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Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

Interplay of vertical and horizontal gauge symmetry for a high-quality axion

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Based on 2503.16648 with L. Di Luzio, G. Landini, F. Mescia

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Motivation — Peccei-Quinn and axions

- **Strong CP problem**:
  - → Standard Model (SM) allows the term

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\bar{\theta}\,G_{\mu\nu}\tilde{G}^{\mu\nu}; \qquad \bar{\theta} = \theta + \arg\det M_q \tag{1}$$

  - → Bound from neutron EDM: $\boxed{\bar{\theta} \lesssim 10^{-10}}$. Why is $\bar{\theta}$ so small?

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- **Axion solution**: via **Peccei-Quinn** (PQ) [1, 2, 3, 4]
  (1) Implement a global $\mathrm{U}(1)_{\mathsf{PQ}}$ anomalous under QCD
  (2) Break $\mathrm{U}(1)_{\mathsf{PQ}}$: low energy theory has a Goldstone boson a — the **axion**

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}(\bar{\theta} + N a/f_a)\, G_{\mu\nu}\tilde{G}^{\mu\nu} \tag{2}$$

  (3) Axion potential $\mathcal{V}_a$ and mass m_a generated by QCD instantons.

$$\mathcal{V}_a(a) \text{ minimum:} \quad \bar{\theta} + N\langle a\rangle/f_a = 0, \qquad m_a f_a \simeq m_\pi f_\pi \tag{3}$$

  (4) Hence $\boxed{\bar{\theta}_{\mathsf{eff}} = 0}$ and strong CP is conserved.

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  - → ~~**PQ**~~: non-renormalizable $\mathcal{O}$ from e.g. gravity (M_{Planck} suppressed)
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- **This talk**: PQ quality from vertical and horizontal symmetry
  - $\rightarrow$ More strongly connected to SM than adding unrelated symmetry
  - $\rightarrow$ Vertical and horizontal symmetry for Yukawa sector: Berezhiani [7, 8, 9]
  - $\rightarrow$ Vertical for quality: Vecchi [10], Babu+ [11]
  - $\rightarrow$ Horizontal for quality: Darmé+ [12, 13]

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  - → Pati-Salam (PS) easier to **work out details**; only R-flavor gauged
  - → **Interesting features**: anomalons, quality mechanism can be probed ...

Constructing the model

- Field content: $G_{\text{PS}} \equiv \mathrm{SU}(4)_{\text{PS}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

Field	Lorentz	$G_{\text{PS}} \times \mathrm{SU}(3)_{f_R}$	copies	$\mathrm{U}(1)_{\text{PQ}}$
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- Not much freedom in choice of irreps for a realistic model. . .

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- **Fermions**:
  - $\rightarrow$ $\overline{Q}_L$ and Q_R contain SM fermions and ν_R (standard in PS)
  - $\rightarrow$ only Q_R transforms under gauged flavor, $\overline{Q}_L$ in 3 copies

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- **Anomalons** Ψ: $\rightarrow$ Needed to cancel anomalies due to $\text{SU}(3)_{f_R}$ factor

  $\rightarrow$ They are SM singlets

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- Scalars with EW VEVs Φ and Σ: contain **SM Higgs**

  $\rightarrow$ terms $\overline{Q}_L\, Q_R\, \Phi$ and $\overline{Q}_L\, Q_R\, \Sigma$ for realistic Yukawa sector

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- Scalar Δ: $\rightarrow$ for ν **Majorana mass** term $Q_R\, Q_R\, \Delta^*$

  $\rightarrow$ involved in PS and flavor breaking

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- Scalar χ: $\rightarrow$ helps Δ with PS and flavor braking

  $\rightarrow$ to **break PQ and rank of PS**: need Δ and χ both

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- Scalar ξ: $\rightarrow$ needed for **accidental PQ** to arise

  $\rightarrow$ connects Δ and χ in scalar potential: $\Delta \chi^2 \xi$

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- PQ accidental in

$$\mathcal{V}_{\mathbb{C}} = \Phi\Sigma^*\xi + \Phi\Sigma^*\left(|\Sigma|^2 + |\Delta|^2 + |\chi|^2 + \xi^2\right) + \Sigma^{*2}\left(\Phi^2 + \Delta^2\right) + \Delta\chi^2\xi + \text{h.c.}$$

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- Breaking: $G_{\text{PS}} \times \mathrm{SU}(3)_{f_R} \xrightarrow{\langle \Delta, \chi, \xi \rangle} G_{\text{SM}} \xrightarrow{\langle \Phi, \Sigma \rangle} \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\text{EM}}$

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- **Procedure** for finding invariant operators $\mathcal{O}$:
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  (1) Necessary condition: trivial under gauge center $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$
  (2) Automatized confirmation (`GroupMath` [14], `LieART` [15, 16], `LiE` [17])
  (3) Explicit index contraction for confirming vacuum contribution $\langle \mathcal{O} \rangle \neq 0$

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- VEVs:

$$\langle \Phi \rangle, \langle \Sigma \rangle \sim v \qquad \sim 10^2\,\text{GeV} \qquad (\not{EW}\ \text{VEVs}), \tag{4}$$

$$\langle \Delta \rangle, \langle \chi \rangle, \langle \xi \rangle \equiv V_\Delta, V_\chi, V_\xi \qquad \sim V \qquad (\not{PS}\ \text{VEVs}), \tag{5}$$

$$\Lambda \equiv M_{\text{Planck}} \qquad \sim 10^{19}\,\text{GeV} \qquad (\text{cutoff}) \tag{6}$$

PQ quality — part 2

- **High-quality condition**: small shift in QCD-generated axion potential

$$\langle \mathcal{O} \rangle \lesssim \bar{\theta}\, \chi^4_{\text{QCD}} \quad \equiv \bar{\theta}\, \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \simeq 10^{-10}\, (76\,\mathrm{MeV})^4. \tag{7}$$

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- Dominant $\not{P}\!\!\not{Q}$ contributions:

$\mathcal{O}$	$\langle \mathcal{O} \rangle$
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PQ quality — part 2

- **High-quality condition**: small shift in QCD-generated axion potential

$$\langle \mathcal{O} \rangle \lesssim \bar{\theta}\, \chi^4_{\text{QCD}} \quad \equiv \bar{\theta}\, \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \simeq 10^{-10}\,(76\,\text{MeV})^4. \qquad (7)$$

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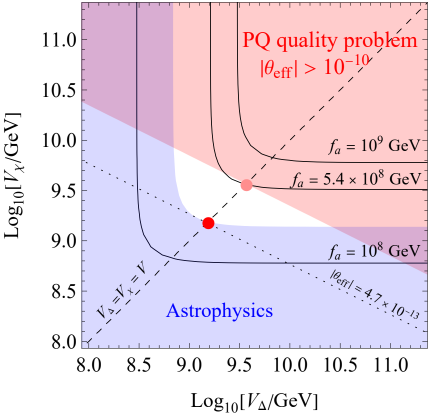
- Axion decay constant f_a:

$$f_a = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2 + 4V_\Delta^2}} \qquad (8)$$

- Constraints: quality and astro



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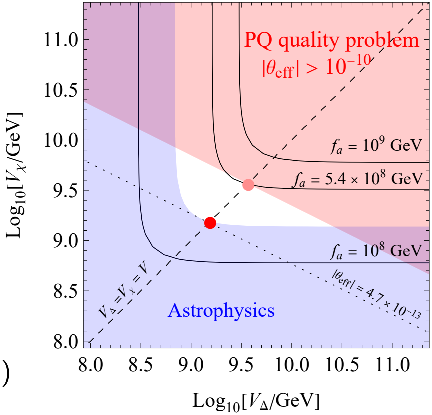
- Window for $V_\Delta = V_\chi \equiv V$:

$$V \in [1.5, 3.7] \times 10^9\,\text{GeV} \quad (8)$$

(larger V degrades quality, but possible)



Axion properties

- Axion a: (mostly) a combination of polar modes in Δ and χ
- Couplings to photons and gluons:

$$\mathcal{L} \supset \frac{\alpha_{\mathrm{EM}} E}{4\pi} \frac{a}{f_a} F\tilde{F} + \frac{\alpha_s N}{4\pi} \frac{a}{f_a} G\tilde{G}. \tag{9}$$

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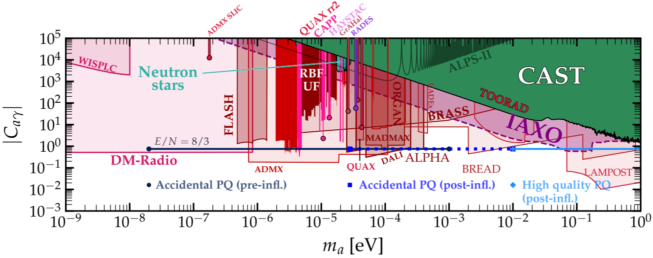
- Mass range: **high-quality** PQ vs **accidental** PQ scenario



high-quality:

$$m_a \gtrsim 10\,\mathrm{meV}$$

Yukawa sector — up, down, charged leptons

- Renormalizable operators for UDE sectors:

$$\mathcal{L}_Y \supset \sum_{\alpha=1}^{N_\Phi} \sum_{I=1}^{3} Y_I^{\Phi^\alpha} \, \overline{Q}_L{}^I Q_R \, \Phi^\alpha + \sum_{\alpha=1}^{N_\Sigma} \sum_{I=1}^{3} 2\sqrt{3}\, Y_I^{\Sigma^\alpha} \, \overline{Q}_L{}^I Q_R \, \Sigma^\alpha \tag{10}$$

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- $\boxed{N_\Phi \geq 1,\ N_\Sigma \geq 2}$ is **realistic** (based on DOF count, not fit)

[considering masses, CKM, PMNS]

$$(M_U)_{IA} = Y_I^{\Phi} v_A^{u\Phi} + \ Y_I^{\Sigma} v_A^{u\Sigma} + \ Y_I^{\Sigma'} v_A^{u\Sigma'}, \qquad (11)$$

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- **Unusual** setup: $\to$ $\boxed{Y_I}$ are **vectors** in family space (not matrices)

$\to$ EW VEVs $\boxed{v_A}$ are R-flavor **vectors** (not numbers)

Yukawa sector — neutrinos and anomalons

- Anomalons Ψ are EW (and SM) singlets $\Rightarrow \nu \leftrightarrow \Psi$ can mix
- Spectrum: (a) terms with Ψ: only from **non-renormalizable** $\mathcal{O}$
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- Dominant operators in every entry (schematically):

$$\begin{array}{c|cccc} & \overline{Q}_L & Q_R & \Psi_\perp & \Psi_0 \\ \overline{Q}_L & \Delta(\Phi^2+\Phi\Sigma+\Sigma^2) & \Phi+\Sigma & \chi(\Phi+\Sigma) & \Delta\chi^*(\Phi^*+\Sigma^*) \\ Q_R & \Phi+\Sigma & \Delta^* & \Delta^*\chi & \Delta^{*2}\Delta\chi \\ \Psi_\perp & \chi(\Phi+\Sigma) & \Delta^*\chi & \Delta^*\chi^2 & \Phi^{*2}+\Sigma^{*2}+\Delta\Delta^{*2}\chi^2 \\ \Psi_0 & \Delta\chi^*(\Phi^*+\Sigma^*) & \Delta^{*2}\Delta\chi & \Phi^{*2}+\Sigma^{*2}+\Delta\Delta^{*2}\chi^2 & \Phi^{*2}+\Sigma^{*2}+\Delta\Delta^{*2}\chi^2 \end{array}$$

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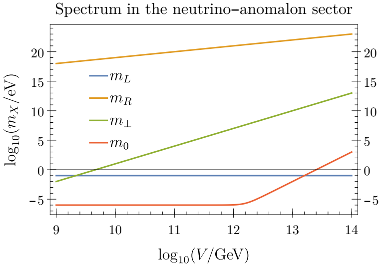
$\rightarrow$ In upper-left 2×2: **see-saw** type I



Anomalon cosmology (in high quality scenario)

- Anomalon mass $< 1\,\mathrm{eV}$: $m_\perp \simeq 10\,\mathrm{meV}$, $m_0 \simeq 1\,\mu\mathrm{eV}$
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  (ii) Conversion of ν_L via mixing $\theta_{\nu\Psi}$: $e^+e^- \to \bar{\nu}_f\nu_f \to \bar{\nu}_f\Psi_m$
  (iii) (generalized) Yukawa interaction: $\nu_L\Psi h$, $\Psi\Psi h$, $\nu_L\Psi hh$, $\Psi\Psi\phi$, etc.
  processes: $ff \xrightarrow{h} \Psi\Psi$, $\phi \to \Psi\Psi$, $\phi\phi \to \Psi\Psi$, etc.

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  processes: $ff \xrightarrow{h} \Psi\Psi$, $\phi \to \Psi\Psi$, $\phi\phi \to \Psi\Psi$, etc.
- **Challenge**: large enough relic abundance for measurable ΔN_{eff}, but no thermalization $\to$ only **(ii)** may be viable, requires dedicated analysis

Conclusions

(A) Quality from vertical and horizontal symmetry:
PQ-breaking scale $\leftrightarrow$ flavor and PS breaking scale

(B) **Technically challenging** to check PQ quality

(C) Presence of **anomalons** (fermions): they mix with neutrinos
$\rightarrow$ UV dynamics of PQ quality potentially testable (e.g. ΔN_{eff})

(D) However, also some **drawbacks** (not discussed):
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Thank you for your attention!

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