The role of the counterterms in the conservation of superhorizon curvature perturbations at one loop

Keisuke Inomata (Johns Hopkins University)

Based on arXiv: 2502.12112 (accepted for PRD)

Planck2025 @Padova, May 29, 2025

Main message

Recently, several papers claim that superhorizon curvature perturbations are not conserved at one loop level.

I point out that the counterterm plays a crucial role in the curvature conservation at one loop, which was overlooked in the recent papers.

$$\mathcal{H}_0 = \frac{1}{2a^2} b^2 \left[\zeta'^2 + (\partial_i \zeta)^2 \right],$$

$$\mathcal{H}_{\text{int},3} = \frac{1}{6} V_{(3)} b^3 \zeta^3, \;\; \mathcal{H}_{\text{int},4} = \frac{1}{24} V_{(4)} b^4 \zeta^4,$$

$$\underline{\mathcal{H}_c = b V_{c,(1)} \zeta + \frac{b^2}{2} V_{c,(2)} \zeta^2}$$

were overlooked!

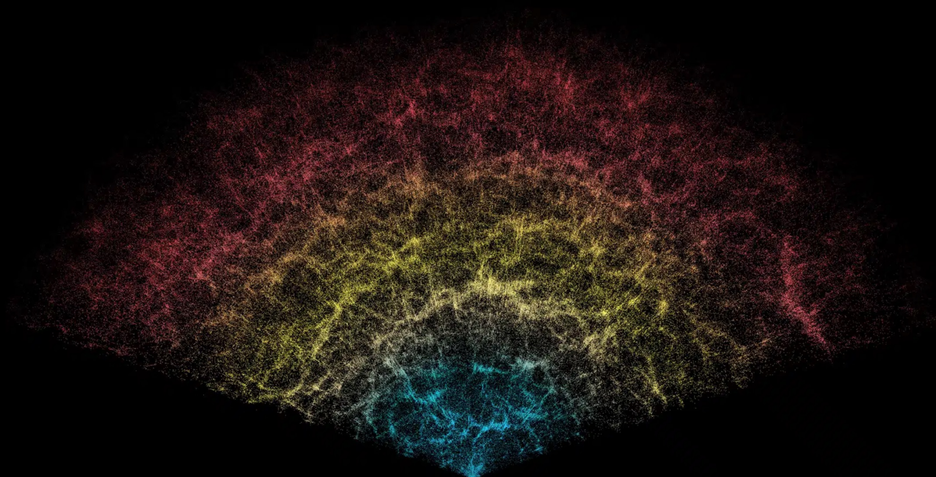
Outline

- **Introduction**
- Role of counterterms in curvature conservation
  - Key logic
  - One loop calculation
- Summary

Cosmological perturbations

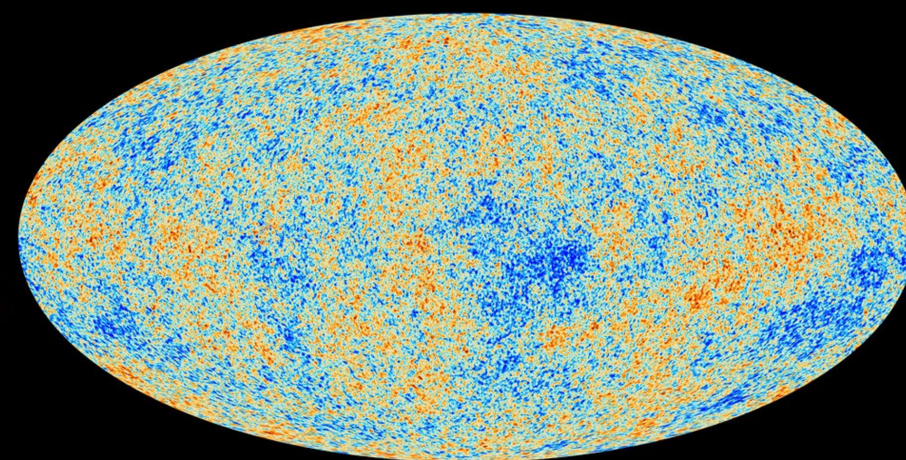
We have observed cosmological density perturbations in the Universe.

(Credit: DESI Collaboration/NOIRLab/NSF/AURA/R. Proctor)



Large Scale Structure

(Credit: ESA and the Planck Collaboration)



CMB anisotropies

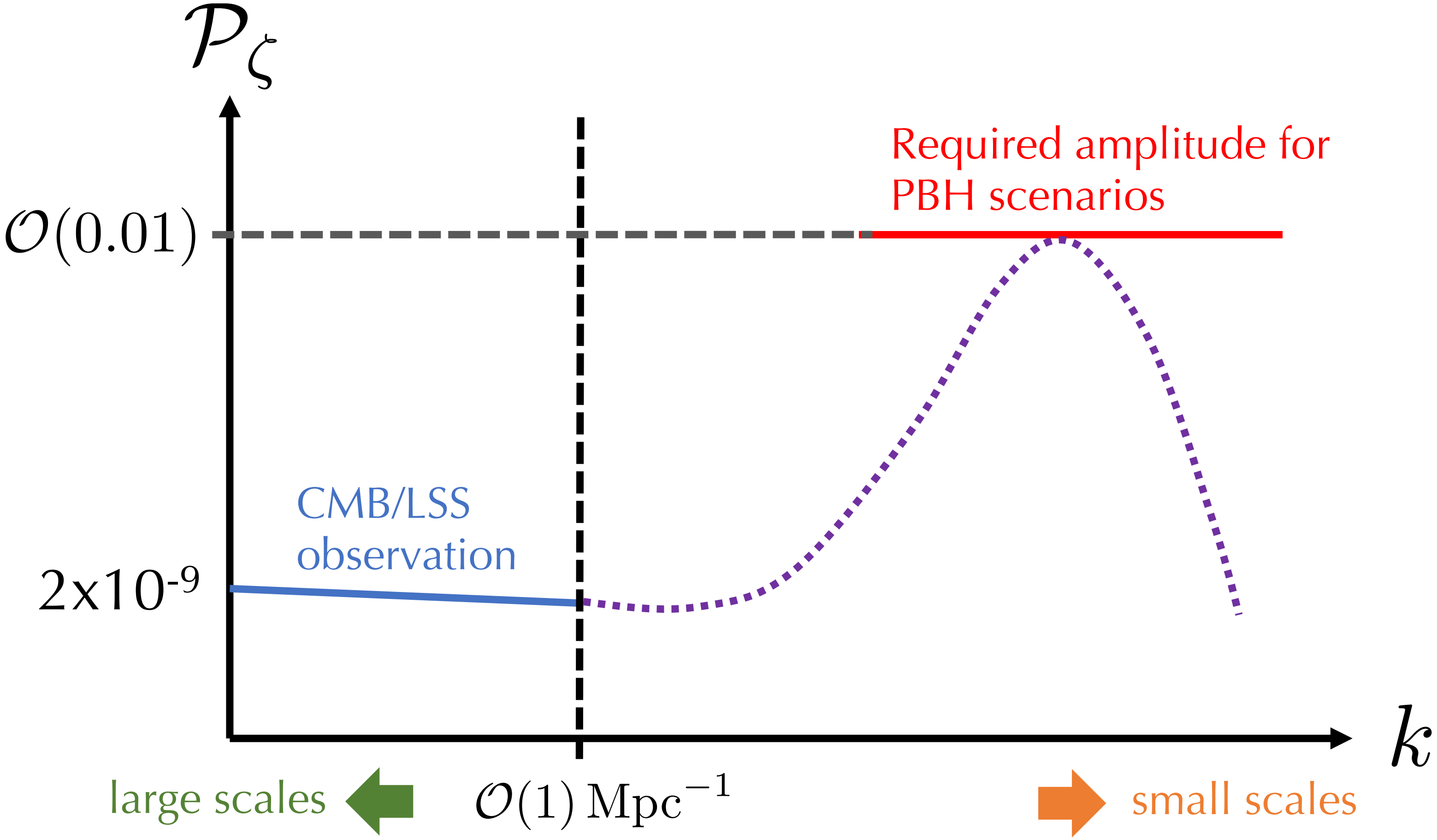
$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \ \text{(Planck 2018)}$$

$$\to \delta\rho/\bar{\rho} \simeq 10^{-5}$$

ζ: curvature perturbation

Large perturbations on small scales

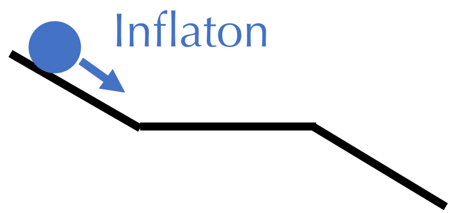
Large-amplitude perturbations can produce primordial black holes (PBHs) and secondary GWs.



Inflaton potentials for large amplification

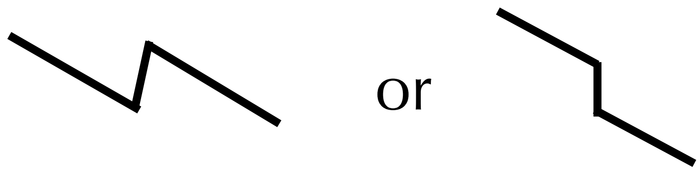
Single field models for large amplification of density perturbations:

flatter region



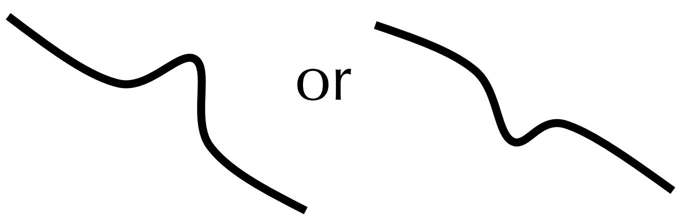
(Starobinsky 1992, Ivanov+ 1994,
Inoue and Yokoyama 2001, Kinney 2005)

step feature



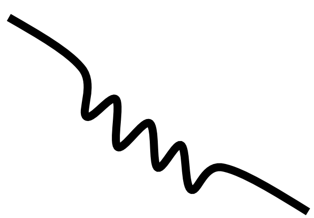
(Kefala et al. 2020,
Inomata+ 2021)

bump/dip feature



(Ozsoy+ 2018,
Mishra and Sahni 2019)

oscillatory feature



(R.G. Cai+ 2019, Zhou+ 2020,
Peng+ 2021)

One loop corrections

Lagrangian: $\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$

E.o.m. for the inflaton fluctuations: (slow-roll-parameter suppressed terms neglected)

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1} \qquad \left(V_{(n)} \equiv \partial^n V/\partial\phi^n\right)$$

beyond liner order corrections

In-in formalism: (Jordan 1986, Calzetta and Hu 1987, Weinberg 2005)

$$\langle\delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta} d\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\left(T e^{-i\int_{-\infty}^{\eta} d\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$\left(H_{{\rm int},n} \equiv \int d^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

The non-linear corrections to the power spectrum can be characterized with loop diagrams.

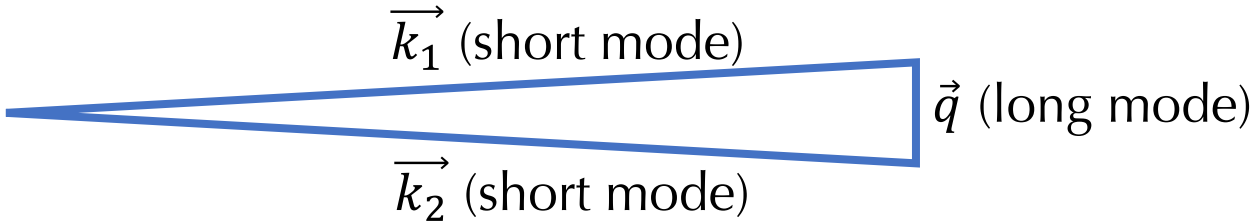
Tree

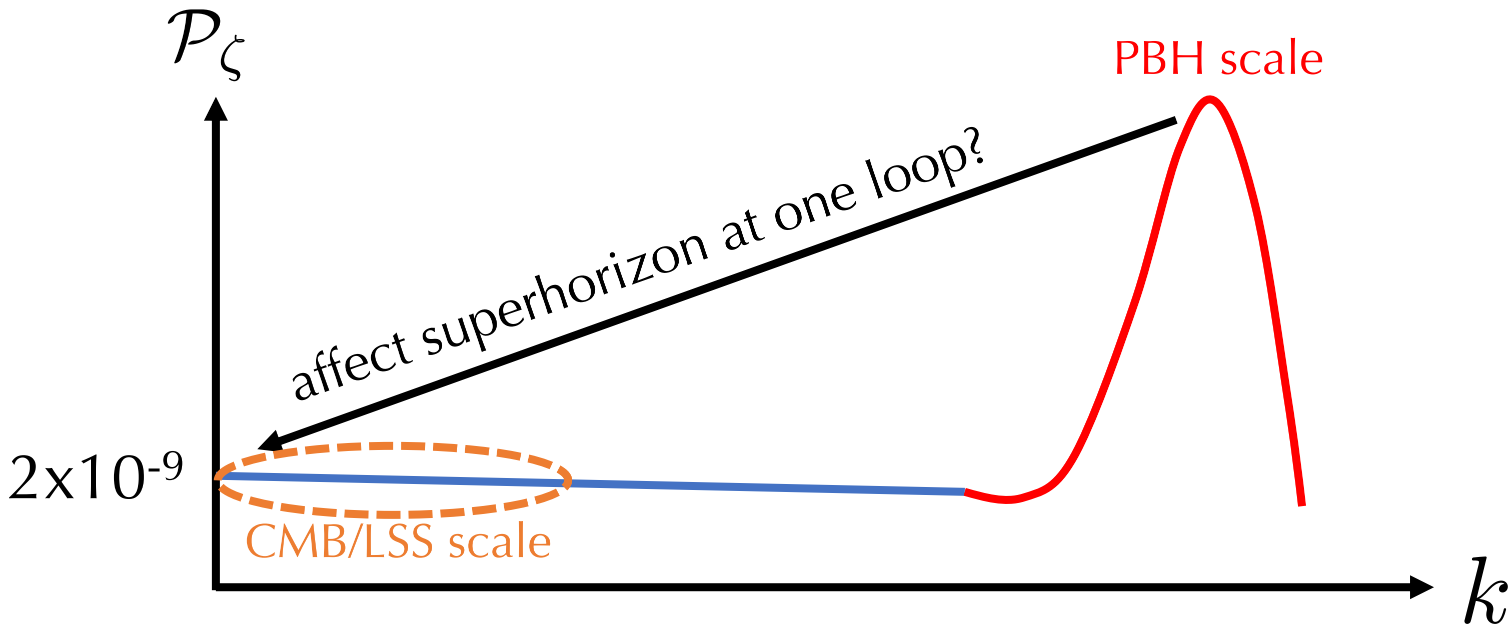
One loop

(We will discuss how to introduce the counterterms later.)

Question this talk addresses

Long mode and short modes are coupled at non-linear level.





(For experts: I will not discuss the IR divergence issue.)

Superhorizon curvature evolves?

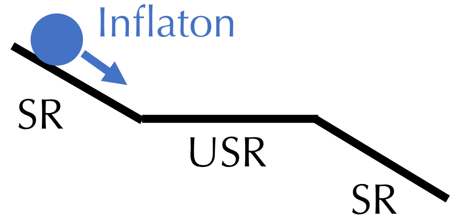
(Note: the conservation of linear ζ is well-known.)

Recent claim:
Superhorizon-**limit** curvature perturbations **are not conserved at one-loop level** in the transitions of slow-roll (SR) → ultra-slow roll (USR) → SR.

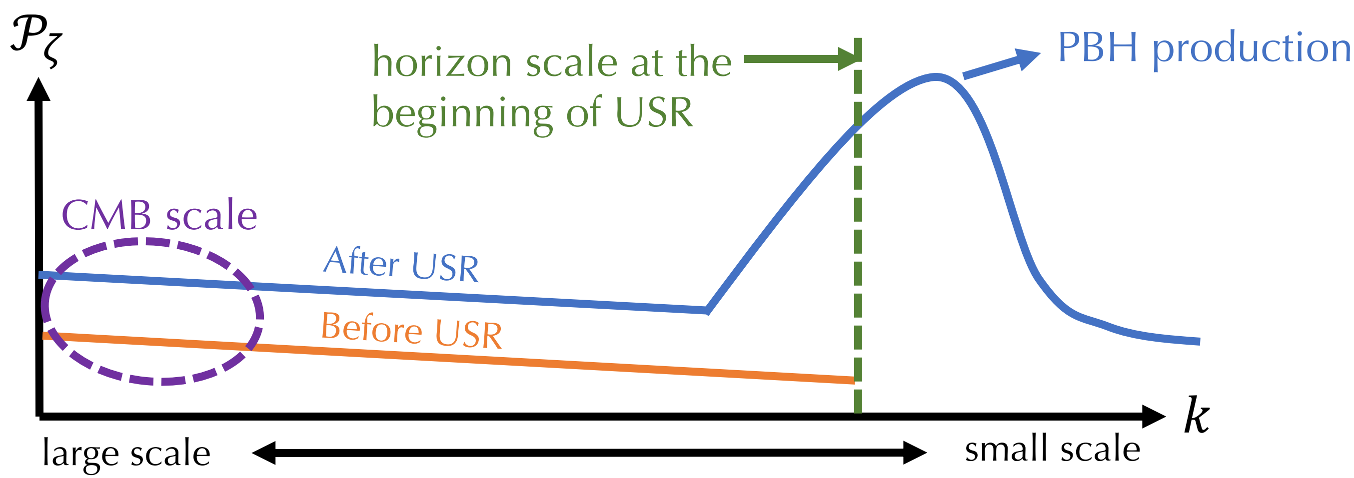
e.o.m.

$$3H\dot{\phi} + V'(\phi) = 0 \text{ (SR)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0 \text{ (USR)}$$



Kristiano & Yokoyama (2022), followed by
Riotto, Choudhury+, Firouzjahi, Motohashi & Tada, Franciolini+, Gianmassimo, Cheng+, Maity+, Davies+ (2023), Saburov & Ketov, Ballesteros & Egea (2024), …



The one loop corrections can be comparable to the tree-level power spectrum in some PBH models. → Break down of perturbation theory? → models constrained?

However, this is inconsistent with the separate Universe picture...

Papers against the recent claim

The papers that have shown the conservation of superhorizon curvature at one loop:

Fumagalli (2023, 2024) and Tada et al. (2023) point out the importance of the boundary terms.

Kawaguchi+ (2024) uses the path integral approach.

Comoving gauge without field redefinition

Inomata (2024) uses the spatially-flat gauge.

Spatially flat gauge

On the other hand, Kristiano & Yokoyama (2022) and some following papers claim non-conservation of curvature by using the **comoving gauge with field redefinition**.

Due to the formalism differences, it has been unclear where this discrepancy comes from.

This talk will clarify what was overlooked in the recent works claiming the non-conservation.

(spoiler: counterterms were overlooked.)

Outline

- Introduction
- **Role of counterterms in curvature conservation**
  - **Key logic**
  - One loop calculation
- Summary

Field redefinition of curvature

$$\zeta_n \equiv -\frac{H}{\dot{\phi}}\delta\phi = -\frac{\delta\phi}{\sqrt{2\epsilon}M_{\rm Pl}}$$

($\delta\phi$: inflaton fluctuation in spatially-flat gauge)
($\epsilon \equiv -\dot{H}/H$)

Redefined curvature (used in the original paper)

The relation between the original and the redefined curvature:

(2nd order, Maldacena 2002)

$$\zeta = \zeta_n - f_2(\zeta_n) = \zeta_n + \frac{1}{2}\frac{\ddot{\phi}_c}{\dot{\phi}_c H}\zeta_n^2 + \frac{1}{4}\frac{\dot{\phi}_c^2}{H^2}\zeta_n^2 + \frac{\zeta_n\dot{\zeta}_n}{H} + \frac{1}{2}\partial^i\zeta_n\partial_i{\chi_\phi}^{(1)} - \frac{1}{2}\partial^{-2}\partial^i\partial^j(\partial_i\zeta_n\partial_j{\chi_\phi}^{(1)})$$
$$-\frac{1}{4H^2}\left(\partial_i\zeta_n\partial^i\zeta_n - \partial^{-2}\partial^i\partial^j(\partial_i\zeta_n\partial_j\zeta_n)\right)$$

(See Jarnhus & Sloth 2007 for 3rd order)

During slow-roll (SR) period, we find

$$\zeta \simeq \zeta_n$$

Since we discuss whether the superhorizon curvature evolves through SR → non-SR → SR, we can use ζ_n instead of ζ.

Point: ζ_n is proportional to $\delta\phi$ in the flat gauge.

Let's first see the Lagrangian in the flat gauge in the following.

Lagrangian in flat gauge

(We take the de-Sitter (or decoupling limit) $\epsilon \to 0$, where metric perturbations are negligible.)

$$S = \int \mathrm{d}\eta\, \mathrm{d}^3x\, a^4 \mathcal{L}, \quad \mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V_b(\phi).$$

$$V_b(\phi) = V(\phi) + V_c(\phi)$$

bare potential → $V_b(\phi)$; tree-level potential → $V(\phi)$; counter potential → $V_c(\phi)$

We express $\phi = \bar{\phi} + \delta\phi$, where the evolution of $\bar{\phi}$ is determined by the tree-level potential:

$$\bar{\phi}'' + 2\mathcal{H}\bar{\phi}' + a^2 V_{(1)}(\bar{\phi}) = 0 \qquad (V_{(n)}(\phi) \equiv d^n V(\phi)/d\phi^n)$$

Substituting $\phi = \bar{\phi} + \delta\phi$, we obtain the Lagrangian for $\delta\phi$:

$$\mathcal{L}_{\delta\phi} = \frac{1}{2a^2}\left[(\delta\phi')^2 - (\partial_i\delta\phi)^2\right] - \sum_{n=2} \frac{1}{n!} V_{b,(n)}(\bar{\phi})\delta\phi^n - V_{c,(1)}(\bar{\phi})\delta\phi$$

Hamiltonian for $\delta\phi$

$$\mathcal{L}_{\delta\phi} = \frac{1}{2a^2}\left[(\delta\phi')^2 - (\partial_i\delta\phi)^2\right] - \sum_{n=2}\frac{1}{n!}V_{b,(n)}(\bar{\phi})\delta\phi^n - V_{c,(1)}(\bar{\phi})\delta\phi$$

From this Lagrangian, we can obtain the free and interaction Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 + \sum_n \mathcal{H}_{\mathrm{int},n}$$

$$\mathcal{H}_0 \equiv \frac{1}{2a^2}\left[(\delta\phi')^2 + (\partial_i\delta\phi)^2\right] + \frac{1}{2}V_{(2)}(\bar{\phi})\delta\phi^2$$

$$\mathcal{H}_{\mathrm{int},n} \equiv \begin{cases} \frac{1}{n!}V_{c,(n)}(\bar{\phi})\delta\phi^n & (n=1,2) \\ \frac{1}{n!}V_{b,(n)}(\bar{\phi})\delta\phi^n = \frac{1}{n!}\left[V_{(n)}(\bar{\phi}) + V_{c,(n)}(\bar{\phi})\right]\delta\phi^n & (\text{others}) \end{cases}.$$

We here change the variable from $\delta\phi$ to ζ_n by using

$$\zeta_n \equiv -\frac{H}{\dot{\bar{\phi}}}\delta\phi = -\frac{\delta\phi}{\sqrt{2\epsilon}M_{\mathrm{Pl}}}$$

Hamiltonian for ζ_n

Then, we obtain

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int,3} + \mathcal{H}_{\rm int,4} + \mathcal{H}_c$$

$$\mathcal{H}_0 = \frac{1}{2a^2} b^2 \left[\zeta'^2 + (\partial_i \zeta)^2\right],$$

$$b(\tau) \equiv -\dot{\phi}(\tau)/H = -\sqrt{2\epsilon(\tau)} M_{\rm Pl}$$

$$\mathcal{H}_{\rm int,3} = \frac{1}{6} V_{(3)} b^3 \zeta^3,\ \ \mathcal{H}_{\rm int,4} = \frac{1}{24} V_{(4)} b^4 \zeta^4,$$

$$\mathcal{H}_c = b V_{c,(1)} \zeta + \frac{b^2}{2} V_{c,(2)} \zeta^2$$

(The subscript n of ζ_n is omitted)

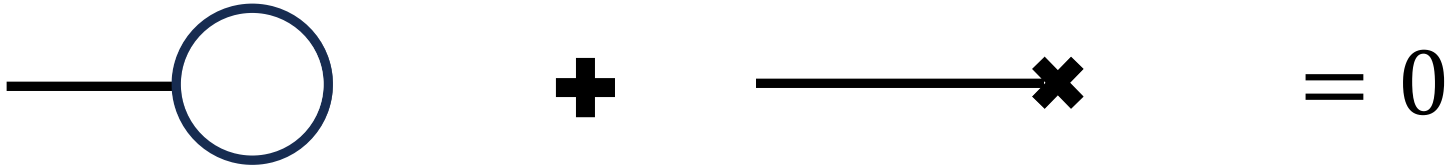
Using these, we can calculate the two-point function:

$$\begin{aligned}
\langle \zeta_{\mathbf{q}}(\tau) \zeta_{\mathbf{q}'}(\tau) \rangle =\ & \langle 0 | \zeta^I_{\mathbf{q}}(\tau) \zeta^I_{\mathbf{q}'}(\tau) | 0 \rangle + 2\,{\rm Im}\left[\int_{\tau_i}^{\tau} {\rm d}\tau'\, \langle 0 | \zeta^I_{\mathbf{q}}(\tau) \zeta^I_{\mathbf{q}'}(\tau) H_{\rm int,4}(\tau') | 0 \rangle\right] \\
& + 2\,{\rm Re}\left[\int_{\tau_i}^{\tau} {\rm d}\tau' \int_{\tau_i}^{\tau'} {\rm d}\tau''\, \langle 0 | \left(H_{\rm int,3}(\tau') \zeta^I_{\mathbf{q}}(\tau) \zeta^I_{\mathbf{q}'}(\tau) - \zeta^I_{\mathbf{q}}(\tau) \zeta^I_{\mathbf{q}'}(\tau) H_{\rm int,3}(\tau')\right) H_{\rm int,3}(\tau'')(\tau'')) | 0 \rangle\right] \\
& + 2\,{\rm Im}\left[\int_{\tau_i}^{\tau} {\rm d}\tau'\, \langle 0 | \zeta^I_{\mathbf{q}}(\tau) \zeta^I_{\mathbf{q}'}(\tau) H_c(\tau') | 0 \rangle\right] \\
=\ & (2\pi)^3 \delta(\mathbf{q} + \mathbf{q}') \frac{2\pi^2}{q^3} \mathcal{P}_\zeta(q, \tau),
\end{aligned}$$

If we neglect $H_{int,4}$ and H_c, we reproduce the results of many previous works.

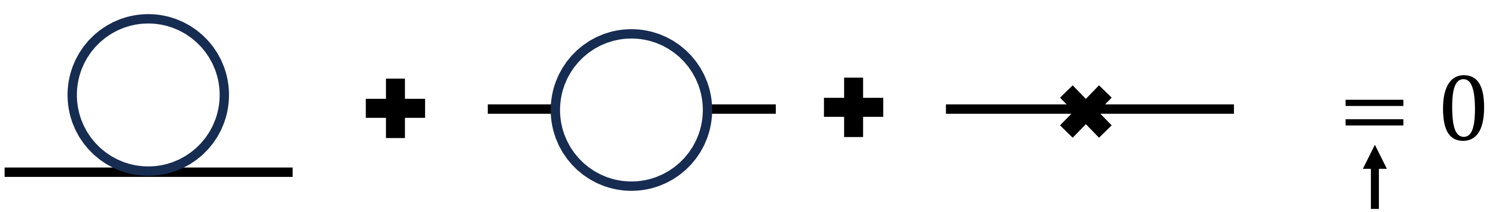
Key logic

Once we fix the linear counterterm ($V_{c,(1)}$) as



(zero tadpole $<\zeta_n> = <\delta\phi> = 0$)

the quadratic counterterm ($V_{c,(2)} \equiv \frac{dV_{c,(1)}}{d\phi}$) cancel the one loops



(superhorizon limit
& negligible IR power)

Outline

- Introduction
- **Role of counterterms in curvature conservation**
  - Key logic
  - **One loop calculation**
- Summary

Determine the counterterms

To determine the counter term, we here impose the zero-tadpole condition, $\langle \zeta_n \rangle = 0$.

$$\langle \zeta(\mathbf{x},\tau)\rangle = -i\int \mathrm{d}^3 y \int_{\tau_i}^{\tau} \mathrm{d}\tau' a^4(\tau')b(\tau')[\zeta(\mathbf{x},\tau),\zeta(\mathbf{y},\tau')] \times \left[V_{c,(1)}(\tau') + \frac{1}{2}b^2(\tau')V_{(3)}(\tau')\sigma_\zeta^2(\tau')\right]$$

(The subscript n of ζ_n is omitted)

$$\langle \zeta(\mathbf{x},\tau)\rangle = 0 \to V_{c,(1)}(\tau') = -\frac{1}{2}b^2(\tau')V_{(3)}(\tau')\sigma_\zeta^2(\tau') \quad \text{(zero-tadpole condition)}$$

$$\zeta^I(\mathbf{x},\tau) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \zeta^I_{\mathbf{k}}(\tau) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[\zeta_k(\tau)a(\mathbf{k}) + \zeta_k^*(\tau)a^\dagger(-\mathbf{k})\right]$$

$$V_{(n)}(\tau) \equiv V_{(n)}(\bar\phi(\tau))$$

$$\sigma_\zeta^2(\tau) \equiv \langle 0|(\zeta^I(\mathbf{x},\tau))^2|0\rangle = \int_{k_{\mathrm{IR}}}^{k_{\mathrm{UV}}\frac{a(\tau)}{a_i}} \mathrm{d}\ln k \frac{k^3}{2\pi^2}|\zeta_k(\tau)|^2.$$

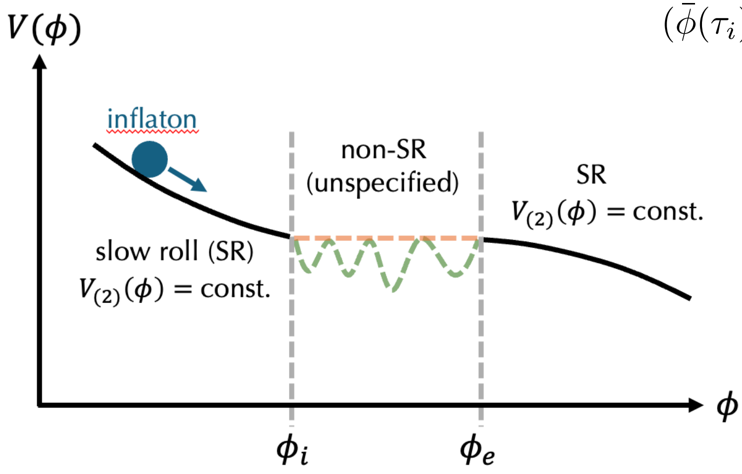
Point: Once we fix $V_{c,(1)}$, $V_{c,(2)} \equiv dV_{c,(1)}/d\phi$ is automatically fixed:

$$V_{c,(2)}(\tau) = \frac{a(\tau)}{\dot{\bar\phi}} \frac{\mathrm{d}V_{c,(1)}(\tau)}{\mathrm{d}\tau} = -\frac{V_{(4)}(\tau)b^2(\tau)\sigma_\zeta^2(\tau)}{2} + \frac{a(\tau)V_{(3)}(\tau)}{2Hb(\tau)}\frac{\mathrm{d}}{\mathrm{d}\tau}\sigma_{\delta\phi}^2(\tau)$$

Substitute the zero-tadpole condition

$(\sigma_{\delta\phi}^2(\tau) \equiv \langle 0|(\delta\phi(\mathbf{x},\tau))^2|0\rangle = b^2(\tau)\sigma_\zeta^2(\tau))$

One loop calculation



$(\bar{\phi}(\tau_i) = \phi_i)$

$$\begin{aligned}\langle \zeta_{\mathbf{q}}(\tau)\zeta_{\mathbf{q}'}(\tau)\rangle =& \langle 0|\zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau)|0\rangle + 2\,\mathrm{Im}\left[\int_{\tau_i}^{\tau} \mathrm{d}\tau'\, \langle 0|\zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau)H_{\mathrm{int},4}(\tau')|0\rangle\right] \\ &+ 2\,\mathrm{Re}\left[\int_{\tau_i}^{\tau}\mathrm{d}\tau' \int_{\tau_i}^{\tau'}\mathrm{d}\tau''\, \langle 0|\left(H_{\mathrm{int},3}(\tau')\zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau) - \zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau)H_{\mathrm{int},3}(\tau')\right) H_{\mathrm{int},3}(\tau'')(\tau''))|0\rangle\right] \\ &+ 2\,\mathrm{Im}\left[\int_{\tau_i}^{\tau}\mathrm{d}\tau'\,\langle 0|\zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau)H_c(\tau')|0\rangle\right] \\ =& (2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}[\mathcal{P}_{\zeta,\mathrm{tr}}(q,\tau) + \mathcal{P}_{\zeta,1\mathrm{vx}}(q,\tau) + \mathcal{P}_{\zeta,2\mathrm{vx}}(q,\tau) + \mathcal{P}_{\zeta,c}(q,\tau)]\end{aligned}$$

After straightforward calculation, we obtain

$$\mathcal{P}_{\zeta,1\mathrm{vx}}(q,\tau) = \frac{q^3}{\pi^2}\int_{\tau_i}^{\tau}\mathrm{d}\tau' b^4(\tau')V_{(4)}(\tau')\mathrm{Im}[\zeta_q(\tau)\zeta_q^*(\tau')]\mathrm{Re}[\zeta_p(\tau)\zeta_p^*(\tau')]\sigma_\zeta^2(\tau')$$

$$\begin{aligned}\lim_{q\to 0}\mathcal{P}_{\zeta,2\mathrm{vx}}(q,\tau) =& \frac{4q^3}{\pi^2}\int_{\tau_i}^{\tau}\mathrm{d}\tau'\int_{\tau_i}^{\tau'}\mathrm{d}\tau'' a^4(\tau')a^4(\tau'')b^3(\tau')b^3(\tau'')V_{(3)}(\tau')V_{(3)}(\tau'')\mathrm{Im}[\zeta_q(\tau)\zeta_q^*(\tau')]\mathrm{Re}[\zeta_q(\tau)\zeta_q^*(\tau'')] \\ &\times\int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{Im}[\zeta_k(\tau')\zeta_k^*(\tau'')]\mathrm{Re}[\zeta_k(\tau')\zeta_k^*(\tau'')].\end{aligned}$$

In the next slides, we will see $\mathcal{P}_{\zeta,c}$.

Cancellation of one loops

$$2\,\mathrm{Im}\left[\int_{\tau_i}^{\tau} \mathrm{d}\tau' \, \langle 0|\zeta^I_{\mathbf{q}}(\tau)\zeta^I_{\mathbf{q}'}(\tau)H_c(\tau')|0\rangle\right] = (2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}\mathcal{P}_{\zeta,c}(q,\tau) \quad \left(H_c = \int \mathrm{d}^3x a^4 \left(bV_{c,(1)}\zeta + \underline{\frac{b^2}{2}V_{c,(2)}\zeta^2}\right)\right)$$

contributes to $\mathcal{P}_{\zeta,c}$

From the zero-tadpole condition, we have obtained

$$V_{c,(2)}(\tau) = -\frac{V_{(4)}(\tau)b^2(\tau)\sigma^2_\zeta(\tau)}{2} + \frac{a(\tau)V_{(3)}(\tau)}{2Hb(\tau)}\frac{\mathrm{d}}{\mathrm{d}\tau}\sigma^2_{\delta\phi}(\tau)$$

$(\sigma^2_{\delta\phi}(\tau) \equiv \langle 0|(\delta\phi(\mathbf{x},\tau))^2|0\rangle = b^2(\tau)\sigma^2_\zeta(\tau))$

After some calculation, we find

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\sigma^2_{\delta\phi}(\tau) = Ha(\tau)b(\tau)^2\mathcal{P}_{\zeta,\mathrm{tr}}(k_{\mathrm{IR}},\tau) - 4a(\tau)\int_{\tau_i}^{\tau}\mathrm{d}\tau' a^4(\tau')b^3(\tau')b^2(\tau)HV_{(3)}(\tau')\int\frac{\mathrm{d}^3k}{(2\pi)^3}\,\mathrm{Im}[\zeta_k(\tau)\zeta^*_k(\tau')]\,\mathrm{Re}[\zeta_k(\tau)\zeta^*_k(\tau')].$$

(see arXiv:2502.08707 for the intermediate steps)

Substituting this into $V_{c,(2)}$, we obtain

$$\mathcal{P}_{\zeta,c}(q,\tau) = \frac{2q^3}{\pi^2}\int_{\tau_i}^{\tau}\mathrm{d}\tau' b^2(\tau')V_{c,(2)}(\tau')\,\mathrm{Im}[\zeta_q(\tau)\zeta^*_q(\tau')]\,\mathrm{Re}[\zeta_q(\tau)\zeta^*_q(\tau')]$$
$$= -\mathcal{P}_{\zeta,1\mathrm{vx}}(q,\tau) - \lim_{q\to 0}\mathcal{P}_{\zeta,2\mathrm{vx}}(q,\tau)$$

(assumed $\mathcal{P}_\zeta(k_{IR})$ is negligibly small.
Namely, $\mathcal{P}_\zeta(k_{IR})$ is not enhanced by the non-SR period.)

The counterterm contribution cancels the other one-loop contributions.

$$\lim_{q\to 0}\langle\zeta_{\mathbf{q}}(\tau)\zeta_{\mathbf{q}'}(\tau)\rangle = \lim_{q\to 0}(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}[\mathcal{P}_{\zeta,\mathrm{tr}}(q,\tau)+\mathcal{P}_{\zeta,1\mathrm{vx}}(q,\tau)+\mathcal{P}_{\zeta,2\mathrm{vx}}(q,\tau)+\mathcal{P}_{\zeta,c}(q,\tau)]$$
$$= \lim_{q\to 0}(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}\mathcal{P}_{\zeta,\mathrm{tr}}(q,\tau)$$

Conservation of curvature at one loop

Comparison with literature

The role of counterterms has already been stressed in Pimentel, Senatore, and Zaldarriaga (2013), though they use comoving gauge without the field redefinition.

The counterterm in comoving gauge is introduced as

$$S_{\rm tad,counter} = \int \mathrm{d}^4x\sqrt{-g}\,\left[g^{00}\delta M^4(t) + \delta\Lambda(t)\right]$$

counterterms

δM^4 and $\delta\Lambda$ are determined by the zero-tadpole condition $\langle\zeta\rangle = 0$.

Point: $\sqrt{-g}$ and g^{00} include not only $\mathcal{O}(\zeta)$ but also $\mathcal{O}(\zeta^n)$.

$\mathcal{O}(\delta M^4\,\zeta^2)$ and $\mathcal{O}(\delta\Lambda\,\zeta^2)$ cancel the other one-loop corrections so that the superhorizon curvature perturbations are conserved at one loop.

See also Inomata (2502.08707) and Fang+ (2025) for the curvature conservation without the zero-tadpole condition ($< \delta\phi > \neq 0$), where the backreaction instead plays a crucial role.

Outline

- Introduction
- Role of counterterms in curvature conservation
  - Key logic
  - One loop calculation
- **<u>Summary</u>**

Summary

We have shown the conservation of superhorizon curvature perturbations at one loop.

We have clarified the counterterm contribution plays a crucial role in the curvature conservation, which was overlooked in the recent papers.

$$\mathcal{H}_0 = \frac{1}{2a^2} b^2 \left[\zeta'^2 + (\partial_i \zeta)^2 \right] ,$$

$$\mathcal{H}_{\text{int},3} = \frac{1}{6} V_{(3)} b^3 \zeta^3 , \quad \mathcal{H}_{\text{int},4} = \frac{1}{24} V_{(4)} b^4 \zeta^4 ,$$

$$\underline{\mathcal{H}_c = b V_{c,(1)} \zeta + \frac{b^2}{2} V_{c,(2)} \zeta^2}$$

were overlooked!

Under the condition $\langle \zeta \rangle = 0$, $V_{c,(1)}$ is fixed and $V_{c,(2)} (\equiv dV_{c,(1)}/d\phi)$ is also fixed.
We have seen $V_{c,(2)} \zeta^2$ cancel the other loop contributions.

A future direction:

The counterterm contribution ($\frac{b^3}{6} V_{c,(3)} \zeta^3$) in the one-loop correction to the bispectrum $\langle \zeta^3 \rangle$.

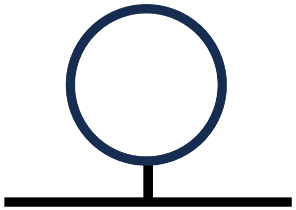
Backup

Case with non-zero tadpole

So far, we have seen the conservation of curvature under $\langle\zeta_n\rangle = \langle\delta\phi\rangle = 0$.

Given that most of the recent works do not take into account the tadpole contribution, it seems that they implicitly or explicitly impose the zero-tadpole condition.

In the case of $\langle\delta\phi\rangle \neq 0$ ($\langle\zeta_n\rangle \neq 0$), we need to be careful about the tadpole contribution and the backreaction.



Tadpole contribution

Short summary of 2502.08707

(Fang+ (2025) also discusses the curvature conservation considering the backreaction in comoving gauge.)

$$\zeta = \delta N = H\frac{\delta\phi}{\langle\dot{\phi}\rangle} = H\frac{\delta\phi}{\dot{\phi} + \langle\delta\dot{\phi}\rangle}$$

← backreaction

$$\langle\zeta_{\mathbf{q}}(\eta)\zeta_{\mathbf{q}'}(\eta)\rangle = \frac{(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}\mathcal{P}_{\delta\phi}(q,\eta)}{\left(\dot{\phi}(\eta) + \langle\delta\dot{\phi}(\eta)\rangle\right)^2} = \frac{(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}\mathcal{P}_{\delta\phi,\mathrm{tr}}(q,\eta)\left(1 + \frac{\mathcal{P}_{\delta\phi,1-\mathrm{loop}}(q,\eta)}{\mathcal{P}_{\delta\phi,\mathrm{tr}}(q,\eta)}\right)}{\left(\dot{\phi}(\eta)\right)^2\left(1 + 2\frac{\langle\delta\dot{\phi}(\eta)\rangle}{\dot{\phi}(\eta)}\right)}$$

After some calculation, we found

$$\lim_{q\to 0}\frac{\mathcal{P}_{\delta\phi,1-\mathrm{loop}}(q,\eta)}{\mathcal{P}_{\delta\phi,\mathrm{tr}}(q,\eta)} = 2\frac{\langle\delta\dot{\phi}(\eta)\rangle}{\dot{\phi}(\eta)}$$

(neglected the IR boundary term)

$$\Rightarrow \quad \lim_{q\to 0}\langle\zeta_{\mathbf{q}}(\eta)\zeta_{\mathbf{q}'}(\eta)\rangle = \frac{(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')\frac{2\pi^2}{q^3}\mathcal{P}_{\delta\phi,\mathrm{tr}}(q,\eta)}{\left(\dot{\phi}(\eta)\right)^2} = \mathrm{const.}$$

Conservation of curvature at one loop

Why controversial?

Separate universe picture (= cosmological principle + causality)
If we consider a very large region (typically larger than Hubble distance), that region can be regarded as a homogeneous and isotropic Universe with the FLRW metric.
(Sasaki & Tanaka 1998, Wands+ 2000)

Separate universe in single-clock inflation
→ Superhorizon-limit curvature perturbations are conserved at non-perturbative level
(Lyth, Malik, and Sasaki, 2004)

Single-clock = The universe evolution is characterized only by ϕ (inflaton field value).

Single-clock during Ultra Slow Roll (USR)?
→ Depends on the scale of the region we consider.

For the superhorizon-limit region, which exits the horizon much before the USR, we can regard that region as a single-clock.

The recent claim violates the separate universe (cosmological principle or causality)?