

PLANCK 2025

Thermal Effects in Freeze-In Dark Matter Production

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arXiv:2506.xxxxx

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DM PRODUCTION MECHANISMS









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Freeze-In (FIMPs)



DM PRODUCTION MECHANISMS







Freeze-in models are sensitive to thermal corrections







How does the thermal plasma affect the freeze-in mechanism?



OUTLINE



How does the thermal plasma affect the freeze-in mechanism?

Thermal masses, modified distribution functions, off-shell effects, thermal widths, **multiple soft scatterings (LPM)**, ...



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VECTORLIKE SCALAR FIMP MODEL









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5 possible realizations of this model: e_L, q_L, e_R, d_R, u_R



VECTORLIKE SCALAR FIMP MODEL







5 possible realizations of this model: e_L, q_L, e_R, d_R, u_R

Parametrized with 4 quantities:

 $y_{\rm DM}$

$$G = Y^2 g_1^2 + C_2(\mathcal{R}_2) g_2^2 + C_2(\mathcal{R}_3) g_3^2$$
$$\delta = \frac{m_F - m_{\rm DM}}{m_{\rm DM}}$$
$$m_F$$



DM RATE EQUATION



CTP Formalism

Fields are defined on a complex time contour that doubly transverses the real-time axis



Advantage: allows for non-equilibrium phenomena



DM RATE EQUATION



CTP Formalism

Fields are defined on a complex time contour that doubly transverses the real-time axis



Equation for the DM production rate from the Kadanoff-Baym and CTP formalism

[Becker, Copello, Harz, Tamarit, 2312.17246]

$$\gamma_{\rm DM} \equiv \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\Pi_s^{\mathcal{A}}(\omega_p, |\vec{p}|)}{\omega_p} f_{\rm B}(\omega_p)$$



PREVIOUS WORK



Calculation with 1PI-resummed propagators provides a more accurate rate for freeze-in

				$(\Omega h^2$	$)^{\rm HTL}/($	$(\Omega h^2)^{\rm fu}$	$^{11} - 1$			
	1.6 - 27%	25%	15%	11%	9%	7%	6%	5%	5%	5%
	1.5 - 26%	21%	14%	11%	8%	7%	6%	5%	4%	4%
	1.4- 26%	20%	14%	10%	8%	6%	5%	4%	4%	2%
	1.3-26%	20%	14%	10%	7%	6%	5%	4%	4%	4%
	1.2- 23%	19%	14%	9%	6%	5%	4%	3%	3%	1%
	1.1- 23%	18%	13%	8%	6%	4%	3%	3%	3%	0%
5	1.0- 22%	18%	12%	8%	7%	4%	3%	3%	3%	0%
	0.9- 22%	17%	12%	8%	6%	4%	3%	3%	3%	0%
	0.8- 21%	16%	12%	7%	6%	6%	4%	3%	4%	0%
	0.7 - 22%	16%	11%	6%	5%	4%	3%	5%	3%	0%
	0.6 - 21%	16%	10%	7%	9%	5%	4%	4%	3%	2%
	0.5 - 20%	15%	9%	6%	6%	7%	5%	7%	3%	1%
	0.4 - 19%	15%	8%	9%	5%	5%	6%	4%	1%	0%
	-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1
				\log_{10}	$_0(m_F/$	$m_{\rm DM}$	(-1)			

 $(\Omega h^2)^{\rm tree}/(\Omega h^2)^{\rm full}-1$

	1.6	-100%	-96%	-81%	-56%	-35%	-22%	-16%	-13%	-12%	-12%
	1.5	-100%	-95%	-79%	-54%	-33%	-21%	-15%	-13%	-12%	-12%
	1.4	-100%	-95%	-78%	-52%	-32%	-20%	-15%	-12%	-11%	-12%
	1.3	-99%	-94%	-76%	-50%	-30%	-19%	-14%	-12%	-11%	-11%
	1.2	-99%	-93%	-74%	-49%	-29%	-18%	-14%	-11%	-11%	-10%
	1.1	-99%	-92%	-72%	-46%	-28%	-17%	-13%	-11%	-10%	-10%
G	1.0	-99%	-91%	-70%	-44%	-25%	-16%	-12%	-10%	-9%	-9%
	0.9	-99%	-89%	-67%	-41%	-23%	-14%	-10%	-9%	-8%	-8%
	0.8	-98%	-87%	-64%	-38%	-21%	-12%	-8%	-7%	-6%	-7%
	0.7	-97%	-85%	-60%	-36%	-19%	-11%	-8%	-5%	-6%	-6%
	0.6	-97%	-82%	-56%	-31%	-14%	-9%	-6%	-4%	-4%	-3%
	0.5	-95%	-78%	-51%	-27%	-13%	-5%	-3%	-1%	-3%	-3%
	0.4	-93%	-73%	-45%	-20%	-10%	-5%	-1%	-2%	-4%	-2%
		-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1
					\log_{10}	$_0(m_F/$	$m_{\rm DM}$	(-1)			

[Becker, Copello, Harz, Tamarit, 2312.17246]



FRAMEWORK



Two key approximations:

• Self-energy truncation:

whether we calculate $\Pi^{\mathcal{A}}_{s}$ at 1-loop, 2-loop...



• Propagator structure:

whether tree-level, HTL, or 1PI propagators are used inside $\Pi^{\mathcal{A}}_{s}$





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FRAMEWORK

PRISMA⁺

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[Becker, Copello, Harz, Tamarit, 2312.17246]

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THE LADDER DIAGRAM







POWER COUNTING



The ladder diagram is a LO process in the coupling constant in the kinematical regime







Discovered by Landau, Pomeranchuk and Migdal (LPM) in the context of bremsstrahlung radiation. [10.1103/PhysRev.103.1811]





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Multiple soft scattering of high-energy gauge bosons with the particles in a hot plasma.





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Studied previously for

fermion self energies

- Leptogenesis [Besak, Bodeker, 1202.1288]
- Fermionic FIMPs [Biondini, Ghiglieri, 2012.09083]

photon self energies

• QGP [Arnold, Moore, Yaffe, hep-ph/0111107]



CALCULATION OF THE LPM RATE



[Anisimov, Besak, Bodeker, 1202.1288]





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n rungs

 \approx



Recursion relation

n+1 rungs





We have calculated the LPM rate in the ultrarelativistic regime. To move to non-relativistic, we use a simple prescription:

$$\gamma_{\rm DM} = \left(\gamma_{\rm DM}^{\rm LPM} - \gamma_{\rm DM}^{\rm LPMBorn}\right) f(m_F) + \gamma_{\rm DM}^{\rm 1PI}$$

[Biondini, Ghiglieri, 2012.09083]









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 $G=1.2,\,\delta=10$





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LPM Rate

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LPM Born rate

Limit of vanishing soft scatterings. Removed to avoid double counting.







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The 1PI rate

Obtained calculating the 1-loop self-energy with 1PI-resummed propagators

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The 1PI rate

Obtained calculating the 1-loop self-energy with 1PI-resummed propagators

Switch-off function

Switches off the unphysical rate. Must satisfy

$$f(\frac{m_F}{T} \to 0) = 1, f(\frac{m_F}{T} \to \infty) = 0$$

JGU C MPA MAINZ PHYSICS ACADEMY







DIFFERENT SWITCH-OFF PROCEDURES



[Ghiglieri, Laine, 1605.07720]

Susceptibility function

- Slower switch-off might lead to DM overestimation
- Doesn't switch-off scatterings fast enough

Thermal function

- + Captures 1PI behaviour
- + Most conservative approach

Smooth interpolation[Ghiglieri, Laine 2110.07149]

- Scatterings must be added manually
- Decay peak is overestimated





Discrepancies of ~30% and ~5% at the Ωh^2 level

THE FINAL DM RATE





We provide a fitting of the LPM in the ultrarelativistic limit $\left.\gamma_{\rm DM}^{\rm LPM}/(y_{\rm DM}^2T^4)\right|_{z=0.01}=a+bG$





 d_R

	1.6	27%	-26%	-23%	-19%	-15%	-12%	-10%	-9%	-9%	-8% -	7.9×10^{2}
	1.5	26%	-25%	-22%	-18%	-14%	-11%	-9%	-8%	-8%	-8% -	1.6×10^{3}
	1.4		-24%	-21%	-17%	-13%	-10%	-9%	-8%	-7%	-7% -	3.7×10^{3}
	1.3		-23%	-20%	-16%	-12%	-10%	-8%	-7%	-7%	-7% -	9.4×10^{3}
	1.2		-23%	-20%	-15%	-11%	-9%	-7%	-7%	-6%	-6% -	2.9×10^{4}
	1.1	23%	-22%	-19%	-14%	-10%	-8%	-7%	-6%	-6%	-5% -	1.1×10^{5}
G	1.0		-21%	-18%	-13%	-10%	-7%	-6%	-5%	-5%	-5% -	5.3×10^{5}
	0.9	21%	-20%	-16%	-12%	-9%	-6%	-5%	-5%	-4%	-6% -	3.7×10^{6}
	0.8	20%	-19%	-15%	-11%	-8%	-6%	-5%	-5%	-5%	-5% -	4.4×10^{7}
	0.7	19%	-18%	-14%	-10%	-7%	-5%	-4%	-3%	-3%	-3% -	1.1×10^{9}
	0.6		-16%	-13%	-9%	-6%	-4%	-4%	-4%	-3%	-3% -	8.5×10^{10}
	0.5	17%	-15%	-11%	-7%	-5%	-4%	-3%	-4%	-2%	-2% -	2.8×10^{13}
	0.4		-13%	-9%	-6%	-4%	-2%	-4%	-2%	-6%	-7% -	9.6×10^{15}
		-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1	
		-	.15	-15	log	(m_E)	m	(-1)	- / -			
					-0810	$\langle n r F \rangle$	T DM	-)				

DM Relic Abundance Ωh^2

The amount of DM that is underestimated if one does not take the LPM into account

- Greater *G* leads to bigger LPM effect
- Smaller δ leads to bigger LPM effect (no decay contribution)
- Five possible realizations of DM mediator



 $m_F[\text{GeV}]$



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											1	
1.6	27%	-26%	-23%	-19%	-15%	-12%	-10%	-9%	-9%	-8% -	7.9×10^{2}	
1.5	26%	-25%	-22%	-18%	-14%	-11%	-9%	-8%	-8%	-8% -	1.6×10^{3}	
1.4	26%	-24%	-21%	-17%	-13%	-10%	-9%	-8%	-7%	-7% -	3.7×10^{3}	
1.3		-23%	-20%	-16%	-12%	-10%	-8%	-7%	-7%	-7% -	9.4×10^{3}	
1.2	24%	-23%	-20%	-15%	-11%	-9%	-7%	-7%	-6%	-6% -	2.9×10^{4}	
1.1	23%	-22%	-19%	-14%	-10%	-8%	-7%	-6%	-6%	-5% -	1.1×10^{5}	7
1.0		-21%	-18%	-13%	-10%	-7%	-6%	-5%	-5%	-5% -	5.3×10^{5}	$_{F}[GeV$
).9	21%	-20%	-16%	-12%	-9%	-6%	-5%	-5%	-4%	-6% -	3.7×10^{6}	m
).8	20%	-19%	-15%	-11%	-8%	-6%	-5%	-5%	-5%	-5% -	4.4×10^{7}	
0.7		-18%	-14%	-10%	-7%	-5%	-4%	-3%	-3%	-3% -	1.1×10^{9}	
0.6	18%	-16%	-13%	-9%	-6%	-4%	-4%	-4%	-3%	-3% -	8.5×10^{10}	
).5	-17%	-15%	-11%	-7%	-5%	-4%	-3%	-4%	-2%	-2% -	2.8×10^{13}	
).4		-13%	-9%	-6%	-4%	-2%	-4%	-2%	-6%	-7% -	9.6×10^{15}	
	-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1		
				\log_{10}	(m_F)	$m_{\rm DM}$	-1)					

5



u_R

	1.6	26%	-25%	-22%	-18%	-14%	-11%	-10%	-9%	-8%	-8% -	1.3×10^{3}	
	1.5		-24%	-21%	-17%	-13%	-10%	-9%	-8%	-8%	-7% -	2.9×10^{3}	
	1.4		-23%	-20%	-16%	-12%	-10%	-8%	-7%	-7%	-7% -	7.3×10^{3}	
	1.3	24%	-22%	-19%	-15%	-12%	-9%	-8%	-7%	-6%	-6% -	2.1×10^{4}	
	1.2	23%	-21%	-18%	-14%	-11%	-8%	-7%	-6%	-6%	-6% -	7.6×10^{4}	
7.	1.1		-20%	-17%	-13%	-10%	-7%	-6%	-5%	-5%	-5% -	3.5×10^{5}	GeV]
0	1.0	21%	-19%	-16%	-12%	-9%	-7%	-5%	-5%	-5%	-4% -	2.3×10^{6}	$m_F[0$
	0.9	20%	-18%	-15%	-11%	-8%	-6%	-5%	-4%	-4%	-4% -	2.5×10^{7}	
	0.8		-17%	-14%	-10%	-7%	-5%	-4%	-4%	-3%	-5% -	5.8×10^{8}	
	0.7		-16%	-13%	-9%	-6%	-4%	-3%	-4%	-5%	-3% -	4.2×10^{10}	
	0.6		-15%	-11%	-8%	-5%	-4%	-3%	-2%	-5%	-2% -	1.7×10^{13}	
	0.5		-13%	-10%	-6%	-4%	-4%	-2%	-2%	-2%	-6% -	$9. \times 10^{15}$	
		-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1		
					log	(m-)	mpy-	- 1)	0.000	1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 -			
					10g ₁₀	(mF)	mDM	- 1)					
						-							

 q_L

	0.8	17%	-16%	-13%	-9%	-6%	-5%	-4%	-3%	-3%	-5%	1.9×10 ¹³
	0.9		-17%	-14%	-10%	-7%	-5%	-4%	-4%	-4%	-3% -	1.2×10^{11}
	1.0	19%	-18%	-15%	-11%	-8%	-6%	-5%	-4%	-4%	-4% -	2.1×10^{9}
5	1.1	20%	-18%	-16%	-12%	-9%	-7%	-5%	-5%	-5%	-5%	9.2×10^{7}
	1.2	20%	-19%	-17%	-13%	-9%	-7%	-6%	-5%	-5%	-5% -	7.7×10^{6}
	1.3		-20%	-18%	-14%	-10%	-8%	-7%	-6%	-6%	-5% -	$1. \times 10^{6}$
	1.4		-21%	-18%	-15%	-11%	-9%	-7%	-7%	-6%	-6%	1.9×10^{5}
	1.5	23%	-22%	-19%	-15%	-12%	-9%	-8%	-7%	-7%	-7% -	4.8×10^{4}
	1.6		-23%	-20%	-16%	-13%	-10%	-9%	-8%	-7%	-7% -	1.5 × 10 [*]



COMPARISON TO SIMPLIFIED METHODS

			$(\Omega$	$(h^2)^{\mathrm{d}}$	ec,th.r	^{n.} /($(2h^2)^{\mathrm{f}}_{a}$	${}^{\mathrm{full}}_{dR}$									(Ωh^2)	$^{2})^{\mathrm{dec}}$	+scat,t	$^{\mathrm{th.m.}}/$	(Ωh^2)	$^{2})_{dR}^{\mathrm{full}}$				
1.6	-100%	-96%	-81%	-56%	-33%	-19%	-12%	-8%	-8%	-7% -	7.9×10^{2}		1.	63	3% -	-31%	-24%	-8%	7%	16%	19%	20%	21%	21% -	7.9×10^{2}	
1.5	-100%	-95%	-79%	-53%	-31%	-17%	-10%	-8%	-6%	-6% -	1.6×10^{3}		1.	5 3	3% -	-32%	-22%	-6%	8%	16%	20%	21%	20%	21% -	1.6×10^{3}	
1.4	-99%	-95%	-77%	-51%	-29%	-15%	-9%	-6%	-5%	-6% -	3.7×10^{3}		1.	4 3	2% -	-30%	-20%	-5%	10%	17%	20%	21%	21%	20% -	3.7×10^{3}	
1.3	99%	-94%	-75%	-48%	-26%	-13%	-7%	-4%	-3%	-3% -	9.4×10^{3}		1.	33	2% -	-30%	-19%	-2%	11%	18%	20%	22%	22%	22% -	9.4×10^{3}	
1.2	99%	-93%	-73%	-46%	-24%	-12%	-5%	-4%	-2%	-2% -	2.9×10^{4}		1.	23	2% -	-29%	-17%	-1%	12%	19%	21%	21%	22%	22% -	2.9×10^{4}	
1.1	99%	-91%	-70%	-42%	-21%	-9%	-4%	-2%	-1%	-1% -	1.1×10^{5}	[\ \	1.	1 3	1% -	-28%	-15%	1%	14%	20%	21%	22%	23%	22% -	1.1×10^5	
1.0	98%	-90%	-67%	-38%	-17%	-7%	-2%	-1%	1%	1% -	5.3×10^{5}	7,0[Ge	U 1.	03	2% -	-27%	-13%	4%	17%	21%	22%	23%	23%	23% -	5.3×10^5	F,0[G(
0.9	-98%	-88%	-63%	-34%	-14%	-4%	0%	2%	2%	1% -	3.7×10^{6}	m_I	0.	93	0% -	-25%	-10%	7%	18%	23%	24%	24%	23%	22% -	3.7×10^{6}	ш Ш
0.8	-98%	-86%	-59%	-30%	-10%	0%	4%	4%	4%	2% -	4.4×10^{7}		0.	82	9% -	-23%	-7%	10%	21%	25%	26%	25%	24%	22% -	4.4×10^{7}	
0.7	97%	-83%	-54%	-25%	-7%	2%	4%	8%	7%	7% -	1.1×10^{9}		0.	7 2	8% -	-20%	-4%	12%	22%	24%	25%	27%	26%	25% -	1.1×10^{9}	
0.6	96%	-79%	-48%	-19%	1%	6%	7%	8%	9%	10% -	8.5×10^{10}		0.	62	6% -	-16%	1%	17%	28%	28%	26%	25%	27%	28% -	8.5×10^{10}	
0.5	94%	-74%	-40%	-13%	3%	11%	12%	12%	12%	12% -	2.8×10^{13}		0.	52	4% -	-12%	6%	21%	28%	31%	30%	28%	26%	27% -	2.8×10^{13}	
0.4	91%	-65%	-31%	-2%	7%	13%	13%	14%	7%	7% -	9.6×10^{15}		0.	42	1%	-6%	12%	29%	29%	31%	28%	28%	19%	19% -	9.6×10^{15}	
	-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1				-	-1	-7/9	-5/9	-1/3	-1/9	1/9	1/3	5/9	7/9	1		
			j	$\log_{10}($	$(m_{F,0})$	$/m_{ m DM}$	(1 - 1)]	$\log_{10}($	$(m_{F,0})$	$/m_{\rm DM}$	(-1)					



0

CONCLUSIONS AND OUTLOOK





- Multiple soft scatterings give up to a 27% (8%) contribution for mass splittings of $\delta = 0.1 (\delta = 10)$
- We calculate for the first time the LPM effect for scalar DM
- We provide the most accurate state-of-art calculation









RATE EQUATION



Equation for the DM production rate from the Kadanoff-Baym and CTP formalism

$$\frac{d}{dt}n_{s,\rm ph}(t) + 3Hn_{s,\rm ph} = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\Pi_s^{\mathcal{A}}(\omega_p, |\vec{p}|)}{\omega_p} f_{\rm B}(\omega_p) \equiv \gamma_{\rm DM}$$



The self-energy is given by the functional derivative of the 2PI effective action

$$\Pi^{ab}(x,y) = iab \frac{\delta \Gamma_2[\Delta,S]}{i\delta \Delta^{ba}(y,x)}$$





STRATEGY OF THE CALCULATION





- 1. In the CTL regime, relate the n-point function to the (n-1)-point function where only one gauge boson was removed
- 2. Define a current by integrating over all external momenta and contracting with external fields
- 3. Integrate out the soft gauge boson background, so that external gauge bosons appear as rungs or in the self-energy insertions

$$\begin{split} \hat{\Pi}_{\phi}(P,k_{||},\vec{k}_{\perp}) &= \frac{1}{\epsilon_{ab}(P,\vec{k})} \left\{ d(r)\mathcal{C}_{\phi}(K,P) \right. \\ &+ ig^2 C_2(r)T \int \frac{\mathrm{d}^2 q_{\perp}}{(2\pi)^2} \mathcal{K}(\vec{q}_{\perp}) \left[\hat{\Pi}_{\phi}(P,k_{||},\vec{k}_{\perp}) - \hat{\Pi}_{\phi}\left(P,k_{||},\vec{k}_{\perp} - \vec{q}_{\perp}\right) \right] \right\} \end{split}$$



KINEMATICS



External and internal momenta are collinear and in the near-lightcone scale.

$$p \sim T, p^2 \sim g^2 T^2, p \cdot k \sim g^2 T^2$$



We can define the lightlike coordinates through a lightlike vector $V^{\mu} = (1, \hat{v})$ $p_{\parallel}, k_{\parallel} \sim \mathcal{O}(T)$ $\vec{p}_{\perp}, \vec{k}_{\perp} \sim \mathcal{O}(gT)$

Finite temperature analogue of Soft Collinear Effective Theory (SCET) $p_{\pm} = p_0 \pm p_{\parallel}$ $p^{\mu} \sim (p_+, p_-, \vec{p}_{\perp})$ $p_+ \sim T, \vec{p}_{\perp} \sim gT, p_- \sim g^2T$





	Y	SU(2)	SU(3)	G	$\mu = M_Z$	$10^4 { m GeV}$	$10^7 { m ~GeV}$	$10^{10} { m GeV}$
e_L	-1/2	2	1	$\frac{g_1^2}{4} + \frac{3g_2^2}{4}$	0.38	0.4	0.46	0.52
q_L	+1/6	2	3	$\frac{g_1^2}{36} + \frac{3g_2^2}{4} + \frac{4g_3^2}{3}$	2.3	1.6	1.2	1.0
e_R	-1	1	1	g_1^2	0.21	0.22	0.24	0.26
u_R	+2/3	1	3	$\frac{4g_1^2}{9} + \frac{4g_3^2}{3}$	2.1	1.3	0.9	0.7
d_R	-1/3	1	3	$\frac{g_1^2}{9} + \frac{4g_3^4}{3}$	2.0	1.2	0.8	0.6





If all masses are thermal masses,

$$k^2 = m_F^2 = (p+q)^2 = m_{\rm DM}^2 + m_f^2 + 2pq$$

then the emission occurs collinearly $\theta \sim g$

 $pq = |\vec{p}| |\vec{q}| \left(1 - \cos \theta\right) \sim g^2 T^2$

The mean free time between collisions is of the same order as particle formation. It is impossible to treat emission and scatterings as independent processes.







PRESCRIPTIONS. RATE WITHOUT SCATTERINGS







PRESCRIPTIONS. RATE WITHOUT LPM.











