Extended EFT of Dark Energy

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May 29th, 2025
PLANCK 2025 Conference, Padua, Italy

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based on Phys. Lett. B 844(2023) 138115, 2304.07344 [hep-th]
& 2502.02401 [hep-th]

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Introduction

- The discovery of the current acceleration of the universe took us by surprise.

- The simplest explanation is a Cosmological Constant (CC) $\sim 10^{-122} M_{\rm Pl}^4$

- Another approach is modified gravity

- Among MoG, are scalar-tensor (ST) theories $\Longrightarrow$ Const. Long Range Force

- Other approaches to modifying gravity are DGP braneworld model and Fierz-Pauli theory of massive gravity

- π interacts with matter with gravitational strength which makes it healthy.

- These theories reduce to ST theory on $\Lambda_{\rm IR} < E$ instead of GR

- The theories become strongly coupled at $\Lambda_{\rm UV}$

Introduction

- Ghost condensation (GC): modification of gravity with no strong coupling in UV

Arkani-Hamed, et al. (2009)

- It is like CC, with excitations around the spontaneously Lorentz-Breaking background that breaks time-translation too

$$\langle \dot{\phi} \rangle = c$$

- Unlike massive gravitons, one gets only an additional scalar field with modified dispersion relation.

- The kinetic term for ghost field appears with a sign opposite to the SM fields

$$\mathscr{L} = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi$$

- Like in Higgs mechanism, higher order term $(\partial\phi)^4$ stabilizes the theory.

Introduction

- It was argued in Arkani-Hamed et al. (2004) that going beyond $\omega^2 \sim k^4$ leads to strong coupling at IR.

- For e.g. if one assumes $E \to sE$ and $\omega^2 \sim k^6$, the scaling of t, x, and π are as follow

$$
\begin{aligned}
t &\to s^{-1} t \\
x &\to s^{-1/3} x \\
\pi &\to s^{0} \pi
\end{aligned}
$$

- Therefore the interacting operator

$$\int \mathrm{d}^3x \; \mathrm{d}t \; M^4 \dot{\pi} (\nabla \pi)^2 \,,$$

scales like $s^{-1/3}$ and therefore becomes relevant at IR.

Introduction

- In cosmological setups however approaching the IR limit is a bit tricky.

- During inflation, if there is a transition from $\omega^2 \sim k^6$ to $\omega^2 \sim k^4$ or $\omega^2 \sim k^2$ before it becomes strongly coupled at Λ_6^{IR} one can have a consistent EFT.

Ashoorioon, et al. (2018)
Ashoorioon, et al. (2021)

- The late-time acceleration of the universe is a recent phenomenon in cosmological time scales, $z \approx 0.7$

- Modes do not really approach the IR limit and therefore an EFT with pure $\omega^2 \sim k^6$ can make sense.

Outline:

- Structure of the ghost condensate with a sextic dispersion relation.

- Avoidance of Strong Coupling in the late time cosmological acceleration

- GC coupled to gravity in Minkowski spacetime

- Coupling GC to Gravity in de Sitter

Structure of the GC with a sextic dispersion relation

- Let us consider the action

$$S = S_0 + \Delta S\,,$$

where

$$S_0 = \int d^4x\sqrt{-g}\,\left[M^4 P(X)\right],$$

$$\Delta S = \int d^4x\sqrt{-g}\left[M^2\left(S_1(\Box\phi)^2 + S_2(\partial_\mu\partial_\nu\phi)^2\right) + S_3\left(\partial_\mu\Box\phi\right)^2 + S_4\left(\partial_\mu\partial_\nu\partial_\rho\phi\right)^2 + \cdots\right].$$

$S_i(X)$ where $X = \partial_\mu\phi\partial^\mu\phi$

- Varying the action, we obtain

$$\partial^\mu\left[P'(X)\partial_\mu\phi\right] = 0\,,$$

when gravity is ignored

$$\phi = ct$$

where c is an arbitrary dimensionless constant.

Structure of the GC with a sextic dispersion relation

- Now looking at $\phi = ct + \pi$

$$S_0 = M^4 \int d^4x\sqrt{-g}\left[\dot{\pi}^2(P'(c^2) + 2c^2P''(c^2)) - P'(c^2)(\partial_i\pi)^2 - 2P''(c^2)c\dot{\pi}(\partial_i\pi)^2 + \frac{1}{2}P''(c^2)(\partial_i\pi)^4 + \cdots\right].$$

- The time and spatial kinetic sign of π appear with the standard sign if

$$P'(c^2) > 0, \qquad P'(c^2) + 2c^2P''(c^2) > 0.$$

- In an expanding background, $P'(\dot{\phi}^2) \propto a(t)^{-3}$ $\Rightarrow$ the coefficient of $(\partial_i\pi)^2$ vanishes for $\dot{\phi}^2 = c_*^2$

- When one considers just the $S_{1,2}$ terms, $\omega^2 \sim k^4$, in absence of gravity.

  - Arkani-Hamed et al. (2004)

- We want to go to $\omega^2 \sim k^6$, so we have to include $S_{3,4}$ terms.

Structure of the GC with a sextic dispersion relation

$$S = \int \mathrm{d}^4x \frac{\sqrt{-g}}{2}\left[M^4\dot{\pi}^2 - \bar{M}^2(\partial_i\partial_j\pi)^2 - \bar{S}(\partial_i\partial_j\partial_k\pi)^2 + \cdots\right]$$

$$\bar{M}^2 = -2M^2(S_1(c_*^2) + S_2(c_*^2)) \geq 0$$
$$\bar{S} = -2(S_3(c_*^2) + S_4(c_*^2)) > 0$$

$$\omega^2 = \frac{\bar{M}^2}{M^4}k^4 + \frac{\bar{S}}{M^4}k^6\,.$$

- To obtain pure $\omega^2 \sim k^6$ one has to either assume

$$\bar{M}^2 = -2M^2(S_1(c_*^2) + S_2(c_*^2)) = 0$$

or

$$\frac{\bar{M}^2}{\bar{S}} \ll k^2 \sim \frac{H^2}{(1+z_c)^2}$$

Viability of $\omega^2 \sim k^6$ Dispersion Relation

- Contemporary acceleration is like a low-scale inflation
- The largest mode that has exit the horizon is $k \simeq \dfrac{H}{1+z_c}$, where z_c is the redshift of domination of DE.
- Like inflation avoidance of strong coupling could be defined as

$$\left| f_{\rm NL} \right| \ll |\zeta|^{-1}$$

where $\zeta = -H\pi$ and

$$f_{\rm NL} \sim \frac{B(k_1, k_2, k_3)}{\Delta_\zeta^4(k)}$$

$$\Delta_\zeta^2(k) = \frac{k^3}{2\pi^2}\langle \zeta\zeta \rangle$$

$$B(k_1, k_2, k_3) = \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle$$

- $u = a(\eta)\pi_c(\eta)$ where $\pi_c = \pi M^2$ satisfies the following equation

$$\left(-\frac{a''(\eta)}{a(\eta)} + \frac{\bar{S}}{M^4}\frac{k^6}{a(\eta)^4} \right) u(\eta) + u''(\eta) = 0.$$

Viability of $\omega^2 \sim k^6$ Dispersion Relation

- In de-Sitter space $a = -\frac{1}{H\eta}$, the solution is

$$u(\eta, k) = -\sqrt{\frac{3}{2}} \frac{M}{\bar{S}^{1/4} k^{3/2} H\eta} \exp\left(\frac{-i\sqrt{\bar{S}}}{3}\left(\frac{H}{M}\right)^2 (k\eta)^3\right).$$

and the two-point function is obtained to be

$$\Delta_\zeta^2(k) = \frac{3\pi}{\sqrt{\bar{S}}}\left(\frac{H}{M}\right)^2.$$

and the three-point function could be computed from the in-in formalism

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = -\frac{H^3}{M^6}\mathrm{Re}\left\{ -2i\langle \pi_c(\tilde{\eta}, \vec{k}_1)\pi_c(\tilde{\eta}, \vec{k}_2)\pi_c(\tilde{\eta}, \vec{k}_3)\int \frac{\mathrm{d}^3 q_1}{(2\pi)^3}\frac{\mathrm{d}^3 q_2}{(2\pi)^3}\frac{\mathrm{d}^3 q_3}{(2\pi)^3}\int \mathrm{d}^3 x \int_{\eta_1}^{\eta_0} \mathrm{d}\eta'\, a(\eta')^4 \right.$$

$$\left. \times \mathscr{H}_{\mathrm{int}} e^{-i(\vec{q}_1+\vec{q}_2+\vec{q}_3)\cdot\vec{x}}\rangle \right\},$$

where $\mathscr{H}_{\mathrm{int}} = -\mathscr{L}_{\mathrm{int}} = \frac{1}{2c_* M^2}\dot{\pi}_c\left(\partial_i \pi_c\right)^2$

one should note that $\eta_0 = -\frac{1}{H}$ and $\eta_1 = -\frac{1+z_c}{H}$

Viability of $\omega^2 \sim k^6$ Dispersion Relation

For $k_1 = k_2 = k_3 = \dfrac{H}{1+z_c}$ and in the limit that $\dfrac{H}{M} \ll 1$, one obtains

$$\left| f_{\rm NL} \right| \ll |\zeta|^{-1} \quad \longrightarrow \quad c_* \bar{S}^{1/4} \gg 10^{-34}$$

which can be satisfied for c_* and $\bar{S} \sim \mathcal{O}(1)$

- When the strong coupling become important, i.e. $\left| f_{\rm NL} \right| \simeq |\zeta|^{-1}$

$$z_c^* \geq 6 \times 10^{33}$$

which mean ~ 77 e-folds from now!

Coupling of the Ghost Condensate to Gravity

- In unitary gauge the modification of the gravity is explicit.

- One can break the time-diff. by choosing the gauge $\phi(t, x) = t$

- One can write down the unitary gauge action with the terms that respect the residual symmetry,

$$t' = t$$
$$x'^i = x^i + \xi^i(t, x)$$

- Looking at $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, one of the invariants is h_{00}

- Restore the full diff. by performing the Stueckelberg trick, i.e. performing a broken ξ^0 diffeomorphism and then promoting it to field π

$$h_{00} \to h_{00} - 2\partial_0 \pi \,, \quad h_{0i} \to h_{0i} - \partial_i \pi \,, \quad h_{ij} \to h_{ij}$$

Coupling of the Ghost Condensate to Gravity

- The kinetic term for π is generated through the term

$$\frac{1}{8}\int M^4 h_{00}^2$$

- Noting that $K_{ij} \to K_{ij} + \partial_i \partial_j \pi$, one can write down the terms lead to the sixth order dispersion relation,

$$S = -\int \mathrm{d}^3x\mathrm{d}t\left(\frac{\sigma_1}{2}\nabla_i K_{jk}\nabla_i K_{jk} + \frac{\sigma_2}{2}(\nabla_i K_{jj})^2 + \frac{\sigma_3}{2}\nabla_i K_{ij}\nabla_l K_{lj} + \frac{\sigma_4}{2}\nabla_i K_{ij}\nabla_j K_{ll}\right)$$

- Reintroducing π in the above Lagrangian one obtains

$$S = \int -\frac{\sigma}{2}\left[(\nabla^3\pi)^2 + \cdots\right]$$

Coupling of the Ghost Condensate to Gravity

- Let us now look at the modification of gravity in the GC at the linearized level

$$ds^2 = (1 + 2\Phi(t, x))dt^2 - (1 - 2\Psi(t, x))\delta_{ij}dx^i dx^j \,,$$

the Einstein-Hilbert contribution enforces that in the Newtonian limit $\omega^2 \ll k^2$

$$\Phi = \Psi$$

the the Lagrangian at the quadratic level is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}M^4(\Phi - \dot{\pi})^2 - \frac{\sigma}{2}\left(k^6\pi^2 + \dots\right).$$

introducing the canonical variables, $M^2\pi \rightarrow \pi_c$, and $\sqrt{2}M_{\text{Pl}}\Phi \rightarrow \Phi_c$ and setting $\mu = \equiv \dfrac{M^2}{\sqrt{2}M_{\text{Pl}}}$, the kinetic matrix of the Lagrangian is

$$\frac{1}{2}\begin{pmatrix} -k^2 + \mu^2 & i\mu\omega \\ -i\mu\omega & \omega^2 - \frac{\sigma}{M^4}k^6 \end{pmatrix} \quad \Rightarrow \quad \omega^2 = \frac{\sigma}{M^4}\left(k^6 - \mu^2 k^4\right)$$

due to mixing with gravity

Coupling of the Ghost Condensate to Gravity

- for $k < \mu$ there is instability.

- The width of instability in ω, gives the timescale t_c, that a change in the gravitational potential happens at scale $\sim \mu^{-1}$

$$\omega_{\text{ins}} = i\frac{1}{3}\sqrt{\frac{\sigma}{6}}\frac{M^4}{M_{\text{Pl}}^3} = i\frac{2}{3}\sqrt{\frac{\sigma}{6}}\frac{\mu^2}{M_{\text{Pl}}} \equiv i\Upsilon\,.$$

- To obtain the modified gravity potential, we need to compute $\langle \Phi_c \Phi_c \rangle$

$$\frac{1}{k^2}\left(-1 + \frac{\frac{\sigma\mu^2}{M^4}k^4}{-\frac{\sigma}{M^4}k^6 + \frac{\sigma\mu^2}{M^4}k^4 + \omega^2}\right).$$

- The correction to the Newtonian potential is $\mathcal{O}(1)$ if

$$\left|\omega - \frac{\sqrt{\sigma}}{M^2}k^3\right| \lesssim \sqrt{\sigma}\frac{\mu^2}{M_{\text{Pl}}} = \frac{3\sqrt{6}}{2}\Upsilon$$

Coupling of the Ghost Condensate to Gravity

- There is an instability timescale, $t_c \sim \Upsilon^{-1} \sim \dfrac{M_{\rm Pl}^3}{\sqrt{\sigma} M^4}$
- This timescale is a factor of $\dfrac{M_{\rm Pl}}{M}$ larger than the corresponding timescale in quartic dispersion relation due to the slowdown of the signal at $r_c \ll t_c$

- This gives rise to the modification of gravity

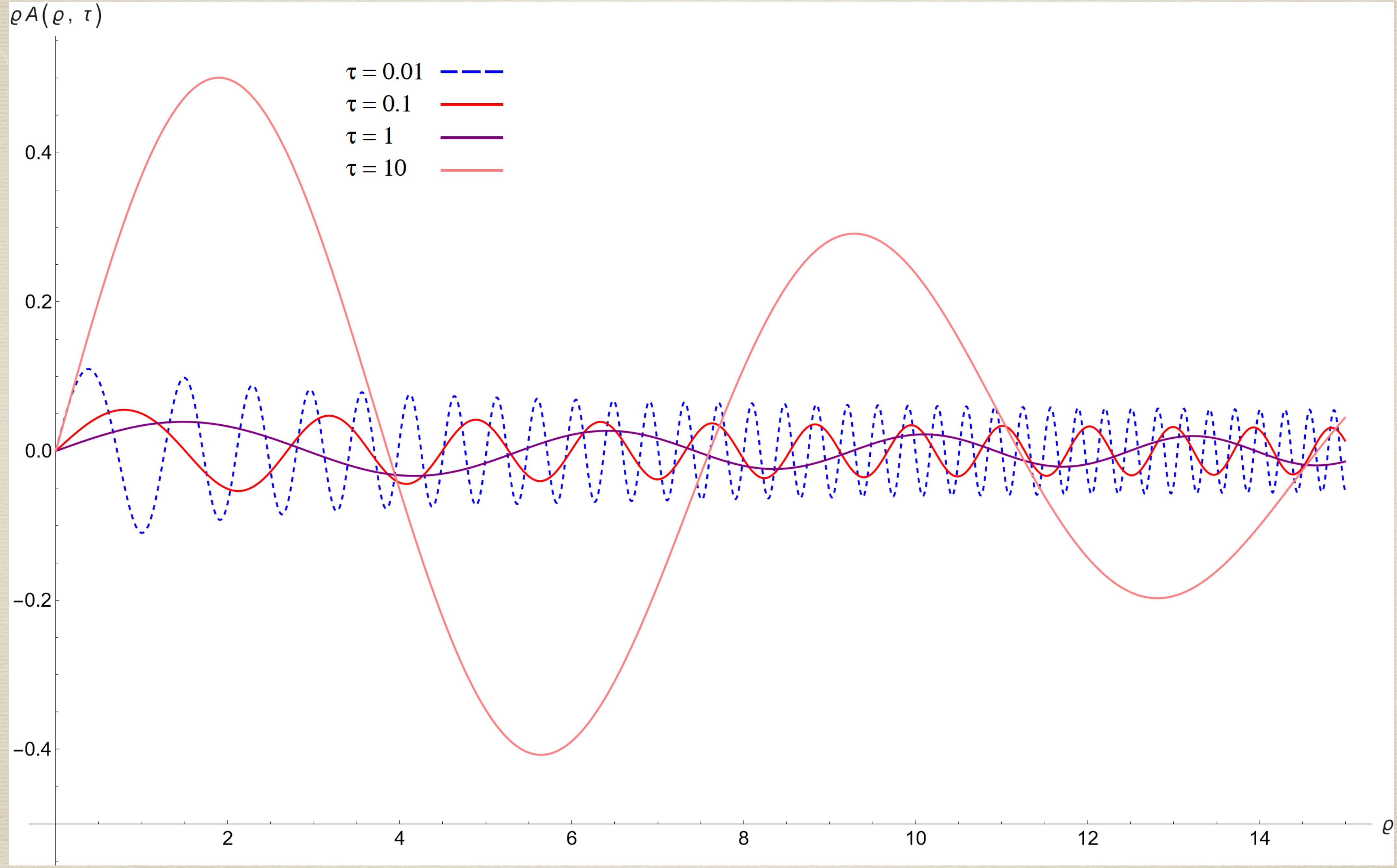
$$\Phi = \Phi_{\rm N} + \Phi_{\rm Mod}$$

$$\Phi_{\rm N} = -\frac{M_{\rm Pl}^{-2}}{r}, \quad \Phi_{\rm Mod} = M_{\rm Pl}^{-2} A(r,t),$$

If we define

$$\kappa \equiv \mu^{-1} k, \quad \Omega \equiv \frac{2}{3\sqrt{6}} \frac{\omega}{\Upsilon}, \quad \varrho \equiv \mu r, \quad \tau \equiv \frac{3\sqrt{6}}{2} \Upsilon t \, .$$

Coupling of the Ghost Condensate to Gravity



Ghost Condensate coupled to Gravity in dS

We start from the action

$$A = A_{\rm EH} + A_{\rm GC}$$

where

$$A_{\rm EH} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}M_{\rm Pl}^2 R$$

and

$$A_{\rm GC} = \int \mathrm{d}^4x \frac{\sqrt{\gamma}}{2}\Big[\frac{M^4}{4}(X-1)^2 - \gamma^{ij}\left(\sigma_1\nabla_i K_{lr}\nabla_j K^{lr} + \sigma_2\nabla_i K\nabla_j K\right) - \sigma_3\nabla_i K^i_j\nabla_l K^{lj} - \sigma_4\nabla^i K_{ij}\nabla^j K\Big],$$

We perturb the metric

$$\mathrm{d}s^2 = (1+2\Phi(x^\mu))\mathrm{d}t^2 - a^2(t)(1-2\Psi(x^\mu))\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

and expand the action $A_{\rm GC}$ in terms of Φ, Ψ and recover the Goldstone boson, π using the Stueckelberg trick

$$\begin{aligned}\mathcal{L}_{\rm GC} = -\frac{1}{2a^3}\Big[&\sigma_1\big[\partial_i(\partial_j^2\pi) + Ha^2\partial_i\left(\Phi-\dot\pi\right) + a^2\partial_i\dot\Psi\big]^2 + \sigma_2\big[\partial_i(\partial_j^2\pi) + 3Ha^2\partial_i(\Phi-\dot\pi) + 3a^2\partial_i\dot\Psi\big]^2\\ &+\sigma_3\big[\partial_i(\partial_j^2\pi) + a^2\partial_i\dot\Psi\big]^2 + \sigma_4\Big[\big(\partial_i(\partial_j^2\pi) + 2Ha^2\partial_i(\Phi-\dot\pi) + 2a^2\partial_i\dot\Psi\big)^2\\ &-\big(Ha^2\partial_i(\Phi-\dot\pi) + a^2\partial_i\dot\Psi\big)^2\Big]\Big] + \frac{1}{2}M^4a^3\left(\Phi-\dot\pi\right)^2 \ .\end{aligned}$$

Ghost Condensate coupled to Gravity in dS

Varying the action A w.r.t. $g^{\mu\nu}$

$$M_{\rm Pl}^2 G^\rho_\lambda - \frac{1}{2} M^4 \delta^\rho_0 \delta_{\lambda 0}(1-X) - \frac{1}{2}\left(\sigma_1 \mathcal{K}_{1\,\lambda}^{\ \rho} + \sigma_2 \mathcal{K}_{2\,\lambda}^{\ \rho} + \sigma_3 \mathcal{K}_{3\,\lambda}^{\ \rho} + \sigma_4 \mathcal{K}_{4\,\lambda}^{\ \rho}\right) = 0\,, \quad (*)$$

where $\mathcal{K}_{i\,\lambda}^{\ \rho}$ are complicated expressions of the extrinsic curvature and its derivative and G^ρ_λ is the Einstein tensor. The off-diagonal component of (*) suggests

$$\Phi = \Psi$$

Varying the action A w.r.t. π and using the three equations from three equations above, one can obtain the EoM for dark energy perturbations, π

$$\left(\frac{\partial}{a}\right)^6 \pi - C_1 \left(\frac{\partial}{a}\right)^6 \dot{\pi} + C_2 \left(\frac{\partial}{a}\right)^4 \pi + C_3 \left(\frac{\partial}{a}\right)^4 \dot{\pi} + C_4 \left(\frac{\partial}{a}\right)^2 \pi + C_5 \left(\frac{\partial}{a}\right)^2 \dot{\pi} + C_6 \left(\frac{\partial}{a}\right)^2 \ddot{\pi}$$
$$+ C_7 \ddot{\pi} - C_8 \dot{\pi} = 0\,,$$

Ghost Condensate coupled to Gravity in dS

where

$$S_2 \equiv 3\sigma_1 + \sigma_2 + 2\sigma_4\,, \quad S_3 \equiv\; 3(\sigma_1 + 3\sigma_2 + \sigma_4)\,, \qquad S_4 \equiv 2S_2 + \sigma_3\,,$$

$$\begin{aligned}
C_1 &= \frac{3S_2S_4H}{12S_1M_{\rm Pl}^2 - 3S_2S_4H^2}\,,\\
C_2 &= \frac{S_2\left(S_4\left(\mu^4 - 3H^2\mu^2\right) - 4M_{\rm Pl}^2\left(3H^2+\mu^2\right)\right) - 3S_3S_4H^4}{12S_1M_{\rm Pl}^2 - 3S_2S_4H^2}\,,\\
C_3 &= \frac{S_2\left(3H^2S_4\left(\mu^2 - 3H^2\right) + 2\mu^2M_{\rm Pl}^2\right) - 3S_3S_4H^4}{3\left(S_2S_4H^3 - 4S_1HM_{\rm Pl}^2\right)}\,,\\
C_4 &= \frac{3S_3S_4H^4\mu^2 + 4S_3H^2\mu^2M_{\rm Pl}^2 - H^2\mu^4S_3S_4}{3S_2S_4H^2 - 12S_1M_{\rm Pl}^2}\,,\\
C_5 &= \frac{S_3\left(S_4\left(9H^5 - 3H^3\mu^2\right) - 2H\mu^2M_{\rm Pl}^2\right) + 12S_4H\mu^2M_{\rm Pl}^2}{12S_1M_{\rm Pl}^2 - 3S_2S_4H^2}\,,\\
C_6 &= \frac{36S_4\mu^2M_{\rm Pl}^2}{4S_1M_{\rm Pl}^2 - S_2S_4H^2}\,,\\
C_7 &= \frac{4\mu^2M_{\rm Pl}^2\left(9S_4H^2\left(\mu^2-3H^2\right) - 2M_{\rm Pl}^2\left(H^2 - 3\mu^2\right)\right)}{S_2S_4H^4 - 4S_1H^2M_{\rm Pl}^2}\,,\\
C_8 &= 4\mu^2M_{\rm Pl}^2\left(\frac{6M_{\rm Pl}^2\left(H^2-3\mu^2\right)}{S_2S_4H^3 - 4S_1HM_{\rm Pl}^2} + \frac{S_4\left(30H^4 - 19H^2\mu^2 + 3\mu^4\right)}{S_2S_4H^3 - 4S_1HM_{\rm Pl}^2}\right).
\end{aligned}$$

Finally the speed of sound of dark energy perturbations is given by $c_s^2 = \dfrac{C_7}{C_4}$

Ghost Condensate coupled to Gravity in dS

The EoM for Φ is

$$\frac{S_1S_3}{6S_2M_{\rm Pl}^2\mu^2}\left(\frac{\partial}{a}\right)^6\Phi-\frac{S_1S_3}{2S_2}\frac{H}{M_{\rm Pl}^2\mu^2}\left(\frac{\partial}{a}\right)^4\dot\Phi+\left[\frac{2S_1}{3S_2H^2}+\frac{S_3H^2}{6M_{\rm Pl}^2\mu^2}\left(\frac{3S_1}{S_2}-1\right)-\frac{S_4}{6M_{\rm Pl}^2}\right]\left(\frac{\partial}{a}\right)^4\Phi$$
$$+\left[\frac{S_3H^4}{2M_{\rm Pl}^2\mu^2}-\frac{2S_1}{3S_2}\frac{\mu^2}{H^2}-\left(1+\frac{S_1}{S_2}\right)\right]\left(\frac{\partial}{a}\right)^2\Phi+\left[\frac{S_3H^3}{2M_{\rm Pl}^2\mu^2}-\frac{1}{3H}\right]\left(\frac{\partial}{a}\right)^2\dot\Phi$$
$$+3H^2\Phi+4H\dot\Phi+\ddot\Phi=0\,.$$

For distances $\gg H^{-1}$

$$\Phi\sim c_1(r)e^{-3Ht}+c_2(r)e^{-Ht}\,.$$

Decomposing Φ

$$\Phi=\Phi_{\rm GR}+\Phi_{\rm GC}$$

where

$$\left(\frac{\nabla}{a}\right)^2\Phi_{\rm GR}=\frac{\rho}{2M_{\rm Pl}^2}\,.$$

Ghost Condensate coupled to Gravity in dS

$$S_1 \to S\,, \quad S_2 \to \frac{5}{3}S\,, \quad S_3 \to 4S\,, \quad S_4 \to \frac{11}{3}S\,. \qquad \alpha = \frac{\mu}{H} \quad \beta = \frac{H}{\gamma} \qquad X = \mu r$$

The time-independent solution ($\partial_T = 0$) satisfies the equation

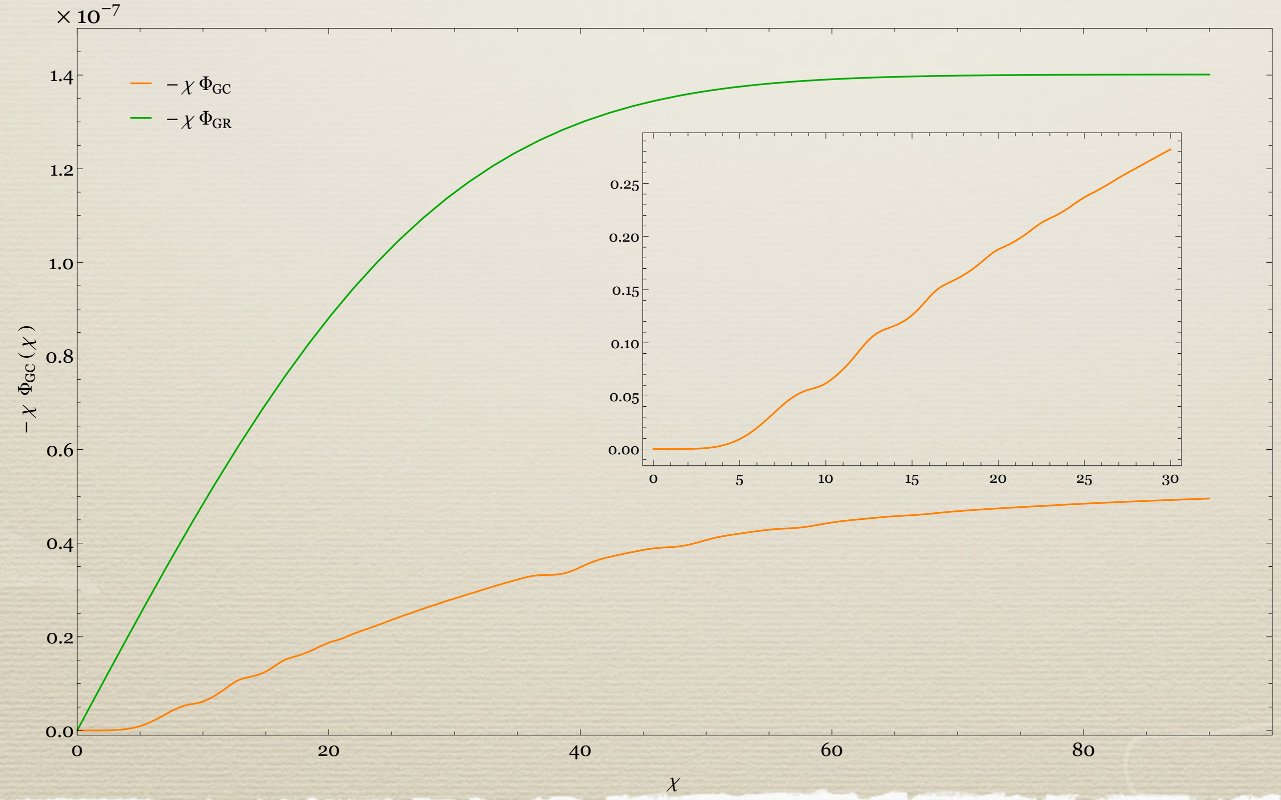
$$\frac{27}{5\beta^2}\partial_X^6\Phi_{\rm GC} + \left(\frac{162}{5X} - \frac{81}{5\alpha^2\beta^2}X\right)\partial_X^5\Phi_{\rm GC} + \left(\frac{2\alpha^4}{5} - \frac{612}{5\alpha^2\beta^2} - \frac{33}{4\beta^2}\right)\partial_X^4\Phi_{\rm GC}$$
$$+ \left(\frac{27X}{\alpha^4\beta^2} - \frac{\alpha^2 X}{3} + \frac{8\alpha^4}{5X} - \frac{828}{5\alpha^2\beta^2 X} - \frac{33}{\beta^2 X}\right)\partial_X^3\Phi_{\rm GC} + \left(\frac{135}{\alpha^4\beta^2} + \frac{2\alpha^4}{5} - \frac{71\alpha^2}{15} + X^2\right)\partial_X^2\Phi_{\rm GC}$$
$$+ \left(\frac{108}{\alpha^4\beta^2 X} + \frac{4\alpha^4}{5X} - \frac{112\alpha^2}{15X} + 5X\right)\partial_X\Phi_{\rm GC} + 3\Phi_{\rm GC}$$
$$= \left(\frac{\alpha^2}{3} - \frac{27}{\alpha^4\beta^2}\right)\frac{X\partial_X\rho}{2\mu^2 M_{\rm Pl}^2} + \left(\frac{17}{5}\alpha^2 - 2\alpha^4 - \frac{27}{\alpha^4\beta^2}\right)\frac{\rho}{2\mu^2 M_{\rm Pl}^2}\,.$$

Ghost Condensate coupled to Gravity in dS

- $\rho = 0$

$\alpha = 4 \qquad \beta = 2$

- $\dfrac{\rho}{m^2} = 10^{-11}\exp(-10^{-3}X^2)$



$\alpha = 4 \qquad \beta = 2$

Ghost Condensate coupled to Gravity in dS

- For tensor perturbations, considering the following metric

$$g_{ij} = -2a^2(t)(\delta_{ij} + \mathscr{H}_{ij}) , \qquad \text{where} \qquad \mathscr{H}_{ii} = 0 , \qquad \partial_i \mathscr{H}_{ij} = 0 .$$

- The term proportional to σ_1, $\gamma^{ij}\nabla_i K_{ml} \nabla_j K^{ml}$, adds the following contribution

$$S_{\sigma_1} = -\frac{\sigma_1}{2} \int \mathrm{d}t \ \mathrm{d}^3x \ a^3 \left[a^{-2} \left(\partial_k \dot{\mathscr{H}}_{ij} \right)^2 \right] .$$

which in turn modifies the speed of GWs as follows,

$$c_T^2 = \frac{1}{1 - 4\sigma_1 \frac{k^2}{M_{\mathrm{Pl}}^2}} .$$

Only when $k \sim \dfrac{M_{\mathrm{Pl}}^2}{\sqrt{|\sigma_1|}}$, one sees a substantial modification to the speed of GWs.

Ghost Condensate coupled to Gravity in dS

- Subluminal GW requires $\sigma_1 \leq 0$

- $H \lesssim k \lesssim M$, where the latter inequality comes from the viability of EFT.

- To see substantial deviation from the luminal propagation of the GWs:

  - For $k \sim M \sim 10^{-3}$ eV, $|\sigma_1| \sim 10^{60}$

  - For $k \sim H \sim 10^{-33}$ eV, $|\sigma_1| \sim 10^{120}$

- From the LIGO-Virgo observation of the binary neutron star merger GW170817 $-3 \times 10^{-15} < c_T - 1 < 7 \times 10^{-16}$ in the frequency range $10 \text{ Hz} \lesssim f \lesssim 10 \text{ kHz}$

$$\Rightarrow \begin{cases} f \sim 10 \text{ Hz} \Rightarrow |\sigma_1| \lesssim 10^{69} \\ f \sim 10 \text{ kHz} \Rightarrow |\sigma_1| \lesssim 10^{63} \end{cases}$$

Conclusion

- GC is an IR modification of gravity with no UV strong coupling in which the additional d.o.f. acquires a non-standard dispersion relation

- We exorcized the sextic dispersion relation, noting that accelerated expansion of the universe is a recent cosmological phenomenon.

- If the GC field is coupled to SM, the axial coupling gives rise to different dispersion relations for particles and antiparticles.

- Also it produces a spin-dependent scale-independent force.

- The instability at the scales $\mu^{-1} \sim \dfrac{M_{\rm Pl}}{M^2}$ occurs over the timescale $t_c \sim \dfrac{M_{\rm Pl}^3}{M^4}$

Conclusion

- The potential Φ acquires oscillatory features in the scales μ^{-1} over the time scales t_c

- For the GC with a sextic dispersion relation, in an expanding background, the EoM of $\Phi_{\rm GC}$ is governed by the spacetime variations of ρ too.

- The potential acquires oscillatory features on top of $\dfrac{1}{X}$ behavior.

- The term in the unitary gauge action induces correction to the GWs perturbation, and modifies the speed of GWs as

$$c_T^2 = \frac{1}{1 - \sigma_1 \frac{k^2}{M_{\rm Pl}^2}}$$

Thanks for your attention!

Coupling of the Ghost Field to the SM particles

- If the ghost field is coupled to the SM fields through the derivative vector coupling:

$$\frac{c_\psi}{F}\bar{\psi}\gamma^\mu\psi\partial_\mu\phi$$

and for ψ, the symmetry $\psi \to e^{ic_\psi\phi/F}\psi$ is broken by the mass term $m_D\psi\psi^c$, then the coupling could be removed.

- However the axial coupling

$$\frac{c_\psi}{F}\bar{\psi}\gamma^\mu\gamma^5\psi\partial_\mu\phi$$

gives rise to different dispersion relation for the particle and anti-particles.

$$\omega = \sqrt{(|p| \pm m_{\rm eff})^2 + m_D^2} \qquad m_{\rm eff} \equiv M^2/F$$

plus and negative signs are respectively for the left-helicity particle and right-helicity antiparticles.

Coupling of the Ghost Field to the SM particles

- If the earth is moving with respect to the background where the ghost field is isotropic, the interaction becomes

$$m_{\text{eff}}\bar{\psi}\vec{\gamma}\gamma^5\psi \cdot \vec{v}_{\text{earth}} \sim \mu\vec{S}\cdot\vec{v}_{\text{earth}}$$

where $|\vec{v}_{\text{earth}}| \sim 10^{-3}$. For electrons, $m_{\text{eff}} \lesssim 10^{-25}$ GeV and for protons and neutrons, $m_{\text{eff}} \lesssim 10^{-24}$ GeV.

- A long range force is expected from the exchange of π, which has the derivative coupling to spin,

$$\Delta\mathscr{L} \sim \frac{1}{F}\vec{S}\cdot\nabla\pi,$$

- In the limit $\omega \to 0$, and assuming that the sources are static in the $\tau \sim \omega^{-1} \sim M^2 r^3$, there is a spin-dependent but distance-independent force with potential

$$V = \frac{M^4}{32\pi\bar{S}F^2}\left[\vec{S}_1\cdot\vec{S}_2 + (\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})\right]r\,.$$