Dark matter in QCD-like theories with a theta vacuum Cosmological and Astrophysical implications

Giacomo Landini

29/05/2025

based on Phys.Rev. D 111 (2025) 6, arXiv [**2405.10367**]

C. García-Cely, **GL**, Ó. Zapata

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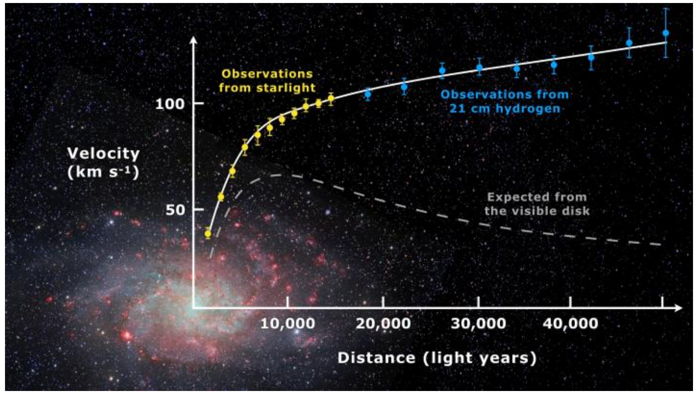
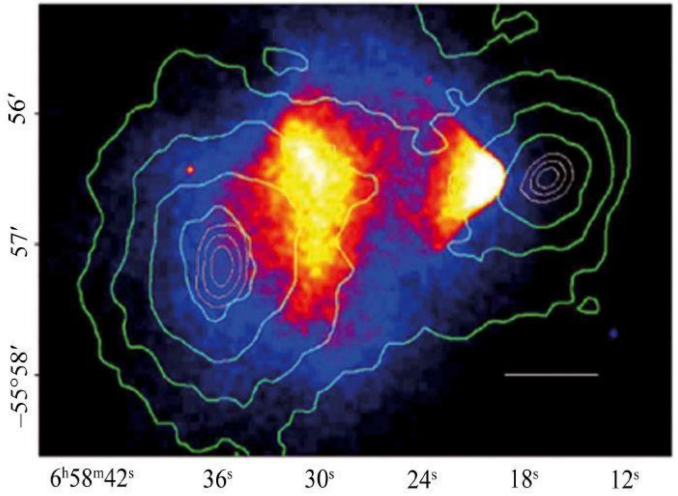
Dark Matter evidence

Dark Matter evidence at very different scales



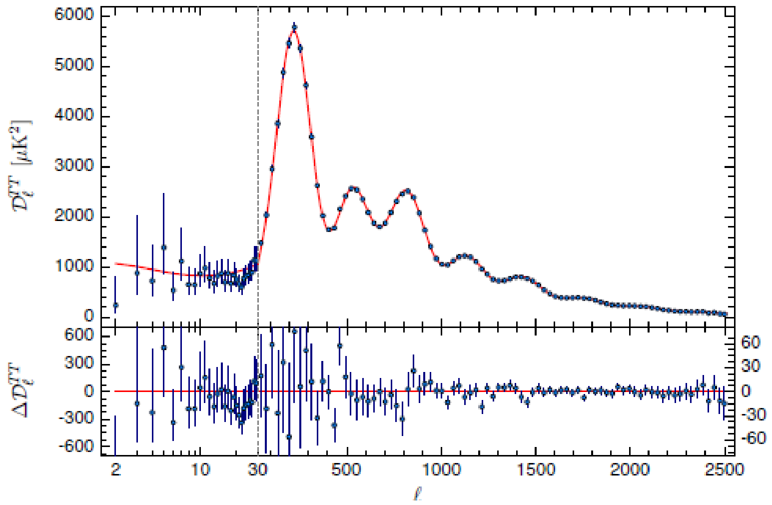
Rotation curves of **Galaxies**

Rubin, Ford (1970)



Galaxy **Clusters**

Clowe *et al.* (2006)
Harvey *et al.* (2015)
Robertson *et al.* (2017)



CMB

PLANCK (2018)

d

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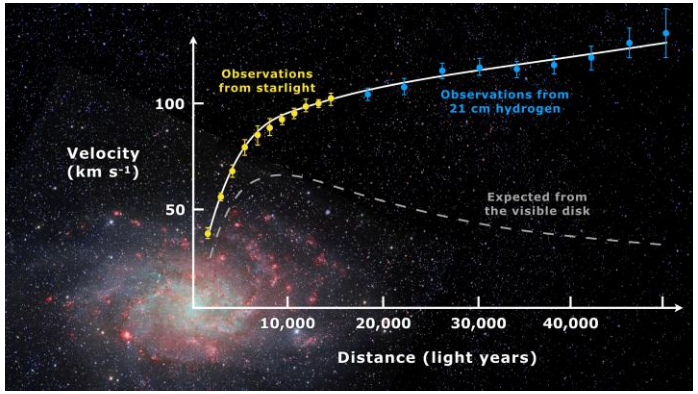

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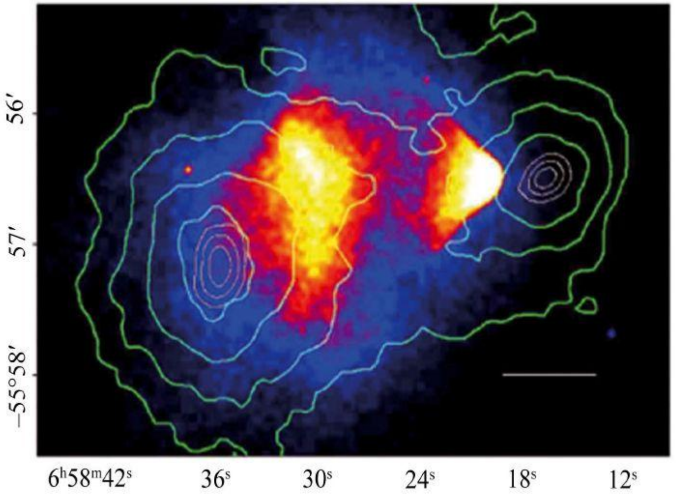

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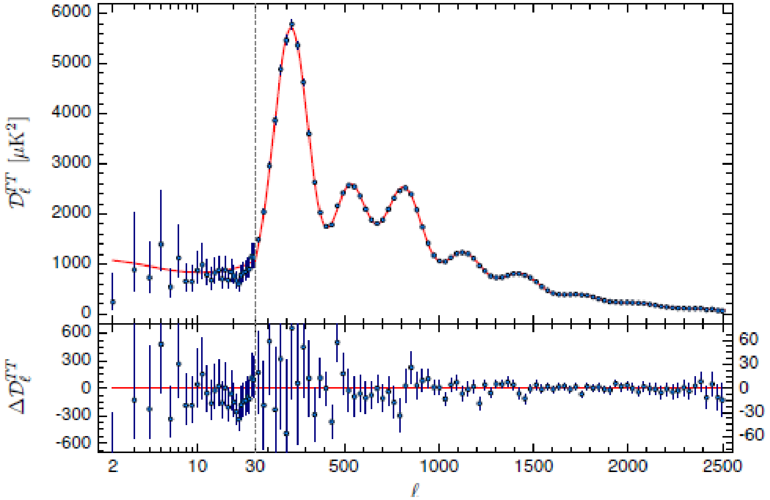

CMB

PLANCK (2018)

Dwarf Galaxies DM-dominated

low DM velocity $v \sim \mathcal{O}(10 - 100)\ \mathrm{Km/sec}$

d

Small scale problems

N-body simulations of collision-less DM **vs** observations

(d < 100 Kpc) Small scales (dwarf galaxies) - low DM velocity

DM-dominated $v \sim \mathcal{O}(10 - 100)\ \mathrm{Km/sec}$

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DM-dominated $v \sim \mathcal{O}(10-100)\ \mathrm{Km/sec}$

<u>ΛCDM simulations</u>

Universality

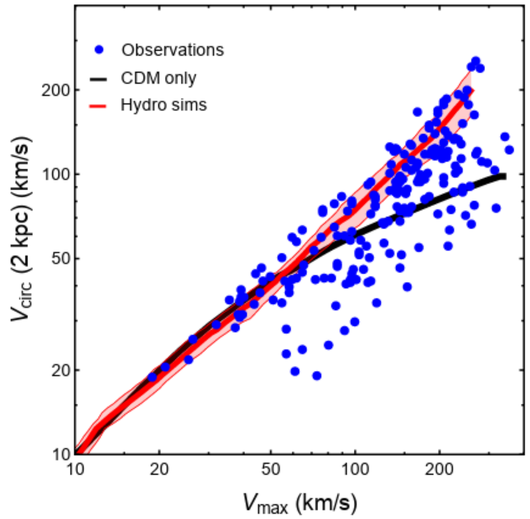
Large central density (*cusp*) $\rho(r) \propto r^{-1}$

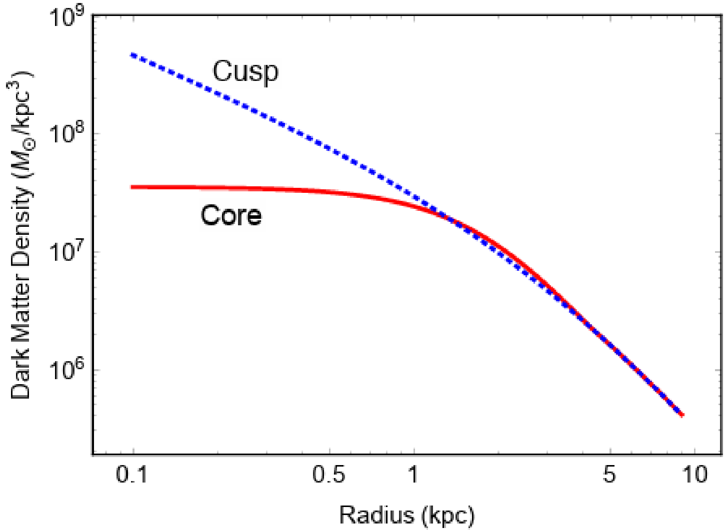
<u>Observations</u>

"Considerable diversity"

Tulin, Yu (2017)

Smaller central density (*core*) $\rho(r) \propto r^{0}$





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N-body simulations of collision-less DM **vs** observations

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ΛCDM simulations

Universality

Large centra

Observations

"Considerable diversity"

e) $\rho(r) \propto r^0$

Possible solutions

Systematic uncertaintes in deriving DM distributions from observations

Inclusion of dissipative baryonic processes in simulations

DM self-interactions

Spergel and Steinhardt (2000)...

Still no general consensus!

Self-interacting Dark Matter (SIDM)

<u>Elastic Dark Matter scatterings</u>

$$\pi_{\mathrm{DM}}\pi_{\mathrm{DM}} \to \pi_{\mathrm{DM}}\pi_{\mathrm{DM}}$$

Spergel and Steinhardt (2000)
Dave *et al*. (2001)
Vogelsberger *et al*. (2001)
...

Reduction of central density at small scales if

$\sigma(v)/m_{\mathrm{DM}} \sim 1 - 10\ \mathrm{cm}^2/\mathrm{g}$ for $v \sim \mathcal{O}(10 - 100)\ \mathrm{Km/sec}$

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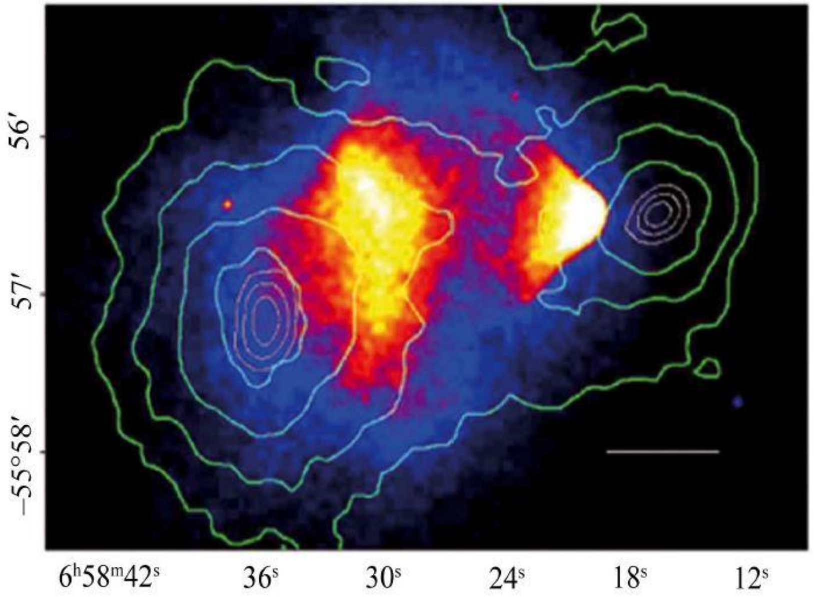

Bullet Cluster and other galaxy clusters

$$\sigma(v)/m_{\rm DM} \lesssim 0.5\ {\rm cm}^2/{\rm g}$$

for $v \sim \mathcal{O}(2000)\ {\rm Km/sec}$

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Need for a **velocity-dependent** self-interaction cross section

$$\sigma(v)/m_{\rm DM} \sim 1 - 10\ {\rm cm}^2/{\rm g}$$

for $v \sim \mathcal{O}(10 - 100)\ {\rm Km/sec}$

and

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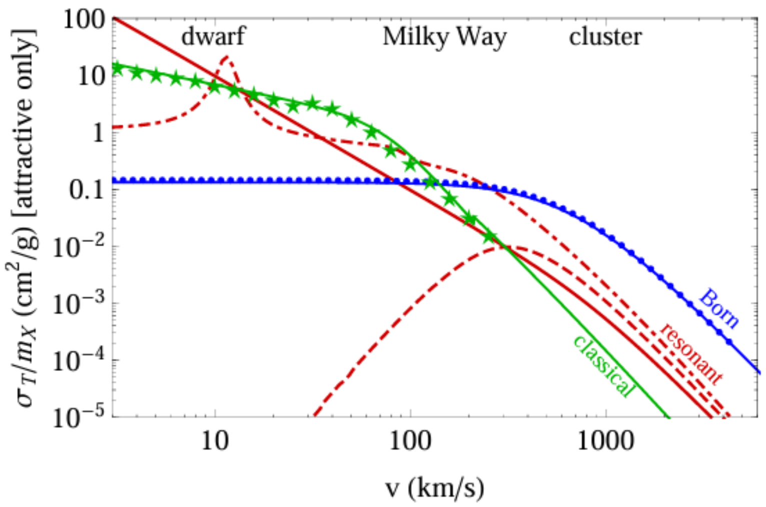
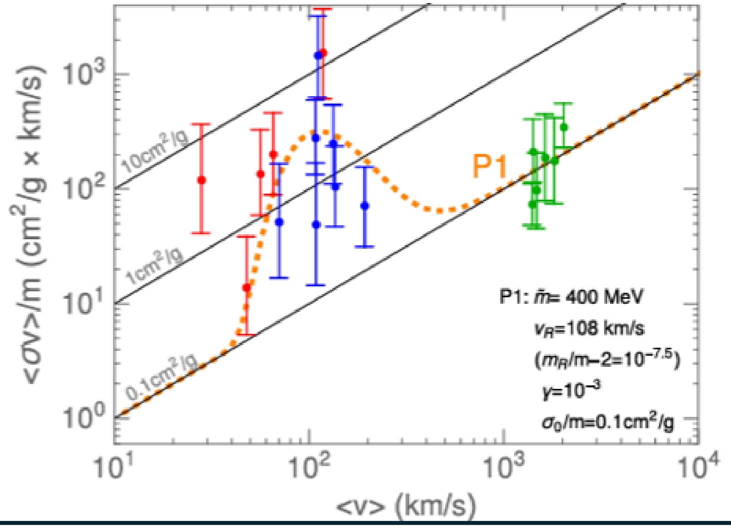
[Tulin, Yu, Zurek] *(2013)*





[Chu, Garcia-Cely, Murayama] *(2018)*

Possible realizations: Light (MeV) mediator, Resonant SIDM,...

SIMP Dark Matter

Dark Matter is in thermal equilibrium with the SM bath in the early Universe

First proposed as *The SIMP Miracle* by Y. Hochberg,E.Kuflik, T.Volanksy,J.G.Wacker (2014)

See also
Y. Hochberg,E.Kuflik,H.Murayama T.Volanksy,J.G.Wacker (2014)
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A.Kamada,H.Kim,T.Sekiguchi (2017)
A.Katz,E.Salvioni,B.Shakya (2020)
Chu, Nikolic, Pradler (2024)
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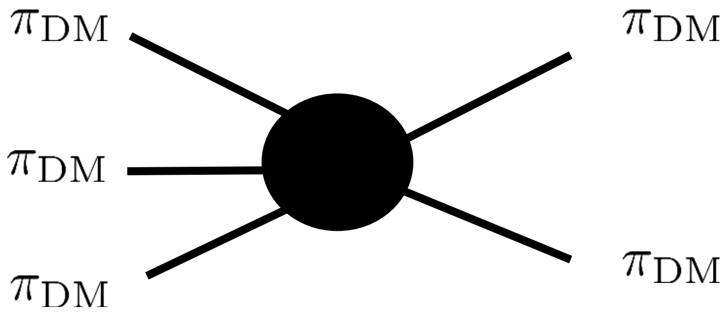
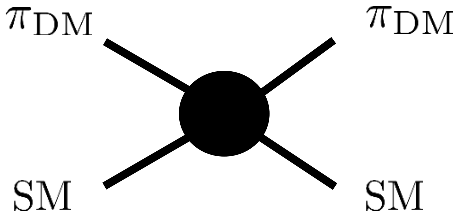


3 DM-to-2 DM



Thermal equilibrium

Strongly coupled theories provide a natural realization of SIMP DM

$$m_{\rm DM} \sim (T_{\rm eq}^2 M_{\rm Pl})^{1/3} \sim \text{ MeV - GeV}$$

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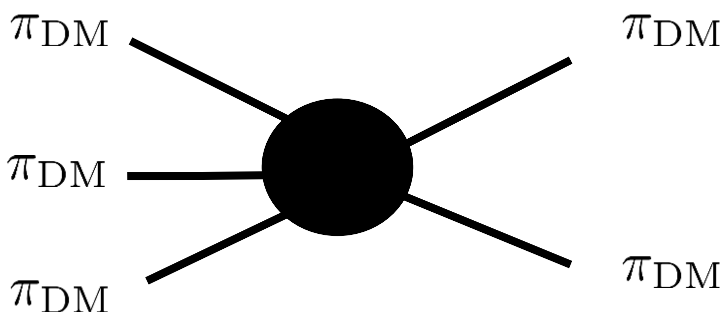

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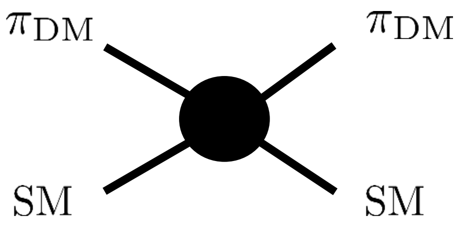

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Strongly coupled theory

$$\sigma/m_{\rm DM} \sim \alpha_{\rm eff}^2/m_{\rm DM}^3$$

$$m_{\rm DM} \sim \ {\rm MeV} - \ {\rm GeV}$$

scale predicted by SIMP DM

$$\sigma(v)/m_{\rm DM} \sim 1 - 10 \ {\rm cm}^2/{\rm g}$$

Typical values required to solve the small-scale problems!

QCD-like theories

We introduce a new dark gauge interaction (e.g a $SU(N_c)$ **sector)**

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

usually ignored

$$M = \mathrm{diag}(m_1, \cdots, m_{N_f}) \quad \text{with} \quad N_f \geq 2$$

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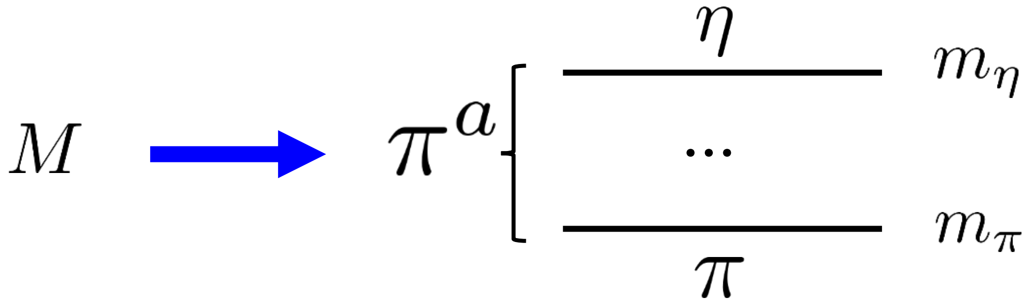
<u>Gauge confinement</u> $\Lambda_D \gg m_q$

$N_f^2 - 1$ pseudo-goldstone bosons $m_\pi \approx \sqrt{m_q \Lambda_D}$



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$M \longrightarrow \pi^a$

η m_η

...

π m_π

It can be a multiplet if some $m_i = m_j$

(flavor symmetry)

Pion-like Dark Matter!

Dark Pion interactions ($\theta = 0$)

The low-energy dynamics of dark pions is described by ChPT

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4} Tr[\partial_\mu U^\dagger \partial^\mu U] + \frac{f_\pi^2}{2} B_0 Tr[M\ U + U^\dagger M^\dagger]\ + \mathcal{L}_{\text{WZW}}$$

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$U = \exp(i\pi^a \lambda^a / f_\pi)$ $\quad \Lambda \sim 4\pi f_\pi / \sqrt{N_c}$ $\quad$ ChPT breaks down for $m_\pi \gtrsim \Lambda$

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First proposed in the context of SIMP DM by
Y. Hochberg,E.Kuflik,H.Murayama
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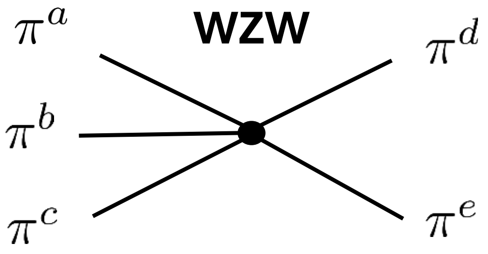
$$\mathcal{L}_{\text{WZW}} = -\frac{N_c}{240\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} Tr[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi]$$

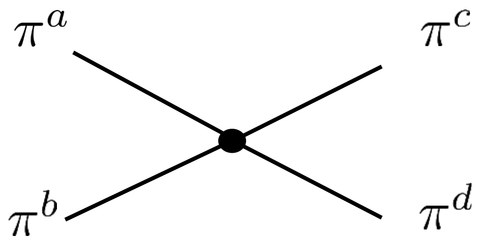
DM number changing processes



SIMP $\langle \sigma_{32} v^2 \rangle \propto \left(\frac{m_\pi^5}{f_\pi^{10}} \right) v^2 \sim \left(\frac{m_\pi^5}{f_\pi^{10}} \right) \left(\frac{T}{m_\pi} \right)^2$

<u>NO cubic vertex</u>

DM self-interactions



$\sigma / m_\pi \propto \frac{m_\pi}{f_\pi^4}$ constant

SIMP DM ($\theta = 0$)

This framework predicts GeV DM

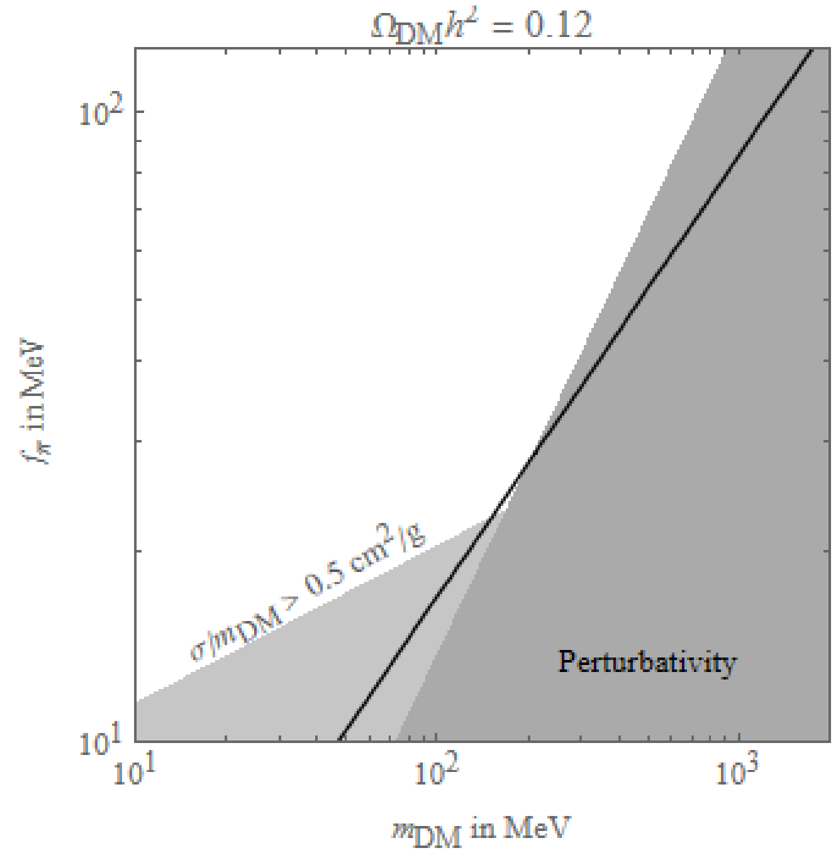
Tension among DM relic and Bullet Cluster bound

Perturbativity breakdown

The self-interactions cross section is constant

$$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$$

No SIDM realization



Dark Pion interactions ($\theta \neq 0$)

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New *odd* interactions induced by θ

$$\mathcal{L}_\theta = \frac{B_0 \theta}{3 f_\pi Tr M^{-1}} \left(d_{abc} \pi_a \pi_b \pi_c - \frac{c_{abcde}}{10 f_\pi^2} \pi_a \pi_b \pi_c \pi_d \pi_e \right)$$

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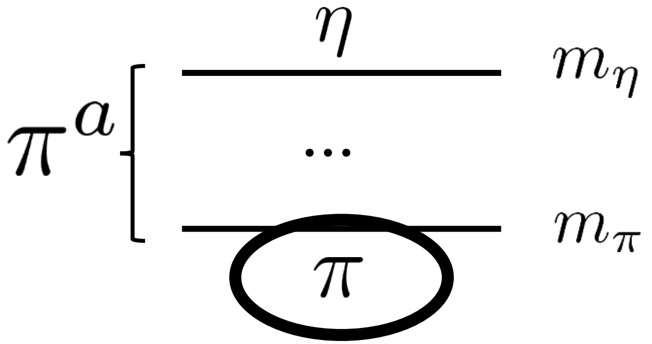
$$d_{abc} = \mathrm{Tr}(\{\lambda_a, \lambda_b\} \lambda_c)/4$$

Dark Pion interactions ($\theta \neq 0$)

Quark masses determine the meson spectrum

$$M = \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & m_{N_f} \end{pmatrix}$$

$$(M_\pi^2)_{ab} \propto \mathrm{Tr}\left[M\{\lambda^a \lambda^b\}\right] \longrightarrow$$



It can be a multiplet if some $m_i = m_j$

(flavor symmetry)

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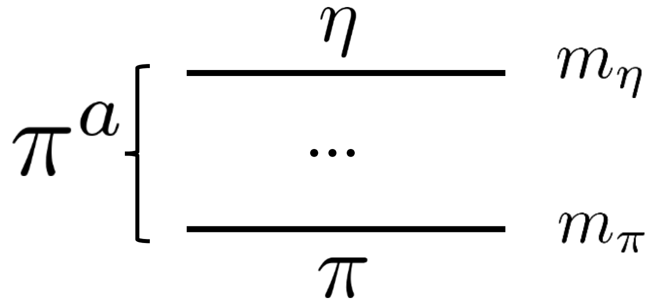
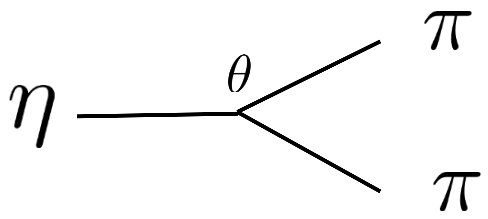


$$d_{abc}\pi_a\pi_b\pi_c \subset \xi\,\eta\pi\pi$$



$$\Gamma(\eta\to\pi\pi) = \frac{\theta^2 B_0^2 \xi}{24\pi f_\pi^2 m_\eta (TrM^{-1})^2}\sqrt{1-\frac{4m_\pi^2}{m_\eta^2}}$$

Dark Pion interactions ($\theta \neq 0$)

Quark masses determine the meson spectrum

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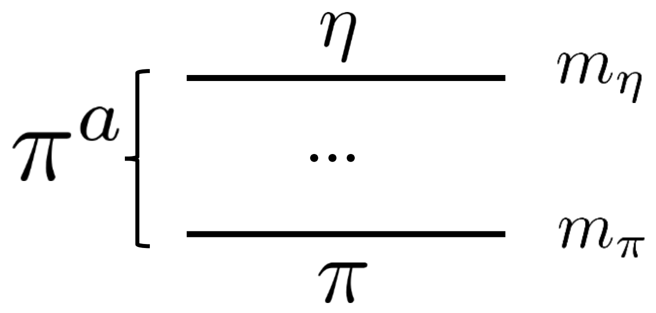

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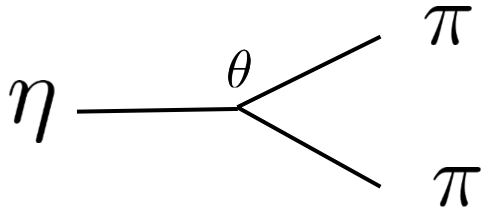

With a proper choice of M the spectrum can account for a resonance

$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \lesssim 0.1$$

v_R may originate from θ

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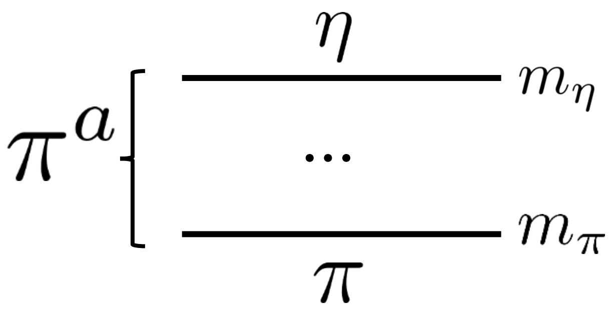
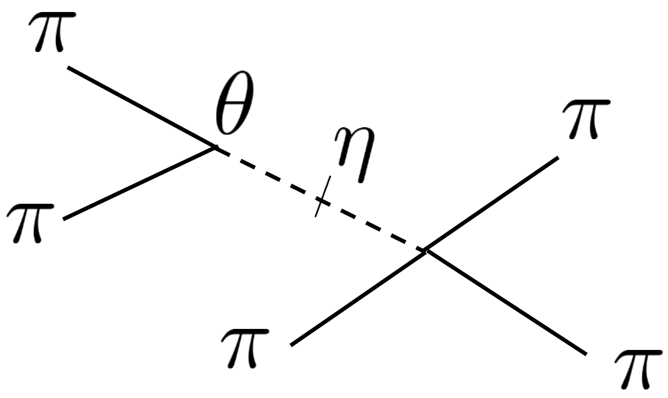


$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$$

θ induces the following resonant interactions

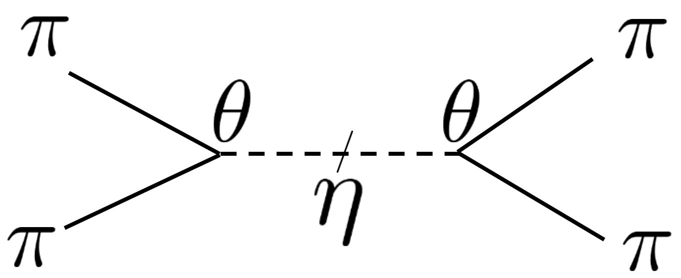
Resonant 3-to-2

Camilo García-Cely, GL, Óscar Zapata [2405.10367]



Early Universe $\rightarrow$ DM relic

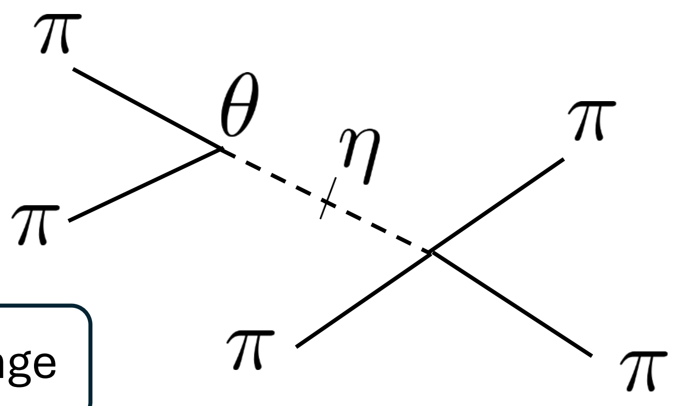
Resonant self-scattering



Today in halos $\rightarrow$ SIDM

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes



$$3\pi \to 2\pi \overset{\text{res}}{\simeq} \pi + 2\pi \to \pi + \eta \to 2\pi$$

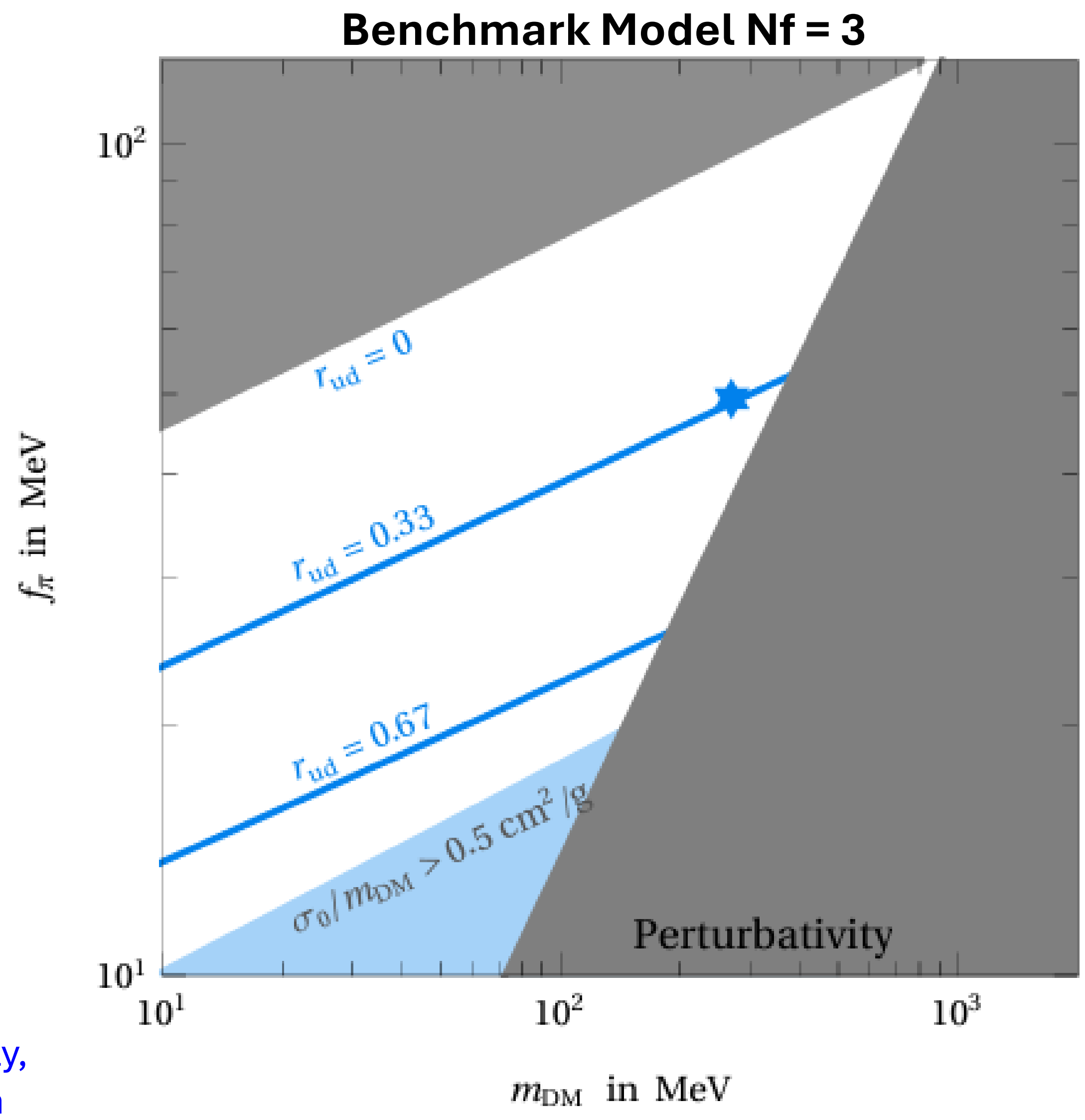
η acts as a **catalyzer**

$\theta \neq 0$ ➡ *On-shell* η resonance exchange

Similar to resonant triple-α reactions in stellar burning $3\alpha \to {}^{12}\mathrm{C}^* \overset{\text{res}}{\simeq} \alpha\alpha \to {}^{8}\mathrm{Be} \dashrightarrow {}^{8}\mathrm{Be}\,\alpha \to {}^{12}\mathrm{C}^*$

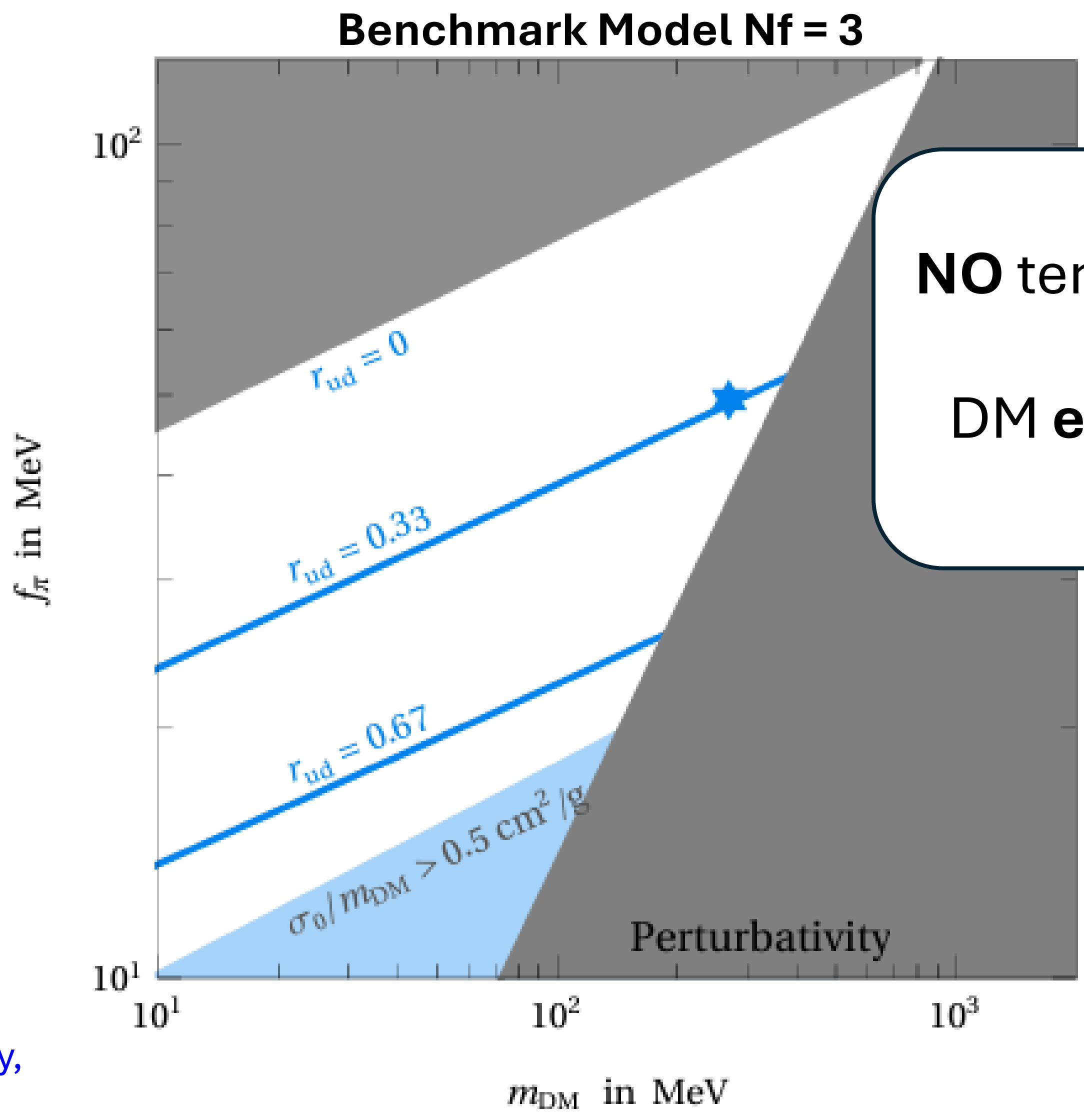
$$\frac{dY}{dz} \simeq -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\mathrm{eq}}}{zH} \left(\frac{Y^3_{\pi_\mathrm{DM}}}{Y^2_{\pi_\mathrm{DM},\mathrm{eq}}} - \frac{Y^2_{\pi_\mathrm{DM}}}{Y_{\pi_\mathrm{DM},\mathrm{eq}}} \right)$$

DM relic abundance with $\theta \neq 0$



Camilo García-Cely, GL, Óscar Zapata [2405.10367]

DM relic abundance with $\theta \neq 0$



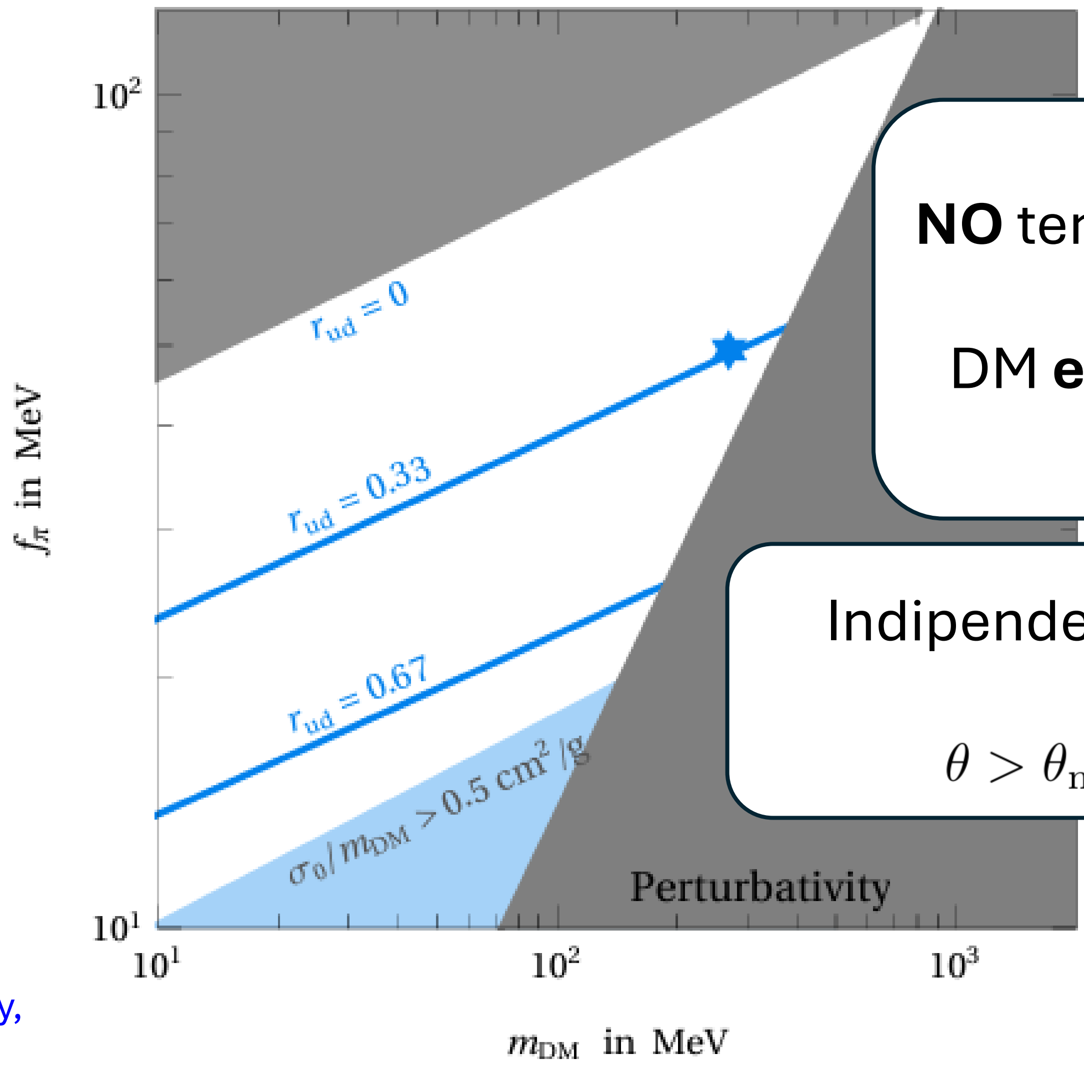
NO tension between relic density and cluster bounds

DM **even far** from perturbation theory breakdown

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

DM relic abundance with $\theta \neq 0$

Benchmark Model Nf = 3



NO tension between relic density and cluster bounds

DM **even far** from perturbation theory breakdown

Indipendent on θ, v_R as long as

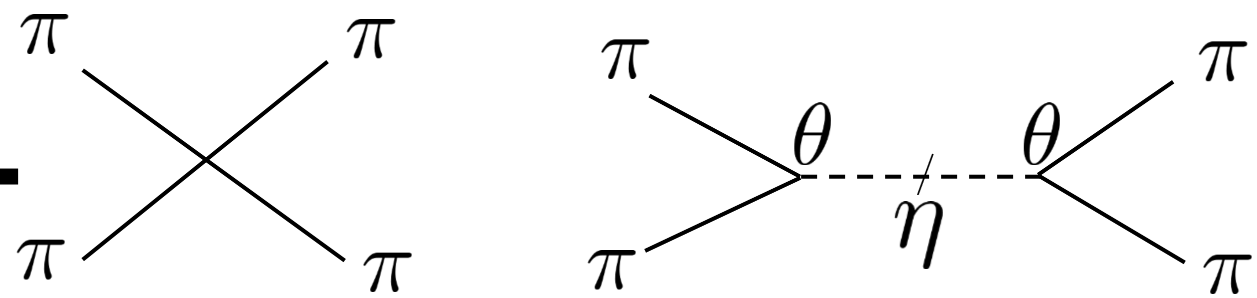
$$\theta > \theta_{\min} \sim 10^{-4}\ ,\ v_R \lesssim 0.1$$

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

SIDM with $\theta \neq 0$

$\theta \neq 0$ induces velocity dependent resonant self-interaction cross section

Already present in Hochberg *et al* (2014)



Camilo García-Cely, GL, Óscar Zapata [2405.10367]

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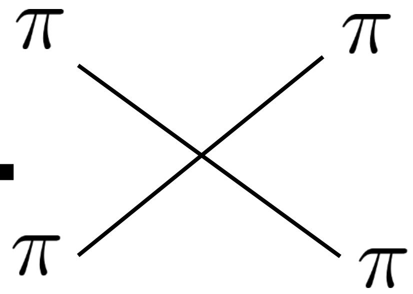
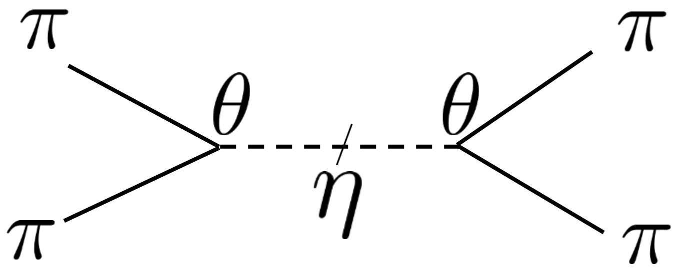




Camilo García-Cely, GL, Óscar Zapata [2405.10367]

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2} \frac{\Gamma^2}{m_\pi^2 (v^2 - v_R^2)^2 + 4\Gamma^2 v^2 / v_R^2}$$

$v =$ DM velocity

<u>constant term</u>

<u>resonant term</u>

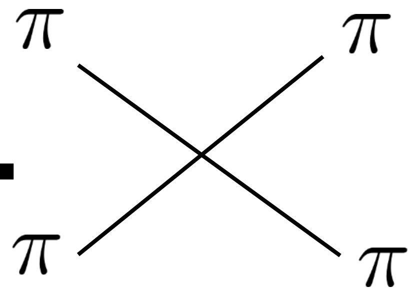
$v \simeq v_R$

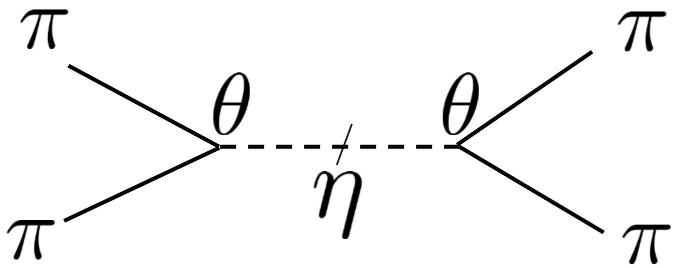
$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \ll 1$$

SIDM with $\theta \neq 0$

$\theta \neq 0$ induces velocity dependent resonant self-interaction cross section

Already present in Hochberg *et al* (2014)





Camilo García-Cely, GL, Óscar Zapata [2405.10367]

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2}\frac{\Gamma^2}{m_\pi^2(v^2 - v_R^2)^2 + 4\Gamma^2 v^2/v_R^2}$$

$v =$ DM velocity

constant term

resonant term

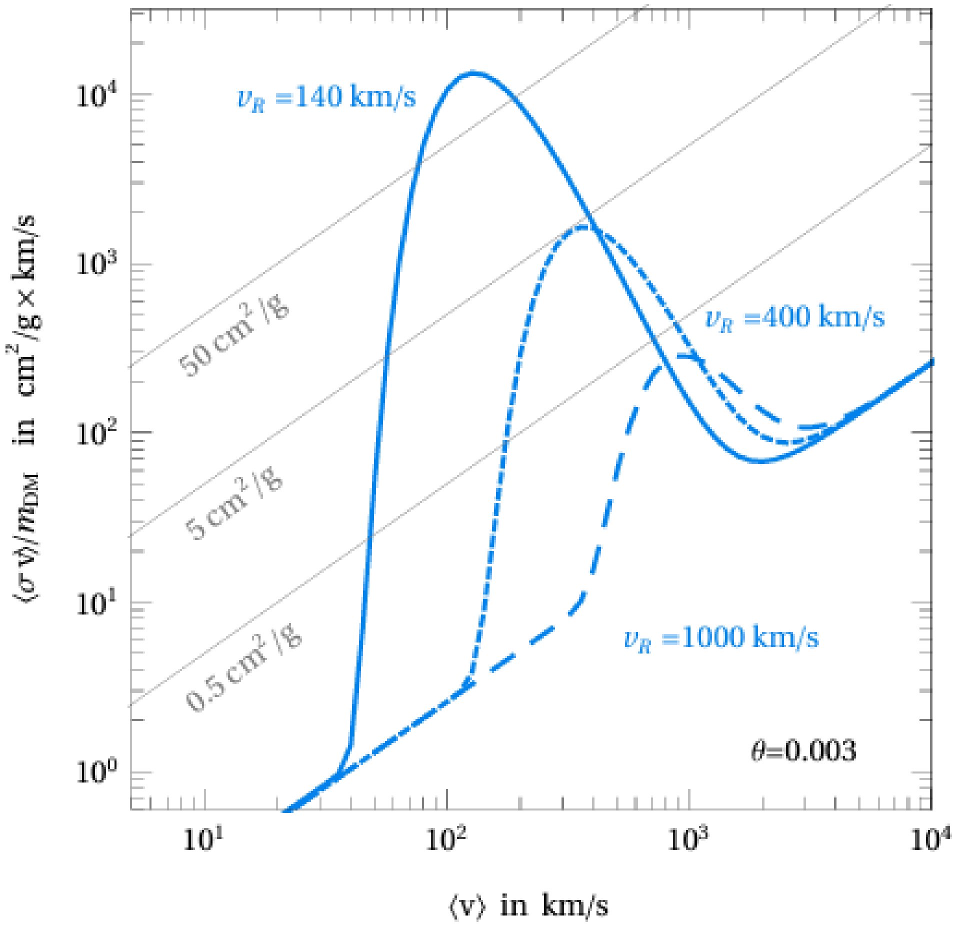
$$v \simeq v_R$$

$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \ll 1$$

$v_R \sim 10 - 100\ \mathrm{km/sec} \sim 10^{-\{5,3\}}$ → resonance at small scales

SIDM with $\theta \neq 0$

Camilo García-Cely, GL, Óscar Zapata [2405.10367]



It works for

$\theta \sim 10^{-\{4,2\}}$

Compatible with relic computation

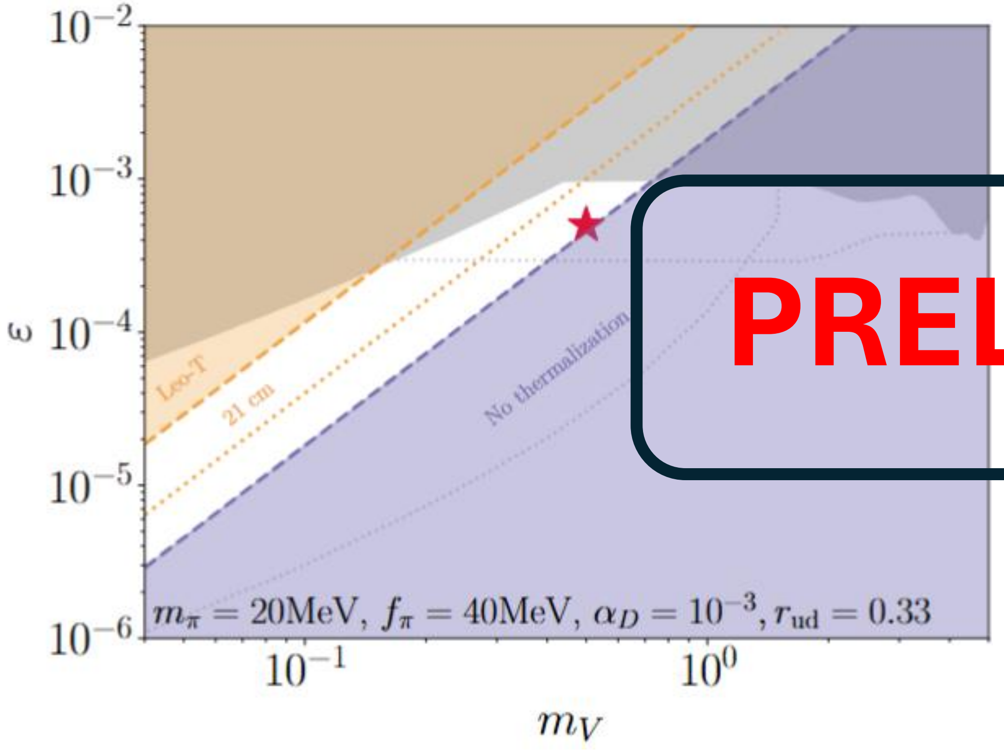
Realization of SIDM for $v_R \sim 10 - 100\ \mathrm{km/sec} \sim 10^{-\{5,3\}}$

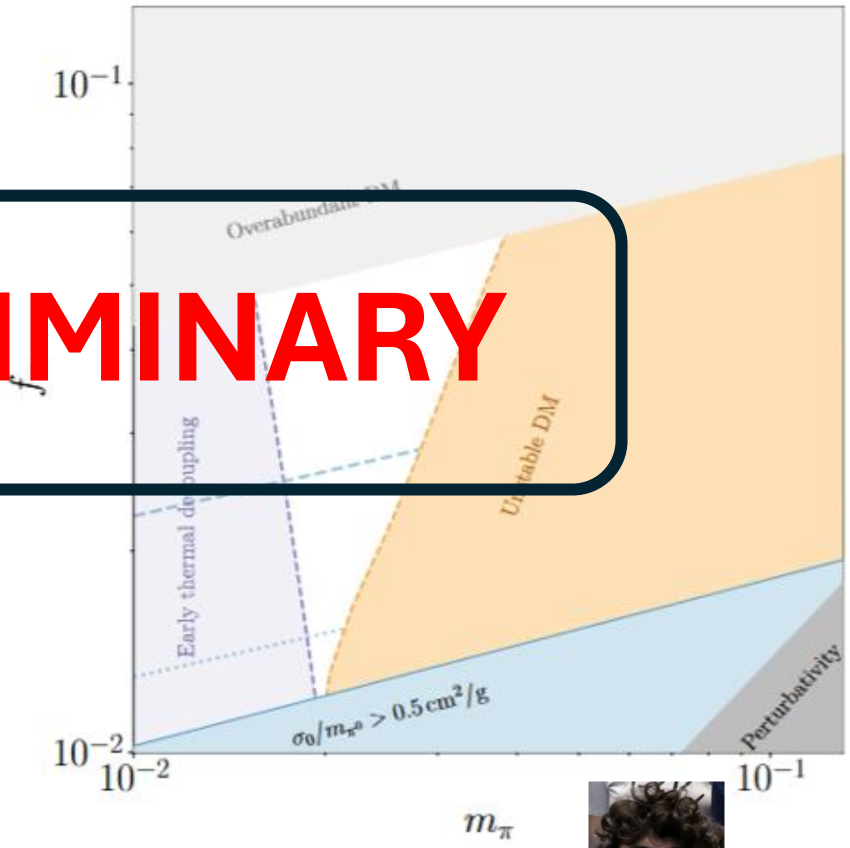
v_R may originate from $\theta \sim 10^{-\{4,2\}}$

Current work

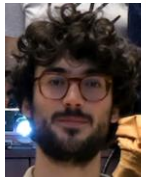
SM portal (thermalization of DM)

$$\mathcal{L}_{\rm DP} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu + \frac{\varepsilon}{2}V_{\mu\nu}B^{\mu\nu}$$





+ large θ , BEQs,...



[2506.xxxxx] in collaboration with C. García-Cely, L. Marsili, O.Zapata

Comments on the results

DM is a pion of a QCD-like dark sector

$\theta \neq 0$

Spectrum with a resonance

$m_\eta \gtrsim 2m_\pi$

We reproduce the **relic abundance** with a resonant 3-to-2 process

avoiding tensions with BC and perturbativity

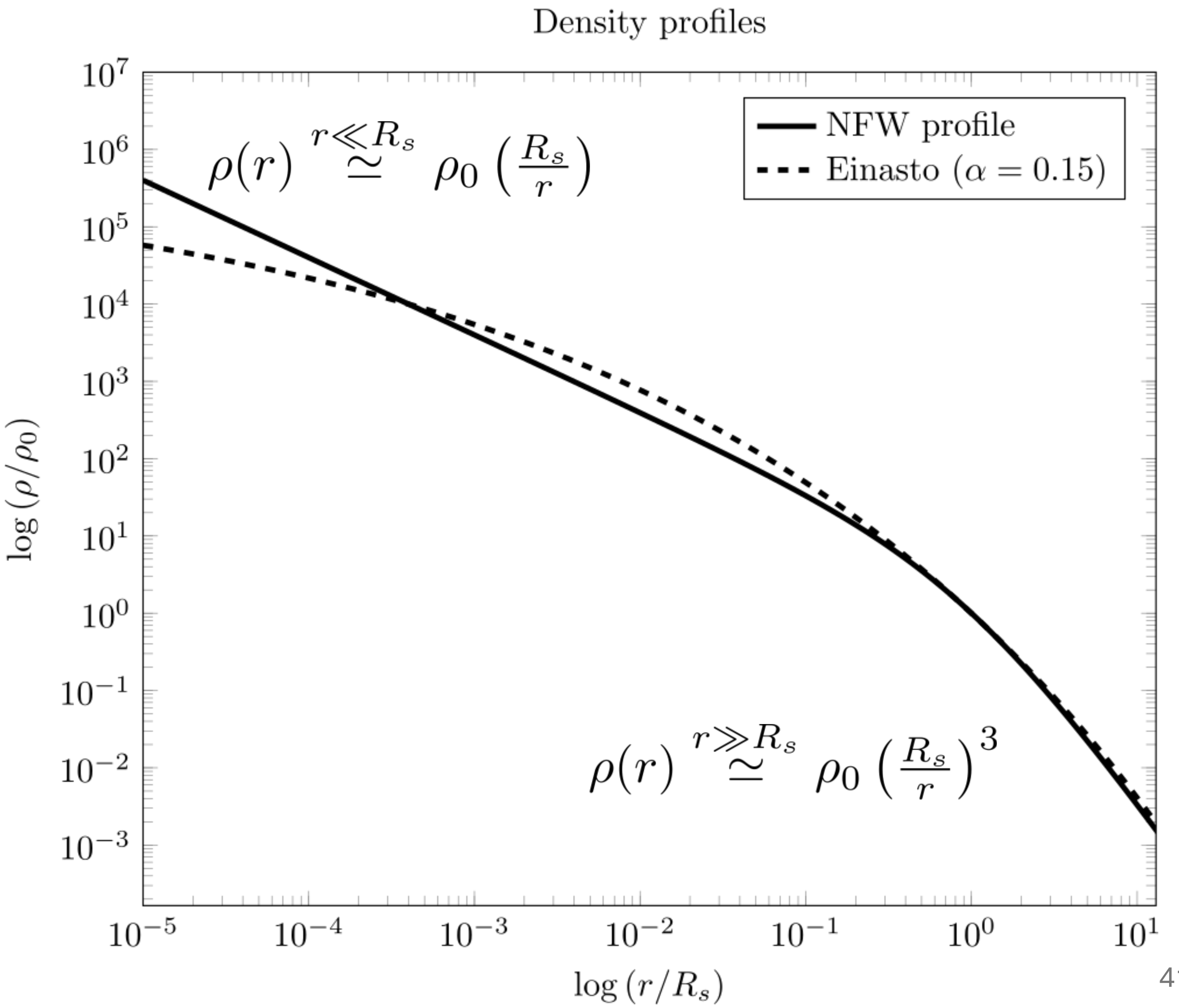
We can solve **small-scale problems** with resonant self-scatterings

No need for light mediator

Backup slides

NFW profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$



Dwarf galaxies

	Name	Discovery	Position (b, ℓ)	Distance in kpc	r_{HL} in pc
classical	Sculptor	1938	$(-83.2^\circ, 287.5^\circ)$	86 ± 6	283 ± 45
	Fornax	1938	$(-65.7^\circ, 237.1^\circ)$	147 ± 12	710 ± 77
	Leo I	1950	$(+49.1^\circ, 226.0^\circ)$	254 ± 15	251 ± 27
	Leo II	1950	$(+67.2^\circ, 220.2^\circ)$	233 ± 14	176 ± 42
	Ursa Minor	1955	$(+44.8^\circ, 105.0^\circ)$	76 ± 3	181 ± 27
	Draco	1955	$(+34.7^\circ, 86.4^\circ)$	76 ± 6	221 ± 19
	Carina	1977	$(-22.2^\circ, 260.1^\circ)$	105 ± 6	250 ± 39
	Sextans	1990	$(+42.3^\circ, 243.5^\circ)$	86 ± 4	695 ± 44

+ ultra-faint dwarf galaxies

Cirelli, Strumia, Zupan (2024)

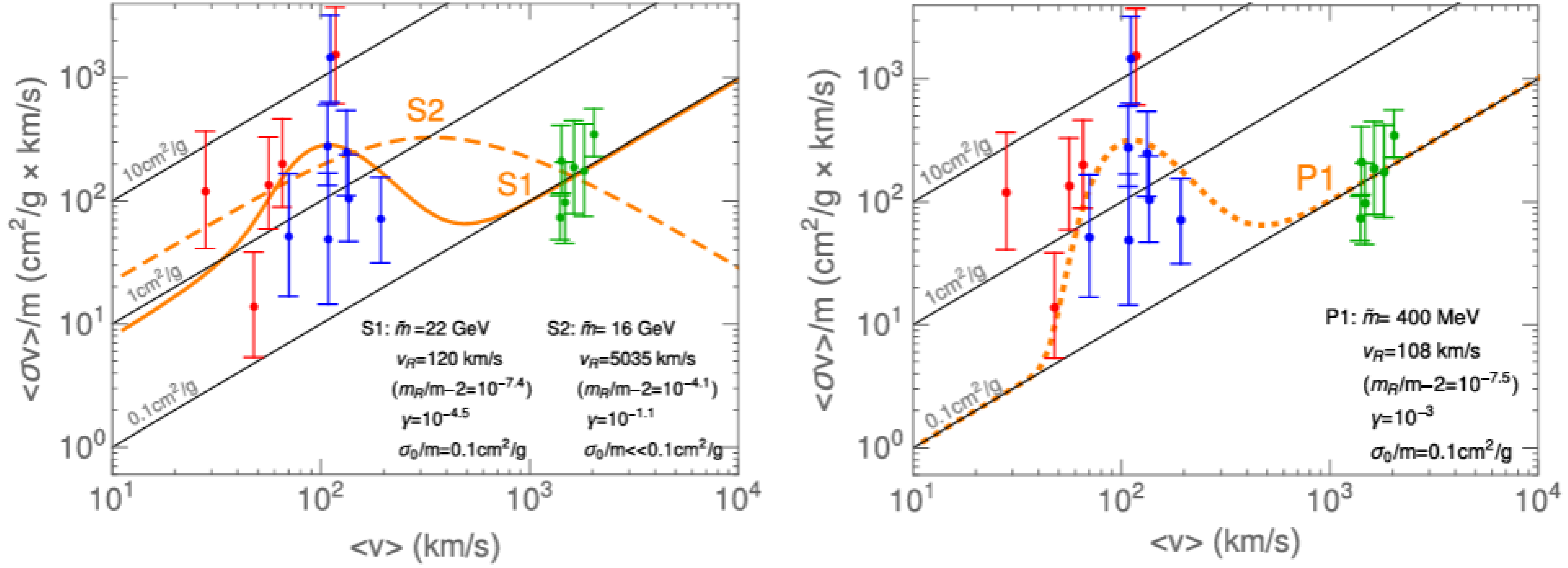
Resonant SIDM

Chu *et al.* (2018)

Dwarf galaxies

Low-surface brightness spiral galaxies

Clusters



$$\langle \sigma v \rangle = \int_0^{v_{\max}} f(v, v_0)\sigma v dv, \qquad f(v, v_0) = \frac{4v^2 e^{-v^2/v_0^2}}{\sqrt{\pi} v_0^3}$$

$$\sigma = \sigma_0 + \frac{4\pi S}{mE(v)} \frac{\Gamma(v)^2/4}{[E(v) - E(v_R)]^2 + \Gamma(v)^2/4}$$

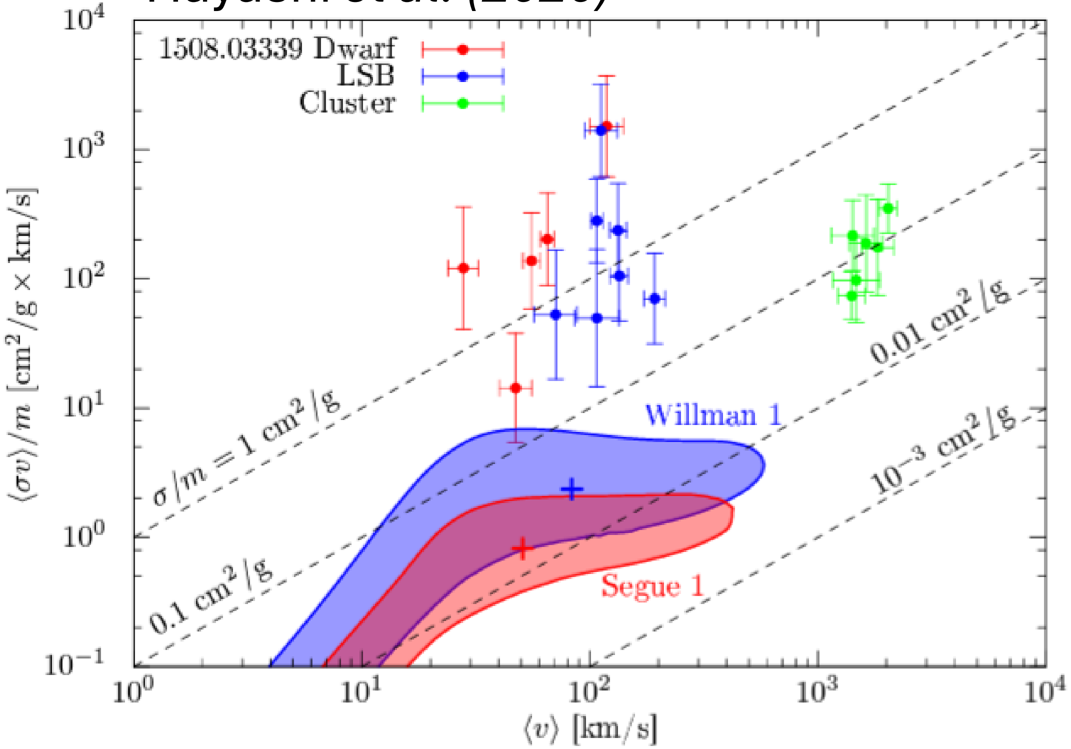
$$\Gamma(v) = m_R \gamma v^{2L+1}$$

Ultra-Faint Dwarf galaxies

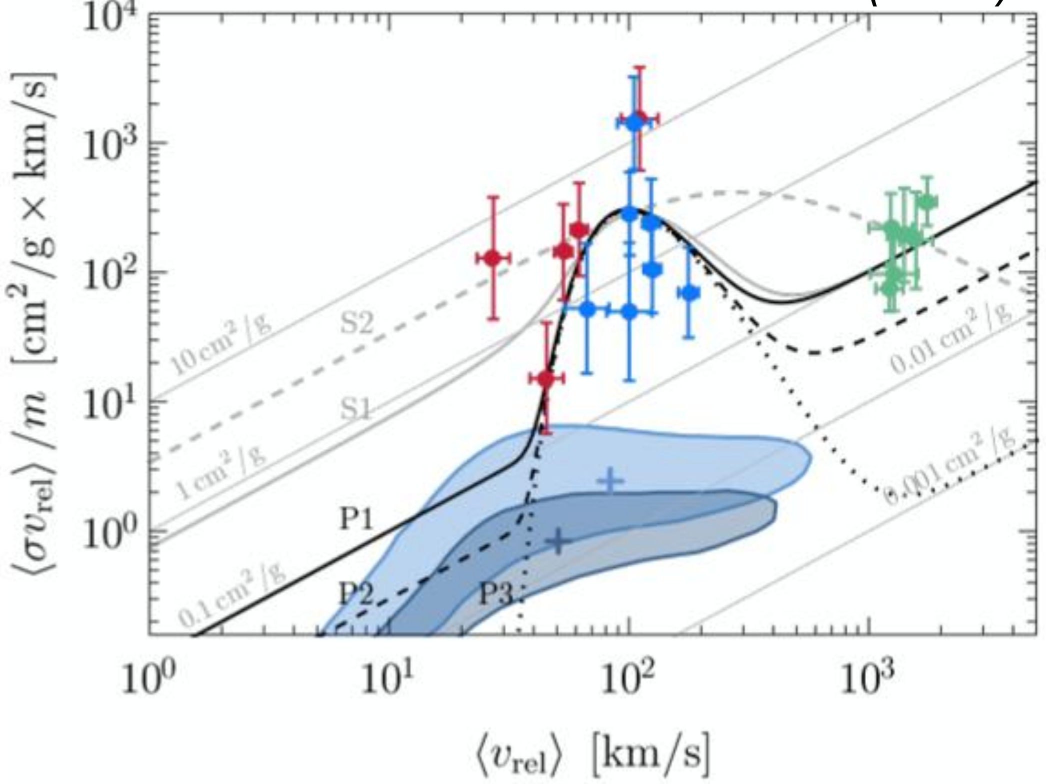
Data from 23 UFDs associated to the Milky Way

DM-dominated (few baryons), $v \sim 10\ \mathrm{Km/sec}$

Hayashi *et al. (2020)*



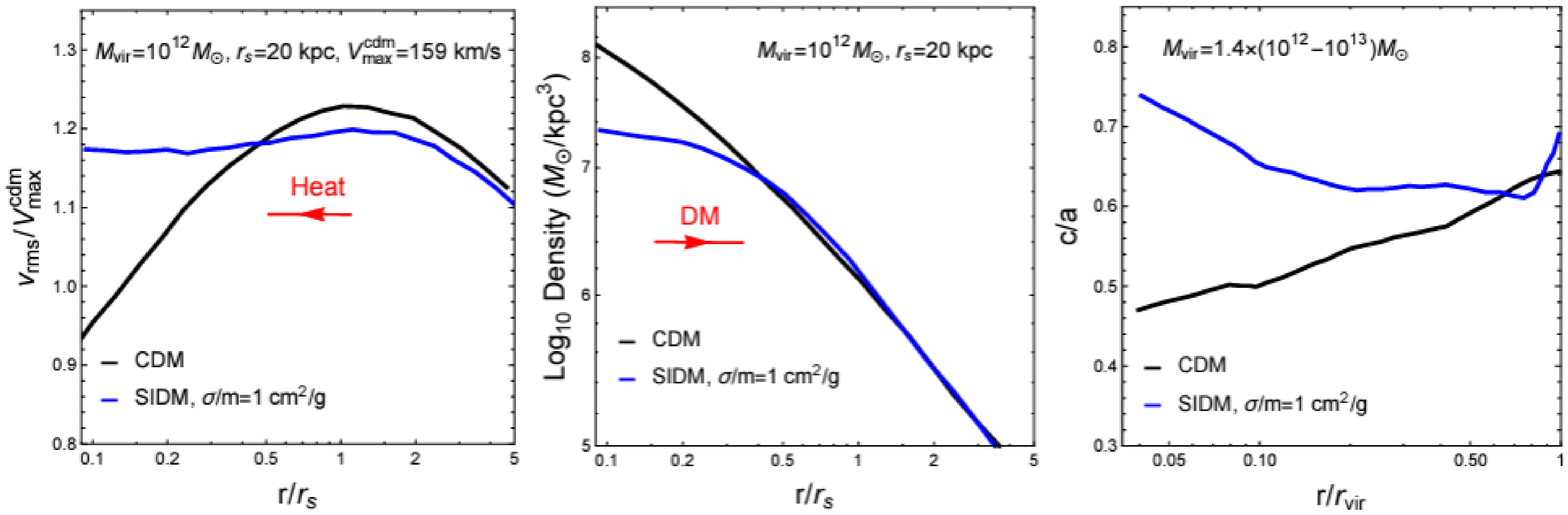
Kamada *et al. (2023)*



Sharp velocity-dependence favour Resonant SIDM models

SIDM vs CDM

Tulin, Yu (2017)



$$R_{\rm scat} = \sigma v_{\rm rel}\rho_{\rm dm}/m \approx 0.1\ {\rm Gyr}^{-1} \times \left(\frac{\rho_{\rm dm}}{0.1\ \ {\rm M_\odot/pc^3}}\right)\left(\frac{v_{\rm rel}}{50\ {\rm km/s}}\right)\left(\frac{\sigma/m}{1\ {\rm cm^2/g}}\right)$$

Cross section: observations vs constraints

Tulin, Yu (2017)

Positive observations	σ/m	$v_{\rm rel}$	Observation	Refs.
Cores in spiral galaxies (dwarf/LSB galaxies)	$\gtrsim 1\ \mathrm{cm}^2/\mathrm{g}$	$30 - 200\ \mathrm{km/s}$	Rotation curves	[102, 116]
Too-big-to-fail problem				
Milky Way	$\gtrsim 0.6\ \mathrm{cm}^2/\mathrm{g}$	$50\ \mathrm{km/s}$	Stellar dispersion	[110]
Local Group	$\gtrsim 0.5\ \mathrm{cm}^2/\mathrm{g}$	$50\ \mathrm{km/s}$	Stellar dispersion	[111]
Cores in clusters	$\sim 0.1\ \mathrm{cm}^2/\mathrm{g}$	$1500\ \mathrm{km/s}$	Stellar dispersion, lensing	[116, 126]
Abell 3827 subhalo merger	$\sim 1.5\ \mathrm{cm}^2/\mathrm{g}$	$1500\ \mathrm{km/s}$	DM-galaxy offset	[127]
Abell 520 cluster merger	$\sim 1\ \mathrm{cm}^2/\mathrm{g}$	$2000 - 3000\ \mathrm{km/s}$	DM-galaxy offset	[128, 129, 130]
Constraints				
Halo shapes/ellipticity	$\lesssim 1\ \mathrm{cm}^2/\mathrm{g}$	$1300\ \mathrm{km/s}$	Cluster lensing surveys	[95]
Substructure mergers	$\lesssim 2\ \mathrm{cm}^2/\mathrm{g}$	$\sim 500 - 4000\ \mathrm{km/s}$	DM-galaxy offset	[115, 131]
Merging clusters	$\lesssim$ few cm^2/g	$2000 - 4000\ \mathrm{km/s}$	Post-merger halo survival (Scattering depth $\tau < 1$)	Table II
Bullet Cluster	$\lesssim 0.7\ \mathrm{cm}^2/\mathrm{g}$	$4000\ \mathrm{km/s}$	Mass-to-light ratio	[106]

TABLE I: Summary of positive observations and constraints on self-interaction cross section per DM mass. Italicized observations are based on *single individual systems*, while the rest are derived from sets of multiple systems. Limits quoted, which assume constant σ/m, may be interpreted as a function of collisional velocity $v_{\rm rel}$ provided σ/m is not steeply velocity-dependent. References noted here are limited to those containing quoted self-interaction cross section values. Further references, including original studies of observations, are cited in the corresponding sections below.

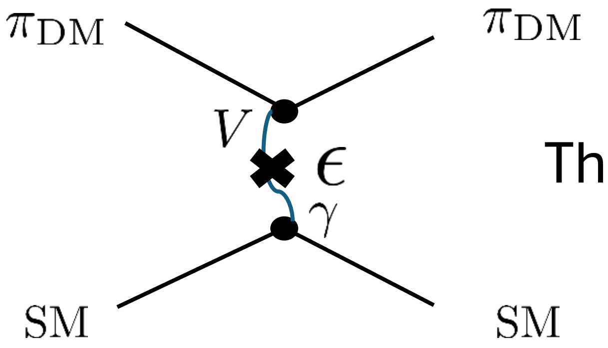
Portals with the SM

Dark photon portal

Hochberg *et al. (2015)*

Gauging a $U(1)_D$ subgroup of unbroken global symmetry $G \to H$ $\quad U(1)_D \supset H$

$$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$$



Thermalization $\quad \epsilon F^{\mu\nu} F'_{\mu\nu}$

Kinetic mixing

Choose $Q_D(q)$
to eliminate axial anomaly

→ Long-lived π^0

ALP portal

Kamada *et al. (2017)*

Hochberg *et al. (2018)*

$$\mathcal{L}_{aq} = -\frac{1}{2} m_a^2 a^2 - \left(\frac{1}{2} m_Q e^{ia/f_{a\pi}} J^{ij} q_i q_j + \text{h.c.} \right) \quad \rightarrow \quad -\frac{1}{2}\left(m_a^2 a^2 + \frac{2 m_\pi^2 f_\pi^2}{f_{a\pi}^2} \right) a^2 + \frac{m_\pi^2}{8 f_{a\pi}^2} a^2 \mathrm{Tr}\pi^2$$

$$\mathcal{L}_{a\gamma} = \frac{1}{4 f_{a\gamma}} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Thermalization $\quad$ $a\pi \rightarrow a\pi$

$$a \leftrightarrow \gamma\gamma$$

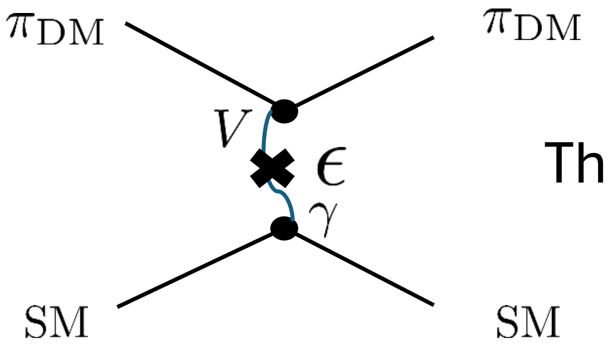
Portals with the SM

Dark photon portal

Hochberg *et al. (2015)*

Gauging a $U(1)_D$ subgroup of unbroken global symmetry $G \to H$ $U(1)_D \supset H$

$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$



Thermalization $\epsilon F^{\mu\nu} F'_{\mu\nu}$

Kinetic mixing

Choose $Q_D(q)$ to eliminate axial anomaly → stable π^0

Make sure that: thermalization is efficient

$$\pi\pi \to \pi V$$
$$\pi\pi \to V \to e^+ e^-$$

are subdominant for DM relic

Allowed by bounds on DP and indirect detection (p-wave)

Portals with the SM

<u>Dark photon portal</u> Work in progress...

- charge assignment 1: $Q_d^{(1)} = \mathrm{diag}\,(+1, +1, -1)$;
- charge assignment 2: $Q_d^{(2)} = \mathrm{diag}\,(+1, -1, -1)$.

NO axial anomaly

O(p^6) operators

$$\frac{1}{(4\pi)^2 f_\pi} i\epsilon_{\mu\nu\rho\sigma} V^{\mu\nu} V^{\rho\sigma} \mathrm{Tr}(Q_d) \mathrm{Tr}(Q_d\, M\, U^\dagger) + h.c.$$

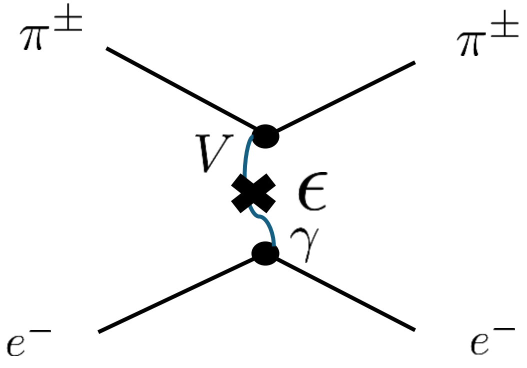
$$\tau_{\rm DM} \sim 1.4{\times}10^{29}\ \mathrm{sec}\ \left(\frac{0.1}{\sin\theta_{\pi\eta}}\right)^2 \left(\frac{10^{-3}}{\alpha_d}\right)^2 \left(\frac{5\times10^{-4}}{\varepsilon}\right)^4 \left(\frac{m_V}{0.5\ \mathrm{GeV}}\right)^8 \left(\frac{20\ \mathrm{MeV}}{m_{\rm DM}}\right)^9 \left(\frac{0.5}{m_{\rm DM}/f_\pi}\right)^6$$

Portals with the SM

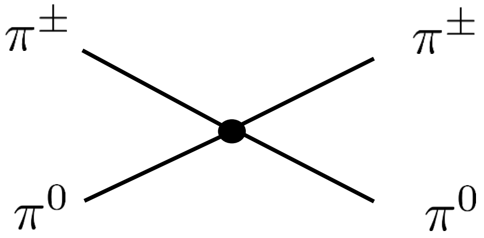
Dark photon portal

Work in progress...

$$\Gamma_{\text{scatt}} = \frac{\sum_{\pi=\pi^\pm} Q_\pi^2}{2} \sum_{f=e^-,e^+} \langle \sigma v(\pi^\pm f \to \pi^\pm f)\, n_f^{\text{eq}} \rangle = \frac{45\zeta(5) e^2 e_d^2 \varepsilon^2 T^5}{2\pi^3 m_V^4} (g_{e^+} + g_{e^-})$$



$$\frac{5\zeta(5)}{4} \frac{T}{m_{K^\pm}} \Gamma_{\text{scatt}} \gtrsim H\big|_{T_{\text{fo}}}$$

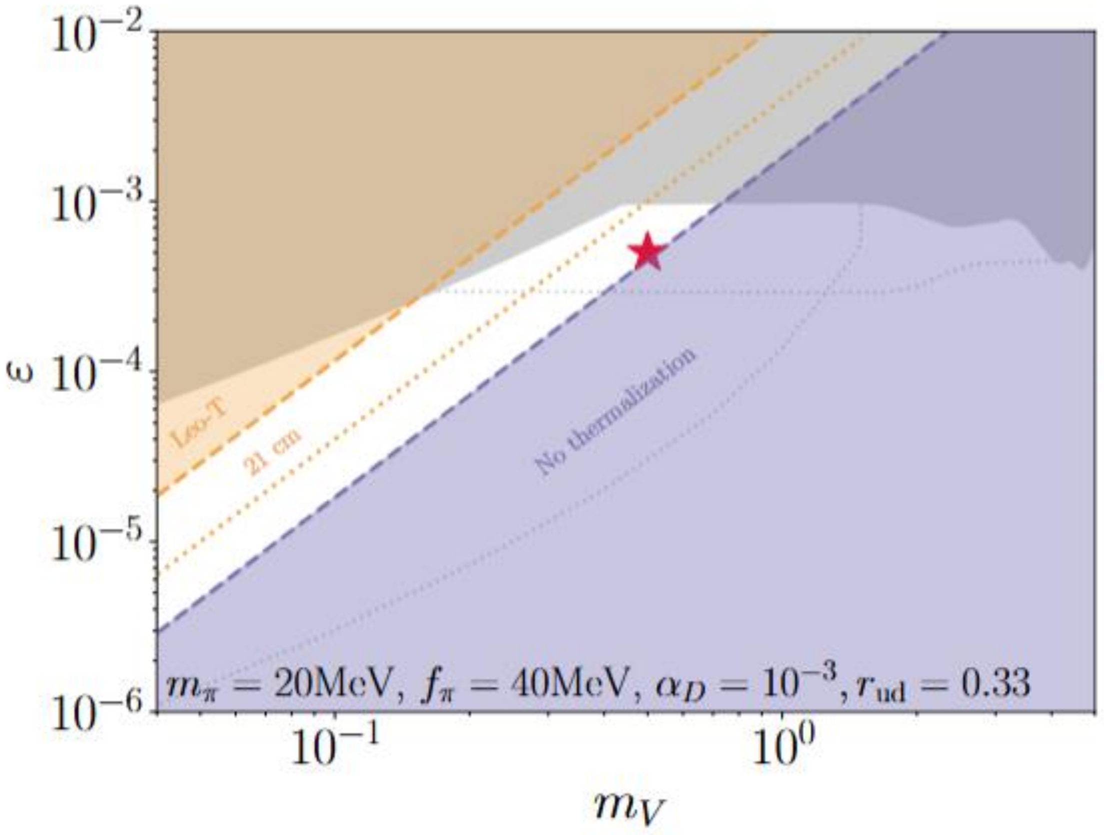


$$n_{\pi^\pm}(T) \langle \sigma v(\pi^\pm \pi^0 \to \pi^\pm \pi^0) \rangle \frac{T}{m_\pi} \gtrsim H\big|_{T_{\text{fo}}}$$

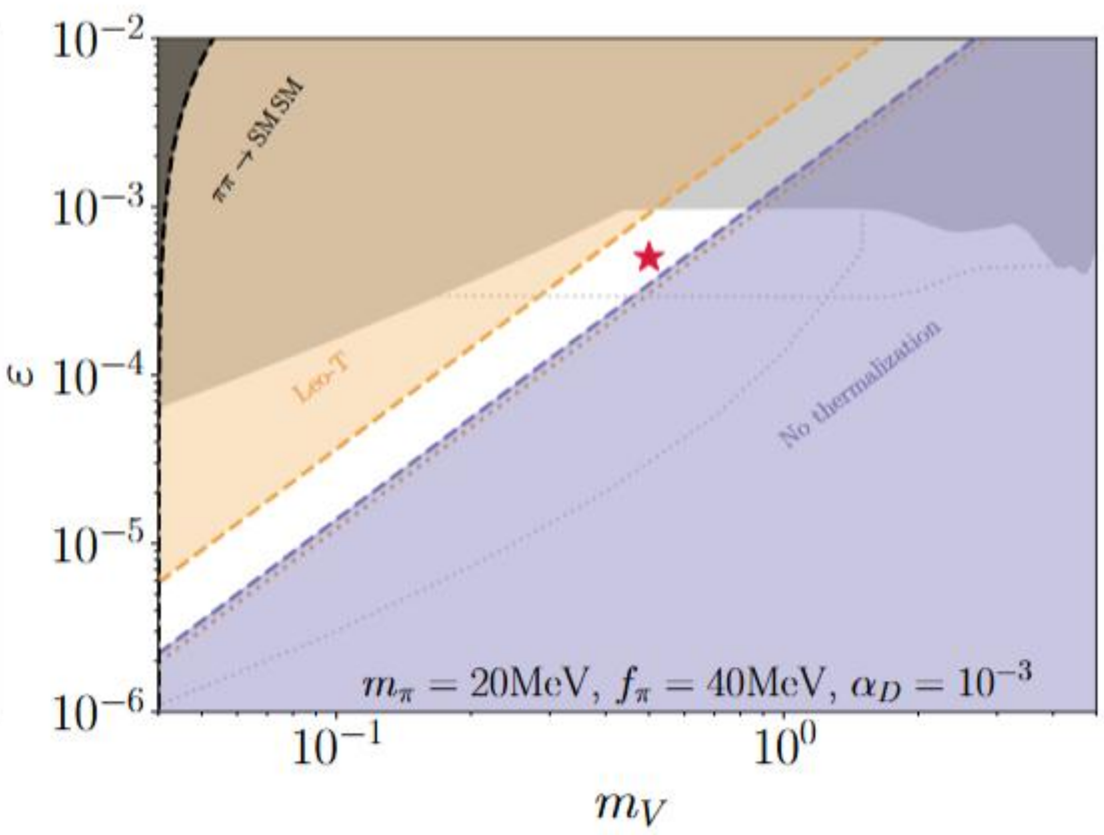
Portals with the SM

<u>Dark photon portal</u> Work in progress...

charge assignment 1



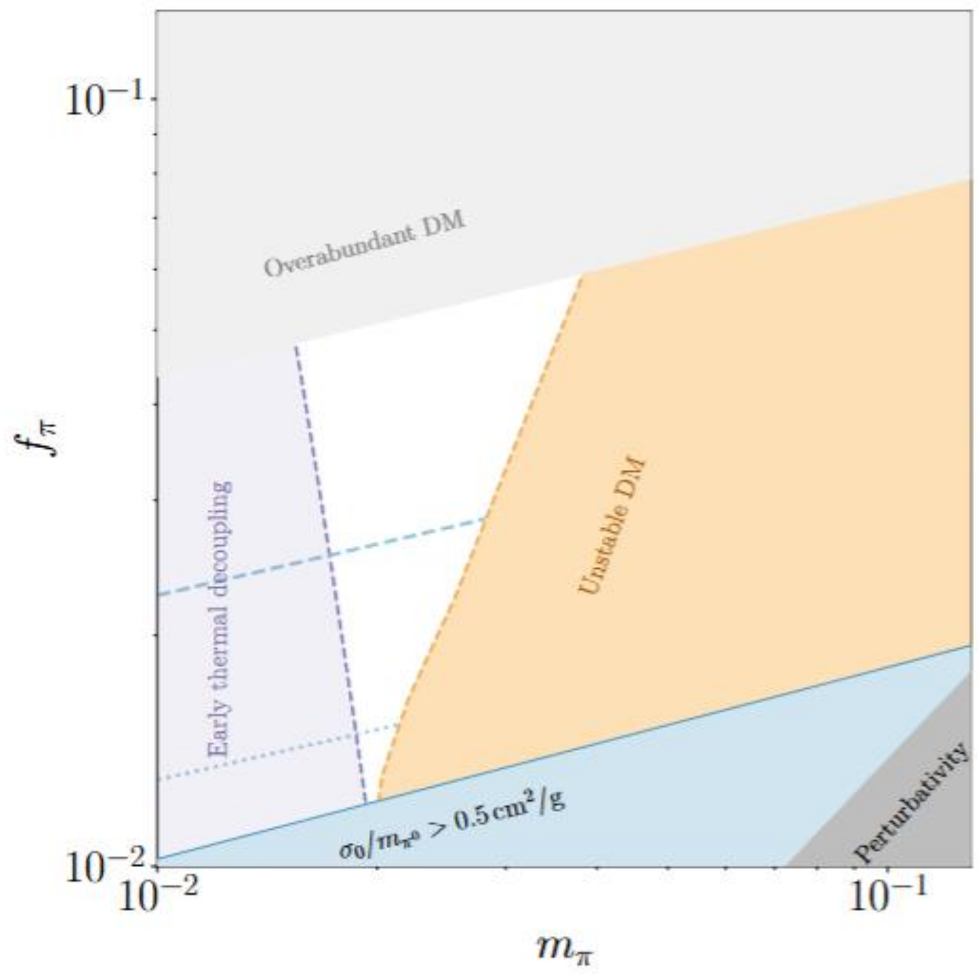
charge assignment 2



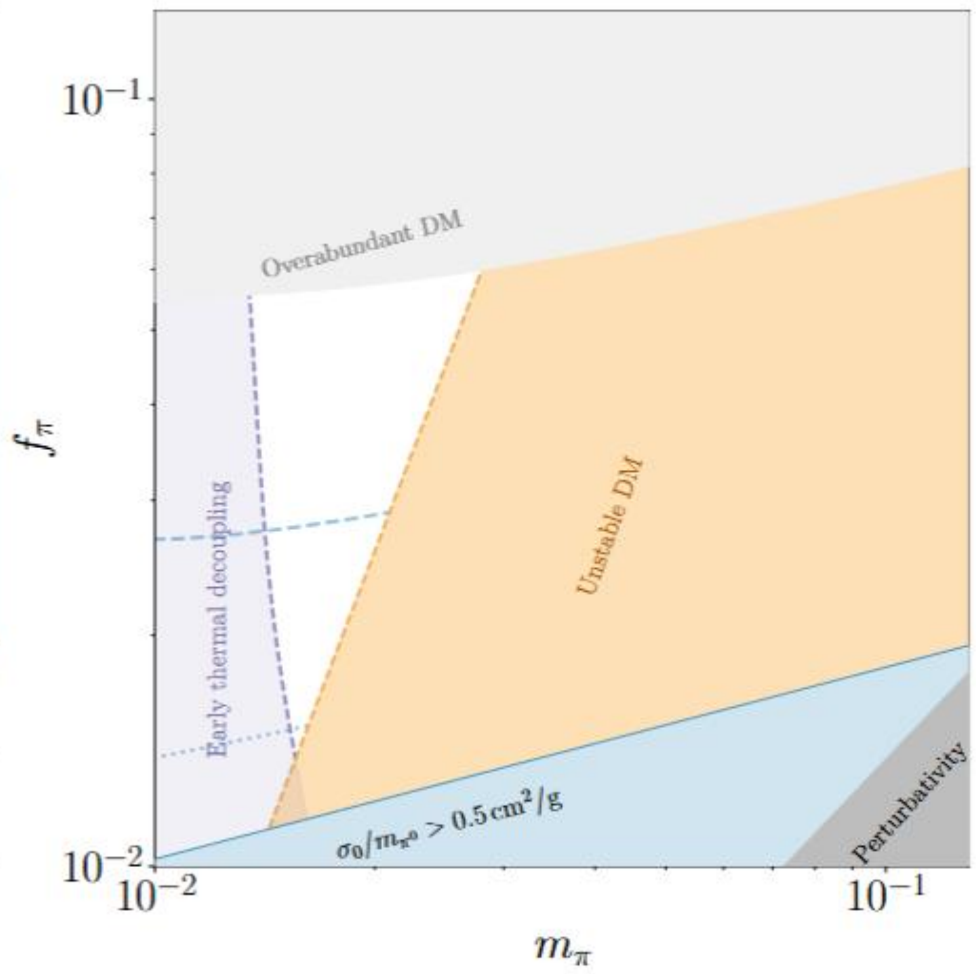
Portals with the SM

<u>Dark photon portal</u> Work in progress...

charge assignment 1



charge assignment 2



Resonance and θ

<u>Example:</u> $M = (m, m, m, 5m)$

$$\longrightarrow \quad \begin{cases} m_\pi^2 = 2B_0 m(1 - 25\theta^2/512) \\ m_\eta^2 = 8B_0 m(1 - 5\theta^2/1024) \end{cases}$$

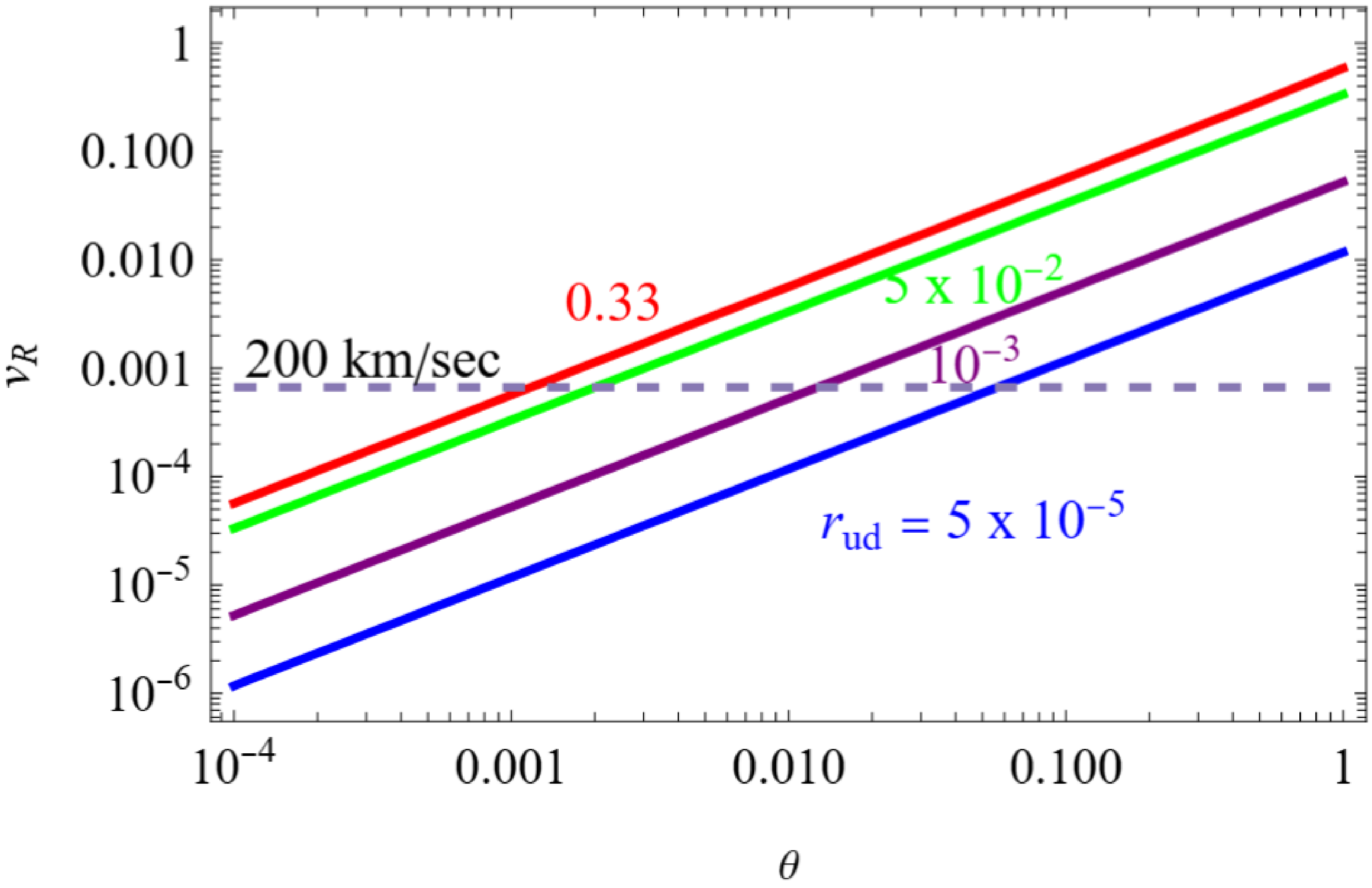
$$\longrightarrow \quad m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \sim 0.4\ \theta$$

It works for DM relic if $10^{-4} \lesssim \theta \lesssim 0.5$

It works for small scale anomalies if $\theta \sim 10^{-3}$

Resonance and θ

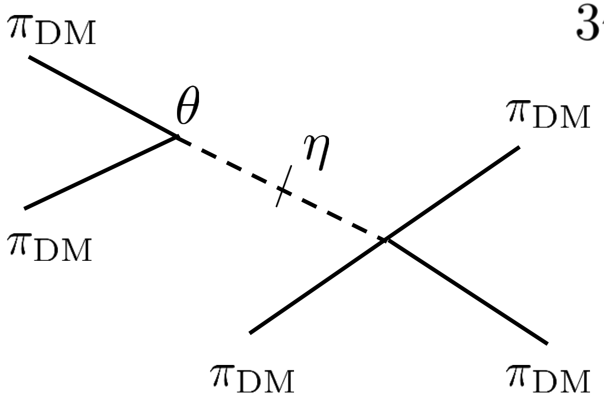
Example:



DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ ➡ *On-shell* η resonance



$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta$ - - - -▸ $\eta\pi \to 2\pi$

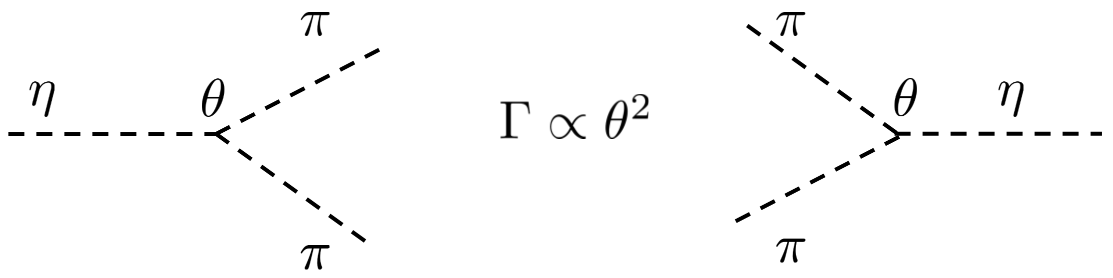
η acts as a **catalyzer**

$$\begin{cases} sHz\dfrac{dY_{\pi_{\rm DM}}}{dz} = +2\gamma_D(\eta\to\pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}-\dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right)+\gamma_2(\eta\pi_{\rm DM}\to\pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}}-\dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \\ sHz\dfrac{dY_\eta}{dz} = -\gamma_D(\eta\to\pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}-\dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right)-\gamma_2(\eta\pi_{\rm DM}\to\pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}}-\dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \end{cases}$$

Neglecting the non-resonant piece of 3-to-2 processes

DM relic abundance with $\theta \neq 0$

Condition for chemical equilibrium



$$\underline{\langle \Gamma(\eta \leftrightarrow \pi\pi) \rangle > H} \longrightarrow \theta > \theta_{\rm min} \sim 10^{-4}$$

evalutated at freeze-out of $\eta\pi \to 2\pi$

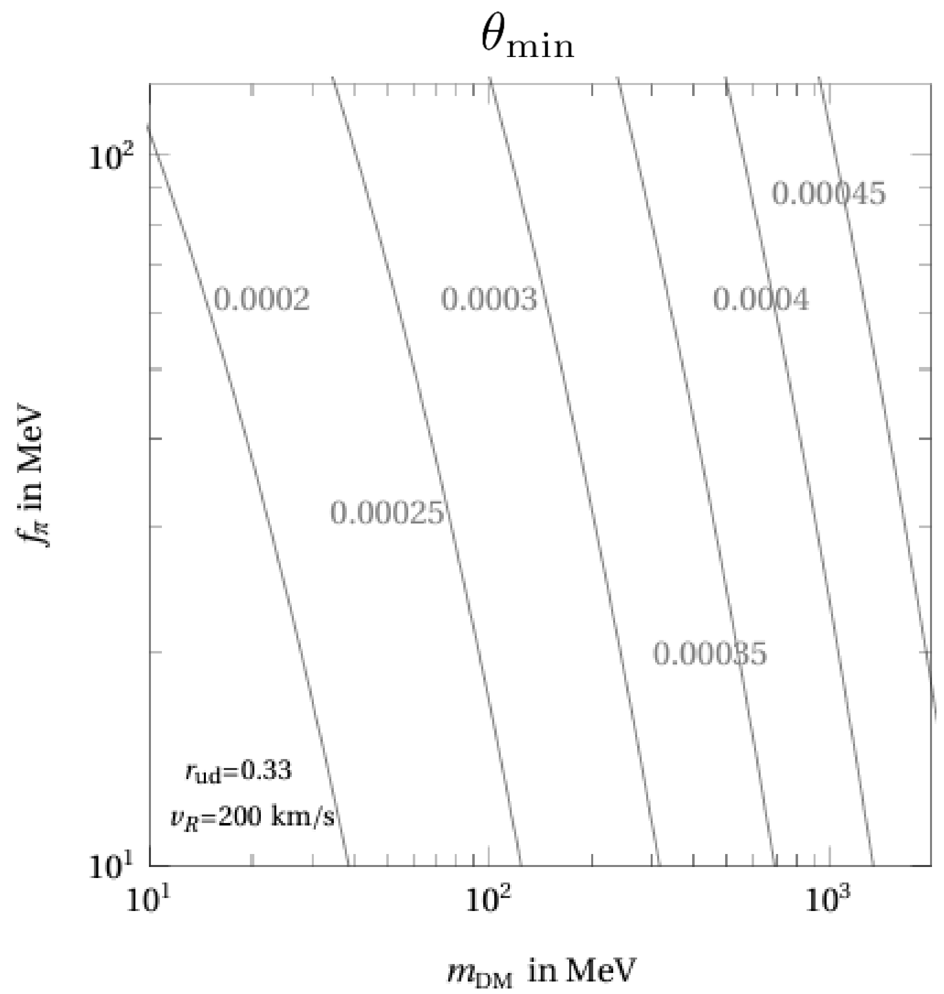
defined as $n_{\eta,\rm eq}(z_{\rm fo})\langle \sigma_{\eta\pi} v\rangle \sim H(z_{\rm fo})$

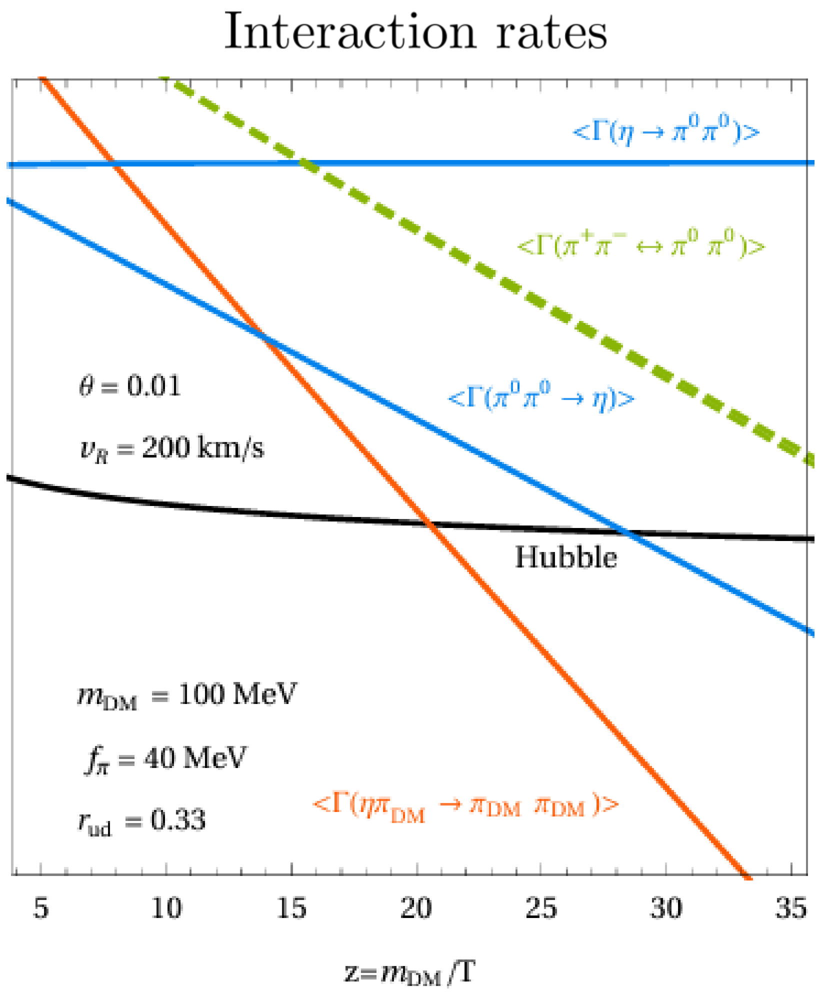
$$\frac{Y_\eta}{Y_{\eta,\rm eq}} = \frac{Y_\pi^2}{Y_{\pi,\rm eq}^2}$$

The Boltzmann equations simplify!

DM relic abundance with $\theta \neq 0$

Condition for chemical equilibrium

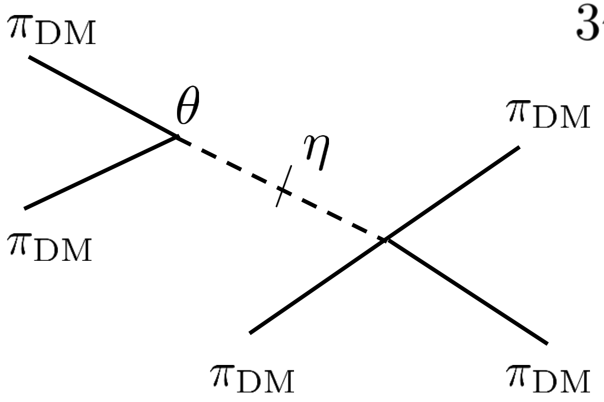




DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ ➡ *On-shell* η resonance



$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta$ - - - ➤ $\eta\pi \to 2\pi$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

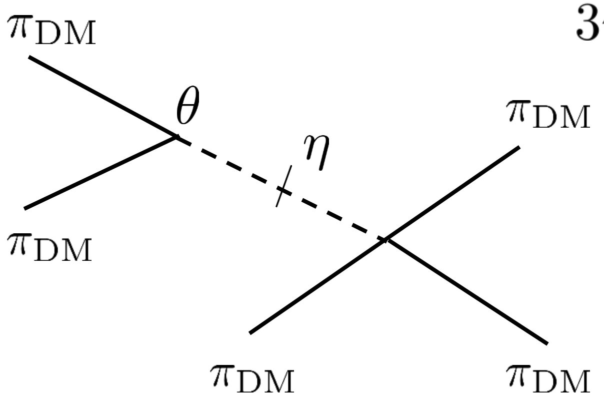
$$\begin{cases} sHz\dfrac{dY_{\pi_{\rm DM}}}{dz} = +2\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\cancel{\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right)} + \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \\[2ex] sHz\dfrac{dY_\eta}{dz} = -\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\cancel{\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right)} - \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \end{cases}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ → *On-shell* η resonance



$$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\rm eq}}{zH} \left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} \right)$$

$$z = m_\pi / T$$

$$\langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$$

$$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

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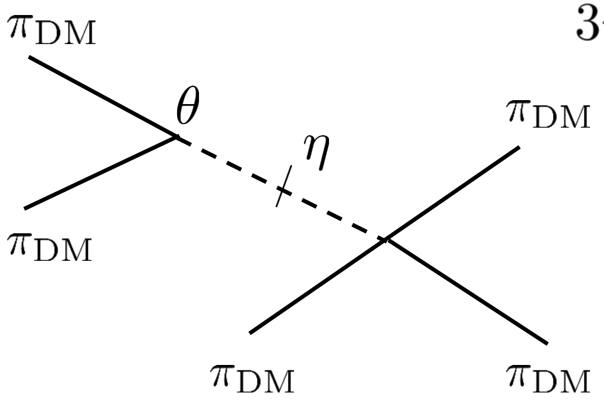


$$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

The relic is fixed by 2-to-2 annihilations

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,{\rm eq}}}{zH} \left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},{\rm eq}}} \right)$$

$$z = m_\pi / T$$

$$\langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$$

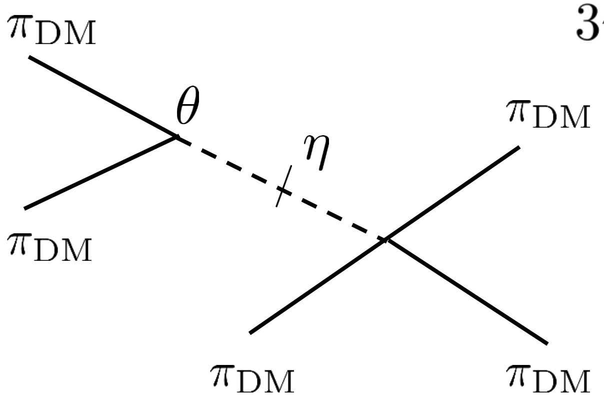
$$Y = Y_{\pi_{\rm DM}} + 2 Y_\eta \simeq Y_{\pi_{\rm DM}}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ ➡ *On-shell* η resonance



$3\pi \to 2\pi \overset{\text{res}}{\simeq} 2\pi \to \eta$ - - - ▸ $\eta\pi \to 2\pi$

if $\theta > \theta_{\min} \sim 10^{-4}$

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{sY_{\eta,\mathrm{eq}}}{zH} \left(\frac{Y^3_{\pi_{\mathrm{DM}}}}{Y^2_{\pi_{\mathrm{DM}},\mathrm{eq}}} - \frac{Y^2_{\pi_{\mathrm{DM}}}}{Y_{\pi_{\mathrm{DM}},\mathrm{eq}}} \right)$$

$z = m_\pi / T$

$\langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$

$$Y = Y_{\pi_{\mathrm{DM}}} + 2Y_\eta \simeq Y_{\pi_{\mathrm{DM}}}$$

We can integrate the Boltzmann Equation (both analytically and numerically)

$Y_{\pi_{\mathrm{DM}}} \simeq Y_{\pi_{\mathrm{DM}},\mathrm{eq}}(z_{\mathrm{fo}})$ defined as $n_{\eta,\mathrm{eq}}(z_{\mathrm{fo}}) \langle \sigma_{\eta\pi} v \rangle \sim H(z_{\mathrm{fo}})$

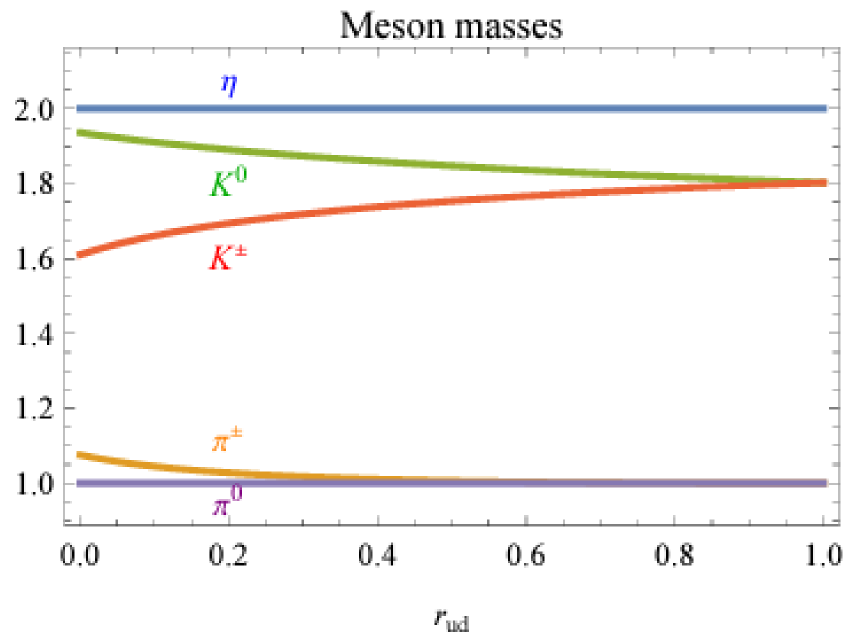
Explicit benchmark model

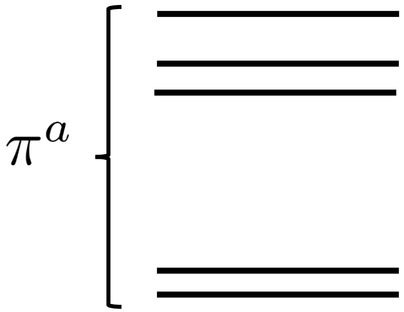
$$M = (m_u, m_d, m_s) \qquad r_{ud} = m_u/m_d$$

$$0 \le r_{ud} \le 1$$

m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$





$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_{\pi^0}$

m_K $\quad n_K \ll n_\pi \sim n_{\mathrm{DM}}$ always negligible

π^a

$m_{\pi^\pm}$

m_{π^0}

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

Explicit benchmark model

$$M = (m_u, m_d, m_s) \qquad r_{ud} = m_u/m_d$$

$$0 \leq r_{ud} \leq 1$$

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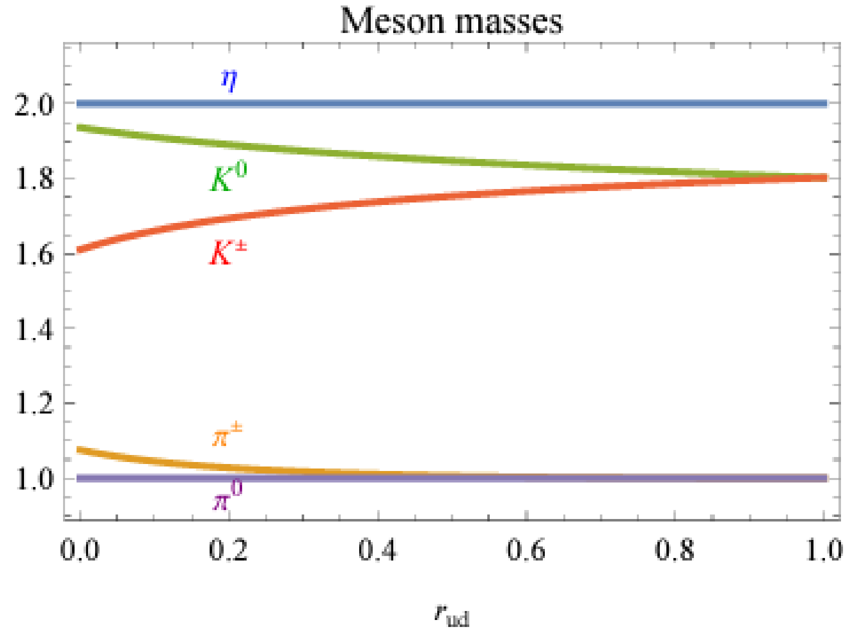


π^a

$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_{\pi^0}$$

m_K

$m_{\pi^\pm}$

m_{π^0}

Contribute to DM in the Early Universe but negligible today $\pi^+\pi^- \to \pi^0\pi^0$

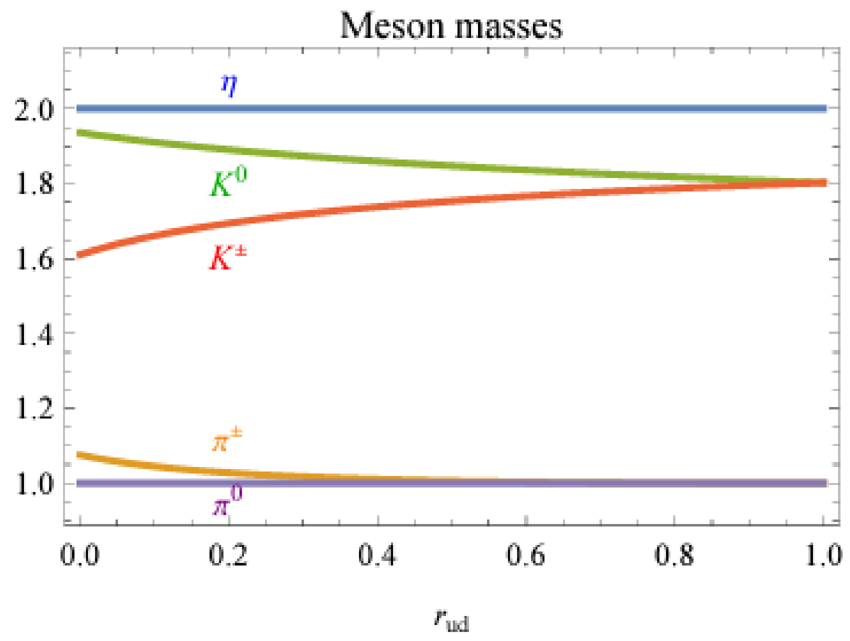
$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

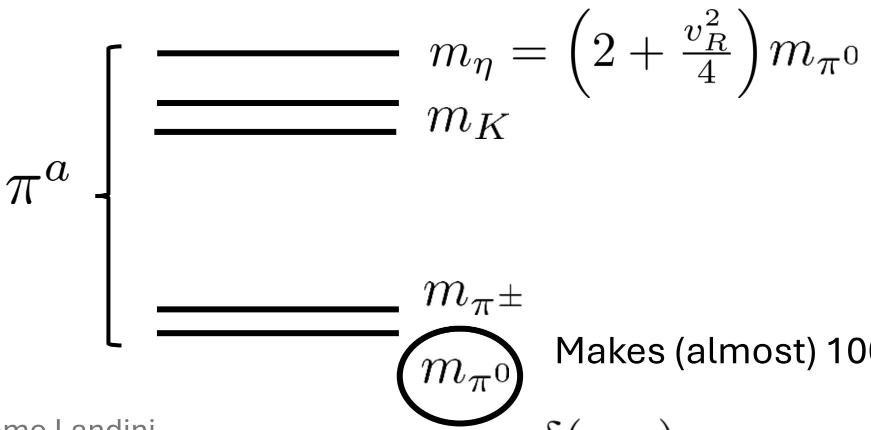
Explicit benchmark model

$$M = (m_u, m_d, m_s) \qquad r_{ud} = m_u/m_d$$
$$0 \leq r_{ud} \leq 1$$

m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$



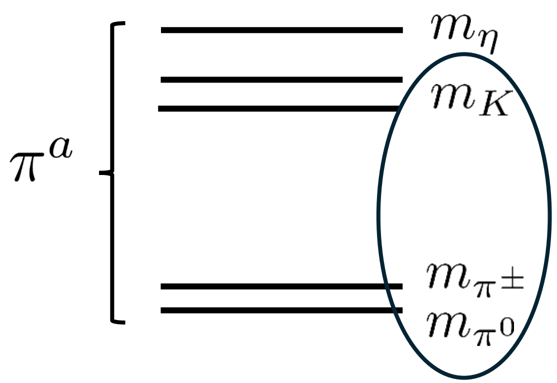


Makes (almost) 100% of DM today

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

Co-annihilations

$$\text{Stable states} = \{\pi^0, \pi^\pm, K\}$$



Stable co-annihilating partners

$$\begin{cases} KK \leftrightarrow \pi\pi \\ \pi^+\pi^- \leftrightarrow \pi^0\pi^0 \end{cases}$$

Keep the species in chemical equilibrium

$$Y_i/Y_{\pi^0} \sim \exp[-(m_i - m_{\pi^0})z/m_{\pi^0}] \qquad i = \pi^\pm, K$$

$m_K \sim 1.7 m_{\pi^0}$ → $z_{\rm fo} \sim 20$ → $Y_K/Y_{\pi^0} \overset{z_{\rm fo}}{\sim} 10^{-6}$ Negligible contribution to DM relic

$m_{\pi^\pm} \lesssim 1.075 m_{\pi^0}$ → $z_{\rm fo} \sim 20$ → $Y_{\pi^\pm}/Y_{\pi^0} \overset{z_{\rm fo}}{\sim} 0.5$ Sizeable contribution to DM relic

$\pi^+\pi^- \xrightarrow{z \gg z_{\rm fo}} \pi^0\pi^0$ → Negligible amount of $\pi^\pm$ today → 100% of DM $\approx \pi^0$

Boltzmann Equation benchmark model

$$\text{Stable states} = \{\pi^0, \pi^\pm, \cancel{K}\}$$

Negligible

$\pi^+\pi^- \to \pi^0\pi^0$ → Chemical equilibrium → $Y_{\pi_{\rm DM}} = Y_{\pi^0} + 2Y_{\pi^\pm}$

$$\dot{n} \; + \; 3Hn = -\left(n_\eta n_{\pi_{\rm DM}}\langle\sigma_{\eta\pi}v\rangle - n^2_{\pi_{\rm DM}}\langle\sigma_{\pi\pi}v\rangle\right) \qquad n = n_{\pi_{\rm DM}} + 2n_\eta$$

$$z = m_\pi/T$$
$$Y = n/s$$

Detailed balance $\quad n^{\rm eq}_\eta\langle\sigma_{\eta\pi}v\rangle = n^{\rm eq}_{\pi_{\rm DM}}\langle\sigma_{\pi\pi}v\rangle$

$+$

Chemical equilibrium $\quad n_\eta/n^2_{\pi_{\rm DM}} = (n_\eta/n^2_{\pi_{\rm DM}})_{\rm eq}$

$\eta \leftrightarrow \pi^0\pi^0$

→ $$\frac{dY}{dz} = -\langle\sigma_{\eta\pi}v\rangle\frac{sY_{\eta,{\rm eq}}}{zH}\left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},{\rm eq}}}\right)$$

Boltzmann Equation benchmark model

All $\eta\pi \to \pi\pi$ involving the different pion species must be taken into account

$$\langle\sigma(\eta\pi^0 \to \pi_{1,2}\pi_{1,2})v\rangle = \frac{529\sqrt{5}\, m_{\pi^0}^2}{5184\pi f_\pi^4}\delta, \quad \langle\sigma(\eta\pi_{1,2} \to \pi^0\pi_{1,2})v\rangle = \frac{49\sqrt{5}\ m_{\pi^0}^2}{2592\pi f_\pi^4}\delta, \quad \langle\sigma(\eta\pi^0 \to \pi^0\pi^0)v\rangle = \frac{\sqrt{5}\, m_{\pi^0}^2}{64\pi f_\pi^4}\delta$$

$$\pi^\pm = \frac{\pi_1 \pm i\pi_2}{\sqrt{2}}$$

$$\langle\sigma_{\eta\pi}v\rangle \simeq \frac{1}{3}\Big(\langle\sigma(\eta\pi^0 \to \pi^0\pi^0)v\rangle + 2\langle\sigma(\eta\pi^0 \to \pi_1\pi_1)v\rangle + 2\langle\sigma(\eta\pi_1 \to \pi^0\pi_1)v\rangle\Big)$$

$$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$$

$$\langle\sigma_{\eta\pi}v\rangle = \frac{445\sqrt{5}m_{\pi^0}^2\,\delta(r_{ud})}{5184\pi f_\pi^4}$$

Boltzmann Equations (numerical)

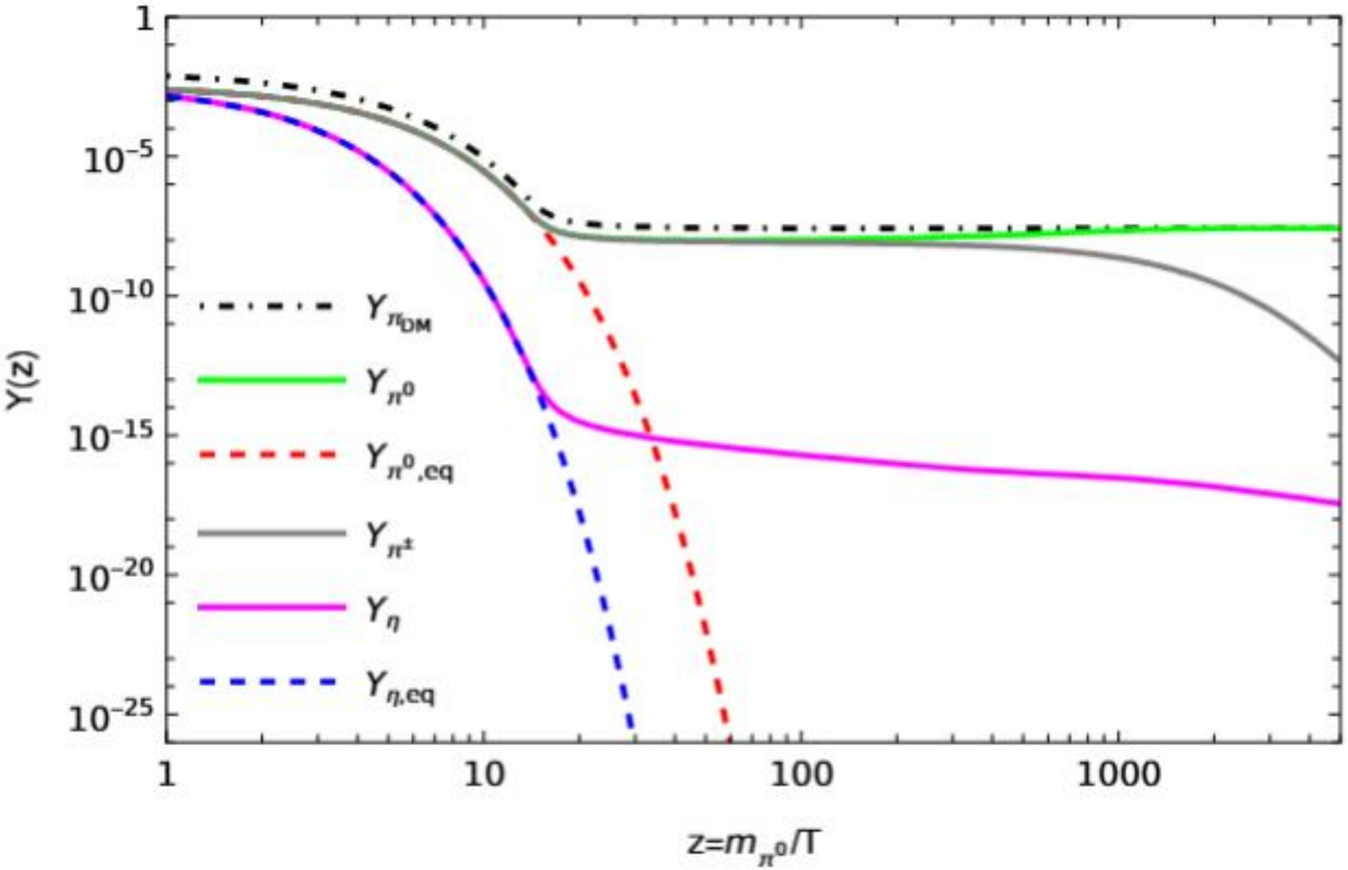
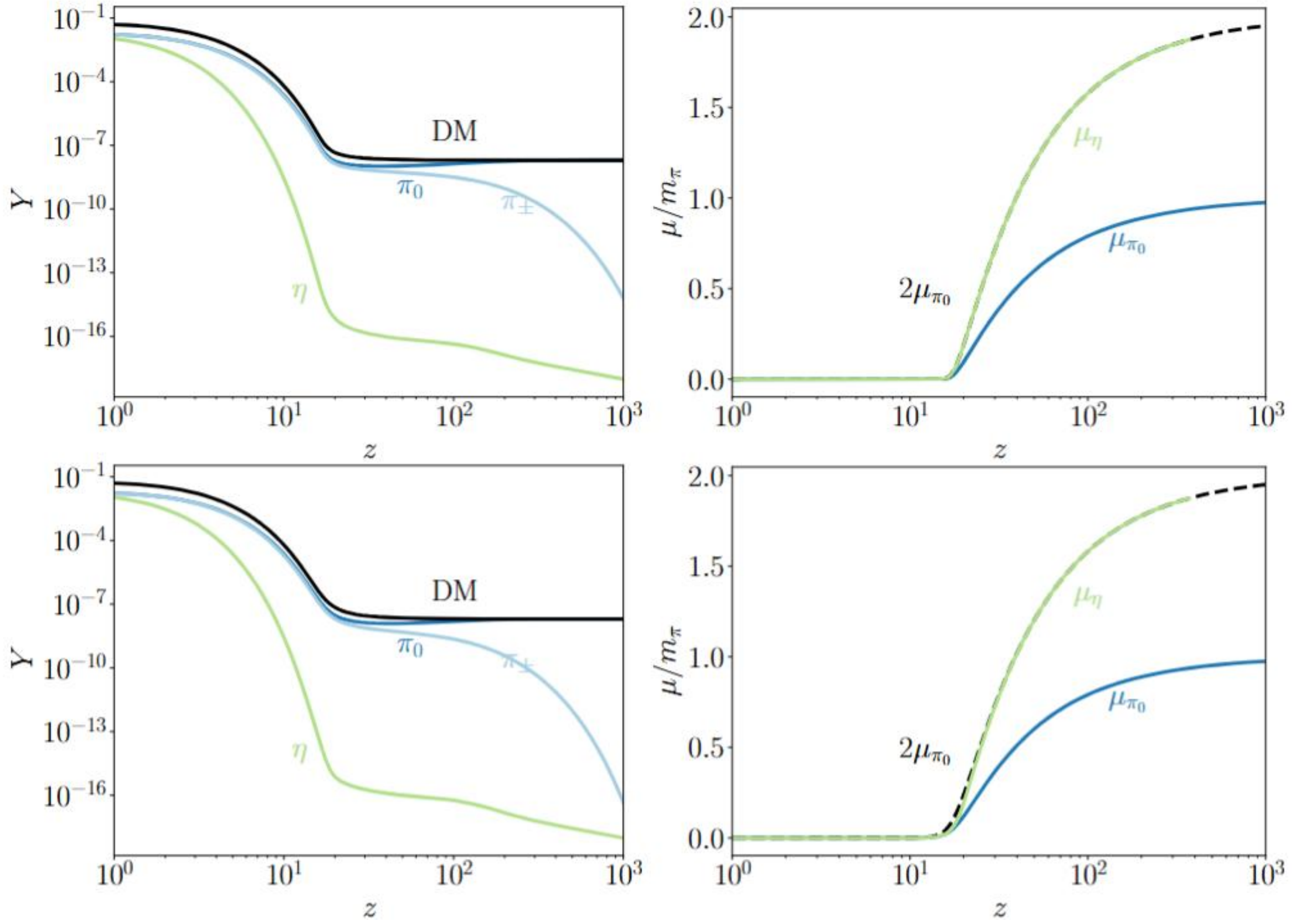
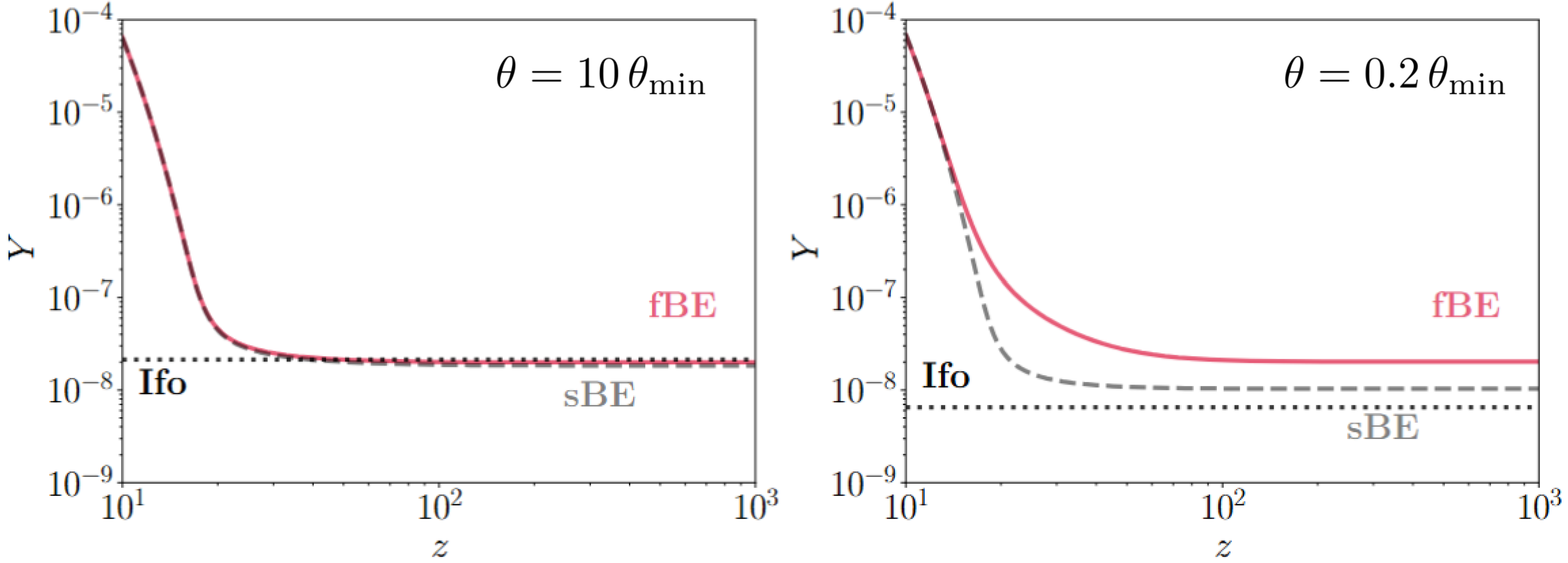


FIG. 1. Yield as function of z for $\pi_{\rm DM}, \pi^0, \pi^\pm, \eta$.

Boltzmann Equations (numerical)



Boltzmann Equations (numerical)



DM self-interactions in halos

$\pi^0\pi^0 \to \pi^0\pi^0$

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2} \frac{\Gamma^2}{m_\pi^2 (v^2 - v_R^2)^2 + 4\Gamma^2 v^2 / v_R^2}$$

$$\sigma_0 = \frac{m_\pi^2}{128\pi f_\pi^4}$$

$\pi^+\pi^- \to \pi^0\pi^0$

Efficient conversions in the Early Universe deplete the $\pi^+\pi^-$ population

Negligible amount of $\pi^\pm$ today in halos

$\pi^0\pi^0 \to \pi^+\pi^-$

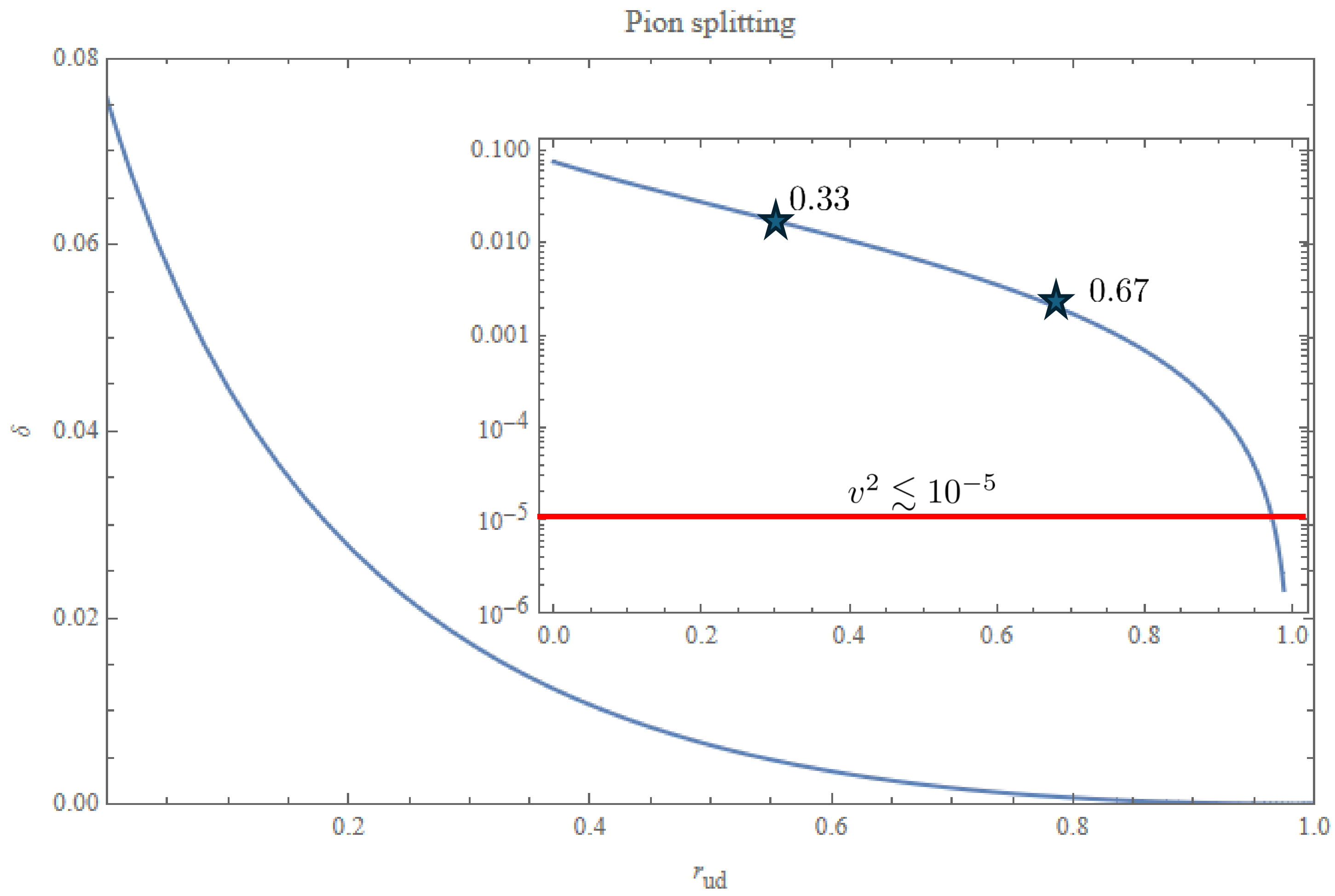
Up-scatterings are kinematically forbidden as $\delta \gg v^2$

$v \lesssim 0.0033 \longrightarrow v^2 \lesssim 10^{-5}$

DM velocity in clusters

$\delta \gtrsim 10^{-5}$ in (almost) all the parameter space (plot)

DM self-interactions in halos

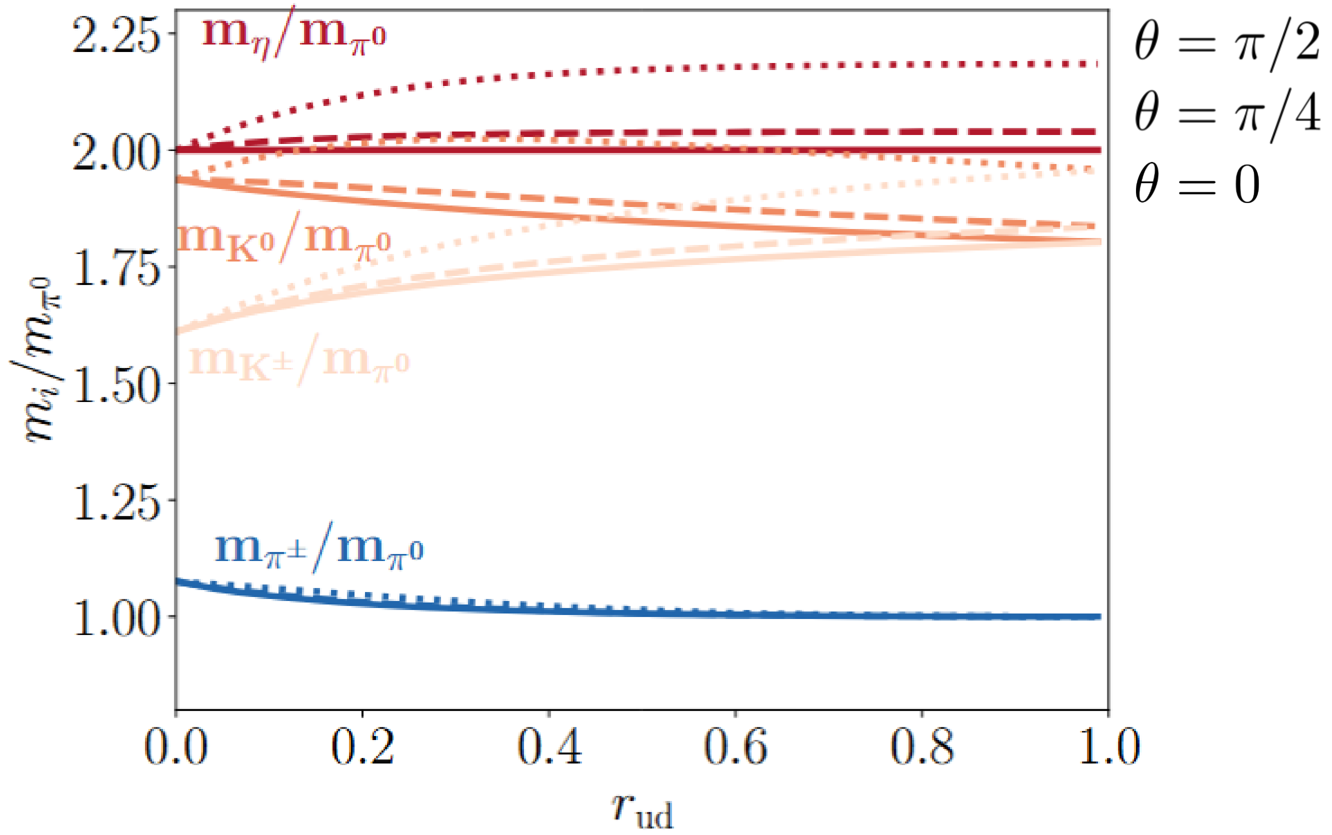


Large theta angle

$$\theta = \sum_i \arcsin\left(\frac{\alpha(\theta)}{m_i}\right)$$

$$m_i \to m_i^{\mathrm{eff}} \equiv \sqrt{m_i^2 - \alpha(\theta)^2}$$

$$\mathcal{L}_{\text{eff, odd terms}} = \frac{B_0\alpha(\theta)}{3f_\pi}\left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e\right)$$



Large theta angle

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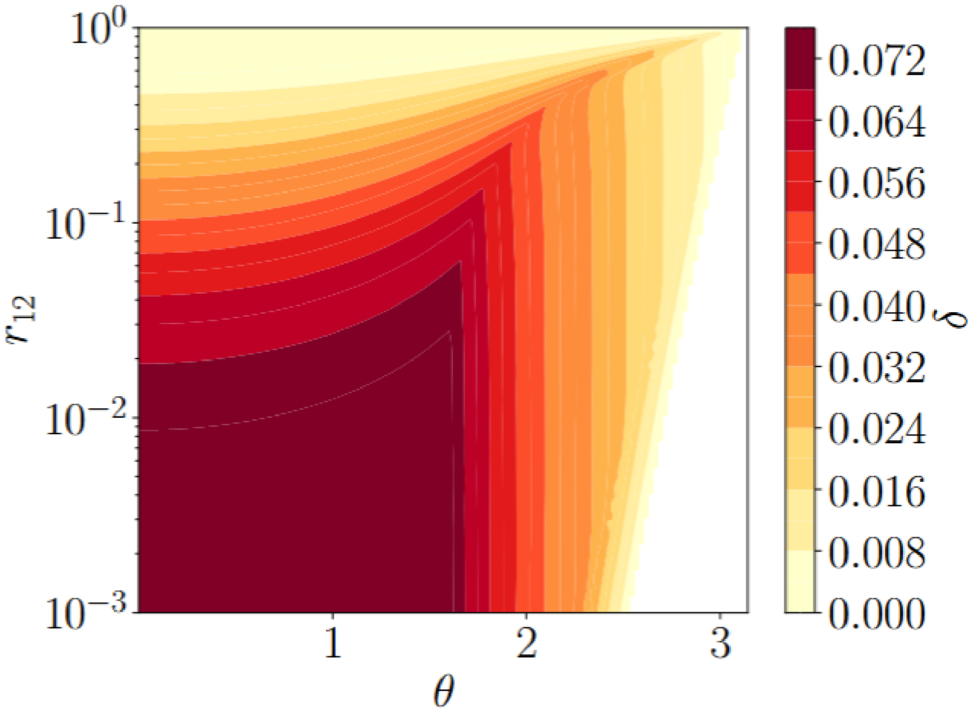
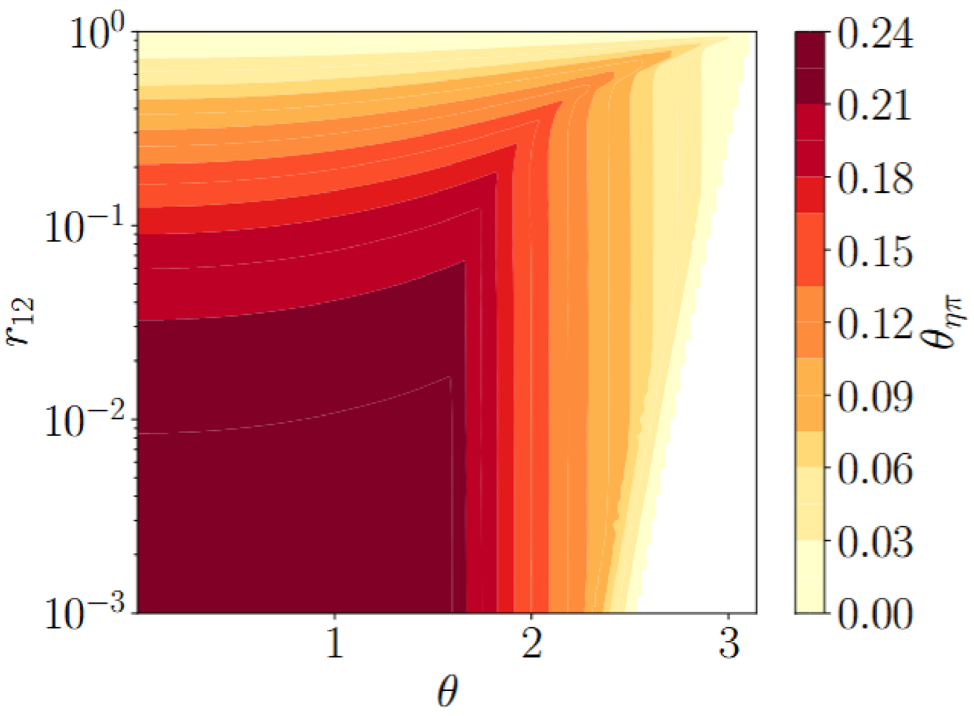
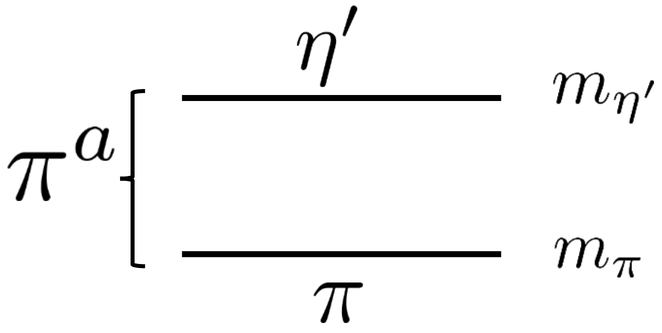


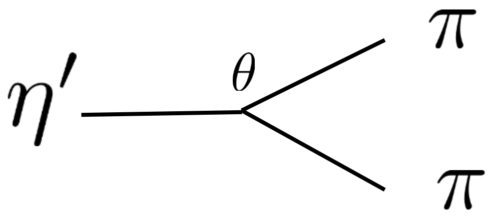


Dark Pion interactions ($\theta \neq 0$)

<u>Alternative choice</u>: it also works with a η'

$$M = \begin{pmatrix} m & & \\ & \ddots & \\ & & m \end{pmatrix} \quad \xrightarrow[+\mathrm{U}(1)_\mathrm{A}\ \text{anomaly}]{(M_\pi^2)_{ab} \propto \mathrm{Tr}\left[M\{\lambda^a \lambda^b\}\right]}$$





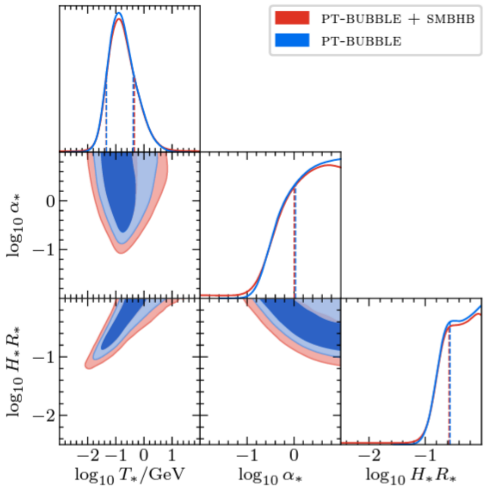
The mass of the η' is determined by U(1) axial anomaly

$\longrightarrow$ practically it is a free parameter

$$m_{\eta'} = \left(2 + \frac{v_R^2}{4}\right) m_\pi$$

Gravitational Waves?

Figure by ***NANOGrav*** *collaboration [2306.16219]*



- Gravitational Waves signal?

Chiral Phase Transition is first-order if $N_f \geq 3$

The PT critical temperature is $T_* \sim f_\pi \sim \mathcal{O}(10-100)\ \mathrm{MeV}$

It could be relevant in view of **PTA signal!**

NANOGrav collaboration 2023

Need to introduce extra d.o.f. to study PT dynamics (e.g. Linear sigma model)

A value $\theta \neq 0$ may deeply alter the PT properties!

SM QCD PT becomes first-order when $\theta \sim \pi$ *Bai, Chen, Korwar (2023)*

DM relic abundance with $\theta \neq 0$

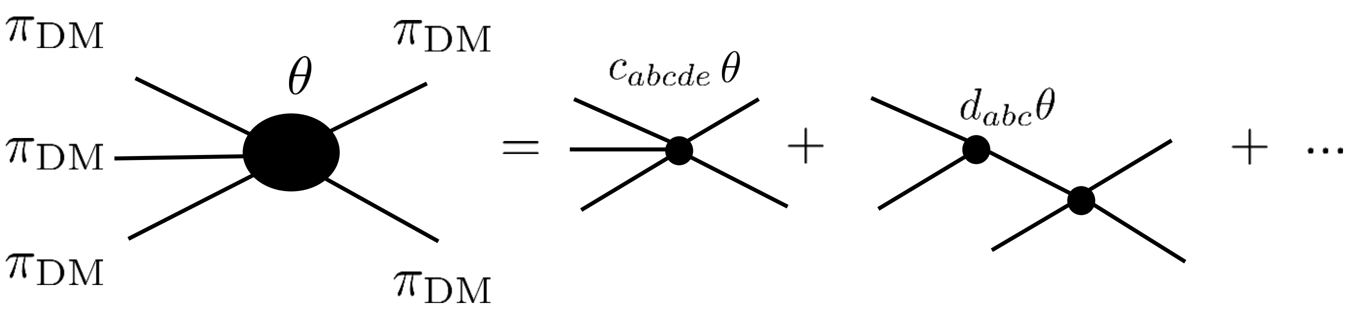
Degenerate quark spectrum gives degenerate pions

$$M = \begin{pmatrix} m & & \\ & \ddots & \\ & & m \end{pmatrix}$$

$$m_\pi^2 = 2B_0 m$$

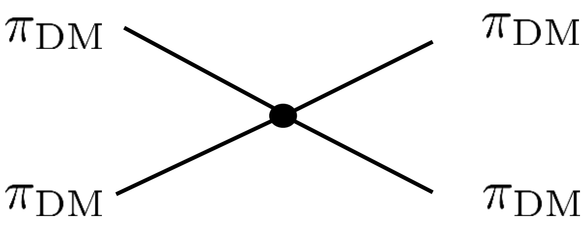
$$\mathcal{L}_\theta = \frac{B_0\theta}{3f_\pi Tr M^{-1}}\left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e\right)$$

DM number changing processes



SIMP $\langle \sigma_{32} v^2 \rangle \propto \theta^2 \frac{m_\pi}{f_\pi^6}$

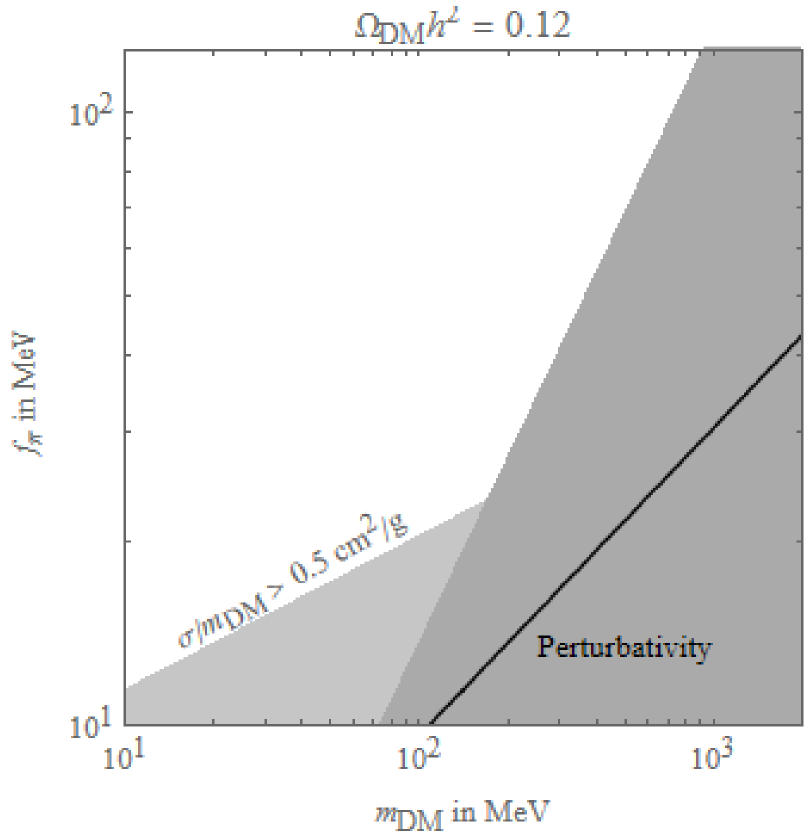
DM self-interactions



$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$ constant

indipendent on θ

DM relic abundance with $\theta \neq 0$



Tension among DM relic and Bullet Cluster bound

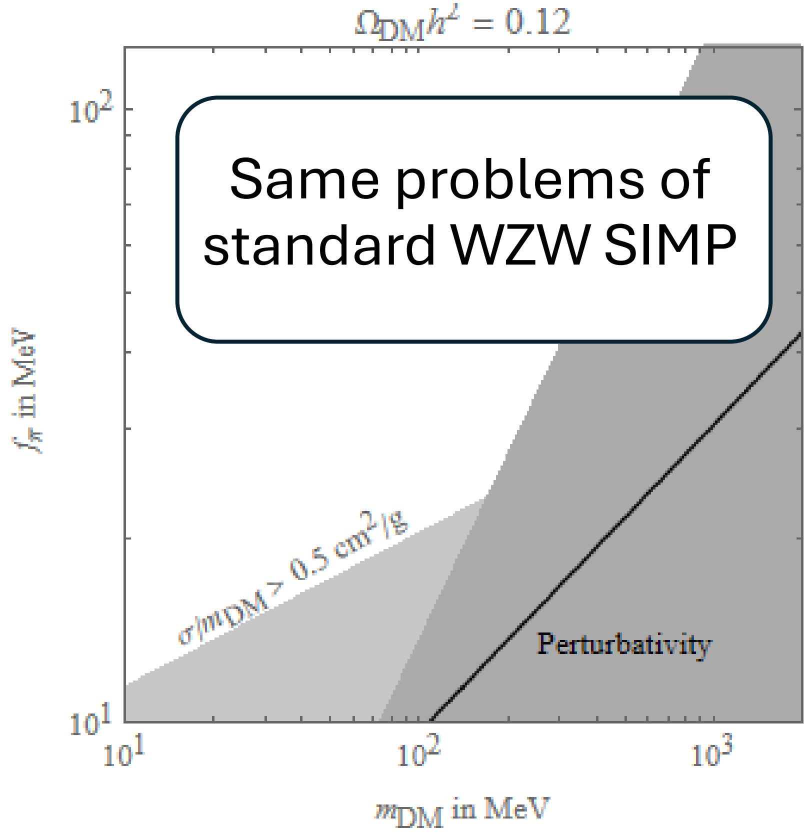
Tension among DM relic and perturbativity

The self-interactions cross section is constant

$$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$$

No SIDM realization

DM relic abundance with $\theta \neq 0$



Tension among DM relic and Bullet Cluster bound

Tension among DM relic and perturbativity

The self-interactions cross section is constant

$$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$$

No SIDM realization

Dark Baryons

Stable because of accidental global $U(1)$

$$m_{\mathcal{B}} \simeq \Lambda$$

$$\overline{\mathcal{B}}\mathcal{B} \to \pi\pi \quad \longrightarrow \quad Y_{\mathcal{B}} \sim \frac{\Lambda}{M_{\rm Pl}}$$

$$\downarrow$$

$$\frac{\Omega_{\mathcal{B}} h^2}{0.11} \simeq \left(\frac{\Lambda}{100\text{TeV}}\right)^2 \ll 1$$

$$\Lambda \sim 4\pi f_\pi/\sqrt{N_c} \sim \mathcal{O}(100\ \text{MeV} - 1\ \text{GeV})$$

Chiral rotation

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

$$q_{L,R} \to e^{\mp i\theta Q/2} q_{L,R} \xrightarrow{\text{anomalous rotation}} \begin{array}{l} \theta \to (1 - TrQ)\theta \\ M \to M_\theta = e^{i\theta Q/2} M e^{i\theta Q/2} \end{array}$$

Choosing $TrQ = 1$ $\longrightarrow$ Remove $F\widetilde{F}$

Choosing $Q = M^{-1}/TrM^{-1}$ $\longrightarrow$ no linear terms in π in the chiral Lagrangian

Chiral rotation

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

More generically one can start from

$$-(\bar{q}_L \underbrace{M e^{i\theta_M}}_{\mathcal{M}} q_R + h.c) + \frac{g^2\theta_F}{32\pi^2}F\widetilde{F}$$

anomalous rotation

$$q_{L,R} \to e^{\mp i\alpha} q_{L,R} \longrightarrow \begin{array}{l} \theta_F \to \theta_F - \alpha N_f \\ \theta_M \to \theta_M + \alpha \end{array}$$

$$\theta \equiv \theta_F + \arg\det\mathcal{M} = \theta_F + N_f\theta_M$$ Invariant

Physical quantity (if all quarks are massive)

$\det\mathcal{M} \neq 0$

Mass spectrum benchmark

$$SU(3)_L \otimes SU(3)_R \xrightarrow{8\pi_a} SU(3)_V$$

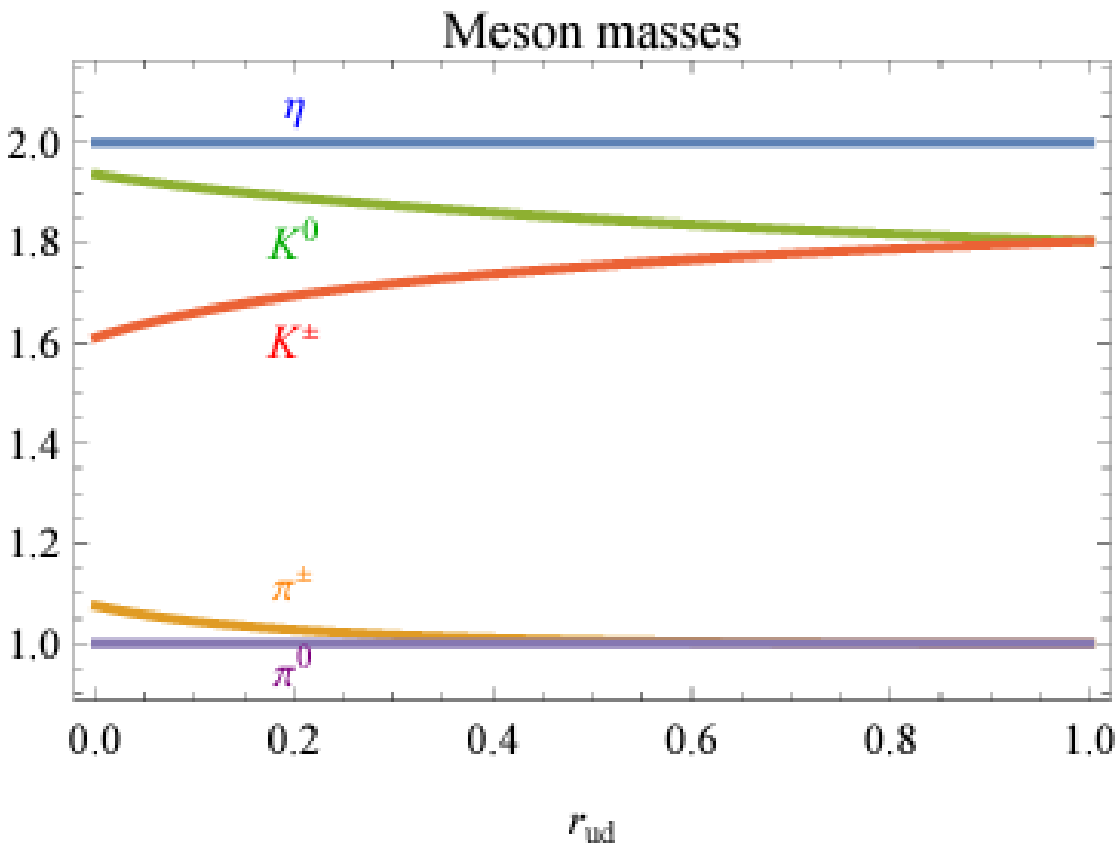
$$\pi^{\pm} = (\pi_1 \pm \pi_2)/\sqrt{2}\ ,\ K^{\pm} = (\pi_4 \pm i\pi_5)/\sqrt{2},\ K^0/\bar{K}^0 = (\pi_6 \pm i\pi_7)/\sqrt{2}$$

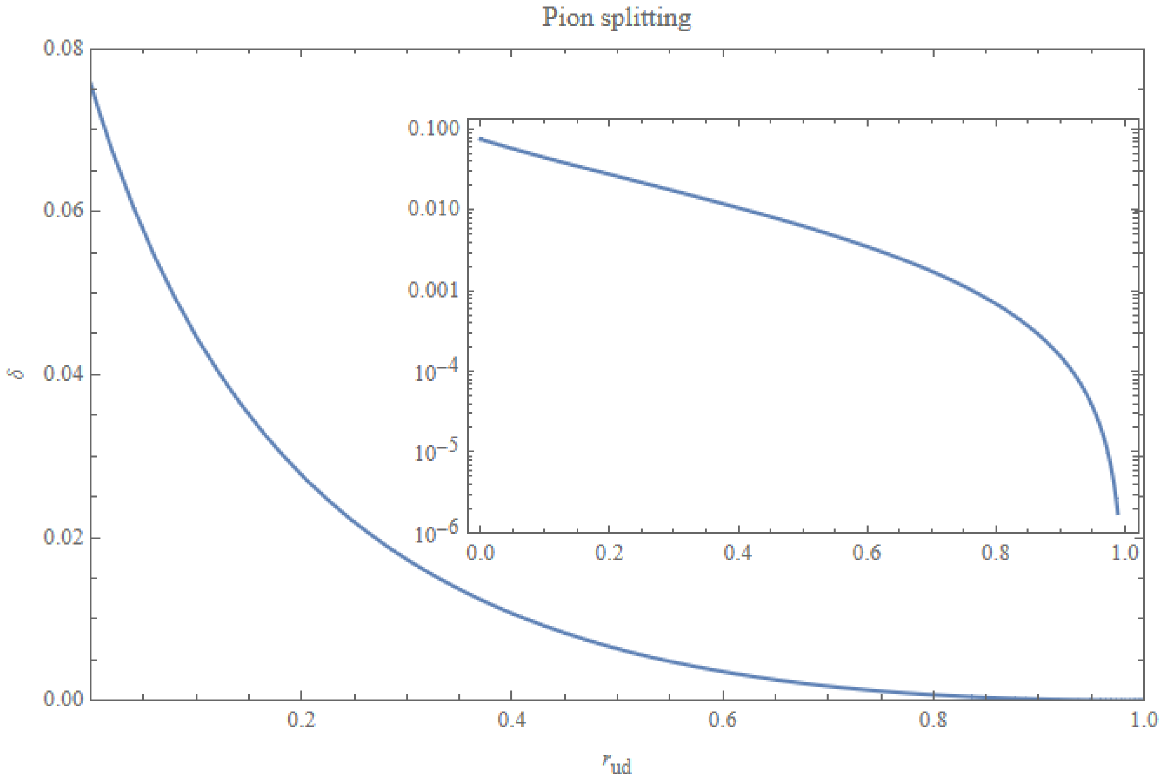
$$\begin{pmatrix} \pi^0 \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\theta_{\eta\pi} & \sin\theta_{\eta\pi} \\ -\sin\theta_{\eta\pi} & \cos\theta_{\eta\pi} \end{pmatrix} \begin{pmatrix} \pi_3 \\ \pi_8 \end{pmatrix}, \qquad \text{with} \qquad \tan(2\theta_{\eta\pi}) = \frac{\sqrt{3}(m_u - m_d)}{(m_u + m_d - 2m_s)}. \tag{B1}$$

The masses squared of the mesons are $m^2_{\pi^\pm} = B_0(m_u + m_d)$, $m^2_{K^\pm} = B_0(m_u + m_s)$, $m^2_{K,\bar{K}^0} = B_0(m_d + m_s)$, while $m^2_{\pi^0}$ and m^2_η are the eigenvalues of

$$\mathcal{M}^2_{\pi^0,\eta} = \begin{pmatrix} B_0(m_u + m_d) & B_0(m_u - m_d)/\sqrt{3} \\ B_0/(m_u - m_d)/\sqrt{3} & B_0(m_u + m_d + 4m_s)/3 \end{pmatrix}. \tag{B2}$$

Mass spectrum benchmark





Cubic interactions

$$\mathcal{L}_{\eta\pi\pi}^{(\mathrm{BM1})} = \frac{B_0\theta}{\sqrt{3}f_\pi \mathrm{Tr}M^{-1}}\cos(3\theta_{\eta\pi})\eta\pi^0\pi^0$$

$$\tan(2\theta_{\eta\pi}) = \frac{\sqrt{3}(m_u - m_d)}{(m_u + m_d - 2m_s)}.$$

$$\Gamma\left(\eta \to \mathrm{DM\,DM}\right) = \frac{\theta^2 B_0^2 \xi}{24\pi f_\pi^2 m_\eta (\mathrm{Tr}M^{-1})^2}\sqrt{1 - \frac{4m_{\mathrm{DM}}^2}{m_\eta^2}}$$

$$\xi = \cos^2 3\theta_{\eta\pi}$$

Details of Symmetry Breaking

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

$$U(1)_V \qquad q_{L,R} \to e^{i\alpha} q_{L,R}$$

$$M \to 0 \quad \longrightarrow \quad \begin{array}{ll} SU(N_f)_L & q_L \to e^{i\alpha_L^a \lambda^a} q_L \\ SU(N_f)_R & q_R \to e^{i\alpha_R^a \lambda^a} q_R \end{array} \quad \xrightarrow{\langle \bar{q}_L q_R \rangle} \quad \begin{array}{c} SU(N_f)_V \\ \alpha_L^a = \alpha_R^a \end{array}$$

$$U(1)_A \qquad q_{L,R} \to e^{\mp i\alpha} q_{L,R}$$

Anomalous! $\longrightarrow$ $N_f \alpha F\widetilde{F}$

Resonances in QCD

$$\frac{m(^8\mathrm{Be}) - 2m(\alpha)}{m(^8\mathrm{Be})} = 0.000012, \qquad \frac{m(^{12}\mathrm{C}^*) - m(^8\mathrm{Be}) - m(\alpha)}{m(^{12}\mathrm{C}^*)} = 0.000026.$$

$$\alpha\alpha \to {}^8\mathrm{Be} \quad \text{followed by} \quad {}^8\mathrm{Be}\,\alpha \to {}^{12}\mathrm{C}^*$$

Important process in stars

Similar to our resonant 3-to-2 processes

Other examples:

$$\frac{m(\phi) - 2m(K^0)}{m(\phi)} = 0.024, \qquad \frac{m(B_{s1}) - m(B^*) - m(K^0)}{m(B_{s1})} = 0.0011,$$

$$\frac{m(D^{0*}) - m(D^0) - m(\pi^0)}{m(D^{0*})} = 0.0035, \qquad \frac{m(\Upsilon(4S)) - 2m(B^0)}{m(\Upsilon(4S))} = 0.0019.$$

Instantons

$$\int d^4x F^a_{\mu\nu} \widetilde{F}^{\mu\nu a} = \int d^4x \partial_\mu K^\mu = \int_{S_3} d\sigma_\mu K^\mu$$

Total derivative

Instantons are (pure-gauge) field configurations which satisfies

$$\int d^4x F^a_{\mu\nu} \widetilde{F}^{\mu\nu a} = \frac{32\pi^2}{g^2}\nu$$

Integer (winding number)

Classical solution to (Euclidean) e.o.m.

Tunnelling among gauge configurations with different winding numbers

Instantons

$$\int d^4x F^a_{\mu\nu}\widetilde{F}^{\mu\nu a} = \int d^4x \partial_\mu K^\mu = \int_{S_3} d\sigma_\mu K^\mu$$

Total derivative

Theta vacuum $$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{in\theta}|n\rangle\,,$$

Vacuum with winding number n

$$\langle\theta_+|\theta_-\rangle_J = \sum_\nu \int \mathcal{D}A\, e^{-\int d^4x\, \frac{1}{4}G\tilde{G} + i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G\tilde{G} + J\text{-term}}\delta\left(\nu - \frac{g_s^2}{32\pi^2}\int d^4x\, G\tilde{G}\right)$$