University of Glasgow
VIA VERITAS VITA

Lepton Flavour Asymmetries: from the early Universe to BBN

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Based on:

Valerie Domcke, Miguel Escudero, MFN, Stefan Sandner [2502.14960] hep-ph

The (flavour) asymmetric Universe

- Matter asymmetry in the early Universe is related to a difference between the number densities of matter and anti-matter.

- In the case of thermal equilibrium, the asymmetry is parameterised in the form of a chemical potential μ:

$$f_{\mathrm{FD}} = \frac{1}{e^{\frac{E \mp \mu}{T}} + 1}$$

$$\xi \equiv \frac{\mu}{T}$$

Comoving chemical potential

$$n = g \int \frac{d^3k}{(2\pi)^3} f(k, T)$$

$$\Delta n = \frac{n - \bar{n}}{T^3} = \frac{1}{6}\left(\xi + \frac{\xi^3}{\pi^2}\right)$$

Comoving asymmetry

for FD distribution

The (flavour) asymmetric Universe

- Baryon asymmetry is small: $$B \sim 10^{-10}$$ $$B, L \equiv \frac{n_{B,L} - n_{\bar{B},\bar{L}}}{s}$$

- Asymmetries in charged leptons similarly constrained by charge neutrality, but neutrino asymmetries could potentially be larger.

- However, if generated before EWPT, then sphalerons erase $B + L$ but conserve $B - L$

$$\longrightarrow L \approx -B \approx -10^{-10}$$

The (flavour) asymmetric Universe

- Baryon a… $B \sim 10^{-10}$ … $\frac{n_{B,L} - n_{\bar{B},\bar{L}}}{s}$
- Asymmet… …out neutrino asymmetr…
- However, … …rve $B - L$

- BUT larger flavour asymmetries are lesser constrained as long as $B - L$ is preserved.

- For lepton flavour asymmetries with total $L \approx 0$, the flavour dependence of sphalerons imposes much weaker bounds:

$$L_\tau \lesssim 10^{-5} \qquad L_e = -L_\mu \lesssim 10^{-3} \longrightarrow \text{Leptoflavourgenesis}$$

… [Mukaida, Schmitz, Yamada, '21]

The (flavour) asymmetric Universe

- If generated much above EWPT, they are strongly bounded $|\xi_{\nu_\alpha}|_{T \gtrsim 10^6\ \mathrm{GeV}} \lesssim 0.01$ [Domcke *et al,* '22]

- However, if generated after EWPT, then lepton flavour asymmetries are essentially unconstrained (up to BBN times).

Production mechanism e.g. via freeze-in leptogenesis, resonant leptogenesis ...

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- Such large lepton flavour asymmetries invoked for BSM model building, including:

  - 1st order QCD phase transition [Gao and Oldengott '21, Gao *et al* '24].

  - Dark matter production e.g. [Stuke *et al*, '11], including sterile neutrino dark matter, see e.g. [Shi and Fuller '99, Shaposnikov and Smirnov '23]

  - Detection prospects of the cosmic neutrino background [Stodolski '75, Duda *et al* '01] and may leave a trace in Gravitational wave backgrounds [Seto '06, Domcke *et al*, '19]

  - EMPRESS Helium anomaly suggest $\xi_{\nu_e}^{\mathrm{BBN}} = 0.043 \pm 0.015$.

The (flavour) asymmetric Universe

- If lepton flavour asymmetries survive until neutrino decoupling and BBN times ($T \lesssim 1$ MeV), then they affect the neutron to proton ratio and $N_{\rm eff}$.

- Assuming thermal equilibrium:

$$p + e^- \rightleftarrows n + \nu_e \qquad p + \overline{\nu}_e \rightleftarrows n + e^+ \longrightarrow \frac{n_n}{n_p} \approx \left.\frac{n_n}{p_n}\right|_{\xi_e=0} e^{-\xi_e}$$

$$N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\nu}{\rho_\gamma} \qquad \Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm SM} = \sum_\alpha \frac{30}{7}\left(\frac{\xi_\alpha}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi_\alpha}{\pi}\right)^4$$

- This translates into the 95% CL bounds:

$$\boxed{|\xi_e|_{T=T_{\rm BBN}} \lesssim 0.03 \qquad \Delta N_{\rm eff} \lesssim 0.23}$$

[Pitrou *et al* '18 …] [Planck '18]

The (flavour) asymmetric Universe

- If le … eV), then

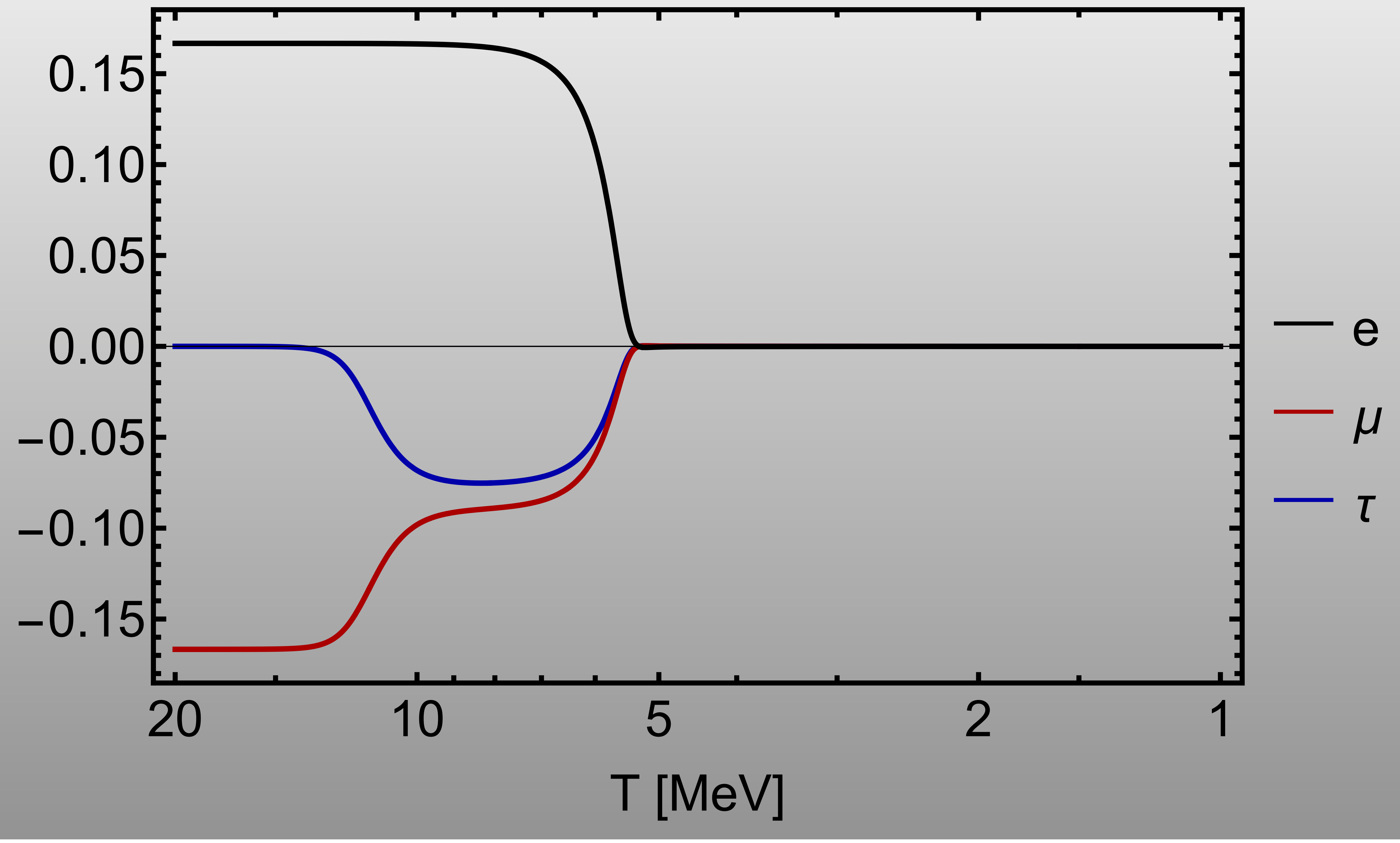
- Assu … $-\xi_e$

N_{ef} … $\Big)^4$

- Thi

- But at 20 MeV neutrino oscillations become effective and may lead to flavour equilibration, such that at BBN times $\xi_\alpha \approx 0$ if $L \approx 0$.

$$\Delta n_\alpha = \frac{(n-\bar{n})_{\alpha\alpha}}{T^3}$$



$$\Delta n^{\text{final}} \approx \frac{\Delta n_e^{\text{ini}} + \Delta n_\mu^{\text{ini}} + \Delta n_\tau^{\text{ini}}}{3} \approx 0$$

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[Pitrou *et al* '18 …] [Planck '18]

The (flavour) asymmetric Universe

- However, pioneering work suggested that flavour equilibration may not be perfect, revealing a strong sensitivity to θ_{13} which was poorly measured at the time: [Dolgov *et al* '02, Pastor *et al* '08, Mangano *et al* '10, Castorina *et al* '12…]

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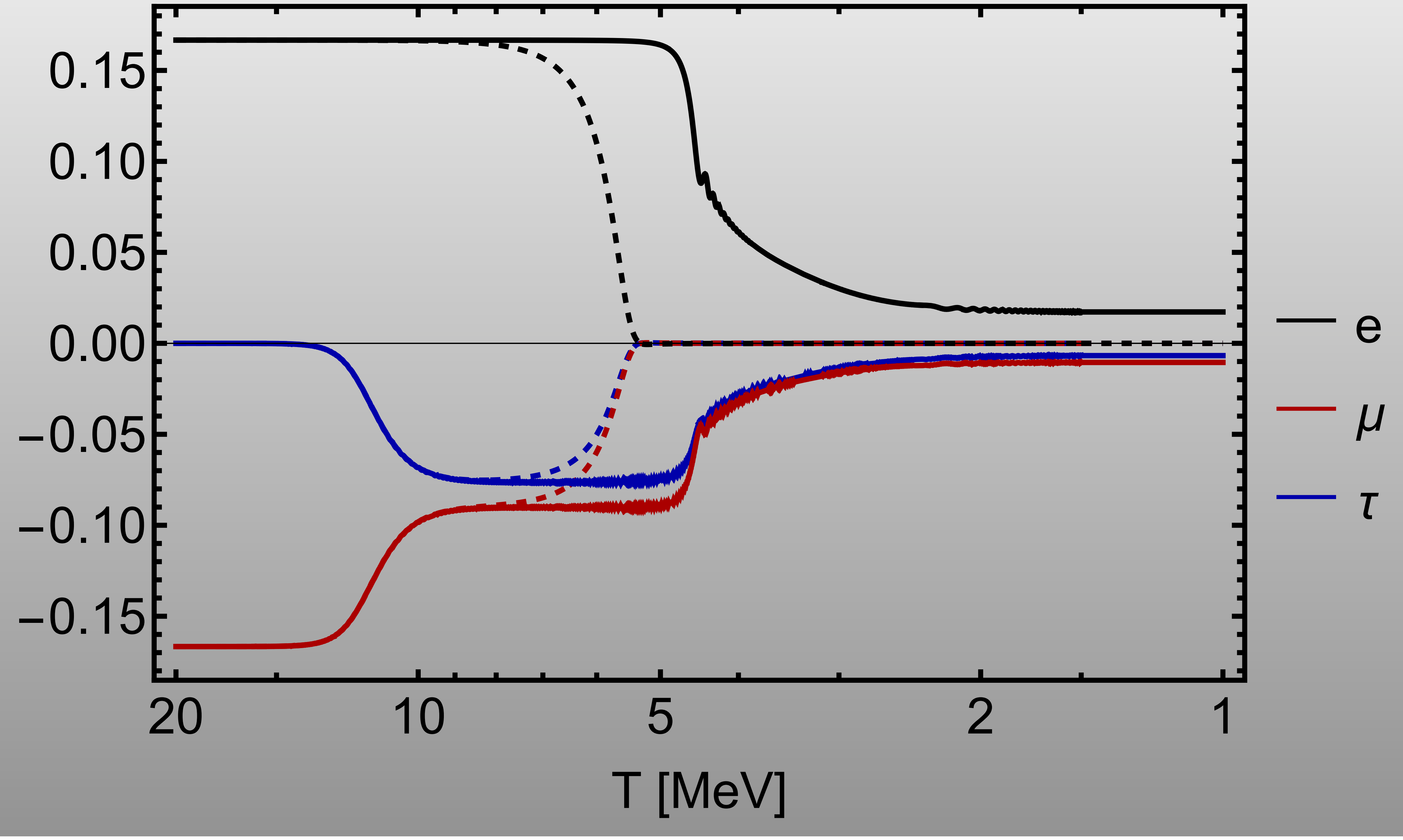

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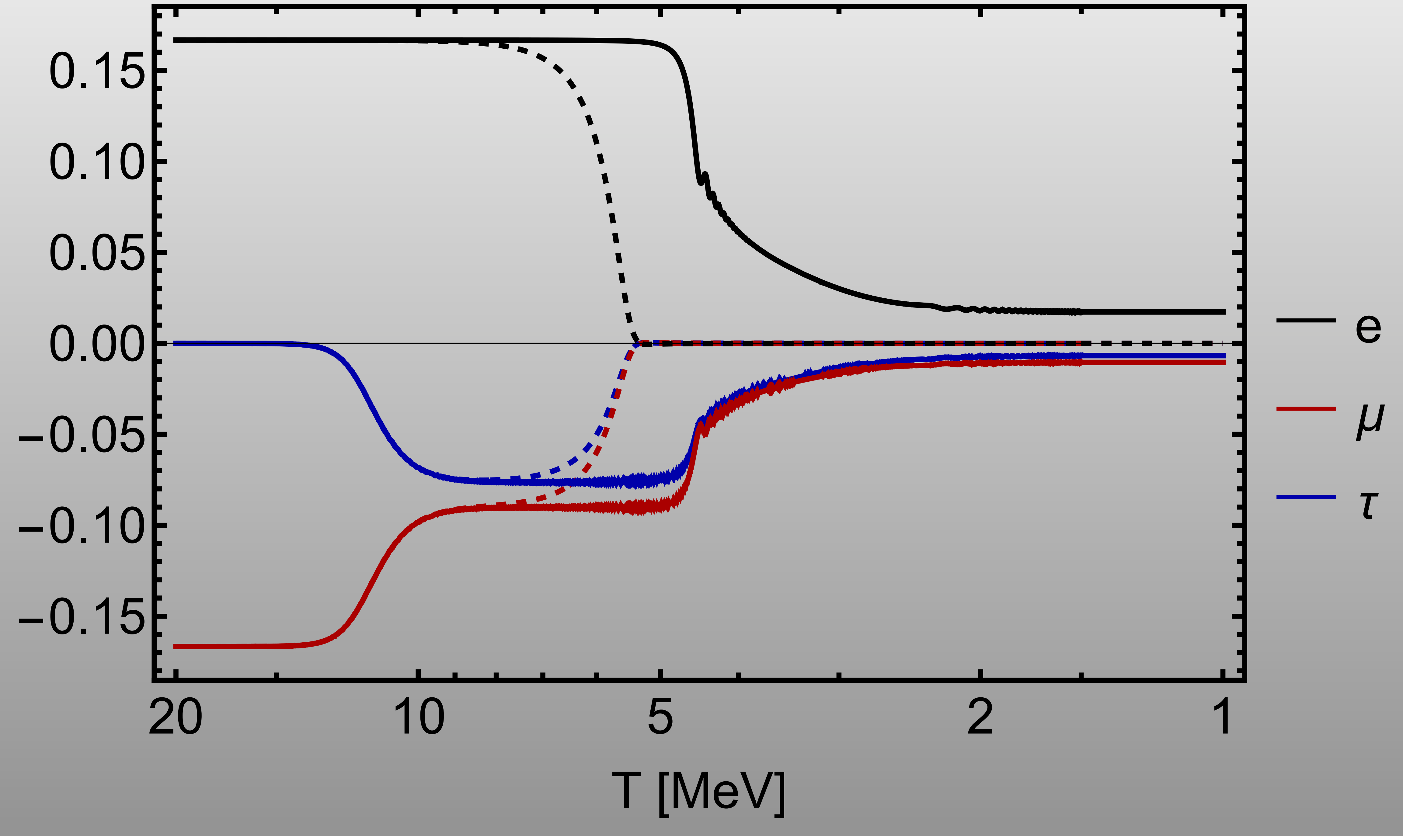
- Can we actually map the asymmetries at 20 MeV to the asymmetries at 1 MeV and systematically explore the parameter space of the $L \approx 0$ case ?

$$\Delta n_\alpha = \frac{(n-\bar{n})_{\alpha\alpha}}{T^3}$$



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[Pitrou *et al* '18 …] [Planck '18]

Quantum Kinetic Equations

$$\boxed{\frac{d\rho}{dt} = -i\,[\mathcal{H}_0 + V_c + V_s, \rho] + \mathcal{I}(\rho)\,, \qquad \frac{d\bar{\rho}}{dt} = +i\,[\mathcal{H}_0 + V_c - V_s, \bar{\rho}] + \bar{\mathcal{I}}(\bar{\rho})\,,}$$

[Sigl and Raffelt '93 …]

$$\rho = \begin{pmatrix} f_{\nu_e} & \rho_{e\mu} & \rho_{e\tau} \\ \rho_{\mu e} & f_{\nu_\mu} & \rho_{\mu\tau} \\ \rho_{\tau e} & \rho_{\tau\mu} & f_{\nu_\tau} \end{pmatrix}$$

- System of integro-differential equations for the neutrino density matrix, including neutrino distribution functions and flavour correlations.
- Need to solve for each momentum mode!

Vaccuum oscillations

$$\mathcal{H}_0 = U_{\mathrm{PMNS}} \frac{1}{2k} \mathrm{diag}(0, \Delta m^2_{21}, \Delta m^2_{31}) U^\dagger_{\mathrm{PMNS}}$$

Matter potential

$$V_c = -2\sqrt{2} G_F \frac{k}{m_W^2} (\mathbb{E} + \mathbb{P})_{\alpha\alpha} T_\gamma^4$$

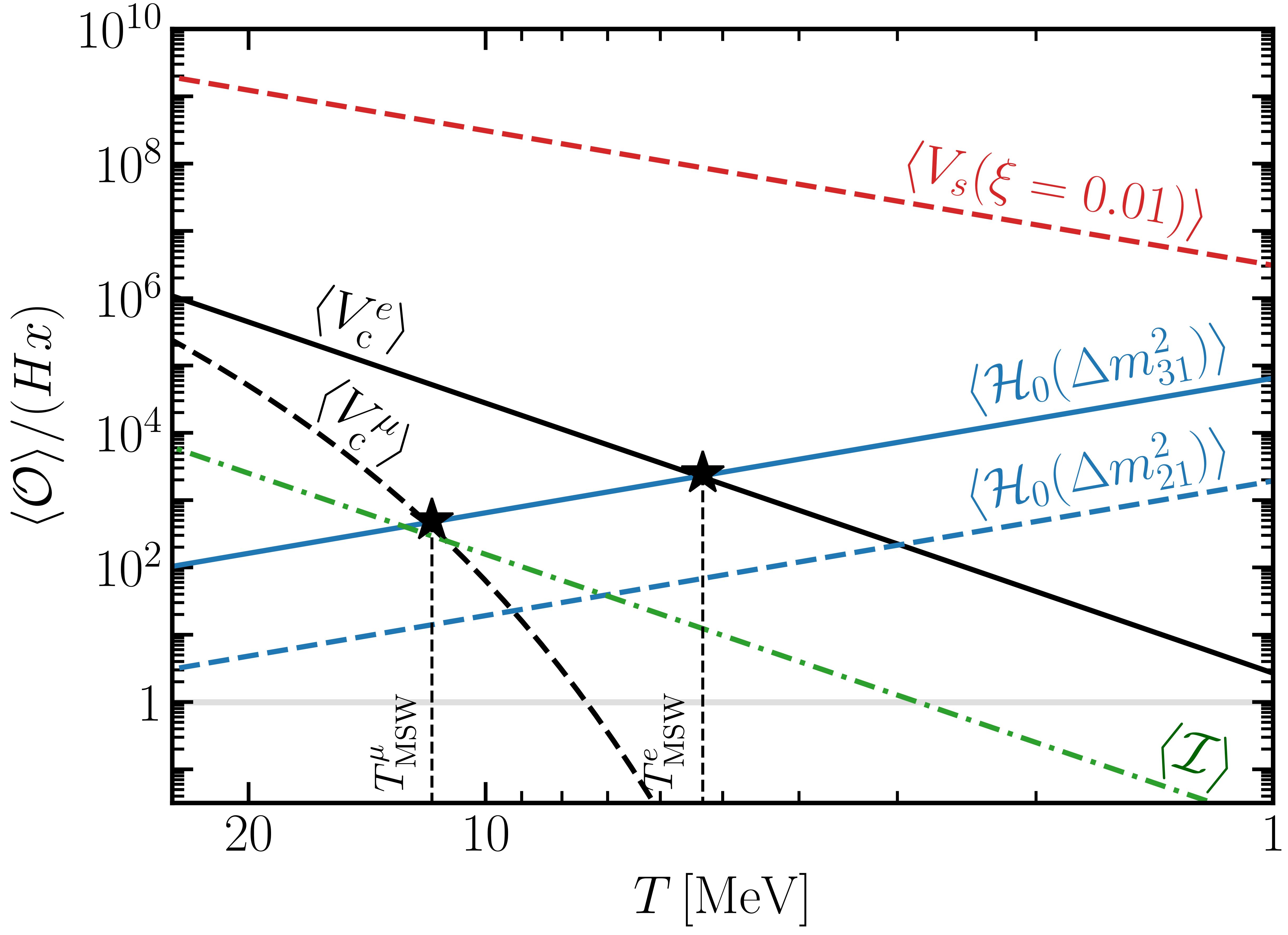
Self-interactions (vanishes if no asymmetries)

$$V_s = \sqrt{2} G_F \int \frac{d^3k'}{(2\pi)^3} (\rho - \bar{\rho})$$

- For initial asymmetries relevant for BBN, V_s always dominates and is momentum independent:

→ All momentum modes become aligned and perform synchronous oscillations

[Pastor, Raffelt, Semikoz, '01]



Dynamics

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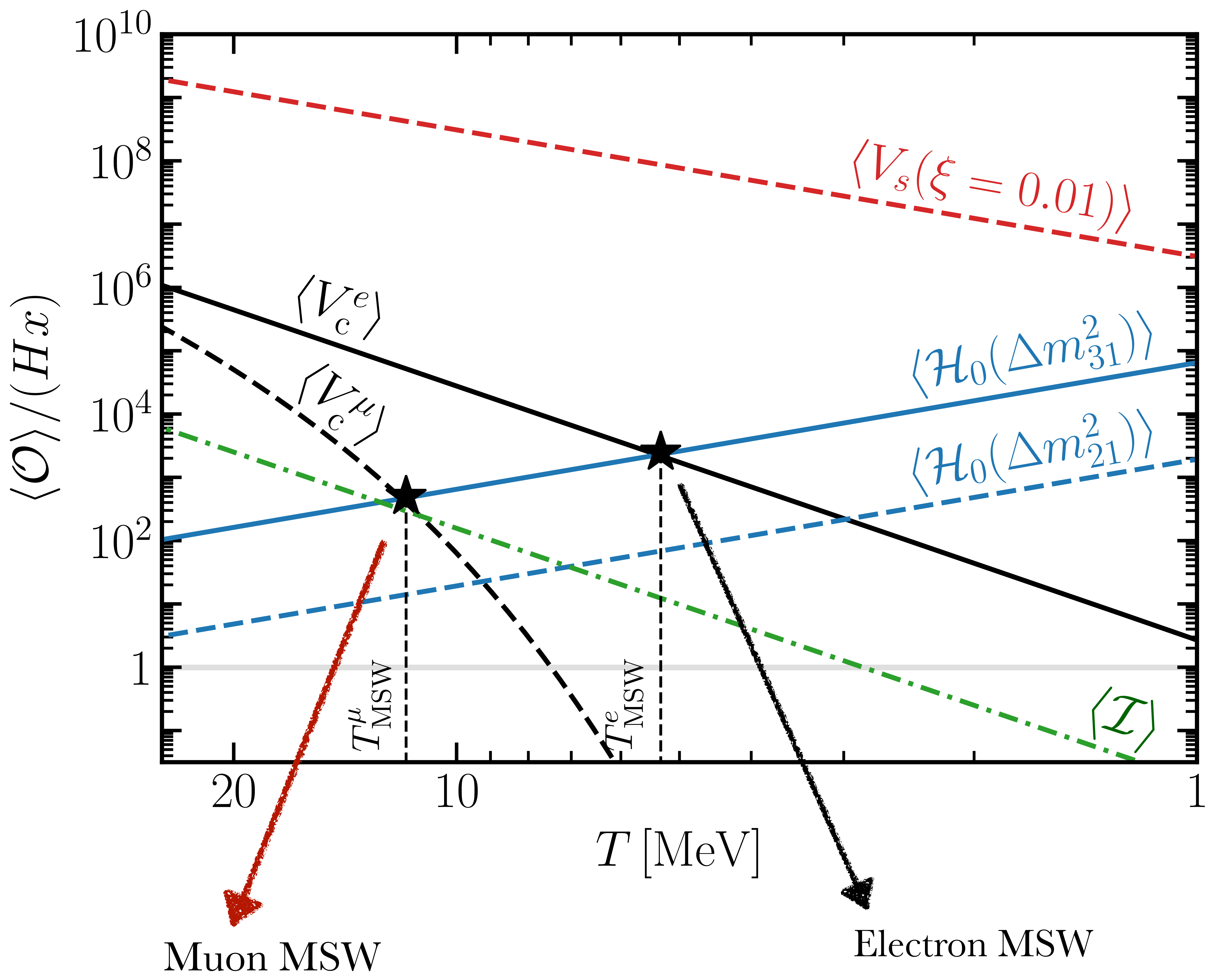
→ All momentum modes become aligned and perform synchronous oscillations

[Pastor, Raffelt, Semikoz, '01]

- Level crossings from V_c to $\mathscr{H}_0$ lead to muon and electron driven MSW transitions:
  - Muon-driven MSW at $T \approx 12$ MeV
  - Electron-driven MSW at $T \approx 5$ MeV
- At low T, collisions become inefficient and neutrinos decouple from the cosmic plasma.



Synchronous oscillations

- Easier to understand synchronous oscillations by using the vector representation of hermitian 3×3 matrices.
- Expanding in the basis of $SU(3)$ Gell-Mann matrices:

$$\rho_0 = \mathrm{Tr}(\rho) \qquad \vec{\lambda} = \lambda_i$$

$$\rho = \frac{1}{3}\rho_0 \mathbb{I} + \frac{1}{2}\vec{\rho}\cdot\vec{\lambda} \qquad \vec{\rho} = \mathrm{Tr}(\rho\lambda_i) \qquad i = 1, ..., 8$$

- Similar for all contributions to the QKEs: $\mathcal{H}_0 \to \vec{\mathcal{H}}_0$, $\quad V_c \to \vec{V}_c$

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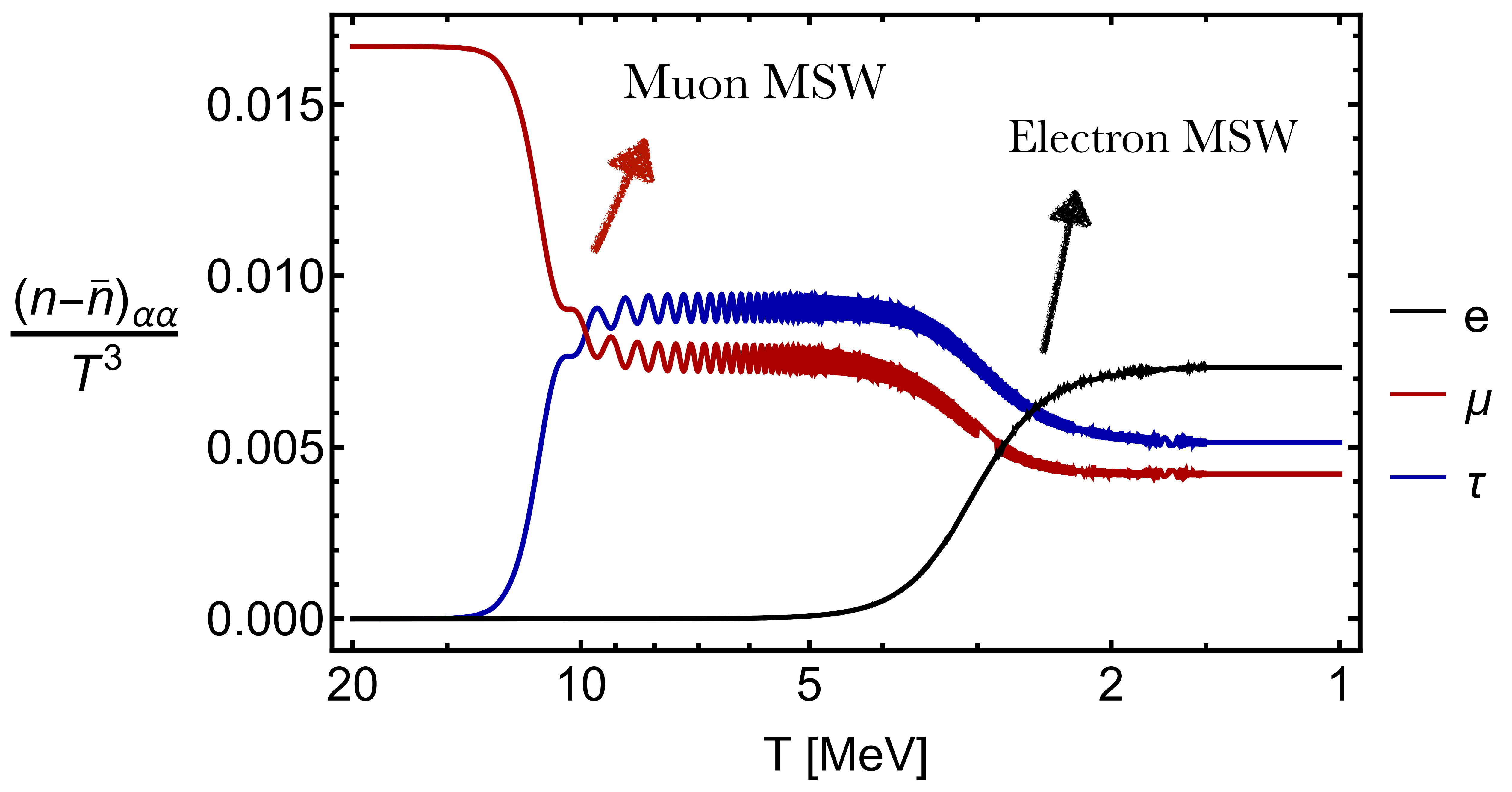
[Pastor, Raffelt, Semikoz '01, Froustey and Pitrou '21 ...]

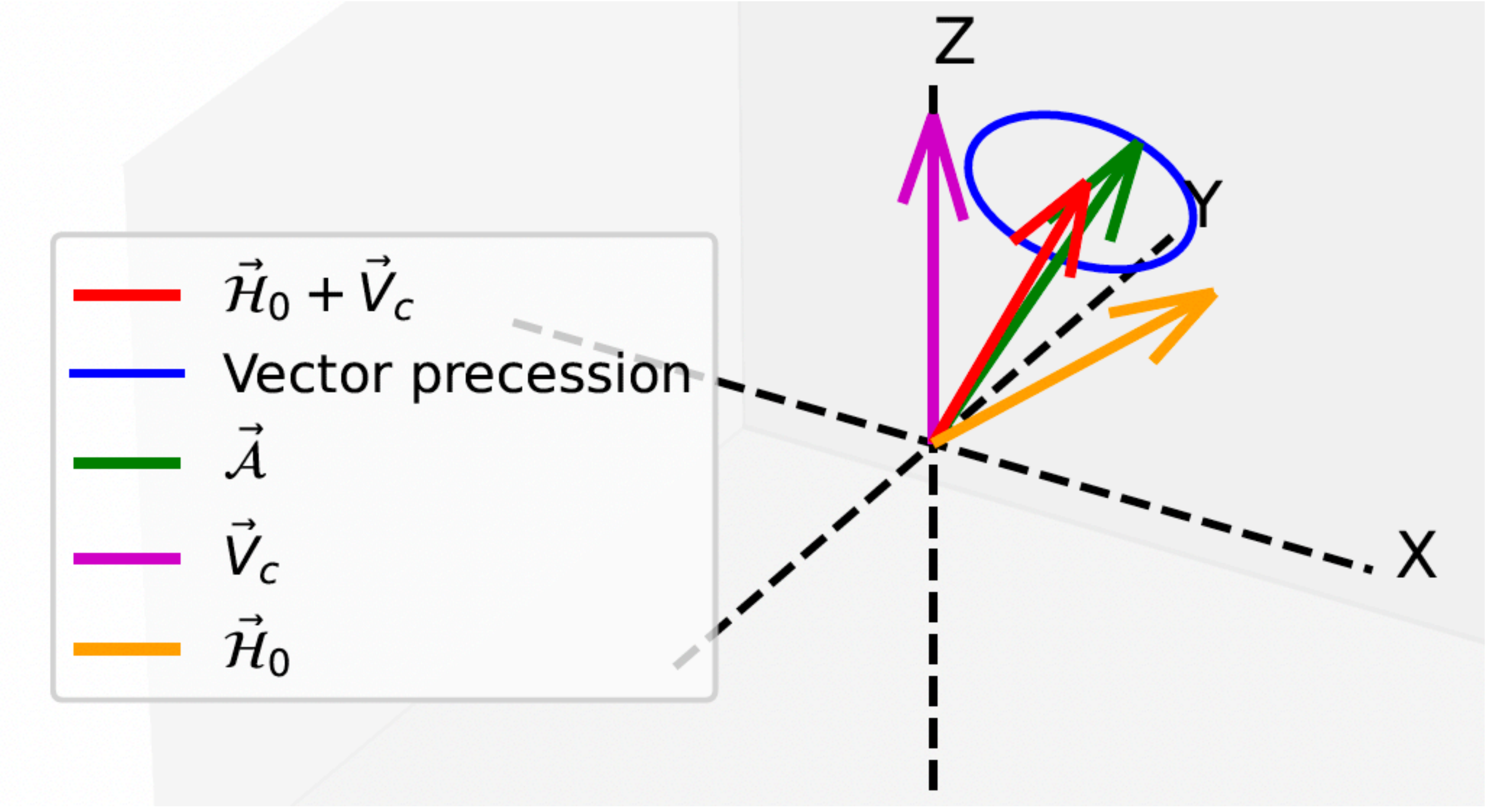
- The evolution of all momentum modes is locked on the evolution of the asymmetry vector, which precesses around $\vec{\mathcal{H}}_0(y_{\text{eff}}) + \vec{V}_c(y_{\text{eff}})$ with effective momentum $y_{\text{eff}} \approx \pi$:

$$\vec{\mathcal{A}} = \int \frac{d^3k'}{(2\pi)^3}(\vec{\rho} - \vec{\bar{\rho}}) \qquad \boxed{\frac{d\vec{\mathcal{A}}}{dt} = \mathcal{F} y_{\text{eff}} \left[\vec{\mathcal{H}}_0(y_{\text{eff}}) + \vec{V}_c(y_{\text{eff}})\right] \times \vec{\mathcal{A}}} \qquad y = \frac{k}{T}$$

Synchronous oscillations

$$\xi_\mu^{\rm ini} = 0.1\,, \quad \xi_e^{\rm ini} = \xi_\tau^{\rm ini} = 0$$





[Pastor, Raffelt, Semikoz '01, Froustey and Pitrou '21 …]

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Momentum average

- Therefore, to good accuracy the system can be described with a momentum average ansatz for the density matrices:

$$\rho(t,k) = r(t)\, f_{\rm FD}(y\,T_{\rm cm}/T_\nu, 0)\,, \quad \bar{\rho}(t,k) = \bar{r}(t)\, f_{\rm FD}(y\,T_{\rm cm}/T_\nu, 0)\,,$$

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- Then we can integrate in momentum and perform the momentum average: $\langle X\rangle = \dfrac{\int dy y^2 X f_{\rm FD}(y\,T_{\rm cm}/T_\nu, 0)}{\int dy y^2 f_{\rm FD}(y\,T_{\rm cm}/T_\nu, 0)}$

$$\frac{dr}{dt} = -i\left[\langle\mathcal{H}_0\rangle + \langle V_c\rangle + V_s, r\right] + \langle\mathcal{I}\rangle\;, \qquad \frac{d\bar r}{dt} = +i\left[\langle\mathcal{H}_0\rangle + \langle V_c\rangle - V_s, \bar r\right] + \langle\overline{\mathcal{I}}\rangle\;,$$

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- Finally, track the energy transfer between neutrinos and the $e^\pm - \gamma$ plasma via the continuity equation (necessary for a precise computation of $N_{\rm eff}$):

$$\frac{d(\varepsilon_\gamma + \varepsilon_e)}{dt} + 4H\varepsilon_\gamma + 3H(\varepsilon_e + p_e) = -\frac{\delta\varepsilon}{\delta t}\,, \qquad \frac{d\varepsilon_\nu}{dt} + 4H\varepsilon_\nu = \frac{\delta\varepsilon}{\delta t}\,,$$

COsmological evolution of lepton FLavour ASYmmetries (COFLASY)

- Code to solve numerically the system of QKEs: https://github.com/mariofnavarro/COFLASY
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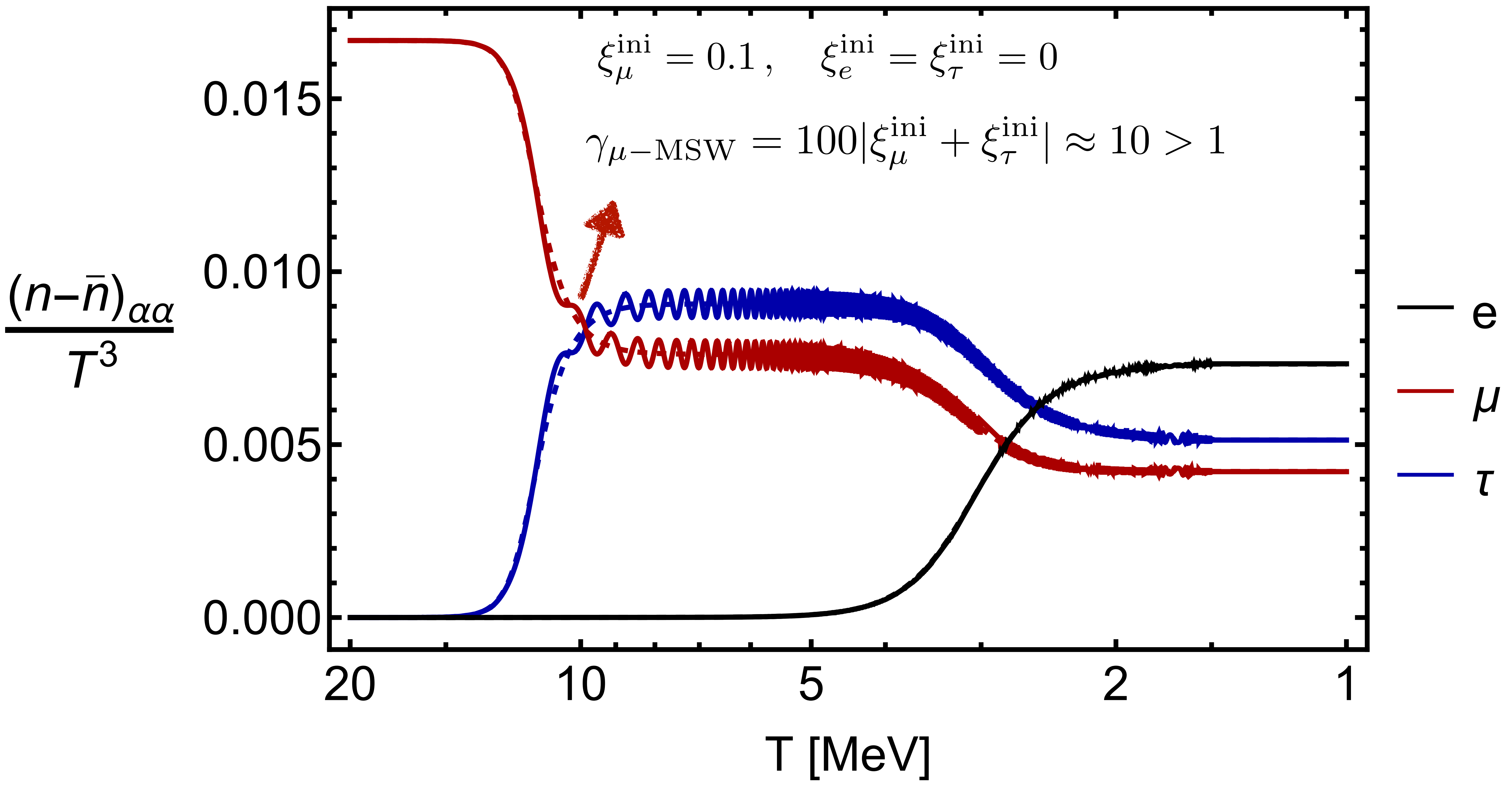
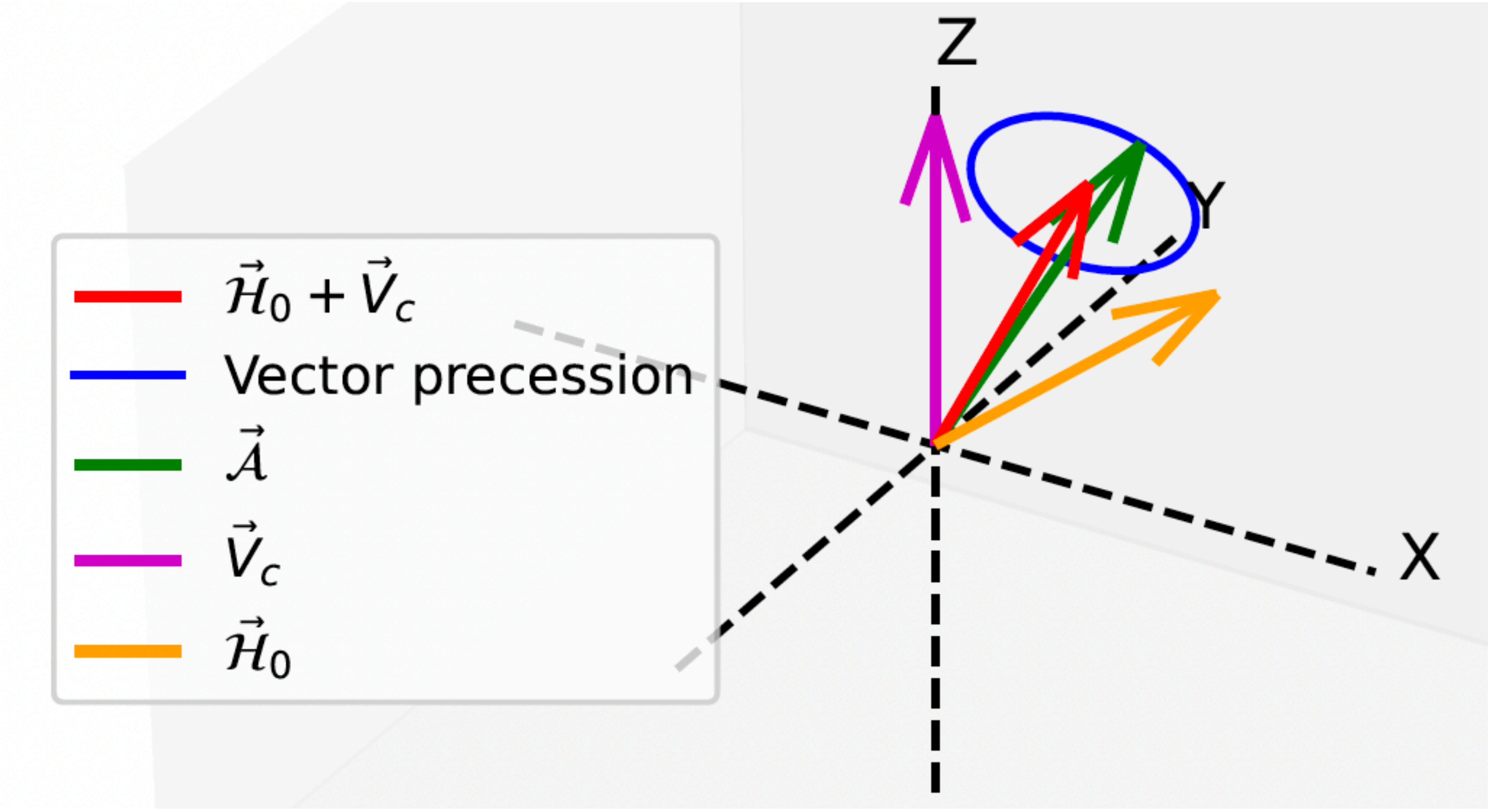
  - Numerical integration and interpolation to include m_e effects (COFLASY-C)
  - Alternatively: damping approximation respecting μ-τ $U(2)$, MB statistics…
- Execution times for large initial asymmetries $\xi_\alpha \gtrsim \mathcal{O}(1)$ →
  - COFLASY-C: $\sim$ 30 seconds
  - COFLASY-M: $\sim$ 3 minutes
  - Momentum-dependent code: $\sim$ several hours or days [Froustey and Pitrou '21, '24]

Adiabatic MSW transition

- If the frequency of synchronous oscillations is fast compared to the MSW transition time scale, the asymmetry vector will follow smoothly (adiabatically) the MSW transition of the Hamiltonian:



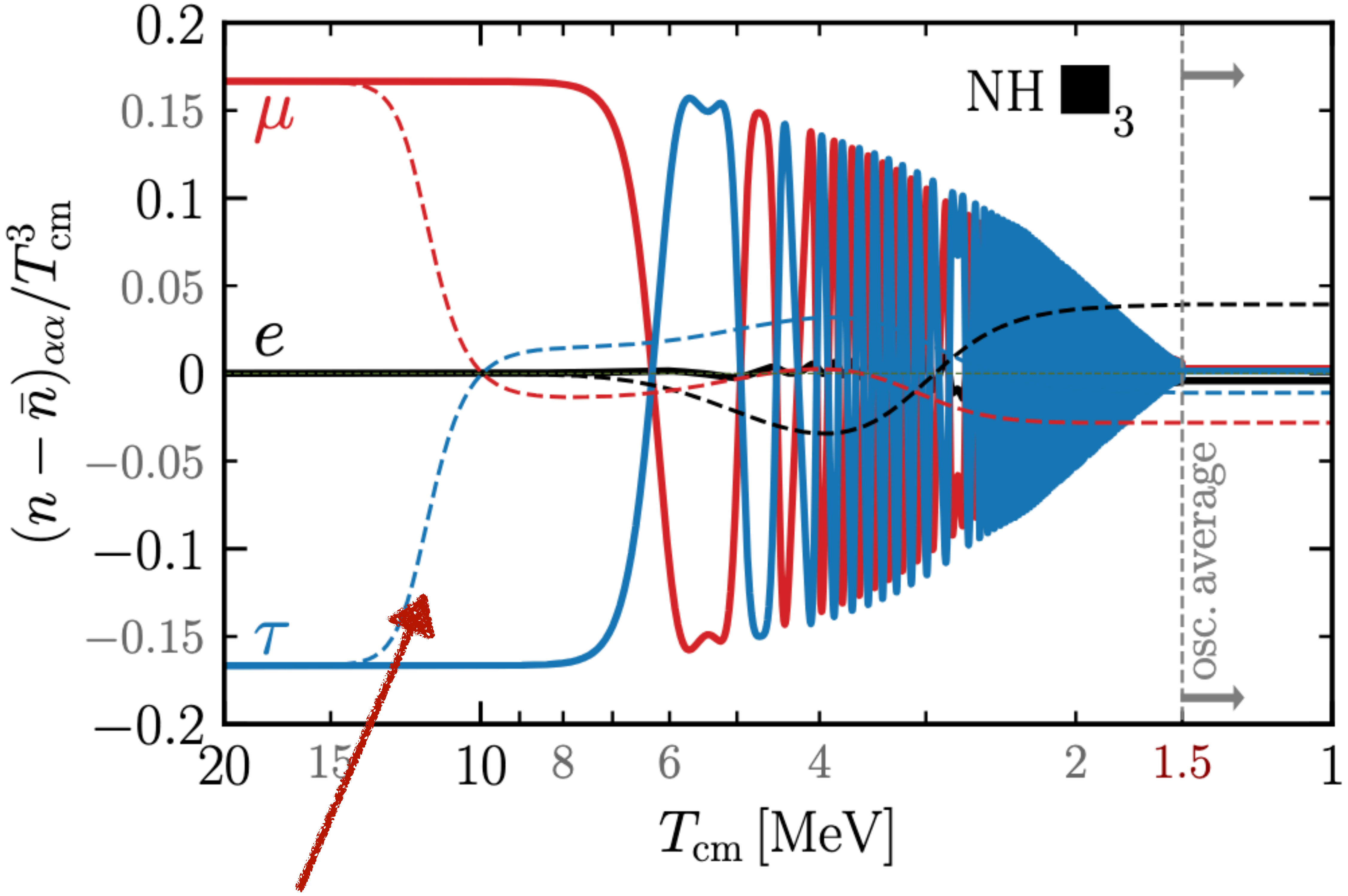


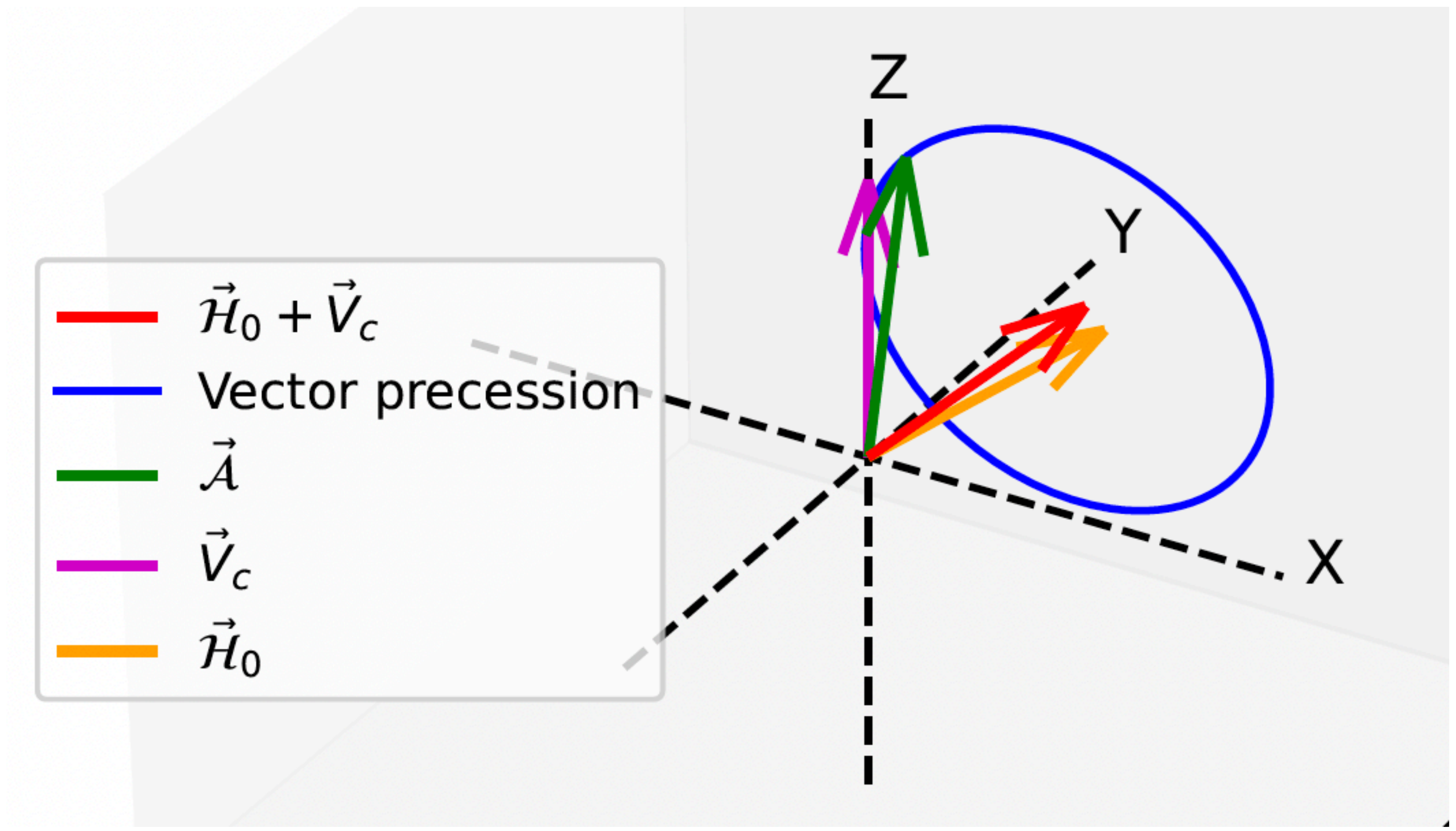
- The self-interactions fully dominate such that $[V_s, r(\bar{r})] \approx 0$, we can set $V_s = 0$ to average the fast oscillations (dashed lines)

Non-adiabatic MSW transition

- If the frequency of synchronous oscillations is slow compared to the MSW transition time scale, the asymmetry vector will be unable to follow smoothly the MSW transition of the Hamiltonian.

$$\xi_\mu^{\rm ini} = -\xi_\tau^{\rm ini} = 0.92\,, \quad \xi_e^{\rm ini} = 0$$



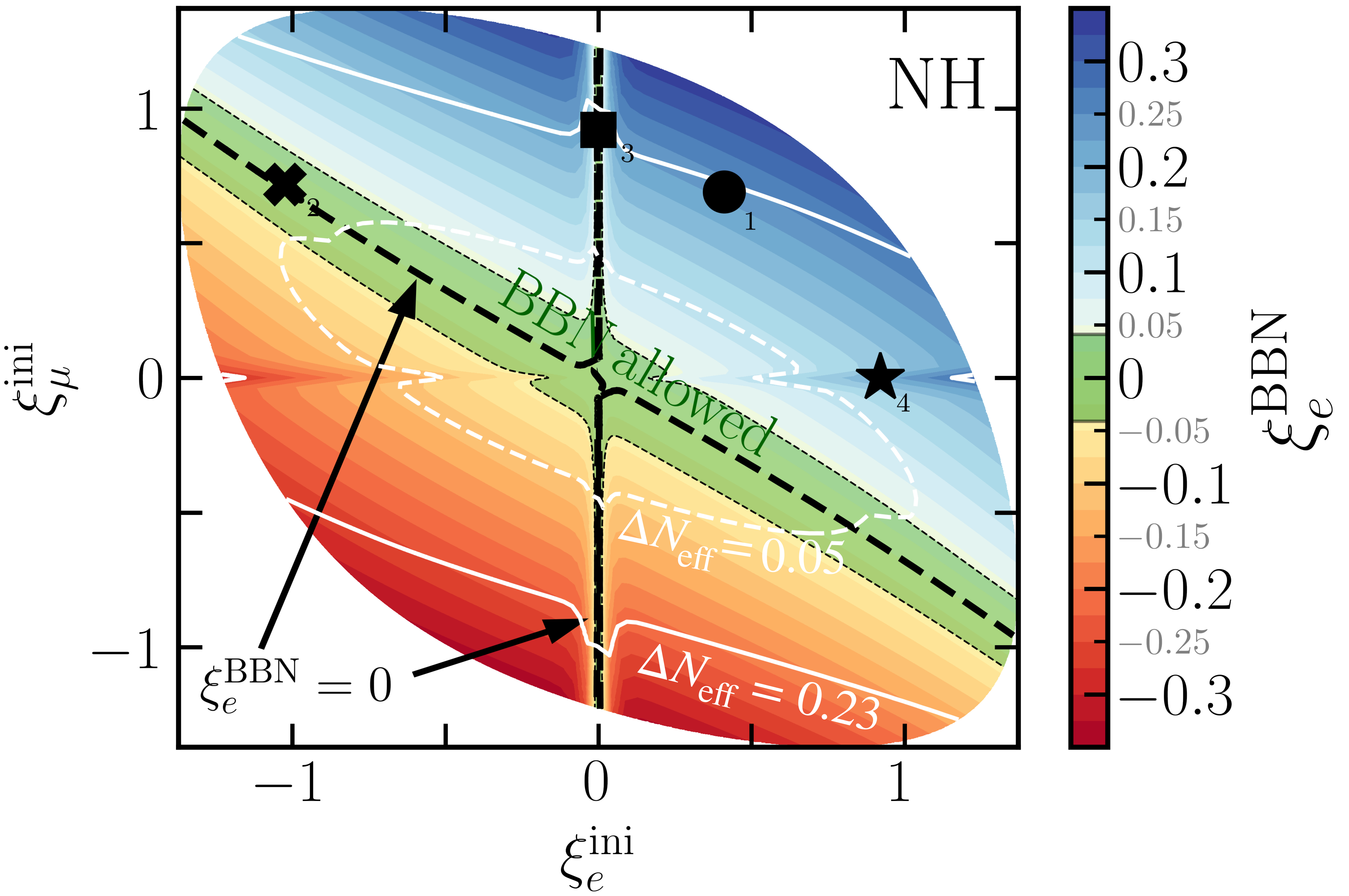


$$\gamma_{\mu-\rm MSW} = 100|\xi_\mu^{\rm ini} + \xi_\tau^{\rm ini}| \approx 0 \ll 1$$

Non-adiabatic behaviour leads to delayed MSW transition

Results : Normal Ordering

$$\Delta n_e^{\text{ini}} + \Delta n_\mu^{\text{ini}} + \Delta n_\tau^{\text{ini}} = 0 \qquad \Delta n_\alpha^{\text{ini}} = \frac{1}{6}\left(\xi_\alpha^{\text{ini}} + \frac{(\xi_\alpha^{\text{ini}})^3}{\pi^2}\right)$$



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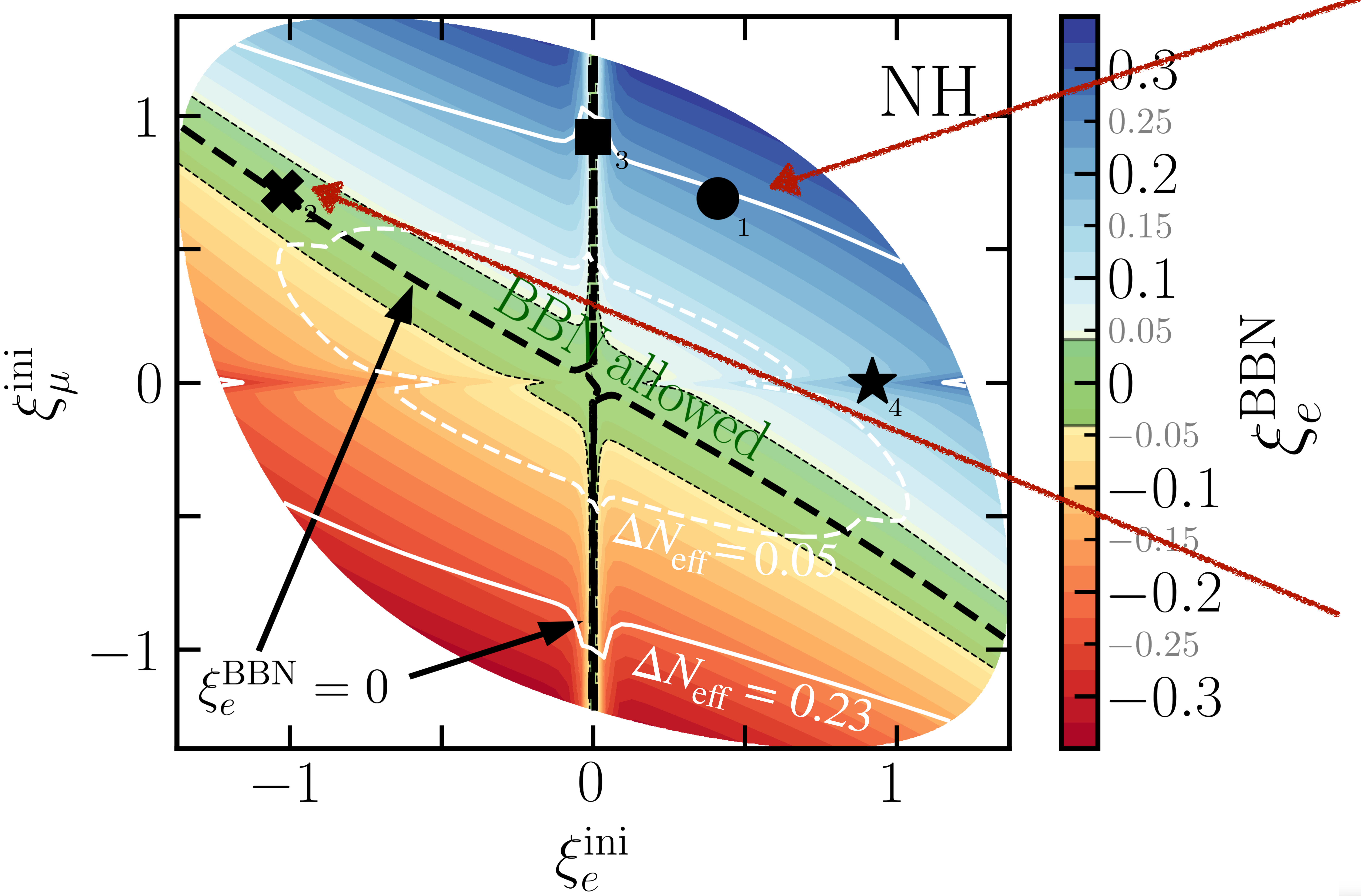
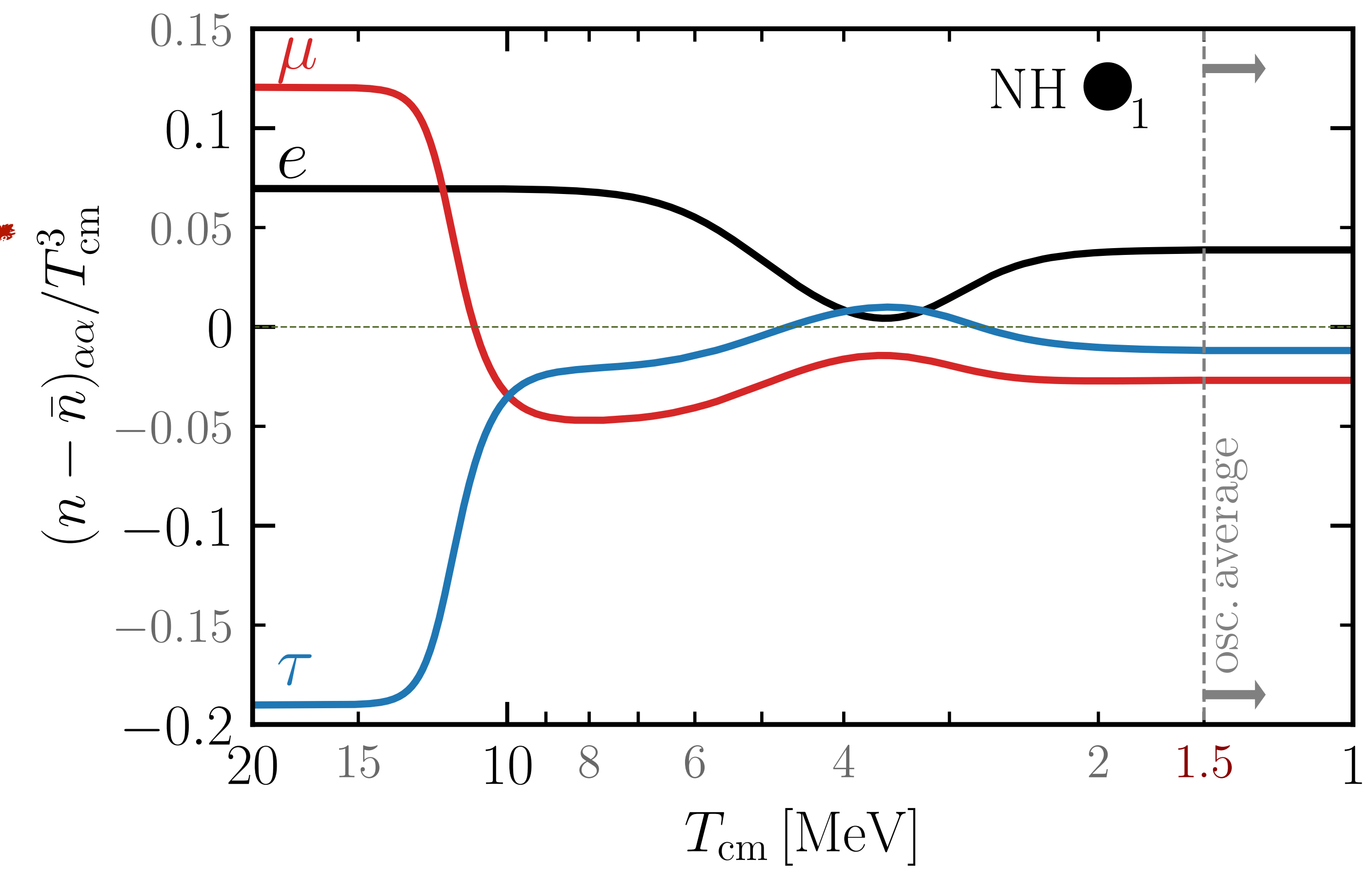
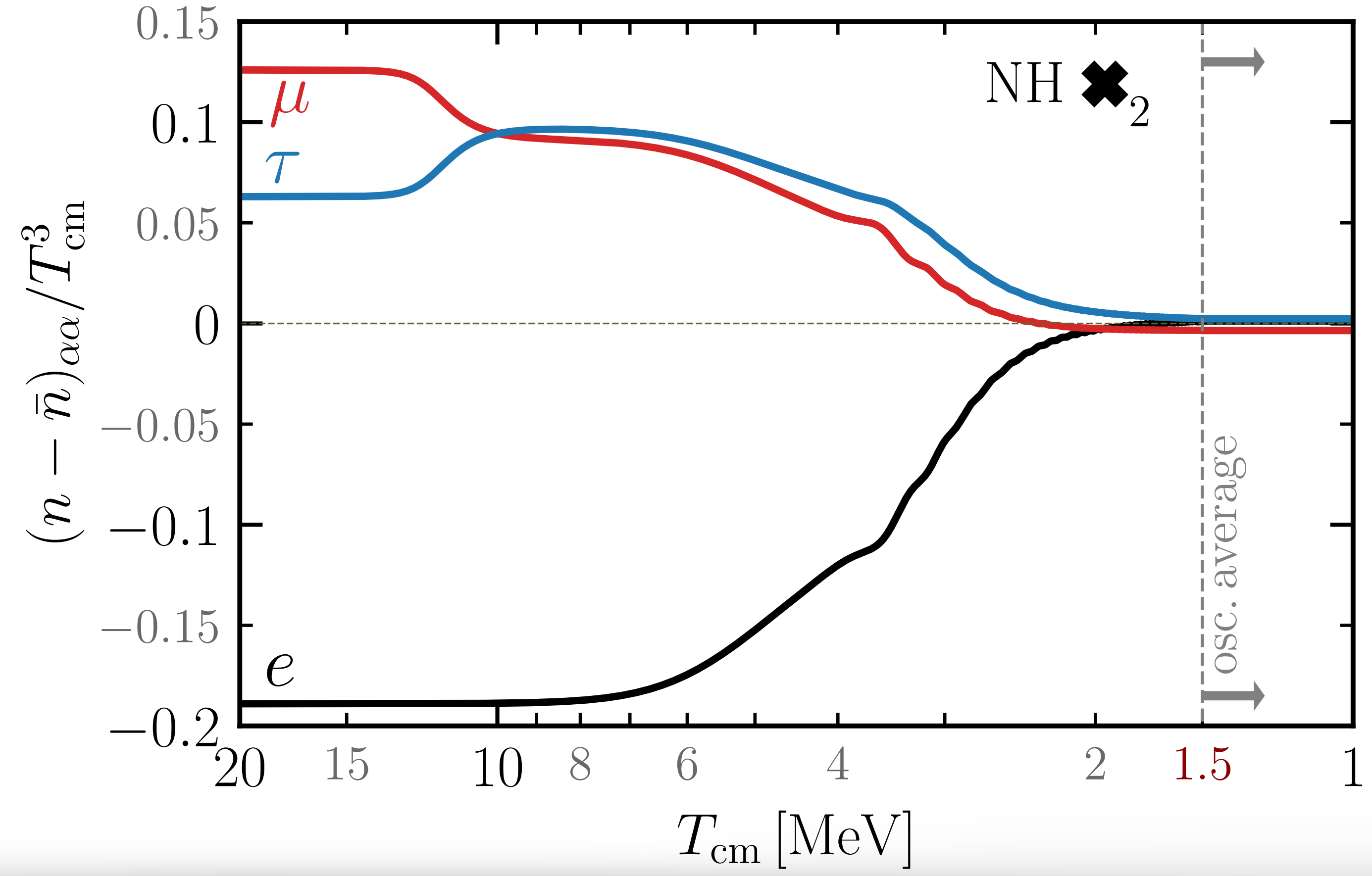




Smooth (adiabatic) behaviour leads to flavour equilibration for:

$$\xi_e^{\rm ini} \approx -\frac{2}{3}\xi_\mu^{\rm ini}\,, \quad \xi_\tau^{\rm ini} \approx -\frac{1}{3}\xi_\mu^{\rm ini}\,,$$



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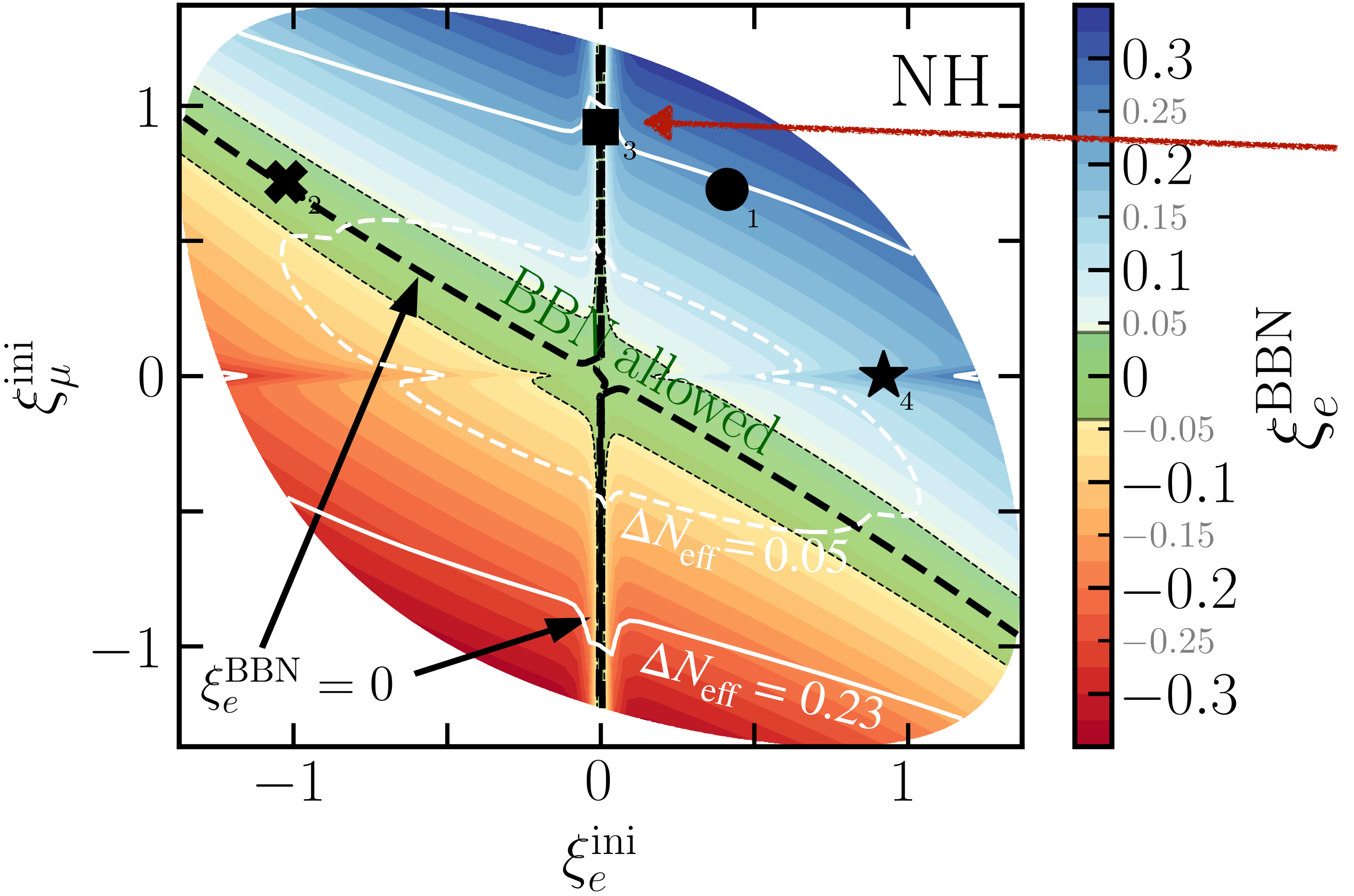
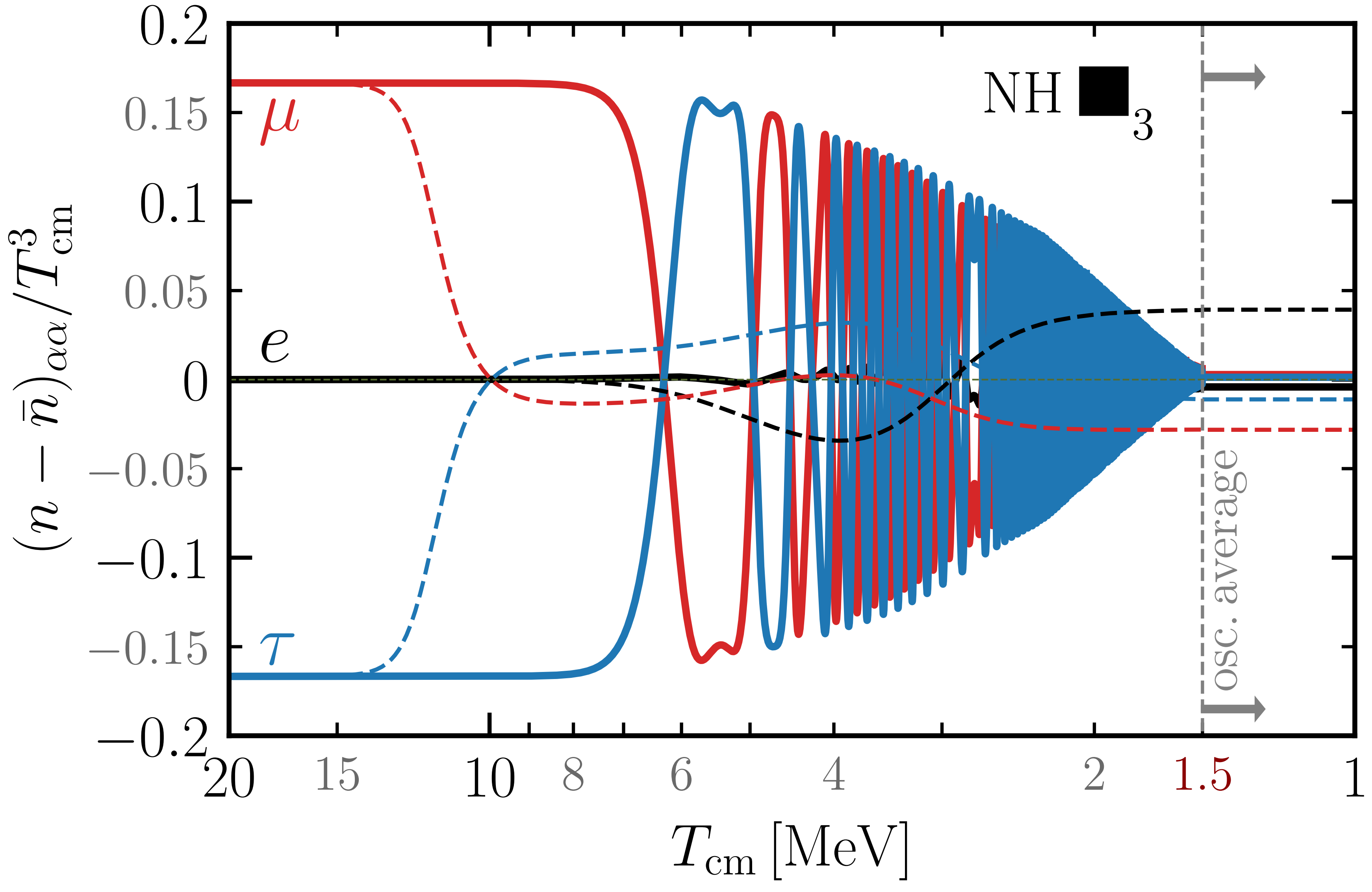


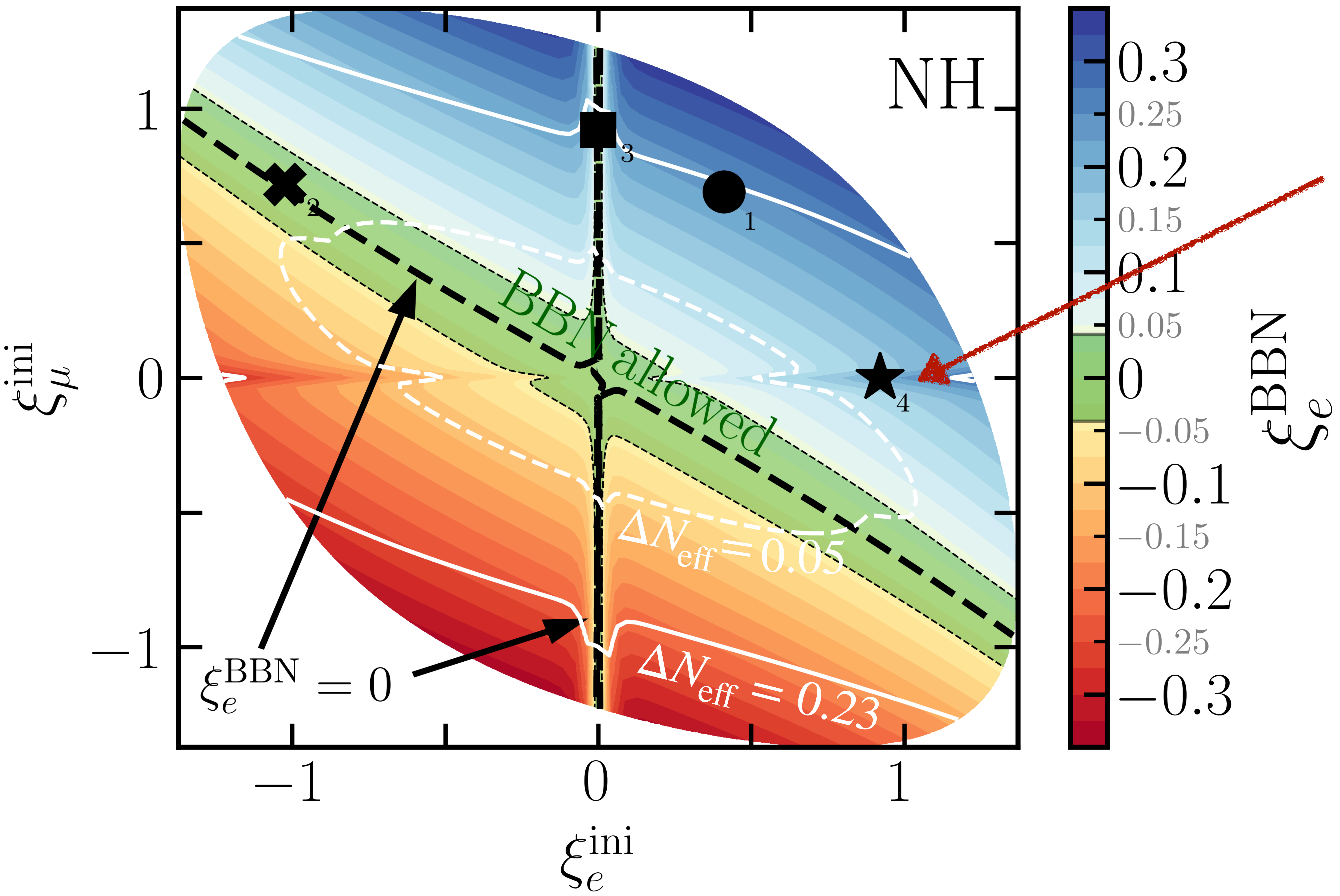


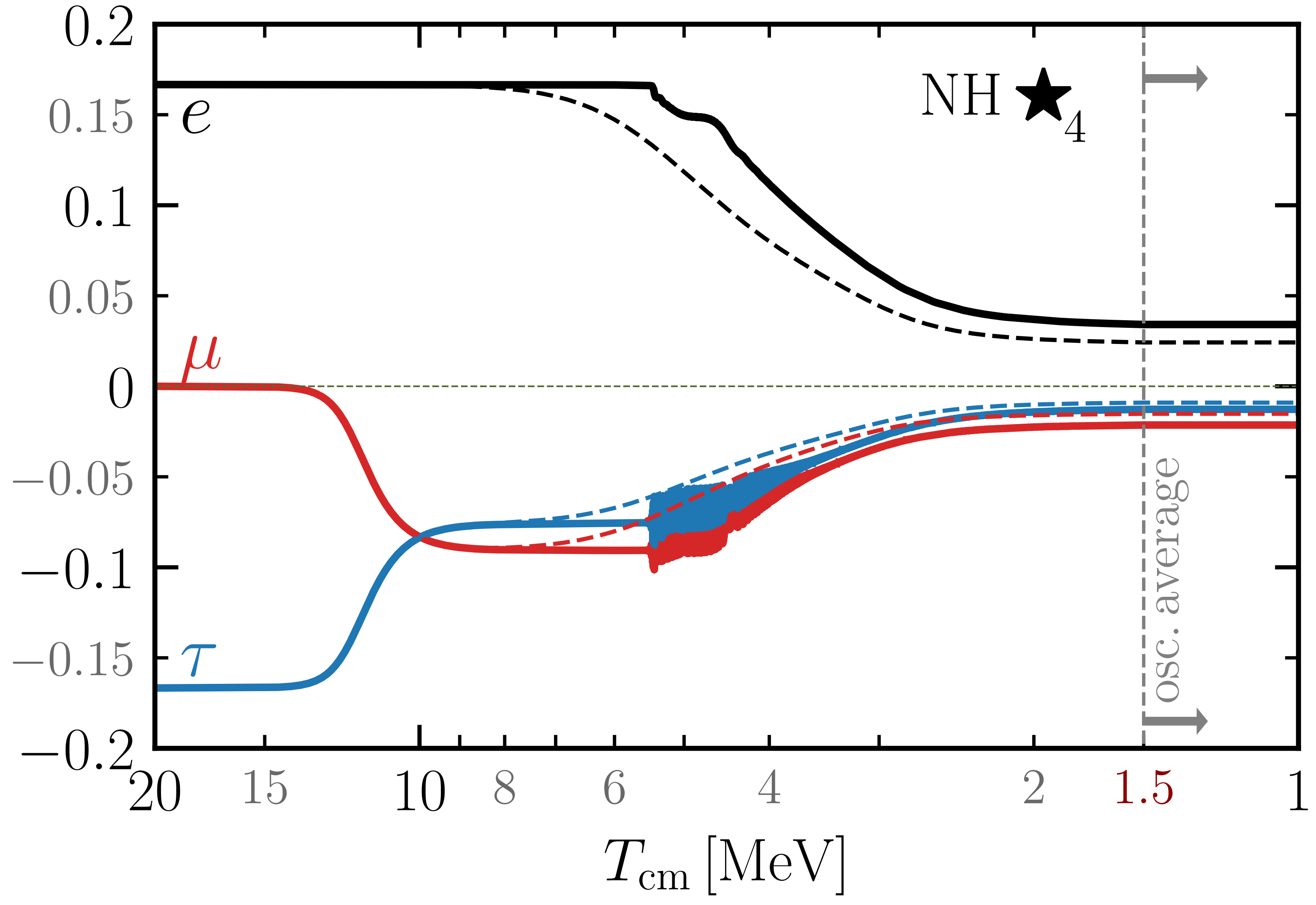
Non-adiabatic muon MSW leads to flavour equilibration:

$$\xi_e^{\rm ini} \simeq 0\,, \quad \xi_\mu^{\rm ini} \simeq -\xi_\tau^{\rm ini}$$

Results : Normal Ordering

$$\Delta n_e^{\rm ini} + \Delta n_\mu^{\rm ini} + \Delta n_\tau^{\rm ini} = 0 \qquad \Delta n_\alpha^{\rm ini} = \frac{1}{6}\left(\xi_\alpha^{\rm ini} + \frac{(\xi_\alpha^{\rm ini})^3}{\pi^2}\right)$$





Non-adiabatic electron MSW leads to less washout:

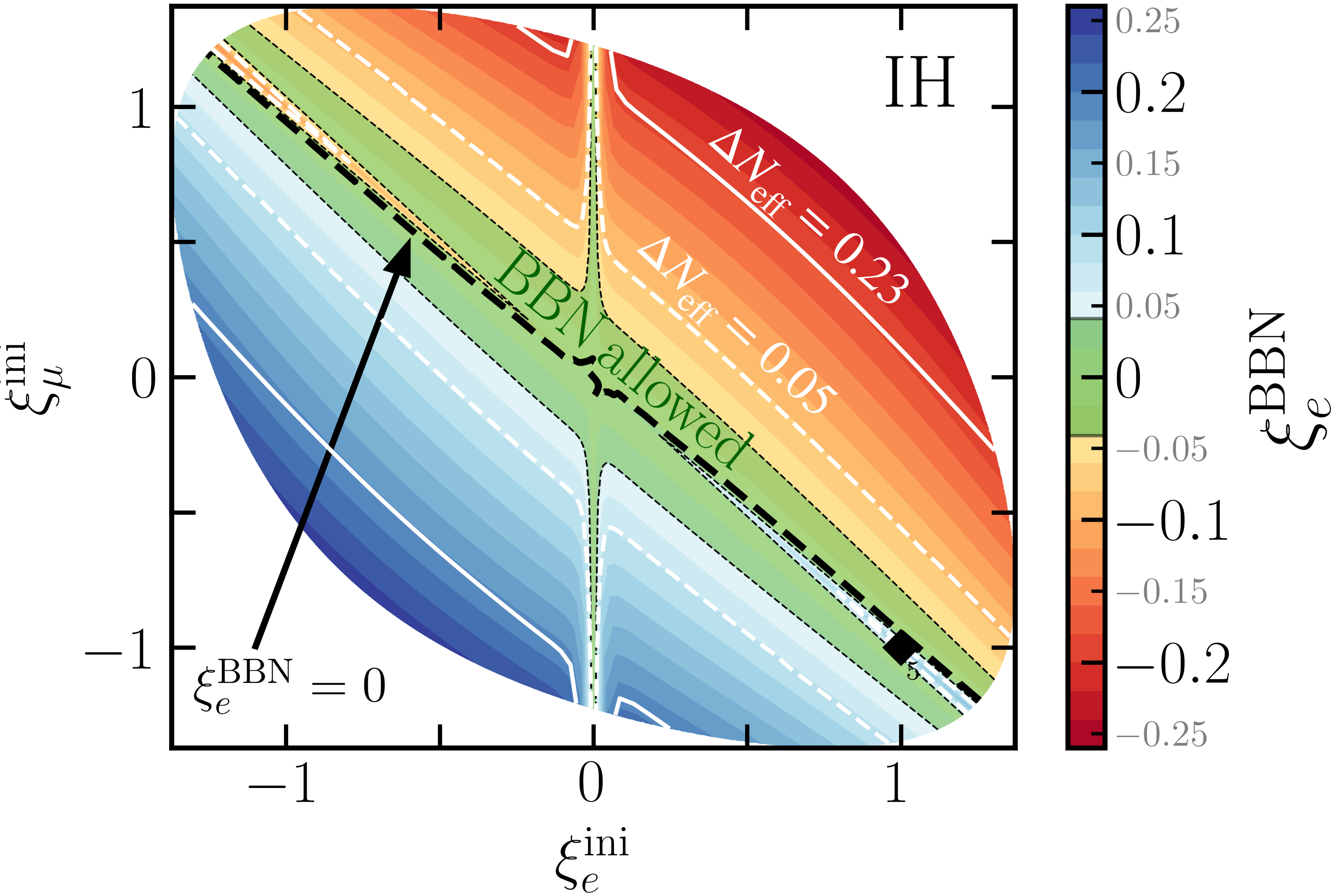
$$\xi_e^{\rm ini} \simeq -\xi_\tau^{\rm ini}\,, \quad \xi_\mu^{\rm ini} \simeq 0$$

$$\gamma_{e-\rm MSW}^{\rm NH} \simeq 485|\xi_e^{\rm ini} + \xi_\tau^{\rm ini}| \approx 0 \ll 1$$

- $\Delta N_{\rm eff}$ not competitive (yet)

Results: Inverted Ordering

$$\Delta n_e^{\rm ini} + \Delta n_\mu^{\rm ini} + \Delta n_\tau^{\rm ini} = 0 \qquad \Delta n_\alpha^{\rm ini} = \frac{1}{6}\left(\xi_\alpha^{\rm ini} + \frac{(\xi_\alpha^{\rm ini})^3}{\pi^2}\right)$$



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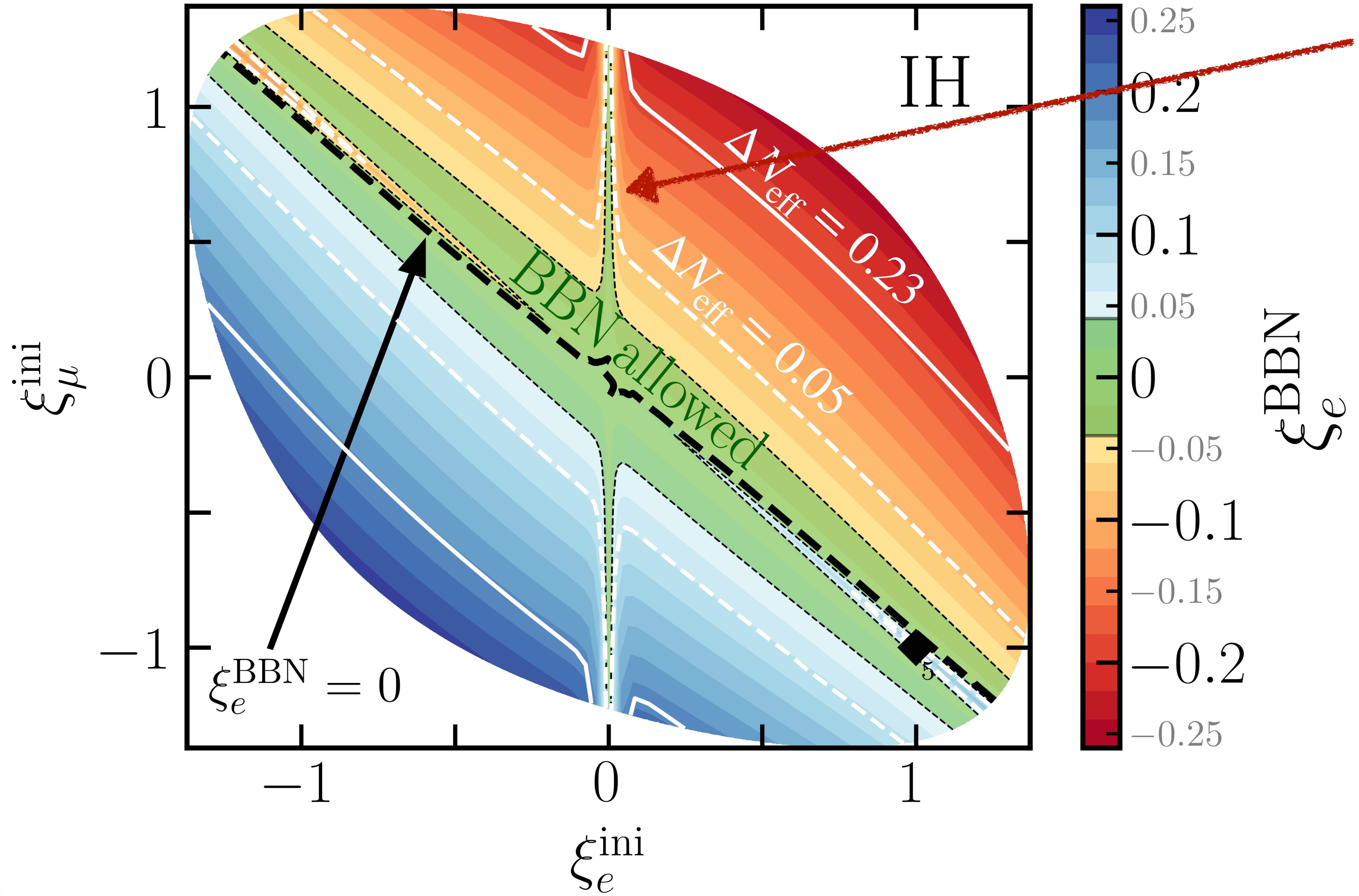

- Non-adiabatic muon MSW as in NH:

$$\xi_e^{\rm ini} \simeq 0\,, \quad \xi_\mu^{\rm ini} \simeq -\xi_\tau^{\rm ini}$$

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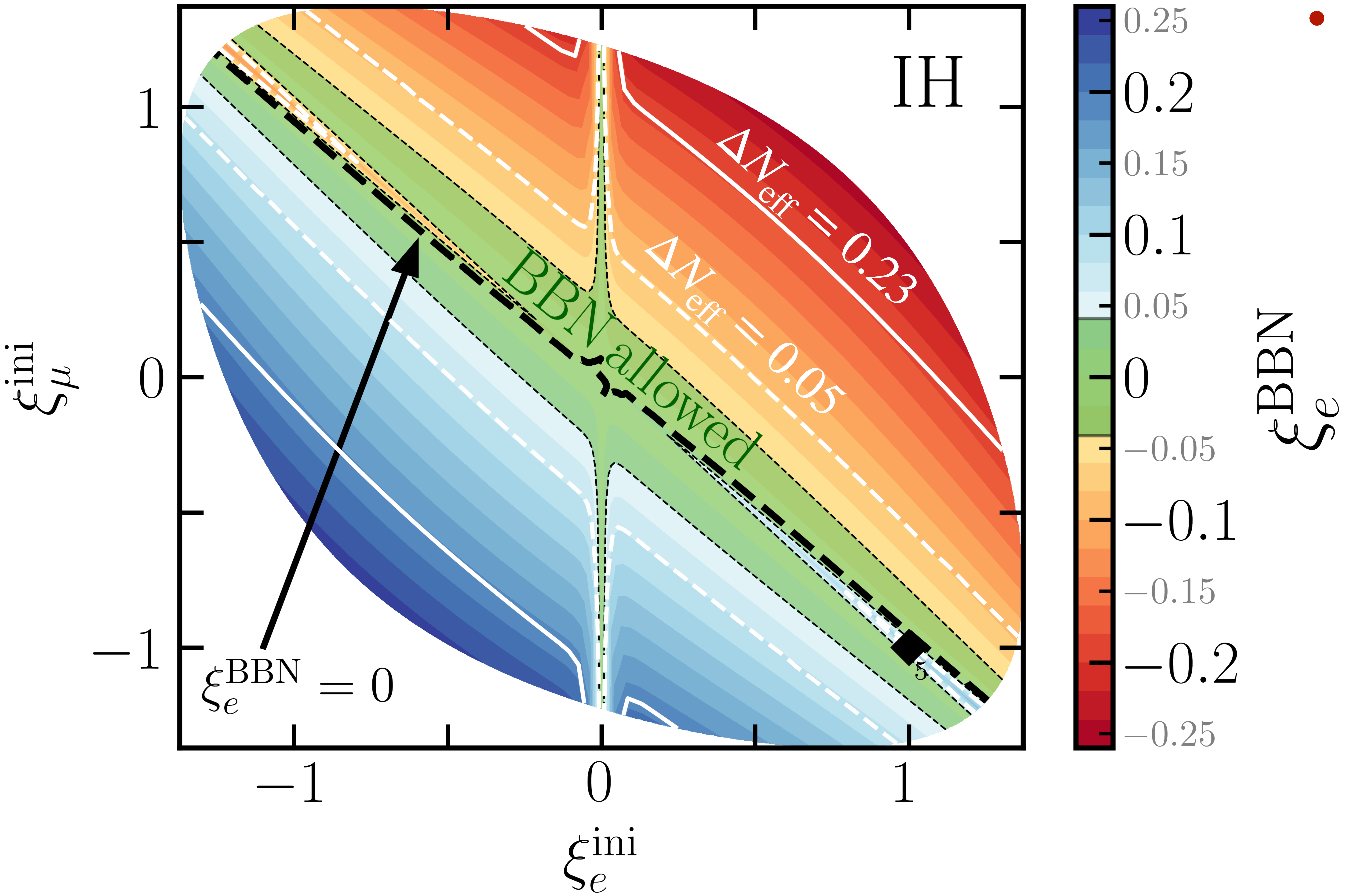
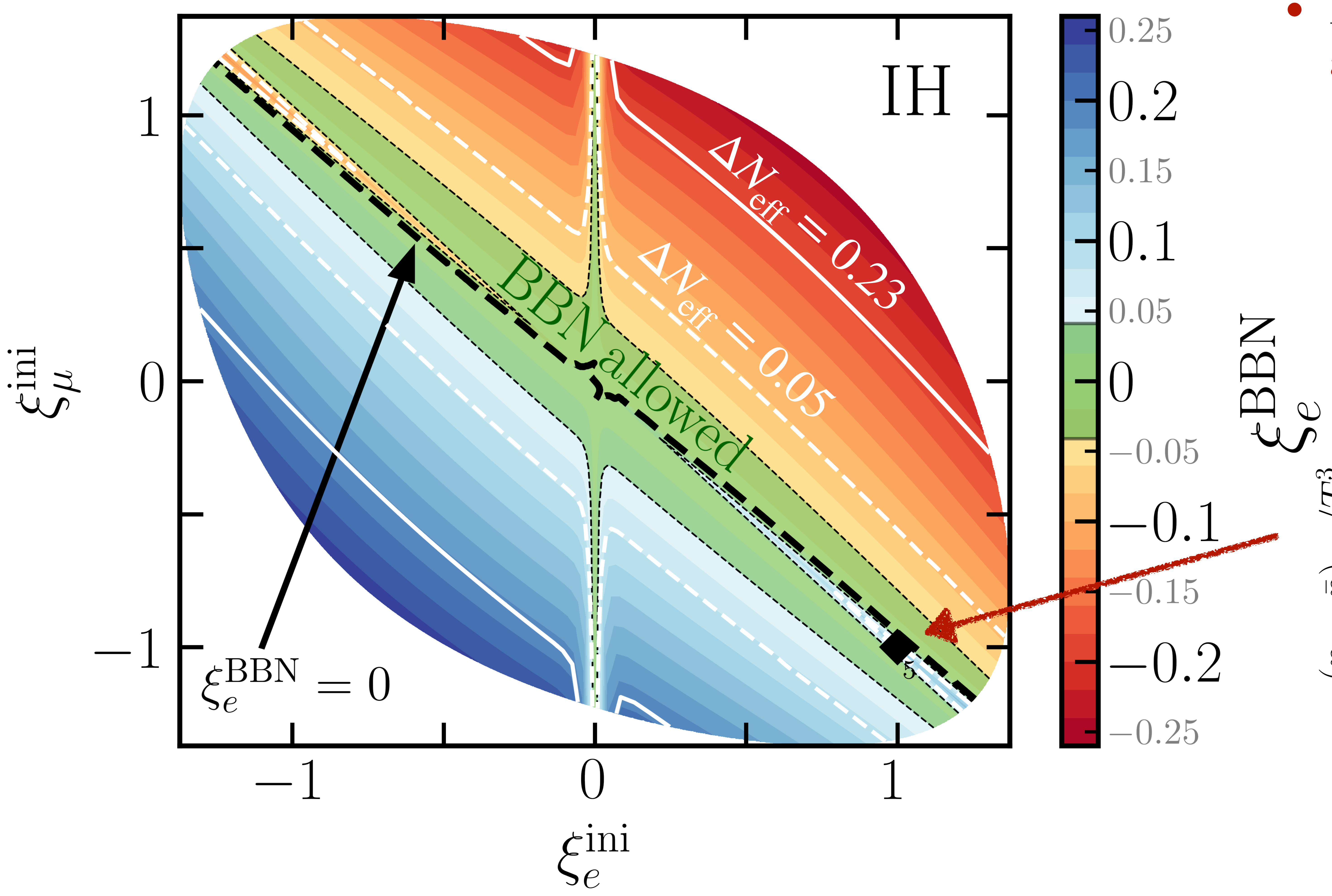


- Adiabatic flavour equilibration generally more efficient than in NH.
- $\Delta N_{\rm eff}$ aligns with BBN band because flavour equilibration is more efficient.

Results: Inverted Ordering

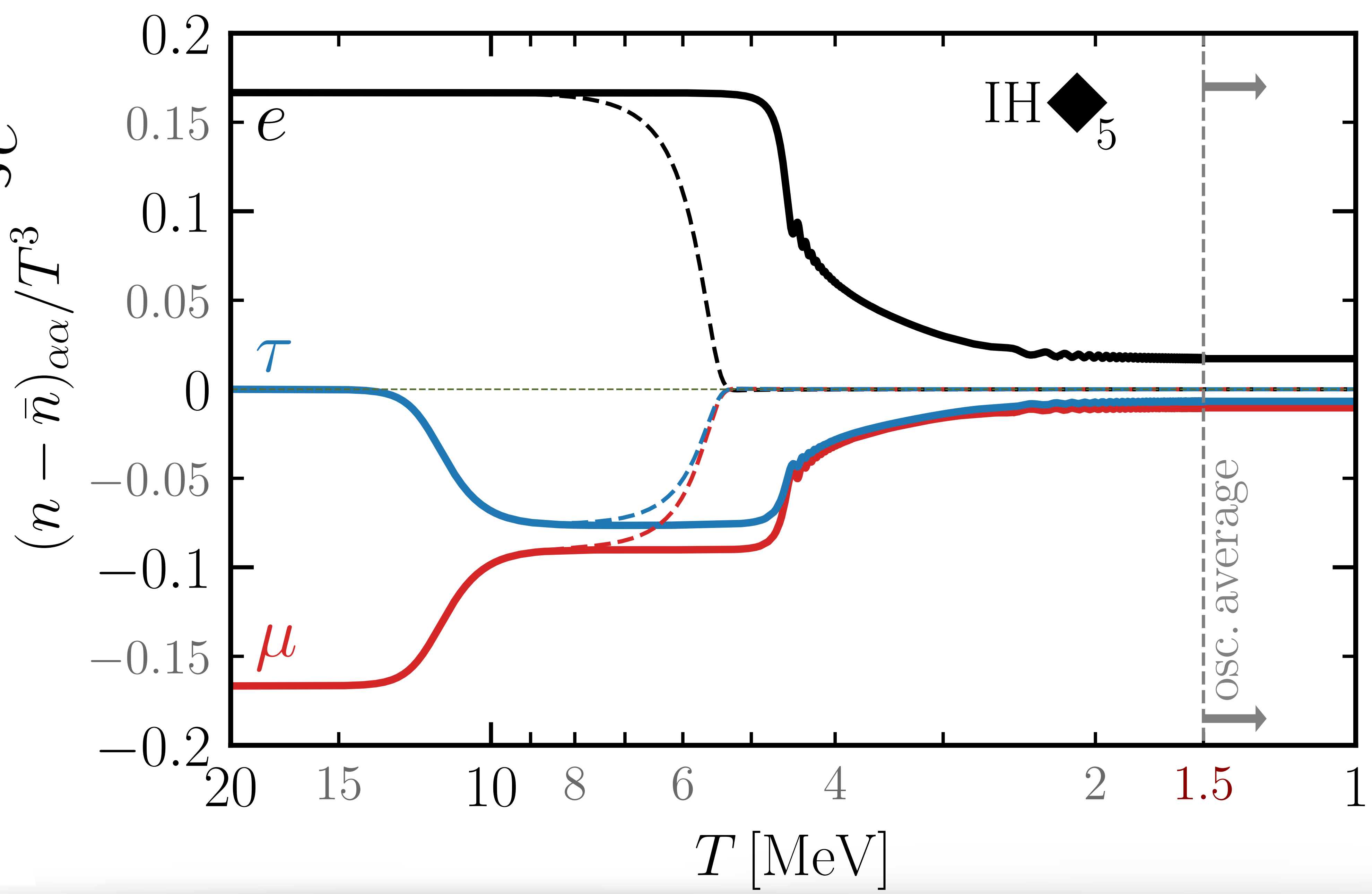
$$\Delta n_e^{\rm ini} + \Delta n_\mu^{\rm ini} + \Delta n_\tau^{\rm ini} = 0 \qquad \Delta n_\alpha^{\rm ini} = \frac{1}{6}\left(\xi_\alpha^{\rm ini} + \frac{(\xi_\alpha^{\rm ini})^3}{\pi^2}\right)$$



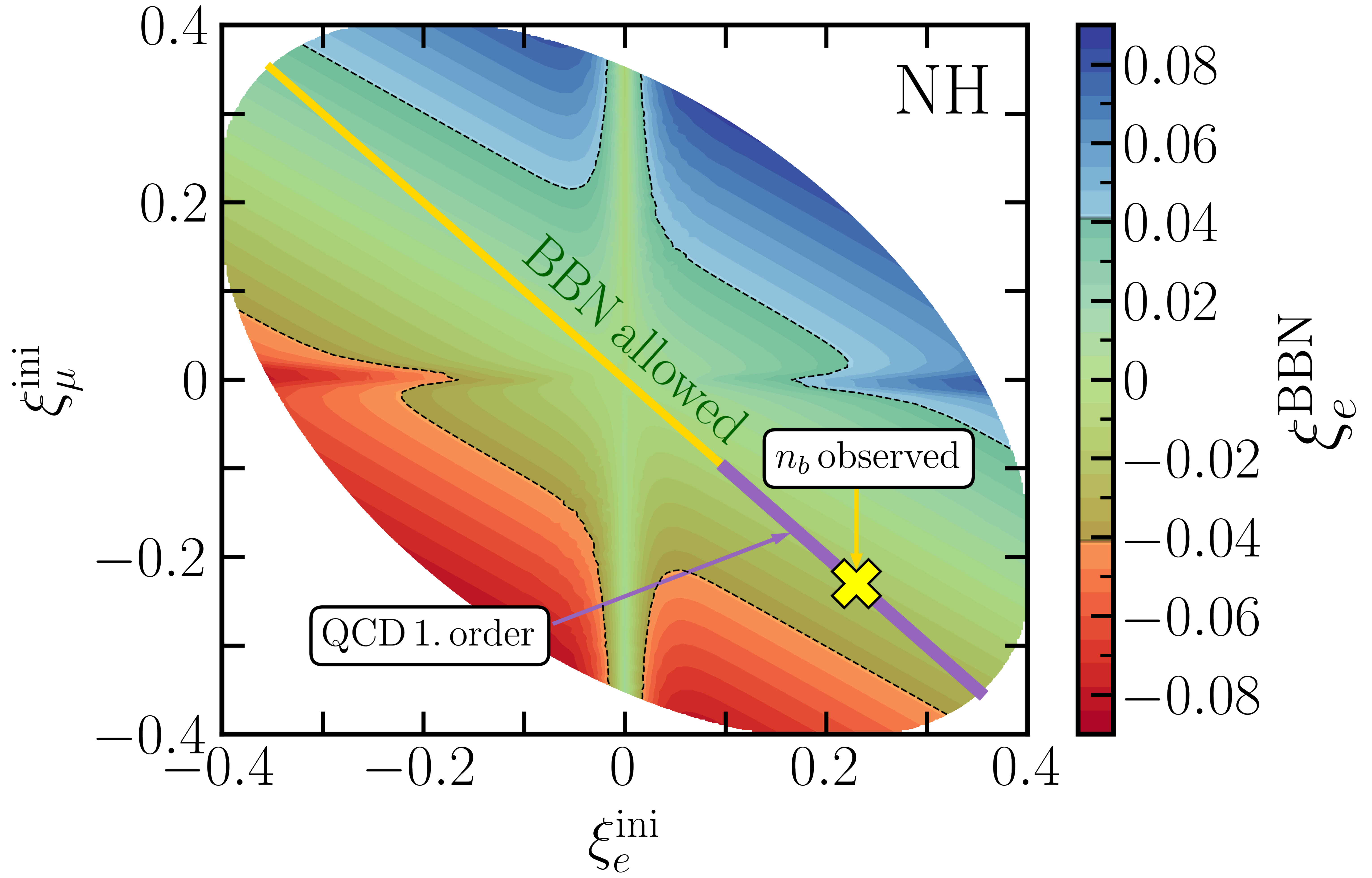
- Adiabatic flavour equilibration generally more efficient than in NH.
- But maximal adiabatic washout close to non-adiabatic e-MSW (now in $\xi_\tau = 0$ direction)

$$\xi_e^{\rm ini} \simeq -\xi_\mu^{\rm ini}\,, \quad \xi_\tau^{\rm ini} \simeq 0$$

$$\gamma_{e-\rm MSW}^{\rm NH} = 100|\xi_e^{\rm ini} + \xi_\mu^{\rm ini}| \approx 0 \ll 1$$



Early Universe implications



1st order QCD phase transition

$$\xi_e^{\rm ini} = -\xi_\mu^{\rm ini} \gtrsim 0.1\,, \quad \xi_\tau \approx 0$$

[Gao and Oldengott, '21]

Leptoflavourgenesis

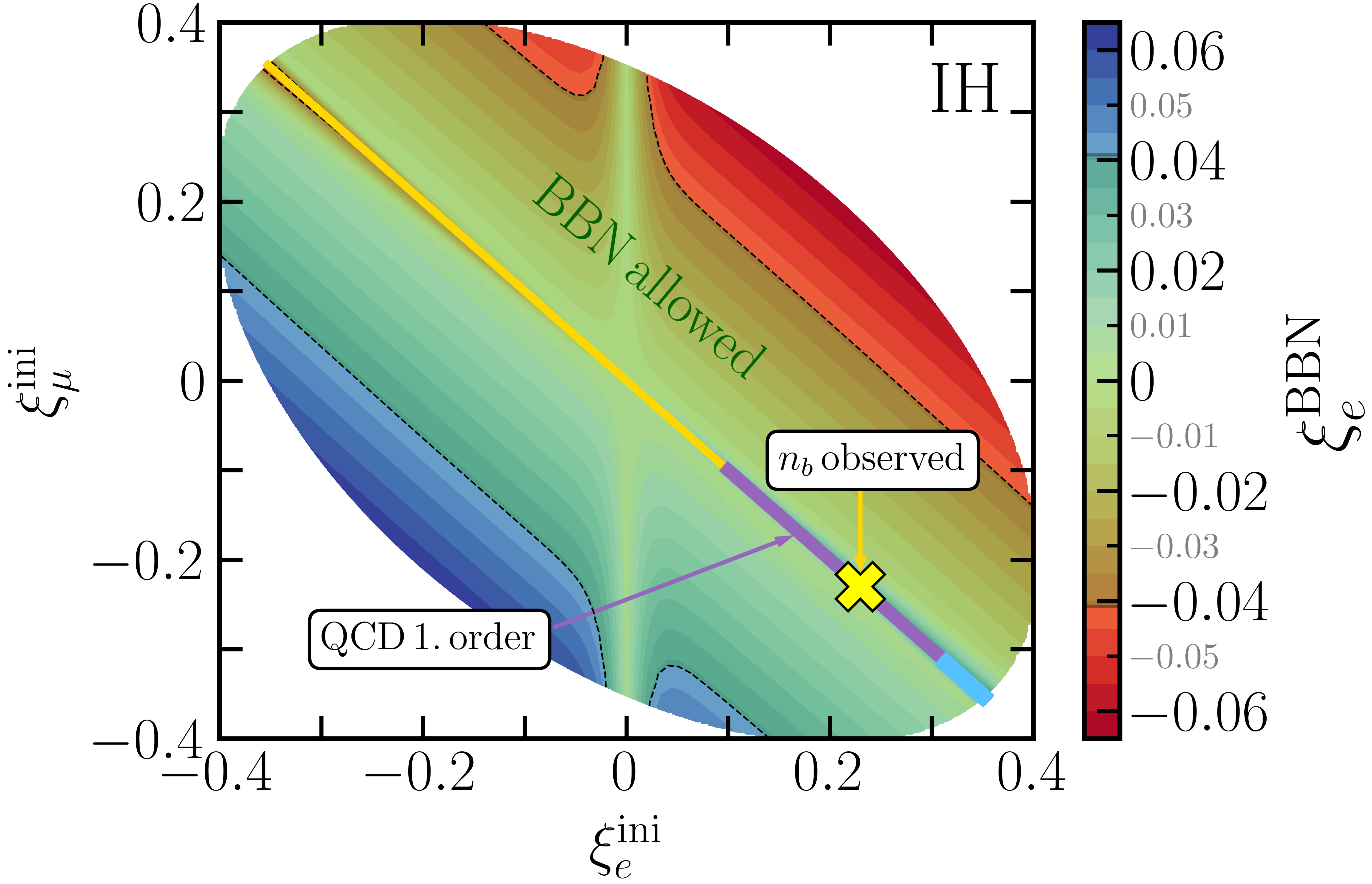
$$\xi_e^{\rm ini} = -\xi_\mu^{\rm ini} \approx 0.23\,, \quad \xi_\tau \approx 0$$

[Mukaida, Schmitz, Yamada, '21]

Our results NH (adiabatic)

$$\xi_e^{\rm ini} = -\xi_\mu^{\rm ini} \lesssim 0.5$$

Early Universe implications



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$$\xi_e^{\rm ini} = -\xi_\mu^{\rm ini} \approx 0.23\,, \quad \xi_\tau \approx 0$$

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Our results IH (non-adiabatic e-MSW)

$$\xi_e^{\rm ini} = -\xi_\mu^{\rm ini} \lesssim 0.3$$

Conclusions

- Lepton flavour asymmetries (LFAs) are well motivated from early Universe scenarios.
- By taking advantage of the dynamics of synchronous oscillations, we developed a consistent momentum-average formalism that captures the non-trivial dynamics of the system.
- This provides a fast and reliable way to study the evolution of primordial LFAs from the onset of neutrino oscillations to BBN times.

→ Public code COFLASY (Mathematica and C++)

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- Systematic study of the total lepton number conserving case $L \approx 0$:

→ Minimal washout factor:

$$\Delta N_{\rm eff} \lesssim 0.14\,(0.125) \sum_\alpha (\xi_\alpha^{\rm ini} + \frac{(\xi_\alpha^{\rm ini})^3}{\pi^2})^2$$

$$\xi_e^{\rm BBN} \lesssim 0.171\,(0.125) \left(\sum_\alpha (\xi_\alpha^{\rm ini} + \frac{(\xi_\alpha^{\rm ini})^3}{\pi^2})^2\right)^{\frac{1}{2}}$$

→ Flavour equilibration more efficient for particular directions in flavour space

→ Leptoflavourgenesis + 1st order QCD phase transition close to BBN limits!

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Thank you!

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Backup: actual equations

$$
\begin{aligned}
&Hxr' + 3Hxr\frac{z'_\nu}{z_\nu} = \\
&\quad - i\left[\frac{\pi^2}{36\zeta(3)}UM^2U^\dagger\frac{1}{z_\nu}\left(\frac{x}{T_{\rm ref}}\right) - 2\sqrt{2}\frac{7\pi^4}{180\zeta(3)}\frac{G_F}{m_W^2}z_\nu z_\gamma^4\left(\frac{T_{\rm ref}}{x}\right)^5(\mathbb{E}+\mathbb{P}) - \frac{3\zeta(3)G_F}{2\pi^2\sqrt{2}}z_\nu^3\left(\frac{T_{\rm ref}}{x}\right)^3\bar{r}, r\right] + \langle\mathcal{I}\rangle\,, \\
&Hx\bar{r}' + 3Hx\bar{r}\frac{z'_\nu}{z_\nu} = \\
&\quad + i\left[\frac{\pi^2}{36\zeta(3)}UM^2U^\dagger\frac{1}{z_\nu}\left(\frac{x}{T_{\rm ref}}\right) - 2\sqrt{2}\frac{7\pi^4}{180\zeta(3)}\frac{G_F}{m_W^2}z_\nu z_\gamma^4\left(\frac{T_{\rm ref}}{x}\right)^5(\mathbb{E}+\mathbb{P}) - \frac{3\zeta(3)G_F}{2\pi^2\sqrt{2}}z_\nu^3\left(\frac{T_{\rm ref}}{x}\right)^3 r, \bar{r}\right] + \langle\bar{\mathcal{I}}\rangle\,.
\end{aligned}
\tag{17}
$$

and

$$
z'_\nu = -\frac{z_\nu}{4}\left[\frac{\mathrm{Tr}[r'+\bar{r}']}{\mathrm{Tr}[r+\bar{r}]} - \frac{\frac{1}{H}\frac{\delta\varepsilon}{\delta t}}{x\varepsilon_\nu}\right]\,, \quad z'_\gamma = \frac{z_\gamma}{x} - \frac{4\varepsilon_\gamma + 3(\varepsilon_e + p_e) - \frac{1}{H}\frac{\delta\varepsilon}{\delta t}}{T_{\rm ref}\frac{\partial}{\partial T_\gamma}(\varepsilon_\gamma + \varepsilon_e)}\,. \tag{18}
$$

$$
x = T_{\rm ref}/T_{\rm cm}\,, \quad y = k/T_{\rm cm}\,,
$$

$$
r^{0\prime} = c^0\,, \quad r^{i\prime} = 2f^{ijk}r^k\left(h_0^j + v_c^j - v_s\bar{r}^j\right) + c^i\,,
$$

$$
z_\gamma = T_\gamma/T_{\rm cm}\,, \quad z_\nu = T_\nu/T_{\rm cm}\,.
$$

$$
\bar{r}^{0\prime} = \bar{c}^0\,, \quad \bar{r}^{i\prime} = -2f^{ijk}\bar{r}^k\left(h_0^j + v_c^j - v_s r^j\right) + \bar{c}^i\,.
$$

Backup: Collisions

- In our momentum average ansatz, collisions can be casted in analytical form in the limit $m_e \to 0$

$$\langle \mathcal{I}_{12\leftrightarrow 34}\rangle = \frac{1}{\int \frac{\mathrm{d}^3\vec{k}_1}{(2\pi)^3} f_1} \int \frac{\mathrm{d}^3\vec{k}_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3\vec{k}_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3\vec{k}_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3\vec{k}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\times S|M|^2 \times F(f_1,f_2,f_3,f_4)\,. \tag{B3}$$

$$\langle \mathcal{I}\rangle^{\nu e\leftrightarrow \nu e} = 4F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}z_\gamma^4 z_\nu(1-\epsilon)\left\{[G^L r G^L(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^L r G^L] - [rG^L(\mathbb{1}-\epsilon\, r)G^L + G^L(\mathbb{1}-\epsilon\, r)G^L r]\right\} \tag{C5}$$

$$\begin{aligned}\langle \mathcal{I}\rangle^{\nu\bar\nu\leftrightarrow e^+e^-} &= F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}\frac{1}{z_\nu^3}\left\{z_\gamma^8[G^L(\mathbb{1}-\epsilon\,\bar r)G^L(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^L(\mathbb{1}-\epsilon\,\bar r)G^L] - z_\nu^8(1-\epsilon)^2[rG^L\bar r G^L + G^L\bar r G^L r]\right\}\\ &+ F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}\frac{1}{z_\nu^3}\left\{z_\gamma^8[G^R(\mathbb{1}-\epsilon\,\bar r)G^R(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^R(\mathbb{1}-\epsilon\,\bar r)G^R] - z_\nu^8(1-\epsilon)^2[rG^R\bar r G^R + G^R\bar r G^R r]\right\}\end{aligned} \tag{C6}$$

$$\langle \mathcal{I}\rangle^{\nu\bar\nu\leftrightarrow \nu\bar\nu} = \frac{1}{4}F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}z_\nu^5\left\{2\mathrm{Tr}[r\bar r][\mathbb{1}-\epsilon\,(r+\bar r)] - (r\bar r+\bar r r)(\mathrm{Tr}[\mathbb{1}]-\epsilon\,\mathrm{Tr}[r+\bar r])\right\}\,, \tag{C7}$$

where as in the main text $T = T_{\rm cm}$, $z_\gamma \equiv T_\gamma/T_{\rm cm}$ and $z_\nu \equiv T_\nu/T_{\rm cm}$. In the limit $F_{\rm FD}\to 1$ and $\epsilon \to 0$, we recover the MB limit outlined in the main text in Eqs. (22), (23), and (24), respectively.

$G^L = \mathrm{diag}(g_L, g_L-1, g_L-1)$ $\quad G^R = \mathrm{diag}(g_R, g_R, g_R)$ $\quad g_L = s_W^2 + 1/2$ and $g_R = s_W^2$

Backup: Initial conditions

- Due to our ansatz for density matrices, any initial conditions is necessarily an approximation to the real system. We choose (to preserve the delicate dynamics of non-daiabatic MSW):

$$r_{\alpha\alpha} = \frac{\int y^2 dy\, f_{\rm FD}(y,\xi_\alpha)}{\int y^2 dy\, f_{\rm FD}(y,0)} = \frac{-2\mathrm{Li}_3(-e^{\xi_\alpha})}{\frac{3\zeta(3)}{2}}\,,$$

$$\bar{r}_{\alpha\alpha} = \frac{\int y^2 dy\, f_{\rm FD}(y,-\xi_\alpha)}{\int y^2 dy\, f_{\rm FD}(y,0)} = \frac{-2\mathrm{Li}_3(-e^{-\xi_\alpha})}{\frac{3\zeta(3)}{2}}\,.$$

- Take into account $T_\nu \neq T_{\rm cm}$ and enforce detailed balance (thermal and chemical eq.)

$$r^{\rm ini}_{\alpha\alpha} = -\frac{4z_\nu^{-3}}{3\zeta(3)}\mathrm{Li}_3\left(-\mathrm{e}^{\xi_\alpha}\right) + \epsilon^\alpha_{\rm db}\,, \quad \bar{r}^{\rm ini}_{\alpha\alpha} = -\frac{4z_\nu^{-3}}{3\zeta(3)}\mathrm{Li}_3\left(-\mathrm{e}^{-\xi_\alpha}\right) + \epsilon^\alpha_{\rm db} \quad \text{(initial conditions)}\,. \tag{31}$$

Backup: $\xi_e^{\rm BBN}$ explain

- Electron neutrino chemical potential connected numerically to helium and deuterium abundances, see e.g. Pitrou *et al* [1801.08023], Froustey and Pitrou [2405.06509]:

The net effect on helium-4 and deuterium abundances coming from an electron (anti)neutrino chemical potential during BBN was estimated numerically in [27]:

$$\frac{Y_{\rm p}}{Y_{\rm p}|_{\xi_e=0}} \simeq e^{-0.96\xi_e^{\rm BBN}} \,, \qquad \frac{{\rm D/H}}{{\rm D/H}|_{\xi_e=0}} \simeq e^{-0.53\xi_e^{\rm BBN}} \,. \quad (16)$$

$$\begin{aligned}\xi_e^{\rm BBN} \sim \left[\frac{6(n_e-\bar{n}_e)}{T_\gamma^3}\right]_{\rm FO} &= \left[\frac{6(n_e-\bar{n}_e)}{z^3 T_{\rm cm}^3}\right]_{\rm FO} \\ &= 6\left[\frac{\eta_e}{z^3}\right]_{\rm FO} \qquad (17) \\ &= 6\left[\frac{\eta_e}{z^3}\right]_{\rm final} \times \left(\frac{z_{\rm final}}{z_{\rm FO}}\right)^3 ,\end{aligned}$$

Backup: Comparison with momentum-dependent

- Dataset of about 2300 points computed with full momentum-dependent code by Froustey and Pitrou [2405.06509]:

$$\delta n_\alpha \equiv \max\left[\left|\frac{(n-\bar{n})_{\alpha\alpha}^{\text{this work}}}{(n-\bar{n})_{\alpha\alpha}^{\text{Froustey+}}}\right|, \left|\frac{(n-\bar{n})_{\alpha\alpha}^{\text{Froustey+}}}{(n-\bar{n})_{\alpha\alpha}^{\text{this work}}}\right|\right] - 1\,\Bigg|_{\text{BBN}},$$

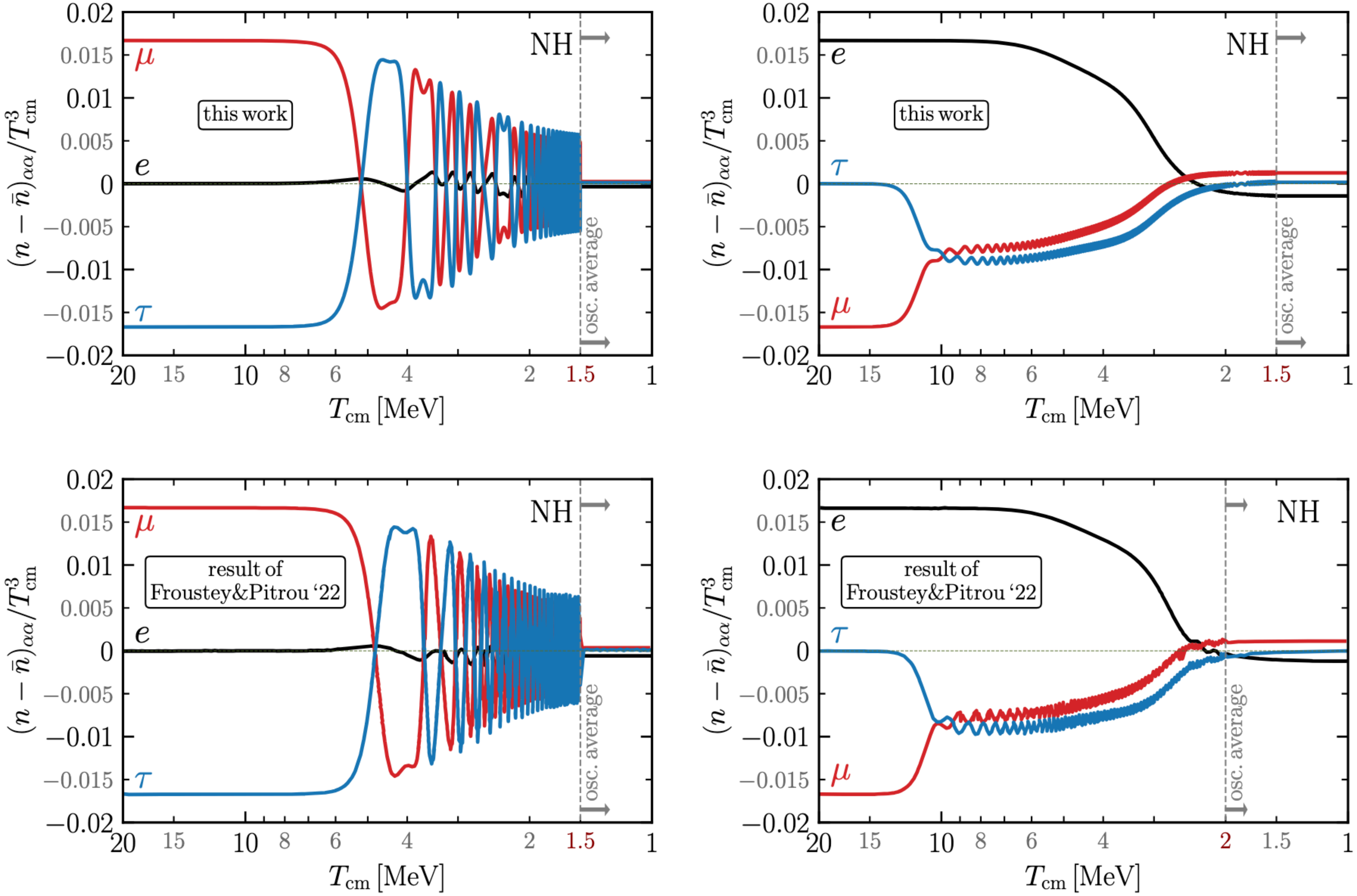
- 85% of points $\longrightarrow$ $\delta n_\alpha \leq 1$ out of this 47%(11%) $\longrightarrow$ $\delta n_\alpha \leq 0.1(0.01)$

- From the remaining 15% which show poor agreement, 83% are excluded

- Focusing on the points allowed (at 95% CL), we find that the deviation from the momentum-dependent code is well modeled by a Gaussian distribution with zero mean and a standard deviation of 0.016:

$$\xi_e^{\text{BBN}} = 0.001 \pm 0.013\ (\text{exp}) \pm 0.016\ (\text{th}) \quad (@\,1\sigma)\,,$$

Backup: Comparison with momentum-dependent



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- 99% of points $\longrightarrow$ $\delta N_{\rm eff} \leq 1$ out of this 60% (1%) $\longrightarrow$ $\delta N_{\rm eff} \leq 0.1(0.01)$

- We conclude that in this case the theoretical uncertainty is subdominant compared to current experimental uncertainty.

Backup: Comparison with momentum-dependent

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- 85% of points $\longrightarrow$ $\delta n_\alpha \leq 1$ out of this 47%(11%) $\longrightarrow$ $\delta n_\alpha \leq 0.1(0.01)$

- Focusing on the remaining 15% which show poor agreement, 83% are excluded, and 43% fall in particular flavour directions:

$$\Delta n_\tau = \Delta n_\mu \simeq -\Delta n_\mu / c \qquad c = 2,\ 1.83$$