A quick guide to fractionally charged particles of the Standard Model

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Motivation

What is the actual Standard Model Group?

Introduction

We know that the Standard Model is based on the gauge group:

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y \,,$$

which describes all fundamental forces in nature apart from gravity. All its elementary matter representations can be grouped in the table below.

	q_L	u_R	d_R	ℓ_L	e_R	H
$SU(3)_c$	3	3	3	1	1	1
$SU(2)_L$	2	1	1	2	1	2
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$

It is common to use the following notation $(R_{SU(3)_c}, R_{SU(2)_L})_Y$
eg. $q_L \in (3,2)_{1/6}$.

Group Theoretics

- The choice of the group and particularly its Lie algebra is very important for interactions between charged particles and mediators in a gauge theory.
- Groups can locally look the same, but it is the **centre** Z (set of elements that commute with the other elements of the group) that can distinguish between them.
- Let us consider a simple example; **SO(3) vs SU(2)**.

$SO(3)$ vs $SU(2)$

Both of these groups are locally the same and their generators satisfy

$$SU(2) : \left[\frac{\sigma_i}{2}\,, \frac{\sigma_j}{2}\right] = i\epsilon_{ijk}\frac{\sigma_k}{2} \;\; ; \;\; SO(3) : [T_i\,, T_j] = i\epsilon_{ijk}T_k \tag{1}$$

Their main difference is the centre, $SU(2)$ has $Z(SU(2)) = Z_2 = \{1, -1\}$ in the fundamental representation, while $SO(3)$ has $Z(SO(3)) = 1$.

$$SU(2) : e^{i\phi\frac{\sigma_3}{2}} \xrightarrow{\phi=0} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = \mathcal{I}\,; \quad e^{i\phi\frac{\sigma_3}{2}} \xrightarrow{\phi=2\pi} \begin{pmatrix} -1 & \\ & -1 \end{pmatrix} = -\mathcal{I}$$

$$SO(3) : e^{i\phi T_3} \xrightarrow{\phi=0,2\pi} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \mathcal{I}$$

So we can obtain $SO(3)$ from $SU(2)$ by removing its centre,

$$SO(3) = \frac{SU(2)}{Z_2}\,. \tag{2}$$

General $SU(N)$

- For an $SU(N)$ group, we can write any of its elements as $e^{i\theta_a T^a}$.
- The centre of $SU(N)$ is $Z(SU(N)) = Z_N = \left\{1, \xi, \xi^2, \ldots, \xi^{N-1}\right\}$ with the first non-trivial generating element being $\xi = e^{2\pi i n_N / N}$ where n_N is the n-ality of the representation, **a positive integer mod** N.
- For $U(N)$ these generating elements are not distinct.
- $U(1)$ is its own centre.

$U(N)$ vs $U(1) \times SU(N)$

- Let us take the composition of $U(1) \times SU(N)$ and compare it to $U(N)$.
- The action of Z_N should leave any $SU(N)$ representations R invariant but there is an extra phase $\theta_0 = -2\pi/(Q_F N)$ coming from $U(1)$.

$$\xi R = e^{2\pi i n_N/N} e^{-2\pi i Q/(Q_F N)} R = R \tag{3}$$

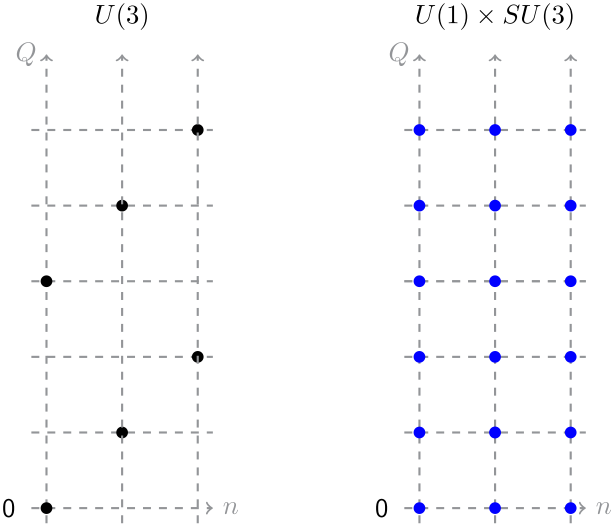
$$2\pi \left(\frac{n_N(R)}{N} - \frac{Q_R}{Q_F N} \right) = 2\pi\mathbb{Z} \tag{4}$$

which is the result of

$$U(N) = \frac{U(1) \times SU(N)}{Z_N} \,. \tag{5}$$

- Taking the quotient reduces the electric spectrum.

$U(3)$ vs $U(1) \times SU(3)$



The Standard Model Group

The SM group can be moded by a discrete symmetry Z_p.

$$G_p \equiv SU(3)_c \times SU(2)_L \times U(1)_Y / Z_p \,, \tag{6}$$

$$Z_p = \begin{cases} Z_1 & \xi^6 \\ Z_2 & \{1\,,\,\xi^3\} \\ Z_3 & \{1\,,\,\xi^2\,,\,\xi^4\} \\ Z_6 & \{1\,,\,\xi\,,\,\xi^2\,,\,\xi^3\,,\,\xi^4\,,\,\xi^5\} \end{cases} \tag{7}$$

where the generating elements are defined as

$$\xi = e^{2\pi i Q_Y}\, e^{2\pi i n_c/3}\, e^{i\pi n_L} \tag{8}$$

and n_c, n_L are the n-alities under $SU(3)_c$, $SU(2)_L$ and Q_Y the hypercharge.

Hypercharge quantisation

The invariance of all SM representations under the action of Z_p leads to quantisation conditions for hyperchage.

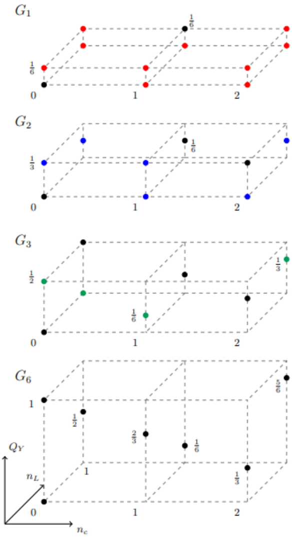
$$G_6: \qquad \xi R = R, \qquad \frac{n_c}{3} + \frac{n_L}{2} + Q_Y = \mathbb{Z}\,, \tag{9}$$

$$G_3: \qquad \xi^2 R = R, \qquad \frac{2n_c}{3} + 2Q_Y = \mathbb{Z}\,, \tag{10}$$

$$G_2: \qquad \xi^3 R = R, \qquad \frac{n_L}{2} + 3Q_Y = \mathbb{Z}\,, \tag{11}$$

$$G_1: \qquad \xi^6 R = R, \qquad 6Q_Y = \mathbb{Z}\,. \tag{12}$$

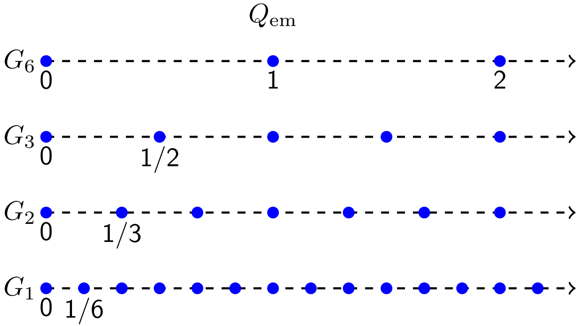
Constituent blocks of allowed hypercharge



Electromagnetic charge spectrum

- There is an apparent electric 1-form symmetry in our theory.
- Taking the quotient introduces a magnetic 1-form symmetry.

$$Q_{em} = T_{3L} + Q_Y \;\; ; \;\; Q_{em} g_{em} = \mathbb{Z} \tag{13}$$



- Notice periodicity of $p/6$ for G_p.

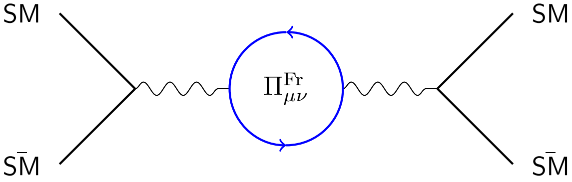
BSM fractional states

$$G_6: \quad \begin{matrix} e_R & \ell_L & d_R \\ (1,1)_{-1}\,, & (1,2)_{-1/2}\,, & (3,1)_{-1/3} \end{matrix}$$

$$\begin{aligned} G_3: &\quad \begin{matrix} \Xi & \Lambda & \Omega \\ (1,1)_{1/2}\,, & (1,2)_{0}\,, & (3,1)_{1/6}\,, \end{matrix} \\ G_2: &\quad \begin{matrix} \Sigma & \Delta & \Theta \\ (1,1)_{1/3}\,, & (1,2)_{1/6}\,, & (3,1)_{0}\,, \end{matrix} \\ G_1: &\quad \begin{matrix} \Phi & \Lambda & \Theta \\ (1,1)_{1/6}\,, & (1,2)_{0}\,, & (3,1)_{0}\,. \end{matrix} \end{aligned}$$

How do these fractional states couple to SM?

- Due to their fractional nature these new fields cannot couple linearly to SM.

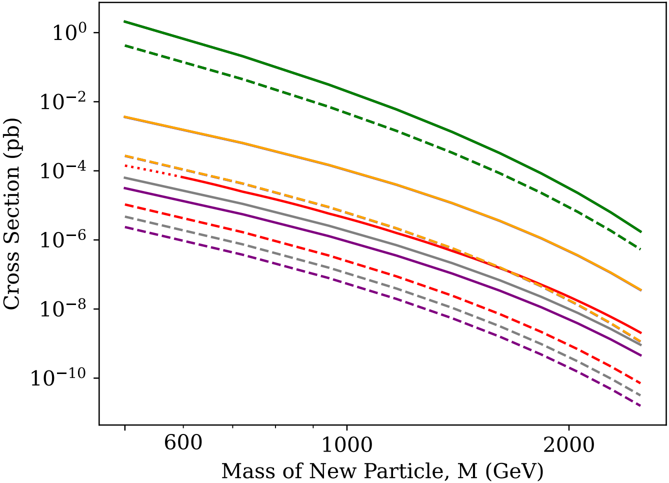


$$\sigma_{q\bar{q}\to \mathsf{Fr}\bar{\mathsf{Fr}}} = \frac{g^4}{s} \mathrm{I}(q)\mathrm{I}(R)\,\mathsf{Im}(i\Pi^{\mu\nu}_{Fr})\,[p_{2\mu}p_{1\nu} + p_{2\nu}p_{1\mu} - g_{\mu\nu}(p_2 \cdot p_1 + m_q^2)]\mathrm{d}(\mathsf{Ad})$$

$$\sum_a T_a(R)T_a(R) \equiv C(R)\mathbb{1} \quad (14)$$

$$C(R)\mathrm{d}(R) = \mathrm{I}(R)\mathrm{d}(\mathrm{Ad}) \quad (15)$$

Hadronic cross section varying with mass



For a more in-depth phenomenological and formal discussion please refer to S. Koren, A. Martin [2406.17850].

Conclusion

- Detection of fractionally charged particles would showcase the actual Standard Model Group.
- More fundamentally, they may help us understand charge quantisation.
- Probing the discrete symmetries of the Standard Model, we can learn more about BSM theories (eg. GUTs).
- There is also an open connection to generalised and higher-form symmetries.
  D. Tong [1705.01853]

Thank you for listening!

Extra material

Applications in SMEFT with fractional states

SMEFT is an effective field theory that describes SM interactions with higher dimension operators.

$$\mathcal{L}_{\mathsf{SMEFT}} = \mathcal{L}_{\mathsf{SM}} + \mathcal{L}_{\mathsf{d=6}} + \ldots \tag{16}$$

$$\mathcal{L}_{d=6} \supset -\frac{2\delta}{v^2}\left(a_c J_c^b J_c^b + a_L J_L^I J_L^I + a_Y J_Y J_Y\right) \tag{17}$$

with b the colour index, I the isospin, the coefficients a_c, a_Y

$$\delta = \frac{a_s \mathrm{d}_L \mathrm{d}_c v^2}{(4\pi)^2 240 M^2}, \qquad a_c = \frac{\mathrm{I}_c g_c^4}{\mathrm{d}_c}, \tag{18}$$

$$a_L = \frac{\mathrm{I}_L g^4}{\mathrm{d}_L}, \qquad a_Y = Q_Y^2 (g_Y)^4 . \tag{19}$$

and the $SU(2)_W, U(1)_Y$ Higgs currents

$$\begin{aligned} J_{L,\mu,a} &= i(H^\dagger\, T_{L,a} D_\mu H - (D_\mu H)^\dagger\, T_{L,a} H) \\ J_{Y,\mu} &= i Q_Y (H^\dagger\, D_\mu H - (D_\mu H)^\dagger\, H). \end{aligned} \tag{20}$$

BSM electroweak precision observables

The ratio of a_L and a_Y takes a discrete set of values for G_p that can be used to infer the quantum numbers of the new particle.
It cannot be determined by a single observable as δ is a free parameter but it can be determined by the correlation between two observables.
Let us define our input scheme:

$$M_W^2 = \frac{g^2 v^2}{4}\left(1 - a_L\delta\right)\,, \tag{21}$$

$$M_Z^2 = \frac{g^2 v^2}{4c_w^2}\left(1 - a_L\delta - a_Y\delta\right)\,, \tag{22}$$

$$G_F = \frac{1}{\sqrt{2}v^2}\,, \tag{23}$$

$$s_w \equiv \frac{\sqrt{4\pi\alpha_{em}}}{2M_W(\sqrt{2}G_F)^{1/2}} = s_w(1 + \frac{1}{2}a_L\delta)\,. \tag{24}$$

ρ parameters

We can then substitute these in the expression for two other observables that maximise the range of angles for the correlation of the Wilson coefficients,

$$\rho_{\Gamma 3} \equiv \frac{1}{6} \frac{M_Z^3 \Gamma_W}{M_W^3 \Gamma_Z^{\rm inv}} = (1 + \delta a_Y)\,, \quad (25)$$

$$\rho_{\Gamma 5} \equiv \frac{1}{6} \frac{(1-\bar{s}_w^2) M_Z^5 \Gamma_W}{M_W^5 \Gamma_Z^{\rm inv}} = (1 - \frac{1}{2} 2\bar{t}_w^2 \delta a_L)\,. \quad (26)$$