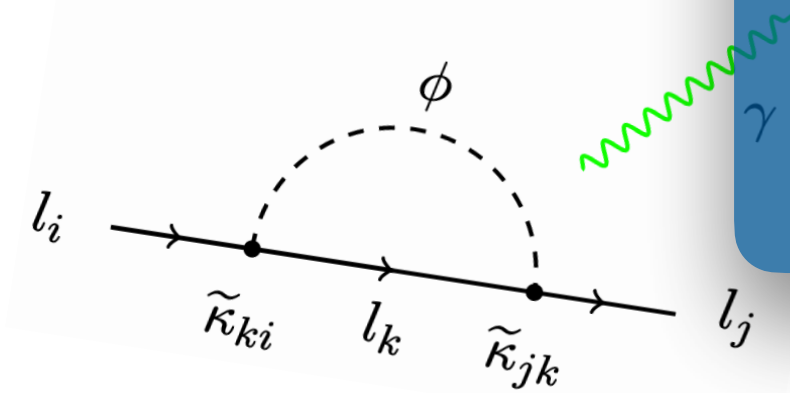




LFV decays in a realistic $U(2)$ model of flavor



Simone Marciano

based on 2505.20281, in collaboration with A. Giarnetti, D. Meloni, M. Rettaroli



PLANCK 2025

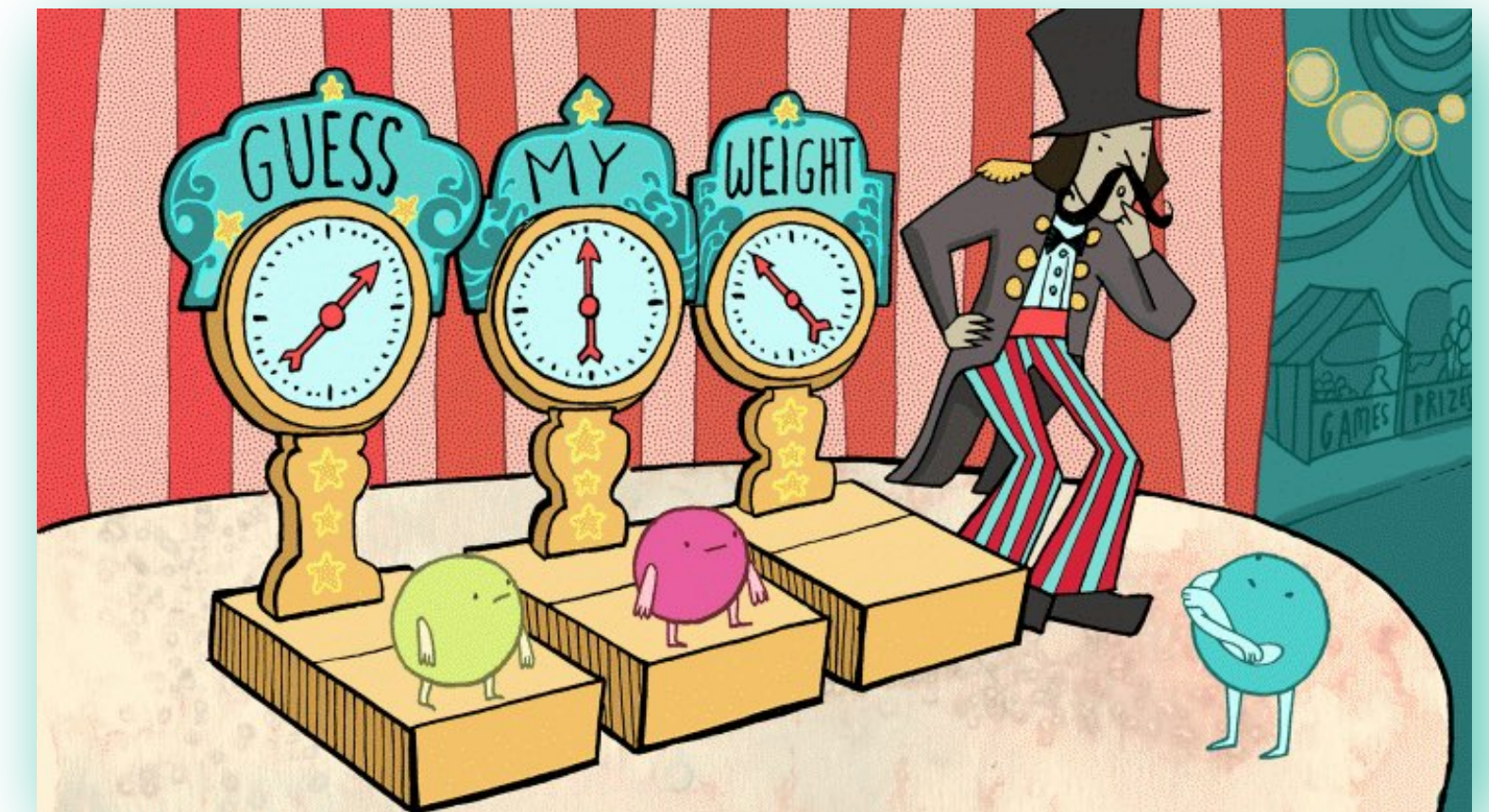
May 29th, 2025 Padova

Neutrinos, an overview

- ✓ Neutrinos are assumed to be massless particles in the SM
 - No right-handed neutrino ν_R
 - Minimal lepton sector
 - Accidental lepton number conservation
 - ✓ There are three different neutrino *flavors*: **electron-**, **muon-** and **tauon-** neutrinos
 - Their mass eigenstates do not correspond to the flavor ones
 $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$
 - Mass eigenstates evolve in time via the Schrödinger equation
 $|\nu_i(t)\rangle = e^{-im_i t} |\nu_i(0)\rangle$
 - Flavor neutrino states **oscillate** $\langle \nu_\alpha(t) | \nu_\beta(0) \rangle \neq \delta_{\alpha\beta}$
- ➡ **Neutrinos must have a non zero mass!**



Quantum Mechanics
at work!



Neutrino mixing parameters

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{3\ell}^2 \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

mass splittings

matter potential

Nu IT 6.0 (2024)					
IC19 without SK atmospheric data		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2 \theta_{12}$	0.307 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.011 \end{smallmatrix}$	$0.275 \rightarrow 0.345$	0.308 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.011 \end{smallmatrix}$	$0.275 \rightarrow 0.345$
	$\theta_{12}/^\circ$	33.68 $\begin{smallmatrix} 0 & 0.73 \\ 0 & 0.70 \end{smallmatrix}$	$31.63 \rightarrow 35.95$	33.68 $\begin{smallmatrix} 0 & 0.73 \\ 0 & 0.70 \end{smallmatrix}$	$31.63 \rightarrow 35.95$
	$\sin^2 \theta_{23}$	0.561 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.015 \end{smallmatrix}$	$0.430 \rightarrow 0.596$	0.562 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.015 \end{smallmatrix}$	$0.437 \rightarrow 0.597$
	$\theta_{23}/^\circ$	48.5 $\begin{smallmatrix} 0 & 7 \\ 0 & 9 \end{smallmatrix}$	$41.0 \rightarrow 50.5$	48.6 $\begin{smallmatrix} 0 & 7 \\ 0 & 9 \end{smallmatrix}$	$41.4 \rightarrow 50.6$
	$\sin^2 \theta_{13}$	0.02195 $\begin{smallmatrix} 0 & 0.00054 \\ 0 & 0.00058 \end{smallmatrix}$	$0.02023 \rightarrow 0.02376$	0.02224 $\begin{smallmatrix} 0 & 0.00056 \\ 0 & 0.00057 \end{smallmatrix}$	$0.02053 \rightarrow 0.02397$
	$\theta_{13}/^\circ$	8.52 $\begin{smallmatrix} 0 & 11 \\ 0 & 11 \end{smallmatrix}$	$8.18 \rightarrow 8.87$	8.58 $\begin{smallmatrix} 0 & 11 \\ 0 & 11 \end{smallmatrix}$	$8.24 \rightarrow 8.91$
	$\text{CP}/^\circ$	177 $\begin{smallmatrix} 19 \\ 20 \end{smallmatrix}$	$96 \rightarrow 422$	285 $\begin{smallmatrix} 25 \\ 28 \end{smallmatrix}$	$201 \rightarrow 348$
	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $\begin{smallmatrix} 0 & 19 \\ 0 & 19 \end{smallmatrix}$	$6.92 \rightarrow 8.05$	7.49 $\begin{smallmatrix} 0 & 19 \\ 0 & 19 \end{smallmatrix}$	$6.92 \rightarrow 8.05$
$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.534 $\begin{smallmatrix} 0 & 0.025 \\ 0 & 0.023 \end{smallmatrix}$	$+2.463 \rightarrow +2.606$	2.510 $\begin{smallmatrix} 0 & 0.024 \\ 0 & 0.025 \end{smallmatrix}$	$2.584 \rightarrow 2.438$	
IC24 with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2 \theta_{12}$	0.308 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.011 \end{smallmatrix}$	$0.275 \rightarrow 0.345$	0.308 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.011 \end{smallmatrix}$	$0.275 \rightarrow 0.345$
	$\theta_{12}/^\circ$	33.68 $\begin{smallmatrix} 0 & 0.73 \\ 0 & 0.70 \end{smallmatrix}$	$31.63 \rightarrow 35.95$	33.68 $\begin{smallmatrix} 0 & 0.73 \\ 0 & 0.70 \end{smallmatrix}$	$31.63 \rightarrow 35.95$
	$\sin^2 \theta_{23}$	0.470 $\begin{smallmatrix} 0 & 0.017 \\ 0 & 0.013 \end{smallmatrix}$	$0.435 \rightarrow 0.585$	0.550 $\begin{smallmatrix} 0 & 0.012 \\ 0 & 0.015 \end{smallmatrix}$	$0.440 \rightarrow 0.584$
	$\theta_{23}/^\circ$	43.3 $\begin{smallmatrix} 1 & 0 \\ 0 & 8 \end{smallmatrix}$	$41.3 \rightarrow 49.9$	47.9 $\begin{smallmatrix} 0 & 7 \\ 0 & 9 \end{smallmatrix}$	$41.5 \rightarrow 49.8$
	$\sin^2 \theta_{13}$	0.02215 $\begin{smallmatrix} 0 & 0.00056 \\ 0 & 0.00058 \end{smallmatrix}$	$0.02030 \rightarrow 0.02388$	0.02231 $\begin{smallmatrix} 0 & 0.00056 \\ 0 & 0.00056 \end{smallmatrix}$	$0.02060 \rightarrow 0.02409$
	$\theta_{13}/^\circ$	8.56 $\begin{smallmatrix} 0 & 11 \\ 0 & 11 \end{smallmatrix}$	$8.19 \rightarrow 8.89$	8.59 $\begin{smallmatrix} 0 & 11 \\ 0 & 11 \end{smallmatrix}$	$8.25 \rightarrow 8.93$
	$\text{CP}/^\circ$	212 $\begin{smallmatrix} 26 \\ 41 \end{smallmatrix}$	$124 \rightarrow 364$	274 $\begin{smallmatrix} 22 \\ 25 \end{smallmatrix}$	$201 \rightarrow 335$
	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $\begin{smallmatrix} 0 & 19 \\ 0 & 19 \end{smallmatrix}$	$6.92 \rightarrow 8.05$	7.49 $\begin{smallmatrix} 0 & 19 \\ 0 & 19 \end{smallmatrix}$	$6.92 \rightarrow 8.05$
$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.513 $\begin{smallmatrix} 0 & 0.021 \\ 0 & 0.019 \end{smallmatrix}$	$+2.451 \rightarrow +2.578$	2.484 $\begin{smallmatrix} 0 & 0.020 \\ 0 & 0.020 \end{smallmatrix}$	$2.547 \rightarrow 2.421$	

Neutrinos in focus: entering the precision Era



ICECUBE GEN2



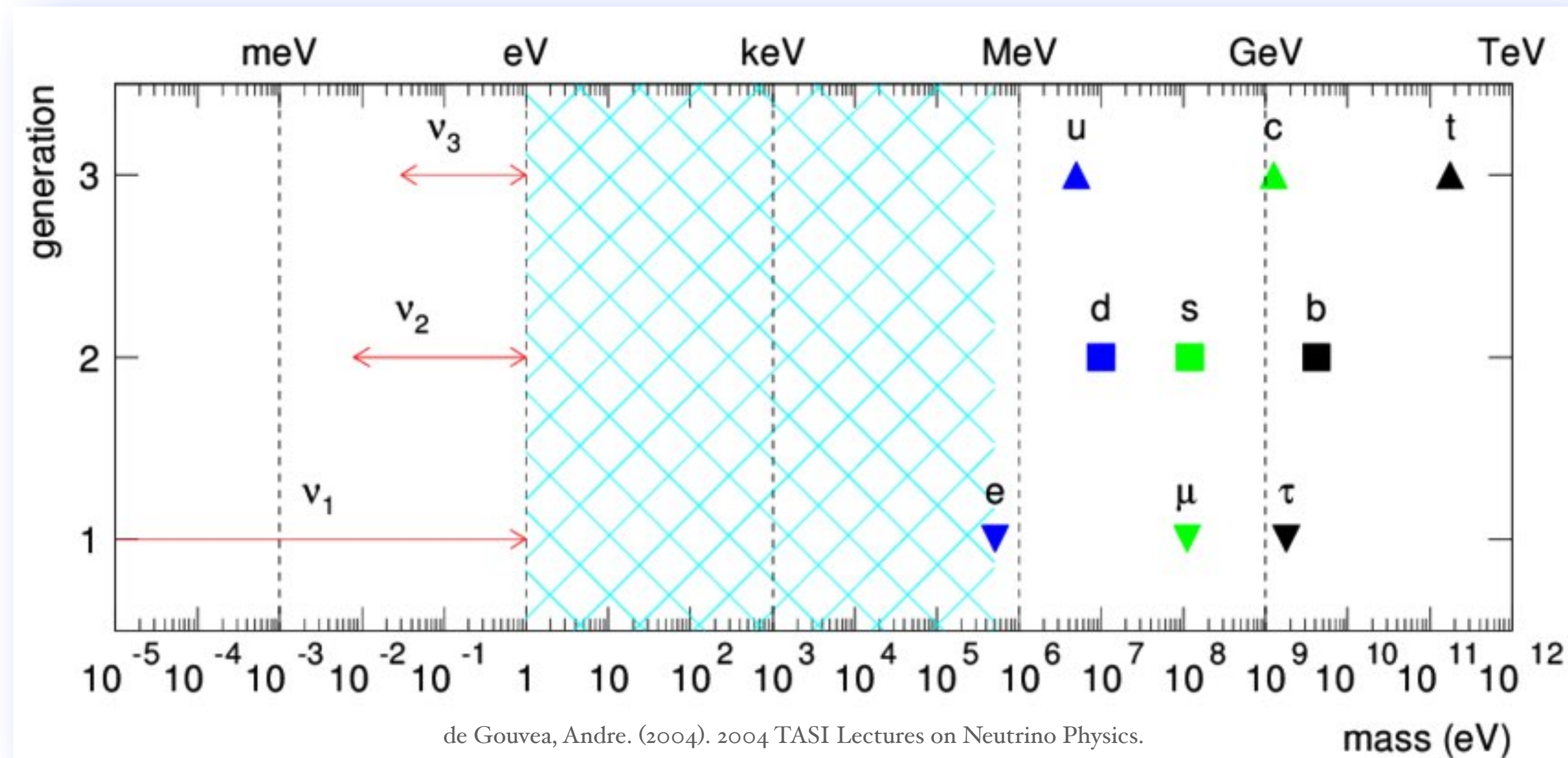
江门中微子实验

Jiangmen Underground Neutrino Observatory

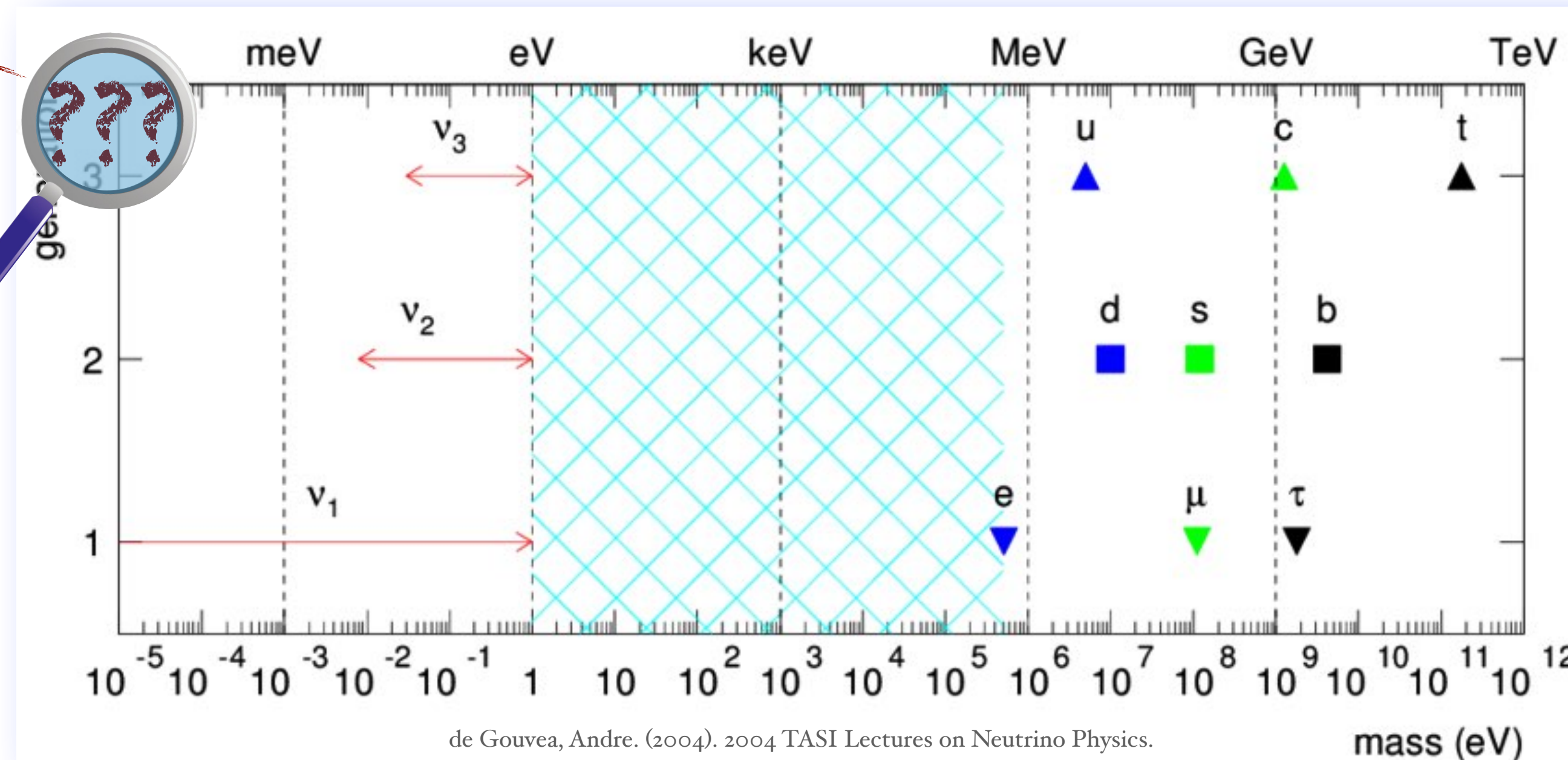


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Flavor puzzle and neutrino masses



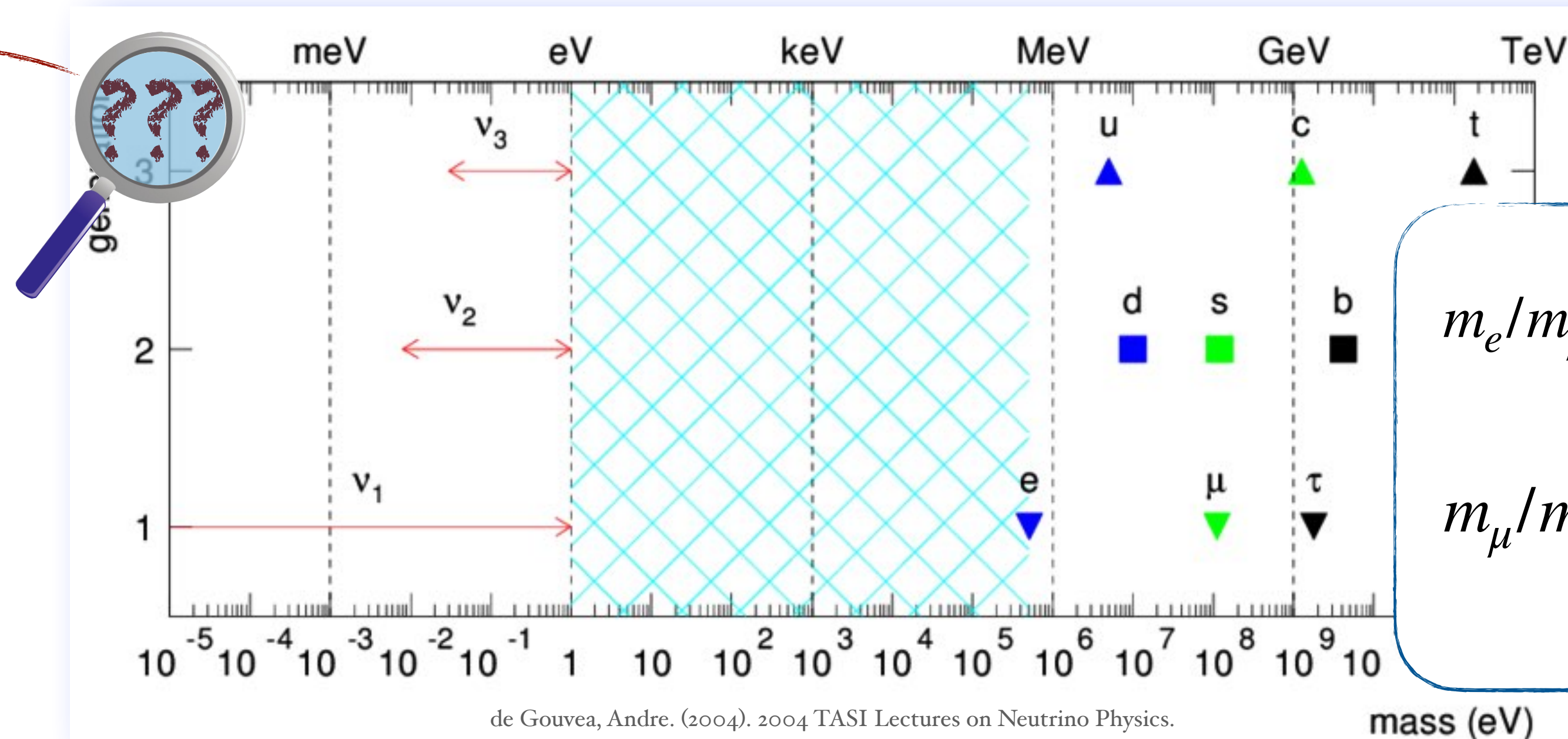
Flavor puzzle and neutrino masses



$$\sum_i^3 m_i < 0.115 \text{ eV}$$

$$\sqrt{\sum_i^3 m_i^2 |U_{ei}^2|} < 0.8 \text{ eV}$$

Flavor puzzle and neutrino masses

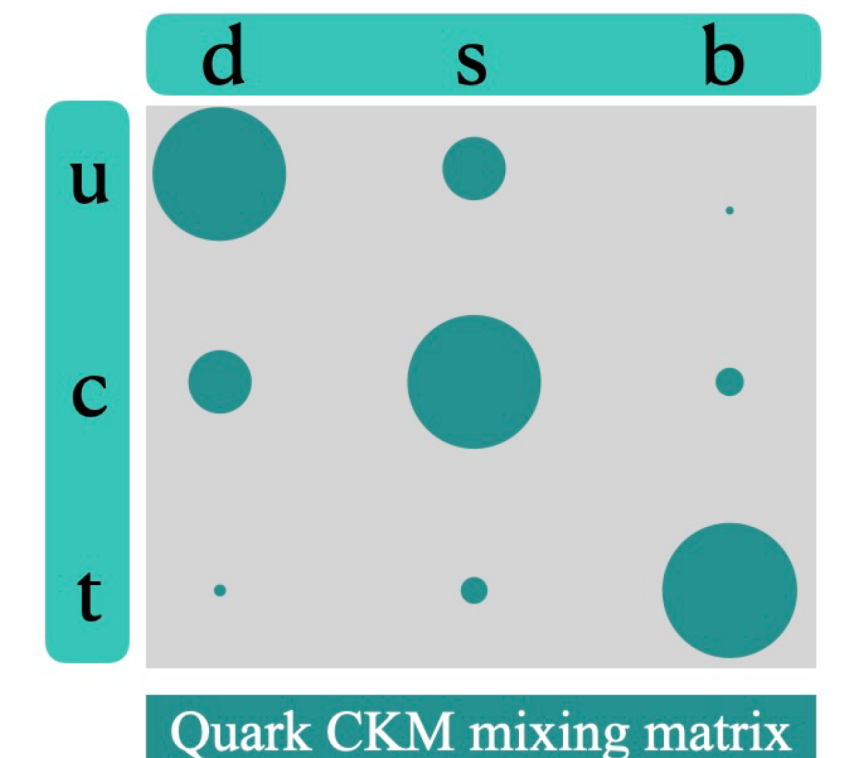
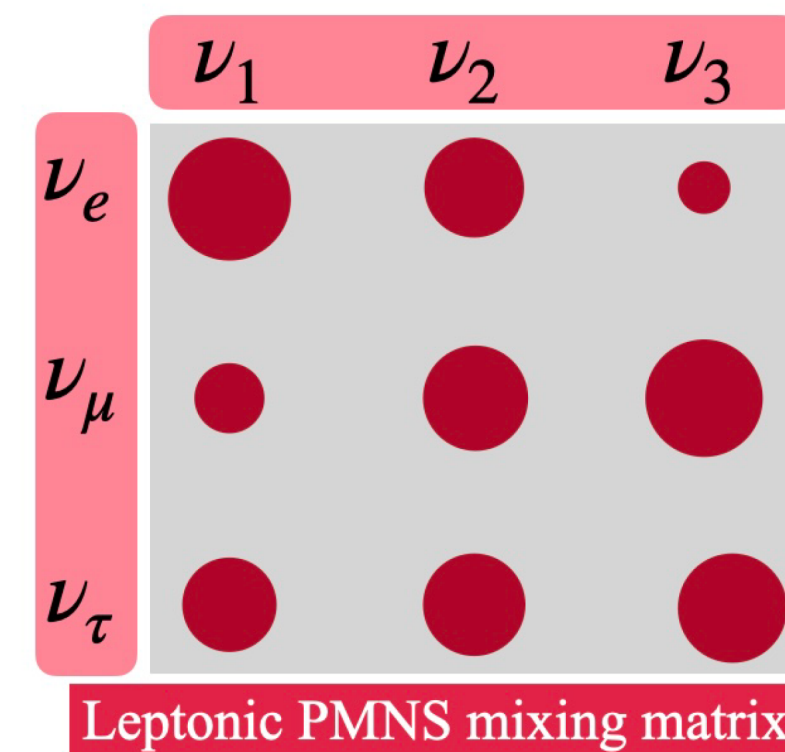


$$m_e/m_\mu \approx \frac{1}{200}$$

$$m_\mu/m_\tau \approx \frac{1}{17}$$

$$\sum_i^3 m_i < 0.115 \text{ eV}$$

$$\sqrt{\sum_i^3 m_i^2 |U_{ei}^2|} < 0.8 \text{ eV}$$



Symmetry **2022**, 14(1), 56;

A realistic model of flavor

- ➡ Introduce some new gauge singlet scalar Φ , called the *flavon*, and a global U(1) symmetry
- ➡ Assume that the SM fermions, the Higgs and the flavon are charged under the new U(1)
- ➡ Effective operator is allowed:

$$c_{ij} \left(\frac{\Phi}{\Lambda} \right)^{n_{ij}} \overline{\Psi}_{L,i} H \Psi_{R,j} + \text{h.c.} \rightarrow c_{ij} \underbrace{\left(\frac{v_{\Phi}}{\sqrt{2}\Lambda} \right)^{n_{ij}}}_{y_{ij}} \overline{\Psi}_{L,i} H \Psi_{R,j} + \text{h.c.}$$

$U(1)_{FN}$ symmetry
breaking scale

- ➡ Hierarchical pattern determined by the charge assignment

$$n_{ij} = -\frac{1}{q(\Phi)} \left[q(\overline{\Psi}_{L,i}) + q(\Psi_{R,j}) + Q(H) \right]$$

U(2) model of flavor applied to quarks and charged leptons

$U(2)$ is isomorphic to $SU(2) \otimes U(1)$

Fields / Representations	$SU(2)_F$	$U(1)_F$
$L_a, D_a, Q_a, U_a, E_a \quad (a = 1, 2)$	2	1
Q_3, U_3, E_3	1	0
L_3, D_3	1	1
$\phi_a \quad (a = 1, 2)$	2	-1
χ	1	-1

$$\langle \phi \rangle = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix}$$

$$\langle \chi \rangle = \varepsilon_\chi \Lambda$$

$$\begin{aligned}
 y_u &\sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}, & y_d &\sim y_e \sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}, & V_{ub} &\sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi}, \\
 y_c &\sim \varepsilon_\phi^2, & y_s &\sim y_\mu \sim \frac{\varepsilon_\phi^2 \varepsilon_\chi}{\sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}}, & V_{cb} &\sim \varepsilon_\phi, \\
 y_t &\sim 1, & y_b &\sim y_\tau \sim \sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}, & V_{us} &\sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi^2}.
 \end{aligned}$$

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$L_a, D_a, Q_a, U_a, E_a \quad (a = 1, 2)$	2	1
Q_3, U_3, E_3	1	0
L_3		
$\phi_a \quad (a = 1, 2)$		

$$\langle \phi \rangle = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix}$$

$$\varepsilon_\chi \Lambda$$

These predictions reproduce the experimental values by taking:

$$\varepsilon_\phi \sim \lambda^2, \quad \varepsilon_\chi \sim \lambda^{2\div 3}, \quad \lambda \approx 0.1$$

$$\begin{aligned} y_c &\sim \varepsilon_\phi^2, & y_s \sim y_\mu &\sim \frac{\varepsilon_\phi^2 \varepsilon_\chi}{\sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}}, & V_{cb} &\sim \varepsilon_\phi, \\ y_t &\sim 1, & y_b \sim y_\tau &\sim \sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}, & V_{us} &\sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi^2}. \end{aligned}$$

U(2) applied to neutrinos

Ziegler and Linster's assumption

- (N_1, N_2) is an SU(2) doublet
- N_3 is a singlet
- All positive U(1) charges

Majorana mass matrix

$$m_\nu^M \sim \frac{v^2}{M} \begin{pmatrix} \varepsilon_\chi^4/\varepsilon_\phi^2 & \varepsilon_\chi^2/\varepsilon_\phi^2 & \varepsilon_\chi^3/\varepsilon_\phi \\ \varepsilon_\chi^2/\varepsilon_\phi^2 & 1/\varepsilon_\phi^2 & \varepsilon_\chi/\varepsilon_\phi \\ \varepsilon_\chi^3/\varepsilon_\phi & \varepsilon_\chi/\varepsilon_\phi & \varepsilon_\chi^2 \end{pmatrix} \sim \begin{pmatrix} \varepsilon^2 & 1 & \varepsilon^2 \\ 1 & 1/\varepsilon^2 & 1 \\ \varepsilon^2 & 1 & \varepsilon^2 \end{pmatrix}$$

Clearly **ruled out** because it implies a

$$\text{NO with } \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \simeq \varepsilon^8$$

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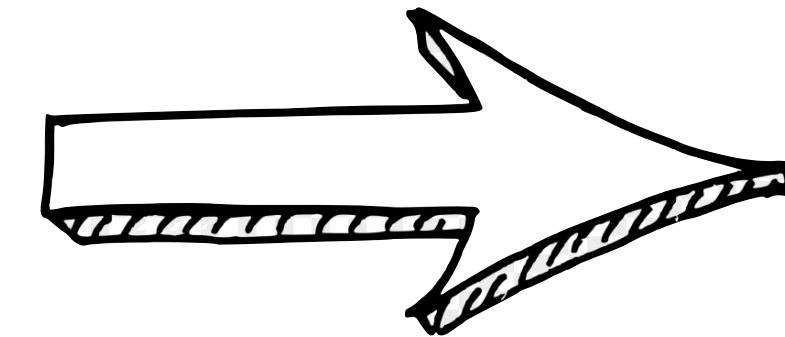
$$m_\nu^M \sim \frac{v^2}{M} \begin{pmatrix} \varepsilon_\chi^4/\varepsilon_\phi^2 & \varepsilon_\chi^2/\varepsilon_\phi^2 & \varepsilon_\chi^3/\varepsilon_\phi \\ \varepsilon_\chi^2/\varepsilon_\phi^2 & 1/\varepsilon_\phi^2 & \varepsilon_\chi/\varepsilon_\phi \\ \varepsilon_\chi^3/\varepsilon_\phi & \varepsilon_\chi/\varepsilon_\phi & \varepsilon_\chi^2 \end{pmatrix} \sim \begin{pmatrix} \varepsilon^2 & 1 & \varepsilon^2 \\ 1 & 1/\varepsilon^2 & 1 \\ \varepsilon^2 & 1 & \varepsilon^2 \end{pmatrix}$$

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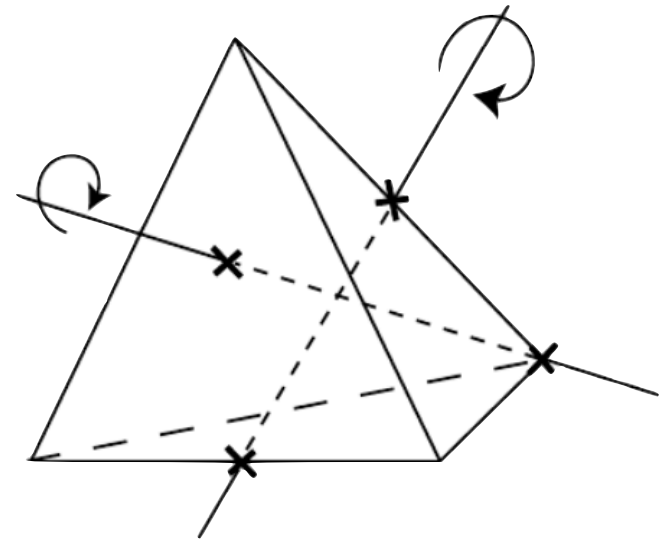
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Their Approach

$U(2)$



$D_6 \otimes U(1)$



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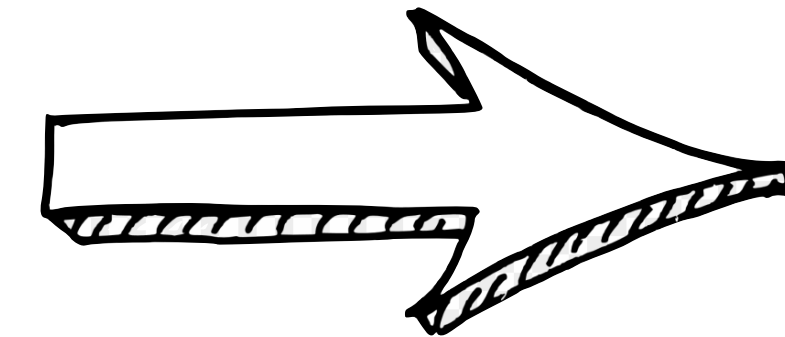
$$m_\nu^M \sim \frac{v^2}{M} \begin{pmatrix} \varepsilon_\chi^4/\varepsilon_\phi^2 & \varepsilon_\chi^2/\varepsilon_\phi^2 & \varepsilon_\chi^3/\varepsilon_\phi \\ \varepsilon_\chi^2/\varepsilon_\phi^2 & 1/\varepsilon_\phi^2 & \varepsilon_\chi/\varepsilon_\phi \\ \varepsilon_\chi^3/\varepsilon_\phi & \varepsilon_\chi/\varepsilon_\phi & \varepsilon_\chi^2 \end{pmatrix} \sim \begin{pmatrix} \varepsilon^2 & 1 & \varepsilon^2 \\ 1 & 1/\varepsilon^2 & 1 \\ \varepsilon^2 & 1 & \varepsilon^2 \end{pmatrix}$$

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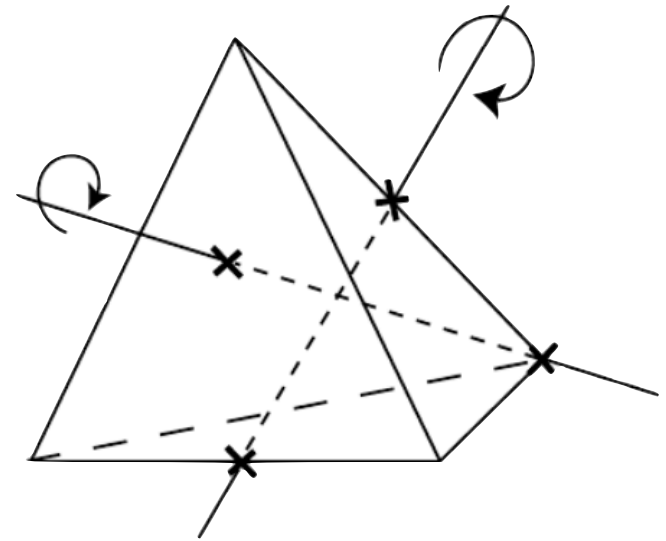
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Our Approach

Model S



Model D



Model T



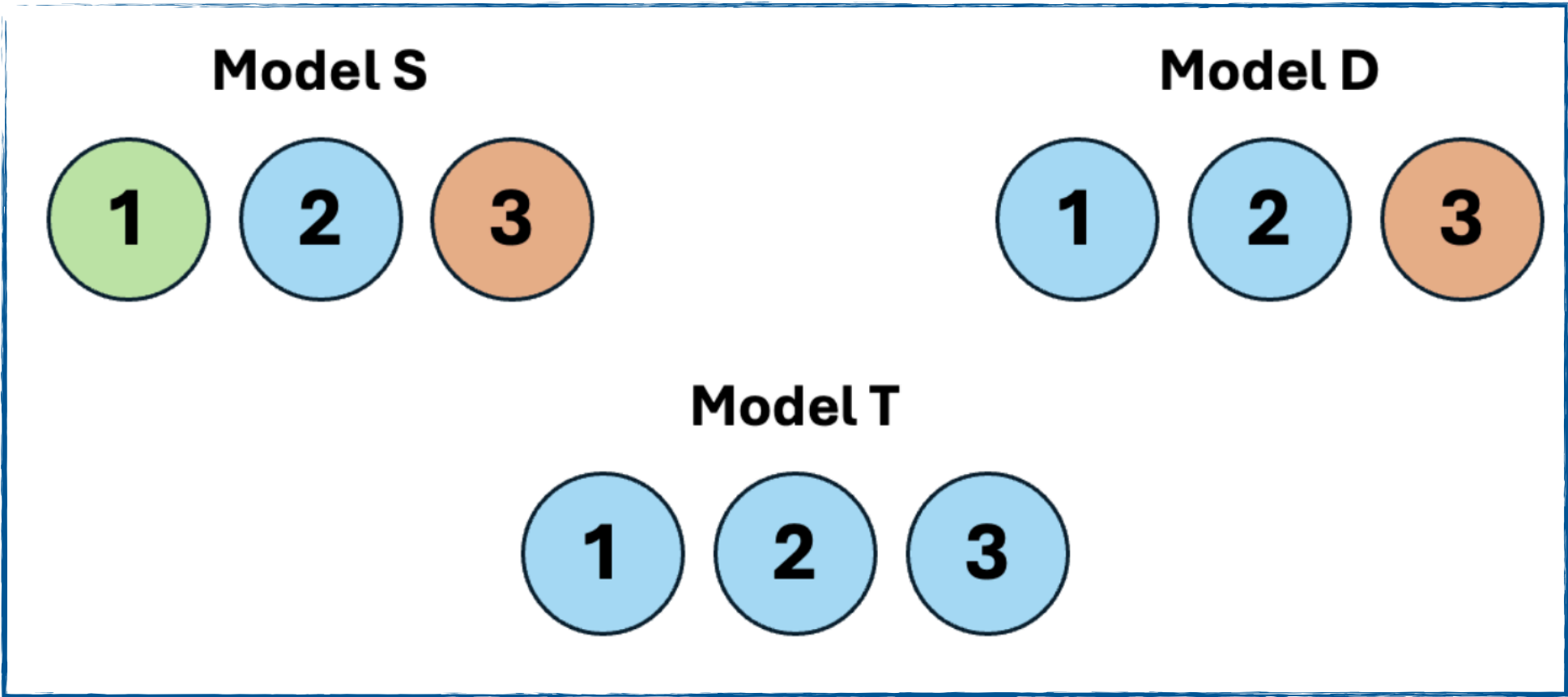
U(2) applied to neutrinos: Version 2.0

Pattern	Charges	LO mass matrix in terms of λ
S1 A S2 A D1 A D2 A	(1, 0, -2) (1, 1, -2) (1, -2) (2, -2)	$\begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \end{pmatrix}$
S3 A S4 A	(2, 1, -2) (2, 2, -2)	$\begin{pmatrix} \lambda^{12} & \lambda^8 & \lambda^8 \\ \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^8 & \lambda^4 & \lambda^4 \end{pmatrix}$
D3 A D4 A	(0, 1) (1, 0)	$\begin{pmatrix} \lambda^4 & 1 & \lambda^4 \\ 1 & 1/\lambda^4 & 1 \\ \lambda^4 & 1 & \lambda^4 \end{pmatrix}$
S1 B S2 B	(1, 0, -2) (1, 1, -2)	$\begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^5 \\ \lambda^4 & \lambda^4 & \lambda^5 \\ \lambda^5 & \lambda^5 & \lambda^6 \end{pmatrix}$
D1 B D2 B	(1, -2) (2, -2)	$\begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^7 \\ \lambda^6 & \lambda^4 & \lambda^5 \\ \lambda^7 & \lambda^5 & \lambda^6 \end{pmatrix}$
D5 B	(0, 0)	$\begin{pmatrix} \lambda^8 & \lambda^2 & \lambda^7 \\ \lambda^2 & \lambda^4 & \lambda \\ \lambda^7 & \lambda & \lambda^6 \end{pmatrix}$

Two different scenarios

- A-Scenario: $\varepsilon_\phi \sim \lambda^2$ $\varepsilon_\chi \sim \lambda^2$
- B-Scenario: $\varepsilon_\phi \sim \lambda^2$ $\varepsilon_\chi \sim \lambda^3$

Our Approach



The Leptonic Dipole Operator

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \left(\sum_{n=1}^d \sum_i \frac{\mathcal{C}_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)} + \text{H.c.} \right), \quad \text{for } d > 4,$$

$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(\mathcal{C}'_{LR} \mathcal{O}_{LR}^{(6)} + \mathcal{C}'_{RL} \mathcal{O}_{RL}^{(6)} \right), \quad \mathcal{C}'_{RL} = \mathcal{C}'_{LR} = \begin{pmatrix} \mathcal{C}'_{ee} & \mathcal{C}'_{e\mu} & \mathcal{C}'_{e\tau} \\ \mathcal{C}'_{\mu e} & \mathcal{C}'_{\mu\mu} & \mathcal{C}'_{\mu\tau} \\ \mathcal{C}'_{\tau e} & \mathcal{C}'_{\tau\mu} & \mathcal{C}'_{\tau\tau} \end{pmatrix},$$

$$\mathcal{O}_{LR}^{(6)} = \frac{v}{\sqrt{2}} \bar{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

Input values and upper bounds

Anomalous magnetic moment of the lepton ℓ
 $(\Delta a_\ell \equiv a_\ell^{\text{exp}} - a_\ell^{\text{SM}})$

**Tree-level radiative LFV
decays Branching Ratios**

$$\Delta a_\ell = \frac{4m_\ell}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \Re [\mathcal{C}'_{\ell\ell}]$$

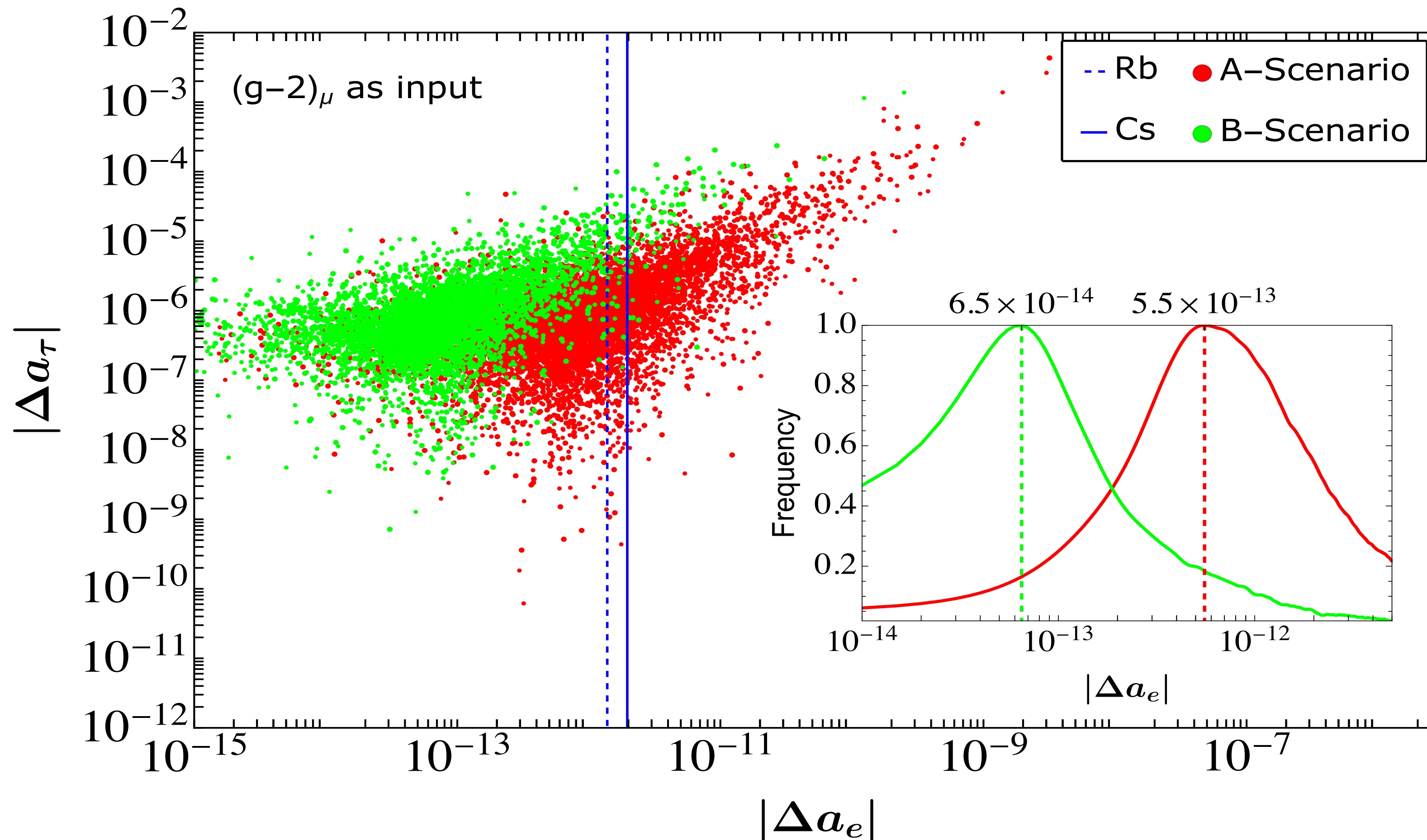
$$\mathcal{B}(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{m_{\ell_\alpha}^3 v^2}{8\pi\Gamma_{\ell_\alpha}} \frac{1}{\Lambda^4} \left(|\mathcal{C}'_{\alpha\beta}|^2 + |\mathcal{C}'_{\beta\alpha}|^2 \right)$$

Observables	Exp.-SM/Bound		Wilson Coef. in $1/\Lambda^2$ [TeV ⁻²]
Δa_μ	249×10^{-11}	Muon g-2	$\Re [C'_{\mu\mu}] = 1.0 \times 10^{-5}$
$\mathcal{B}(\mu \rightarrow e\gamma)$	$< 4.2 \times 10^{-13}$	MEG	$ C'_{e\mu(\mu e)} < 2.1 \times 10^{-10}$
$\mathcal{B}(\tau \rightarrow \mu\gamma)$	$< 4.2 \times 10^{-8}$	BaBar/BELLE	$ C'_{\mu\tau(\tau\mu)} < 2.65 \times 10^{-6}$
$\mathcal{B}(\tau \rightarrow e\gamma)$	$< 3.3 \times 10^{-8}$	BaBar/BELLE	$ C'_{e\tau(\tau e)} < 2.35 \times 10^{-6}$

Predictions on the $(g - 2)_{e,\tau}$

$$\Delta a_e^{\text{Rb}} = a_e^{\text{exp}} - a_e^{\text{SM,Rb}} = (+4.8 \pm 3.0) \times 10^{-13},$$

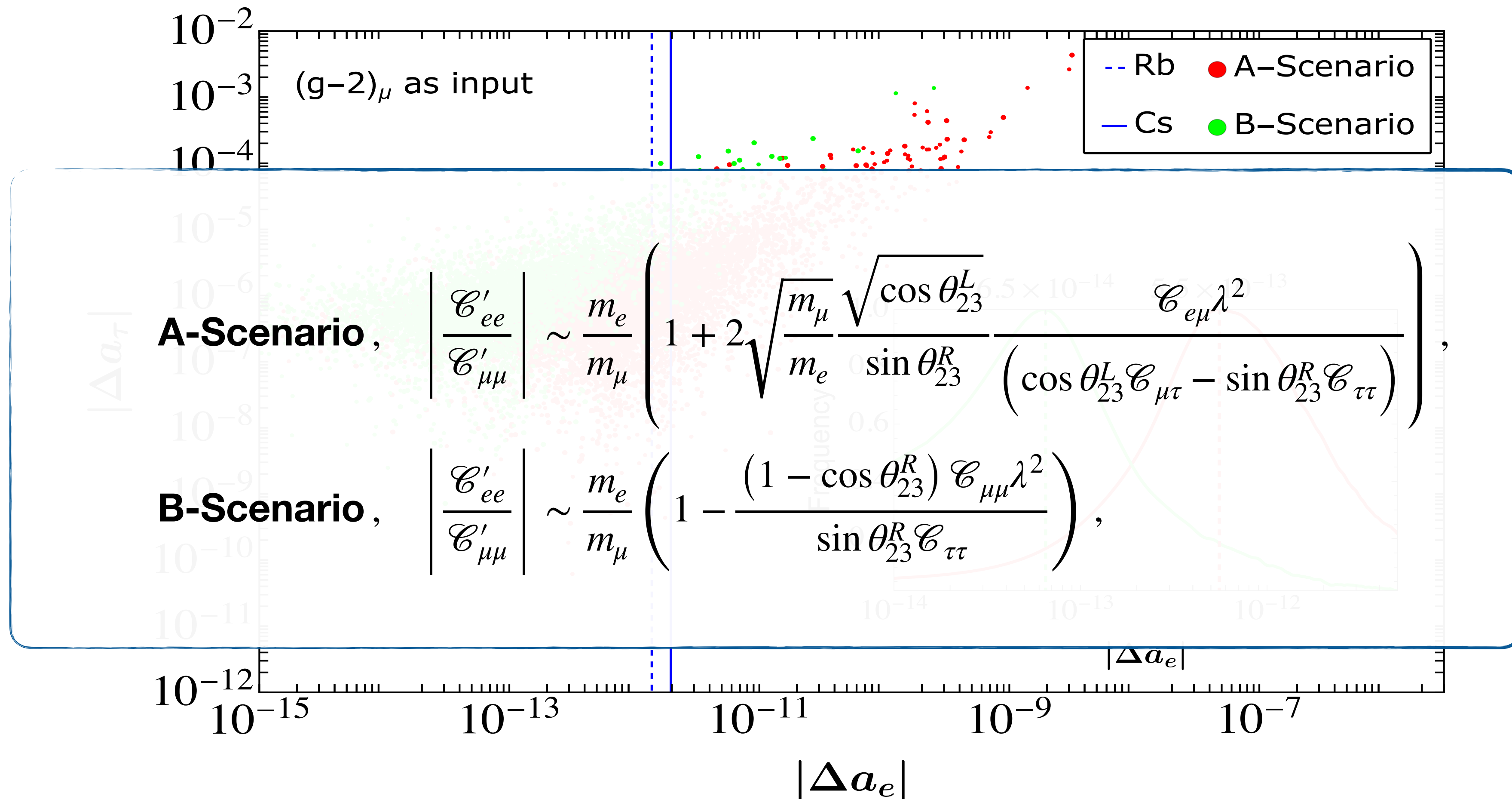
$$\Delta a_e^{\text{Cs}} = a_e^{\text{exp}} - a_e^{\text{SM,Cs}} = (-8.8 \pm 3.6) \times 10^{-13}.$$



Predictions on the $(g - 2)_{e,\tau}$

$$\Delta a_e^{\text{Rb}} = a_e^{\text{exp}} - a_e^{\text{SM,Rb}} = (+4.8 \pm 3.0) \times 10^{-13},$$

$$\Delta a_e^{\text{Cs}} = a_e^{\text{exp}} - a_e^{\text{SM,Cs}} = (-8.8 \pm 3.6) \times 10^{-13}.$$



$\mu \rightarrow e\gamma$ in light of $(g - 2)_\mu$

With $\mathcal{O}(1)$ WCs, $\mathcal{C}'_{e\mu(\mu e)}$ is suppressed by at most an $\mathcal{O}(10^{-2})$ with respect to $\mathcal{C}'_{\mu\mu}$

A-Scenario

$$\left| \frac{\mathcal{C}'_{e\mu}}{\mathcal{C}'_{\mu\mu}} \right| \sim \frac{1}{\sqrt{\cos \theta_{23}^L}} \sqrt{\frac{m_e}{m_\mu}} + \cot \theta_{23}^R \frac{\mathcal{C}_{e\mu} \lambda^2}{\cos \theta_{23}^L \mathcal{C}_{\mu\tau} - \sin \theta_{23}^L \mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^4),$$

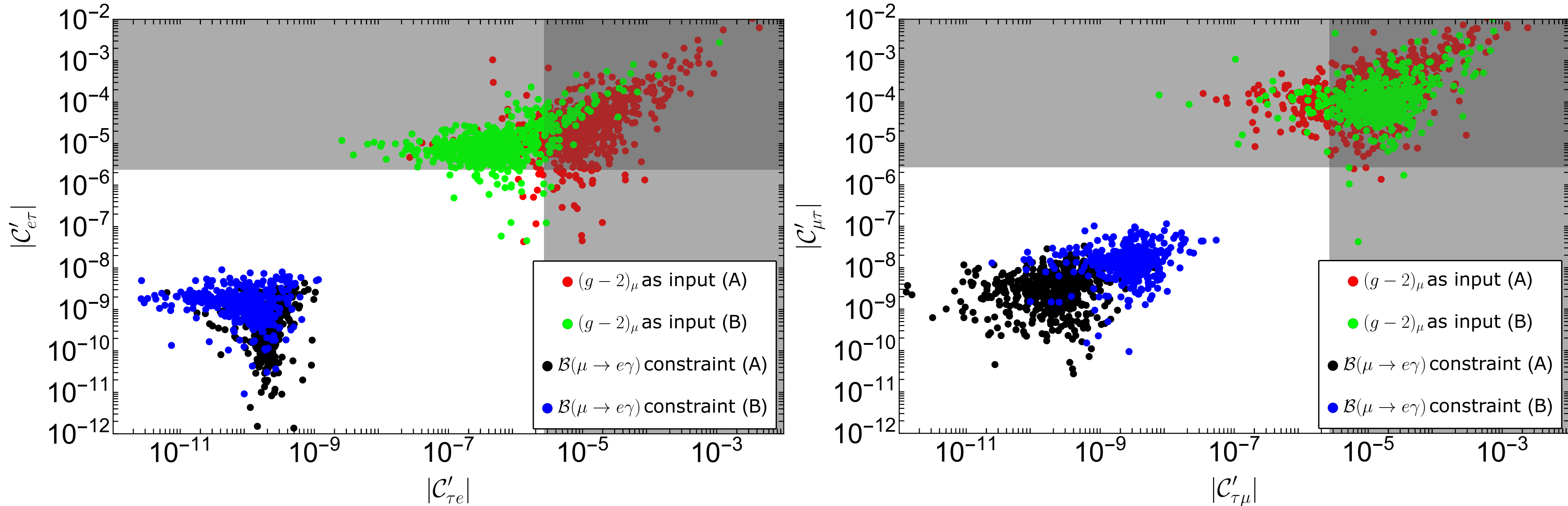
$$\left| \frac{\mathcal{C}'_{\mu e}}{\mathcal{C}'_{\mu\mu}} \right| \sim \sqrt{\cos \theta_{23}^L} \sqrt{\frac{m_e}{m_\mu}} + \cot \theta_{23}^R \frac{\mathcal{C}_{e\mu} \lambda^2}{\cos \theta_{23}^L \mathcal{C}_{\mu\tau} - \sin \theta_{23}^L \mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^2 \sqrt{m_e/m_\mu});$$

B-Scenario

$$\left| \frac{\mathcal{C}'_{e\mu}}{\mathcal{C}'_{\mu\mu}} \right| \sim \frac{1}{\sqrt{\cos \theta_{23}^L}} \sqrt{\frac{m_e}{m_\mu}} + \cot \theta_{23}^R \frac{\mathcal{C}_{e\mu} \lambda^2}{\cos \theta_{23}^L \mathcal{C}_{\mu\tau} - \sin \theta_{23}^L \mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^4),$$

$$\left| \frac{\mathcal{C}'_{\mu e}}{\mathcal{C}'_{\mu\mu}} \right| \sim \sqrt{\cos \theta_{23}^L} \sqrt{\frac{m_e}{m_\mu}} + \cot \theta_{23}^R \frac{\mathcal{C}_{e\mu} \lambda^2}{\cos \theta_{23}^L \mathcal{C}_{\mu\tau} - \sin \theta_{23}^L \mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^2 \sqrt{m_e/m_\mu}).$$

LFV decays in light of $(g - 2)_\mu$



Within an $U(2)$ flavor model, the current experimental limits on the branching ratios of LFV decays are not compatible with the observed anomalous Δa_μ

Conclusions

- ➔ **Neutrinos:** A Majorana mass term without invoking D_6
- ➔ **Fit:** Successful fit to the low-energy neutrino observables
- ➔ **LFV and the $(g - 2)_\mu$:** Incompatibility in SMEFT
(UV completion?)
- ➔ **Final remark:** Fully consistent if $(g - 2)_\mu$ is resolved



S. Marignano, IFIC - CSIC - UV



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Thank you for the attention

S. Marignano, IFIC - CSIC - UV

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Backup slides

The curious case of Mr. Neutrino



1930

Fermi's theory of
weak interactions
and beta decay



1956



The Standard
Model of Particle
Physics

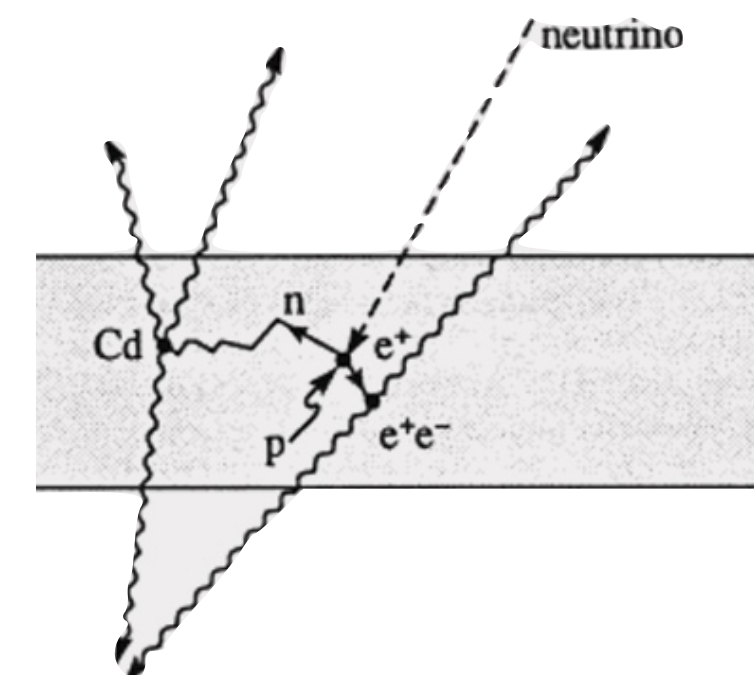


1934

Pauli's hypothesis:
"what if the missing energy
is carried off by an otherwise
invisible particle?"



Discovery of the
neutrino by
Cowan & Reines

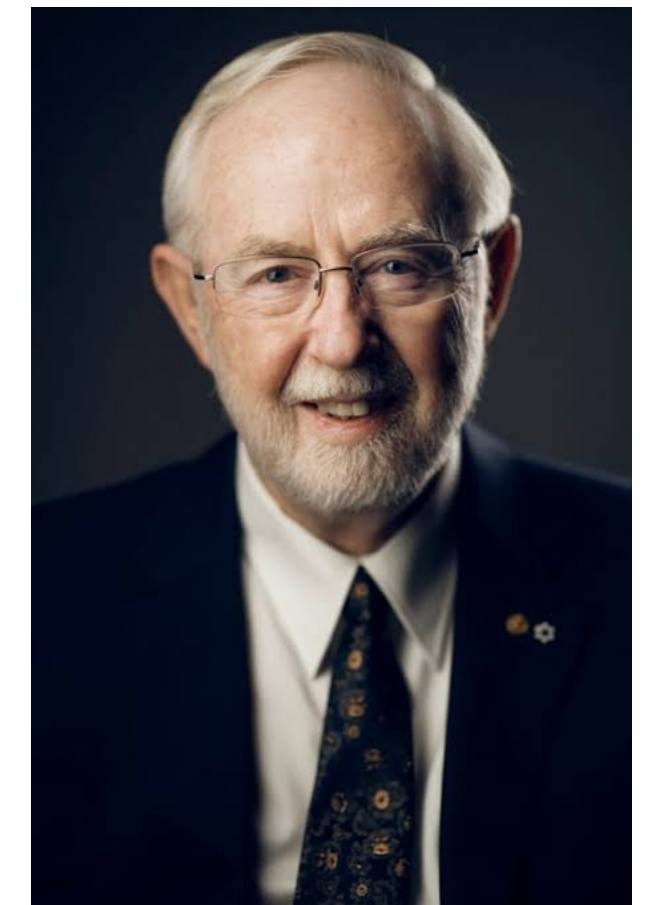
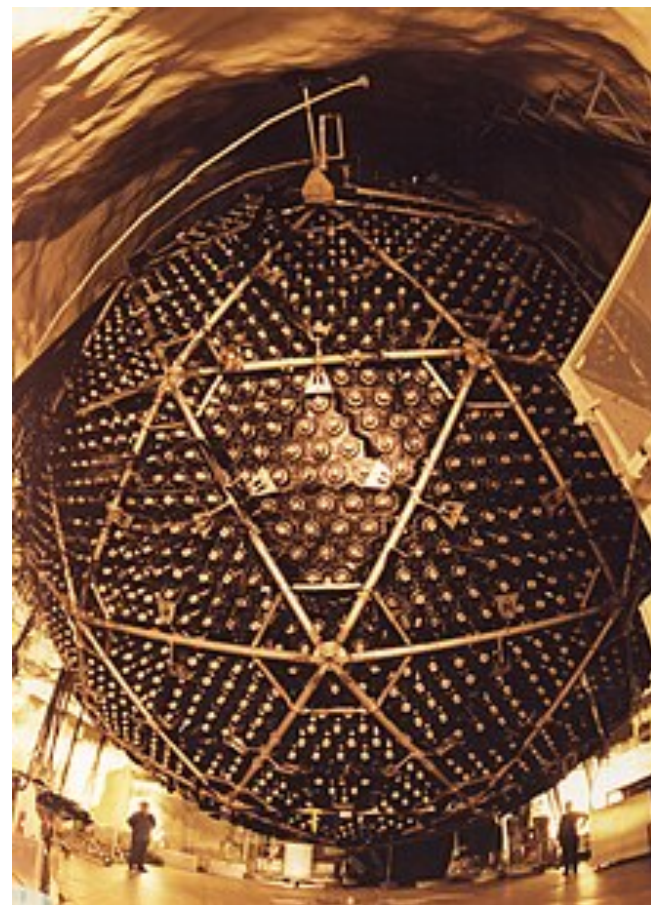
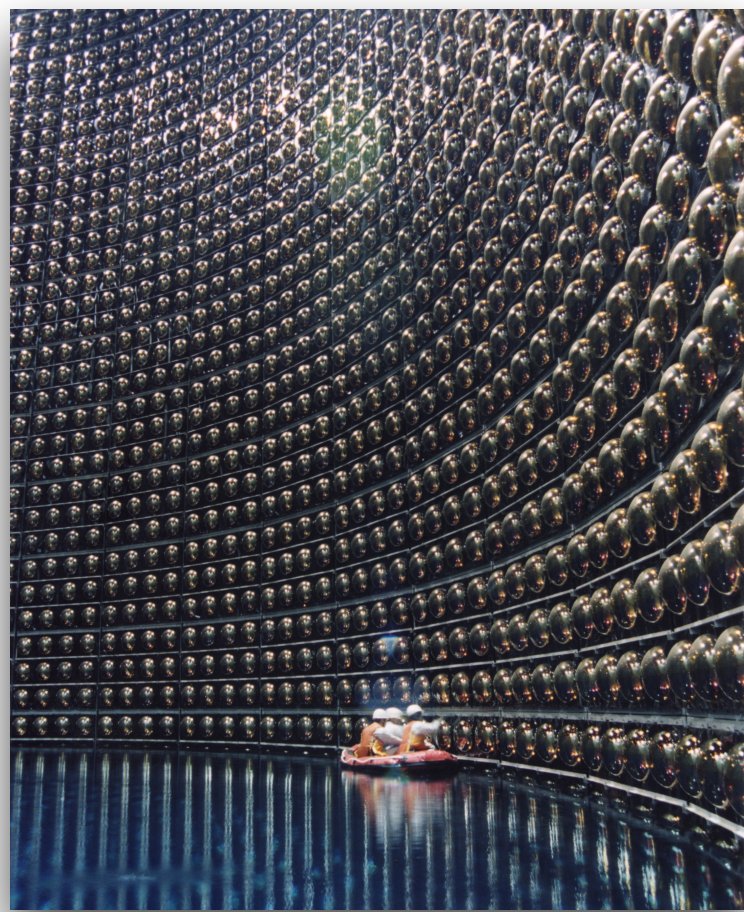


~ 1960/1970



Neutrinos, an overview

- ✓ Neutrinos are assumed to be massless particles in the SM
 - No right-handed neutrino ν_R
 - Minimal lepton sector
 - Accidental lepton number conservation
- ✓ There are three different neutrino *flavors*: **electron-**, **muon-** and **tauon-** neutrinos

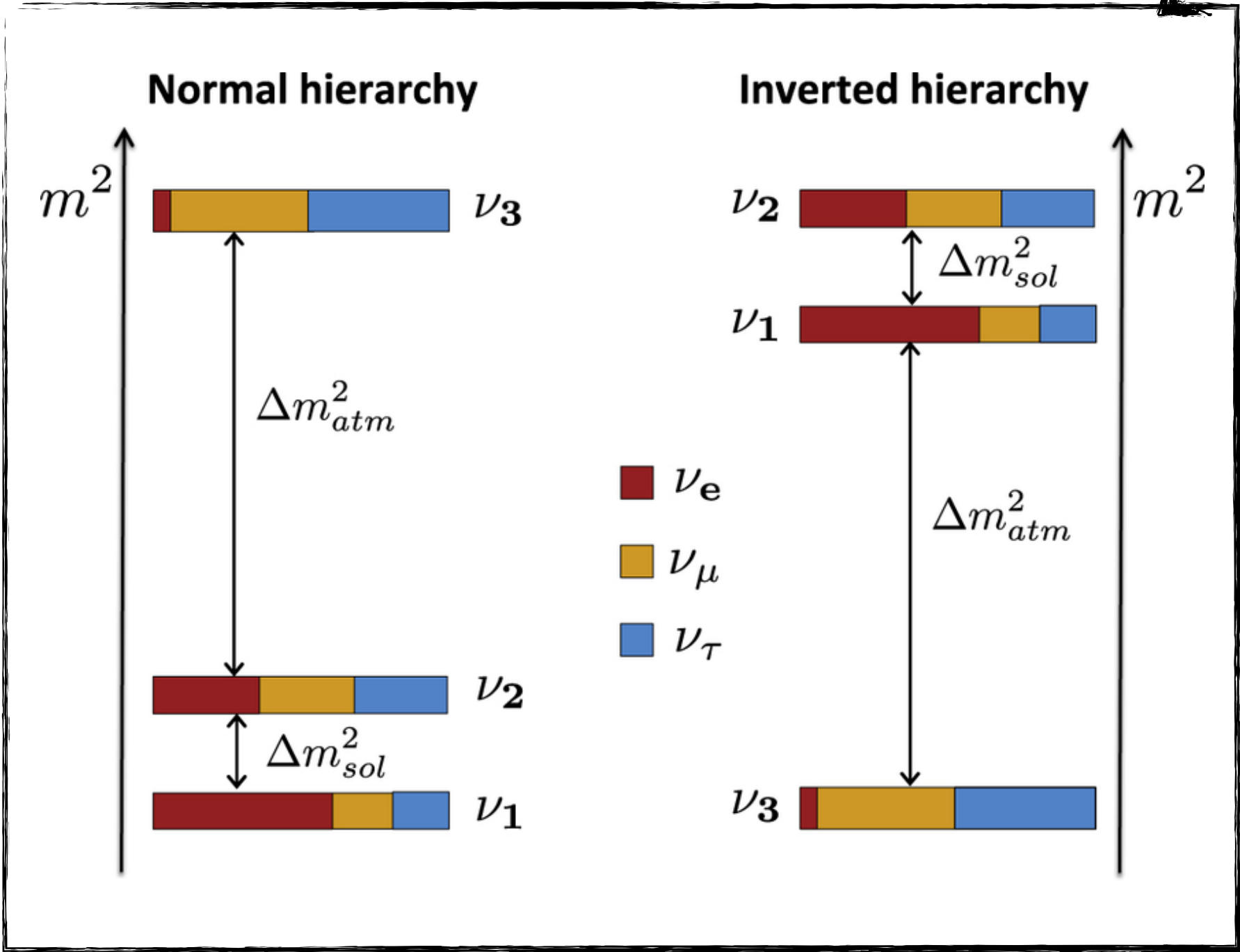


Physics Nobel 2015

Neutrino mixing parameters

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



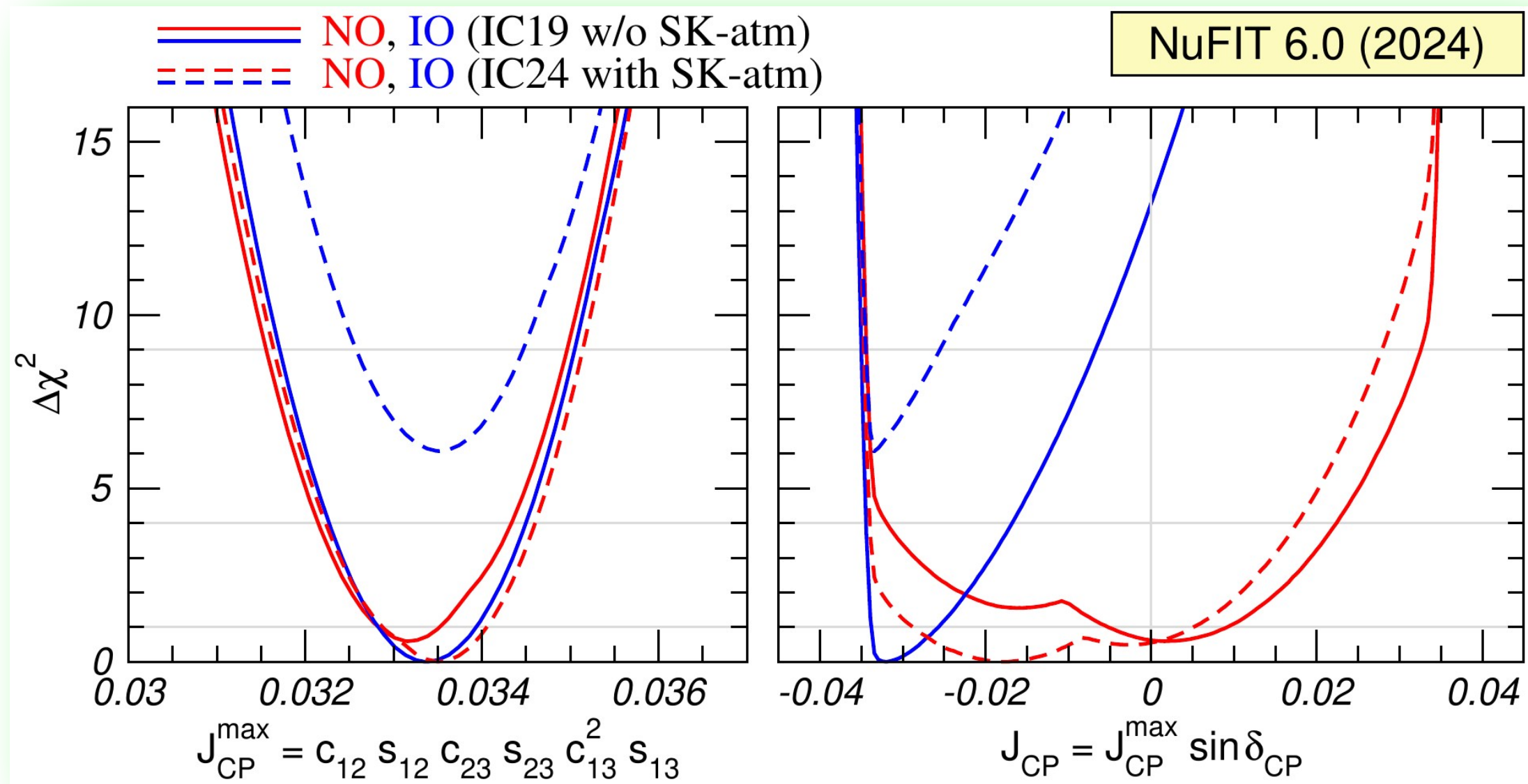
Nu IT 6.0 (2024)					
without SK atmospheric data		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$ 3σ range		bfp $\pm 1\sigma$ 3σ range	
	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345
	$\theta_{12}/^\circ$	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95
	$\sin^2 \theta_{23}$	0.561 $^{+0.012}_{-0.015}$	0.430 $\rightarrow$ 0.596	0.562 $^{+0.012}_{-0.015}$	0.437 $\rightarrow$ 0.597
	$\theta_{23}/^\circ$	48.5 $^{+0.7}_{-0.9}$	41.0 $\rightarrow$ 50.5	48.6 $^{+0.7}_{-0.9}$	41.4 $\rightarrow$ 50.6
	$\sin^2 \theta_{13}$	0.02195 $^{+0.00054}_{-0.00058}$	0.02023 $\rightarrow$ 0.02376	0.02224 $^{+0.00056}_{-0.00057}$	0.02053 $\rightarrow$ 0.02397
	$\theta_{13}/^\circ$	8.52 $^{+0.11}_{-0.11}$	8.18 $\rightarrow$ 8.87	8.58 $^{+0.11}_{-0.11}$	8.24 $\rightarrow$ 8.91
CP/ $^\circ$	177 $^{+19}_{-20}$	96 $\rightarrow$ 422	285 $^{+25}_{-28}$	201 $\rightarrow$ 348	
IC19	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.534 $^{+0.025}_{-0.023}$	+2.463 $\rightarrow$ +2.606	2.510 $^{+0.024}_{-0.025}$	2.584 $\rightarrow$ 2.438
with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$ 3σ range		bfp $\pm 1\sigma$ 3σ range	
	$\sin^2 \theta_{12}$	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345
	$\theta_{12}/^\circ$	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95
	$\sin^2 \theta_{23}$	0.470 $^{+0.017}_{-0.013}$	0.435 $\rightarrow$ 0.585	0.550 $^{+0.012}_{-0.015}$	0.440 $\rightarrow$ 0.584
	$\theta_{23}/^\circ$	43.3 $^{+1.0}_{-0.8}$	41.3 $\rightarrow$ 49.9	47.9 $^{+0.7}_{-0.9}$	41.5 $\rightarrow$ 49.8
	$\sin^2 \theta_{13}$	0.02215 $^{+0.00056}_{-0.00058}$	0.02030 $\rightarrow$ 0.02388	0.02231 $^{+0.00056}_{-0.00056}$	0.02060 $\rightarrow$ 0.02409
	$\theta_{13}/^\circ$	8.56 $^{+0.11}_{-0.11}$	8.19 $\rightarrow$ 8.89	8.59 $^{+0.11}_{-0.11}$	8.25 $\rightarrow$ 8.93
CP/ $^\circ$	212 $^{+26}_{-41}$	124 $\rightarrow$ 364	274 $^{+22}_{-25}$	201 $\rightarrow$ 335	
IC24	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.513 $^{+0.021}_{-0.019}$	+2.451 $\rightarrow$ +2.578	2.484 $^{+0.020}_{-0.020}$	2.547 $\rightarrow$ 2.421

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

reactor/interference



Nu IT 6.0 (2024)					
atmospheric data		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$		bfp $\pm 1\sigma$	
		3 σ range		3 σ range	
	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345
	$\theta_{12}/^\circ$	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95
	$\sin^2 \theta_{23}$	0.561 $^{+0.012}_{-0.015}$	0.430 $\rightarrow$ 0.596	0.562 $^{+0.012}_{-0.015}$	0.437 $\rightarrow$ 0.597
	$\theta_{23}/^\circ$	48.5 $^{+0.7}_{-0.9}$	41.0 $\rightarrow$ 50.5	48.6 $^{+0.7}_{-0.9}$	41.4 $\rightarrow$ 50.6
without SK	$\sin^2 \theta_{13}$	0.02195 $^{+0.00054}_{-0.00058}$	0.02023 $\rightarrow$ 0.02376	0.02224 $^{+0.00056}_{-0.00057}$	0.02053 $\rightarrow$ 0.02397
	$\theta_{13}/^\circ$	8.52 $^{+0.11}_{-0.11}$	8.18 $\rightarrow$ 8.87	8.58 $^{+0.11}_{-0.11}$	8.24 $\rightarrow$ 8.91
	$\text{CP}/^\circ$	177 $^{+19}_{-20}$	96 $\rightarrow$ 422	285 $^{+25}_{-28}$	201 $\rightarrow$ 348
IC19	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.534 $^{+0.025}_{-0.023}$	CP violation?	2.510 $^{+0.024}_{-0.025}$	2.584 $\rightarrow$ 2.438
atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$		bfp $\pm 1\sigma$	
		3 σ range		3 σ range	
	$\sin^2 \theta_{12}$	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345	0.308 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.345
	$\theta_{12}/^\circ$	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95	33.68 $^{+0.73}_{-0.70}$	31.63 $\rightarrow$ 35.95
	$\sin^2 \theta_{23}$	0.470 $^{+0.017}_{-0.013}$	0.435 $\rightarrow$ 0.585	0.550 $^{+0.012}_{-0.015}$	0.440 $\rightarrow$ 0.584
	$\theta_{23}/^\circ$	43.3 $^{+1.0}_{-0.8}$	41.3 $\rightarrow$ 49.9	47.9 $^{+0.7}_{-0.9}$	41.5 $\rightarrow$ 49.8
with SK	$\sin^2 \theta_{13}$	0.02215 $^{+0.00056}_{-0.00058}$	0.02030 $\rightarrow$ 0.02388	0.02231 $^{+0.00056}_{-0.00056}$	0.02060 $\rightarrow$ 0.02409
	$\theta_{13}/^\circ$	8.56 $^{+0.11}_{-0.11}$	8.19 $\rightarrow$ 8.89	8.59 $^{+0.11}_{-0.11}$	8.25 $\rightarrow$ 8.93
	$\text{CP}/^\circ$	212 $^{+26}_{-41}$	124 $\rightarrow$ 364	274 $^{+22}_{-25}$	201 $\rightarrow$ 335
IC24	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05	7.49 $^{+0.19}_{-0.19}$	6.92 $\rightarrow$ 8.05
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.513 $^{+0.021}_{-0.019}$	+2.451 $\rightarrow$ +2.578	2.484 $^{+0.020}_{-0.020}$	2.547 $\rightarrow$ 2.421

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{3\ell}^2 \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

matter potential

mass splittings

Nu IT 5.3 (2024)					
data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$		bfp $\pm 1\sigma$	
		3 σ range		3 σ range	
atmospheric	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$		0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344
	$\theta_{12}/^\circ$	33.66 $^{+0.73}_{-0.70}$	θ_{23} octant?	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94
atmospheric	$\sin^2 \theta_{23}$	0.572 $^{+0.018}_{-0.023}$	0.407 $\rightarrow$ 0.620	0.578 $^{+0.016}_{-0.021}$	0.412 $\rightarrow$ 0.623
	$\theta_{23}/^\circ$	49.1 $^{+1.0}_{-1.3}$	39.6 $\rightarrow$ 51.9	49.5 $^{+0.9}_{-1.2}$	39.9 $\rightarrow$ 52.1
without SK atmospheric	$\sin^2 \theta_{13}$	0.02396 $^{+0.00056}_{-0.00057}$		0.02396 $^{+0.00069}_{-0.00057}$	0.02049 $\rightarrow$ 0.02420
	$\theta_{13}/^\circ$	8.58 $^{+0.11}_{-0.11}$	8.23 $\rightarrow$ 8.91	8.57 $^{+0.13}_{-0.11}$	8.23 $\rightarrow$ 8.95
	CP/ $^\circ$	232 $^{+39}_{-25}$	139 $\rightarrow$ 350	273 $^{+24}_{-26}$	195 $\rightarrow$ 342
without SK atmospheric	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.511 $^{+0.027}_{-0.027}$	+2.428 $\rightarrow$ +2.597	2.498 $^{+0.032}_{-0.024}$	2.581 $\rightarrow$ 2.409
with SK atmospheric	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344
	$\theta_{12}/^\circ$	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94
	$\sin^2 \theta_{23}$	0.454 $^{+0.019}_{-0.016}$	0.411 $\rightarrow$ 0.606	0.568 $^{+0.016}_{-0.021}$	0.412 $\rightarrow$ 0.611
with SK atmospheric	$\theta_{23}/^\circ$	42.3 $^{+1.1}_{-0.9}$	39.9 $\rightarrow$ 51.1	48.9 $^{+0.9}_{-1.2}$	39.9 $\rightarrow$ 51.4
	$\sin^2 \theta_{13}$	0.02224 $^{+0.00056}_{-0.00057}$	0.02047 $\rightarrow$ 0.02397	0.02222 $^{+0.00069}_{-0.00057}$	0.02049 $\rightarrow$ 0.02420
with SK atmospheric	$\theta_{13}/^\circ$	8.58 $^{+0.11}_{-0.11}$	8.23 $\rightarrow$ 8.91	8.57 $^{+0.13}_{-0.11}$	8.23 $\rightarrow$ 8.95
	CP/ $^\circ$	232 $^{+39}_{-25}$	139 $\rightarrow$ 350	273 $^{+24}_{-26}$	195 $\rightarrow$ 342
with SK atmospheric	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.505 $^{+0.024}_{-0.026}$	+2.426 $\rightarrow$ +2.586	2.487 $^{+0.027}_{-0.024}$	2.566 $\rightarrow$ 2.407

The atmospheric angle is almost maximal, but only small sensitivity to its octant

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{solar}$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{3\ell}^2 \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

matter potential

mass splittings

Nu IT 5.3 (2024)					
		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344
	$\theta_{12}/^\circ$	33.66 $^{+0.73}_{-0.70}$	31.60 $\rightarrow$ 35.94	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94
	$\sin^2 \theta_{23}$	0.572 $^{+0.018}_{-0.023}$	0.407 $\rightarrow$ 0.620	0.578 $^{+0.016}_{-0.021}$	0.412 $\rightarrow$ 0.623
	$\theta_{23}/^\circ$	49.1 $^{+1.0}_{-1.3}$	39.6 $\rightarrow$ 51.9	49.5 $^{+0.9}_{-1.2}$	39.9 $\rightarrow$ 52.1
	$\sin^2 \theta_{13}$	0.02203 $^{+0.00056}_{-0.00058}$	0.02029 $\rightarrow$ 0.02391	0.02219 $^{+0.00059}_{-0.00057}$	0.02047 $\rightarrow$ 0.02396
	$\theta_{13}/^\circ$	8.54 $^{+0.11}_{-0.11}$	8.19 $\rightarrow$ 8.89	8.57 $^{+0.11}_{-0.11}$	8.23 $\rightarrow$ 8.90
	$\text{CP}/^\circ$	197 $^{+41}_{-25}$	108 $\rightarrow$ 404	286 $^{+27}_{-32}$	192 $\rightarrow$ 360
	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.511 $^{+0.027}_{-0.027}$	+2.428 $\rightarrow$ +2.597	2.498 $^{+0.032}_{-0.024}$	2.581 $\rightarrow$ 2.409
		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2 \theta_{12}$	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344	0.307 $^{+0.012}_{-0.011}$	0.275 $\rightarrow$ 0.344
	$\theta_{12}/^\circ$	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94	33.67 $^{+0.73}_{-0.71}$	31.61 $\rightarrow$ 35.94
	$\sin^2 \theta_{23}$	0.454 $^{+0.019}_{-0.016}$	0.411 $\rightarrow$ 0.606	0.568 $^{+0.016}_{-0.021}$	0.412 $\rightarrow$ 0.611
	$\theta_{23}/^\circ$	42.3 $^{+1.1}_{-0.9}$	39.9 $\rightarrow$ 51.1	48.9 $^{+0.9}_{-1.2}$	39.9 $\rightarrow$ 51.4
	$\sin^2 \theta_{13}$	0.02224 $^{+0.00056}_{-0.00057}$	0.02047 $\rightarrow$ 0.02397	0.02222 $^{+0.00069}_{-0.00057}$	0.02049 $\rightarrow$ 0.02420
	$\theta_{13}/^\circ$	8.58 $^{+0.11}_{-0.11}$	8.23 $\rightarrow$ 8.91	8.57 $^{+0.13}_{-0.11}$	8.23 $\rightarrow$ 8.95
	$\text{CP}/^\circ$	232 $^{+39}_{-25}$	139 $\rightarrow$ 350	273 $^{+24}_{-26}$	195 $\rightarrow$ 342
	$\frac{m_{21}^2}{10^{-5} \text{ eV}^2}$	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03	7.41 $^{+0.21}_{-0.20}$	6.81 $\rightarrow$ 8.03
	$\frac{m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	+2.505 $^{+0.024}_{-0.026}$	+2.426 $\rightarrow$ +2.586	2.487 $^{+0.027}_{-0.024}$	2.566 $\rightarrow$ 2.407

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\Re \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \right] + 2\Im \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right) \right]$$

ν_e appearance

$$P(\nu_\mu \rightarrow \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

ν_μ disappearance

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\Re \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \right] + 2\Im \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right) \right]$$

ν_e appearance

$$P(\nu_\mu \rightarrow \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

ν_μ disappearance

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

No dependence on the sign of the mass splitting

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\Re \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \right] + 2\Im \left[\sum_{i>j}^3 U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right) \right]$$

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The atmospheric angle θ_{23} is almost maximal,
but only small sensitivity to its octant

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The CP - violating phase δ appears only in the subleading contribution

The AxiFlavon

Coupling	$m_a^{\max}/C$ [eV]	$m_a^{\max, \text{U}(2)}$ [eV]	$f_a^{\min, \text{U}(2)}$ [GeV]	Constraint
$C_{\mu e}$	$2.1 \cdot 10^{-3}$	78	$7.3 \cdot 10^4$	$\mu \rightarrow ea$ [30]
C_{bs}^V	$9.1 \cdot 10^{-2}$	16	$3.6 \cdot 10^5$	$B^+ \rightarrow K^+ a$ [28]
C_{sd}^V	$1.7 \cdot 10^{-5}$	0.58	$9.8 \cdot 10^6$	$K^+ \rightarrow \pi^+ a$ [29]
C_{ee}^A	$3.1 \cdot 10^{-3}$	0.014	$4.1 \cdot 10^8$	WD Cooling [33]
C_N	$3.5 \cdot 10^{-3}$	0.0092	$6.2 \cdot 10^8$	SN1987A [34]

The AxiFlavon

