Higher dimensional operators at finite temperature affect gravitational wave predictions

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Do we understand all of the relevant theory uncertainties?

FOPT Thermodynamics are hard to compute

Effective potential $V(v)$ determines equation of state:

$$p(T) = p_+ - T\Delta V(v_{\text{min}}) \qquad L^3 V(v) = -\int_{\text{1PI}} \mathcal{D}\phi_4 \; e^{-S_E[\phi_4 + v_4]}$$

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- Dimensional reduction technique is common starting point for sophisticated effective theory / lattice computations

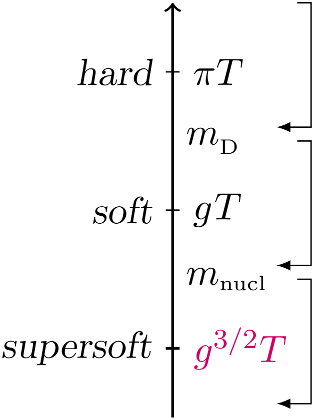
  $\Rightarrow$ **What are the uncertainties from dimensional reduction?**

L^3 = Space-time volume / $v_4 = T^{1/2} v$ / $\Delta V = V(v) - V(0)$

Understanding the basics of dimensional reduction

Abelian Higgs model:

$$\mathcal{L}_E = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + D_\mu\phi_4^\dagger D_\mu\phi_4 + \mu^2\phi_4^\dagger\phi_4 + \lambda(\phi_4^\dagger\phi_4)^2 \quad D_\mu = \partial_\mu - \mathrm{i}\, gB_\mu$$



Full theory w. discrete energies $\omega = 2n\pi T$

1) Effective theory of static zero modes
$(\phi_4 \to T^{\frac{1}{2}}\phi,\ g \to T^{-\frac{1}{2}}g_3,\ \dots)$

Effective potential $V(v)$

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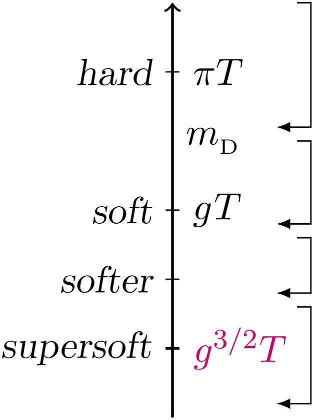


Full theory w. discrete energies $\omega = 2n\pi T$

1) Effective theory of static zero modes
$(\phi_4 \to T^{\frac{1}{2}}\phi,\ g \to T^{-\frac{1}{2}}g_3,\ \dots)$

2) Effecive theory without B_0 zero modes
$(g_3 \to \overline{g}_3,\ \dots)$

Effective potential $V(v)$

High T expansion induces uncertainties

- A priori EFT construction yields infinite tower of operators
- Can truncate tower if all scales small compared to T
  (High T expansion)
- NLO is state of the art:

$$\Rightarrow \mathcal{L}_{\text{soft}} = \frac{1}{4}F_{ij}F_{ij} + D_i\phi^\dagger D_i\phi + \mu_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2 + \frac{1}{2}(\partial_i B_0)^2 + \kappa_3 B_0^4 + h_3(\phi^\dagger\phi)B_0^2 + \mathcal{O}\big(g^6\big)$$

What about higher order contributions?

Correct matching safeguards gauge invariance

- We use field redefinitions to construct **complete** $d = 6$ operator bases for Abelian Higgs model soft and softer EFTs
- We compute complete leading **gauge-invariant** $d = 6$ matching contributions for the first time

$\Rightarrow$ Leading ϕ^6 contributions:

$$\mathcal{L}_{\text{soft}} \supset c_6 \phi^6 \qquad c_6 = \frac{\zeta(3) g^6}{32\pi^4}\left(1 - \frac{31}{30} x_{\text{LO}} + 5 x_{\text{LO}}^2 + \frac{20}{3} x_{\text{LO}}^3\right) + \mathcal{O}(g^8)$$

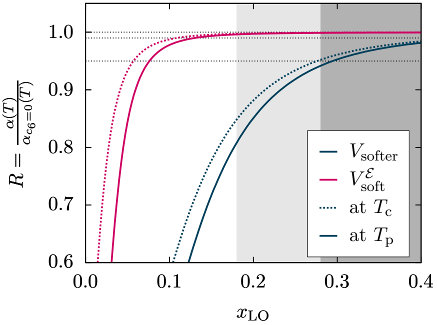
$$\mathcal{L}_{\text{softer}} \supset \bar{c}_6 \phi^6 \qquad \bar{c}_6 = \frac{\sqrt{3}\, g^3}{8\pi}(1 - x_{\text{LO}}) + \mathcal{O}(g^4)$$

$x_{\text{LO}} = \lambda / g^2$

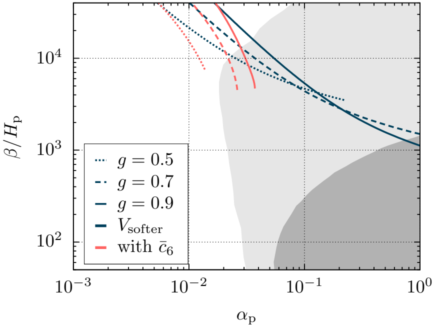
Softer theory is not suitable for strong transitions

$$\Delta V_{\text{softer}} = \overline{g}_3^6 \left(\frac{\overline{y}}{2}\varphi^2 + \frac{\overline{x}}{4}\varphi^4 + \frac{\overline{c}_6}{8}\varphi^6 - \frac{1}{6\pi}\varphi^3 \right)$$

v^6 impact on transition strength



v^6 impact on α/β



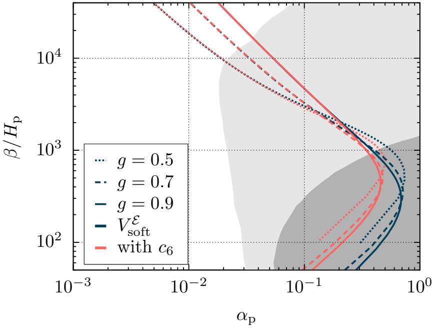
- Large $\overline{c}_6$ correction suggests EFT breakdown
- Transition strength α negative for small $x_{\text{LO}} \Rightarrow$ unphysical regime

$\varphi = v/\overline{g}_3$, $\overline{x} = \overline{\lambda}_3/\overline{g}_3^2$, $\overline{y} = \overline{\mu}^2/\overline{g}_3^4$,

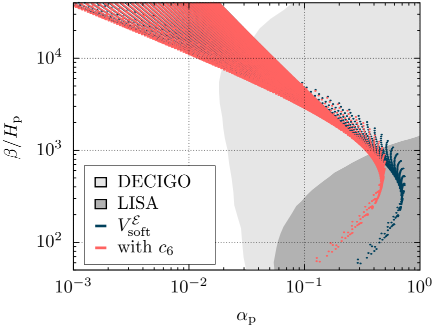
Soft theory is better but not perfect

$$\Delta V^{\mathcal{E}}_{\text{soft}} = \mathcal{E}^2 g_3^6 \left(\frac{y}{2}\varphi^2 + \frac{x}{4}\varphi^4 + \frac{c_6}{8}\varphi^6 - \frac{1}{6\pi}\varphi^3 \right)$$

x scan for fixed g



full x and g parameter scan



- c_6 weakens transition

  $\Rightarrow$ Significant uncertainties in the LISA region

$\varphi = v/g_3$, $\bar{x} = \lambda_3/g_3^2$, $\bar{y} = \mu^2/g_3^4$, $\mathcal{E} = 3/2$

Summary and Outlook

- We constructed complete gauge-invariant $d = 6$ bases operators for Abelian Higgs model soft and softer EFTs
- We examined impact on strong phase transitions + GW production:
  1. Softer EFT becomes unreliable
  2. Higher dimensional operators induce significant corrections
     $\Rightarrow$ Also relevant for lattice computations

Future avenues: Match higher-dimensional operators for SU(N)

4D Lattice simulations?

Thank you for your attention!

Backup slides

$\Rightarrow$ Latent heat determines available energy budget:

$$\alpha = \frac{(T\partial_T - 3)\Delta V}{3\,\partial_T p_+} \qquad \Delta V = V(v_{\text{min}}) - V(0)$$

$\Rightarrow$ Duration of the phase transition:

$$\frac{\beta}{H} = T\partial_T S_{\text{nucl}}[v_b] \qquad S_{\text{nucl}}[v] = \int_{\boldsymbol{x}} \left[\frac{1}{2}(\partial_i v)^2 + V(v)\right]$$