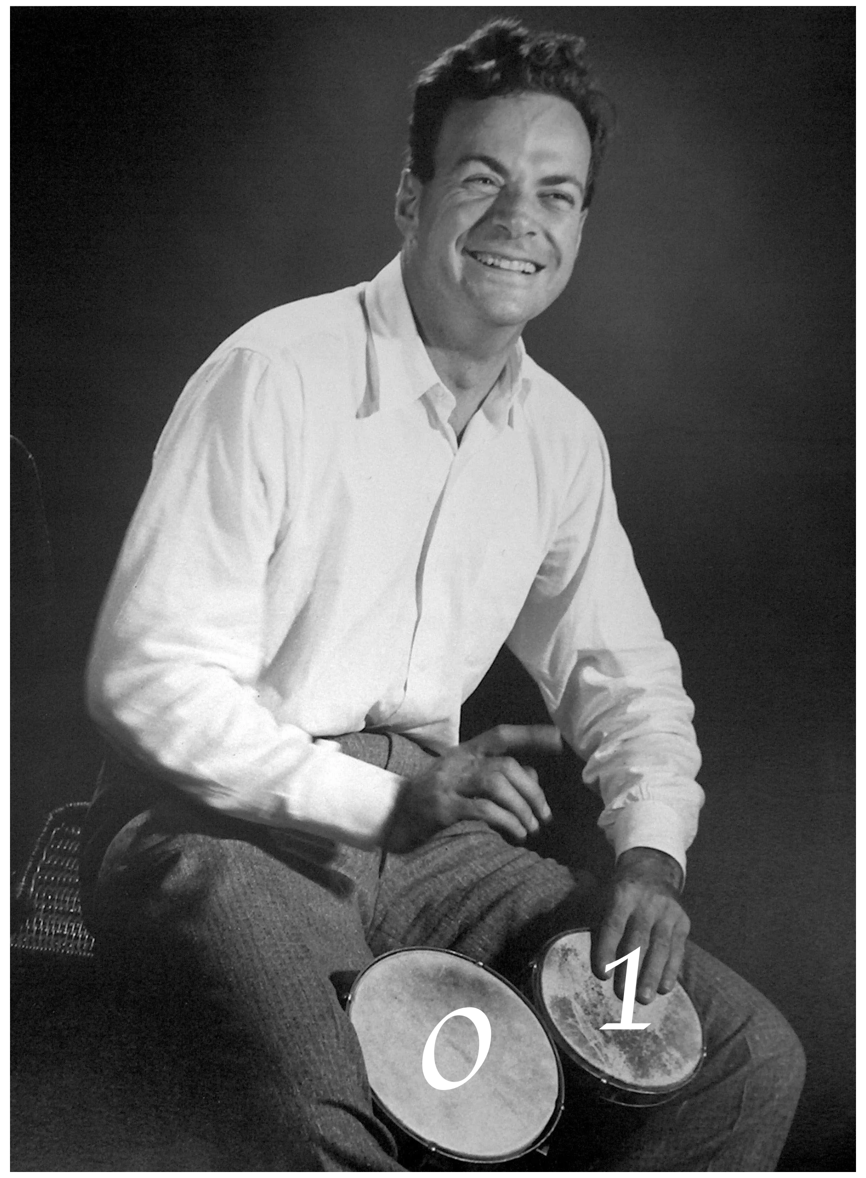
$N = 40$: $2^{40} \approx 10^{12}$ ⟶ 32×10^{12} bytes = **4 Tb**



- Exponentially large Hilbert space
- Exact encoding of large systems (e.g. complex molecules) is hopeless
- Traditional methods (Quantum Monte Carlo, QFT, perturbative methods) have limitations
- Classical methods (DMRG) work well in 1D

Quantum simulation

Feynman 1982;
Lloyd Science 1996



"Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws"

Analog quantum simulation

Single purpose quantum simulator

Digital quantum simulation

Universal quantum computer

Requirements

- Encode Hilbert space
- Prepare/evolve quantum states
- Measure quantum props

Digital quantum simulation

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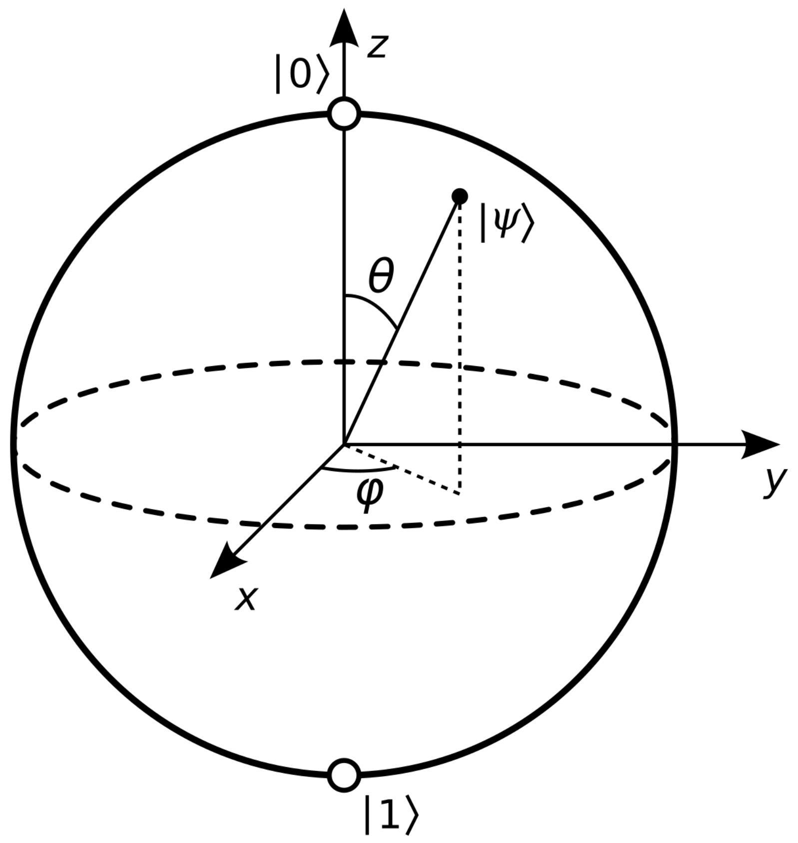
Ingredients

- Bit of quantum information: **qubit**

$$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\, e^{i\phi}\,|1\rangle$$



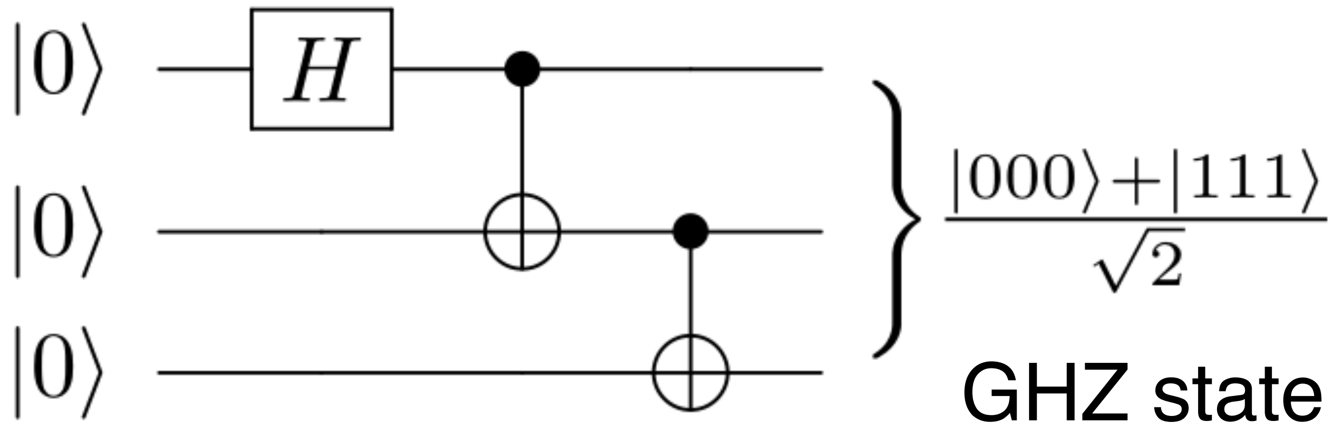
- Decompose quantum dynamics

$$e^{-iHt} \approx (e^{-iH_1\Delta t}e^{-iH_2\Delta t}e^{-iH_3\Delta t}\ldots)^N$$

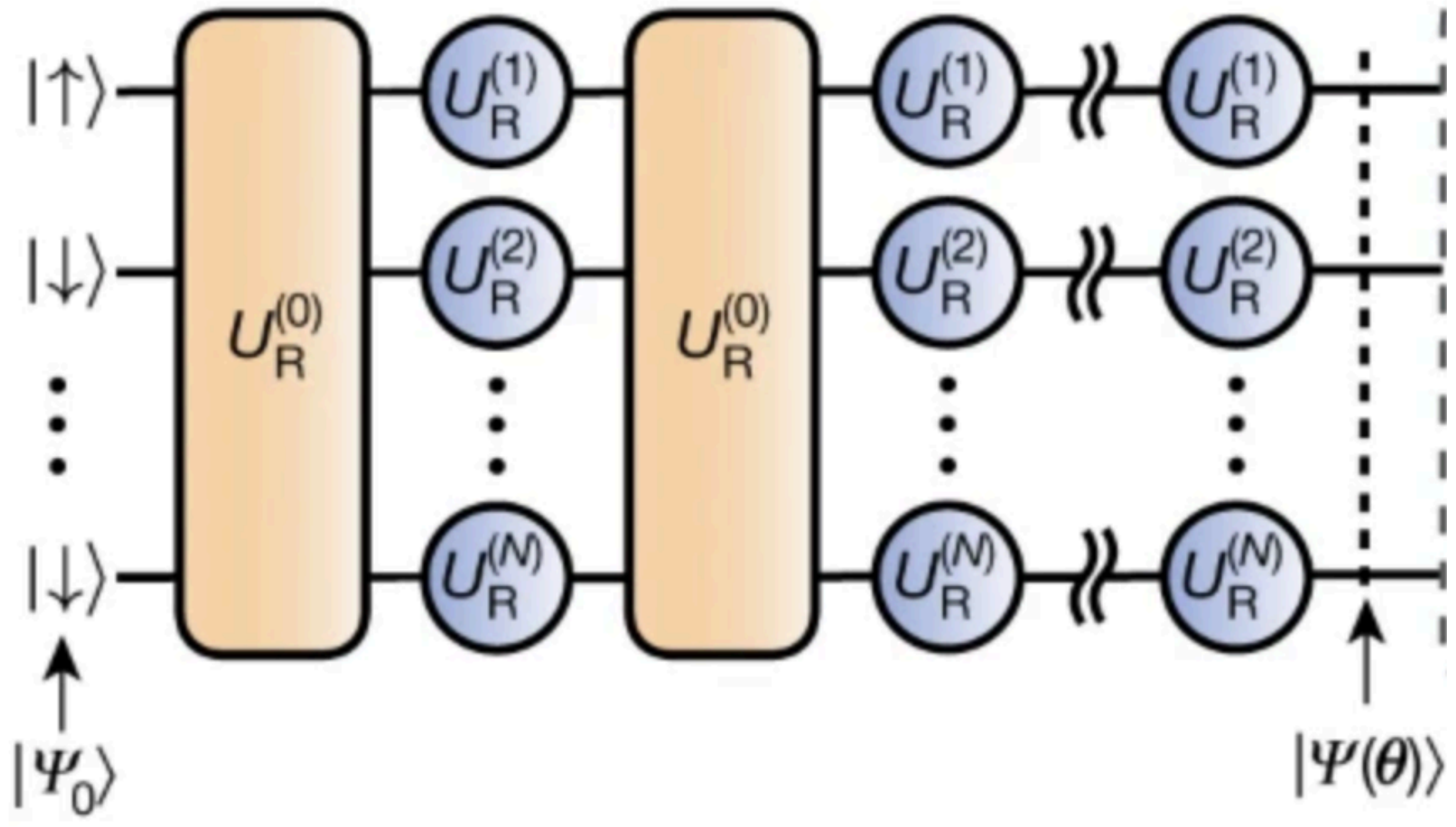
$$H = H_1 + H_2 + H_3 + \ldots$$

- Operations to control quantum info

Single-qubit rotations
Phase gate
Entanglement (CNOT)



- .. or any unitary operation



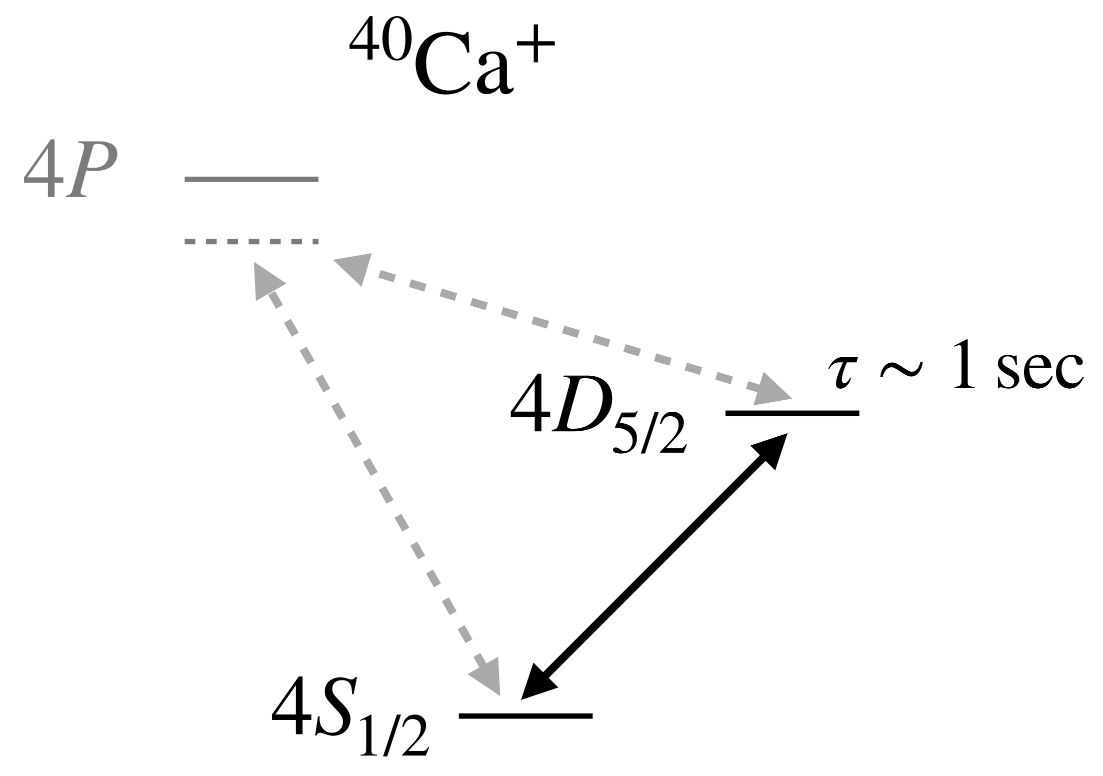
Zoller Nature 2019

Digital quantum simulation: ions

Blatt Sci.Rep. 2008

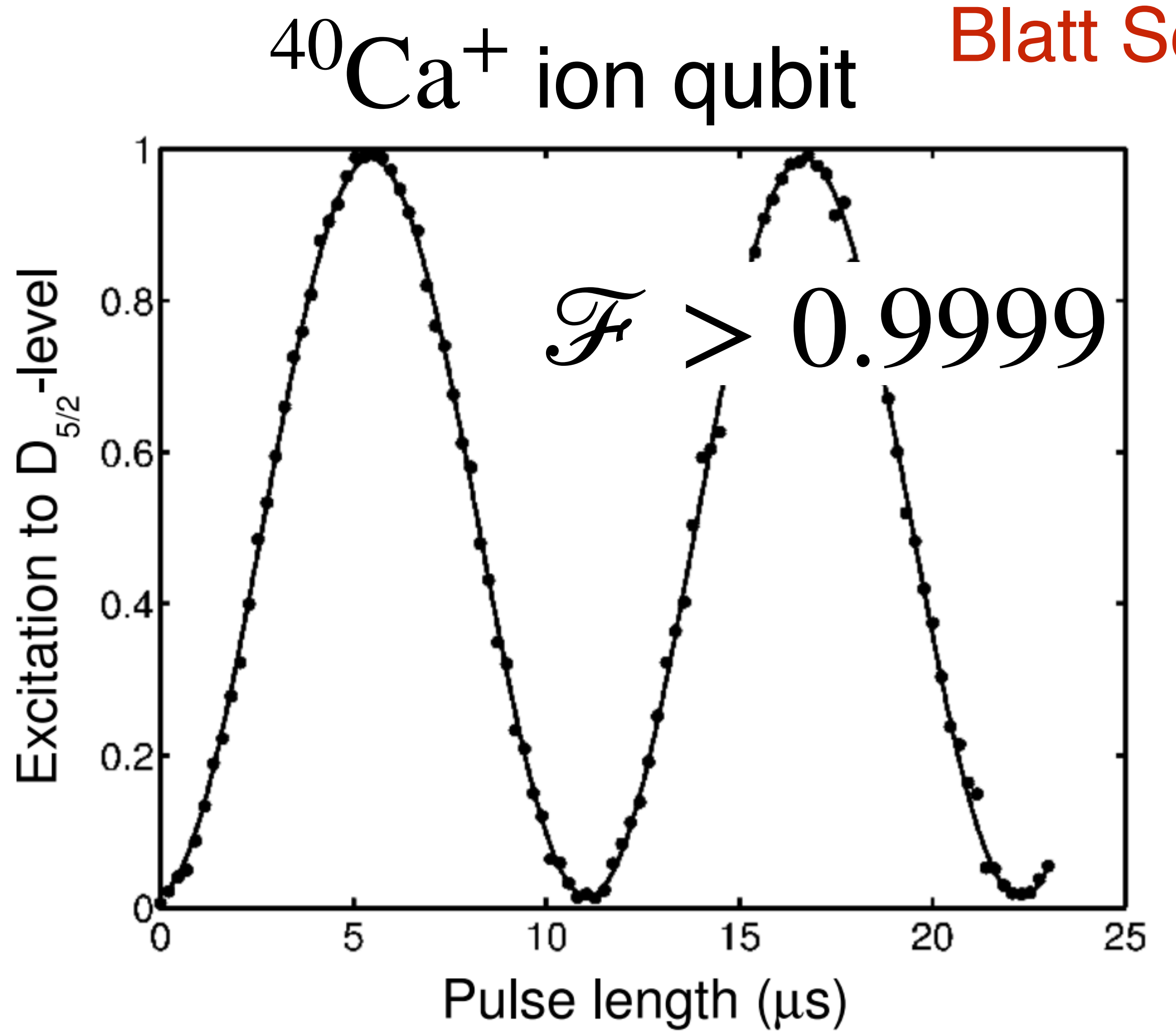
Trapped ions

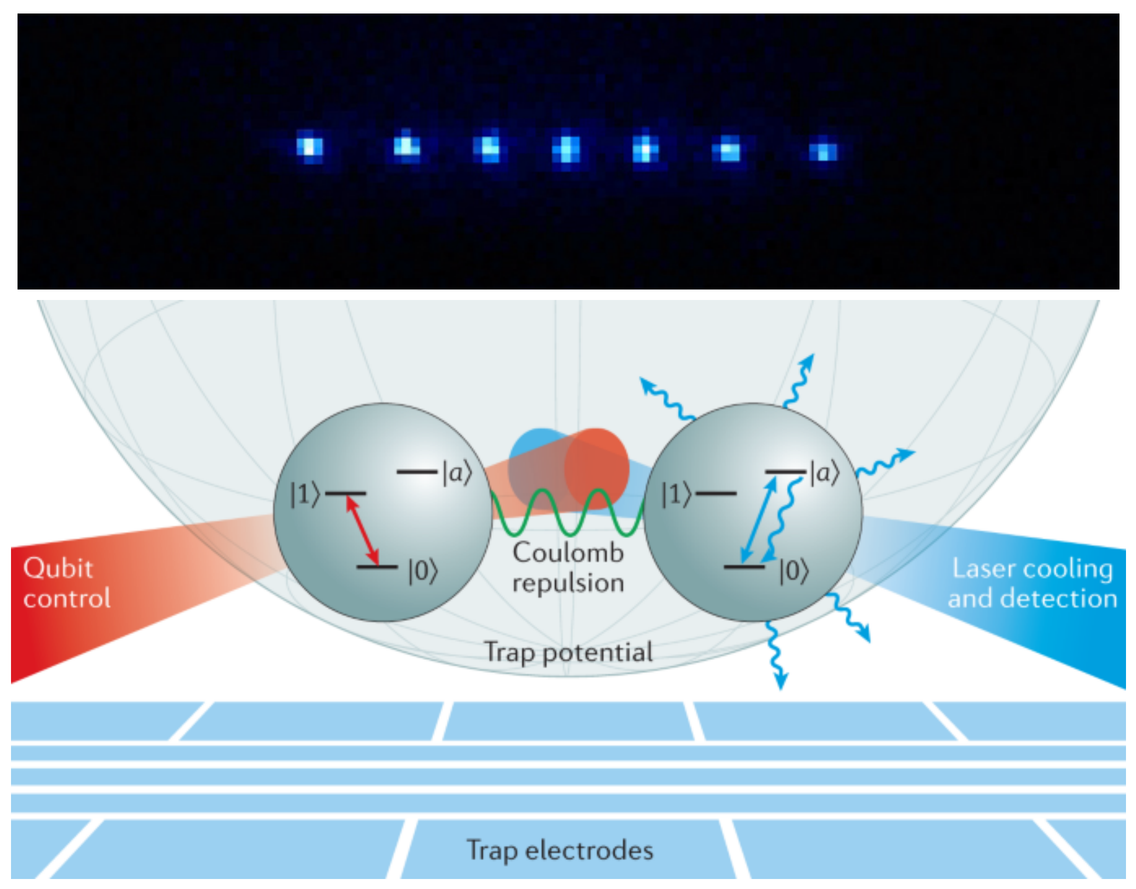
- Coulomb stabilized ion string (up to ~100 ions) in a trap
- Qubit encoded in two atomic levels
- Single-qubit operations via optics



$$\hat{H}_{AL} = -\hat{\mathbf{d}} \cdot \mathbf{E} - \hat{\mathbf{Q}} \cdot \nabla \mathbf{E} + \dots$$

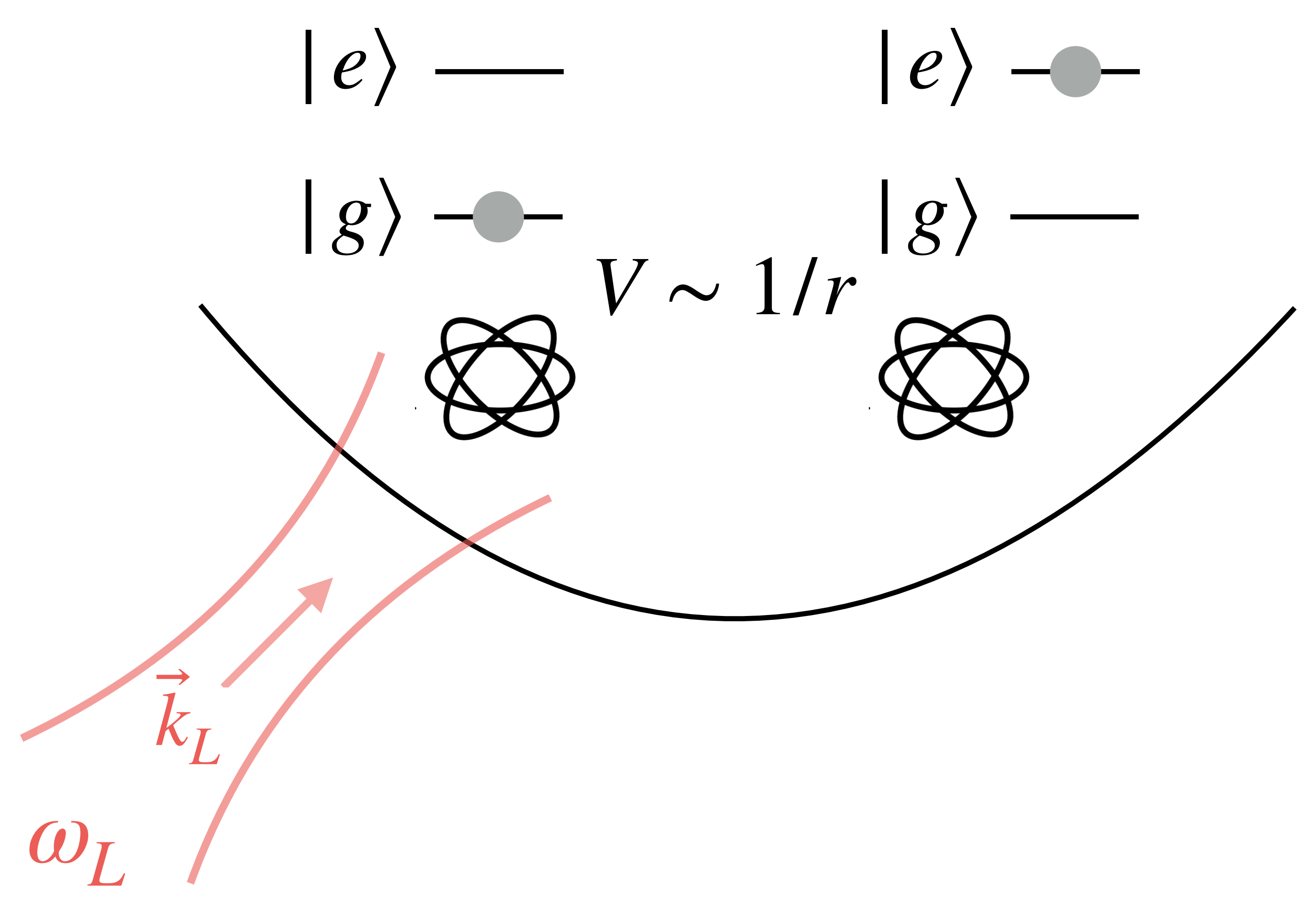
Dipole Quadrupole





- Two-qubit operations mediated by phonons (up to 20 ions)

$$\hat{H}_{\text{int}} = \eta\Omega |e\rangle\langle g| \hat{a} + \text{h.c.}$$



Digital quantum simulation: QED and beyond

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Long-range spin model

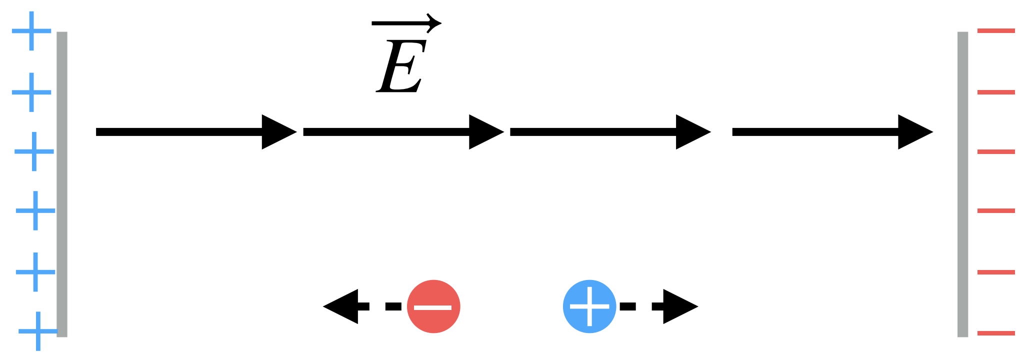
$$\hat{H} = \sum_{i<j} J_{ij} \frac{\hat{\sigma}_i^z \hat{\sigma}_j^z}{|i-j|^\alpha} + h \sum_i \hat{\sigma}_i^x$$

↕ Kogut & Susskind 1975

1+1 QED

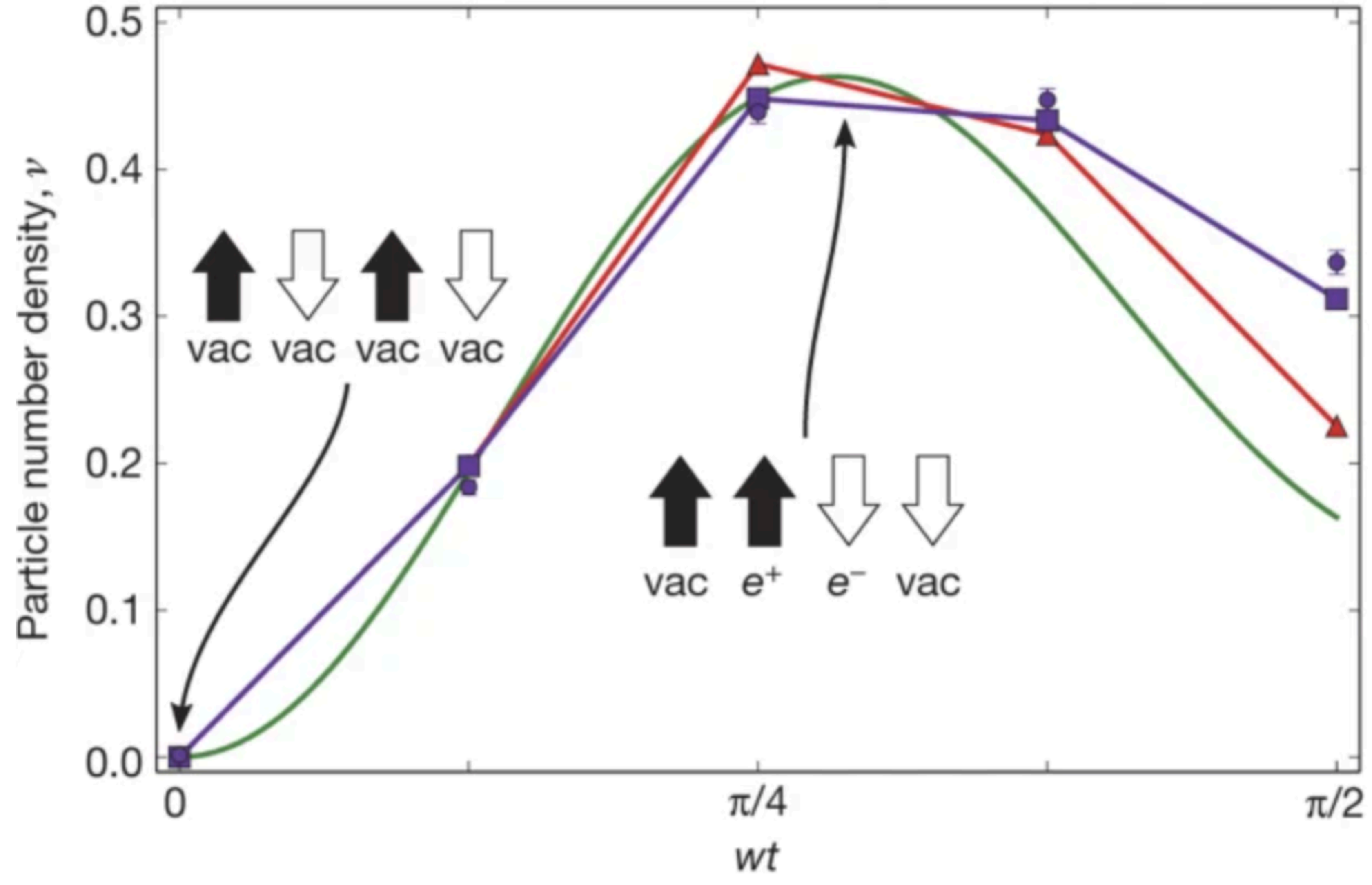
$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

Schwinger mechanism



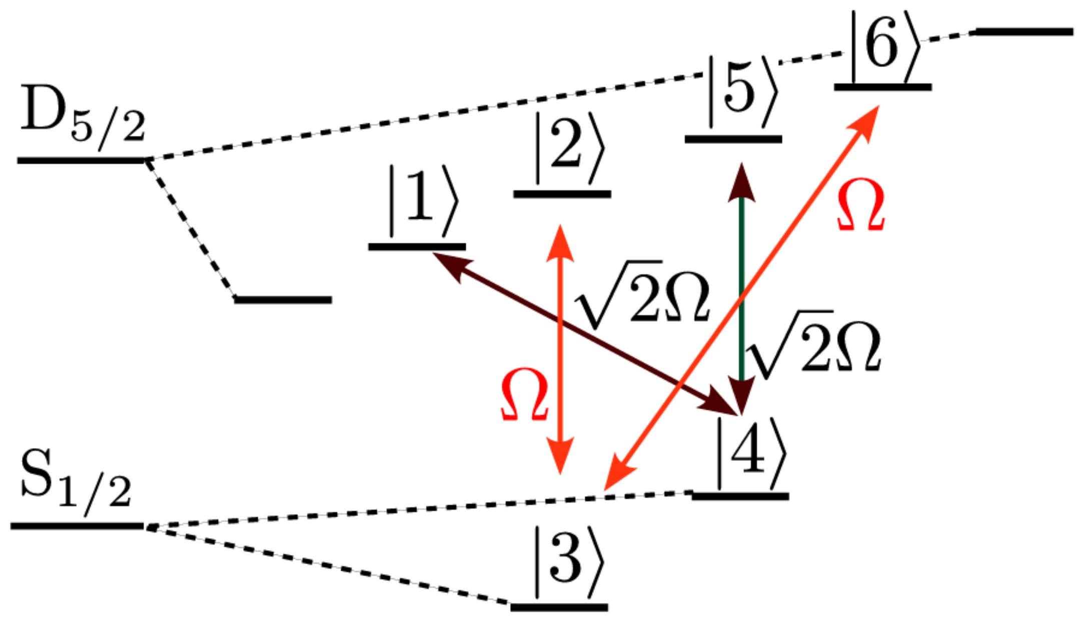
- Pair creation $E > 2mc^2$
- Particle-antiparticle dynamics
- Capacitor discharge

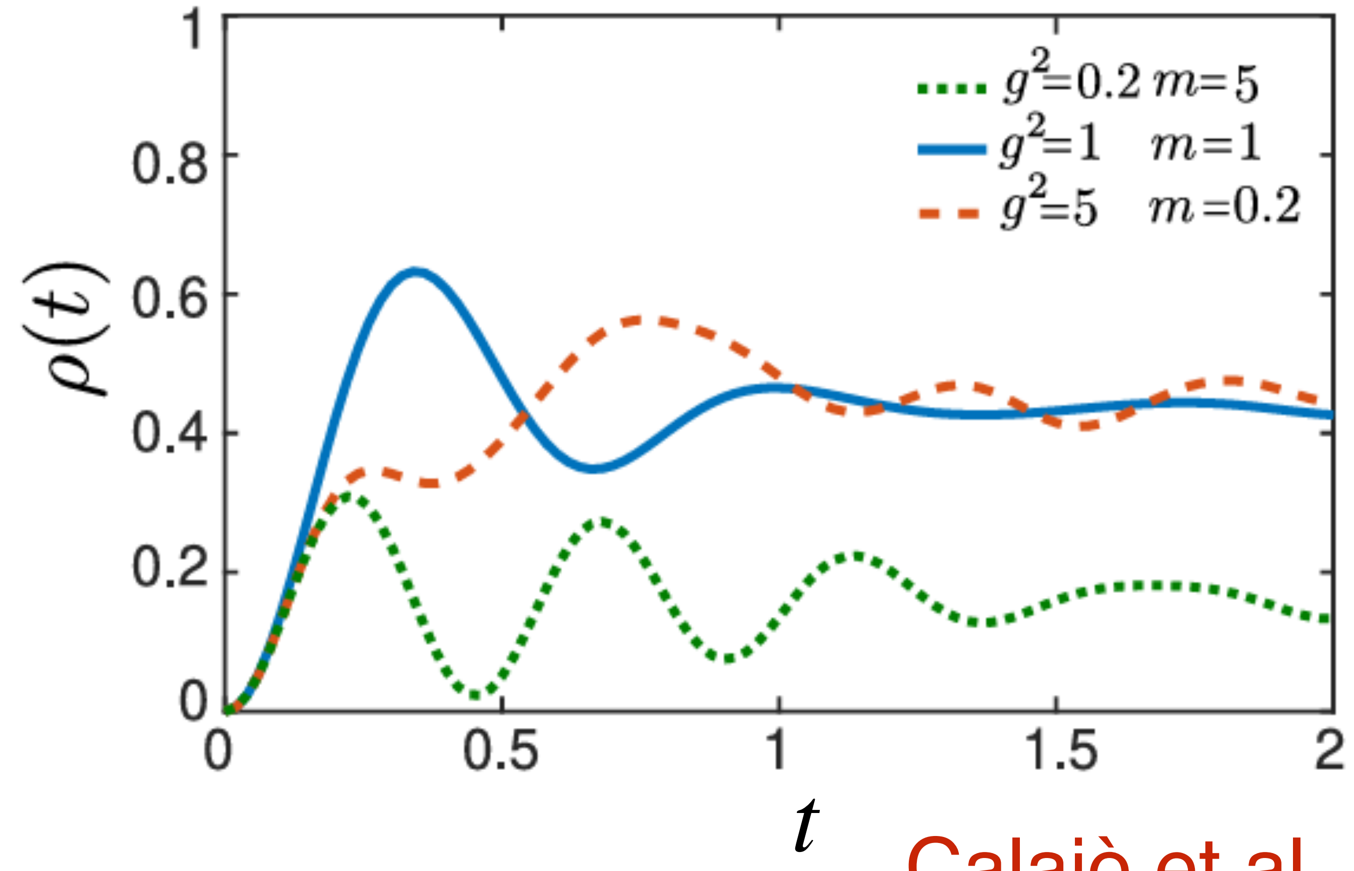
Blatt/Zoller Nature 2016



What about non-Abelian SU(2) Yang-Mills?

$$\hat{H}_0 = \frac{c\hbar}{2a_0} \sum_n \sum_{a,b=r,g} \left[-i \hat{\psi}^\dagger_{na} \hat{U}^{ab}_{n,n+1} \hat{\psi}_{n+1b} + \mathrm{H.c.} \right] + m_0 c^2 \sum_{na} (-1)^n \hat{\psi}^\dagger_{na} \hat{\psi}_{na} + g_0^2 \frac{c\hbar}{2a_0} \sum_n \hat{E}^2_{n,n+1}$$





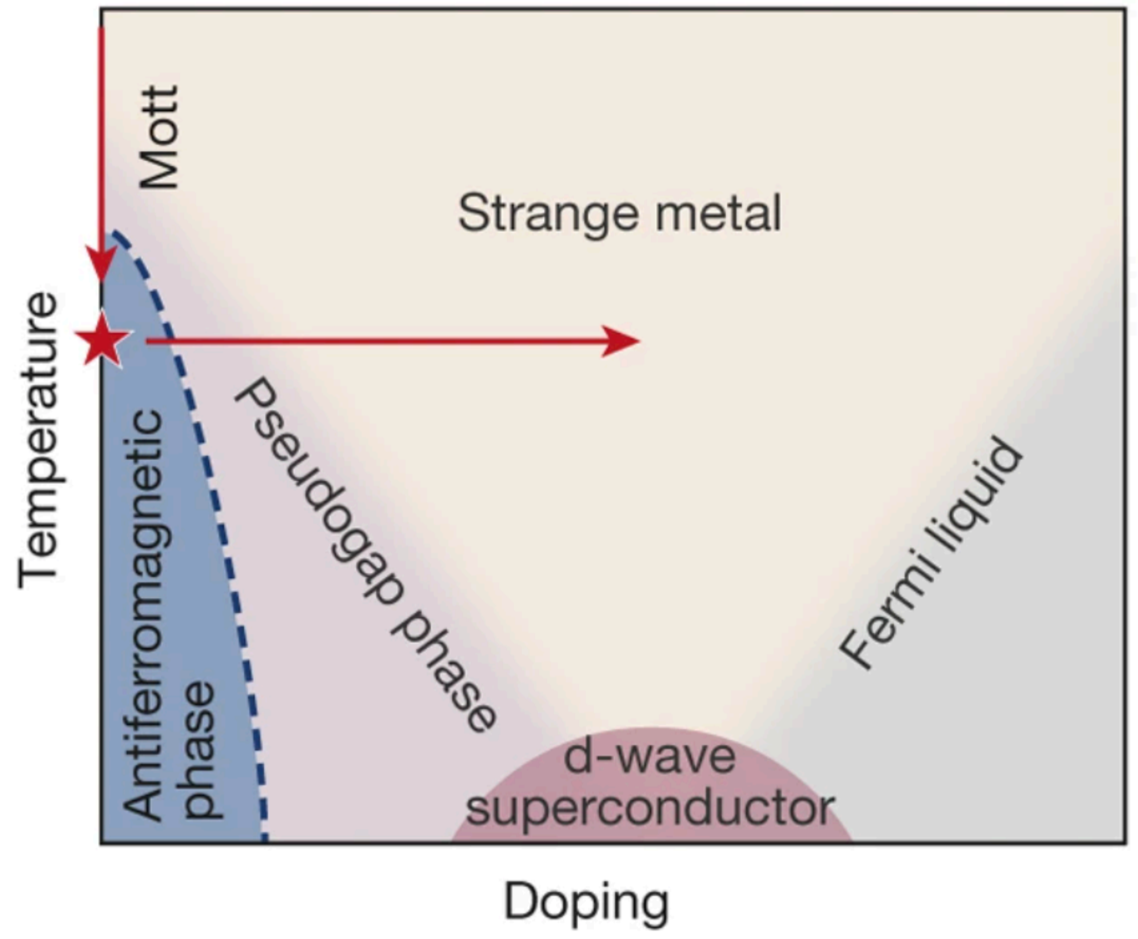
Calajò et al. (UNIPD), arXiv:2403.07110

Analog quantum simulation

E.g. Fermi-Hubbard model

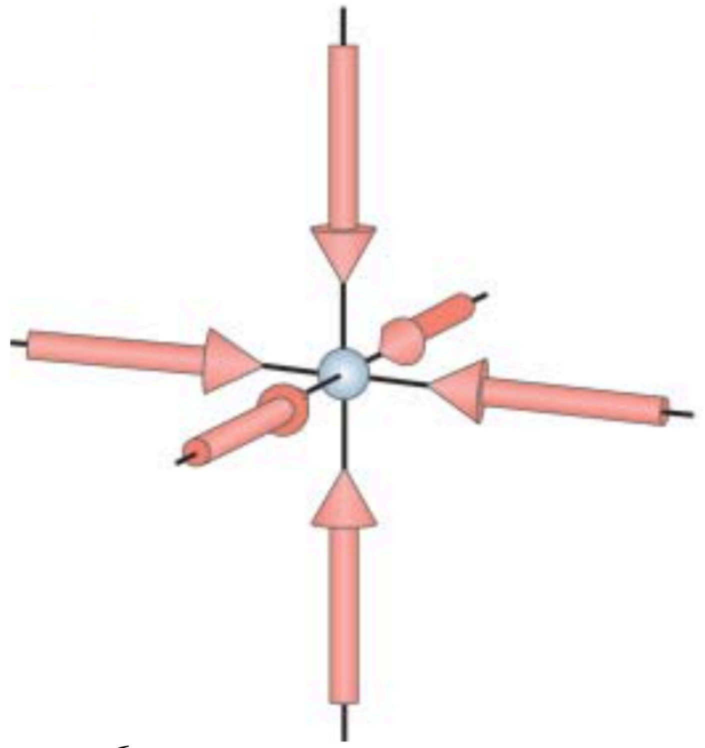
$$\hat{H} = -J \sum_{\langle ij \rangle,\sigma} \hat{c}^\dagger_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

- Unsolved in $d = 2$
- Candidate to describe high-Tc SC
- Simplest model for strongly-correlated fermionic systems

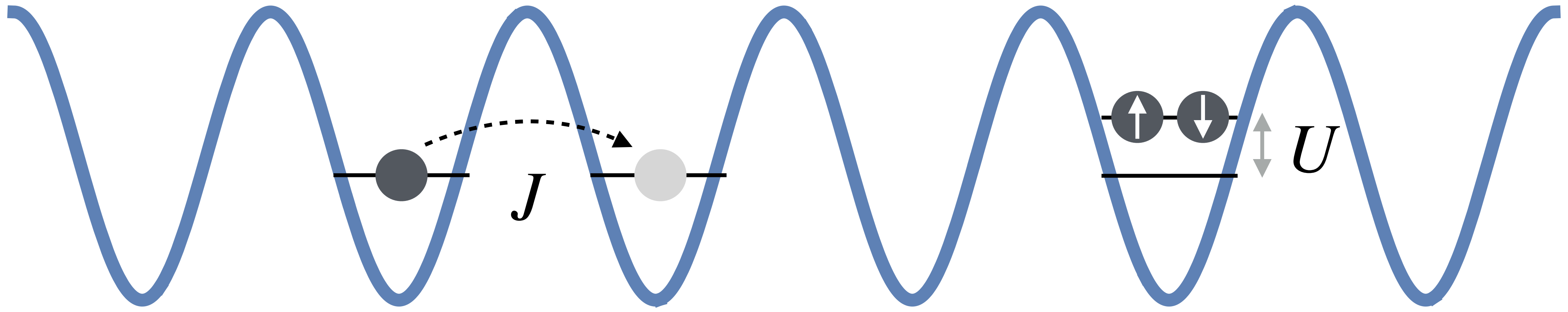


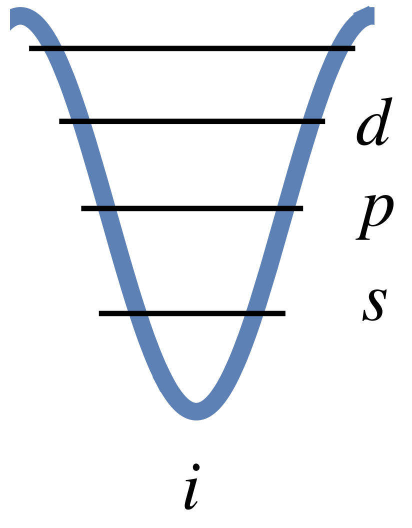
The ultracold atoms toolbox

- Identify fermionic (alkali) atoms ^{6}Li, ^{40}K at nanoKel
- Trap in a (optical) lattice geometry $V(\mathbf{r}) = -\alpha'(\omega)\,|E(\mathbf{r})|^2$



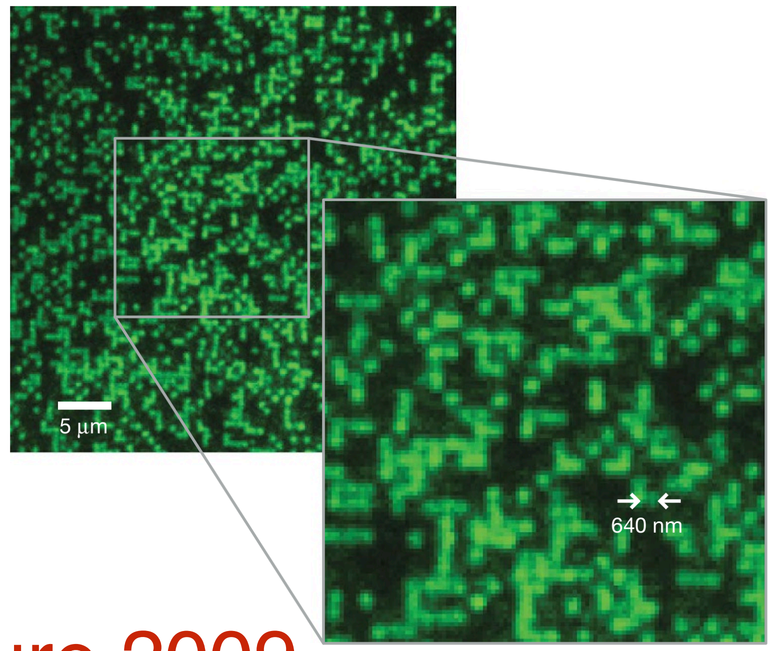
$V_{\text{int}}(r) \sim C_6/r^6$





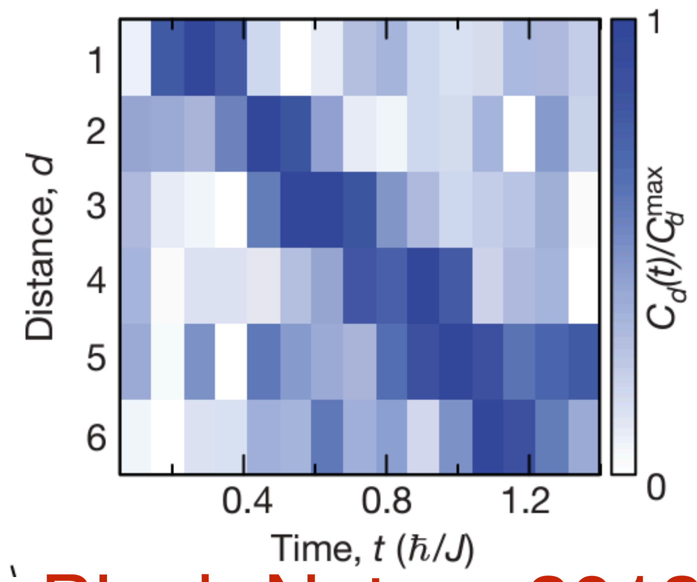
- Measure quantum states

Microscope: single-atom resolution



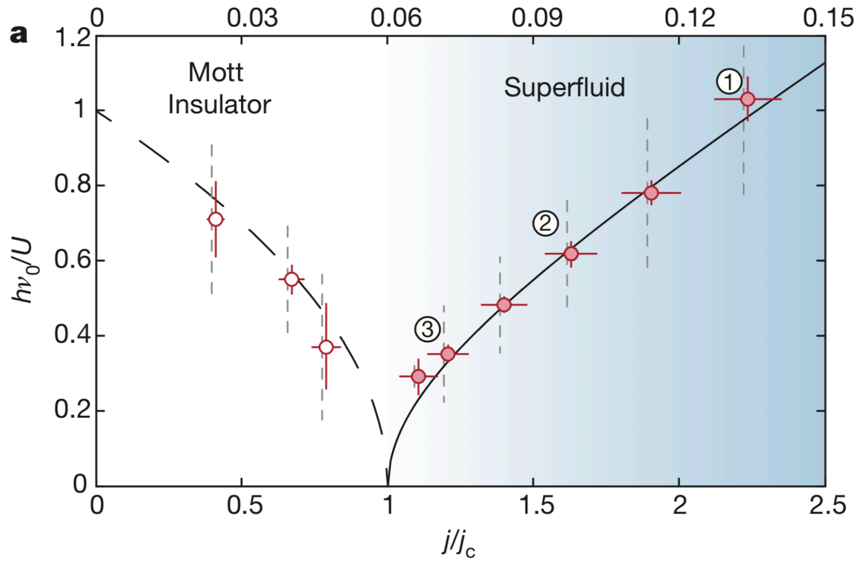
Greiner Nature 2009

Dynamics: correlations spreading



Bloch Nature 2012

Spectroscopy: Measure gaps, dispersion, states



Bloch Nature 2012

Analog quantum simulation

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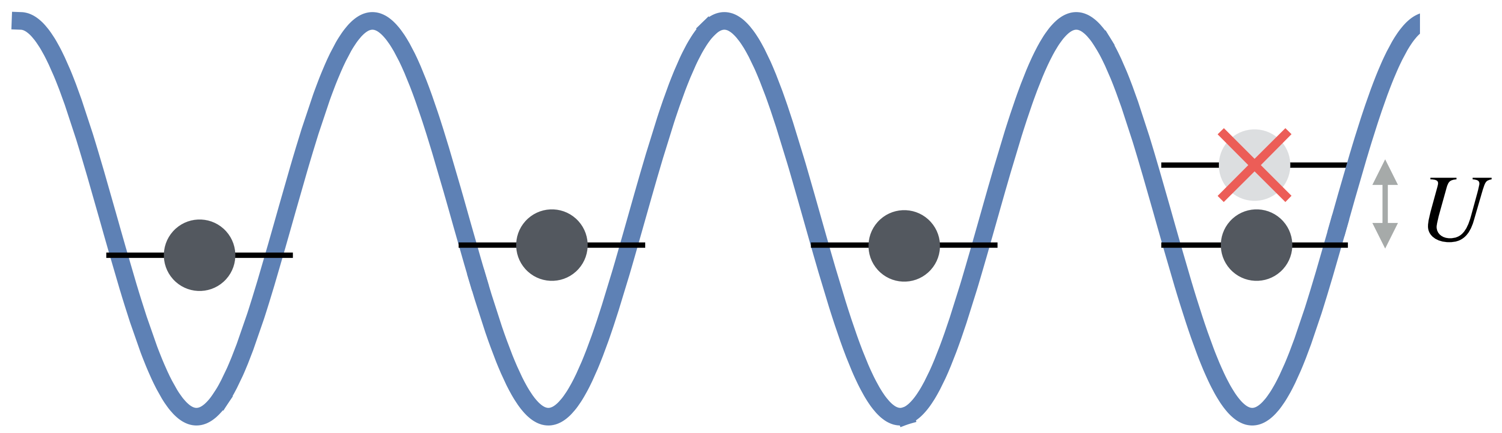
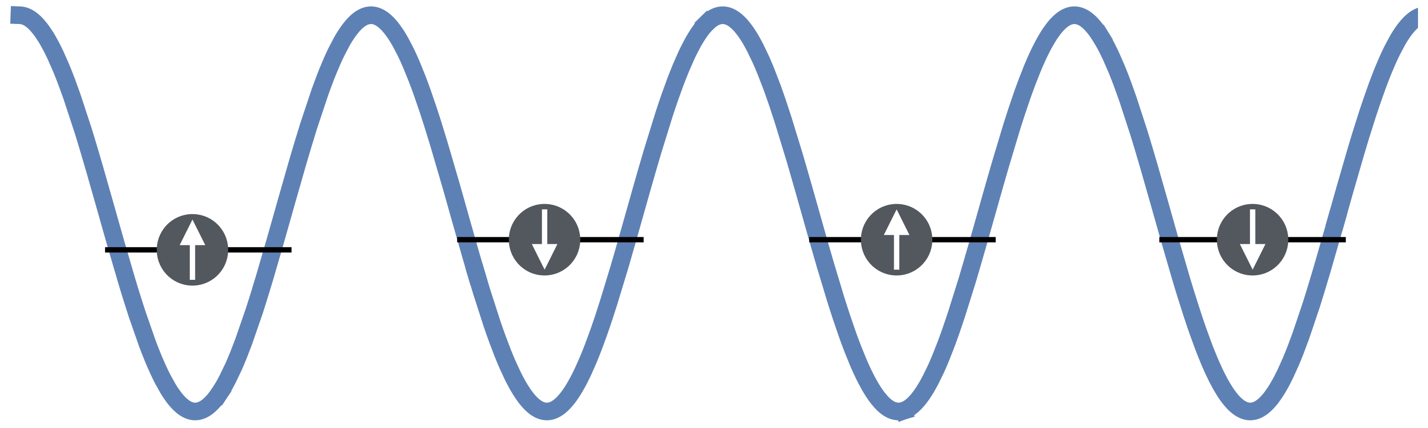
E.g. Fermi-Hubbard model

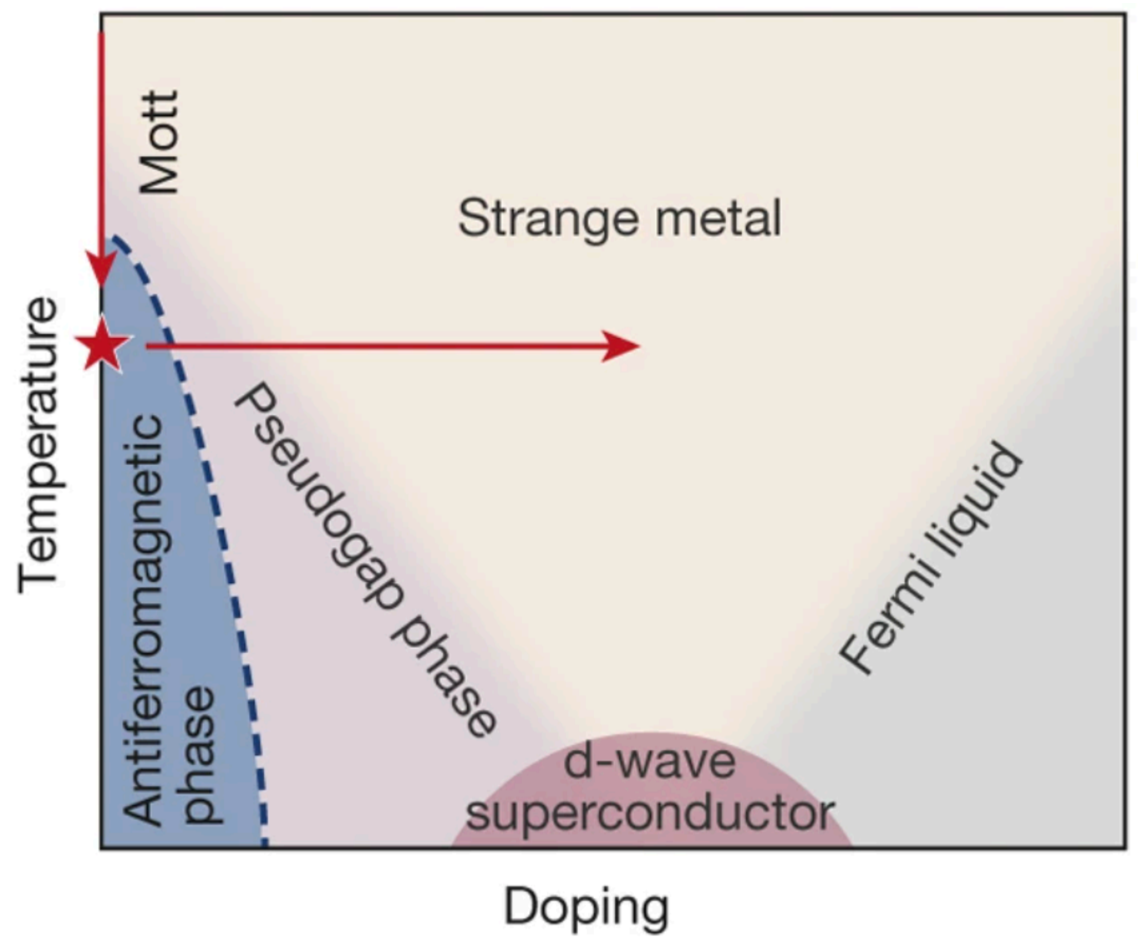
$$\hat{H} = -J \sum_{\langle ij \rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$



- $T \sim U$: Mott insulator physics

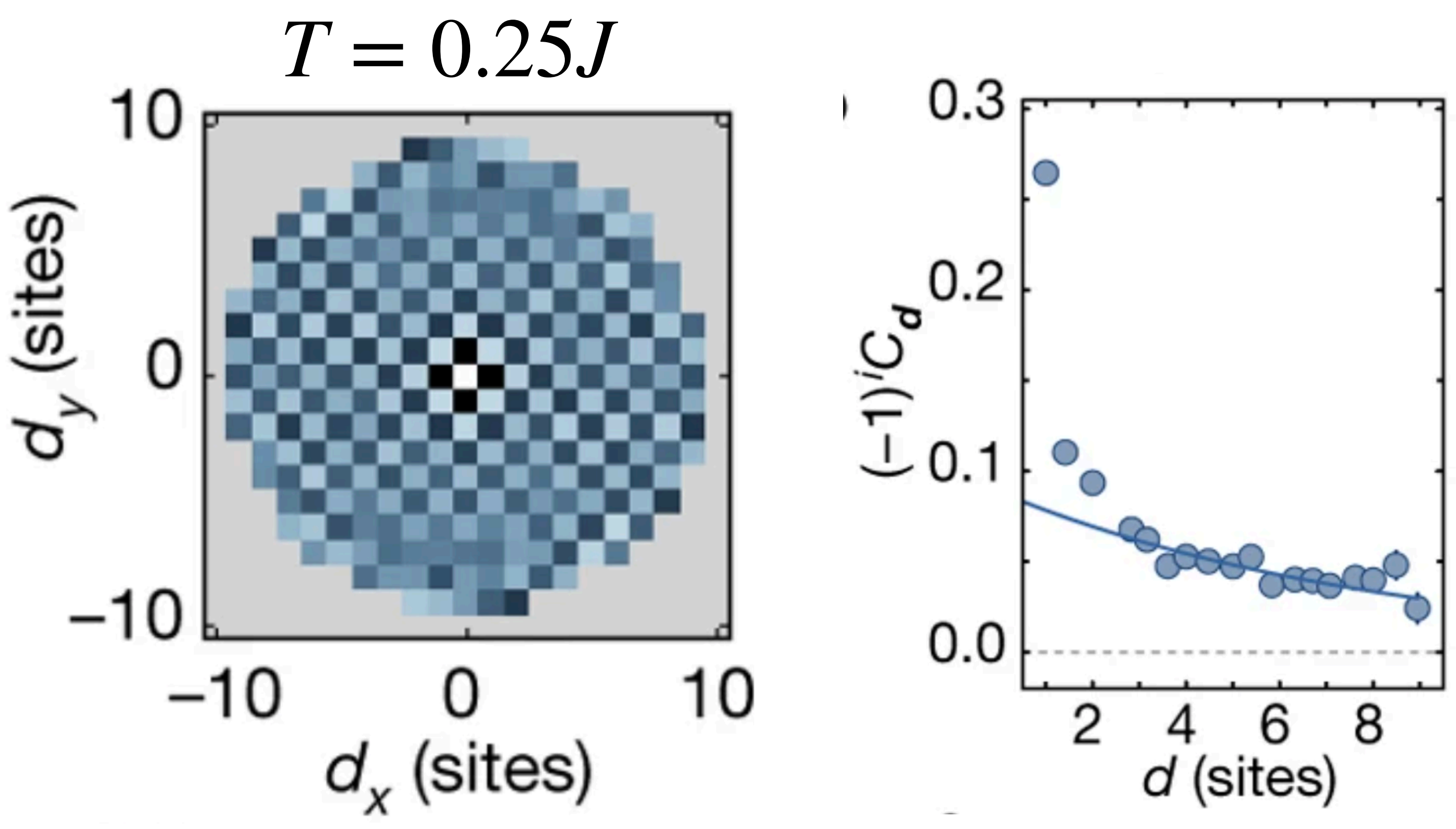


- $T \sim J^2/U$: Antiferromagnetism



- $T \sim T_c(?)$: superconductivity, non-Fermi liquid, new quasiparticles and critical properties

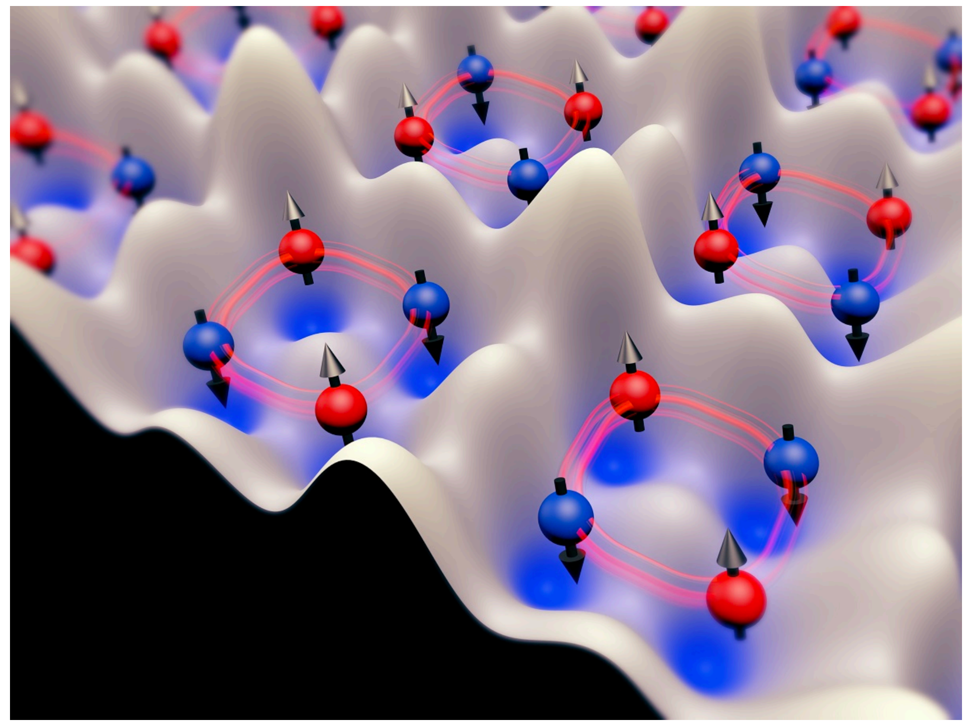
Greiner Nature 2017



Simulation with cold atoms

Fermi-Hubbard model

$$\hat{H} = -J \sum_{\langle ij \rangle, \sigma} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$



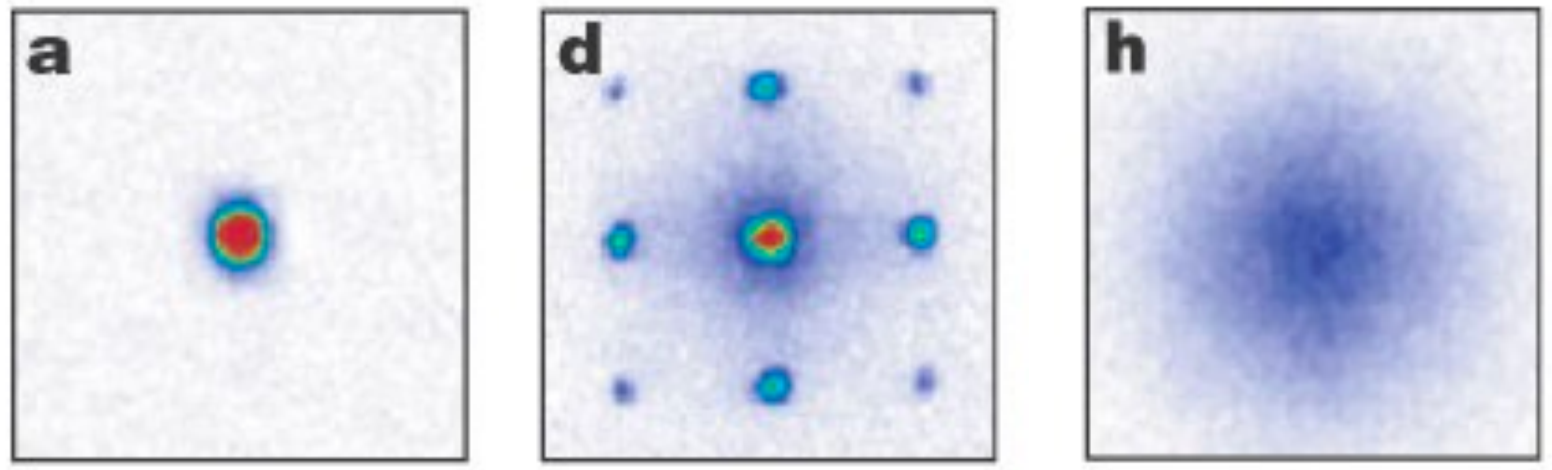
Bose-Hubbard model

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{a}^{\dagger}_{i} \hat{a}_{j} + \frac{U}{2} \sum_{i} \hat{n}_{i}(\hat{n}_{i} - 1)$$

Quantum spin model

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{\sigma}^{z}_{i} \hat{\sigma}^{z}_{j} + h \sum_{i} \hat{\sigma}^{x}_{i}$$

Jaksch PRL 1999
Greiner Nature 2002

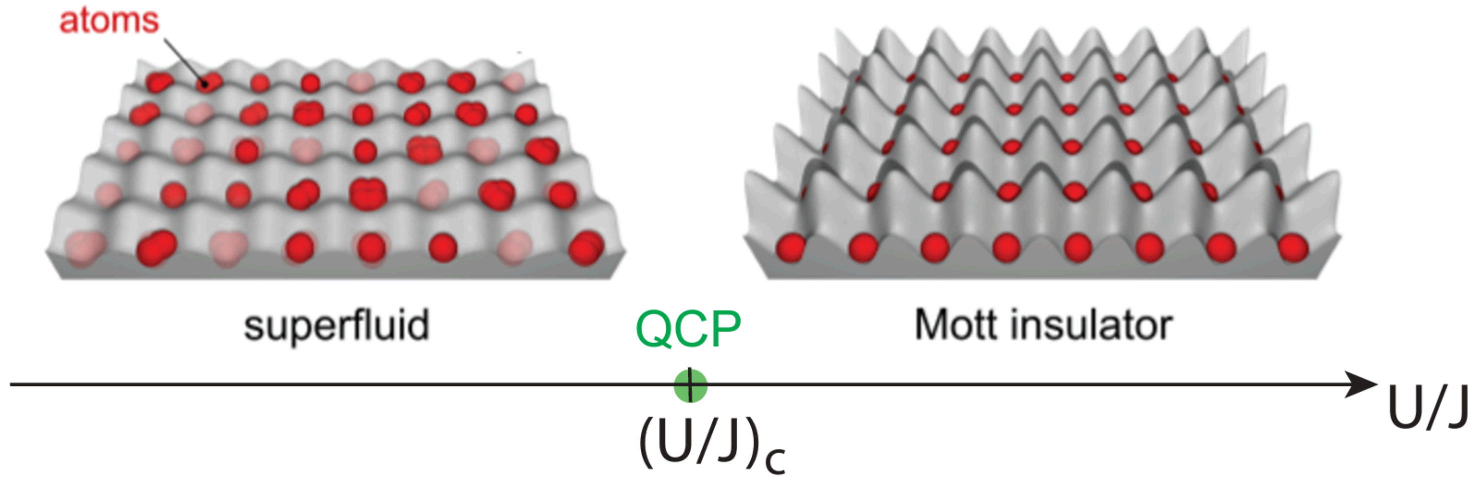


Superfluid

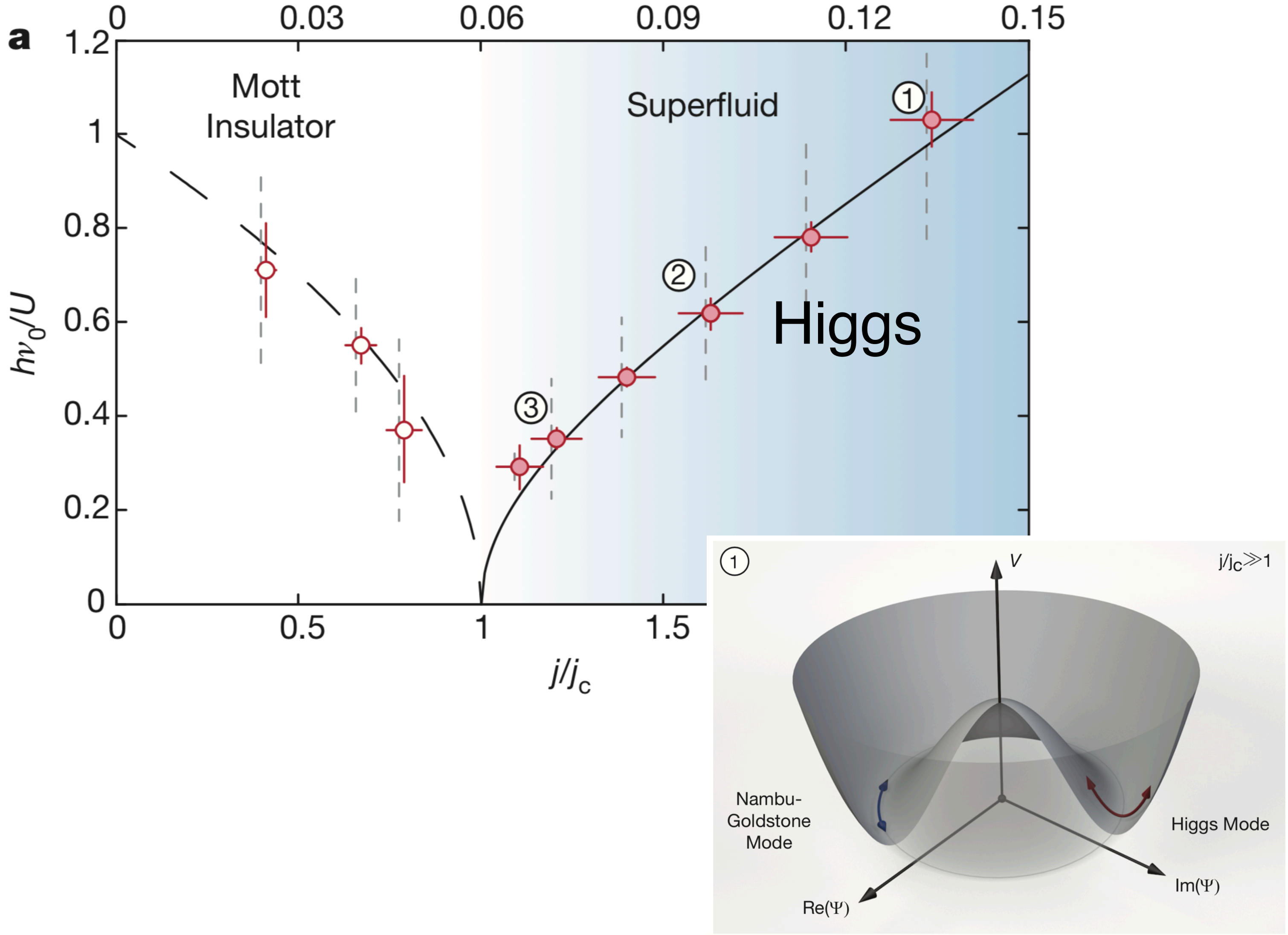
$J \gg U$

Mott insulator

$U \gg J$



Bloch Nature 2012



Achievements

- Strongly-correlated dynamics
- SU(N) models
- Topological phases
- Lattice gauge theories
- Synthetic dimensions
- Quantum magnetism
- Dipolar quantum phases

….

The rising star: Rydberg atoms

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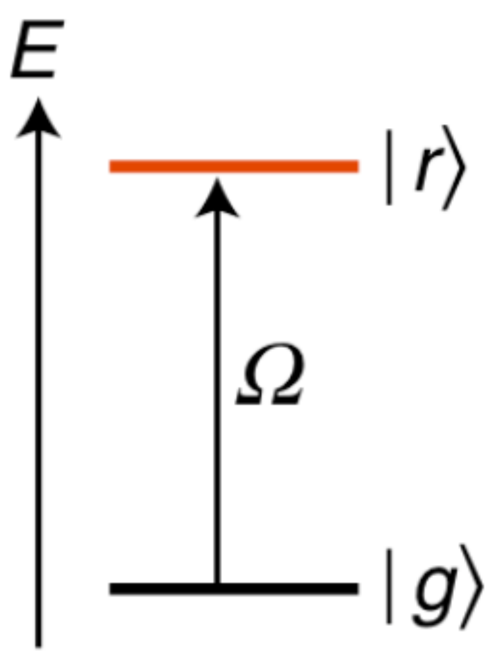
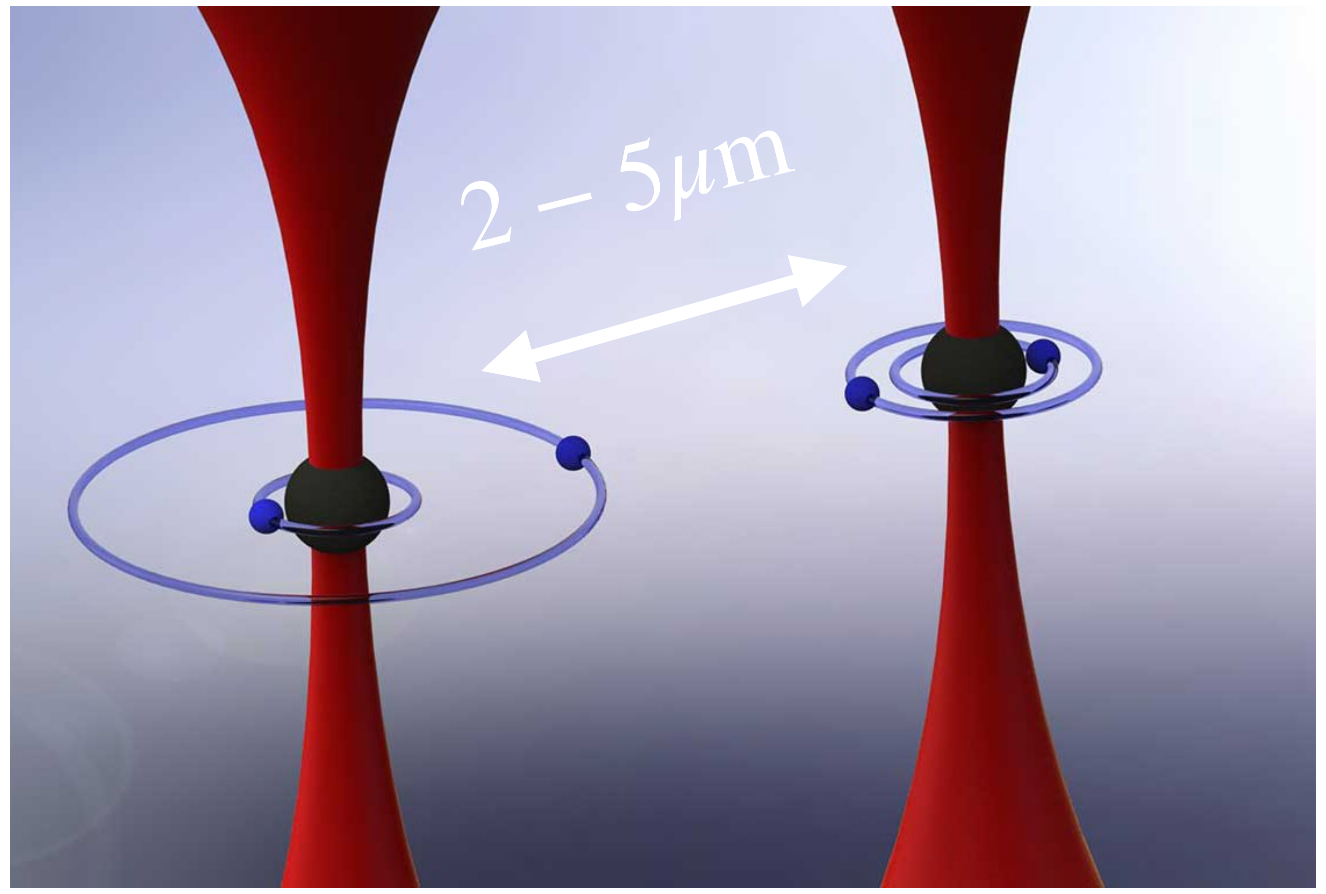
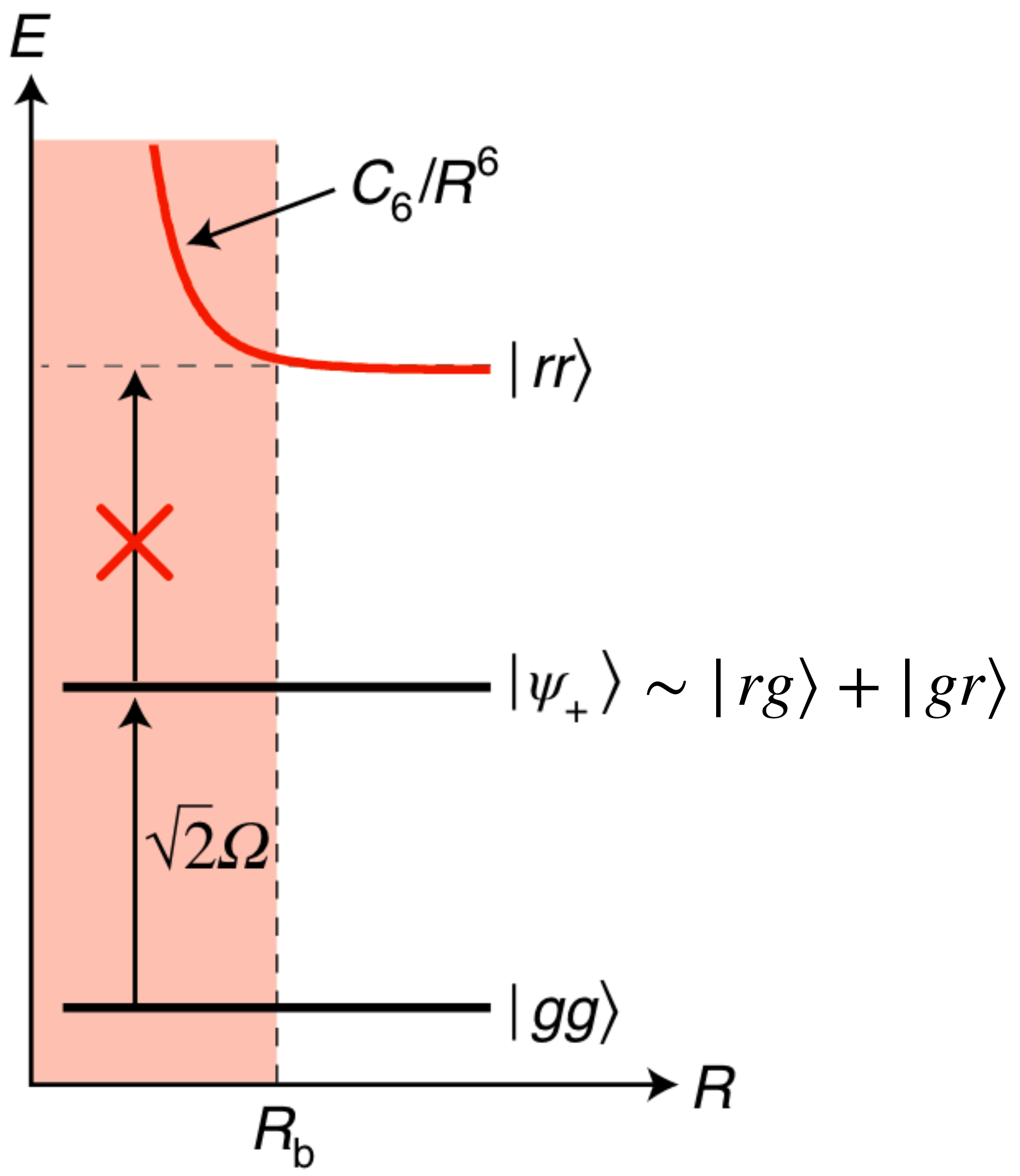


Rydberg level
$|nS\rangle, |nP\rangle$, n > 50

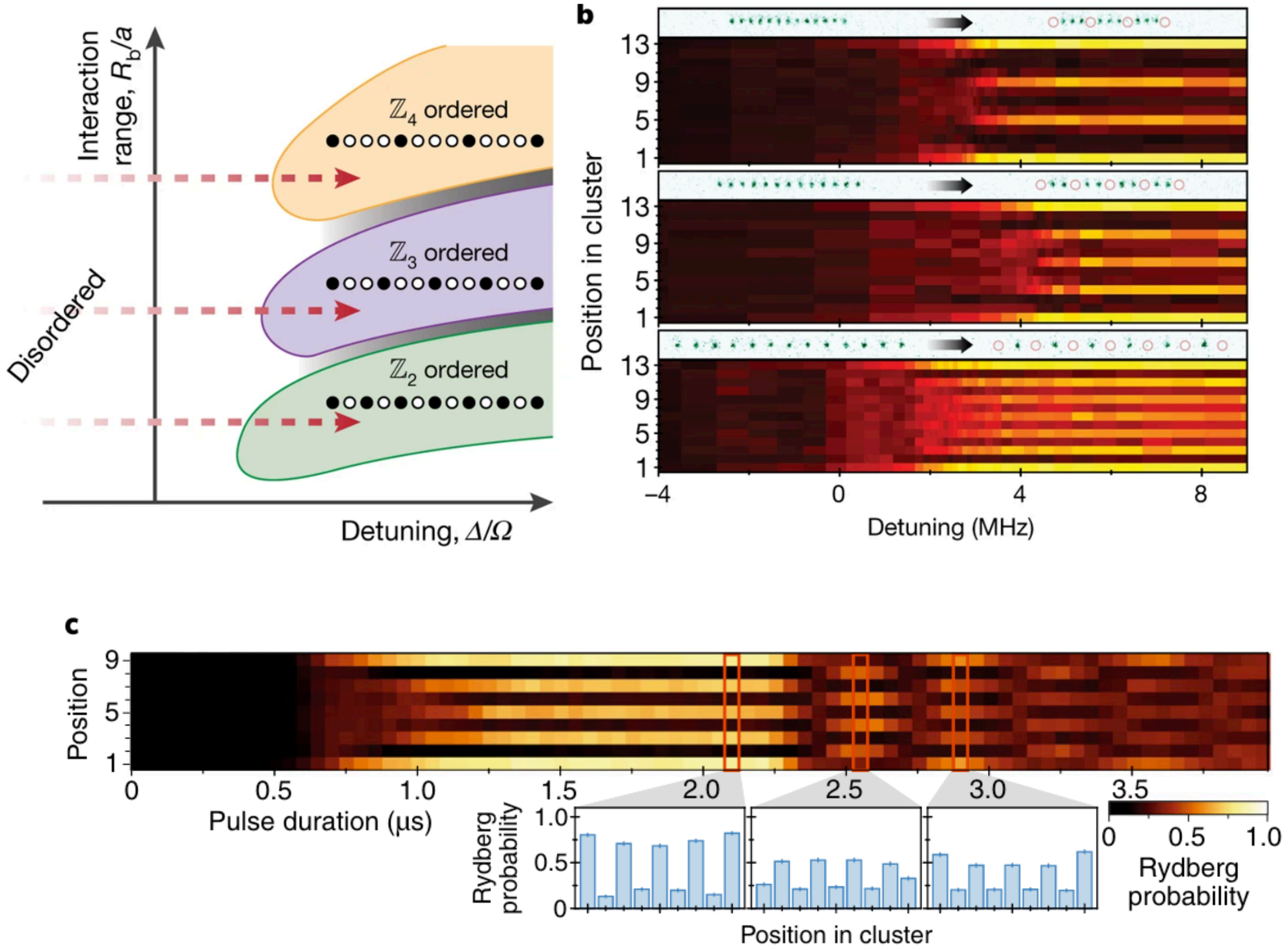
Atomic ground state



Rydberg blockade



Lukin Nature 2017



$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i n_i + \sum_{i<j} V_{ij} n_i n_j, \text{ with } V_{ij} = \frac{C_6}{R_{ij}^6}$$

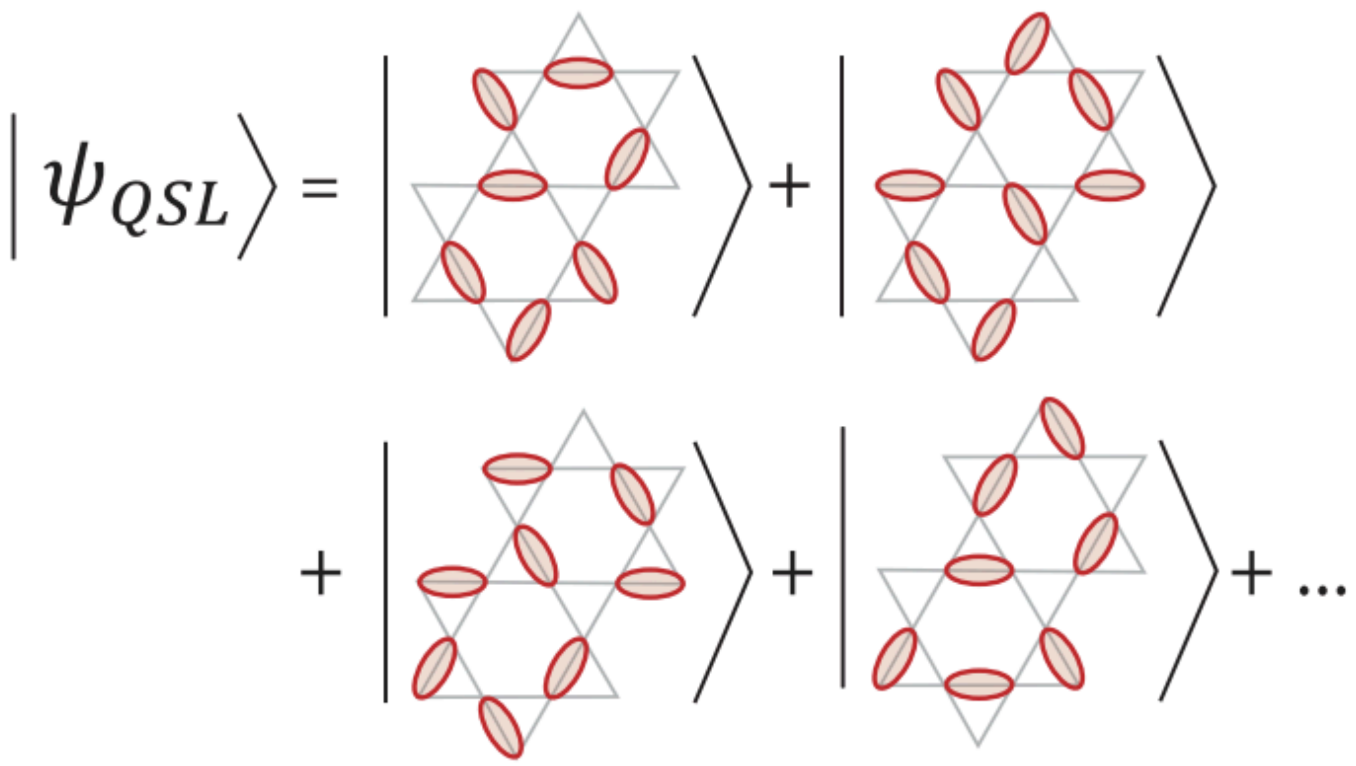
MHz interactions

Topological quantum phases

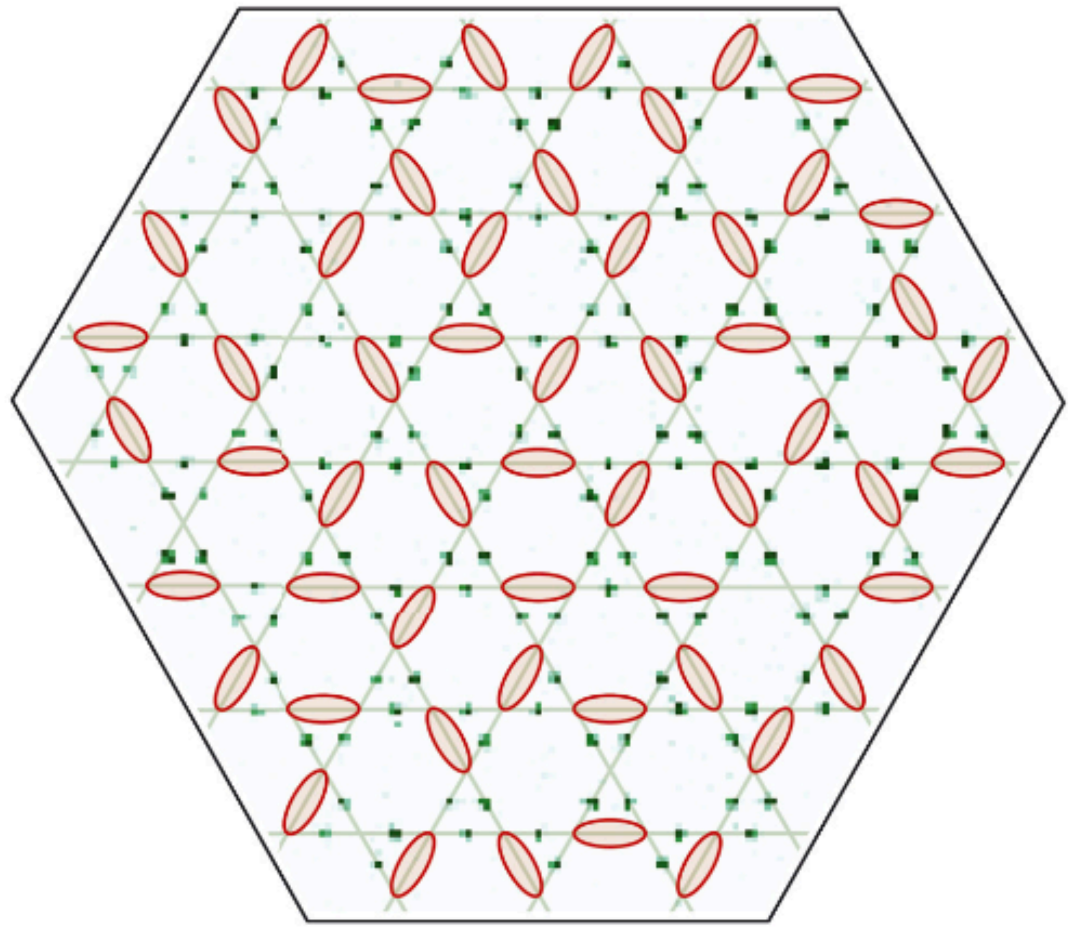
Phases with no local order:
defies Landau paradigm of
phase transitions

Lukin Science 2021

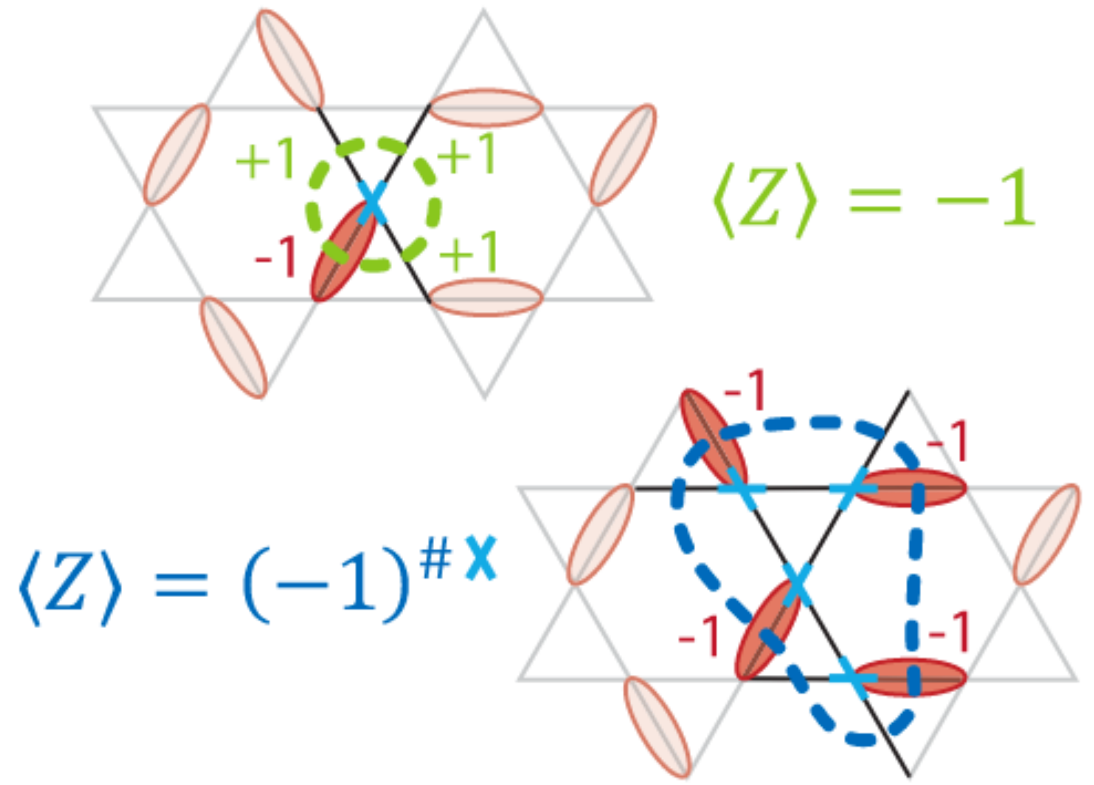
TOPOLOGICAL SPIN LIQUID



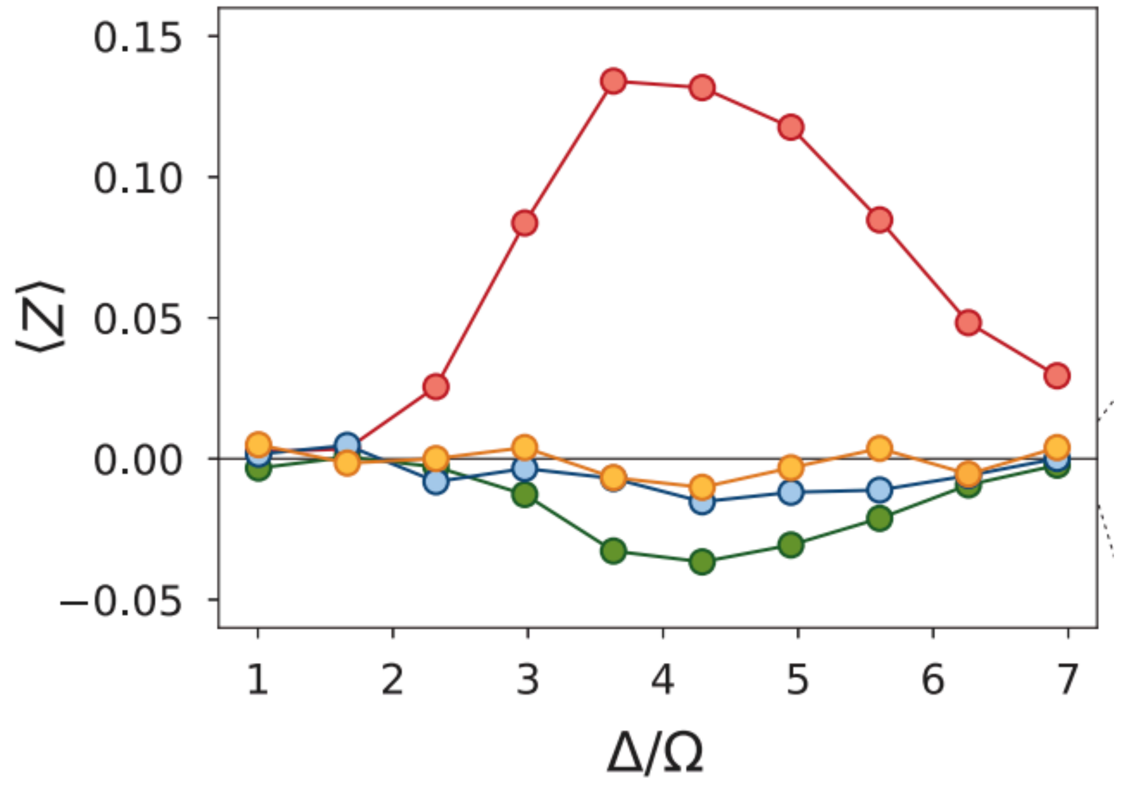
Single shot
measurement



String operator



String operator
measurement

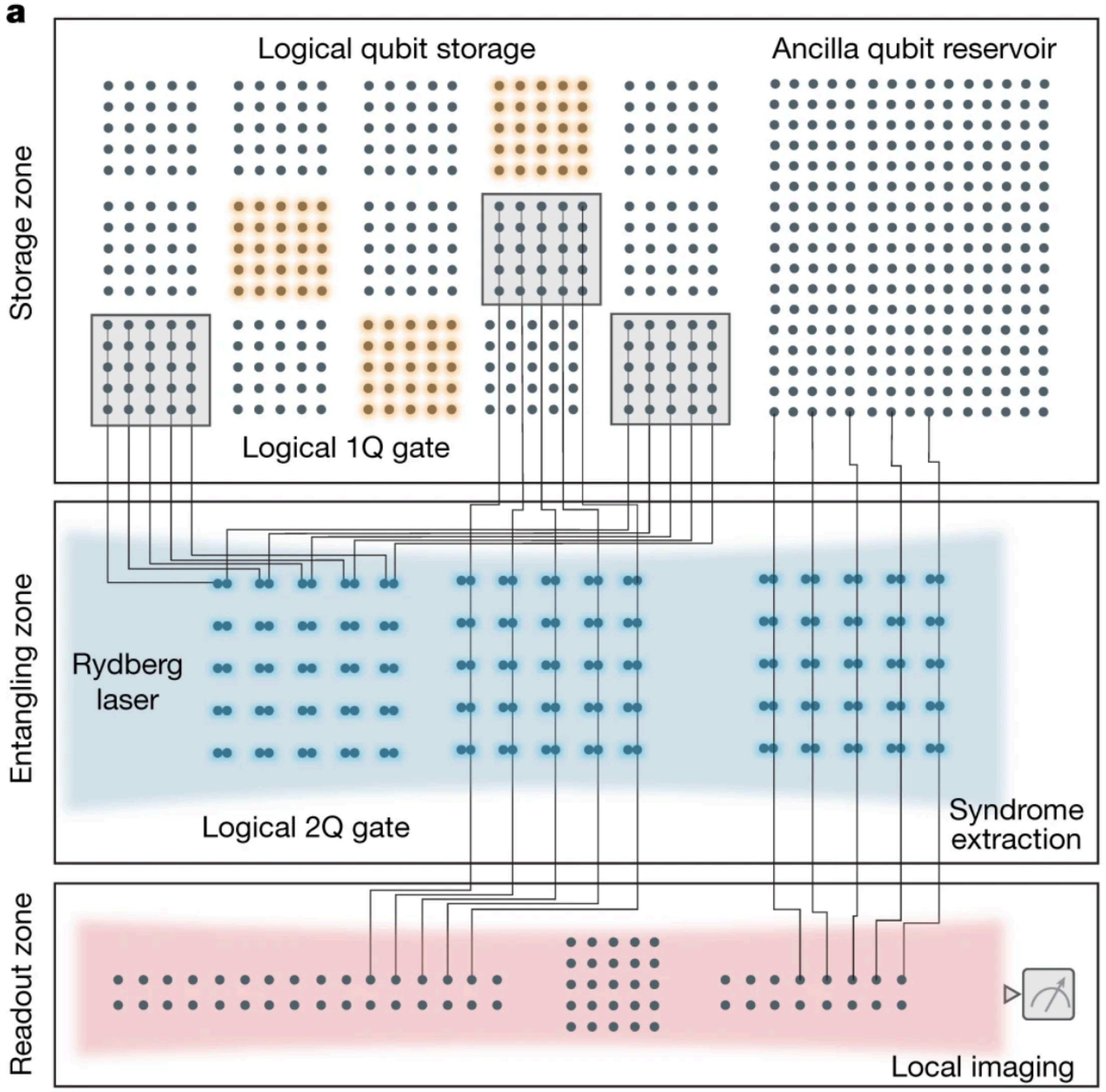


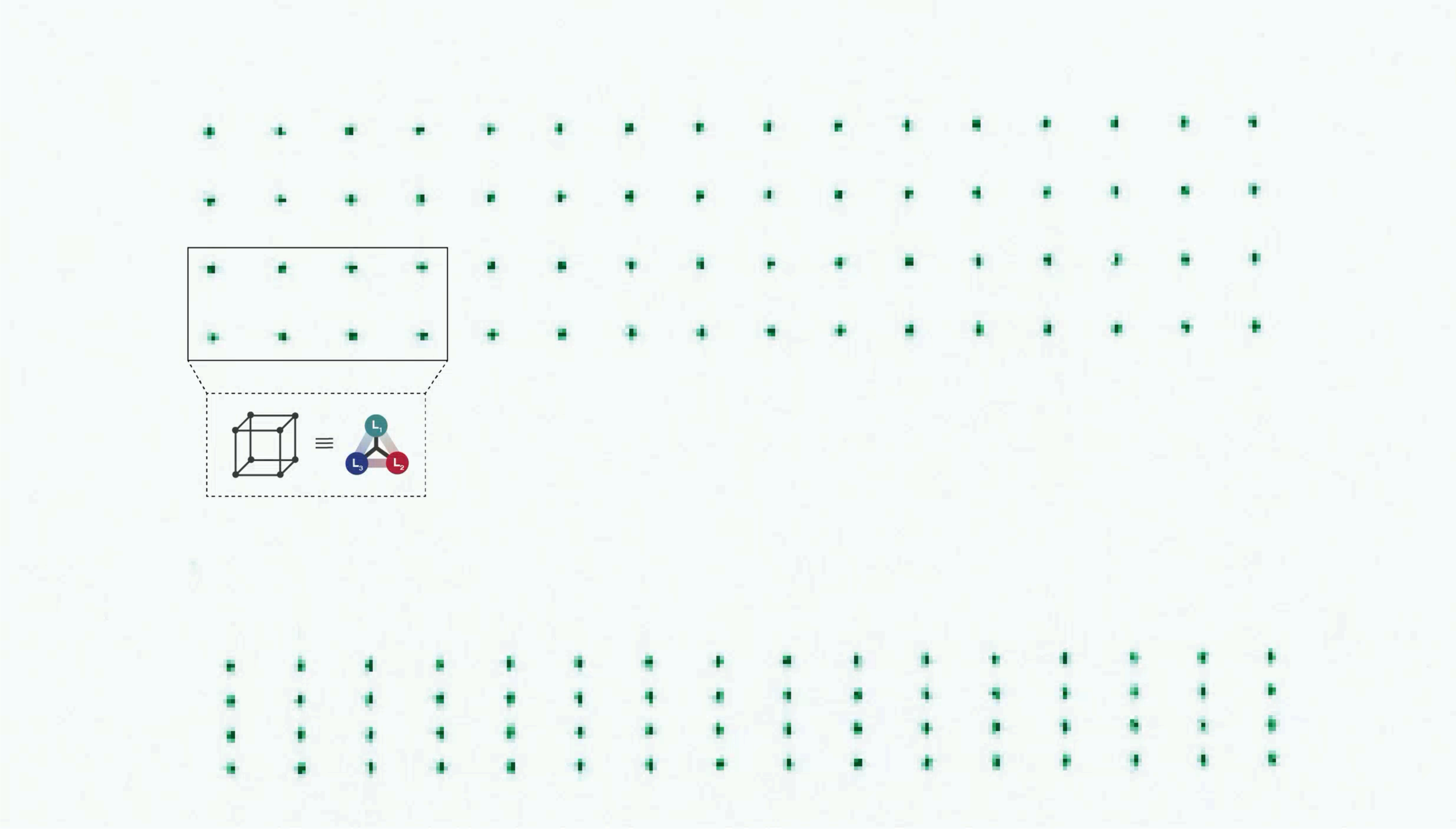
Logical quantum processor

Rydberg blockade

Lukin Nature 2024

→
- Entanglement generation
- Two-qubit gates
- Error correction





Research interests

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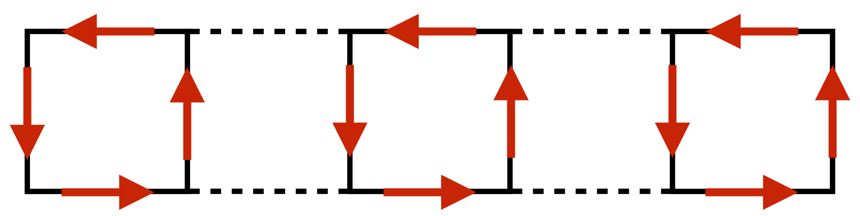
Many-body phases and dynamics

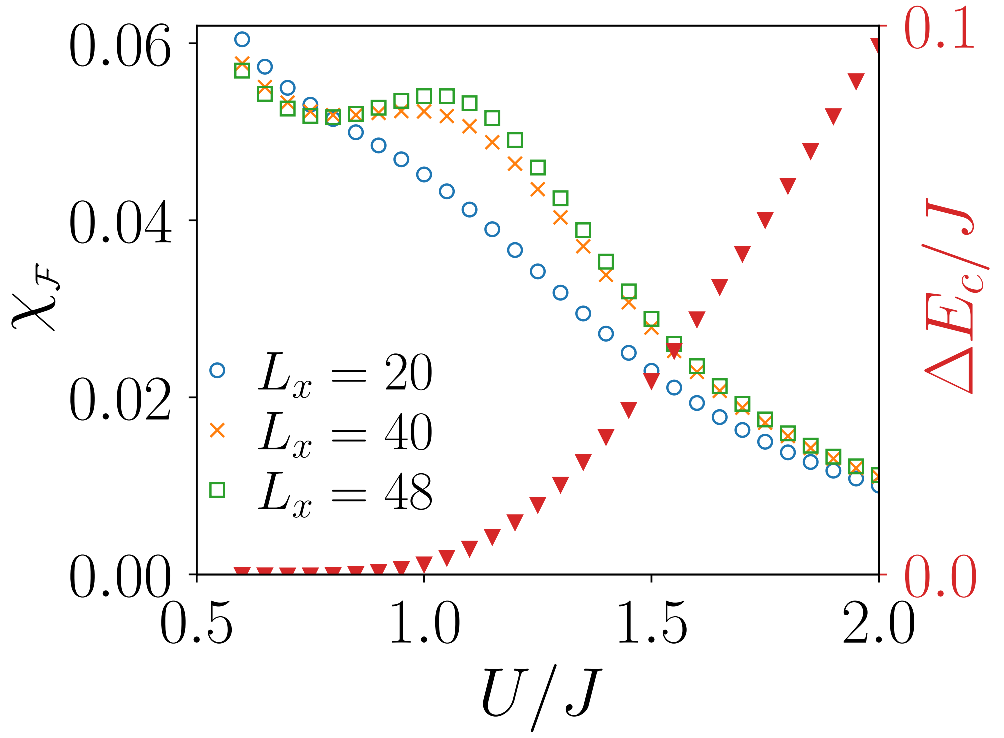
Insulating phase with vortices

Di Liberto and Goldman PRR 2023

Di Liberto et al. Nature Comm. 2020



DMRG numerical simulation



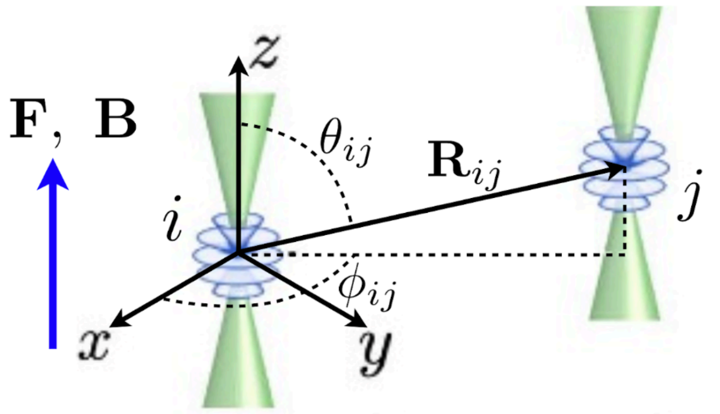
Quantum engineering

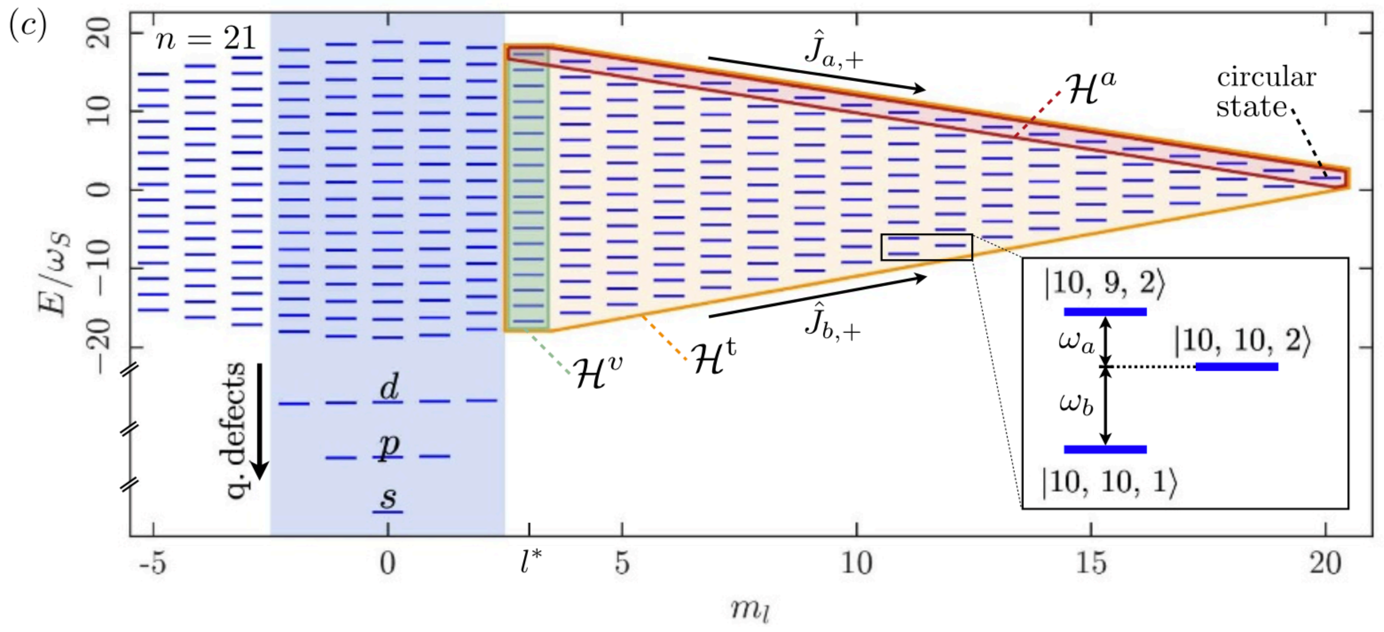
(Large) spin-J models with Rydberg atoms (beyond spin-1/2)

with Zoller's group, Quantum Science and Technologies (2023)

$J \gg 1$

$$\hat{H}^a = \sum_i \left\{ -\Delta_a \hat{J}^{(i)}_{a,z} + \chi \left(\hat{J}^{(i)}_{a,z}\right)^2 + \sum_\kappa \left[\lambda_\kappa \left(\hat{J}^{(i)}_{a,+}\right)^\kappa + \text{H.c.} \right] \right\} + \frac{1}{2} \sum_{i \neq j} V_{ij} \left[\hat{J}^{(i)}_{a,z} \hat{J}^{(j)}_{a,z} - \frac{1}{4} \left(\hat{J}^{(i)}_{a,+} \hat{J}^{(j)}_{a,-} + \text{H.c.} \right) \right].$$



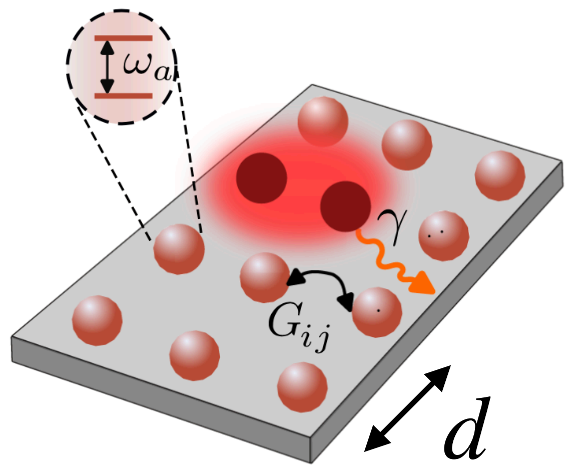


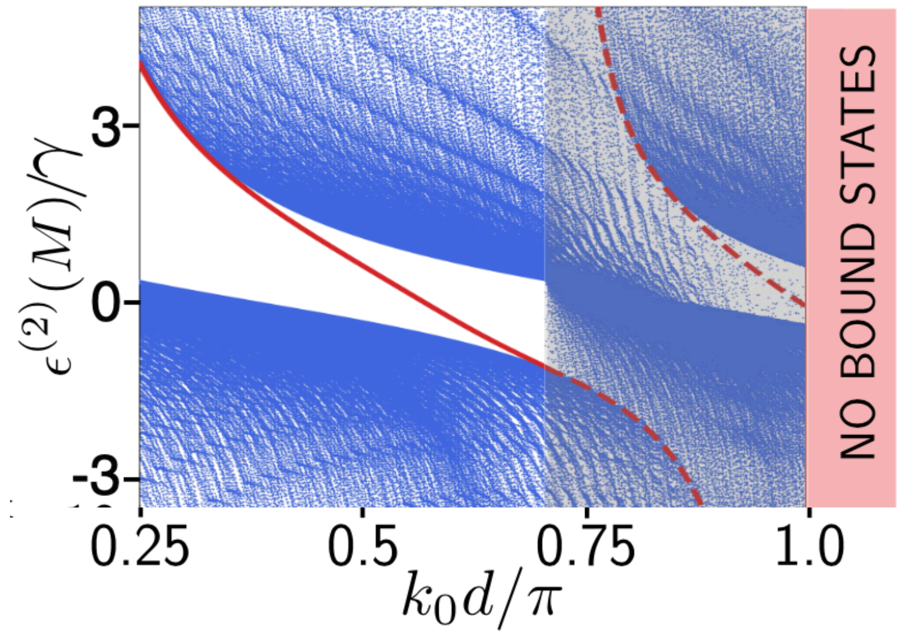
Two-body bound states

Strong light-matter coupling via waveguide

Tecer, **Di Liberto**, Silvi, Montangero, Romanato, Calajò

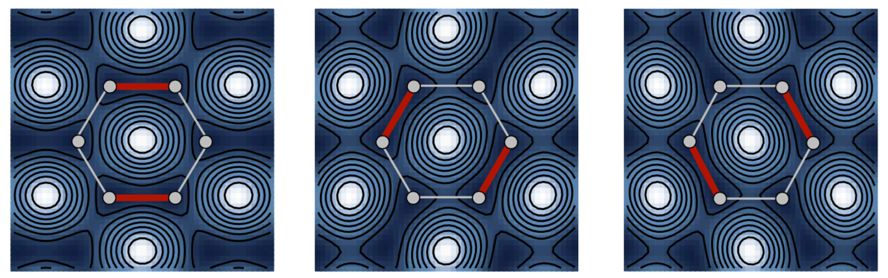
Phys. Rev. Lett. (2024)

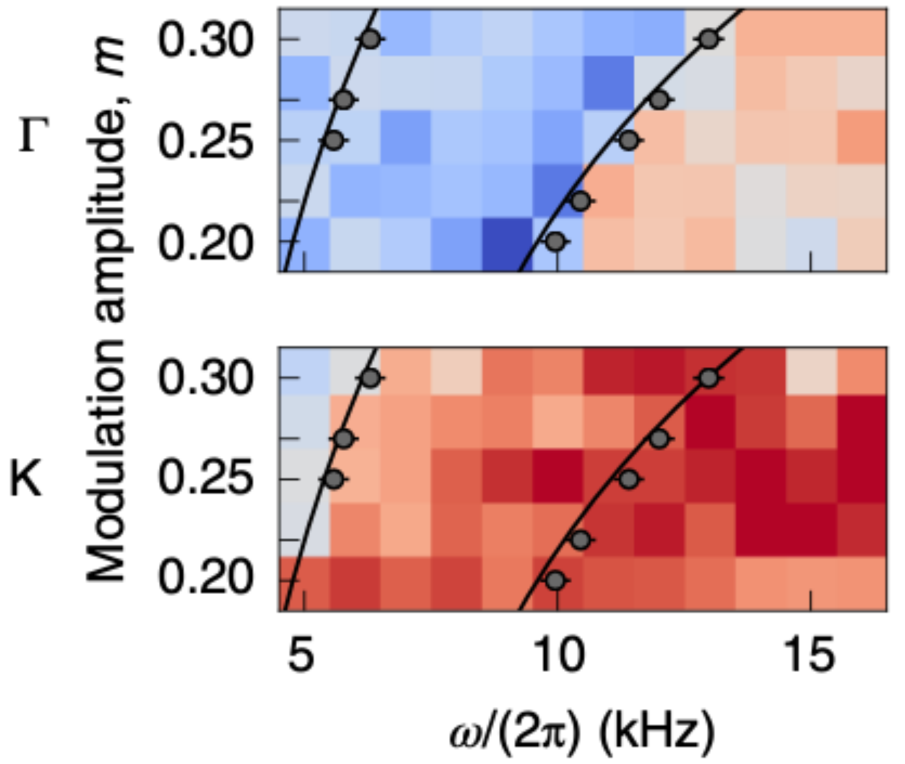


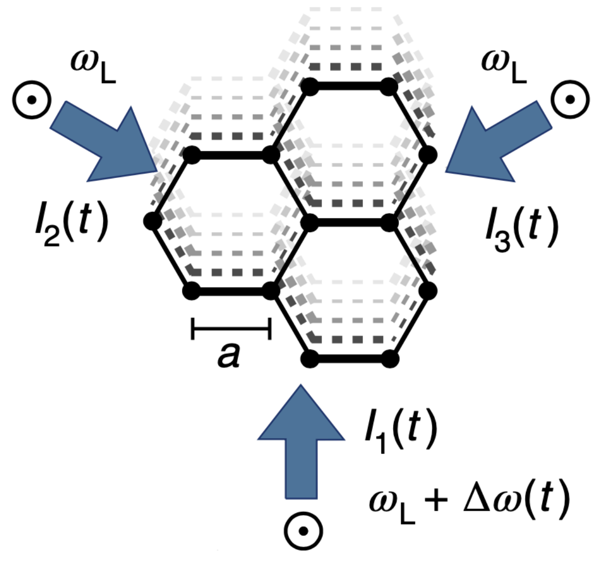


Anomalous Floquet topological insulator

Aidelsburger (**Di Liberto**) Nature Physics 2020





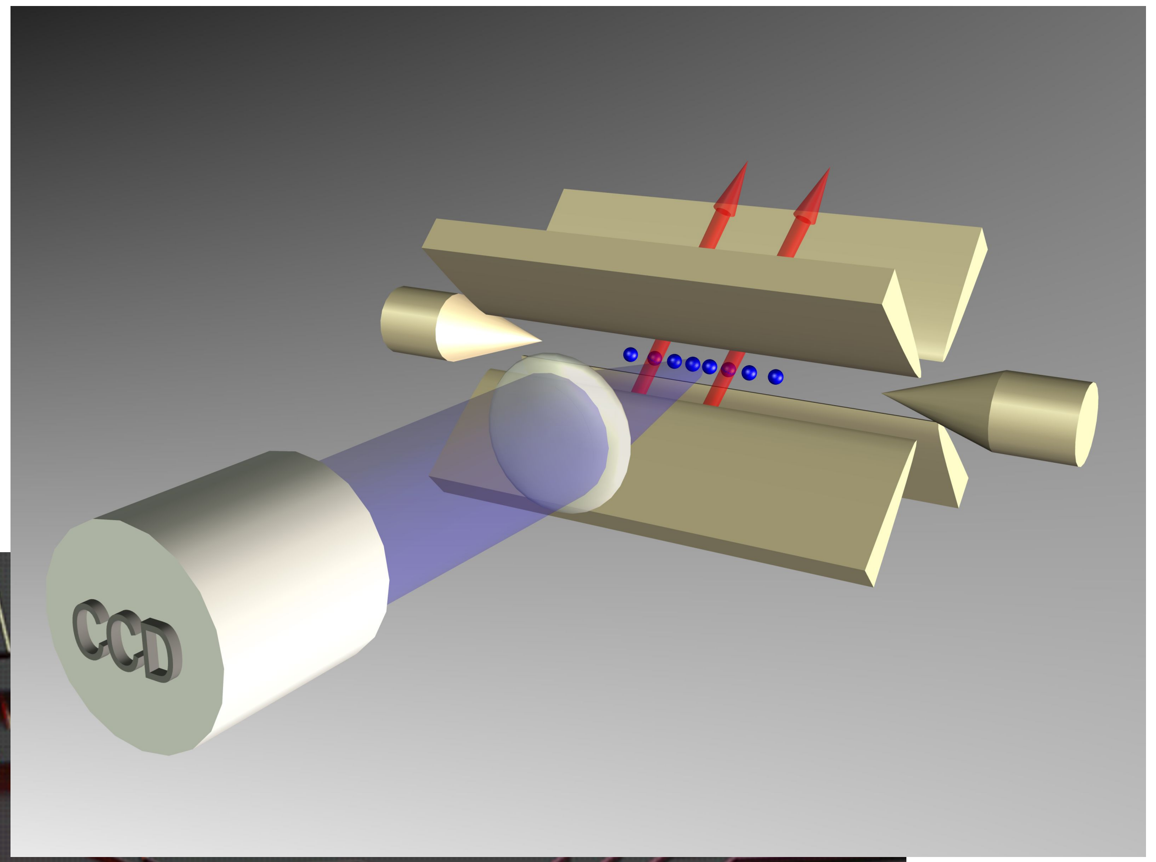


Quantum simulation in Padova

Università degli Studi di Padova

Quantum Computing and Simulation Center

- 7 Faculties
- ~ 10 Postdocs
- ~ 20 PhD students



08
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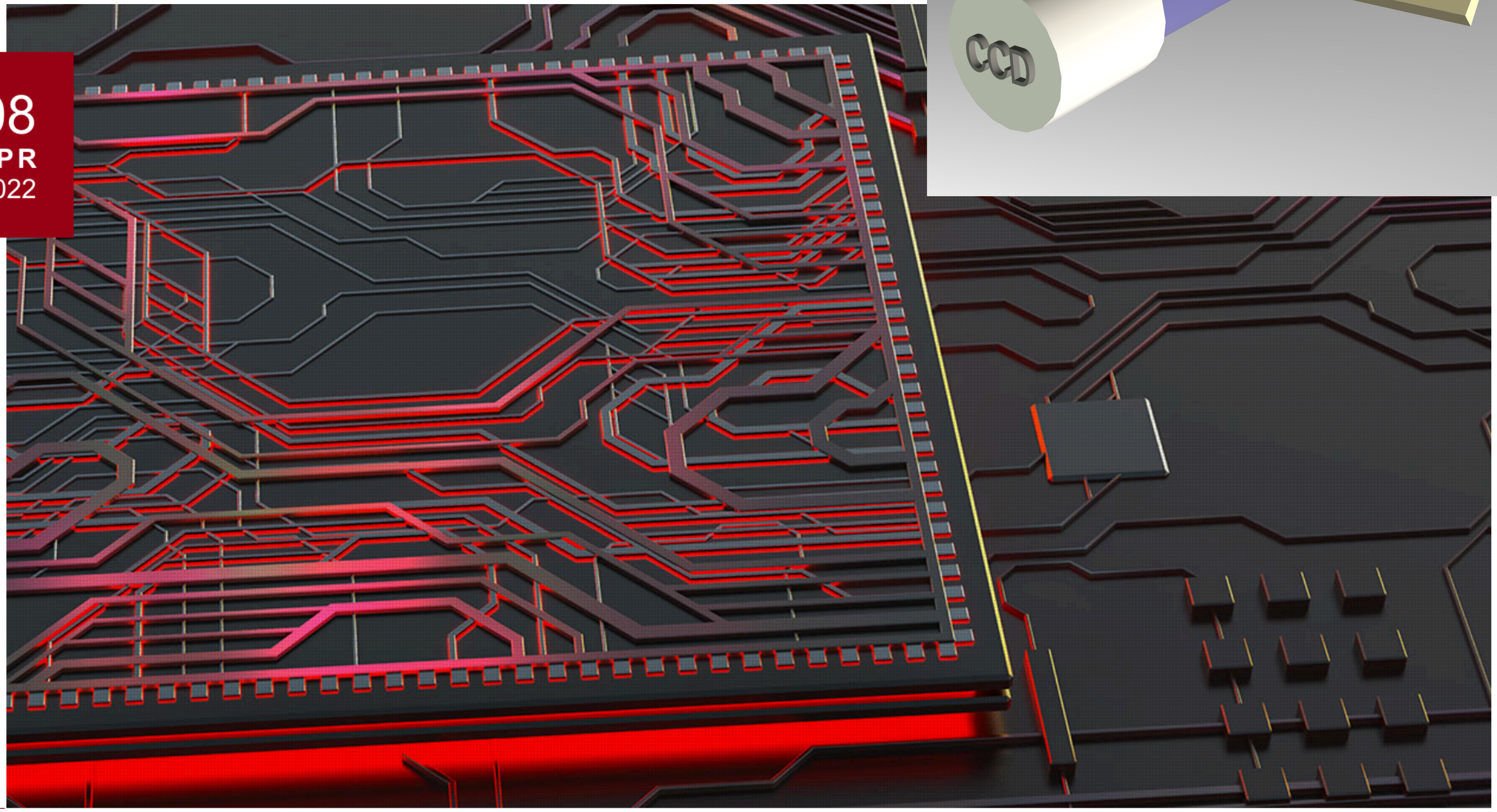
2022

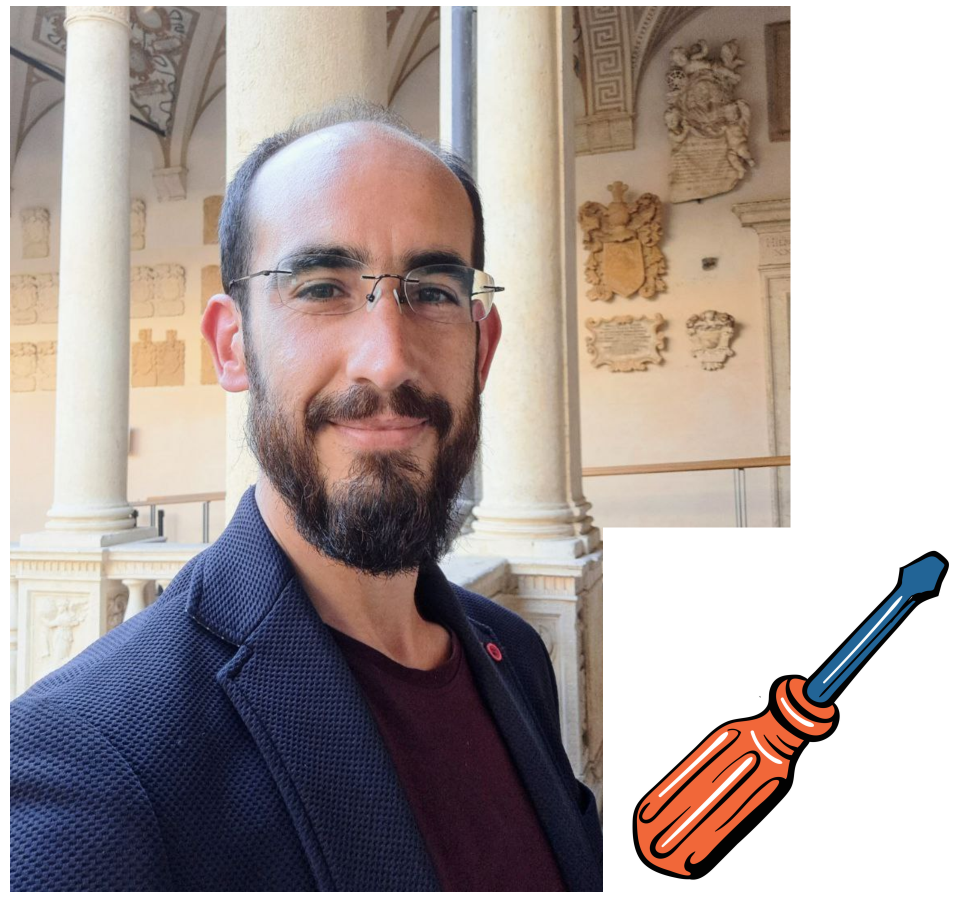
PADOVA E LA NUOVA SFIDA DEL COMPUTER QUANTISTICO

Convegno incentrato sul progetto “Quantum Computing and Simulation Center” (QCSC)

Altro



Simone Montangero



Carmelo Mordini

Acknowledgements

QUANTUM
Information and Matter

T-NiSQ
Tensor Networks in Simulation of Quantum Matter

PADUA TECH

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MIUR

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