Cosmological selection of a small weak scale from large vacuum energy: a minimal approach

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based on 2407.15935

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The Hierarchy Problem

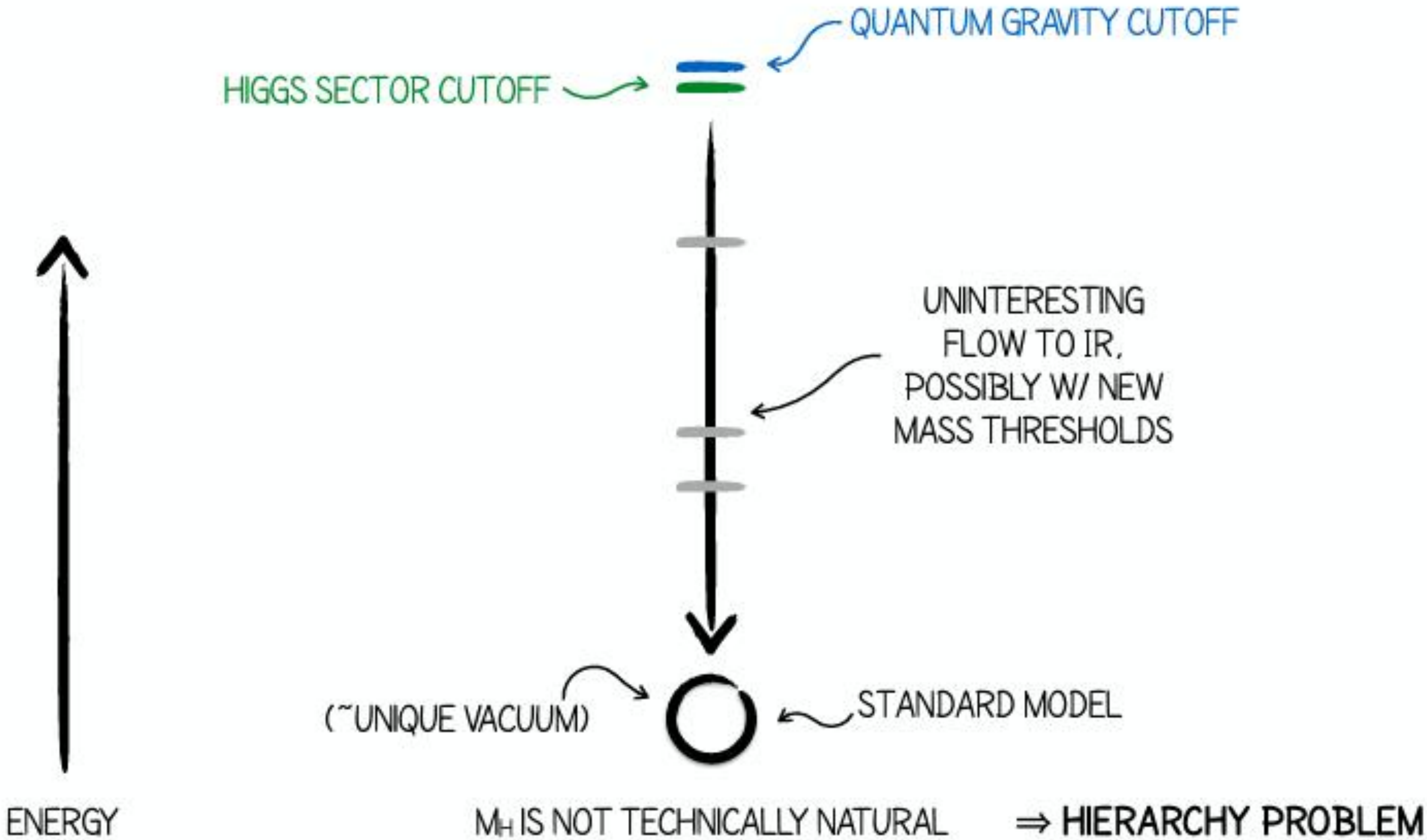
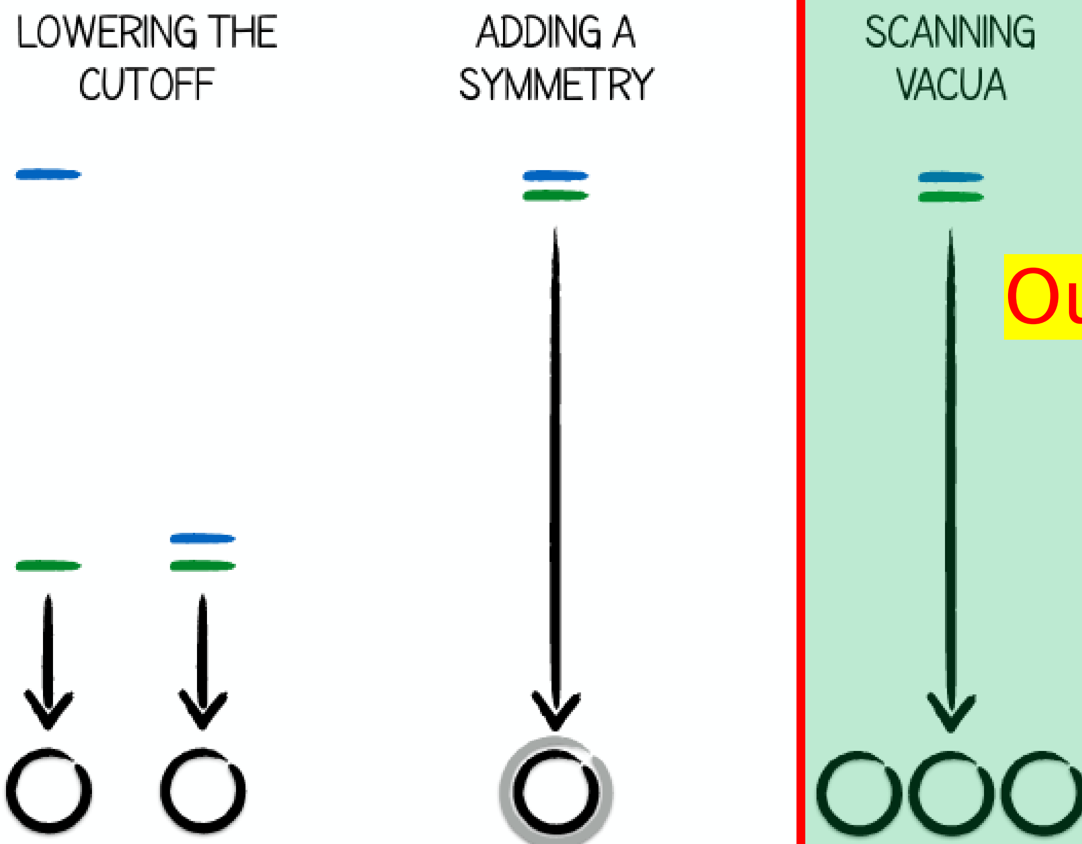


Image Credit: N. Craig, PiTP 2017 Lect.Notes.

The Hierarchy Problem: Solutions

Image Credit: N. Craig, PiTP 2017 Lect.Notes.

Clean Ideas. But so far Null Results at LHC!!!

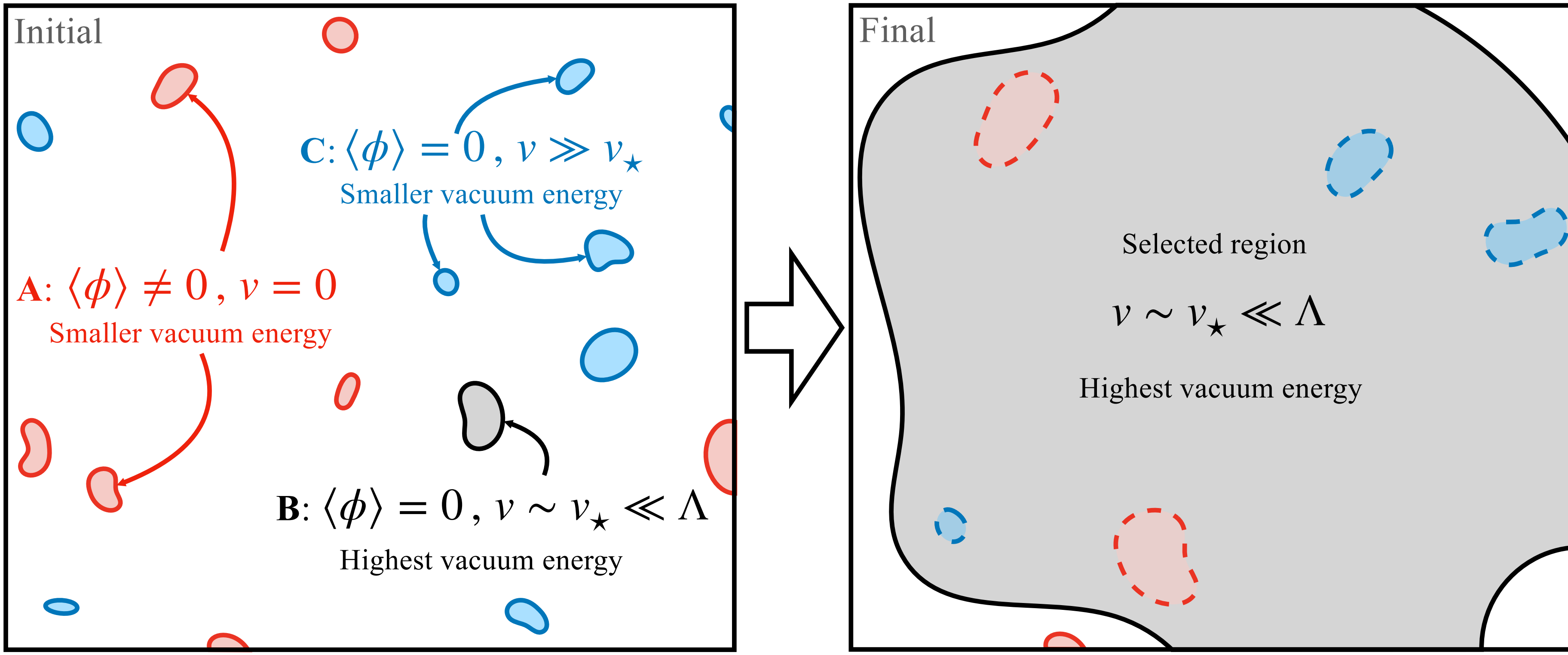


Eg: Technicolor, Extra-dimensions

Eg: SUSY, Composite Higgs

Eg: Relaxion, Sliding Naturalness, Nnaturalness, SOL, Anthropics, etc.

Main Idea

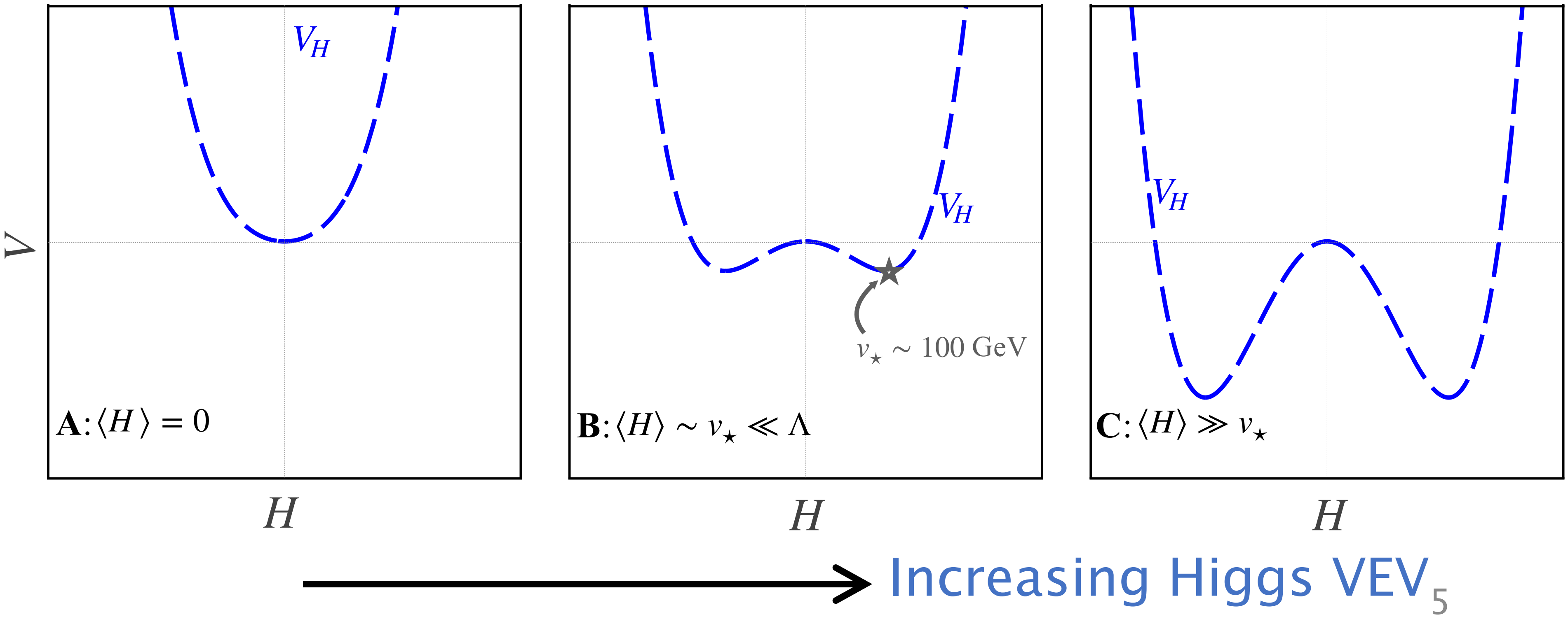


Eternally Inflating Universe

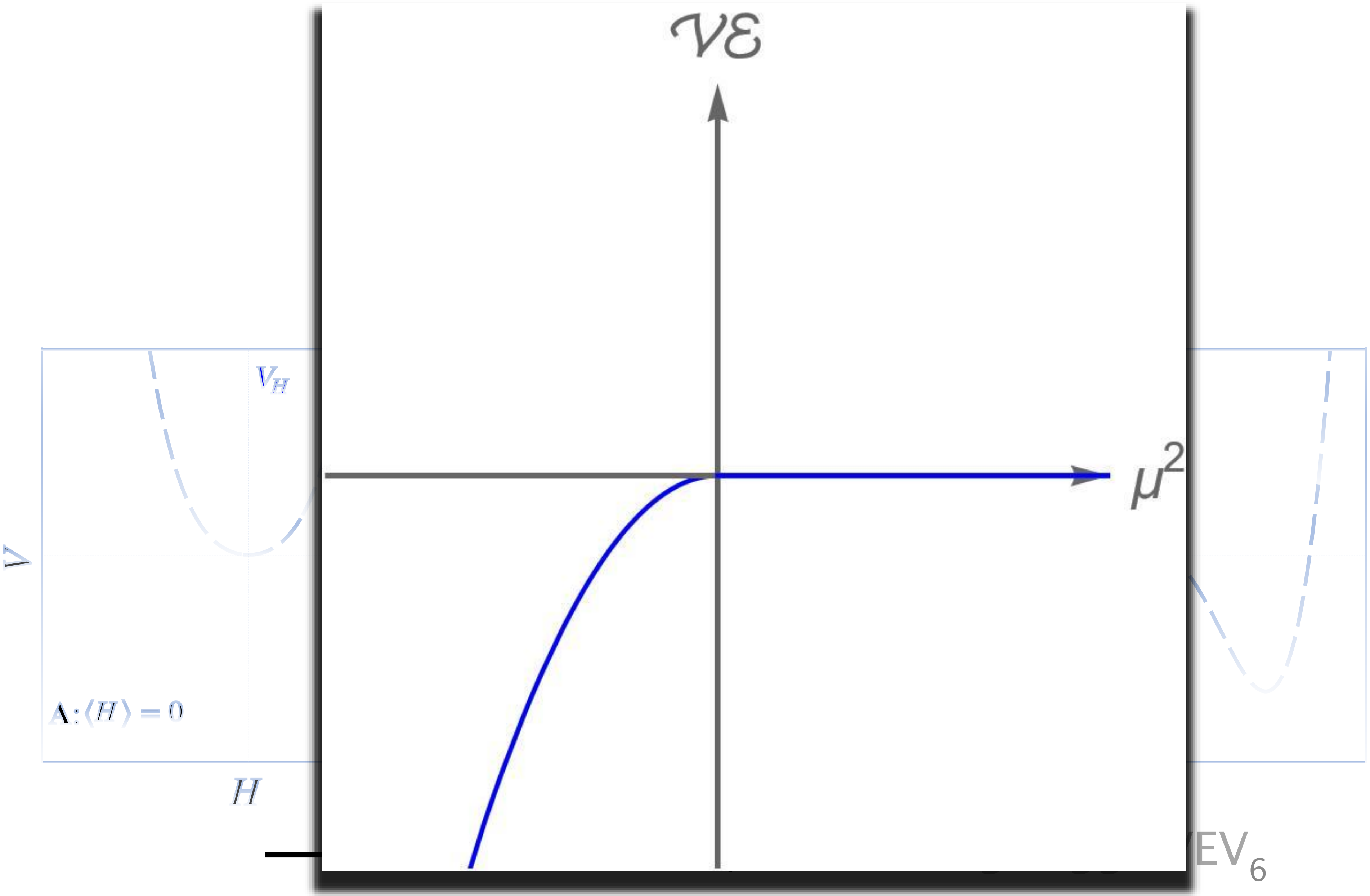
Different causally disconnected patches have different Higgs VEVs (v)

The Standard Model Higgs

$$V_H(H) = \mu^2|H|^2 + \lambda|H|^4$$



The Standard Model Higgs



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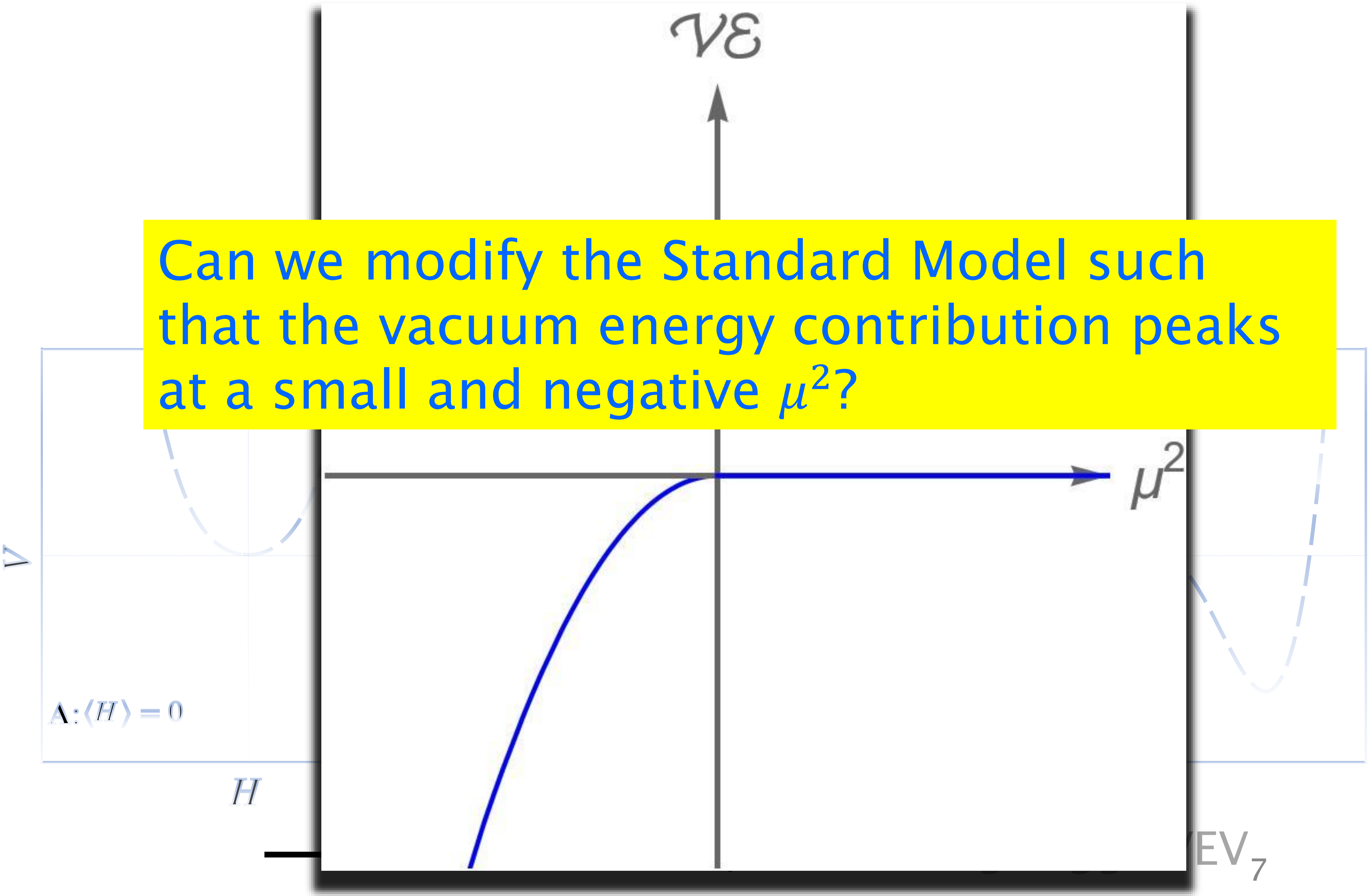


Can we modify the Standard Model such that the vacuum energy contribution peaks at a small and negative μ^2?

Our Model

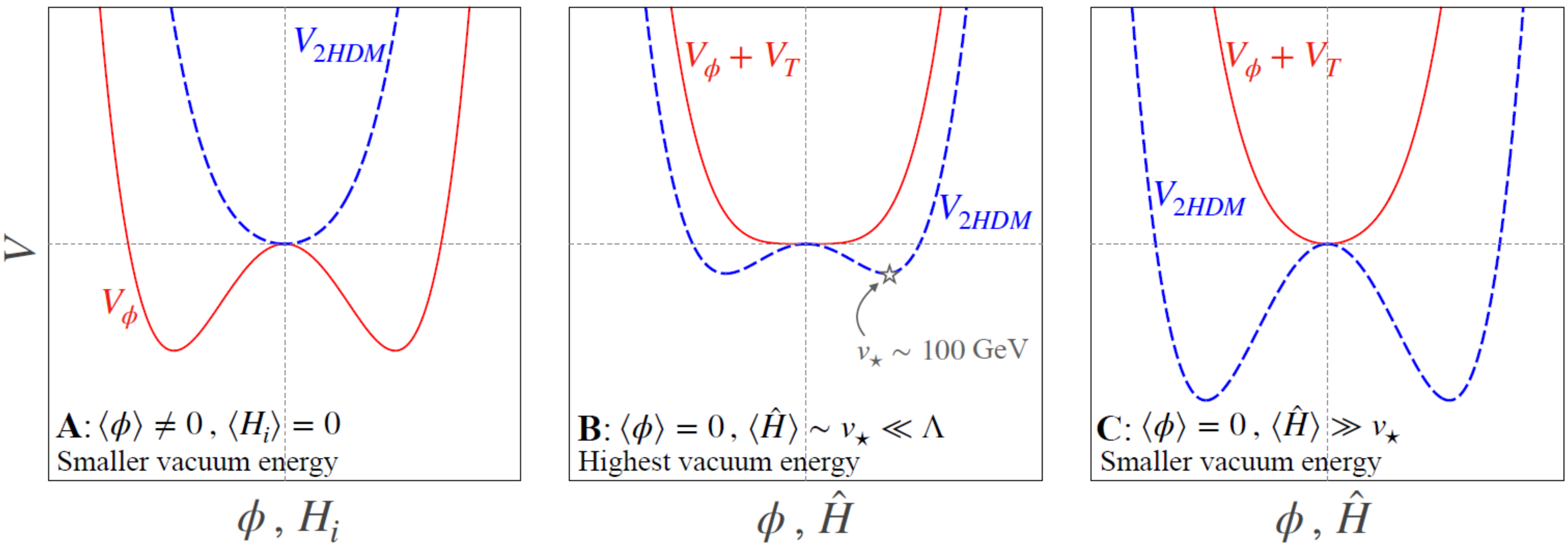
$$V_{\text{2HDM}}(H_1, H_2) = \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2$$
$$+ \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_2^\dagger H_1)(H_1^\dagger H_2)$$
$$+ \frac{1}{2}\left(\lambda_5 (H_1^\dagger H_2)^2 + \lambda_5^* (H_2^\dagger H_1)^2\right)$$

$$V_\phi(\phi) = \mu_\phi^2 f^2 \left(-\frac{1}{2}\left(\frac{\phi}{f}\right)^2 + \lambda_\phi \left(\frac{\phi}{f}\right)^4 + \ldots \right)$$

$$V_T(\phi, H_1, H_2) = \frac{\mu_\phi^2 f^2}{2} \left(\kappa \left(\frac{H_1^\dagger H_2}{\mu_\phi f} \right) + h.c. \right) \left(\frac{\phi}{f}\right)^2$$

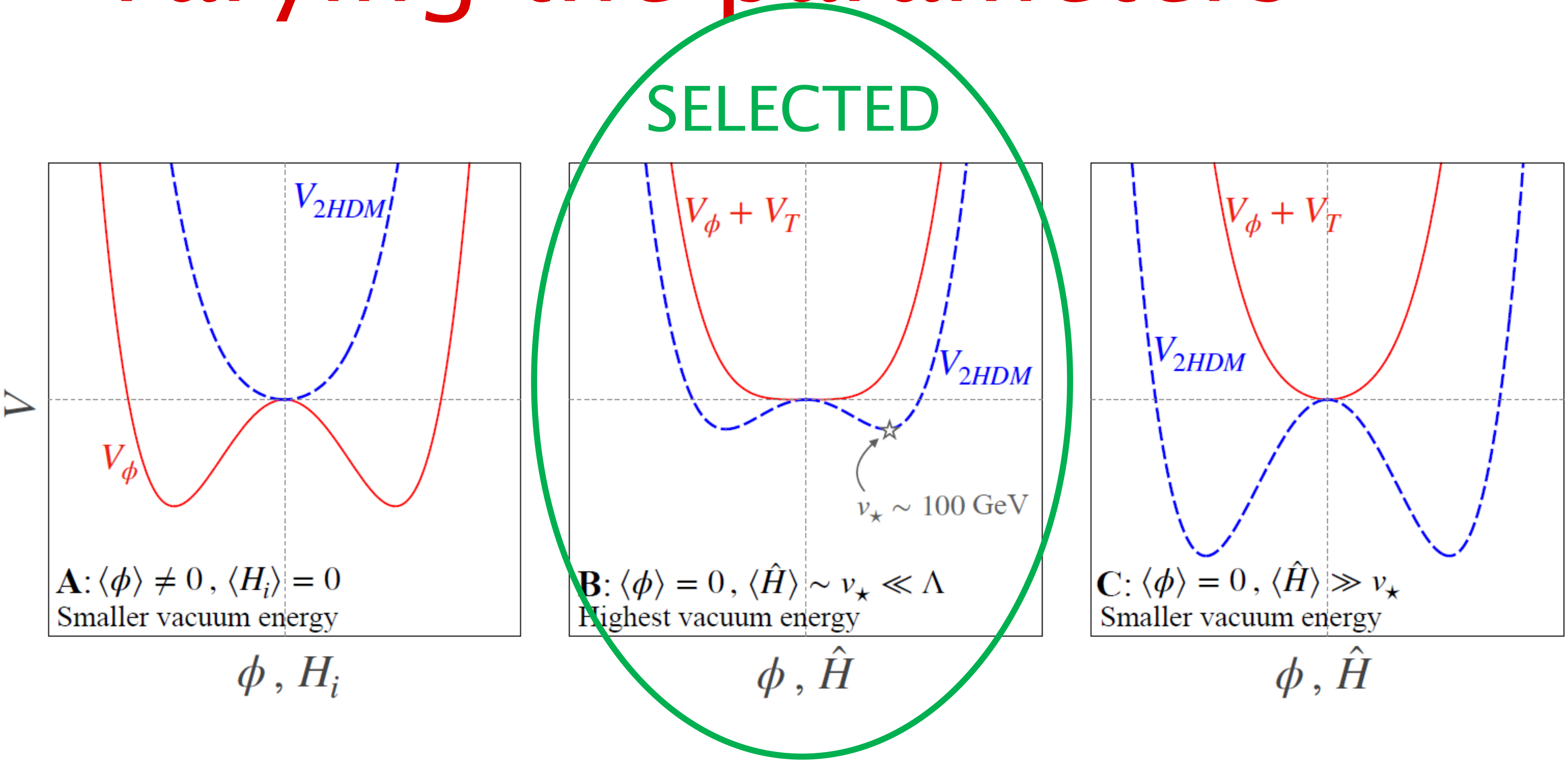
$$V(\phi, H_1, H_2) = V_{\text{2HDM}} + V_\phi + V_T$$

Maximizing Vacuum Energy: Varying the parameters



$\widehat{H}$ is the linear combination of CP even Higgs that gets the vev.

Maximizing Vacuum Energy: Varying the parameters



$\widehat{H}$ is the linear combination of CP even Higgs that gets the vev.

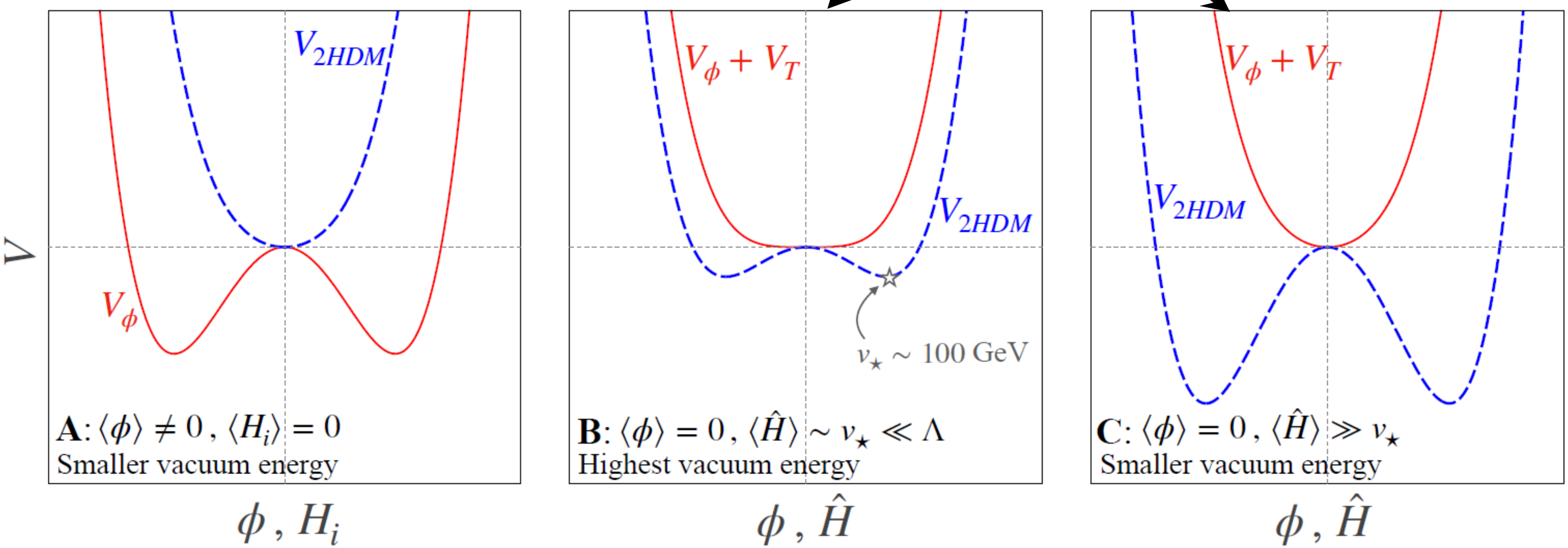
Maximizing Vacuum Energy: Varying the parameters

Class-I	**Class-II**	**Class-III**
EW symmetry preserved: $\langle H_1 \rangle = \langle H_2 \rangle = 0$ $\langle \phi \rangle \neq 0$	EW symmetry broken and $\langle \phi \rangle \neq 0$	EW symmetry broken and $\langle \phi \rangle = 0$

Maximizing Vacuum Energy: Varying the parameters

Class-III

Class-I



What about Class-II minima in which both the Higgs and ϕ get VEVs?

$$\mathcal{VE}^{II} < \mathcal{VE}^{I} < \mathcal{VE}^{III,max}$$

ALWAYS

when

$$\kappa^2 > 4\lambda_\phi(\lambda_{345} + 2\sqrt{\lambda_1\lambda_2})$$

- Class-II minima always have smaller VE than class-I minima

- As we vary the parameters across different universes, the Class-III minima having maximum vacuum energy get selected. In these minima, there is a small but finite EW VEV $\Rightarrow$ Solving the hierarchy problem.

- In the selected universe, Class-II minima do not coexist with the Class-III minima. No possibility of tunneling in the selected universe.

Maximizing Vacuum Energy: Varying μ_1^2 and μ_2^2

Desired class of minima is selected if the quartics satisfy the following conditions:

Potential bounded from below: $\lambda_3 + \lambda_4 - |\lambda_5| + 2\sqrt{\lambda_1\lambda_2} \geq 0$

Class-III minima exist: $\lambda_4 - |\lambda_5| < 0$

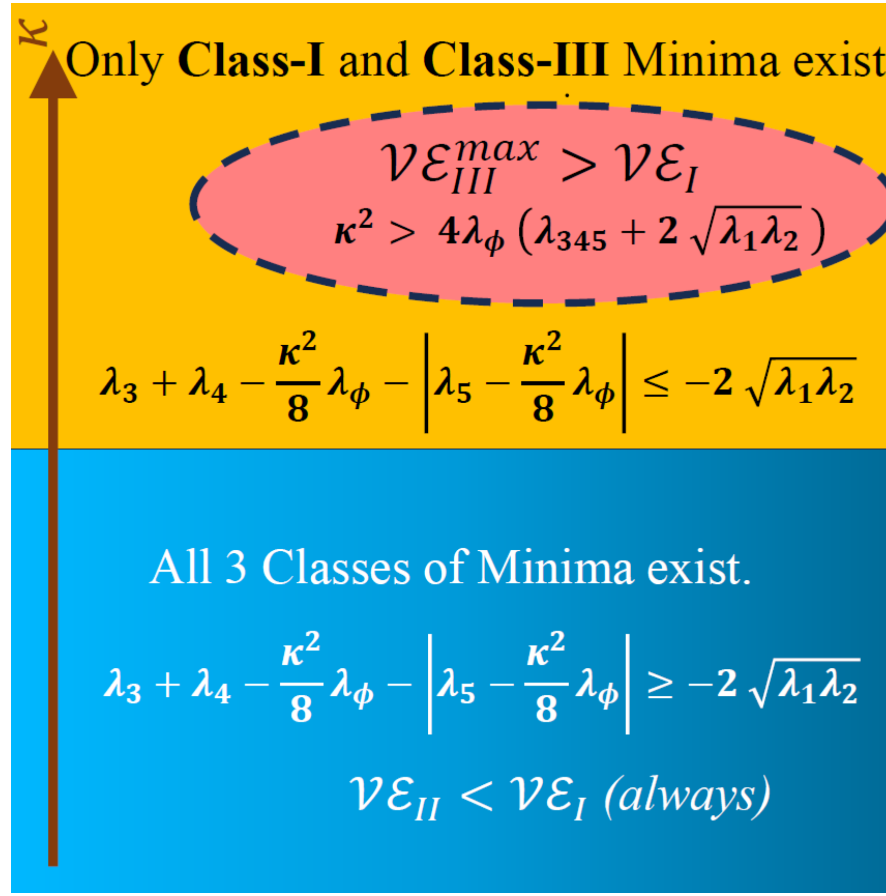
Class-II minima do not co-exist with class-III: $\lambda_3 + \lambda_4 - \dfrac{\kappa^2}{8\lambda_\phi} - \left|\lambda_5 - \dfrac{\kappa^2}{8\lambda_\phi}\right| \leq -2\sqrt{\lambda_1\lambda_2}$

Vacuum energy of class-III > class-I: $\kappa^2 > 4\lambda_\phi(\lambda_{345} + 2\sqrt{\lambda_1\lambda_2})$

where $\lambda_{345} = \lambda_3 + \lambda_4 - |\lambda_5|$

Maximizing Vacuum Energy: Varying the Quartics

By varying the quartics, Class-III minima is always "SELECTED" during inflation.



Thus, ALL the previous conditions are automatically satisfied by requiring the maximal Vacuum energy!

$$\boxed{v_\star^2 = \frac{\mu_\phi f}{\kappa s_{\beta_\star} c_{\beta_\star}} \quad \& \quad \tan^2\beta_\star = \sqrt{\frac{\lambda_1}{\lambda_2}}}$$

Maximizing Vacuum Energy

$$v_\star^2 = \frac{\mu_\phi f}{\kappa s_{\beta\star} c_{\beta\star}} \quad \& \quad \tan^2\beta_\star = \sqrt{\frac{\lambda_1}{\lambda_2}}$$

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Maximizing Vacuum Energy

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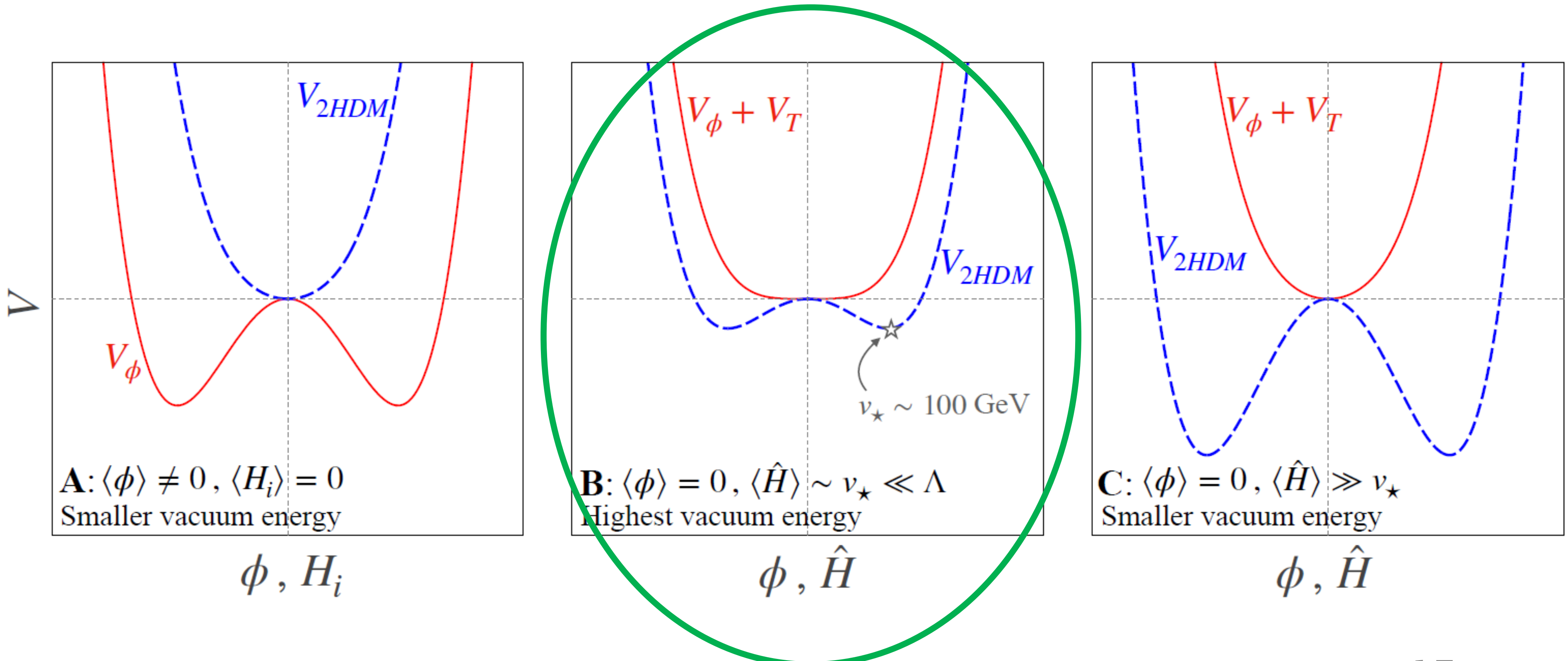

The Cosmological Set-up

$$\mathcal{VE}^{(ijk)} = \mathcal{VE}_{\mathcal{H}}(\{\alpha_H^{(i)}\}, P(\phi, H_1, H_2)) + \mathcal{VE}_{\chi}(\{\beta_\chi^{(j)}\}, P(\chi))) + (\Lambda_{cc}^{(k)})^4$$

Higgs sector

Inflaton sector

Cosmological Constant

$$\{\alpha_H^{(i)}\} = \{\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \kappa, |\mu_\phi^2|, \lambda_\phi\}$$

Inflationary Hubble

For $P(\phi, H_1, H_2)$ to sharply peak at the classical minima: $\frac{H_I^4}{v_\star^4} \ll 1, \quad \frac{H_I^4}{\mu_\phi^2 f^2} \ll 1, \quad \frac{f^2}{M_{pl}^2} \ll 1$

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Inflationary dynamics would result in the multiverse being dominated by the vacuum state where each of the above terms—and in particular the Higgs contribution is maximized.

$$\Lambda_{\text{cutoff}} \sim \sqrt{H_I M_{pl}} \sim 10^{10}\ \text{GeV} \sqrt{H_I/v_\star}$$

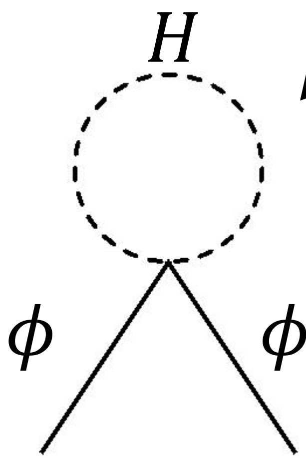
For $P(\phi, H_1, H_2)$ to sharply peak at the classical minima:

$$\frac{H_I^4}{v_\star^4} \ll 1, \qquad \frac{H_I^4}{\mu_\phi^2 f^2} \ll 1, \qquad \frac{f^2}{M_{pl}^2} \ll 1$$

An Aside

Problem with 1 Higgs and 1 ϕ :

$$V_T(\phi, H) = \frac{\mu_\phi^2 f^2}{2} \kappa \left(\frac{|H|^2}{\mu_\phi f} \right) \left(\frac{\phi}{f} \right)^2$$



$$V_{1-\text{loop}} \sim \frac{\kappa \mu_\phi}{f} \frac{\Lambda^2}{16\pi^2} \phi^2$$

Spoils the Triggering mechanism!

Solution: Add another Higgs doublet.

Our Model

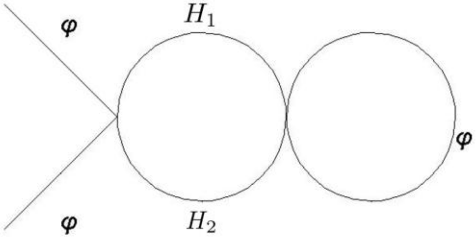
$$V_{\rm 2HDM}(H_1,H_2) = \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_2^\dagger H_1)(H_1^\dagger H_2) + \frac{1}{2}\left(\lambda_5 (H_1^\dagger H_2)^2 + \lambda_5^* (H_2^\dagger H_1)^2\right)$$

$$V_\phi(\phi) = \mu_\phi^2 f^2 \left(-\frac{1}{2}\left(\frac{\phi}{f}\right)^2 + \lambda_\phi \left(\frac{\phi}{f}\right)^4 + \dots \right)$$

$$V_T(\phi, H_1, H_2) = \frac{\mu_\phi^2 f^2}{2}\left(\kappa\left(\frac{H_1^\dagger H_2}{\mu_\phi f}\right) + h.c.\right)\left(\frac{\phi}{f}\right)^2$$

$$\text{Approximate } \mathbb{Z}_2 : H_1 \to -H_1 \Longrightarrow \Delta V_\phi^{2-loop} \sim \kappa^2 \frac{\mu_\phi^2}{f^2}\frac{\mu_\phi^2}{(16\pi^2)^2}\phi^2$$

Effective Triggering possible now!



Predictions: 2HDM Pheno

$$\tan^2\beta_* = \sqrt{\frac{\lambda_1}{\lambda_2}}$$

125 GeV

$$\cos\alpha = \sqrt{\frac{m_h^2 - m_H^2\tan^2\beta}{(m_h^2 - m_H^2)(1+\tan^2\beta)}}$$

Decays of CP-even Higgs bosons

Can be directly measured at the LHC

Charged Higgs/Pseudoscalar decays

Requiring maximal vacuum energy gives **a precise, falsifiable prediction.**

Predictions: 2HDM Pheno

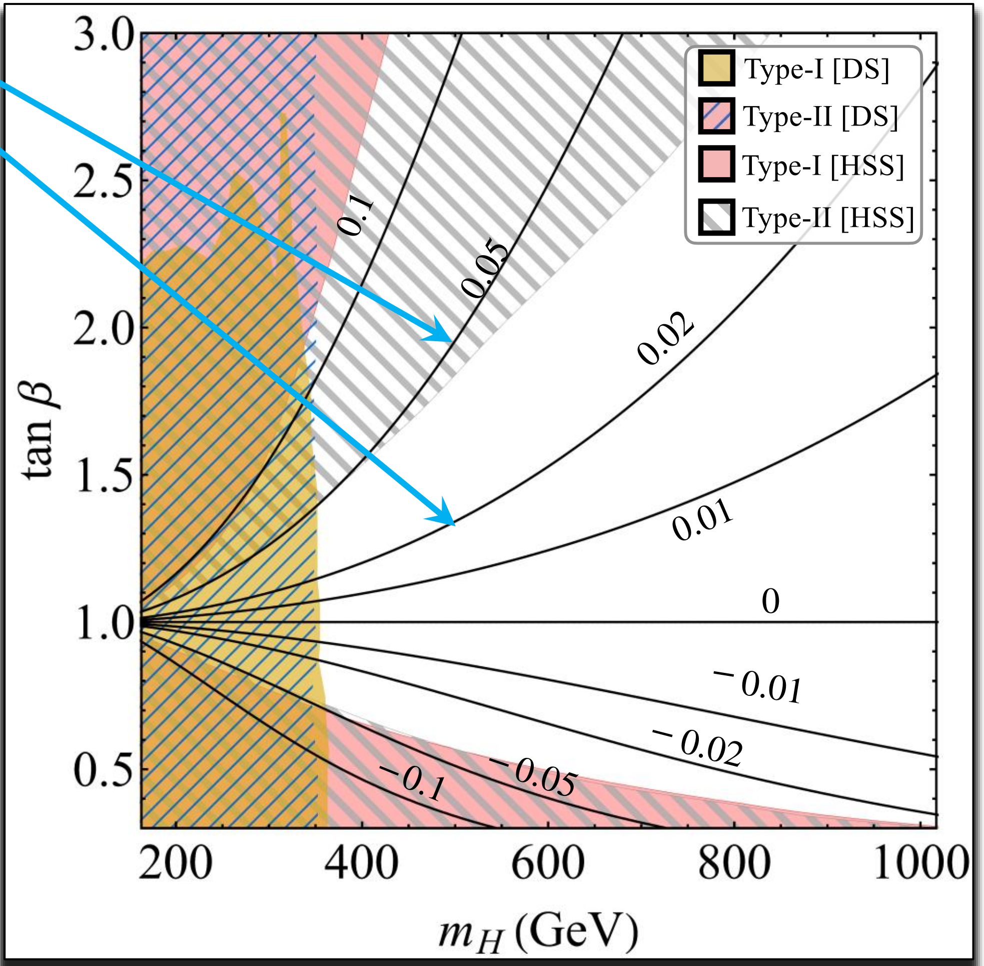
Precise, falsifiable prediction: $\cos\alpha = \sqrt{\dfrac{m_h^2 - m_H^2 \tan^2\beta}{(m_h^2 - m_H^2)(1+\tan^2\beta)}}$

Contours of $\cos(\alpha - \beta)$

Deviations in couplings of the lighter Higgs from SM values. (Higgs Signal Strength)

Direct Search bounds: $H \to \gamma\gamma$
$H/A \to \tau\tau$

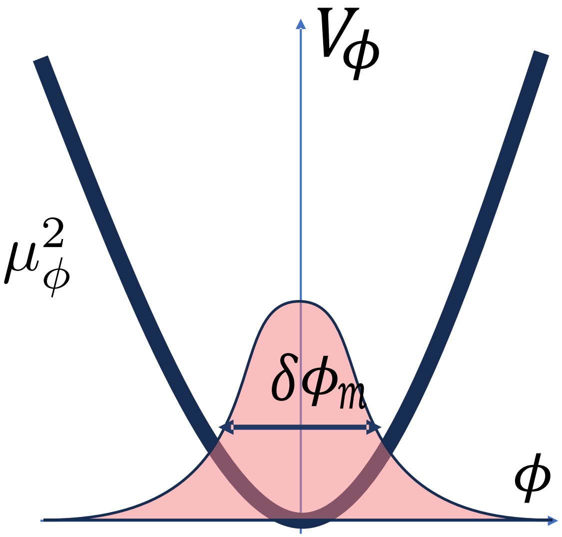
Charged Higgs bounds come mainly from $b \to s\,\gamma$ processes and can always be evaded by choosing $m_{H^+} \gtrsim 650$ GeV.



Pheno of ϕ: The 2 regimes

$$V_\phi = m_\phi^2 \phi^2 + \lambda_\phi \frac{\mu_\phi^2}{f^2} \phi^4$$

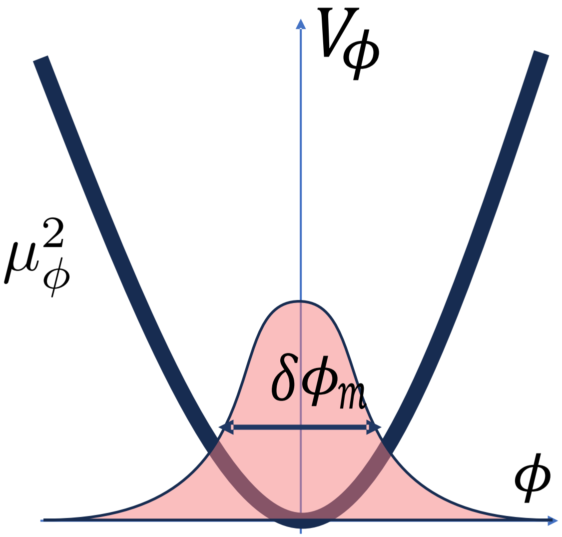
$$\text{where} \quad m_\phi^2 = \left(-\mu_\phi^2 + \kappa \frac{\mu_\phi}{f} v_\star^2 s_{\beta_\star} c_{\beta_\star}\right) \equiv \epsilon^2 \mu_\phi^2$$



To obtain $P(\phi)$, we solve the modified Fokker-Planck equation (volume-weighted):

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left[\frac{H_I^3(\phi)}{8\pi^2}\frac{\partial P}{\partial \phi} + \frac{V'(\phi)}{3H_I(\phi)} P\right] + 3H_I(\phi) P \Rightarrow \delta\phi_m$$

Pheno of ϕ: The 2 regimes



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The 2 scales: $\delta\phi_m \quad \& \quad \delta\phi_q \sim \frac{\epsilon f}{\sqrt{\lambda_\phi}}$

Quadratic Regime ($\rho_\phi \sim a^{-3}$): $\delta\phi_m \sim \frac{H_I^2}{m_\phi} \quad (\text{for} \;\; \delta\phi_m \ll \delta\phi_q)$

Quartic Regime ($\rho_\phi \sim a^{-4}$): $\delta\phi_m \sim \frac{H_I}{\lambda_\phi^{1/4}}\sqrt{\frac{f}{\mu_\phi}} \quad (\text{for} \;\; \delta\phi_m \gg \delta\phi_q)$

Pheno of ϕ: The 2 regimes

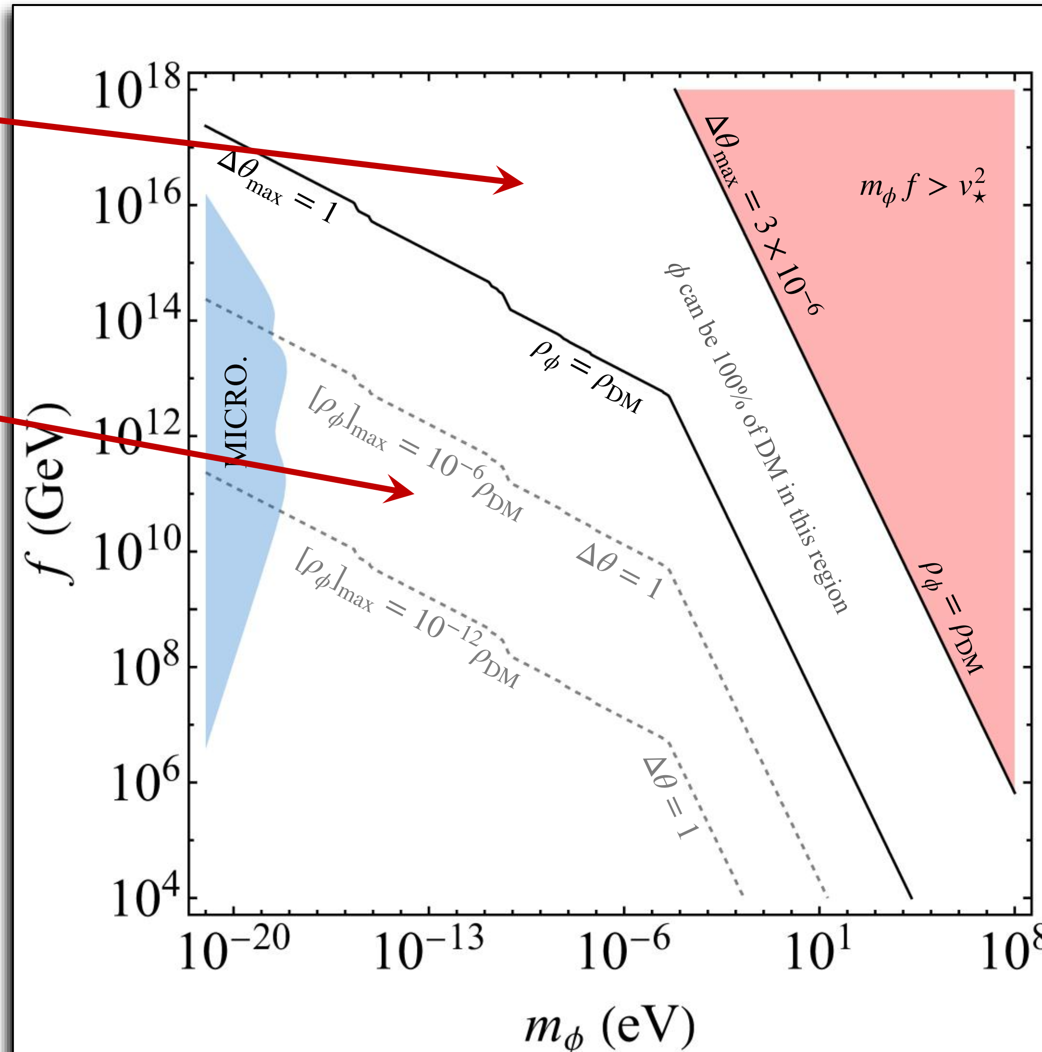
$$\mathcal{L}_{\phi^2 SM} = -c_e \frac{\phi^2}{f^2} m_e \bar{e} e - c_q \frac{\phi^2}{f^2} m_q \bar{q} q + c_\gamma \frac{\phi^2}{f^2} \frac{\alpha_{em}}{4\pi} F_{\mu\nu} F^{\mu\nu} + c_g \frac{\phi^2}{f^2} \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a,\mu\nu}$$

"Dark Matter" band:
ϕ can explain the obs. dark matter density here.

ϕ can explain only a fraction of obs. dark matter density here.

Experiments:
1. Violation of eq. principle (eg: MICROSCOPE)

2. Time variation of fundamental constants in atomic physics exps.



Several recent works

1. G. Dvali and A. Vilenkin, "Cosmic attractors and gauge hierarchy," (2004)

2. G. Dvali, "Large hierarchies from attractor vacua," (2006)

3. P. W. Graham, D. E. Kaplan, and S. Rajendran, "Cosmological Relaxation of the Electroweak Scale," (2015)

4. N. Arkani-Hamed, T. Cohen, R. T. D'Agnolo, A. Hook, H. D. Kim, and D. Pinner, "Solving the Hierarchy Problem at Reheating with a Large Number of Degrees of Freedom," (2016)

5. C. Cheung and P. Saraswat, "Mass Hierarchy and Vacuum Energy,"(2018)

6. G. F. Giudice, A. Kehagias, and A. Riotto, "The Selfish Higgs,"(2019)

7. A. Strumia and D. Teresi, "Relaxing the Higgs mass and its vacuum energy by living at the top of the potential," (2020)

8. C. Csaki, R. T. D'Agnolo, M. Geller, and A. Ismail, "Crunching Dilaton, Hidden Naturalness," (2020)

9. V. Domcke, K. Schmitz, T. You, "Cosmological relaxation through the dark axion portal", (2020)

10. M. Geller, Y. Hochberg, and E. Kuflik, "Inflating to the Weak Scale," (2019)

11. N. Arkani-Hamed, R. T. D'Agnolo, and H. D. Kim, "The Weak Scale as a Trigger," (2020)

12. G. F. Giudice, M. McCullough, and T. You, "Self-Organised Localisation," (2021)

13. R. Tito D'Agnolo and D. Teresi, "Sliding Naturalness," (2021)

14. R. Tito D'Agnolo and D. Teresi, "Sliding Naturalness: Cosmological selection of the weak scale" (2022)

15. M. Detering, T. You," Vacuum Metastability from Axion-Higgs Criticality", (2024)

Mostly from the last decade

Key Features of our model

- A **generic PNGB potential** for ϕ; NO clockwork needed.
- ϕ-field value **never exceeds the cutoff** f,let alone the Planck scale.
- Unlike the anthropic argument for weak scale, our mechanism **doesn't restrict the variation of other model parameters** as the Higgs VEV is varied.
- Maximizing the vacuum energy **automatically selects** regions with desirable properties.
- **Precise, falsifiable 2HDM prediction** that can be tested in present and future colliders.
- ϕ can account for the **observed DM density** and can be probed in exps. looking for **violation of equivalence principle** and **variation of fundamental constants**.
- Compatible with the "**stationary measure**" during eternal inflation. Also, compatible with **Weinberg's anthropic argument for Λ_{cc}**. (described in details in our paper.)

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Thank You !!!

BACKUP SLIDES

Volume of desired region

$$\lambda_* = \{i_\star, j_\star, k_\star, m_\star\}$$

Higgs Sector Parameters

Inflaton Sector Parameters

Cosmological Constant

Highest Vacuum of the Inflaton

$$\begin{cases} \dfrac{dP_\star}{dt} = -\displaystyle\sum_{\lambda}^{\lambda\neq\lambda_\star} \Gamma_{\lambda_\star\to\lambda} P_\star + \sum_{\lambda}^{\lambda\neq\lambda_\star} \Gamma_{\lambda\to\lambda_\star} P_\lambda + 3H_\star P_\star \\ \dfrac{dP_\lambda}{dt} = -\displaystyle\sum_{\mu}^{\mu\neq\lambda} \Gamma_{\lambda\to\mu} P_\lambda + \sum_{\mu}^{\mu\neq\lambda,\lambda_*} \Gamma_{\mu\to\lambda} P_\mu + \Gamma_{\lambda_\star\to\lambda} P_\star + 3H_\lambda P_\lambda \end{cases}$$

After a long time

$$P_\star(t) = P_\star(0) e^{3H_\star t}, \qquad P_\lambda(t) = \frac{\Gamma_{\lambda_\star\to\lambda}}{H_\star} P_\star(t)$$

Volume of desired region

Universes that permit a slow-roll & reheating phase:

$$\lambda_\star \to \lambda_{slow}$$

$$\mathcal{V}(t) \quad = \quad \sum_{\lambda_{slow}} P_\star(0) e^{3H_\star(t-t_+)} \Gamma_{\lambda_\star \to \lambda_{slow}} e^{N_{slow}} r_V(t_{age}) \Theta(t - t_+)$$

$$t_+ = t_{age} + t_{slow}$$

- Decoupling between the inflaton and Higgs sector results

$$\{i_\star, j_\star, k_\star, m_\star\} \to \{i_\star, j_{slow}, k_{slow}, m_{slow}\}$$

with i_* remaining same.

Volume of desired region

$$\{i_\star, j_\star, k_\star, m_\star\} \to \{i', j_\star, k', m_\star\} \to \{i', j_{slow}, k'_{slow}, m_{slow}\}$$

Volume of regions that undergo slow-roll and reheating but with $v \neq v_*$:

$$P'(t) = \sum_{\lambda'} P'_\lambda(t) = \sum_{\lambda'} \frac{\Gamma_{\lambda_\star \to \lambda'}}{H_\star} P_\star(t)$$

This volume is exponentially smaller than P_*

$$\mathcal{V}'(t) \quad = \quad \sum_{\lambda'_{slow}} P'(t-t_+)\Gamma_{\lambda_\star \to \lambda_{slow}} e^{N'_{slow}} r_V(t_{age})\Theta(t-t'_+),$$

Stationary measure:

$$\mathcal{V}(t)_{stationary} = \mathcal{V}(t)|_{t \to t+t_+}, \quad \mathcal{V}'(t)_{stationary} = \mathcal{V}'(t)|_{t \to t+t'_+}$$

$$\lim_{t\to\infty} \frac{\mathcal{V}'(t)_{stationary}}{\mathcal{V}(t)_{stationary}} \ll 1$$

Origin of the Landscape

- N heavy real scalars S_i

$$V_S = \sum_{i=1}^{N} \left(-m_i^2 S_i^2 + \lambda_i S_i^4\right) + \sum_{\text{all perm.}} \left(\prod_{i=1}^{N} \frac{c_{p_1..p_N}}{\Lambda^{(\sum p_i - 4)}} S_i^{p_i} + \text{h.c.} \right),$$

No. of vacuum configurations $\sim Exp(N)$.

2HDM parameters:

$$\alpha_H^{(j)} = \sum_i \frac{c_{ij}}{\Lambda^{5-d_j}} \langle S_i \rangle + \cdots$$

Each vacuum configuration has a different value of parameters.

Maximizing Vacuum Energy: Varying μ_1^2 and μ_2^2

Potential bounded from below:

$$\lambda_3 + \lambda_4 - |\lambda_5| + 2\sqrt{\lambda_1 \lambda_2} \geq 0 \quad \text{for} \quad \lambda_4 - |\lambda_5| \leq 0$$
$$\lambda_3 \geq -2\sqrt{\lambda_1 \lambda_2} \quad \text{for} \quad \lambda_4 - |\lambda_5| > 0$$

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EW symmetry preserved: $\langle H_1 \rangle = \langle H_2 \rangle = 0$ $\langle \phi \rangle \neq 0$	EW symmetry broken and $\langle \phi \rangle \neq 0$	EW symmetry broken and $\langle \phi \rangle = 0$

Class-I minima

EW symmetry preserved: $\langle H_1 \rangle = \langle H_2 \rangle = 0$

- The Trigger term is not effective in this scenario.
- ϕ has a Mexican hat potential in this scenario which is minimized at $\langle \phi \rangle = \pm \frac{f}{\sqrt{4\lambda_\phi}}$.
- Vacuum Energy:

$$\mathcal{V}\mathcal{E}^{I}_{\mathcal{H}} = -\frac{\mu_\phi^2 f^2}{16\lambda_\phi}$$

Class-II minima

EW symmetry broken and $\langle\phi\rangle \neq 0$

- Minimization w.r.t. ϕ, $i.e. \frac{\partial V}{\partial \phi} = 0$ gives:

$$\hat{\phi}^2 = \frac{f^2}{4\lambda_\phi} - \frac{\kappa f}{4\lambda_\phi \mu_\phi} \left(H_1^\dagger H_2 + h.c.\right)$$

Class-II minima

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$$\hat{\phi}^2 = \frac{f^2}{4\lambda_\phi} - \frac{\kappa f}{4\lambda_\phi \mu_\phi} \left(H_1^\dagger H_2 + h.c.\right)$$

- Substituting $\hat{\phi}$:

$$V_H(\phi, H_1, H_2)|_{\phi\to\hat{\phi}} = -\frac{\mu_\phi^2 f^2}{16\lambda_\phi} + \hat{V}_{2HDM}(H_1, H_2).$$

where $\hat{V}_{2HDM}$ is our original 2HDM potential with quartics replaced by $\lambda_4 \to \lambda_4 - \frac{\kappa^2}{8\lambda_\phi}$; $\lambda_5 \to \lambda_5 - \frac{\kappa^2}{8\lambda_\phi}$ and an additional $\frac{\kappa\mu_\phi f}{8\lambda_\phi} H_1^\dagger H_2 + h.c.$

Class-II minima

EW symmetry broken and $\langle\phi\rangle \neq 0$

- Minimization w.r.t. ϕ, $i.e. \frac{\partial V}{\partial \phi} = 0$ gives:

$$\hat{\phi}^2 = \frac{f^2}{4\lambda_\phi} - \frac{\kappa f}{4\lambda_\phi \mu_\phi} (H_1^\dagger H_2 + h.c.)$$

- Substituting $\hat{\phi}$:

$$V_H(\phi, H_1, H_2)|_{\phi\to\hat{\phi}} = -\frac{\mu_\phi^2 f^2}{16\lambda_\phi} + \hat{V}_{2HDM}(H_1, H_2).$$

where $\hat{V}_{2HDM}$ is our original 2HDM potential with quartics replaced by $\lambda_4 \to \lambda_4 - \frac{\kappa^2}{8\lambda_\phi}$; $\lambda_5 \to \lambda_5 - \frac{\kappa^2}{8\lambda_\phi}$ and an additional $\frac{\kappa\mu_\phi f}{8\lambda_\phi} H_1^\dagger H_2 + h.c.$

- Runaway: $\lambda_3 + \lambda_4 - \frac{\kappa^2}{8\lambda_\phi} - \left|\lambda_5 - \frac{\kappa^2}{8\lambda_\phi}\right| \leq -2\sqrt{\lambda_1\lambda_2}$

$$\text{and } \lambda_4 - |\lambda_5| < 0.$$

Class-II minima

EW symmetry broken and $\langle\phi\rangle \neq 0$

- Minimization w.r.t. ϕ, $i.e. \dfrac{\partial V}{\partial \phi} = 0$ gives:

$$\hat{\phi}^2 = \frac{f^2}{4\lambda_\phi} - \frac{\kappa f}{4\lambda_\phi \mu_\phi}\left(H_1^\dagger H_2 + h.c.\right)$$

- Substituting $\hat{\phi}$:

$$V_H(\phi, H_1, H_2)\big|_{\phi\to\hat{\phi}} = -\frac{\mu_\phi^2 f^2}{16\lambda_\phi} + \hat{V}_{2HDM}(H_1, H_2).$$

where $\hat{V}$ is our original 2HDM potential with quartics replaced

by

$\mathcal{VE}^I$

Always ≤ 0.

-

$$\mathcal{VE}^{II} < \mathcal{VE}^I$$

and $\lambda_4 - |\lambda_5| < 0$.

Class-III minima

EW symmetry broken and $\langle\phi\rangle = 0$

- The Trigger term can change the shape of the ϕ-potential from a Mexican hat to one in which $\langle\phi\rangle = 0$, when

$$-\mu_\phi^2 + \frac{\kappa\mu_\phi}{f}\langle(H_1^\dagger H_2 + h.c.\rangle) \geq 0.$$

- When $\langle\phi\rangle = 0$, the trigger term becomes ineffective giving no additional contribution to the 2HDM potential.

Class-III minima

EW symmetry broken and $\langle\phi\rangle = 0$

- With $\langle\phi\rangle = 0$, we can minimize the V_{2HDM} to get 2 sub-classes of minima:

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \end{pmatrix}$$

with $u, v_1, v_2 > 0$.

The first possibility is realized when $\lambda_4 - |\lambda_5| \leq 0$
And the second when $\lambda_4 - |\lambda_5| > 0$.

Class-III minima

EW symmetry broken and $\langle \phi \rangle = 0$

- With $\langle \phi \rangle = 0$, we can minimize the V_{2HDM} to get 2 sub-classes of minima:

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

The second possibility implies $\langle H_1^\dagger H_2 \rangle = 0$ and thus is inconsistent with the condition for $\langle \phi \rangle = 0$, i.e.

$$-\mu_\phi^2 + \frac{\kappa \mu_\phi}{f} \langle (H_1^\dagger H_2 + h.c.) \rangle \geq 0.$$

Class-III minima

EW symmetry broken and $\langle \phi \rangle = 0$

- With $\langle \phi \rangle = 0$, we can minimize the V_{2HDM} to get 2 sub-classes of minima:

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

$$\cancel{\langle H_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \end{pmatrix}}$$

Thus, in our case, only the first possibility is realized with ξ minimized at $\langle 2\xi + \arg(\lambda_5) \rangle = \pi$.

Selects the vacuum preserving the $U(1)_{EM}$.

Maximizing Vacuum Energy: Class-III minima

EW symmetry broken and $\langle \phi \rangle = 0$

- The vacuum energy is given by

$$\mathcal{VE_H}^{III} = -\frac{1}{4}(\lambda_1\, c_\beta^4 + \lambda_2\, s_\beta^4 + \lambda_{345}\, s_\beta^2\, c_\beta^2)\, v^4,$$

where $\lambda_{345} = \lambda_3 + \lambda_4 - |\lambda_5|$ and $\tan\beta = \frac{v_1}{v_2}$ and $v = \sqrt{v_1^2 + v_2^2}$.

- Also the consistency condition for $\langle \phi \rangle = 0$:

$$-\mu_\phi^2 + \frac{\kappa \mu_\phi}{f}\langle (H_1^\dagger H_2 + h.c.\rangle) \geq 0 \implies v^2 \geq \frac{\mu_\phi f}{\kappa s_\beta c_\beta}$$

- Thus,

$$\mathcal{VE_H}^{III} \leq -\frac{1}{4}(\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2)\,\frac{\mu_\phi^2 f^2}{\kappa^2 s_\beta^2 c_\beta^2}$$

Maximizing Vacuum Energy: Class-III minima

EW symmetry broken and $\langle\phi\rangle = 0$

- Thus, $\mathcal{VE_H}^{III} \leq -\frac{1}{4}(\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2)\frac{\mu_\phi^2 f^2}{\kappa^2 s_\beta^2 c_\beta^2}$

- Maximizing w.r.t. β, we get maximum V.E. for $\tan^2\beta_* = \sqrt{\frac{\lambda_1}{\lambda_2}}$ and

$$\boxed{\mathcal{VE_H}^{III,max} = -\frac{\mu_\phi^2 f^2}{4\kappa^2}(\lambda_{345} + 2\sqrt{\lambda_1\lambda_2})}$$

Technically Natural

Prediction: $\boxed{v_\star^2 = \frac{\mu_\phi f}{\kappa s_{\beta\star} c_{\beta\star}} \quad \& \quad \tan^2\beta_\star = \sqrt{\frac{\lambda_1}{\lambda_2}}}$