Spectral behavior of scalar fluctuations produced by gravity during Reheating

Simon Cléry, TUM

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* *Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production,* Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

<u>*See also*</u>

* *Boltzmann or Bogoliubov? Approaches Compared in Gravitational Particle Production,* Kaneta, Lee, Oda, **2206.10929**
* *A New Window into Gravitationally Produced Scalar Dark Matter,* Garcia, Pierre, Verner, **2305.14446**
* *Cosmological gravitational particle production and its implications for cosmological relics,* Kolb and Long, **2312.09042**

Technische Universität München

SFB 1258 | Neutrinos Dark Matter Messengers

Scalar field in curved space-time

Consider a spectator scalar field in a classical gravitational background

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_P^2\left(1+\xi\,\frac{\chi^2}{M_P^2}\right)R + \underbrace{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2}_{\text{spectator scalar}} + \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{inflaton (background)}}\right]$$

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➜ ξ non-minimal coupling of the spectator scalar field

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In a classical Cosmological background $\quad \mathrm{g}_{\mu\nu}^{\mathrm{FLRW}}(x) = a^2(\eta)\,\mathrm{diag}(1,-1,-1,-1)$

$\boxed{R(\eta) = -6a''/a^3}$ gravitational "potential" term

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$$X_{\vec{k}} = a\chi_{\vec{k}} \longrightarrow X''_{\vec{k}} + \left[k^2 + \boxed{a^2m_\chi^2 + \frac{a^2R}{6}(1+6\xi)}\right]X_{\vec{k}} = 0$$

$$m_{\mathrm{eff}}^2(\eta) = m_\chi^2 + (\tfrac{1}{6}+\xi)R(\eta)$$

➜ Time-dependent effective mass which sources gravitational effects

➜ For ξ = -1/6 (conformally coupled), no gravitational effective mass

Modes equation in cosmological background

$$X''_{\vec{k}} + \underbrace{\left[k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6}(1+6\xi)\right]}_{\omega_k(\eta)} X_{\vec{k}} = 0$$

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Time-dependent frequency through the background evolution

➜ observers at different times may decompose operators onto different bases of mode functions and ladder operators

➜ vacuum can be further populated by scalar excitations throughout background evolution

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Already outlined by Schrödinger in 1939, as a potentially **“alarming phenomenon”** for expanding Universe
The Proper Vibrations of the Expanding Universe, Physica (1939)

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➜ vacuum can be further populated by scalar excitations throughout background evolution

$$\begin{aligned} X(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \left(\boxed{a_{\boldsymbol{k}}^{\mathrm{IN}} X_{\vec{k}}^{\mathrm{IN}}(\eta)} + a_{-\boldsymbol{k}}^{\mathrm{IN}\dagger} X_{\vec{k}}^{\mathrm{IN}*}(\eta) \right) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \\ &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \left(\boxed{a_{\boldsymbol{k}}^{\mathrm{OUT}} X_{\vec{k}}^{\mathrm{OUT}}(\eta)} + a_{-\boldsymbol{k}}^{\mathrm{OUT}\dagger} X_{\vec{k}}^{\mathrm{OUT}*}(\eta) \right) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} . \end{aligned}$$

Bogoliubov transformation

SU(1,1) Bogoliubov transformation from ***in*** to ***out*** base

$$X^{\mathrm{IN}}_{\vec{k}}(\eta) = \boxed{\alpha_k}\, X^{\mathrm{OUT}}_{\vec{k}}(\eta) + \boxed{\beta_k}\, X^{\mathrm{OUT}*}_{\vec{k}}(\eta)$$

$$a^{\mathrm{IN}}_{\boldsymbol{k}} = \alpha^*_k\, a^{\mathrm{OUT}}_{\boldsymbol{k}} - \beta^*_k\, a^{\mathrm{OUT}\dagger}_{-\boldsymbol{k}} \,,$$

$$\boxed{\eta \to +\infty}$$

➜ find “in” mode functions at late asymptotic times and project on the “out” base to “count” excitations

$$\langle 0^{\mathrm{IN}} | N^{\mathrm{OUT}} | 0^{\mathrm{IN}} \rangle = V \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} |\beta_k|^2 \longrightarrow a^3 n = \int \frac{\mathrm{d}k}{k} \boxed{\frac{k^3}{2\pi^2} |\beta_k^2|}$$

spectrum for scalar modes sourced by gravity

$$\beta_k = i \left(X^{\mathrm{OUT}\prime}_k X^{\mathrm{IN}}_k - X^{\mathrm{IN}\prime}_k X^{\mathrm{OUT}}_k \right) \simeq \lim_{\eta \to +\infty} \tilde{\beta}_k(\eta)$$

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$$n_k = |\beta_{\vec{k}}|^2 = \frac{1}{2\omega_k} |\omega_k X^{\mathrm{IN}}_{\vec{k}} - i X^{\mathrm{IN}\prime}_{\vec{k}}|^2$$

$$\tilde{\alpha}'_k(\eta) = \tilde{\beta}_k(\eta) \frac{\omega'_k}{2\omega_k} e^{2i\int^\eta \omega_k(\tau)d\tau}$$

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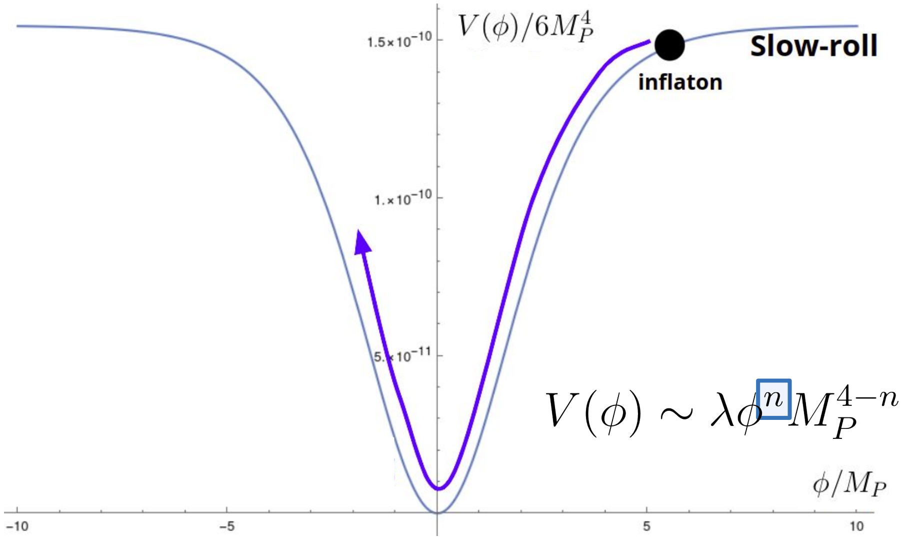
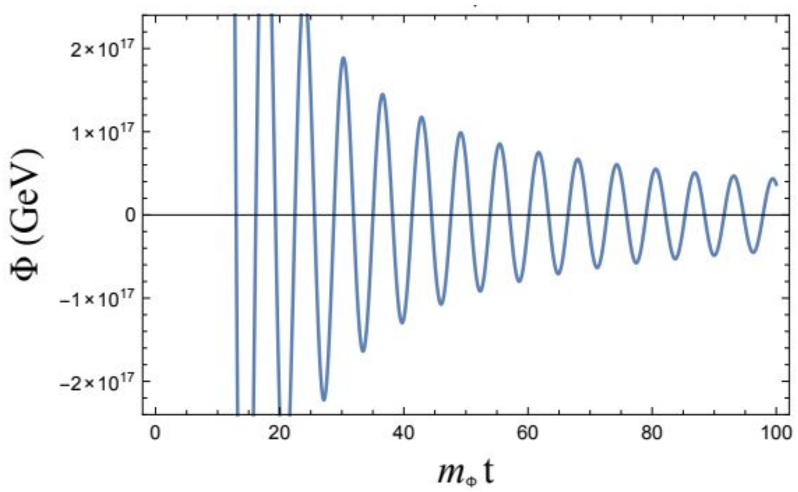
$\eta \to +\infty$

Inflaton oscillations during Reheating





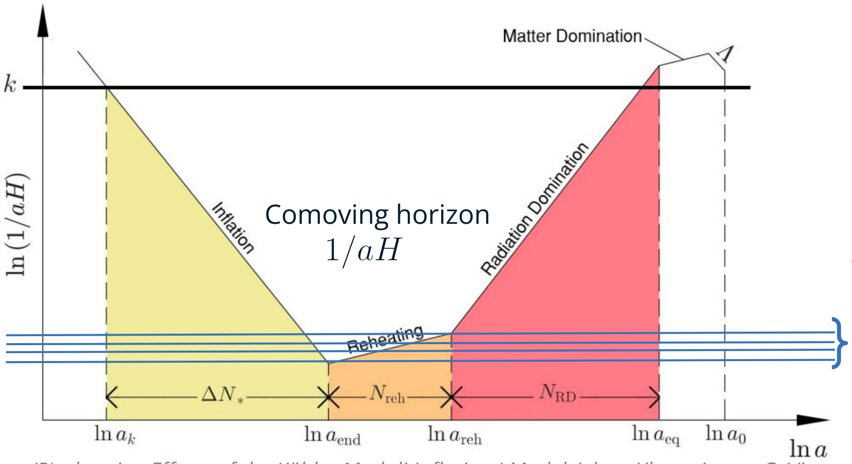
EOM : $\ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi(t)) = 0$

Redshifted envelop and frequency of the oscillations depend on the shape of the potential near the minimum

Couplings of the inflaton with the other fields induce transfer of energy during the oscillations : reheating

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle}{\frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle} = \boxed{\frac{n-2}{n+2}}$$

Long wavelength modes during Reheating



From *(P)reheating Effects of the Kähler Moduli Inflation I Model*, Islam Khan, Aaron C. Vincent and Guy Worthey, **2111.11050**

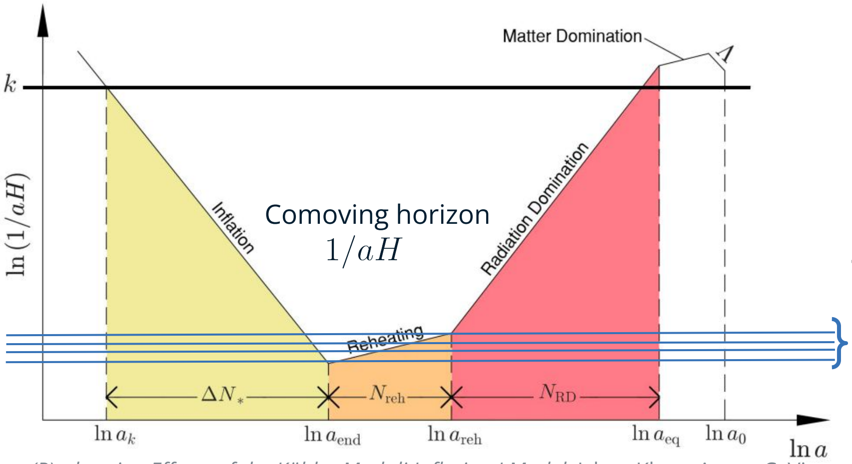
Super-horizon and reentry during reheating

$$k^2 < \left(\frac{a''}{a} - a^2 m_\chi^2\right)\Big|_{a_{\rm e}} = a_{\rm e}^2(2H_{\rm e}^2 - m_\chi^2)$$

➜ occupation number is affected by the equation of state during reheating $\boxed{w_\phi}$

➜ fast inflaton oscillations do not affect the occupation number of the long-wavelength modes

Long wavelength modes during Reheating



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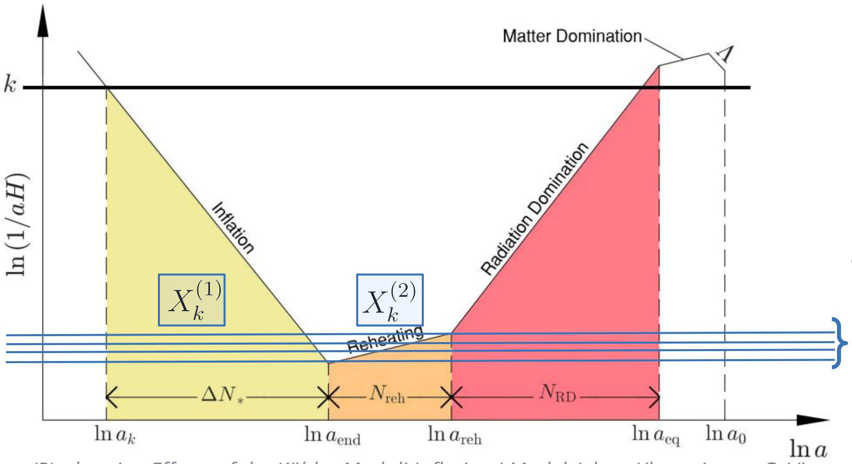
Solve numerically mode equations

Long wavelength modes during Reheating



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$$\beta_k = i\left(X_k^{(2)\prime} X_k^{(1)} - X_k^{(1)\prime} X_k^{(2)}\right)$$

Analytical derivation of the spectrum

Long wavelength modes during Reheating

$$X''_{\vec{k}} + \left[k^2 + a^2 \cancel{m_\chi^2} + \frac{a^2 R}{6}(1 + \cancel{6\xi})\right] X_{\vec{k}} = 0$$ from now consider **only minimal gravitational coupling**

Long wavelength modes during Reheating

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$$\boxed{|\beta_k|^2_{\rm IR} = \frac{\mathcal{D}}{2\pi}\left(\frac{k_{\rm e}}{k}\right)^{(2\bar{\nu}+3)}},$$

$$\boxed{a^3 \left.\frac{dn_\chi}{d\ln k}\right|_{\rm IR} = 2\mathcal{D}\frac{k_{\rm e}^3}{(2\pi)^3}\left(\frac{k_{\rm e}}{k}\right)^{\boxed{2\bar{\nu}}}}$$

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w_ϕ	$\bar{\nu}$
0	3/2
1/3	1/2
1/2	3/10
3/5	3/14
2/3	1/6
5/7	3/22
3/4	3/26
4/5	3/34
9/10	3/74

$$\boxed{\bar{\nu} = \frac{3}{2}\frac{(1 - w_\phi)}{(1 + 3w_\phi)}}$$

Spectral behavior of gravitationally produced ***massless*** *scalar perturbations (IR)*

Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

➜ spectral behavior in the IR varies from k^{-3} for $w_\phi = 0$ to a flat spectrum in the limit $w_\phi \to 1$

Long wavelength modes during Reheating

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we can determine the effect of a dominant mass term

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$$\frac{k_m}{k_\mathrm{e}} = \left(\sqrt{\frac{2}{|3w_\phi - 1|}} \frac{m_\chi}{H_\mathrm{e}}\right)^{\frac{1+3w_\phi}{3(1+w_\phi)}}$$

below this comoving scale, mass term dominates at horizon reentry

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➜ at smaller comoving scales , we obtain a flat spectrum for massive scalar modes whatever the EoS w_ϕ

$$\boxed{|\beta_k|^2 \propto \left(k_e/k\right)^3}$$

Long wavelength modes during Reheating

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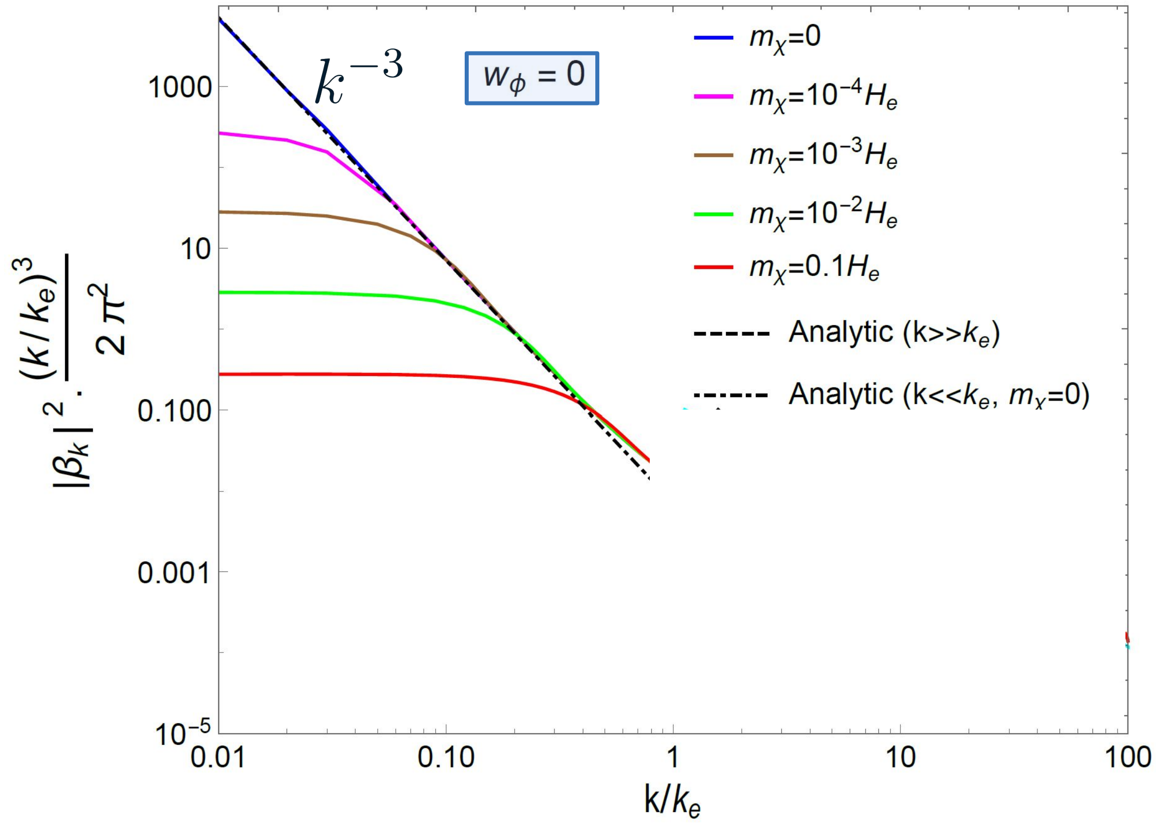
$$\boxed{|\beta_k|^2 \propto \left(k_\mathrm{e}/k\right)^3}$$

➜ at large mass, $(m_\chi/H_\mathrm{e}) > 3/2$, exponentially suppressed spectrum $\beta_k \propto e^{-\frac{m_\chi}{H_e}\frac{k^2}{(a_e H_e)^2}}$

Long wavelength modes during Reheating



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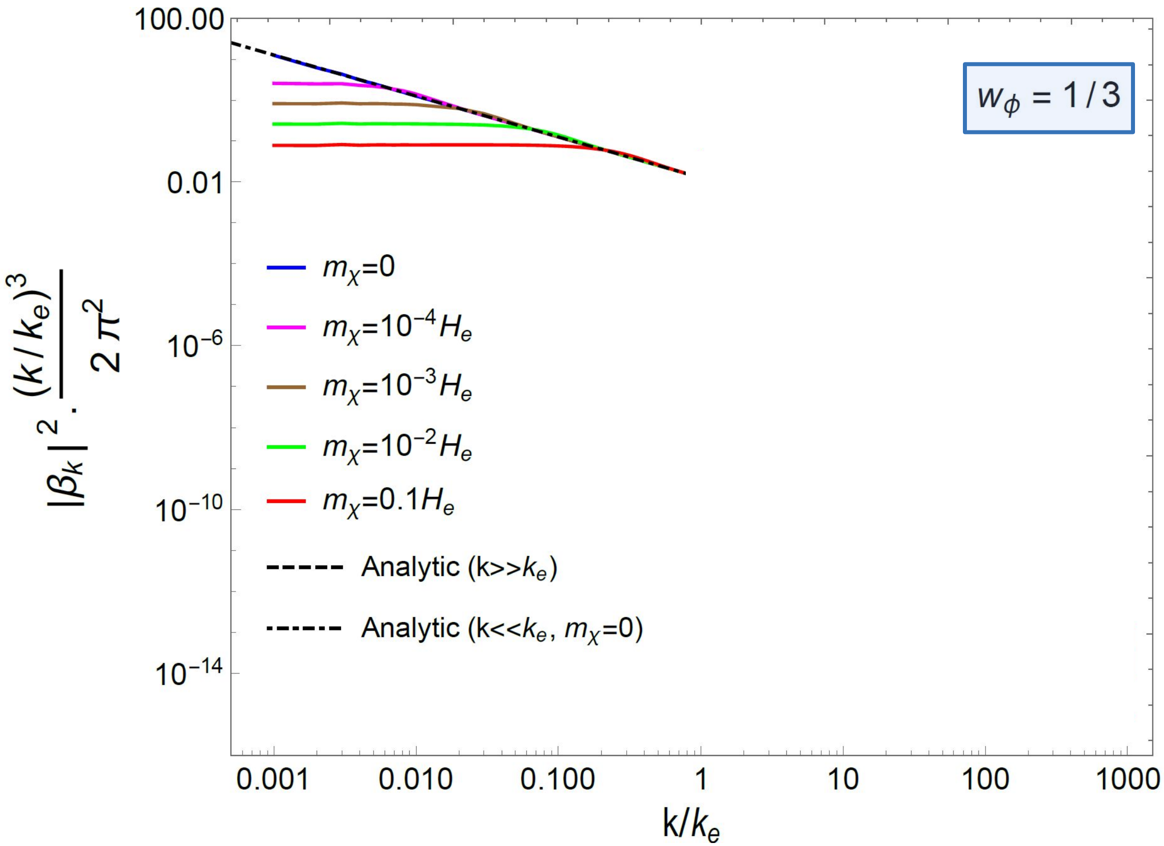

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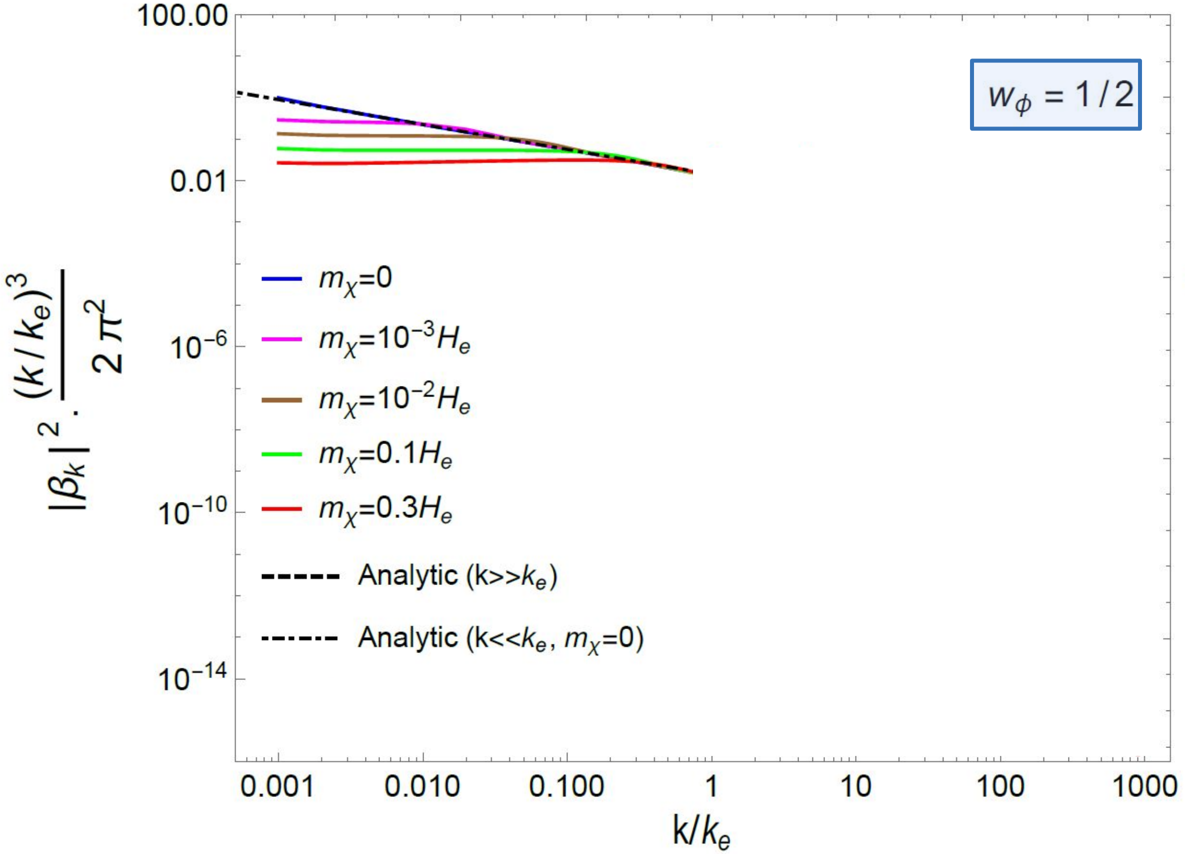

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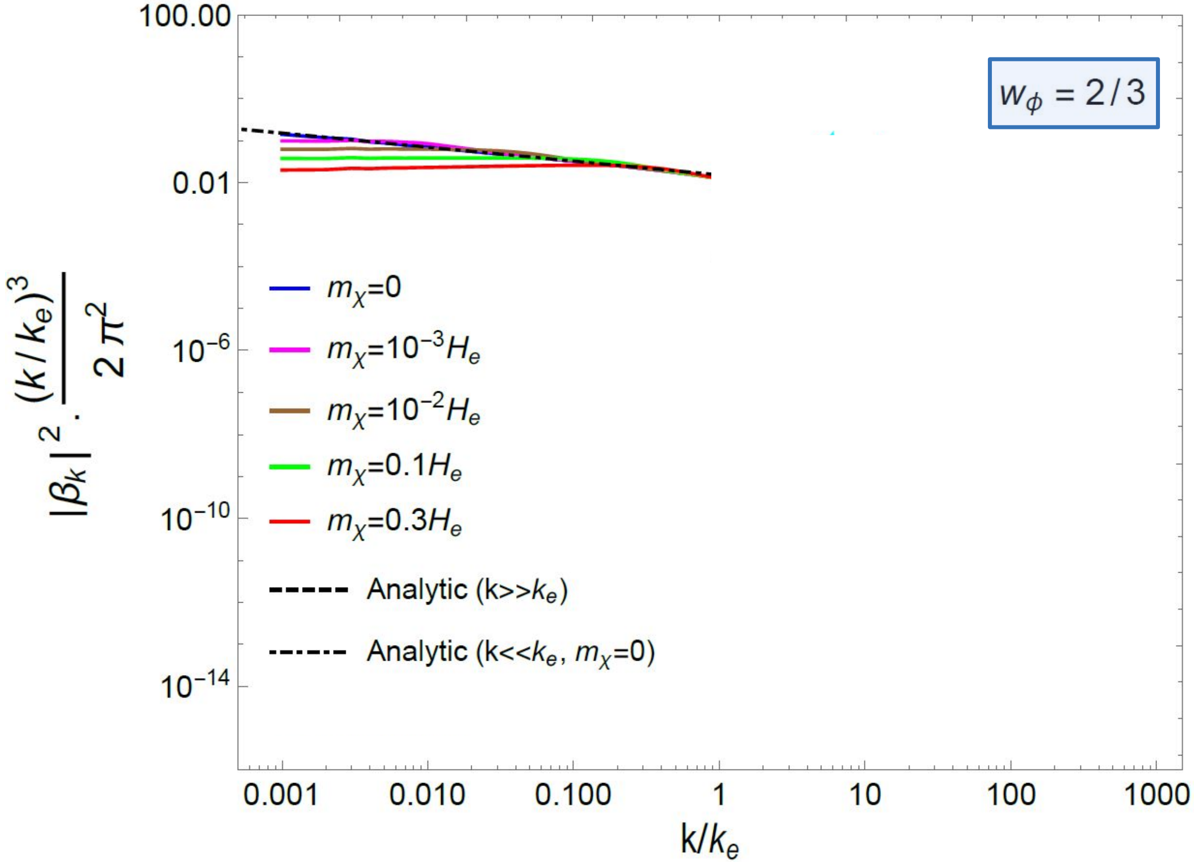

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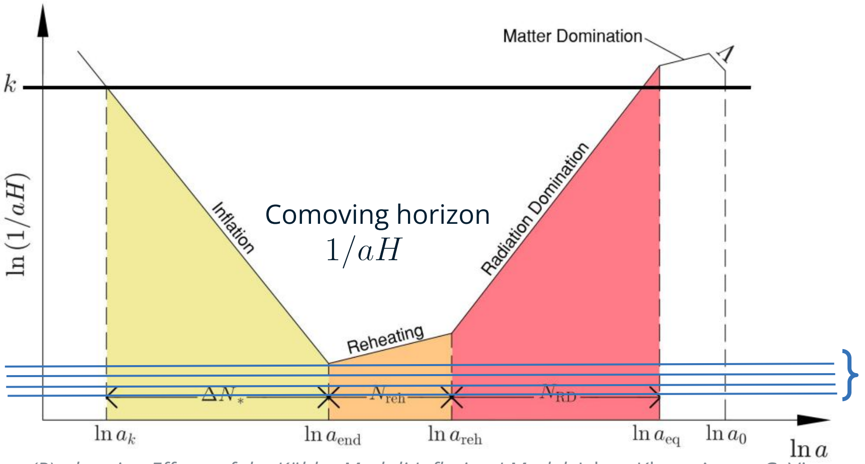
Short wavelength modes during Reheating



Sub-horizon modes

$$k > a_{\rm e} H_{\rm e}$$

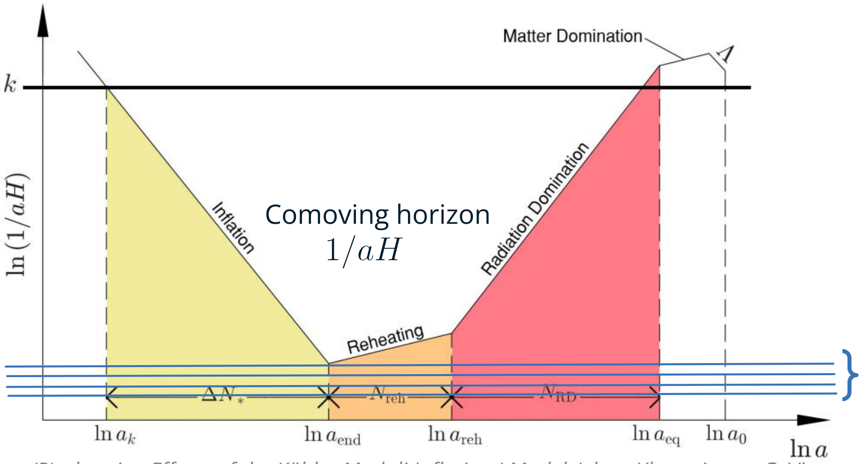
From *(P)reheating Effects of the Kähler Moduli Inflation I Model*, Islam Khan, Aaron C. Vincent and Guy Worthey, **2111.11050**

➜ no violation of adiabaticity in the evolution of mode frequency during inflation and reheating

➜ expect small occupation number in the UV spectrum induced by gravity

➜ fast inflaton oscillations affect the occupation number of the short-wavelength modes

Short wavelength modes during Reheating



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$$|\tilde{\beta}_k(\eta)| \ll 1$$

$$\boxed{\frac{\omega_k'(\eta)}{\omega_k^2(\eta)} \ll 1}$$

$$\beta_k(\eta) \simeq \int_{\eta_{\rm e}}^{\eta} d\eta' \,\frac{\omega_k'}{2\omega_k} e^{-2i\Omega_k(\eta')}$$

$$\Omega_k(\eta') = \int_{\eta_{\rm e}}^{\eta'} \omega_k(\eta) d\eta$$

➜ advantage to compute the Bogoliubov coefficient without solving exactly the mode equations

Inflaton fast oscillations and Fourier modes

Post-inflation background oscillations

$$\boxed{\phi(t) = \Phi(t)\mathcal{P}(t) = \Phi(t)\sum_{\nu\neq 0}\mathcal{P}_\nu e^{i\nu\omega t}}$$

➜ a decaying amplitude $\Phi(t)$ and a quasi-periodic part $\mathcal{P}(t)$ both depends on w_ϕ

➜ develop the oscillating part in Fourier modes $\mathcal{P}_\nu$

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$$H(a) \simeq \boxed{\bar{H}}\left(1 + \frac{\boxed{\mathcal{P}\sqrt{6(1-\mathcal{P}^{2n})}}}{2(n+1)}\left(\frac{\phi_{\rm e}}{M_P}\right)\left(\frac{a}{a_{\rm e}}\right)^{-\frac{3}{n+1}}\right) \qquad \bar{H} = H_{\rm e}\left(a/a_{\rm e}\right)^{-\frac{3n}{n+1}}$$

➜ extract slowly varying amplitude and fast oscillations of all background quantities

➜ higher order oscillating terms are suppressed during reheating by the decay of the inflaton amplitude

Inflaton fast oscillations and Fourier modes

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$$\frac{\dot{\omega}_k(t)}{\omega_k(t)} = \frac{a^2}{\omega_k^2}\left[Hm_\chi^2 - 2H^3 - 3H\dot{H} - \frac{1}{2}\ddot{H}\right]$$

➜ adiabatic variation of modes frequency in terms of the inflaton slowly decaying amplitude and fast oscillating part

Short wavelength modes during Reheating

$$\beta_k \simeq \frac{1}{2}\sum_{\nu,l\neq 0}\int_{t_{\rm e}}^{t} dt' \left(\frac{t_{\rm e}}{t'}\right)^3 \left[\mathcal{N}_0 e^{i(\nu+l)\omega t'}\left(\frac{t'}{t_{\rm e}}\right)^{\frac{1}{n}} + \mathcal{N}_1 e^{i\nu\omega t'} + \mathcal{N}_2 + \mathcal{N}_3 e^{i(\nu+l)\omega t'}\left(\frac{t_{\rm e}}{t'}\right)^{\frac{1}{n}}\right] \times \frac{e^{-2i\Omega_k(t')}}{\left(\frac{k^2}{a^2}+m_\chi^2\right)}$$

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$$\mathcal{N}_0 = 3\sqrt{6}nH_\mathrm{e}^3\left(\mathcal{P}^{2n-1}\right)_\nu \left(\sqrt{1-\mathcal{P}^{2n}}\right)_l \left(\frac{M_P}{\phi_\mathrm{e}}\right)$$

$$\mathcal{N}_1 = \frac{9}{n+1}H_\mathrm{e}^3\ \mathcal{P}_\nu^{2n}\,; \quad \mathcal{N}_2 = \frac{7n-11}{n+1}H_\mathrm{e}^3\,;$$

$$\mathcal{N}_3 = \frac{3\sqrt{6}H_\mathrm{e}^3\mathcal{P}_\nu\left(\sqrt{1-\mathcal{P}^{2n}}\right)_l}{2(n+1)^2}(4n-5)\left(\frac{\phi_\mathrm{e}}{M_P}\right)$$

Short wavelength modes during Reheating

Case $w_\phi < 1/3$

$$\beta_k \simeq \frac{1}{2}\sum_{\nu,l\neq 0}\int_{t_e}^{t} dt' \left(\frac{t_e}{t'}\right)^3 \left[\mathcal{N}_0 e^{i(\nu+l)\omega t'}\left(\frac{t'}{t_e}\right)^{\frac{1}{n}} + \cancel{\mathcal{N}_1 e^{i\nu\omega t'}} + \mathcal{N}_2 + \cancel{\mathcal{N}_3 e^{i(\nu+l)\omega t'}}\left(\frac{t_e}{t'}\right)^{\frac{1}{n}}\right] \times \frac{e^{-2i\Omega_k(t')}}{\left(\frac{k^2}{a^2}+m_\chi^2\right)}$$

use stationary phase approximation

integrate by parts and take the large momentum contribution

Short wavelength modes during Reheating

Case $w_\phi < 1/3$

$$\beta_k \simeq \frac{1}{2}\sum_{\nu,l\neq 0}\int_{t_e}^{t} dt' \left(\frac{t_e}{t'}\right)^3 \left[\mathcal{N}_0 e^{i(\nu+l)\omega t'}\left(\frac{t'}{t_e}\right)^{\frac{1}{n}} + \cancel{\mathcal{N}_1 e^{i\nu\omega t'}} + \mathcal{N}_2 + \cancel{\mathcal{N}_3 e^{i(\nu+l)\omega t'}}\left(\frac{t_e}{t'}\right)^{\frac{1}{n}}\right] \times \frac{e^{-2i\Omega_k(t')}}{\left(\frac{k^2}{a^2}+m_\chi^2\right)}$$

use stationary phase approximation

integrate by parts and take the large momentum contribution

$$|\beta_k|^2_{\mathrm{UV},w_\phi<\frac{1}{3}} = \begin{cases} (\bar{\mathcal{N}}_0)^2 \boxed{k^{\frac{9(w_\phi-1)}{2-6w_\phi}}} + \underbrace{\bar{\mathcal{N}}_0\bar{\mathcal{N}}_2 k^{\frac{45w_\phi-21}{4(1-3w_\phi)}}\cos\psi}_{\text{interference term}} & w_\phi \leq 1/9 \\ (\bar{\mathcal{N}}_2)^2 \boxed{k^{-6}} + \underbrace{\bar{\mathcal{N}}_0\bar{\mathcal{N}}_2 k^{\frac{45w_\phi-21}{4(1-3w_\phi)}}\cos\psi}_{\text{interference term}} & w_\phi > 1/9 \end{cases}$$

➜ recover the well known $k^{-9/2}$ behavior for $w_\phi = 0$

Short wavelength modes during Reheating

Case $w_\phi \geq 1/3$

$$\beta_k \simeq \frac{1}{2}\sum_{\nu,l\neq 0}\int_{t_e}^{t} dt' \left(\frac{t_e}{t'}\right)^3 \left[\mathcal{N}_0 e^{i(\nu+l)\omega t'}\left(\frac{t'}{t_e}\right)^{\frac{1}{n}} + \mathcal{N}_1 e^{i\nu\omega t'} + \mathcal{N}_2 + \mathcal{N}_3 e^{i(\nu+l)\omega t'}\left(\frac{t_e}{t'}\right)^{\frac{1}{n}}\right] \times \frac{e^{-2i\Omega_k(t')}}{\left(\frac{k^2}{a^2}+m_\chi^2\right)}$$

➜ no stationary phase within integration range: extract only the large momentum contribution

$$|\beta_k|^2_{\mathrm{UV},w_\phi\geq\frac{1}{3}} \simeq \frac{1}{16 f^2(w_\phi)} \boxed{\left(\frac{a_e}{k}\right)^6} \times \sum\sum\left[\mathcal{N}_0+\mathcal{N}_1+\mathcal{N}_2+\mathcal{N}_3\right]^2$$

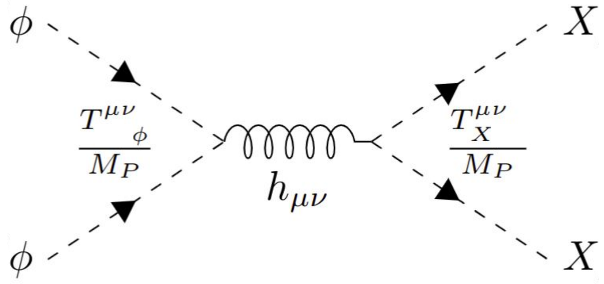
➜ on the whole range $1/9 < w_\phi \leq 1$ spectrum independent of w_ϕ in the UV and $\propto k^{-6}$

Effective graviton portal from perturbative computation

Graviton portal from effective gravitational interaction to small metric perturbations

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2h_{\mu\nu}/M_P$$

$$\boxed{\mathcal{L}_{\text{min.}} = -\frac{1}{M_P} h_{\mu\nu} \left(T^{\mu\nu}_{\phi} + T^{\mu\nu}_{X} \right)}$$



$$T^{\mu\nu}_0 = \partial^\mu S \partial^\nu S - g^{\mu\nu} \left[\frac{1}{2} \partial^\alpha S \partial_\alpha S - V(S) \right]$$

➜ consider massless gravitons (perturbations) coupled to stress-energy and compute the amplitude of the process

$$\boxed{\sum_{\nu=1}^{\infty} |\overline{\mathcal{M}_\nu}|^2 = \frac{1}{2} \times \sum_{\nu=1}^{\infty} \frac{\rho_\phi^2}{M_P^4} |\mathcal{P}_\nu^{2n}|^2}$$

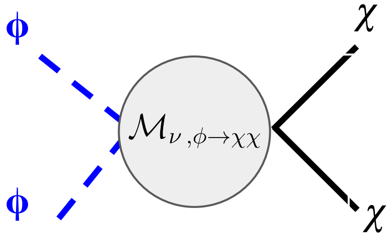
Spin-2 Portal Dark Matter, Bernal, Dutra, Mambrini, Olive, Peloso, Pierre, **1803.01866**

Gravitational Production of Dark Matter during Reheating, Mambrini, Olive, **2102.06214**

Gravitational portals in the early Universe, SC, Mambrini, Olive, Verner, **2112.15214**

Boltzmann approach

Transition amplitude computed perturbatively



Inflaton as a coherently oscillating homogeneous condensate

$$f_\phi(k',t) = (2\pi)^3 n_\phi(t)\delta^{(3)}(\vec{k'})$$

Post-inflation background oscillations

$$\boxed{\phi(t) = \Phi(t)\mathcal{P}(t) = \Phi(t)\sum_{\nu\neq 0}\mathcal{P}_\nu e^{i\nu\omega t}}$$

➜ each Fourier mode can contribute to the transition amplitude

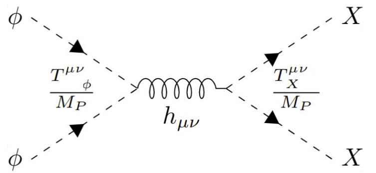
$$\frac{\partial f_\chi}{\partial t} - H|\vec{p}|\frac{\partial f_\chi}{\partial|\vec{p}|} = C\left[f_\chi(|\vec{p}|,t)\right]$$

$$\dot{n}_\chi + 3Hn_\chi = R^{(\mathrm{N})}_{\phi\phi\to\chi\chi}$$

$$\frac{d\rho_\phi}{dt} + 3H(1+w_\phi)\rho_\phi \simeq -(1+w_\phi)\Gamma_\phi\rho_\phi$$

Short wavelength modes from Boltzmann approach

$$\frac{\partial f_\chi}{\partial t} - H|\vec{p}|\frac{\partial f_\chi}{\partial |\vec{p}|} = \sum_{\nu=1}^{\infty} \frac{\pi |\overline{\mathcal{M}_\nu}|^2}{2(p^0)^2}\delta\left(\frac{\nu\omega}{2}-p^0\right)(1+2f_\chi(p))$$



$$f_\chi(k,a) = \frac{9\pi}{4|3w_\phi - 1|}\left(\frac{\bar{\alpha} m_\phi^{\mathrm{e}}}{H_{\mathrm{e}}}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}}\left(\frac{k_{\mathrm{e}}}{k}\right)^{\frac{9(1-w_\phi)}{2(1-3w_\phi)}}\sum_{\nu=1}^{\infty}|\mathcal{P}_\nu^{2n}|^2\left(\frac{\nu}{2}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}}\theta\left(\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}}\bar{\alpha}}\right)^{\frac{1}{1-3w_\phi}} - 1\right)$$

$$\times\,\theta\left(\left(\frac{a}{a_{\mathrm{e}}}\right)\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}}\bar{\alpha}}\right)^{\frac{1}{3w_\phi-1}} - 1\right)$$

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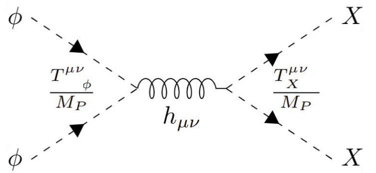

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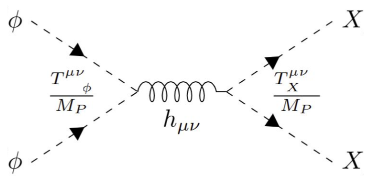
$$\boxed{\times\,\theta\left(\left(\frac{a}{a_{\mathrm{e}}}\right)\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}}\bar{\alpha}}\right)^{\frac{1}{3w_\phi-1}}-1\right)}$$

➜ for $w_\phi < 1/3$ same stationary phase contribution as in the Bogoliubov approach

Short wavelength modes from Boltzmann approach

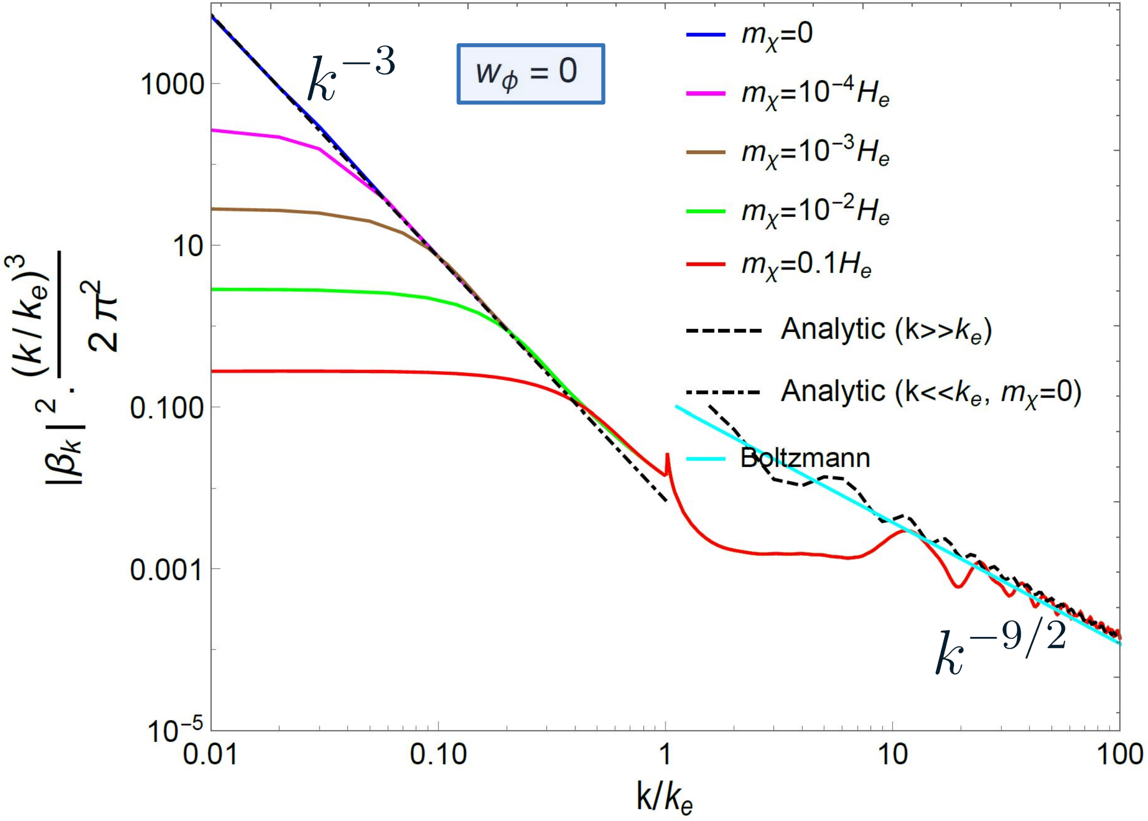
$$\frac{\partial f_\chi}{\partial t} - H|\vec{p}|\frac{\partial f_\chi}{\partial|\vec{p}|} = \sum_{\nu=1}^{\infty} \frac{\pi|\overline{\mathcal{M}_\nu}|^2}{2(p^0)^2}\delta\left(\frac{\nu\omega}{2} - p^0\right)(1 + 2f_\chi(p))$$



$$f_\chi(k,a) = \frac{9\pi}{4|3w_\phi - 1|}\left(\frac{\bar{\alpha} m_\phi^{\mathrm{e}}}{H_{\mathrm{e}}}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}} \boxed{\left(\frac{k_{\mathrm{e}}}{k}\right)^{\frac{9(1-w_\phi)}{2(1-3w_\phi)}}} \sum_{\nu=1}^{\infty} |\mathcal{P}_\nu^{2n}|^2 \left(\frac{\nu}{2}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}} \theta\left(\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}} \bar{\alpha}}\right)^{\frac{1}{1-3w_\phi}} - 1\right)$$
$$\boxed{\times\, \theta\left(\left(\frac{a}{a_{\mathrm{e}}}\right)\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}} \bar{\alpha}}\right)^{\frac{1}{3w_\phi - 1}} - 1\right)}$$

➜ recover the well known $k^{-9/2}$ behavior for $w_\phi = 0$

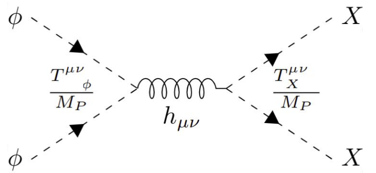
Spectrum for w = 0



Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

Short wavelength modes from Boltzmann approach

$$\frac{\partial f_X}{\partial t} - H|\vec{p}|\frac{\partial f_X}{\partial |\vec{p}|} = \sum_{\nu=1}^{\infty} \frac{\pi |\overline{\mathcal{M}_\nu}|^2}{2(p^0)^2}\delta\left(\frac{\nu\omega}{2} - p^0\right)(1 + 2f_X(p))$$



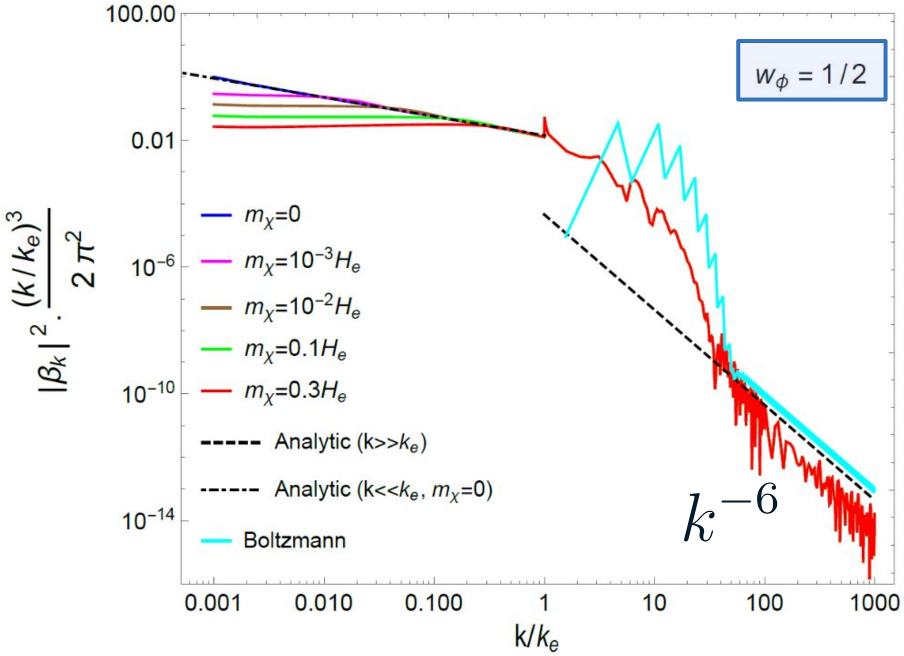
$$f_X(k,a) = \frac{9\pi}{4|3w_\phi - 1|}\left(\frac{\bar{\alpha} m_\phi^{\mathrm{e}}}{H_{\mathrm{e}}}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}}\left(\frac{k_{\mathrm{e}}}{k}\right)^{\frac{9(1-w_\phi)}{2(1-3w_\phi)}} \sum_{\nu=1}^{\infty} |\mathcal{P}_\nu^{2n}|^2 \left(\frac{\nu}{2}\right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}} \theta\left(\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}} \bar{\alpha}}\right)^{\frac{1}{1-3w_\phi}} - 1\right)$$

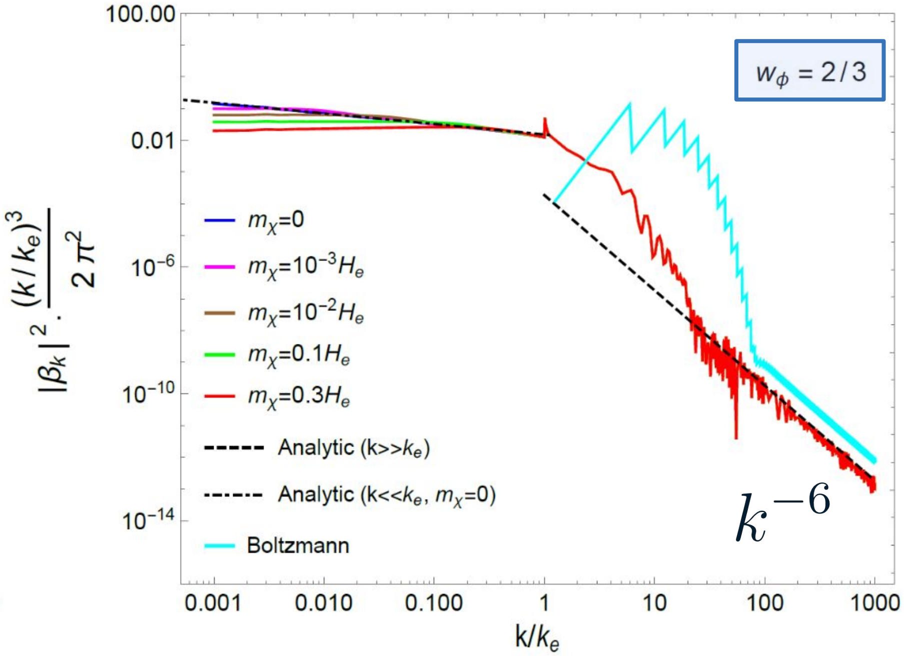
$$\times\, \theta\left(\left(\frac{a}{a_{\mathrm{e}}}\right)\left(\frac{2k}{\nu a_{\mathrm{e}} m_\phi^{\mathrm{e}} \bar{\alpha}}\right)^{\frac{1}{3w_\phi - 1}} - 1\right)$$

$$\nu\bar{\alpha}/2 \geq (k/a_{\mathrm{e}} m_\phi^{\mathrm{e}})$$

➜ for $w_\phi > 1/3$ negative spectral index but only suppressed higher inflaton Fourier modes contribute

Spectrum for higher EoS w > 1/3





Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

Conclusion

- Generalize results of scalar gravitational production by including the Reheating dynamics
- Increasing EoS during Reheating leads to flatter spectrum of light scalar fluctuations (flat spectrum in the limit of kination)
- Oscillatory features in the UV tail can be computed using Bogoliubov transformation
- UV tail in the spectrum is independent of the EoS for a $w > 1/9$
- Agreement for the UV tail power-law behavior between the non-perturbative Bogoliubov approach and the solution to the Boltzmann equation from perturbative gravitational portal

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Thank you !

Backup

Conformal invariance and gravitational production

Cosmological spacetimes (FLRW) are related to Minkowski by a time-dependent conformal transformation

$$g_{\mu\nu}^{\mathrm{FLRW}}(\eta) = a^2(\eta)\eta_{\mu\nu}$$

Under a generic conformal transformation of the metric

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$\delta \mathcal{S} = \frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu} = \int d^4x\sqrt{-g}\,\boxed{T^{\mu}{}_{\mu}}\,\delta\Omega(x)$$

For non-minimally coupled scalar

$$T^{\mu}{}_{\mu} = (6\xi - 1)\left(g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \chi\Box\chi\right) + m_\chi^2\chi^2$$

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$$T^{\mu}{}_{\mu,\,A} = 0$$

For a massless spin-1 vector field

$$T^{\mu}{}_{\mu,\,1/2} \propto m$$

For a massive spin-1/2 fermion field

Asymptotic adiabatic modes and mixing of frequencies

How to track the excitations of the fields due to expansion ?

➜ consider asymptotic early and late times, for which the comoving frequency is slowly varying

$$X_{\vec{k}}^{\mathrm{IN}}(\eta) \sim \frac{\mathrm{e}^{-\mathrm{i}\int^{\eta}\mathrm{d}\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}$$

$$X_{\vec{k}}^{\mathrm{OUT}}(\eta) \sim \frac{\mathrm{e}^{-\mathrm{i}\int^{\eta}\mathrm{d}\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}$$

$\boxed{\frac{\omega_k'(\eta)}{\omega_k^2(\eta)} \ll 1}$ $(\eta \to \pm\infty)$ asymptotic adiabatic condition

➜ at intermediate times, write the mixing of positive and negative frequency modes

$$X_{\vec{k}}^{\mathrm{IN}}(\eta) = \boxed{\alpha_k(\eta)}\frac{1}{\sqrt{2\omega_k(\eta)}}e^{-i\int^{\eta}\omega_k(\tau)d\tau} + \boxed{\beta_k(\eta)}\frac{1}{\sqrt{2\omega_k(\eta)}}e^{i\int^{\eta}\omega_k(\tau)d\tau}$$

$\boxed{|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1}$ CCR preserved by EOM

Mode functions during and after inflation (IR)

Massless scalar mode in de Sitter

$$X_k^{(1)}(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left[1 - \boxed{\frac{i}{k\eta}}\right] \simeq -\frac{i}{\sqrt{2}k^{\frac{3}{2}}\eta}e^{-ik\eta}$$

Massless scalar mode during Reheating

$$X_k^{(2)}(\eta) = \sqrt{\frac{\bar{\eta}}{\pi}}e^{i\left(3\bar{\mu}\frac{k}{k_e}+\frac{\pi}{4}\right)} \times K_{\bar{\nu}}(ik\bar{\eta})$$

K are modified Bessel function with

$$\boxed{\bar{\nu} = \frac{3}{2}\frac{(1-\omega_\phi)}{(1+3w_\phi)}} \qquad \bar{\mu} = \frac{(1+w_\phi)}{(1+3w_\phi)}$$

Massive scalar mode in de Sitter

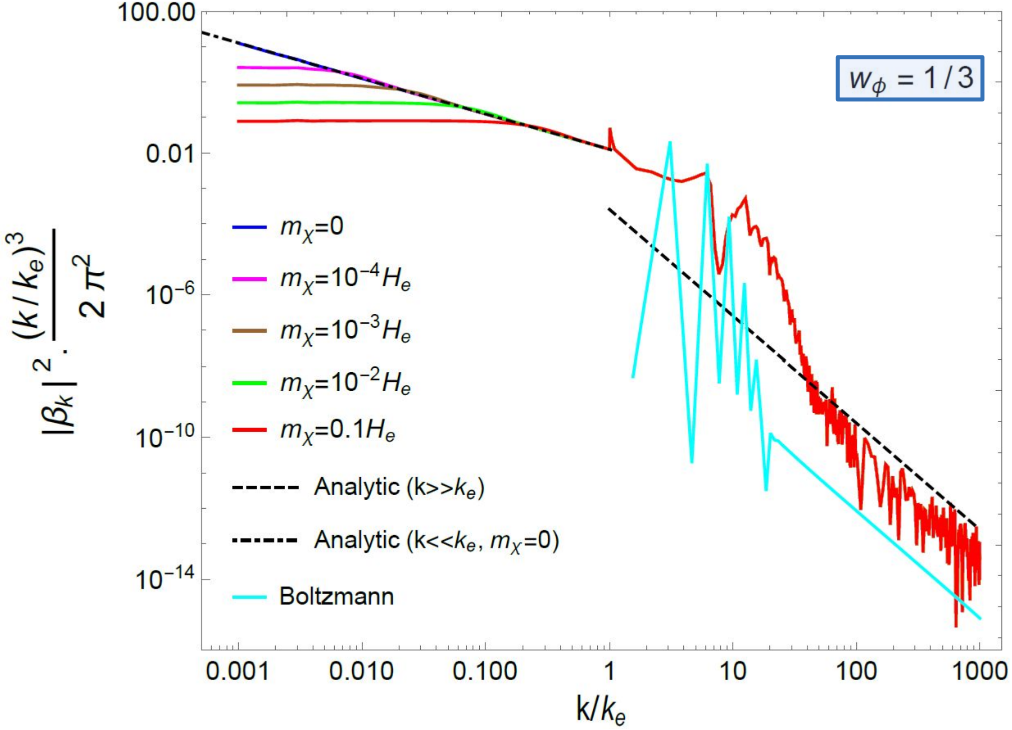
$$X_k^{(1)}(\eta) = \frac{\sqrt{-\pi\eta}}{2}e^{i(\pi/4+\pi\bar{\nu}_1/2)}H_{\bar{\nu}_1}^{(1)}(k|\eta|)$$

H are modified Hankel function with $\bar{\nu}_1 = \sqrt{\frac{9}{4} - \boxed{\frac{m_\chi^2}{H_e^2}}}$

No generic solution for arbitrary EoS and for massive scalar
➜ Use a WKB approximation :

$$X_k^{(2)}(\eta) \simeq \frac{e^{-i\Omega_k(\eta)}}{\sqrt{2\omega_k(\eta)}}$$

Short wavelengths spectrum for w = 1/3

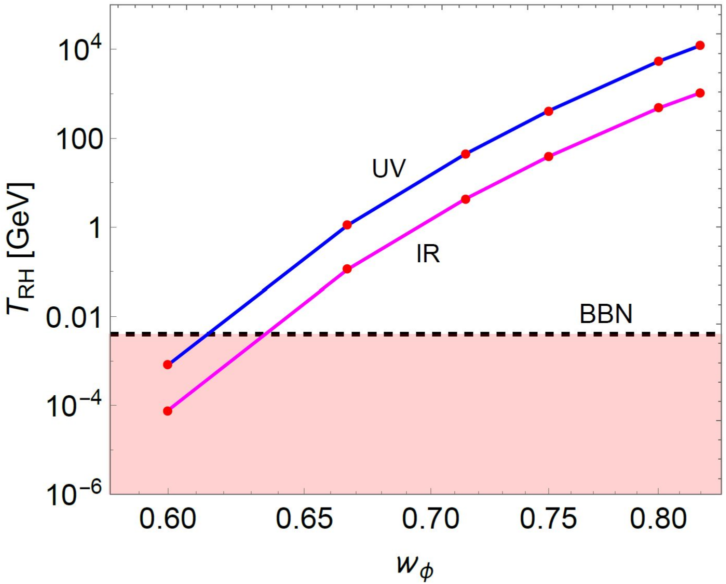


$$f_\chi^{w_\phi=1/3}(k,a) = 3\pi\left(\frac{H_e}{m_\phi^e\bar{\alpha}}\right)^2\left[1-\left(\frac{a_e}{a}\right)^3\right]\boxed{\sum_{\nu=1}^{+\infty}\frac{|\mathcal{P}_\nu^{2n}|^2}{\nu^2}\delta\left(\frac{k}{k_e}-\frac{\nu\bar{\alpha}m_\phi^e}{2H_e}\right)}$$

UV and IR modes contributions to Reheating

Determine the contribution to radiation bath from gravitational production at the end of reheating

$$\rho_R a^4 = \int_{k_{\mathrm{RH}}}^{k_{\mathrm{e}}} \frac{k^3}{2\pi^2} |\beta_k|^2_{\mathrm{IR}} dk + \int_{k_{\mathrm{e}}}^{k_{\mathrm{Planck}}} \frac{k^3}{2\pi^2} |\beta_k|^2_{\mathrm{UV}} dk$$



Bogoliubov		Boltzmann	
w_ϕ	$\mathrm{T_{RH}}$ (GeV)	w_ϕ	$\mathrm{T_{RH}}$ (GeV)
$3/5$	1.12×10^{-3}	$3/5$	9.40×10^{-4}
$2/3$	2.17	$2/3$	0.97
$5/7$	58.26	$5/7$	11.76
$3/4$	3.54×10^2	$3/4$	2.42×10^2
$4/5$	5.84×10^3	$4/5$	2.54×10^3

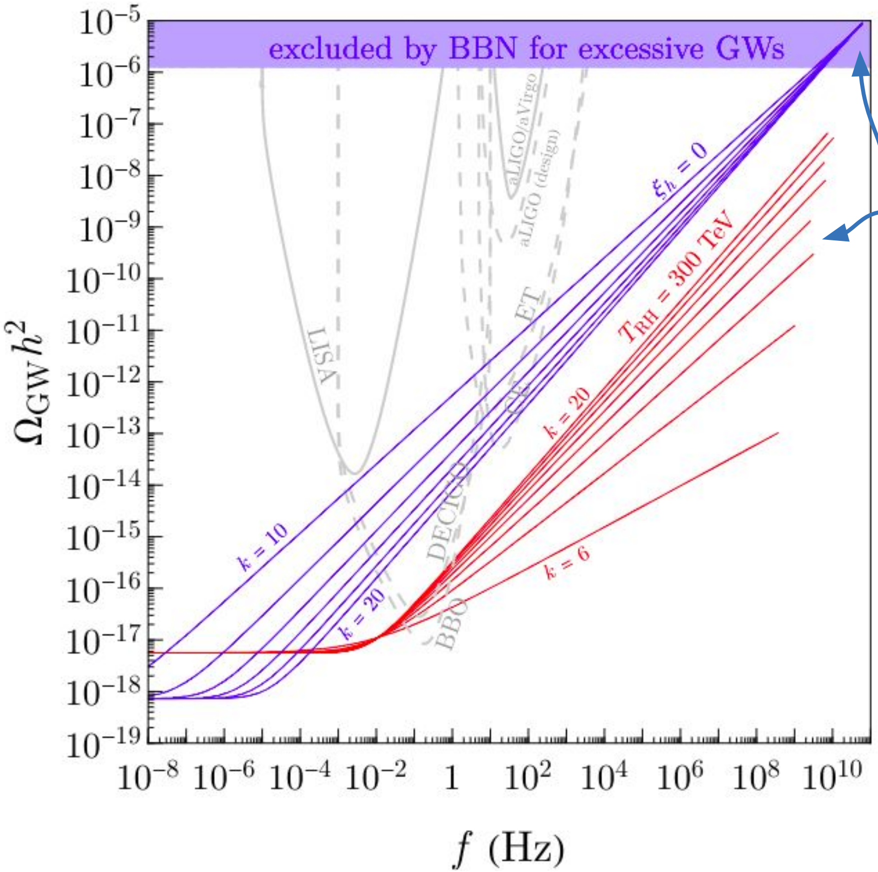
Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

Primordial GWs constraints

➜ Primordial GWs re-entering the horizon during reheating, can be enhanced.

$$\boxed{\Omega^0_{\rm GW} h^2 \propto f^{\frac{k-4}{k-1}}}$$

➜ The slope of this spectrum depends on inflaton potential

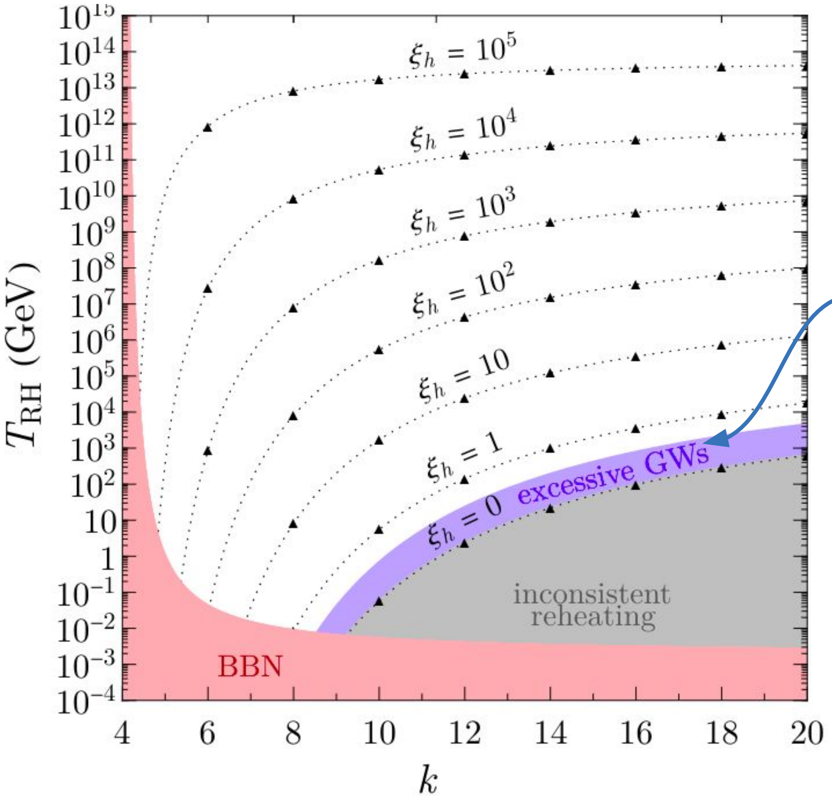


Largest enhancement for modes re-entering the horizon right after inflation

Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, ***SC****, Co, Mambrini, Olive,* **2210.05716**

Primordial GWs strength as function of its frequency f. Blue curves fix ξ_h = 0 and Red curves fix $T_{\rm RH} = 300\,{\rm TeV}$ for k in [6,20]. The sensitivity of several future GWs experiments are shown.

Primordial GWs constraints



Reheating temperature from gravitational portals as function of k, for different ξ_h

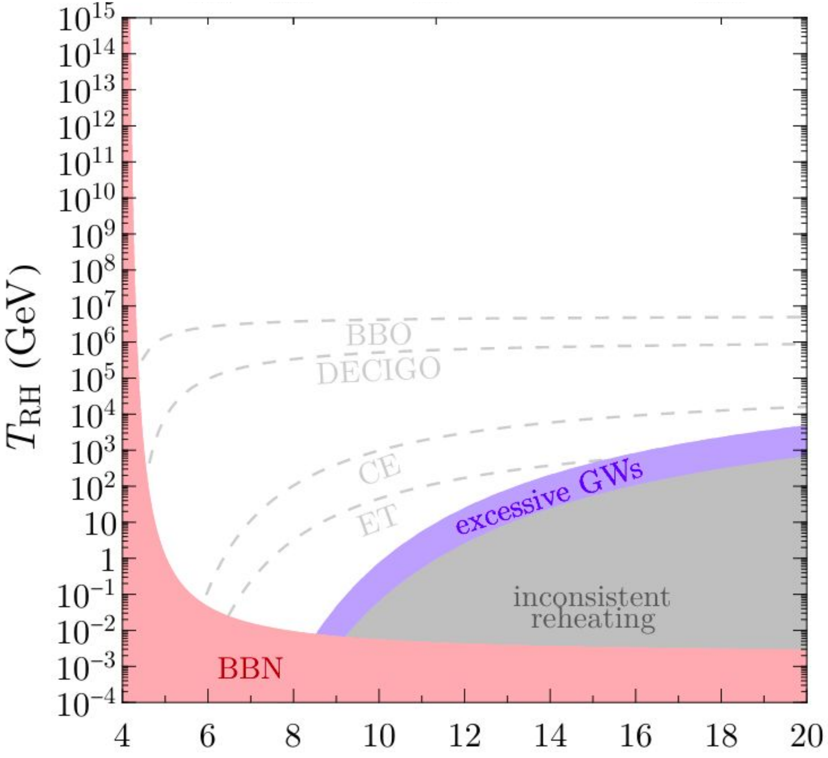
➜ GWs leave the same imprint as free-streaming dark radiation on CMB

➜ The case of minimal gravitational reheating is excluded by the CMB + BBN bound of $\Omega^0_{\rm GW} h^2 \lesssim 10^{-6}$, from excessive GWs as dark radiation

➜ The constraint is relaxed when radiation production is increased by non-minimal gravitational interactions $\xi_h > 0$

*Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, **SC**, Co, Mambrini, Olive,* **2210.05716**

Primordial GWs constraints



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➜ The constraint is relaxed when radiation production is increased by non-minimal gravitational interactions $\xi_h > 0$

➜ An important part of the parameter space for reheating could be probed by future GWs experiments

Done more generically in *Measuring Inflaton Couplings via Primordial Gravitational Waves*, Barman, Ghoshal, Grzadkowski, Socha, **2305.00027**

Bogoliubov approach with non-minimal coupling

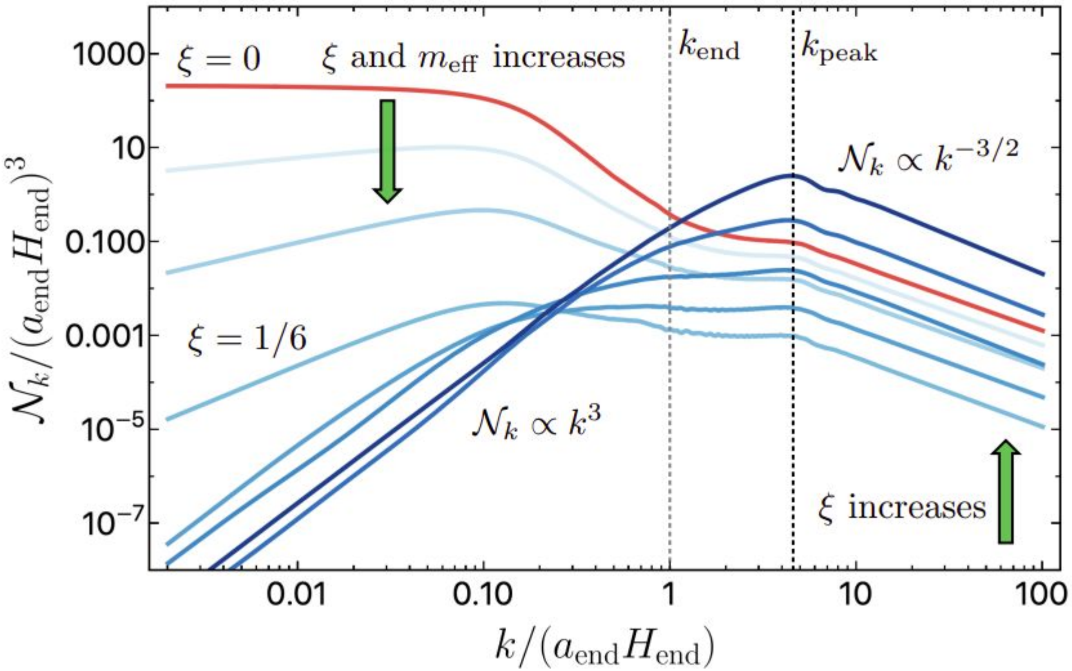
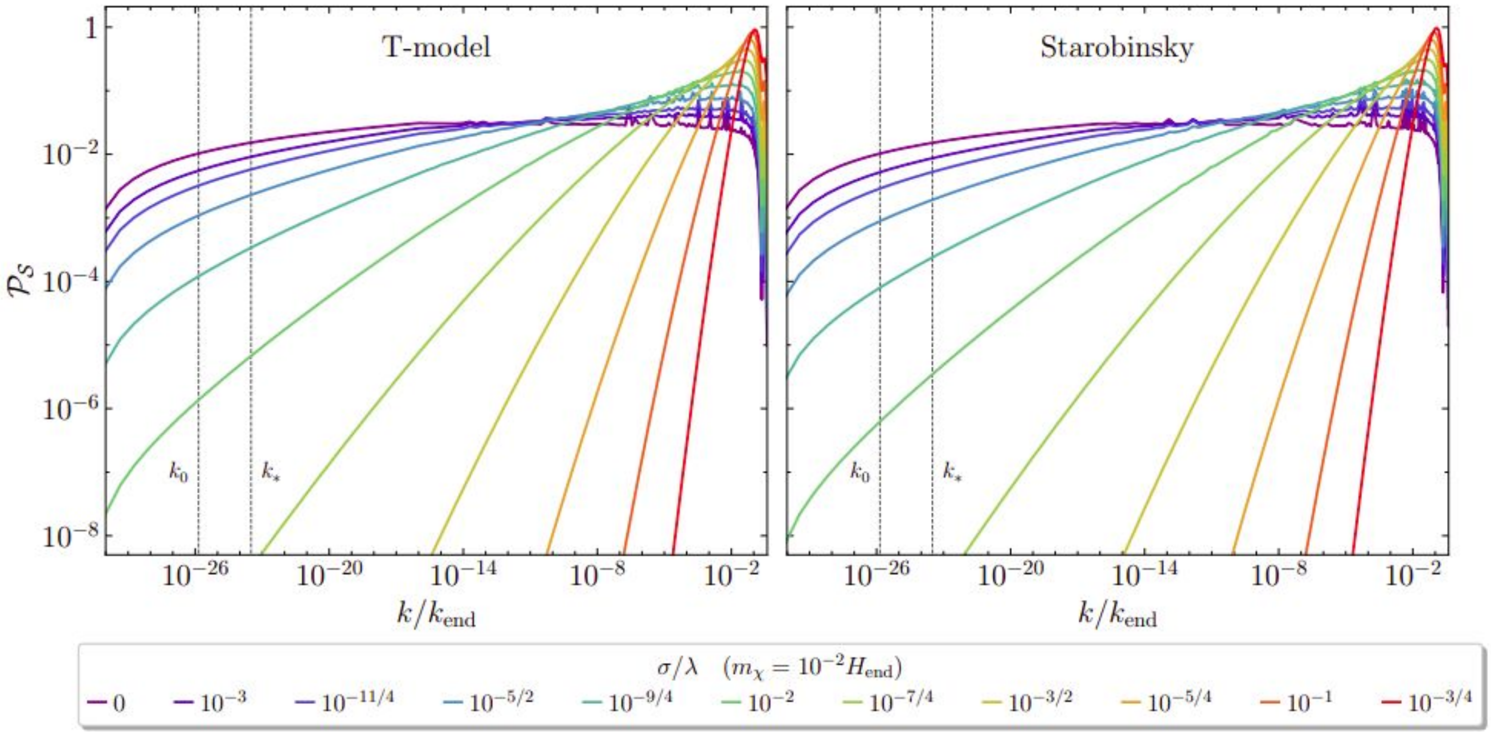


Diagram illustrating the dependence of the produced comoving number density spectrum Nk on non-minimal coupling ξ as a function of rescaled horizon modes momenta

From *A New Window into Gravitationally Produced Scalar Dark Matter,* Garcia, Pierre, Verner, **2305.14446**

Isocurvature perturbations



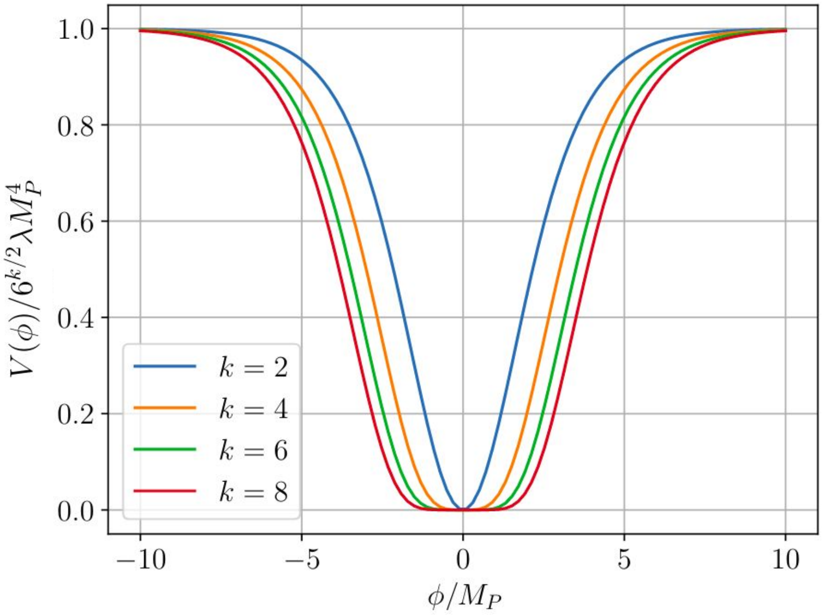
DM isocurvature power spectrum for different inflaton-DM couplings with mχ/Hend = 10^{-2}

From *Isocurvature Constraints on Scalar Dark Matter Production from the Inflaton,* Garcia, Pierre, Verner, **2303.07459**

Starobinksy and α-attractor models

Inflation driven by an homogeneous scalar field ϕ in the potential

$$\boxed{V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k}$$



Inflaton potential for T-models and for different values of k.

$$\lambda \simeq \frac{18\pi^2 A_{S*}}{6^{k/2} N_*^2}$$

➜ determined by the CMB scalar power spectrum amplitude A_S

➜ PLANCK measurements give $\boxed{\lambda \sim 10^{-11}}$ for $k = 2$

Reheating and Post-inflationary Production of Dark Matter, Garcia, Kaneta, Mambrini, Olive, **2004.08404**

Universality Class in Conformal Inflation, Kallosh and Linde, **1306.5220**

Inflaton perturbations from gravity

Looking at inflaton perturbations during inflation

Mode equation in (quasi)-de Sitter background

$$X'' + \left(k^2 + a^2 m_\phi^2 - 6\epsilon_H a^2 H^2 - \frac{a''}{a}\right) X = 0 \longrightarrow v'' + \left(k^2 - \boxed{\frac{1}{\eta^2}\left(\nu_\phi^2 - \frac{1}{4}\right)}\right) v = 0$$

$$\boxed{X \equiv a\delta\phi}$$

$$\nu_\phi \simeq \tfrac{3}{2} + 3\epsilon_V - \eta_V$$

$$v(k,\eta) = \sqrt{-\eta}A(k)H^{(1)}_{\nu_\phi}(-k\eta) + \sqrt{-\eta}B(k)H^{(2)}_{\nu_\phi}(-k\eta)$$

$$\lim_{k \ll aH} v(k,\eta) \propto \frac{1}{\sqrt{2k}}(-k\eta)^{\frac{1}{2}-\nu_\phi}$$

➜ Near constant superhorizon (IR) modes of inflaton perturbations

$$\boxed{|\delta\phi_k| \simeq \frac{H}{\sqrt{2k^3}}\left(\frac{k}{aH}\right)^{3/2-\nu_\phi}} \quad (k \ll aH)$$

Inflation fluctuations and Cosmological perturbations

Scalar perturbations during slow-roll

$$\phi(x,t) = \bar{\phi}(t) + \boxed{\delta\phi(x,t)} \longrightarrow \langle 0|\delta\phi_k \delta\phi_{k'}|0\rangle \propto \frac{2\pi^2}{k^3} \underbrace{\boxed{\left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n_s-1}}}_{\mathcal{P}_\phi}$$

$(k \ll aH)$

Super-horizon scales

$$\mathcal{R} = -\frac{aH}{\bar{\phi}'}\delta\phi \longrightarrow \mathcal{P}_\mathcal{R} = \boxed{\mathcal{A}_S} \left(\frac{k}{k_*}\right)^{\boxed{n_s-1}}$$

near **scale-invariant** spectrum

$$\mathcal{A}_S \simeq \frac{V(\phi_*)}{24\pi^2 M_P^4 \epsilon_{V*}} \qquad n_s - 1 = 2\eta_V - 6\epsilon_V$$

Tensor perturbations during slow-roll

$$ds^2 = a^2(\eta)\left[d\eta^2 - (\delta_{ij} + \boxed{\tilde{h}_{ij}^{TT}})dx^i dx^j\right] \longrightarrow \mathcal{P}_T \equiv \sum_{+,\times} \frac{k^3}{2\pi^2}|\tilde{h}|^2 = \boxed{\mathcal{A}_T} \left(\frac{k}{k_*}\right)^{\boxed{n_T}}$$

Predicted amplitude of primordial GW spectrum

$$r \equiv \frac{\mathcal{A}_T}{\mathcal{A}_S} = 16\epsilon_{V*}$$

Parametric resonances

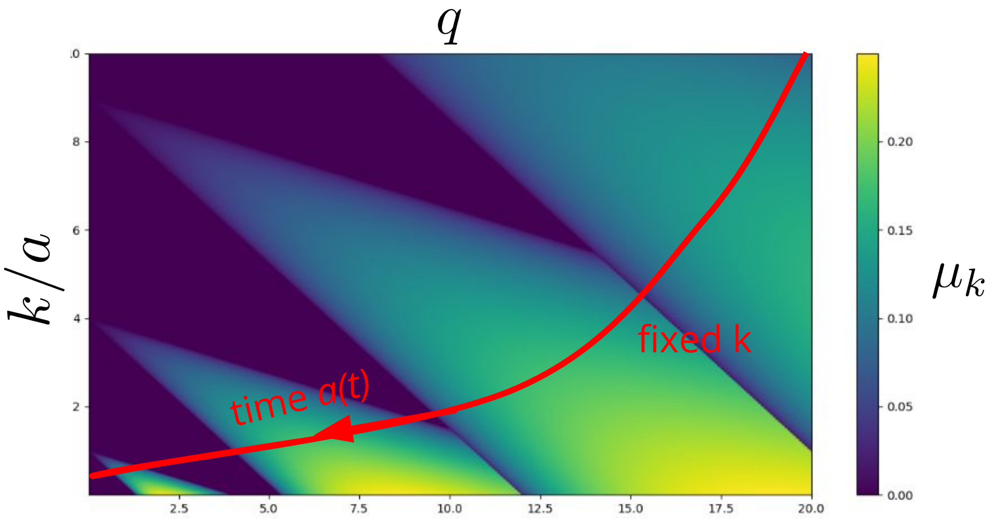
Time dependent background coupled to fields can lead to parametric resonance or tachyonic instability

$$\mathcal{L} \supset \sigma\phi^2\chi^2 + \lambda\phi^k M_P^{4-k}$$

$$\chi_k'' + \left(\frac{k^2}{m_\phi^2 a^2} + 2q - 2q\cos(2z)\right)\chi_k = 0$$

EOM for Fourier modes in the oscillating background

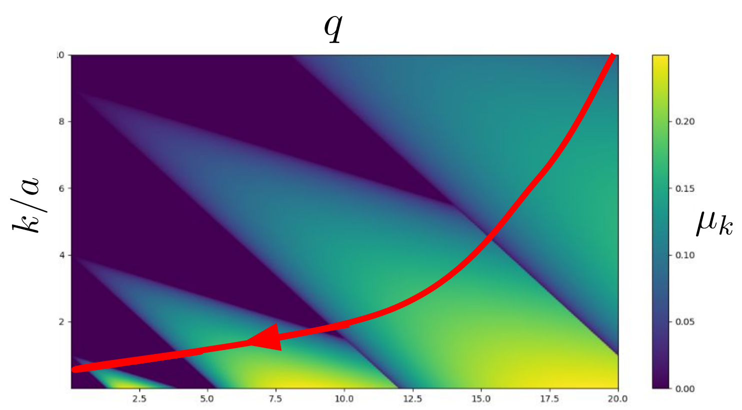
$$q \equiv \frac{\sigma\phi_0^2}{2m_\phi^2} \sim \frac{\sigma}{\lambda}$$



Instabilities in the colored regions
➜ increasing occupation number of the modes

Freeze-in from preheating, Garcia, Kaneta, Mambrini, Olive, Verner, **2109.13280**

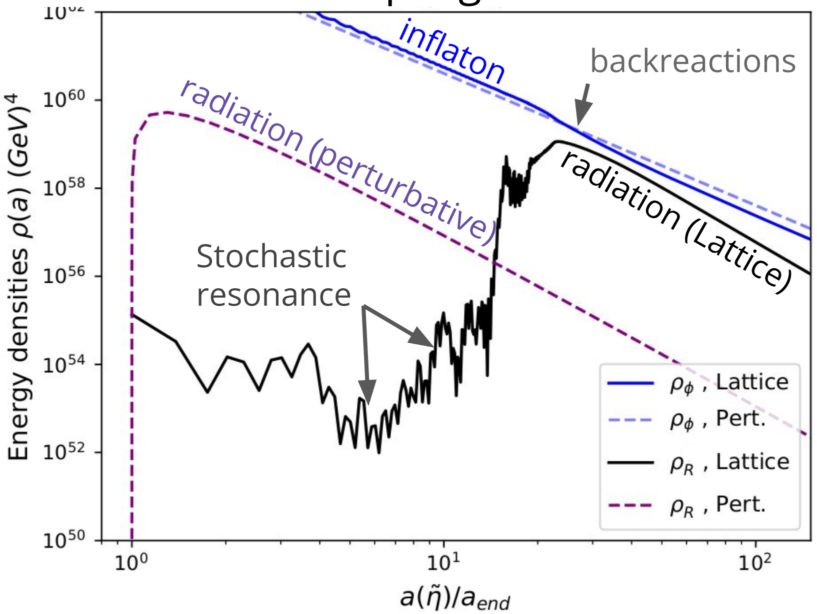
Preheating through non-perturbative processes



Instabilities in the colored regions
➜ number of occupation increasing

$$\chi_k \propto \exp\left[\mu_k t\right]$$

coupling σ ~10^{-6}



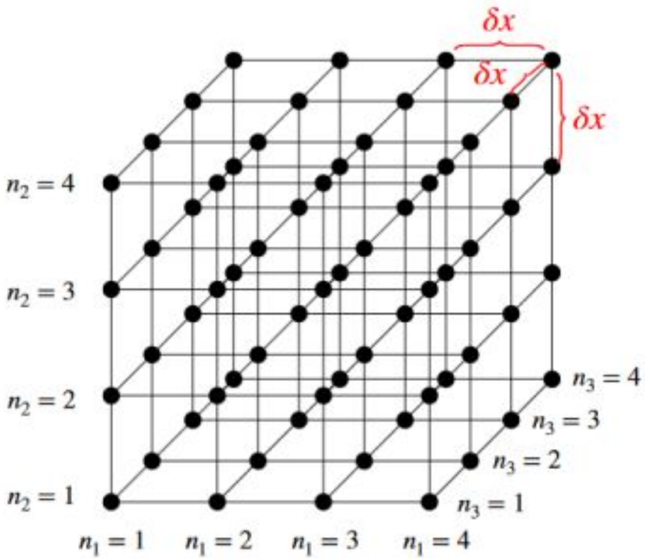
Preheating corresponds to resonances and exponential production

For large couplings, reach a regime of large backreactions of the fields on the background ➜ Lattice

Freeze-in from preheating, Garcia, Kaneta, Mambrini, Olive, Verner, **2109.13280**

Numerical Lattice simulations

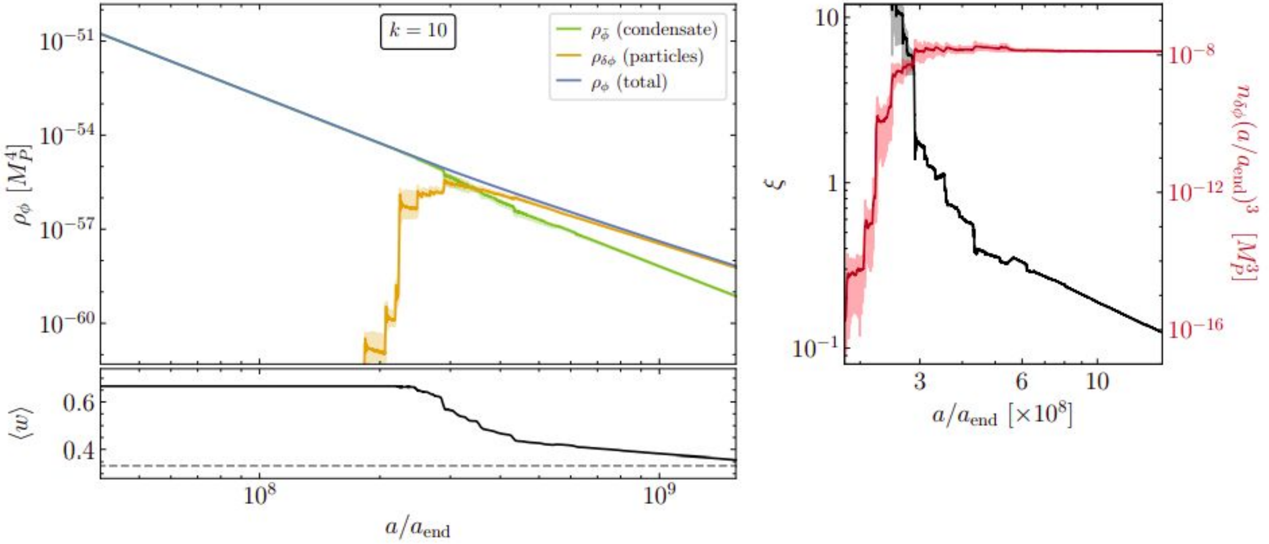
CosmoLattice



The art of simulating the early Universe, Figueroa, Florio, Torrenti, Valkenbug, **2006.15122**

CosmoLattice: A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe, Figueroa, Florio, Torrenti, Valkenbug, **2102.01031**

Inflation self-fragmentation



k	y_{eff}	μ_{eff}	σ_{eff}	T_{RH}
4	1.61×10^{-1}	3.57×10^{10} GeV	3.57×10^{-6}	1.14×10^{13} GeV
6	1.58×10^{-2}	1.84×10^{5} GeV	5.37×10^{-10}	1.19×10^{10} GeV
8	1.32×10^{-3}	6.33×10^{-1} GeV	9.59×10^{-15}	1.50×10^{7} GeV
10	3.62×10^{-5}	1.49×10^{-6} GeV	6.47×10^{-20}	1.80×10^{4} GeV

From Garcia , Gross, Mambrini , Olive, Pierre, and Yoon, **2308.16231**

Weak Field Gravity and gravitons

In the weak field limit of the gravity EFT, we assume that we can expand the local metric field around Minkowski

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu} \quad \text{and} \quad |h_{\mu\nu}| \ll M_P$$

We are free to fix the gauge as $\partial_\mu h^\mu{}_\nu - \frac{1}{2}\partial_\nu h^\lambda{}_\lambda = 0$ (harmonic or de Donder gauge)

Linearized Einstein equations reduce to the wave equation $\Box \bar{h}_{\mu\nu} = -\frac{1}{M_P} T_{\mu\nu}$

Expanding EH action with the matter action at second order in the metric perturbation we have the following Lagrangian density for the canonical gravitons

$$\sqrt{-g}\mathcal{L} = \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \frac{1}{4}\partial_\lambda h \partial^\lambda h \boxed{- \frac{1}{M_P} h^{\mu\nu} T_{\mu\nu}}$$

One can extract the Green function, and the Feynman propagator for the massless graviton

$$iD^{\alpha\beta\gamma\delta} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 + i\epsilon} e^{-q\cdot x} P^{\alpha\beta\gamma\delta}$$

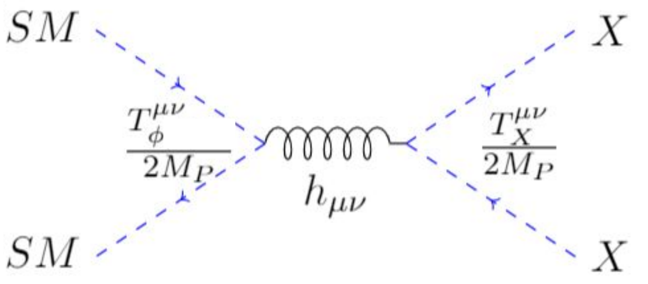
$$\boxed{P^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\gamma} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]}$$

Gravitational portals

$$R_j(T) = \beta_j \frac{\boxed{T^8}}{M_P^4}$$

for spin j = 0, ½ DM final state

See *Spin-2 Portal Dark Matter*, Nicolás Bernal, Maíra Dutra, Yann Mambrini, Keith Olive, Marco Peloso, **1803.01866**

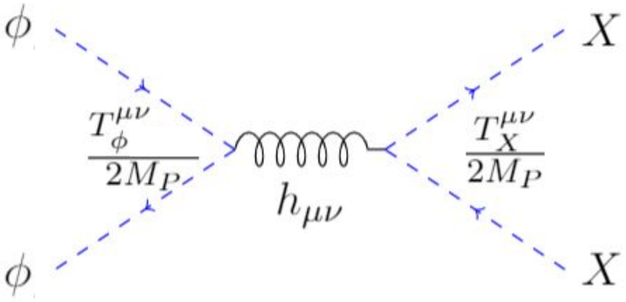


$$R_0^{\phi^k} = \boxed{\frac{2 \times \rho_\phi^2}{16\pi M_P^4}} \sum_{n=1}^{\infty} |\mathcal{P}_n^k|^2 \left[1 + \frac{2m_X^2}{E_n^2}\right]^2 \sqrt{1 - \frac{4m_X^2}{E_n^2}}$$

spin 0

$$R_{1/2}^{\phi^k} = \frac{2 \times \rho_\phi^2 \boxed{m_X^2}}{4\pi M_P^4 \, m_\phi^2} \sum_{n=1}^{+\infty} |\mathcal{P}_n^k|^2 \frac{m_\phi^2}{E_n^2} \left[1 - \frac{4m_X^2}{E_n^2}\right]^{3/2}$$

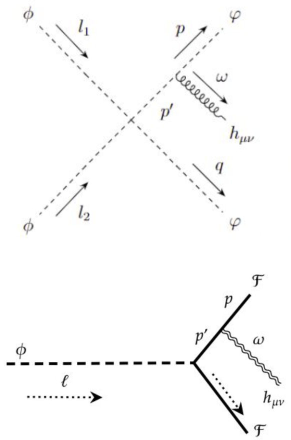
spin ½

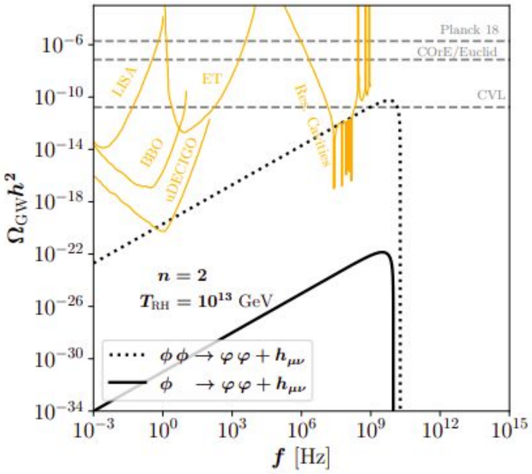


Gravitational portals in the early Universe, **SC,** Mambrini, Olive, Verner, **2112.15214**

Observable signal of effective gravitational interaction

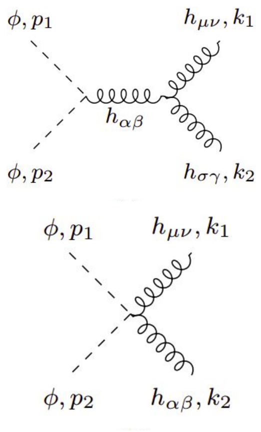
➔ Look at particle origin for stochastic GWs background that generates a spectrum at high frequencies, and depends on the details of reheating

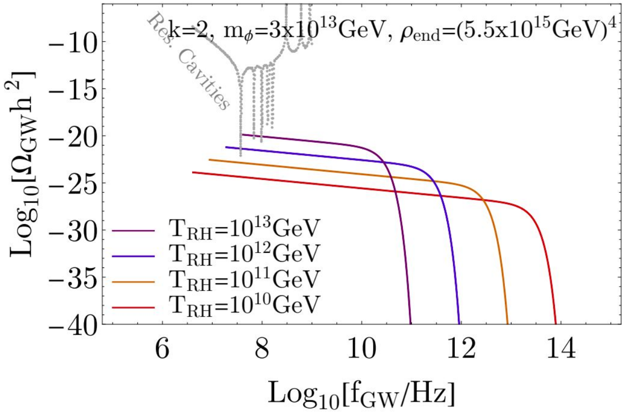




Graviton bremsstrahlung

Probing Reheating with Graviton Bremsstrahlung, Bernal, **SC,** Mambrini and Xu, **2311.12694**

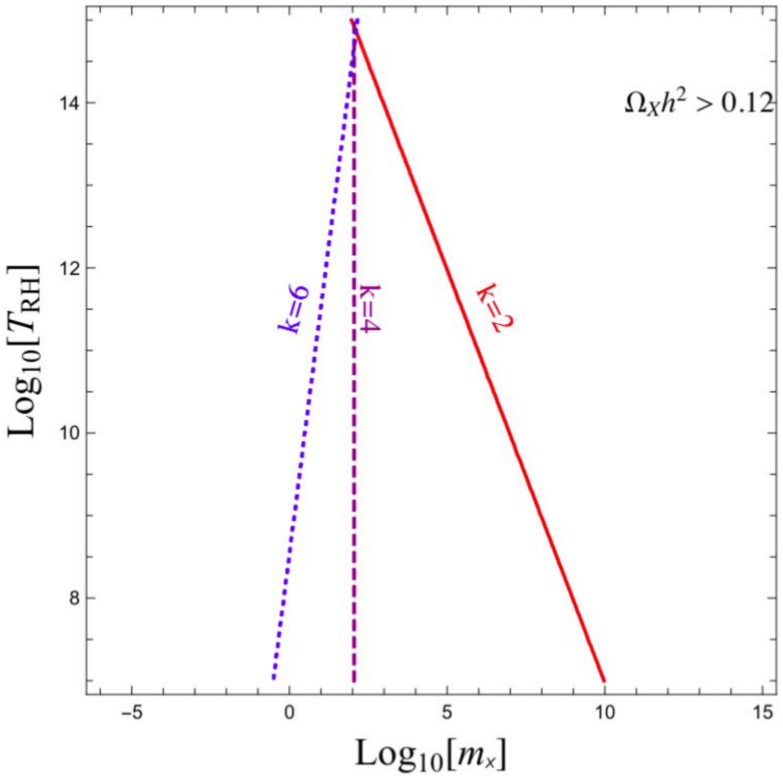




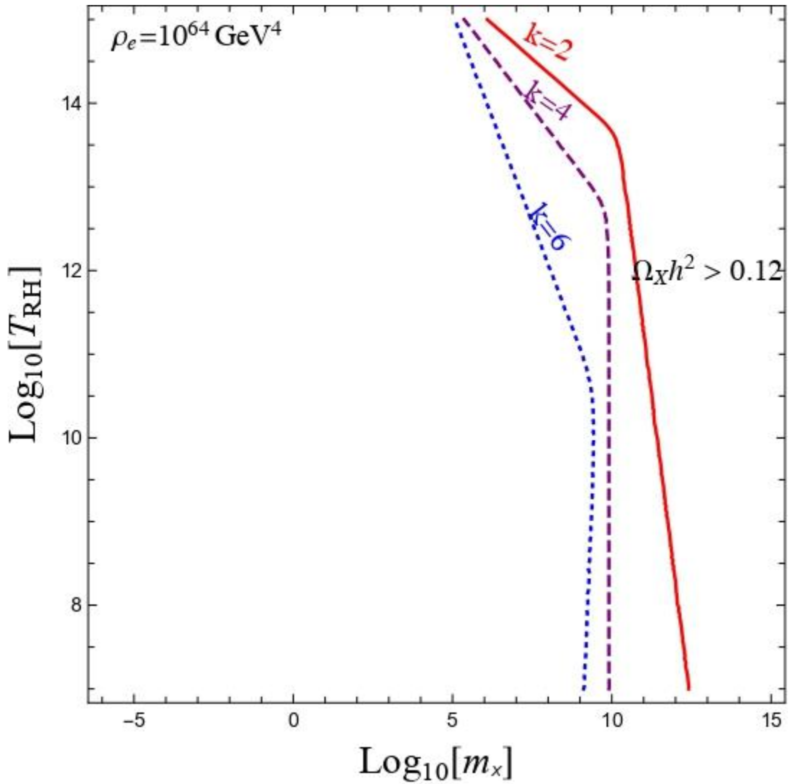
Direct gravitons production

Minimal production of prompt gravitational waves during reheating, Choi, Ke, Olive, **2402.04310**

Dark Matter gravitational production during reheating



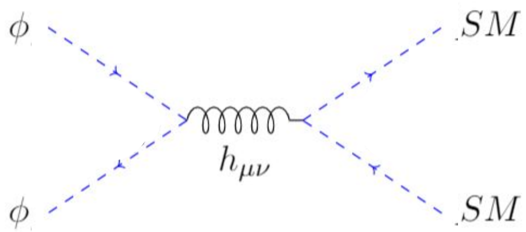
$\Omega_X h^2 = 0.12$ in the case of a spin 0 DM all contributions added

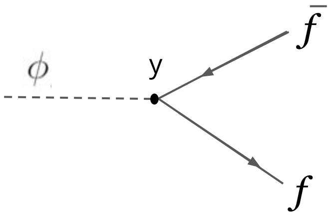


$\Omega_X h^2 = 0.12$ in the case of a spin ½ DM, all contributions added

Gravitational portals in the early Universe, **SC,** Mambrini, Olive, Verner, **2112.15214**

Radiation production





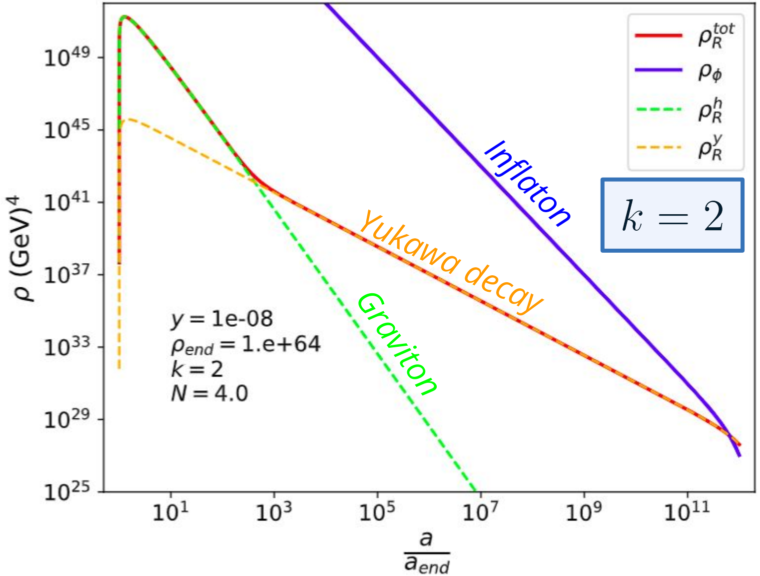
Gravitational effects provide a maximum temperature

	$k=2$	$k=4$	$k=6$
$T_{\max}$	1.0×10^{12} GeV	7.5×10^{11} GeV	6.5×10^{11} GeV

which is unavoidable and model independent !

BUT late time reheating is still given by the decay

➜ No gravitational reheating when $\boxed{k=2}$



Evolution of energy densities of the inflaton (blue), radiation from Yukawa decay (orange) and graviton exchange (green)

Gravitational portals in the early Universe, **SC,** Mambrini, Olive, Verner, **2112.15214**

Non minimal coupling to gravity

The natural generalization of this minimal interaction is to introduce non-minimal couplings to gravity of the form :

$$\mathcal{L}_{\text{non-min.}} = \boxed{-\frac{M_P^2}{2}\Omega^2\tilde{R}} + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X$$

in the Jordan frame

with

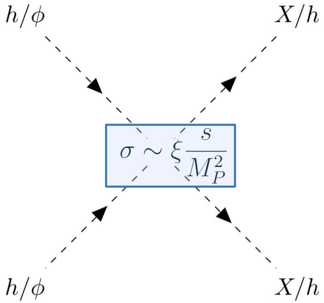
$$\Omega^2 \equiv 1 + \underbrace{\frac{\xi_\phi \phi^2}{M_P^2}}_{\text{inflaton}} + \underbrace{\frac{\xi_h h^2}{M_P^2}}_{\text{SM}} + \underbrace{\frac{\xi_X X^2}{M_P^2}}_{\text{DM}}$$

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}$$

$$\mathcal{L}_{\text{non-min.}} = -\sigma^\xi_{hX} h^2 X^2 - \sigma^\xi_{\phi X}\phi^2 X^2 - \sigma^\xi_{\phi h}\phi^2 h^2$$

in the Einstein frame

Non-minimal couplings induce leading-order interactions in the small fields limit, involved in radiation and DM production



Reheating and Dark Matter Freeze-in in the Higgs-R² Inflation Model, Aoki, Lee, Menkara, Yamashita, **2202.13063**

Gravitational Portals with Non-Minimal Couplings, **SC,** Mambrini, Olive, Shkerin, Verner, **2203.02004**

Non-minimal coupling : the small-field limit

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}K^{ij}g^{\mu\nu}\partial_\mu S_i \partial_\nu S_j - \frac{V_\phi + V_h + V_X}{\Omega^4}\right]$$

in Einstein frame

with

$$\Omega^2 \equiv 1 + \frac{\xi_\phi \phi^2}{M_P^2} + \frac{\xi_h h^2}{M_P^2} + \frac{\xi_X X^2}{M_P^2}$$

and

$$K^{ij} = 6\frac{\partial \log\Omega}{\partial S_i}\frac{\partial \log\Omega}{\partial S_j} + \frac{\delta^{ij}}{\Omega^2}$$

non-canonical kinetic term

Impossible to make a field redefinition to the canonical form, unless all three non-minimal couplings vanish

$$\frac{|\xi_\phi|\phi^2}{M_P^2}, \quad \frac{|\xi_h|h^2}{M_P^2}, \quad \frac{|\xi_X|X^2}{M_P^2} \ll 1$$

Small-field limit: expand the action in powers of M_P^{-2} obtain canonical kinetic term and leading-order interactions induced by the non-minimal couplings

At the end of inflation we have $\boxed{\phi_{\rm end} \sim M_P}$ and the inflaton field is decreasing during the reheating

$$\boxed{|\xi_\phi| \lesssim 1}$$

Gravitational Portals with Non-Minimal Couplings, SC, Mambrini, Olive, Shkerin, Verner, **2203.02004**

➜ Small field approximation is valid if : $\sqrt{|\xi_S|} \lesssim M_P/\langle S \rangle$ with $S = \phi, h, X$

➜ At the end of inflation we have $\phi_{\rm end} \sim M_P$ and the inflaton field is decreasing during the reheating

$$\boxed{|\xi_\phi| \lesssim 1}$$

➜ Perturbative computations involve effective couplings in the Einstein frame that depend on all ξ, the small value of ξ_φ can be compensated by ξ_h. Current constraints on ξ_h from collider experiments is $\xi_h < 10^{15}$

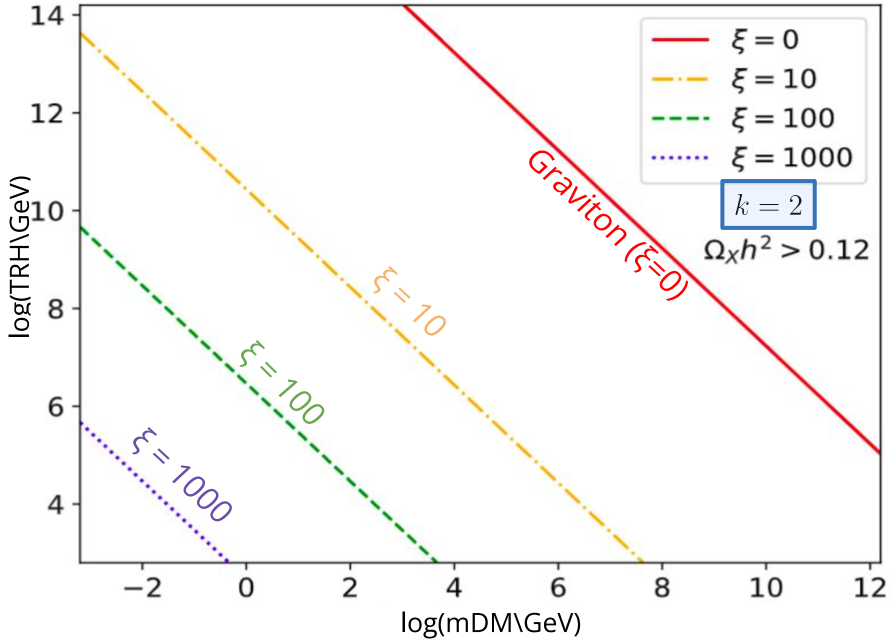
See *Cosmological Aspects of Higgs Vacuum Metastability,* Tommi Markkanen, Arttu Rajantie, Stephen Stopyra, (2018)

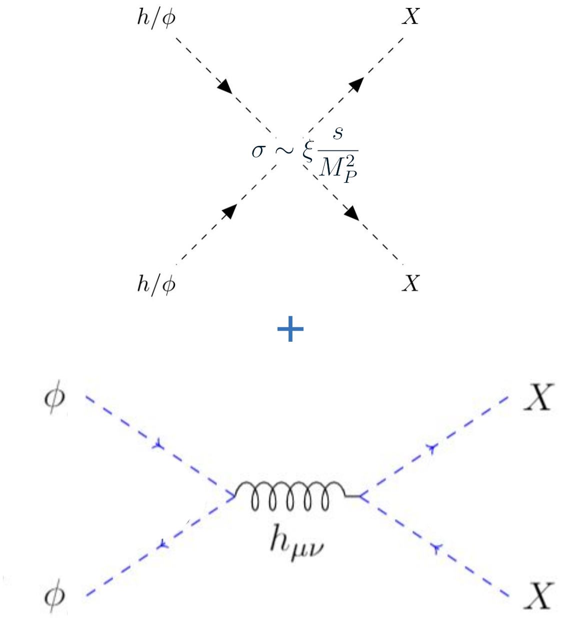
➜ To prevent the EW vacuum instability at inflation scale, we can stabilize through effective Higgs mass from the non-minimal coupling : $\xi_h > 10^{-1}$

➜ In the case of Higgs inflation, large ξ_h is fixed, or can consider conformal coupling for inflation at the pole

See Bezrukov and Shaposhnikov, *Phys. Lett. B (2008), and Higgs inflation at the pole,* **SC,** Lee, Menkara **2306.07767**

Non-minimal production of Dark Matter

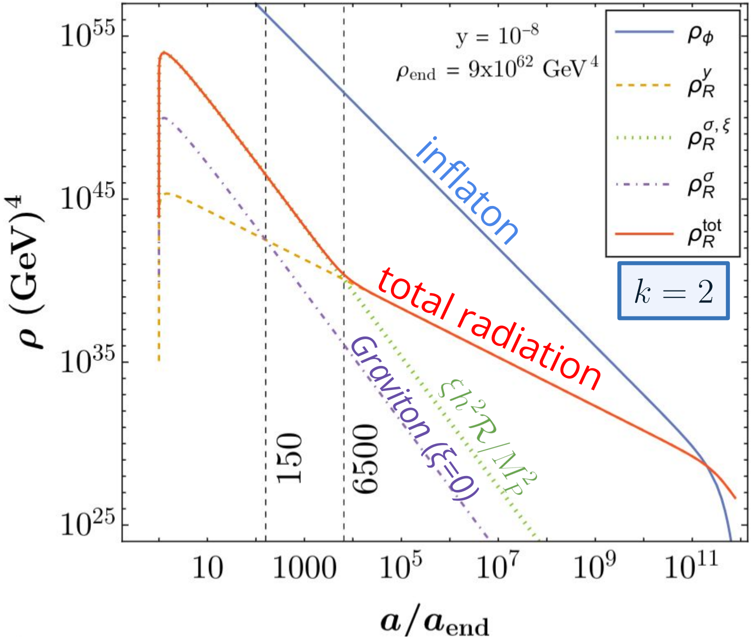


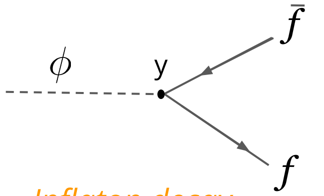


Contours respecting $\Omega_X h^2 = 0.12$ *for spin 0 DM, for different values of* $\xi_h = \xi_X = \xi$.
Both minimal and non-minimal contributions are added.

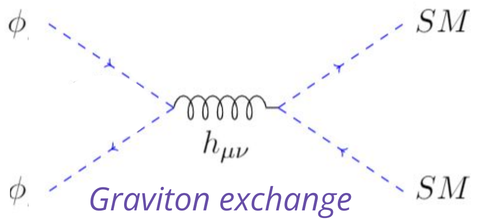
Gravitational Portals with Non-Minimal Couplings, **SC,** Mambrini, Olive, Shkerin, Verner, **2203.02004**

Radiation perturbative production

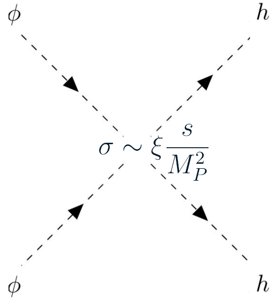




Inflaton decay



Graviton exchange

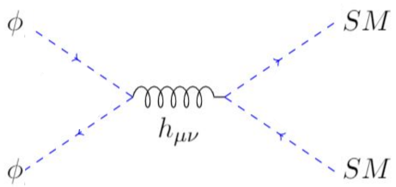


Non-minimal coupling

Energy densities of inflaton (blue), total radiation (red), radiation from inflaton decay (orange), from scattering mediated by graviton (purple) and from non-minimal coupling (green), with ξ = 2

Gravitational Portals with Non-Minimal Couplings, **SC,** Mambrini, Olive, Shkerin, Verner, **2203.02004**

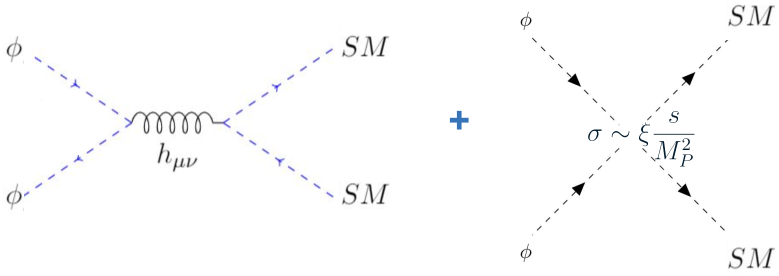
Gravitational reheating



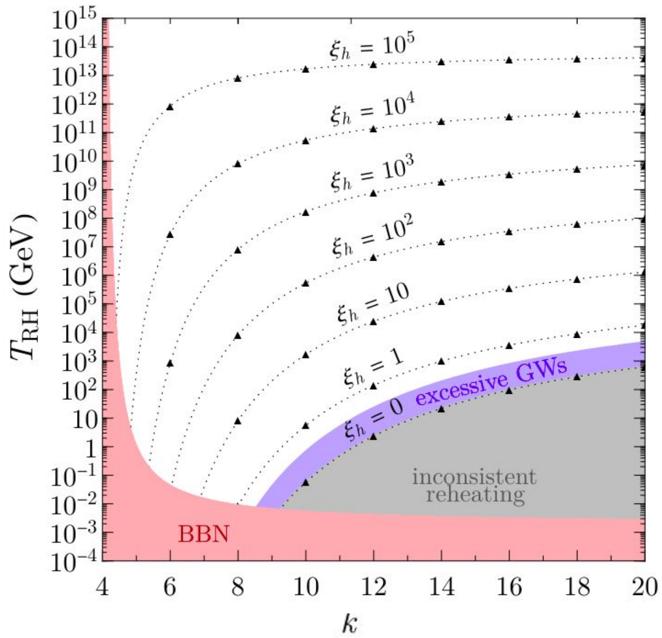
➜ Graviton exchange processes can be sufficient to reheat entirely, for sufficiently steep inflaton potential : $k > 9$

Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, ***SC,*** *Co, Mambrini, Olive,* **2210.05716**

$$\boxed{\rho_\phi(a) \propto \left(\frac{a_{\rm end}}{a}\right)^{\frac{6k}{k+2}} \qquad \rho_R(a) \propto \left(\frac{a_{\rm end}}{a}\right)^4}$$



➜ Requirement of large k can be relaxed adding the non-minimal contribution to radiation production (but still need $k > 4$)



Reheating temperature from gravitational portals as function of k, for different ξ_h