Bubble wall velocities in phase transitions with the fluid Ansatz

Gláuber Carvalho Dorsch

in collab. with T. Konstandin, E. Perboni and D. Pinto

2312.02354
2412.09266

Padova, 27th May 2025

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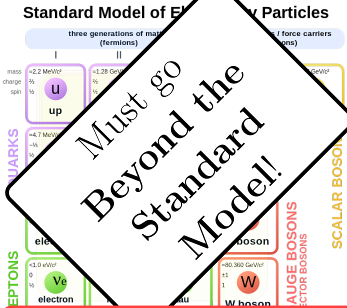
SM built in the past ~ 50 years
in dialogue with colliders/detectors

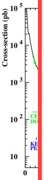


Must go Beyond the Standard Model!

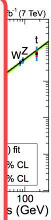
Im... ssful...
...but incomplete!

Dark sector
Neutrino masses
Gravity

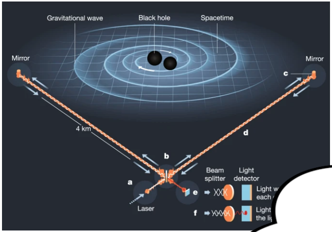
Baryogenesis
Hierarchies
and others...

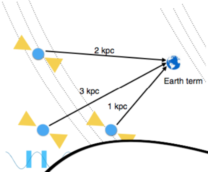


Colliders have become reticent...



Gravitational wave detection!

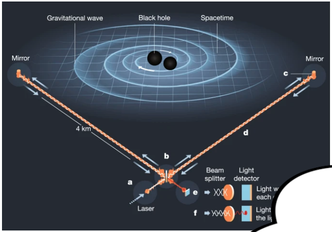


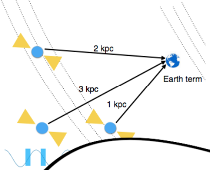


Access to new cosmic messengers!

Can we extract information on particle physics from gravitational waves?

Gravitational wave detection!

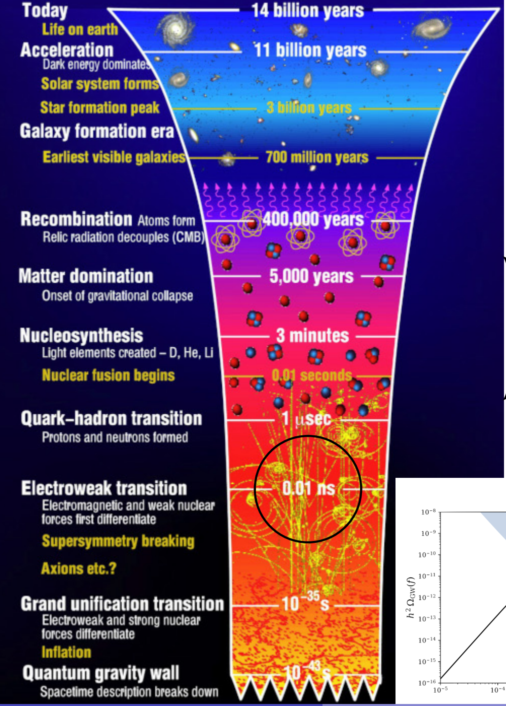




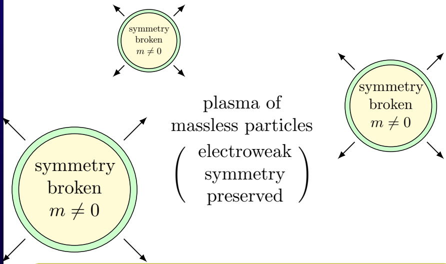
Access to new cosmic messengers!

Can we extract information on particle physics from gravitational waves?

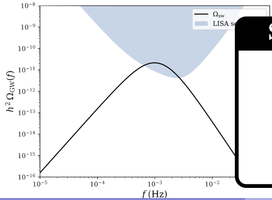
YES!



Cosmological phase transition!



Bubble collisions $\Longrightarrow$ GWs!



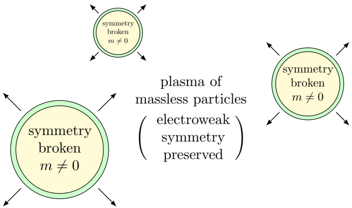
Spectrum depends on

transition temperature
energy released
duration
bubble wall velocity!

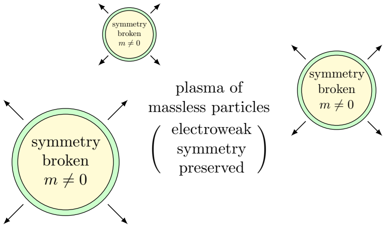
Outline

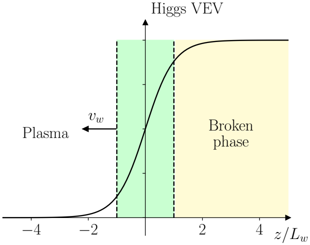
Bubble wall velocities in phase transitions with the fluid Ansatz

- Bolzmann equation and the *fluid Ansatz*
- Linearization, solutions and the problem of the singularity
- Improving the linearization procedure:
  getting rid of the singularity
- Results
- Conclusions and outlook



Boltzmann equation and the *fluid Ansatz*





Energy-momentum conservation

$$\phi' \Box \phi + \phi' \frac{\partial V(T,\phi)}{\partial \phi} + \sum_i \frac{g_i}{2} \phi' \frac{\partial m_i^2}{\partial \phi} \int \frac{d^3 p}{(2\pi)^3 E_i} \delta f_i(p,x) = 0$$

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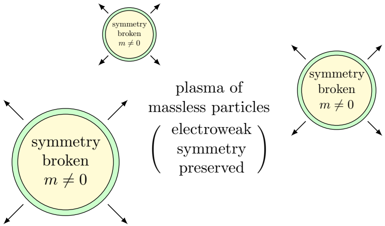

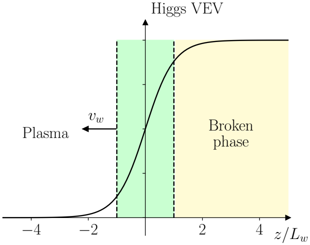

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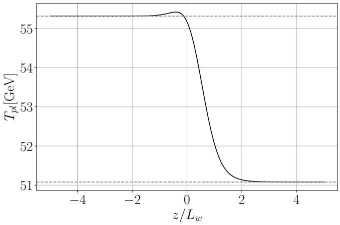
$$\underbrace{\frac{dV}{dz}}_{\text{inner pressure}} - T'\frac{\partial V}{\partial T}$$

passage of the bubble
heats up the plasma
(equilibrium effect)



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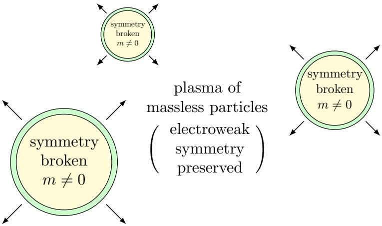

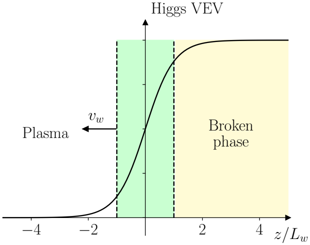

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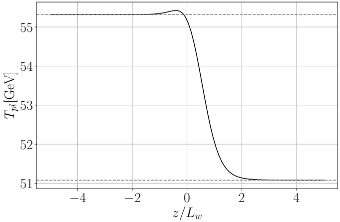
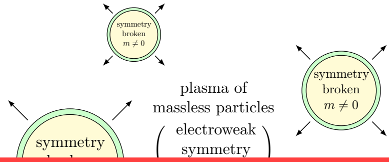
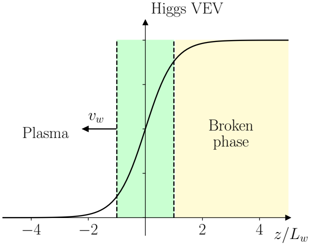
non-equilibrium effects

passage of the bubble heats up the plasma (equilibrium effect)



Boltzmann equation and the *fluid Ansatz*





Non-equilibrium → Boltzmann eq.

$$p^\mu \partial_\mu f_i + \underbrace{\frac{\partial^\mu m^2}{2}}_{\text{source}} \partial_{p^\mu} f_i + \underbrace{\mathcal{C}[f]}_{\text{collision}} = 0$$

Fluid *Ansatz*

$$f_i = \frac{1}{e^{\beta(p^\mu u_\mu - \delta_i)} \pm 1)}, \qquad \delta = q^{(0)} + q^{(1)}_\mu p^\mu + q^{(2)}_{\mu\nu} p^\mu p^\nu + \ldots$$

$q^{(0)}$ → chemical potential

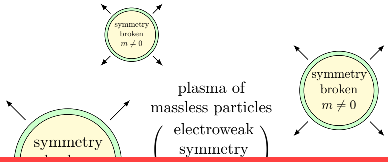
$q^{(1)}_\mu$ → temperature & velocity fluctuations

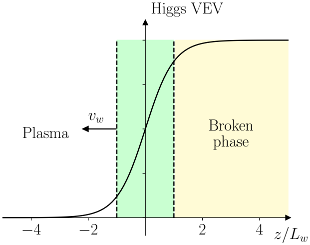
$q^{(2)}_{\mu\nu}$ → dissipative effects

Clear physical interpretation

For other approaches see e.g.
*S. De Curtis et al., JHEP **03** (2022) 163*
*Cline and Laurent, PRD **106** (2022) 2*
*Ekstedt et al., JHEP **04** (2025) 101*

Boltzmann equation and the *fluid Ansatz*





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Clear physical interpretation

Linearized system

$$A \cdot q' + \Gamma \cdot q = S$$

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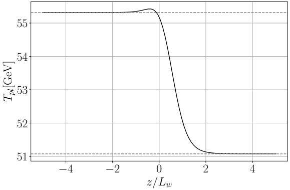
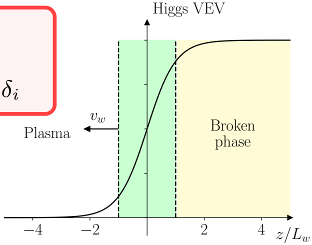
Boltzmann equation and the *fluid Ansatz*



Include the background variation into δ_i



...tzmann eq.

$$p^\mu \partial_\mu f_i + \underbrace{\frac{\partial^\mu m^2}{2}}_{\text{source}} \partial_{p^\mu} f_i + \underbrace{\mathcal{C}[f]}_{\text{collision}} = 0$$

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Clear physical interpretation

First attempt: **constant background!**

For other approaches see e.g.
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First attempt: constant background

GD, S. Huber, T. Konstandin
*JCAP **08** (2021) 020*
*JCAP **04** (2022) 04, 010*

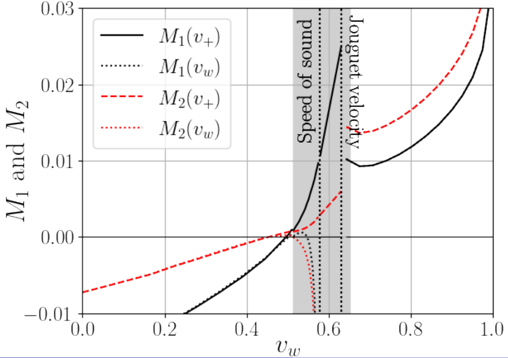
GD, D. Pinto
*JCAP **04** (2024) 027*

Linearized system

$$A \cdot q' + \Gamma \cdot q = S$$

$$A_{\text{light}} \cdot q'_{\text{light}} + \Gamma_{\text{light}} \cdot q_{\text{light}} = 0$$

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \int \left(\phi' \Box \phi + \phi' \frac{\partial V(T,\phi)}{\partial \phi} + \sum_i \frac{g_i}{2} \phi' \frac{\partial m_i^2}{\partial \phi} \int \frac{d^3 p}{(2\pi)^3 E_i} \delta f_i \right) \begin{pmatrix} 1 \\ \tanh z \end{pmatrix} = 0$$



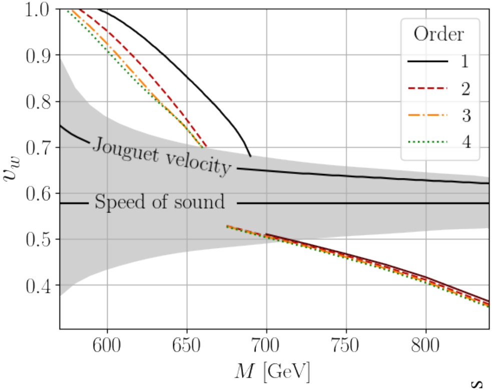
Deflagration

$v_+ < v_- = v_w$

$\phi^6/8M^2$ operator

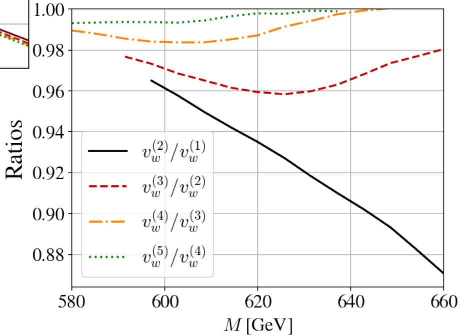
$M = 700,\ L_w T = 7.4266$

Results: wall velocity and convergence



*GD, D. Pinto, JCAP **04** (2024) 027*

Only tops



$$\delta = q^{(0)} + q^{(1)}_{\mu} p^{\mu} + q^{(2)}_{\mu\nu} p^{\mu} p^{\nu} + \dots$$

Why does a singularity appear at $v_w \to c_s$?

Linearized system

$$A \cdot q' + \Gamma \cdot q = S$$

$$A_{\text{light}} \cdot q'_{\text{light}} + \Gamma_{\text{light}} \cdot q_{\text{light}} = 0$$

Some of these eqs. describe energy-momentum

Energy-momentum conservation

Some combinations of eqs. have vanishing Γ

$$\chi \cdot A \cdot \Delta q = \chi \cdot \int S dz \quad \text{for some } \chi$$

For light species

$$A \sim \begin{pmatrix} v_w & 1/3 \\ 1/3 & v_w/3 \end{pmatrix} \implies \text{singular at } v_w = 1/\sqrt{3}$$

Δq_{light} blows up if rhs $\neq 0$!

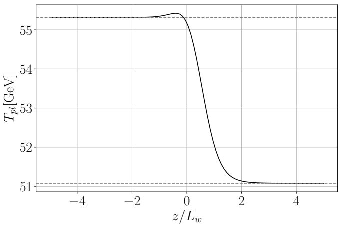
Improving the linearization procedure

Getting rid of the singularity

Fluid *Ansatz*

$$f_i = \frac{1}{e^{\beta(z)(p^\mu u_\mu(z) - \delta_i)} \pm 1)}$$

Spatially-dependent background!



Find $\beta(z)$ and $u_\mu(z)$ such that energy-momentum conservation is satisfied for this background!

*GD, T. Konstandin, E. Perboni and D. Pinto, JCAP **04** (2025) 033*

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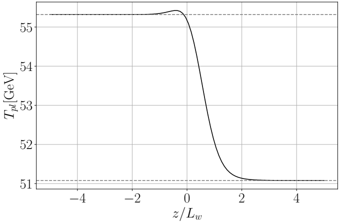

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Plugging back into Boltzmann

$$p^\mu \partial_\mu f_i + \frac{\partial^\mu m^2}{2} \partial_{p^\mu} f_i + \mathcal{C}[f] = 0$$

These terms will give $\partial_z \beta$ and $\partial_z u_\mu$
Will contribute as new source terms!

Linearize over spatially-varying background

$$A \cdot q' + \Gamma \cdot q = S_{\text{old}} + S_{\text{new}}$$

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Improving the linearization procedure

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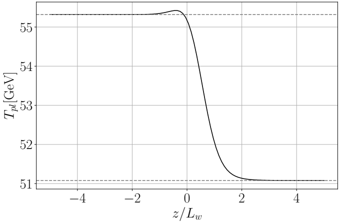

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Find $\beta(z)$ and $u_\mu(z)$ such that energy-momentum conservation is satisfied for this background!

Plugging back into Boltzmann

$$p^\mu \partial_\mu f_i + \frac{\partial^\mu m^2}{2} \partial_{p^\mu} f_i + \mathcal{C}[f] = 0$$

Corresponding energy-momentum eqs.

$\chi \cdot A \cdot \Delta q = 0$ (no source of E-M!)

These terms will give $\partial_z \beta$ and $\partial_z u_\mu$
Will contribute as new source terms!

Linearize over spatially-varying background

$$A \cdot q' + \Gamma \cdot q = S_{\text{old}} + S_{\text{new}}$$

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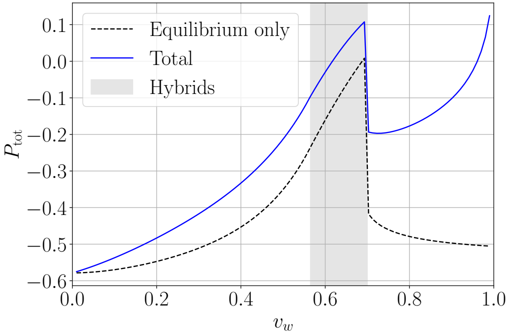
Results: non-singular pressure and wall velocities



Non-singular across sound speed

Non-equilibrium relevant!

Detonations are possible

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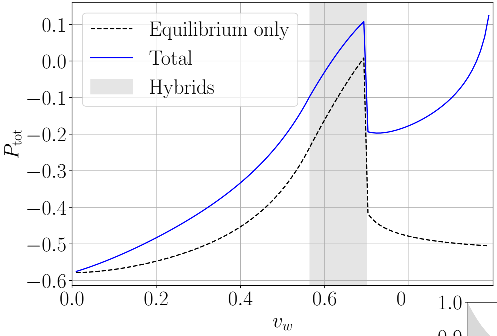

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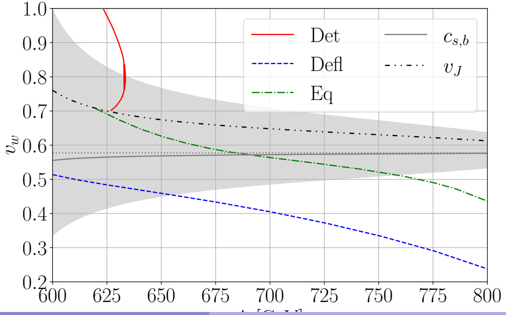

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Conclusions and outlook

- Cosmological phase transitions:
  interface between particle physics & gravitational waves
- Accurate estimate of spectrum $\Leftrightarrow$ wall velocity
  (non-equilibrium!)
- Improved *fluid Ansatz*
  Allows for (relatively) simple physical interpretation
- Non-equilibrium effects are relevant!
  Detonations are possible (but seem to be fine-tuned!)

- Comparison between *fluid Ansatz* and other approaches

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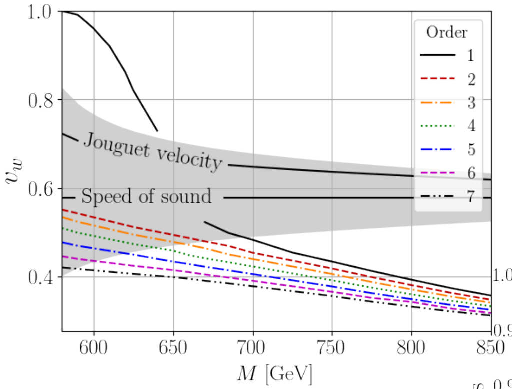
THANK YOU!

Appendices

Results: wall velocity and convergence



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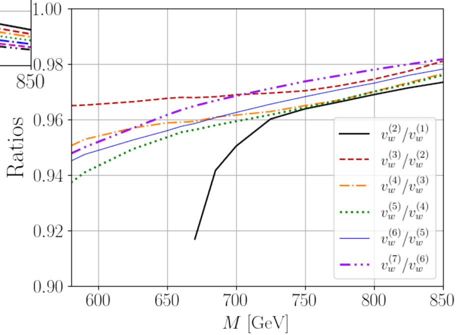
Tops + W's



Breakdown of the linearization procedure

Energy-momentum

$$\gamma_+^2 v_+^2 \omega_+ - V_+ = \gamma_-^2 v_-^2 \omega_- - V_-$$

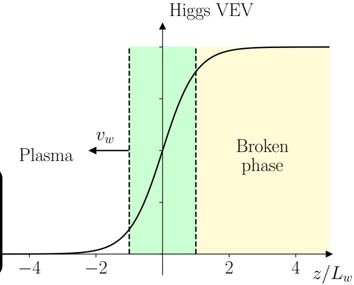
$$\gamma_+^2 v_+ \omega_+ = \gamma_-^2 v_- \omega_-$$

Bag approximation

$$V_\pm = \epsilon_\pm - \frac{a_\pm}{3} T_\pm^4 \quad \text{and} \quad \omega_\pm = \frac{4 a_\pm}{3} T_\pm^4$$

Solutions

$$v_+ = \frac{1}{1+\alpha}\left[X_+ \pm X_- \sqrt{1 + \frac{2}{3}\frac{\alpha}{X_-^2} + \frac{\alpha^2}{X_-^2}}\right]$$



$$X_\pm = \frac{v_-}{2} \pm \frac{1}{6 v_-}$$

$$\text{Linear regime } \frac{\alpha}{X_-^2} \ll 1$$

Checking the vanishing of the source

$$-u_\nu \partial_\mu T^{\mu\nu}_{\text{bg}} = \mathcal{S}_1 = \underbrace{\gamma_w v_w \sum_i \partial_z m_i^2 N_i}_{\text{“old” source}} \underbrace{-\gamma_w \frac{4}{3} a T^4 \left(3 v_w \frac{\partial_z T}{T} + \gamma_w^2 \partial_z v \right)}_{\text{source from background variation}}$$

$$-\overline{u}_\nu \partial_\mu T^{\mu\nu}_{\text{bg}} = \mathcal{S}_2 = \underbrace{-\gamma_w \frac{4}{3} a T^4 \left(\frac{\partial_z T}{T} + \gamma_w^2 v_w \partial_z v \right)}_{\text{source from background variation}}$$