A Systematic Approach to Axion Production at Finite Density

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in collaboration with Konstantin Springmann, Stefan Stelzl and Andreas Weiler

ArXiv:2410.10945 & 2410.19902

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MAX-PLANCK-INSTITUT FÜR PHYSIK

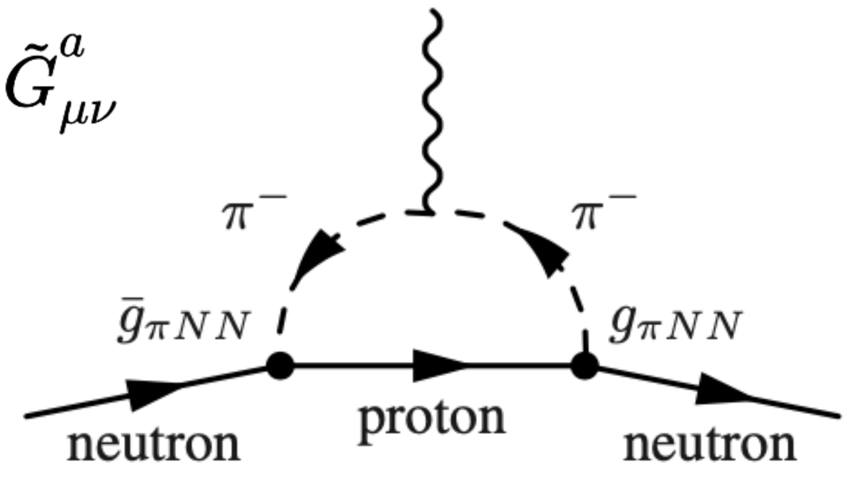
Outline

- Axion Introduction
- Axion-nucleon couplings in Chiral Perturbation Theory
- Density dependence of the axion-nucleon couplings
- Model (in)dependent astrophysical axion bounds

Why Axions?

CP violation in the strong sector

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}\left(i\not{D} - m_q e^{i\theta_q}\right)q - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} - \theta\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$$



Predicts neutron EDM

$$\mathcal{L}_\chi \supset d_n \bar{n}\sigma^{\mu\nu}\gamma_5 n F_{\mu\nu}$$

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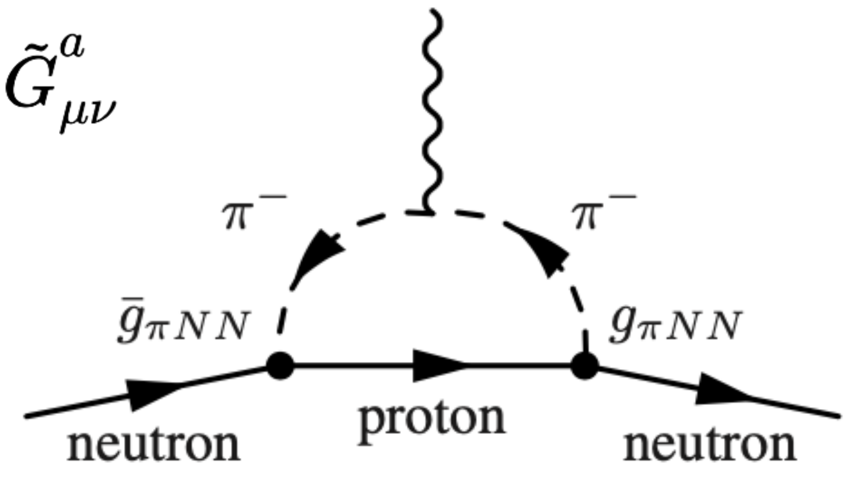

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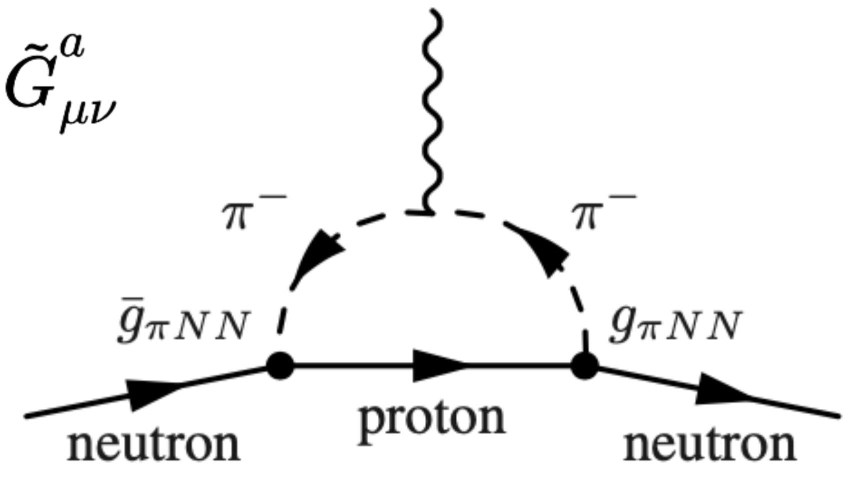

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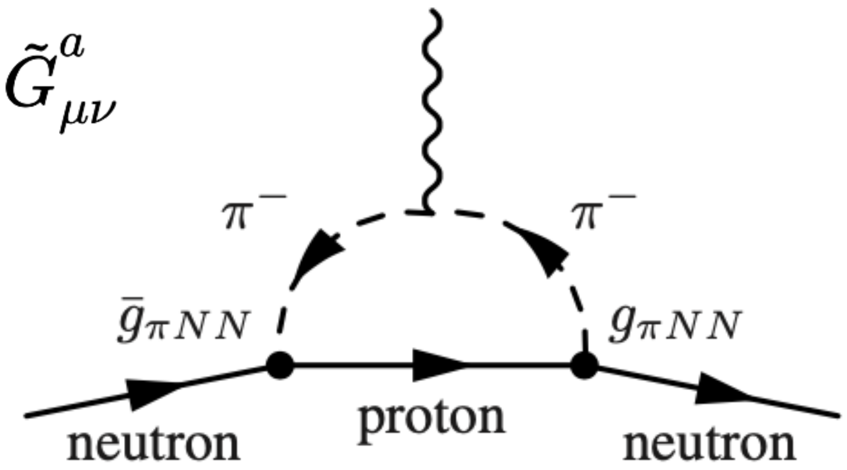

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Why so small???

QCD Axion explains this by promoting θ to a dynamical field $a(x)$

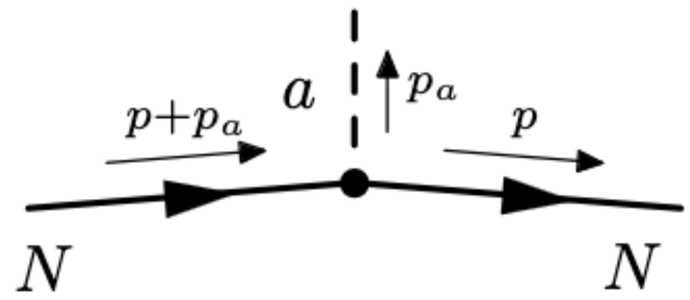
- Axion is very predictive
- Couplings to **nucleons**, photons, electrons, etc. determined by one scale f_a

Axion-Nucleon coupling

EFT valid for $p \ll m_\pi$

$$\mathcal{L} \supset \frac{1}{f_a} \bar{N} c_N S \cdot \partial a N$$

$$N = (p, n)^T$$



$$= \quad -\frac{1}{f_a} c_N S \cdot p_a .$$

Villadoro et.al. 15'

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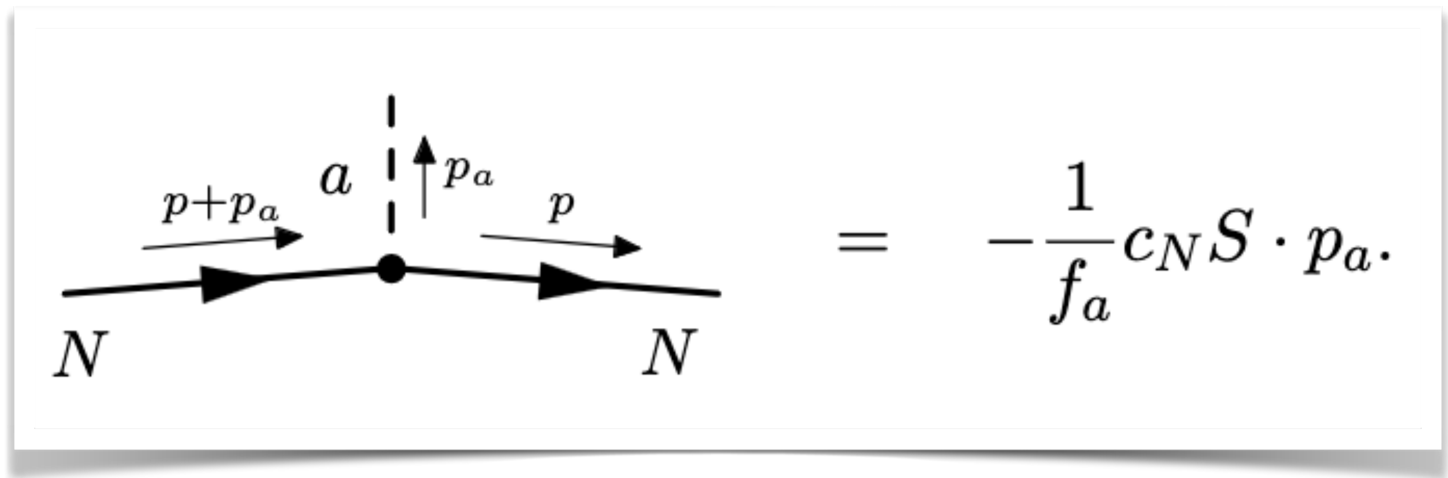

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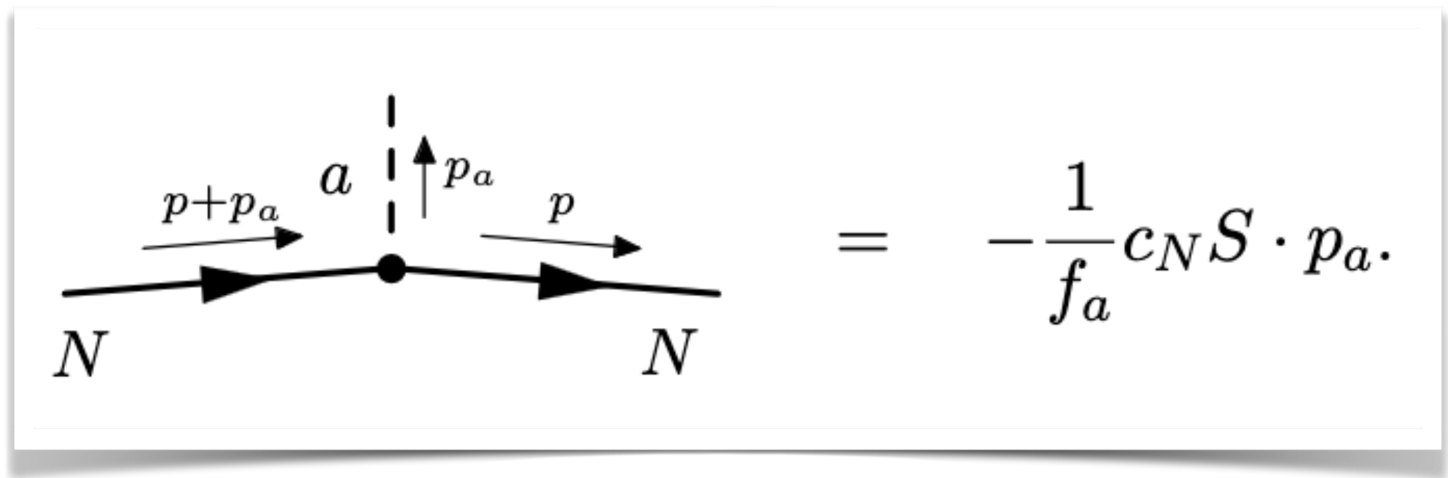

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$$c_p^{\mathrm{KSVZ}} = -0.47(3), \quad c_n^{\mathrm{KSVZ}} = +0.02(3)$$

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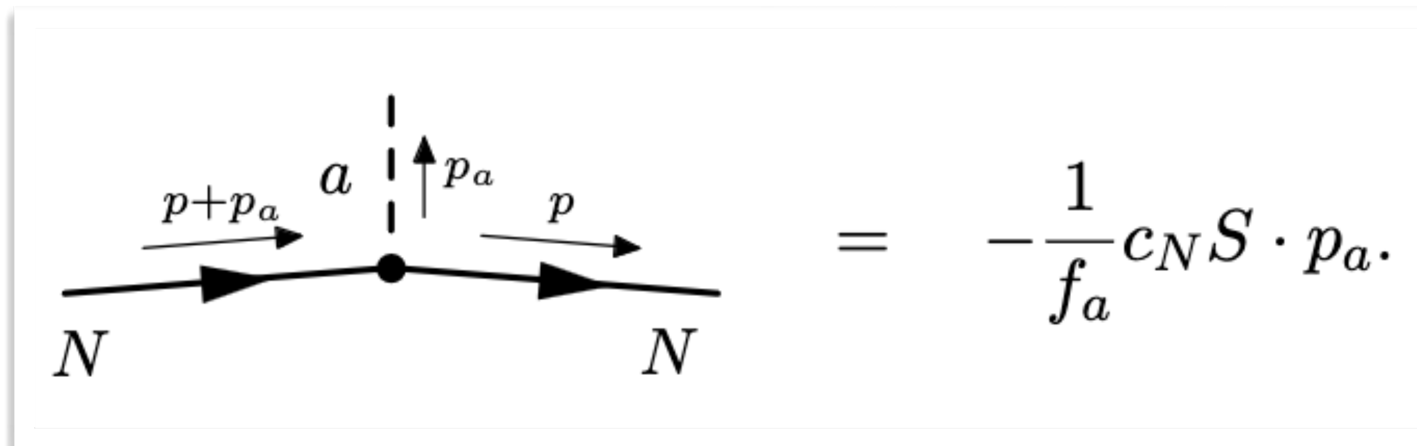

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Compatible with zero due to accidental cancellation

Is this EFT valid in astrophysical environments?

This Hubble Space Telescope image shows Supernova 1987A within the Large Magellanic Cloud

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Not really…

- Typical momenta $k_F \simeq (3\pi^2 n_0)^{1/3} \simeq 260\,\mathrm{MeV}$ $n_0 \simeq 0.16\,\mathrm{fm}^{-3}$

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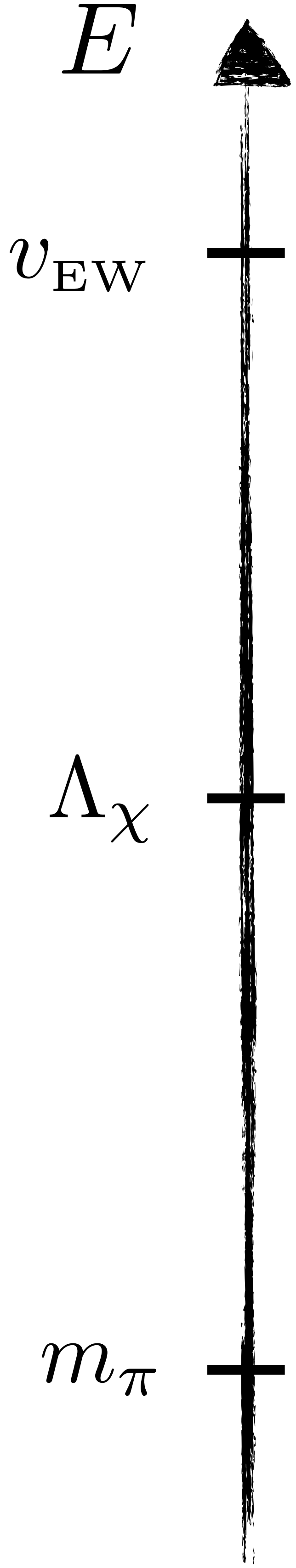
Need to construct EFT of pions and nuclons

Axion EFTs



$\mathscr{L}_{\mathrm{UV}}$: **Model dependent UV Lagrangian**

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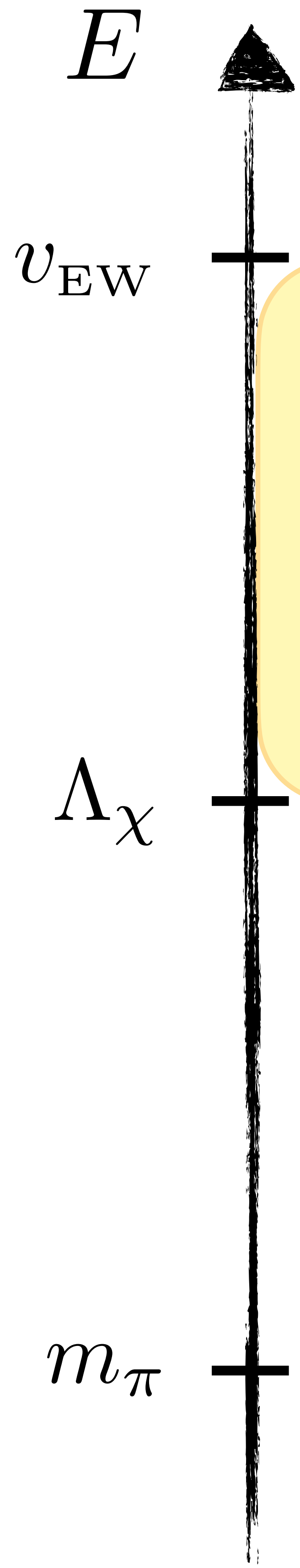

$\mathscr{L}_{\rm UV}$: Model dependent UV Lagrangian

After chiral quark rotation

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm QCD,0} - (\bar{q}_L M_a q_R + \text{h.c.}) + \frac{1}{2}(\partial a)^2 + \frac{\partial_\mu a}{2 f_a} J^\mu_{\rm PQ}$$

$$M_a \equiv e^{\frac{i a(x)}{2 f_a} Q_a} M_q e^{\frac{i a(x)}{2 f_a} Q_a} \qquad Q_a \equiv M_q^{-1} / \text{Tr}(M_q^{-1})$$

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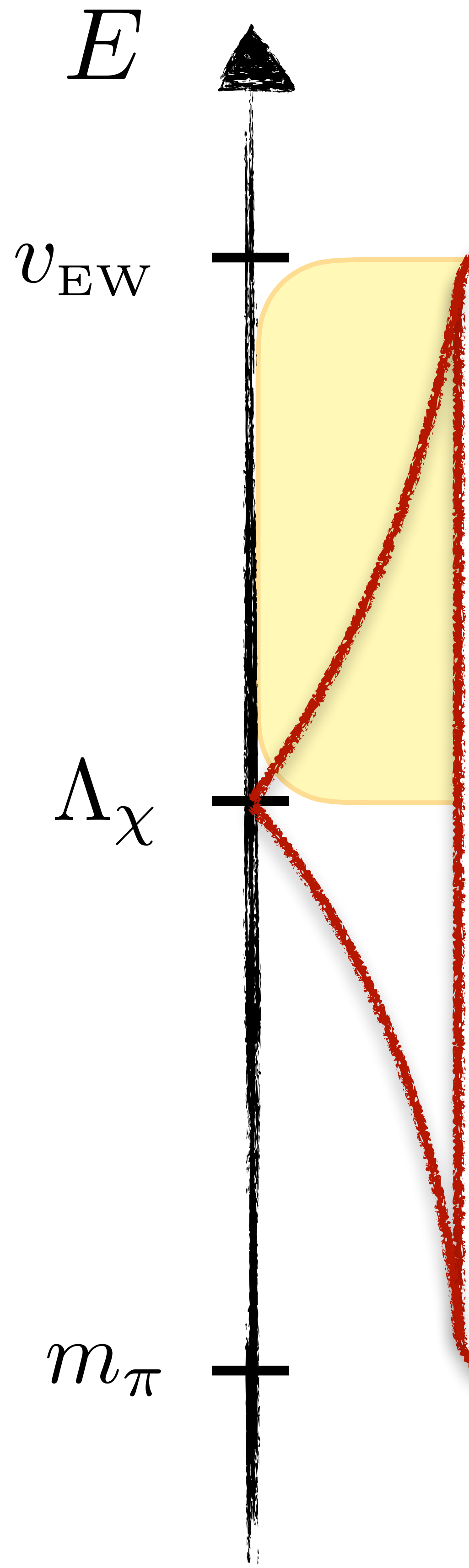


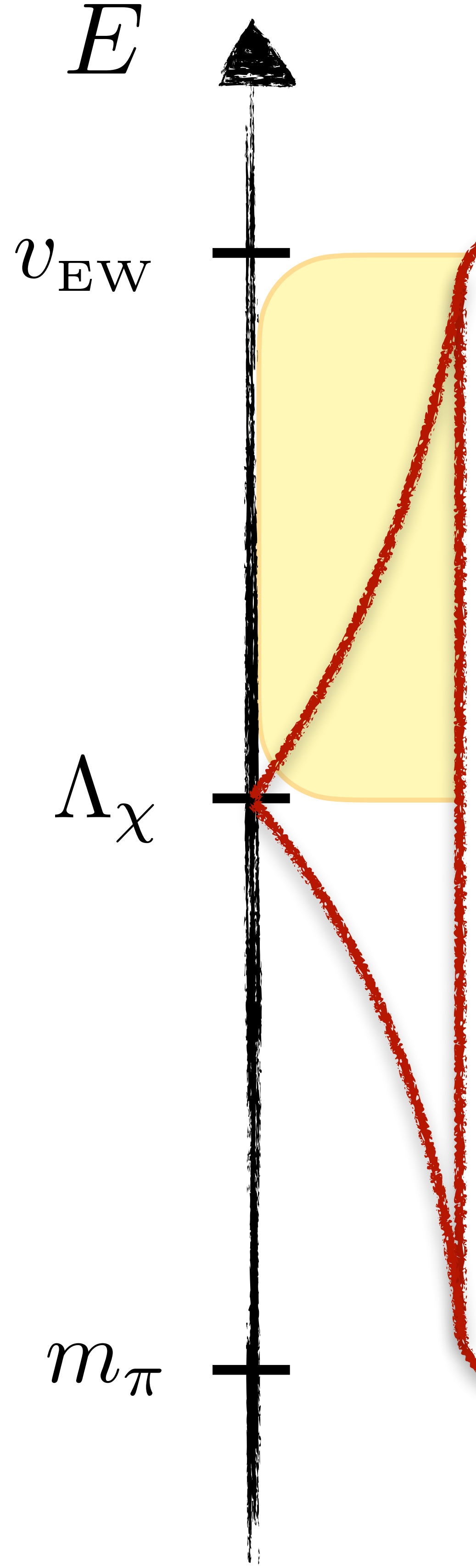
QCD confines: $|\langle \bar{q}_L q_R \rangle| \equiv B f_\pi^2$

$$G_\chi = U(2)_L \times U(2)_R \to U(2)$$

- EFT w/ Mesons and Baryons:

  Chiral Perturbation Theory: spurion analysis

Axion EFTs



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$$p^\mu = m_N v^\mu + k^\mu \qquad \Psi(x) = e^{-i m_N v \cdot x} \left[N_v(x) + H_v(x) \right]$$

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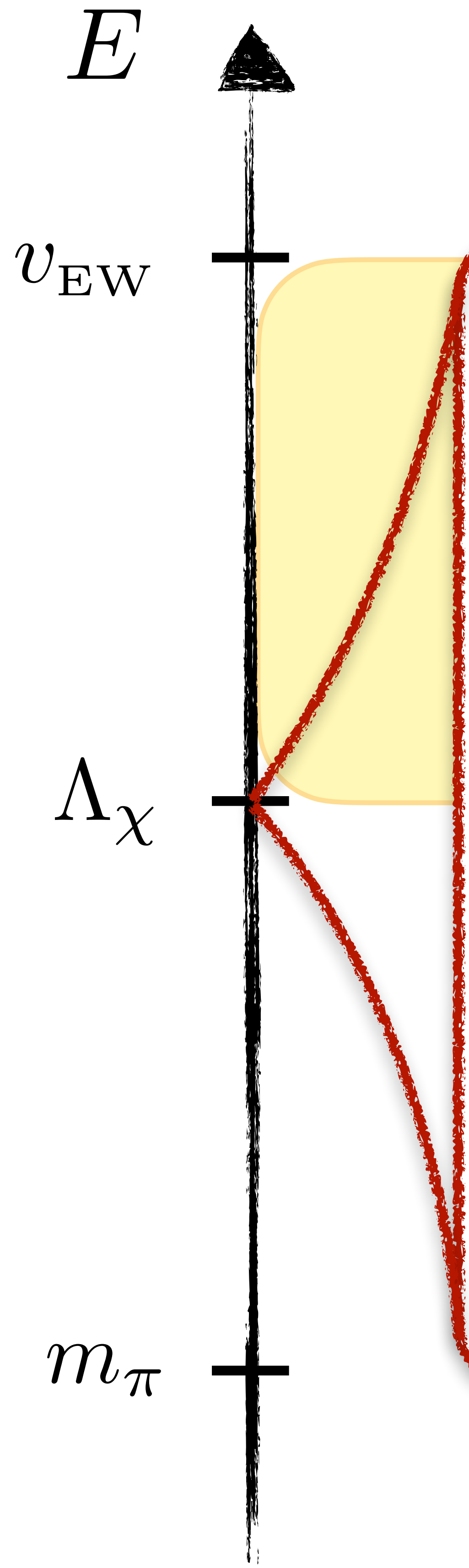

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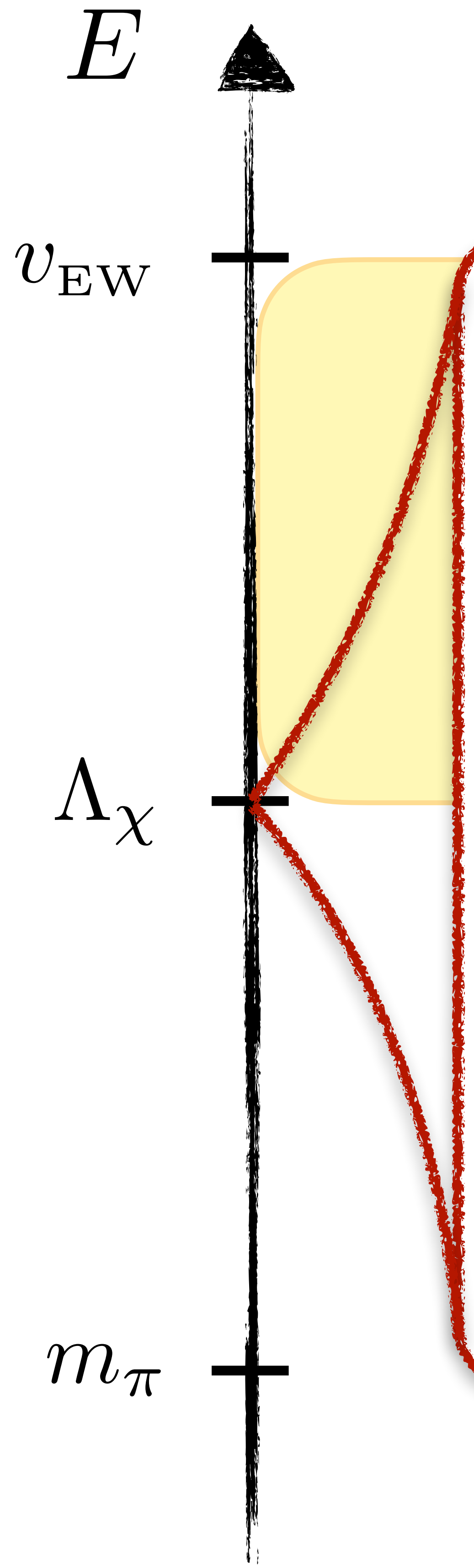

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$1\,\mathrm{GeV}$

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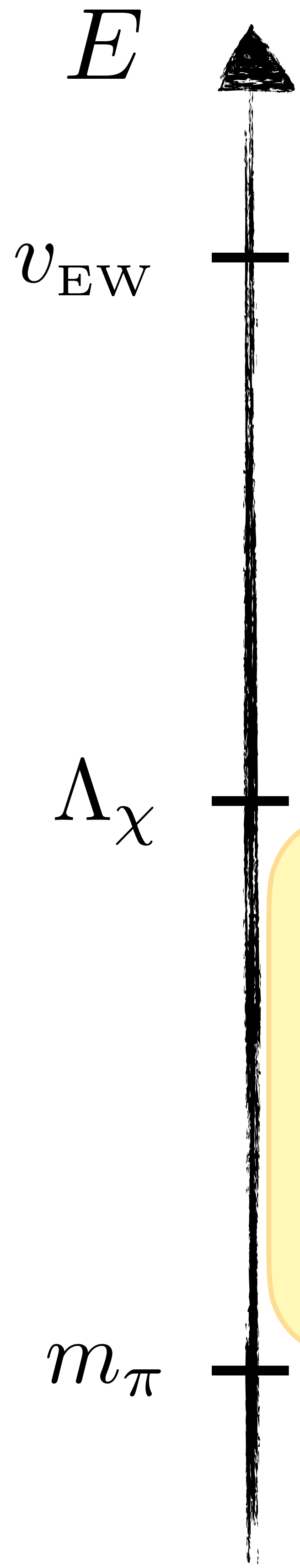
Resonances, e.g.

$\Lambda_\chi \sim (300 - 750)\,\mathrm{MeV}$

Axion EFTs



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$$\hat{u}_\mu = c_{u+d}\left(\frac{\partial_\mu a}{f_a}\right) + \ldots \qquad u_\mu = -\left(\frac{\partial_\mu \pi^a}{f_\pi}\right)\tau^a + c_{u-d}\left(\frac{\partial_\mu a}{f_a}\right)\tau_3$$

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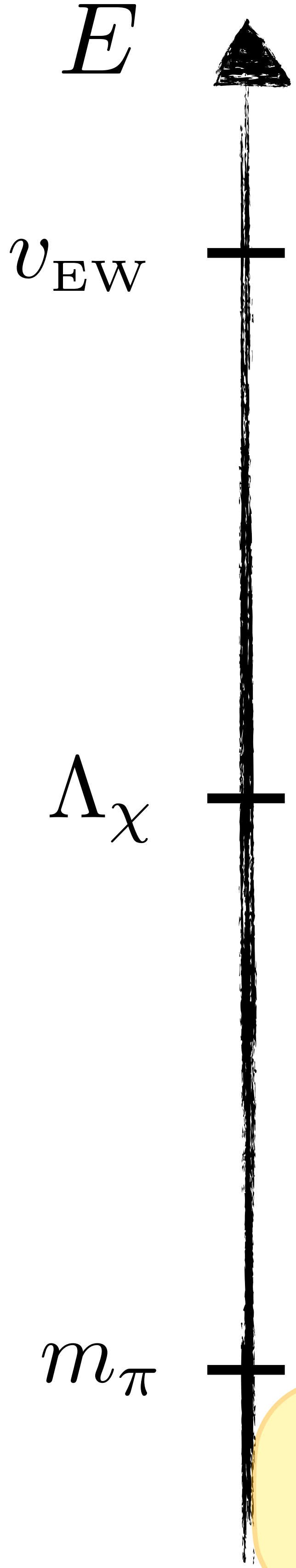

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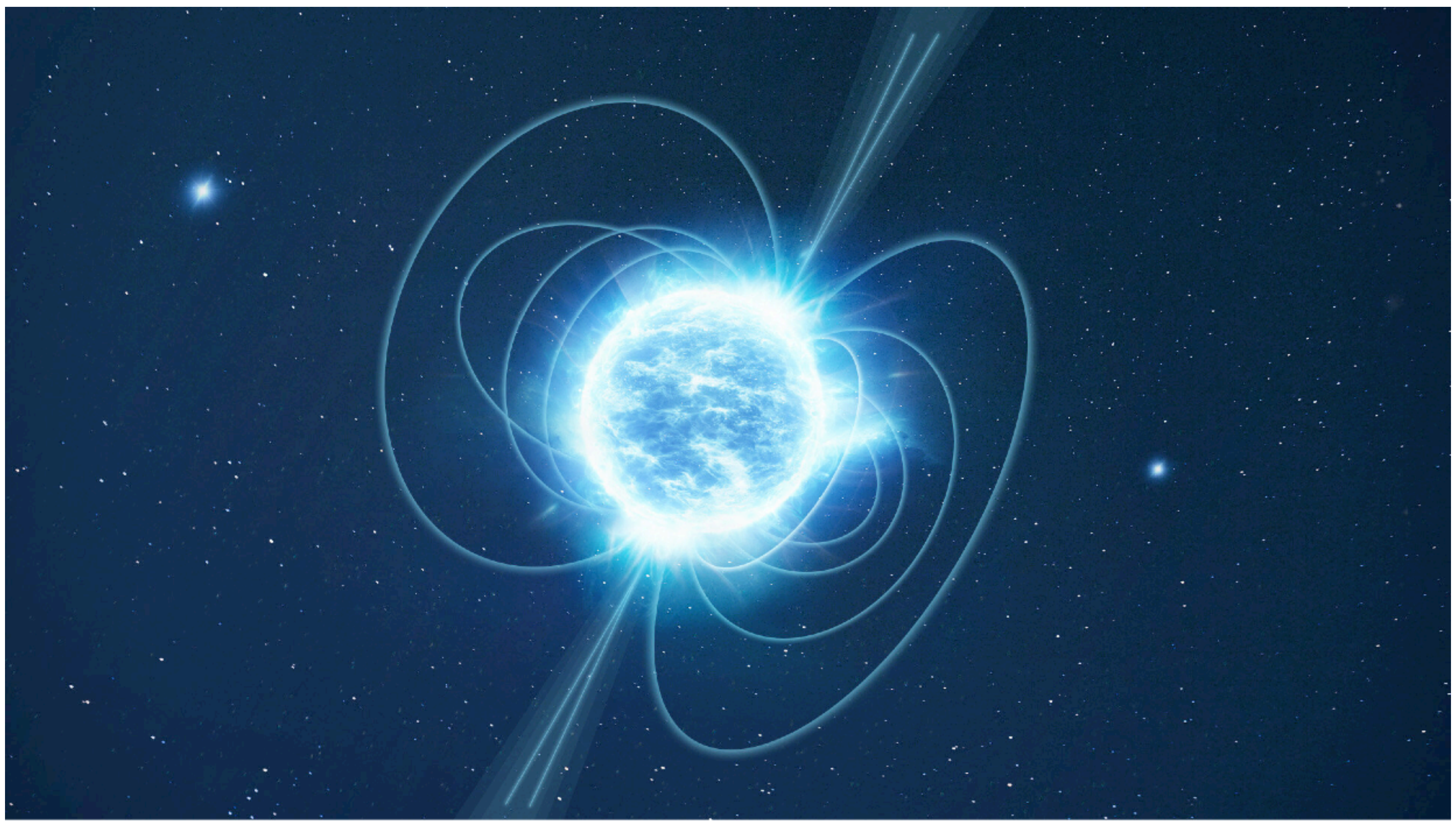
Integrate out pions: theory of baryons and axion

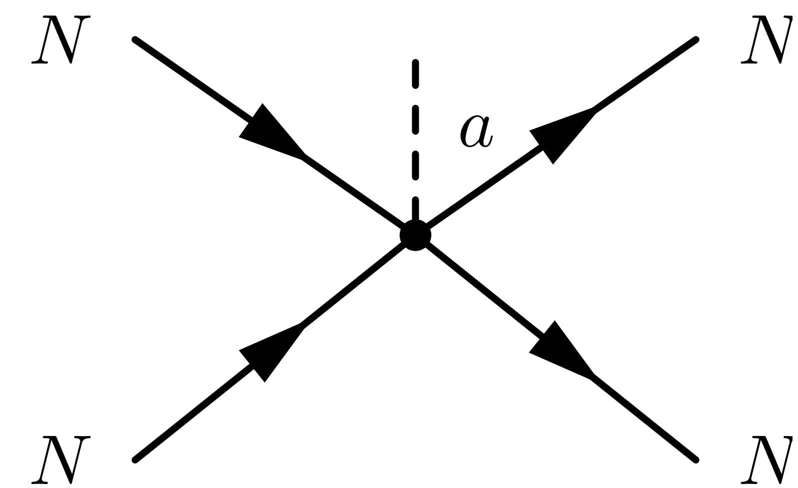
How does a density background change these couplings?



Axion-Nucleon Coupling: Finite density

- **Schematic example:**

$$\mathcal{L}^{(2)}_{\pi NN} = \frac{c_D}{2f_\pi^2\Lambda_\chi}(\bar{N}N)(\bar{N}S\cdot uN)$$



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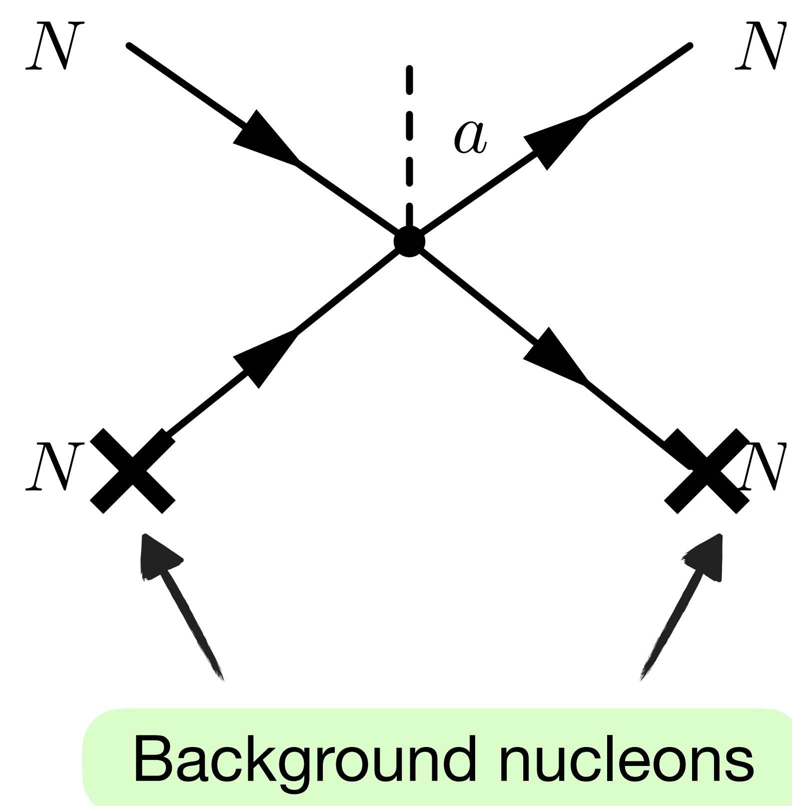

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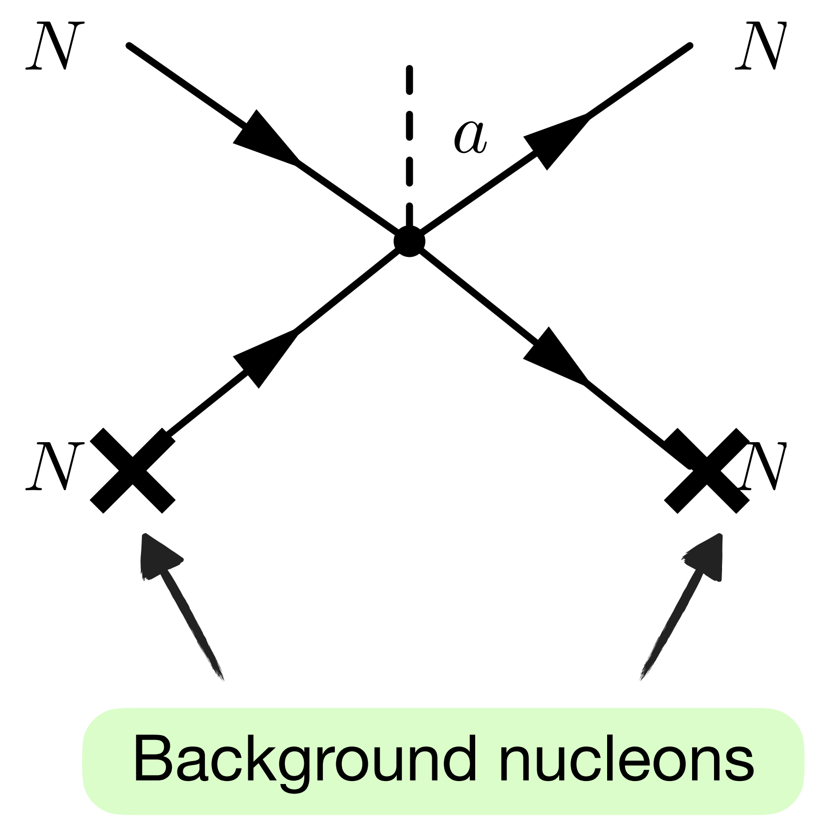
Number density



- **Gives contribution to coupling:** $\sim \dfrac{k_f^3}{(4\pi f_\pi)^2\Lambda_\chi}$

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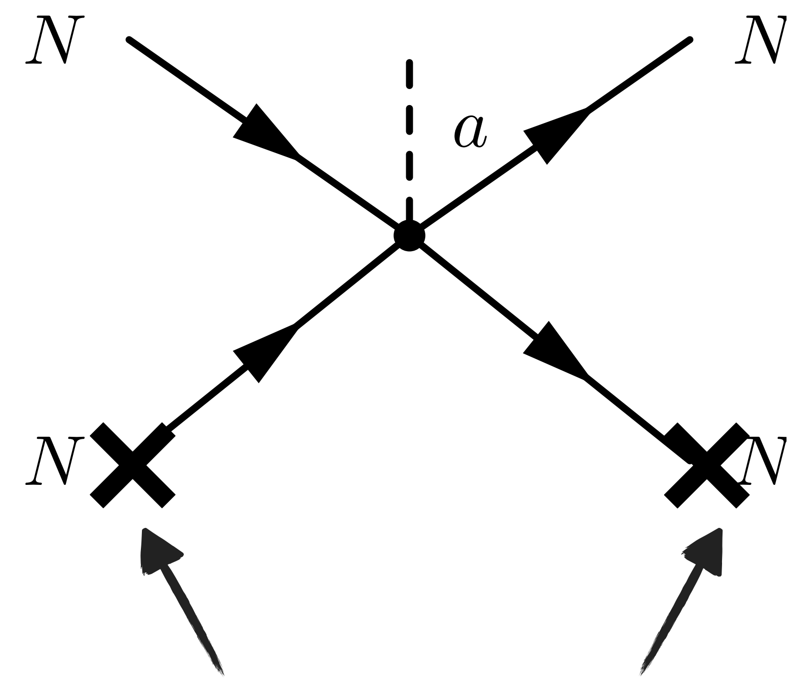


Background nucleons

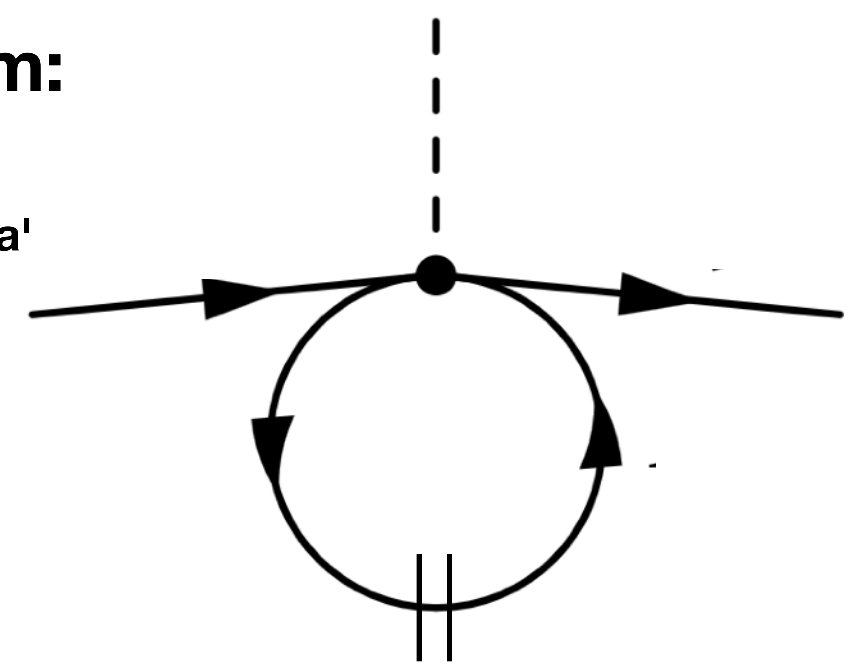
- **Systematically via QFT in Real-Time Formalism:**

Nucleon propagator at finite density

Filled 'Fermi sea'

$$iG(k) = \frac{i}{k^0 + i\epsilon} - 2\pi\delta(k^0)\theta(k_f - |\vec{k}|)$$

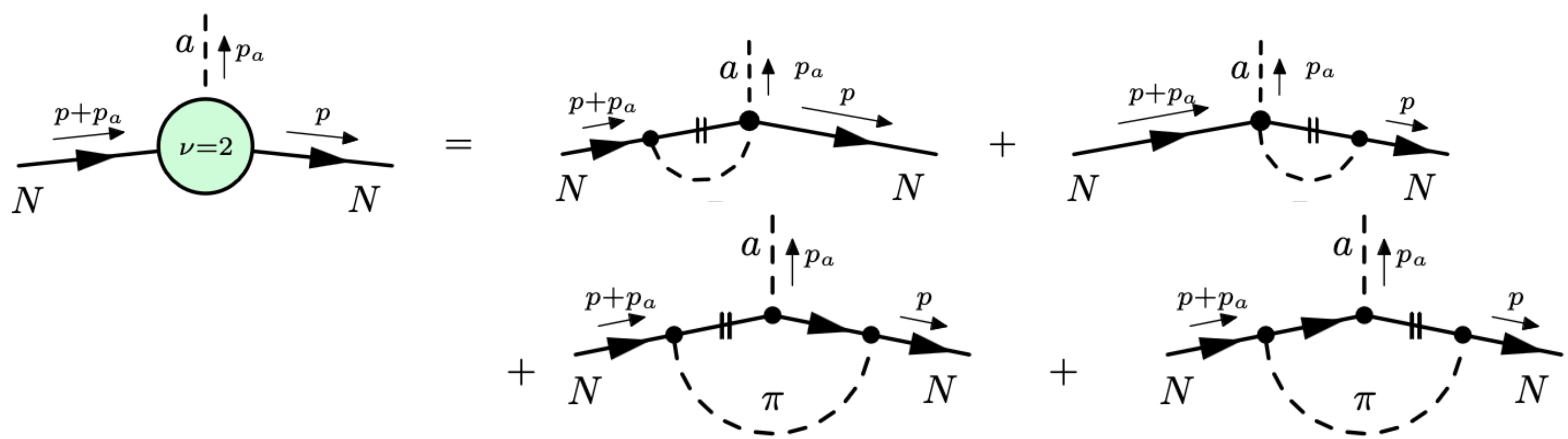
NR fermion propagator



Axion-Nuclon Coupling: Finite density

Get corrections systematically $\left(\frac{p}{4\pi f_\pi}\right)^\nu \to \left(\frac{k_f}{4\pi f_\pi}\right)^\nu$

$$\left(\frac{k_f}{4\pi f_\pi}\right)^2$$



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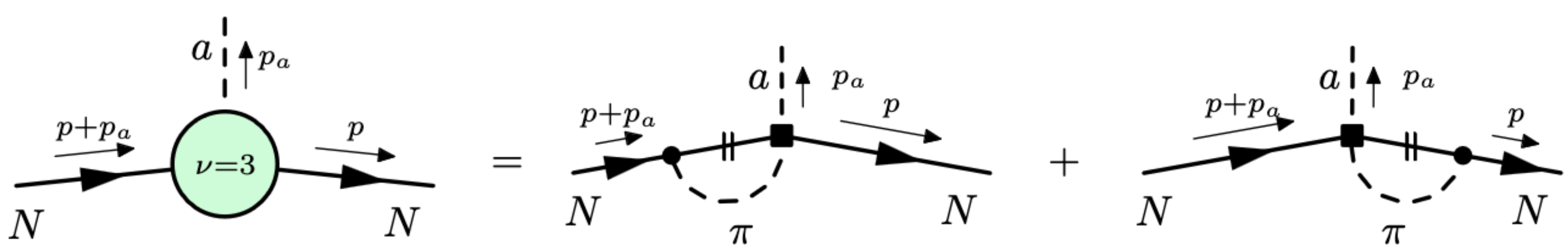
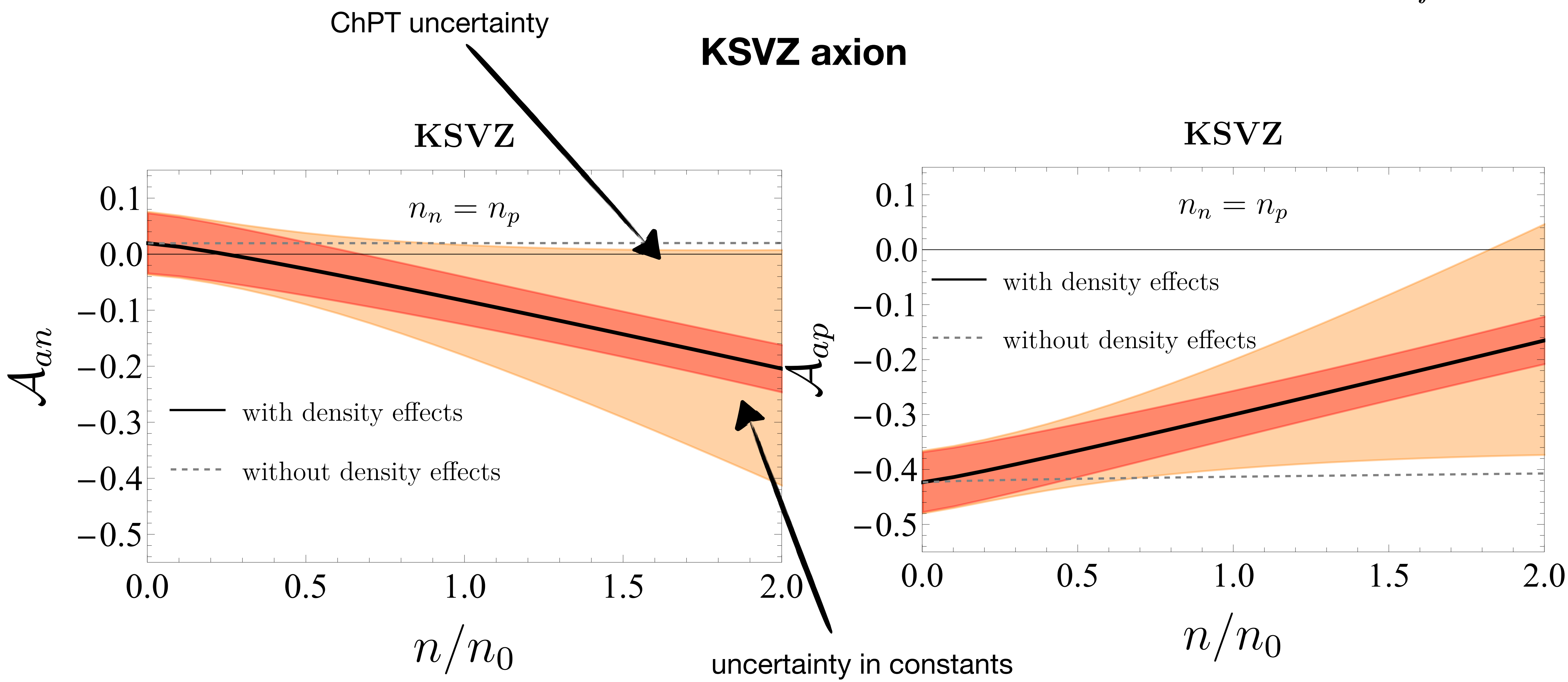


Results

Simplifying assumption:

$p \sim k_f$

KSVZ axion



At nuclear density $\mathcal{A}_{ap}^{\text{KSVZ}}(n_0) = -0.299(43)(98)$ $\mathcal{A}_{an}^{\text{KSVZ}}(n_0) = -0.083(43)(98)$

vs. vacuum $\mathcal{A}_{ap}^{\text{KSVZ}} = -0.42(5)$ $\mathcal{A}_{an}^{\text{KSVZ}} = 0.02(5)$

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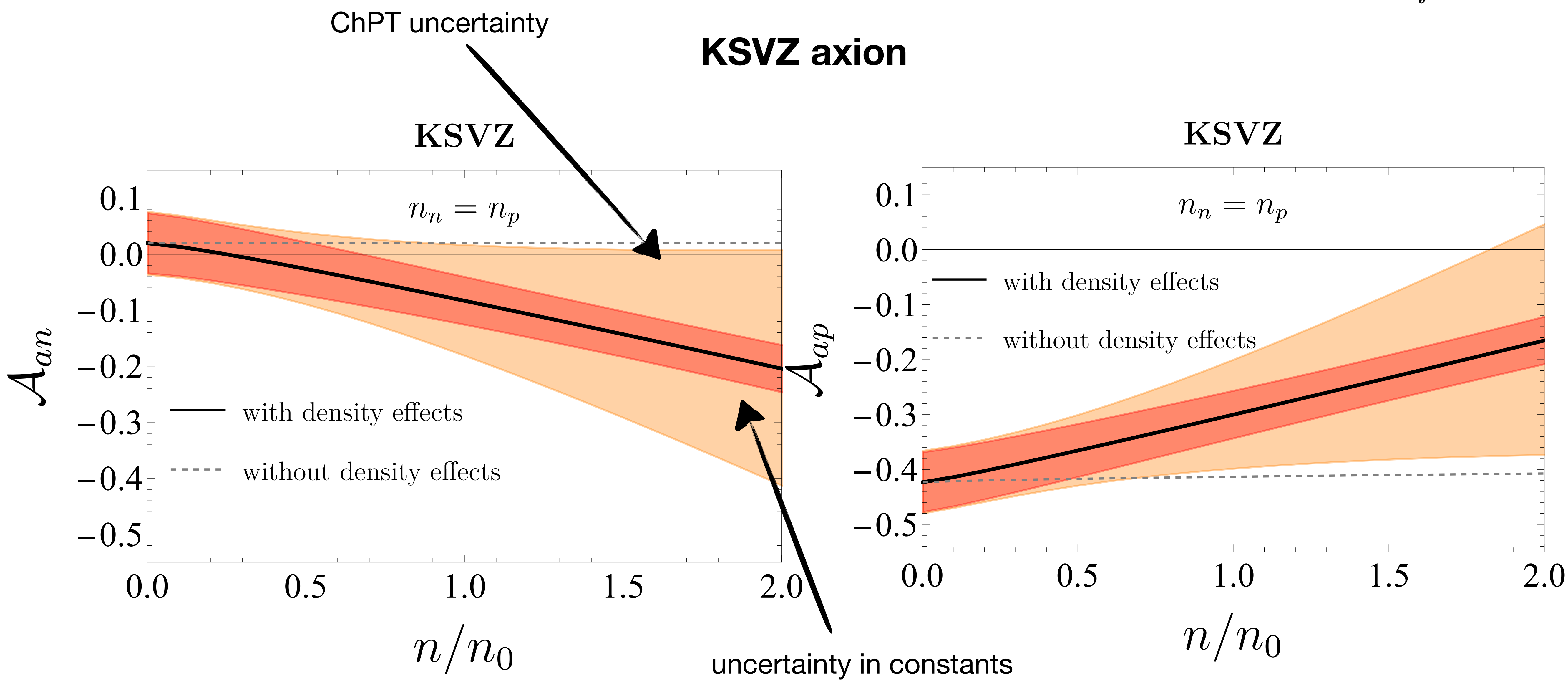

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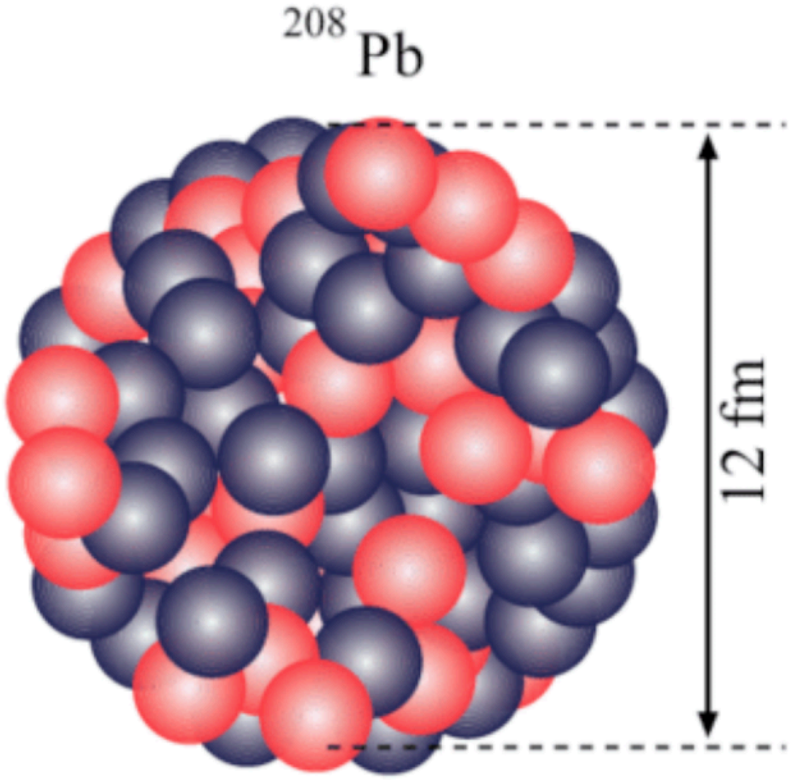
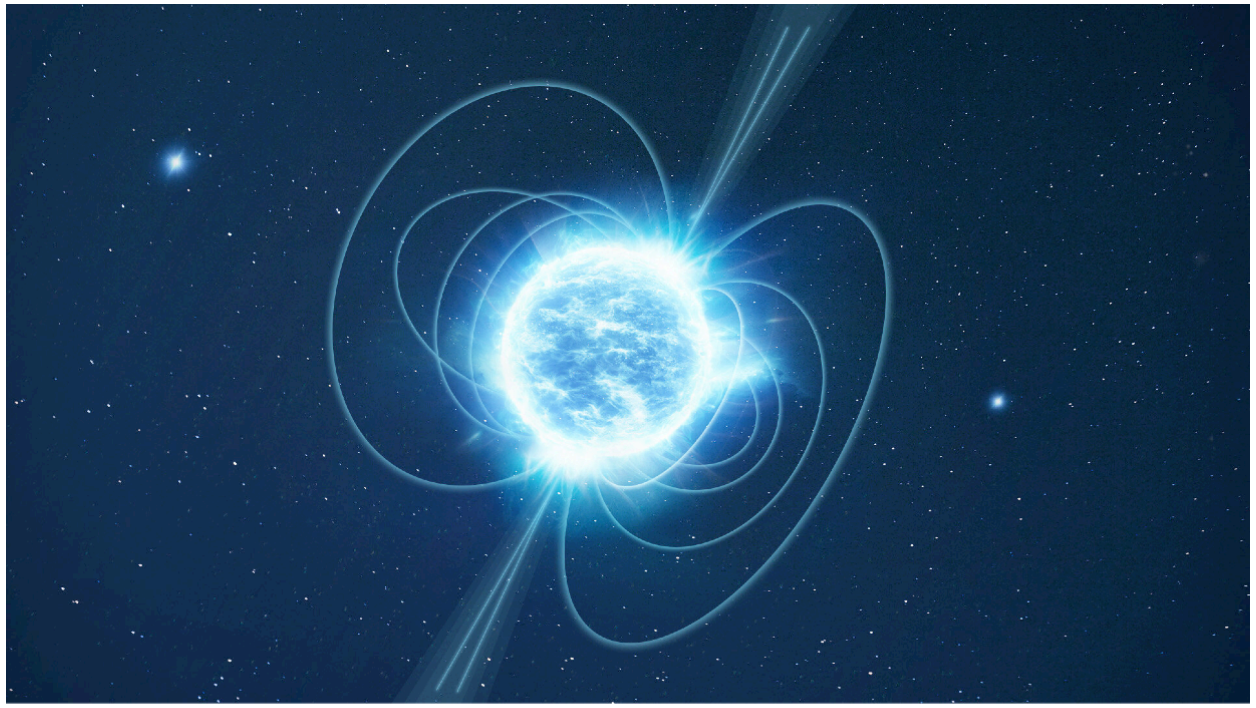
Accidental cancellation is lifted!

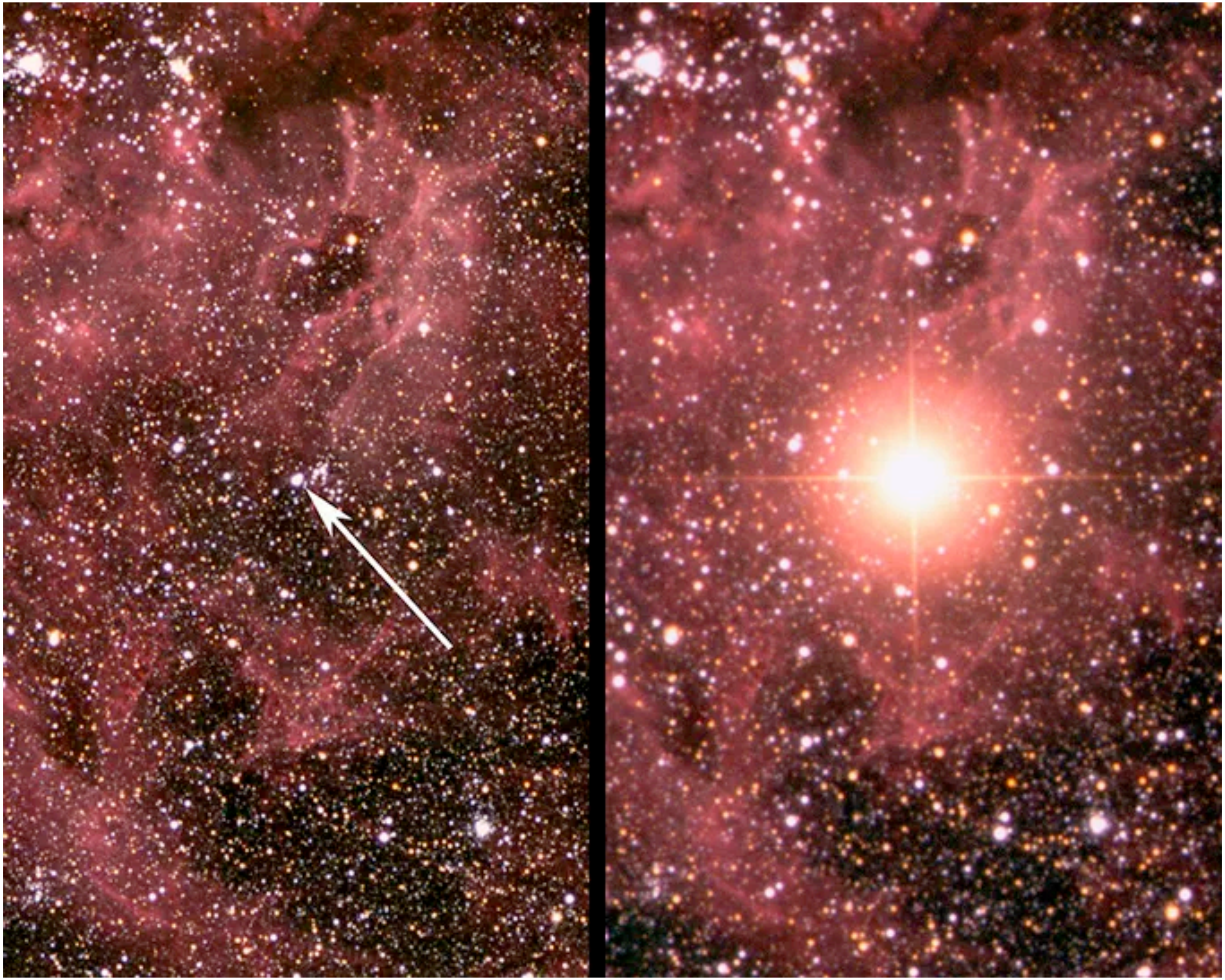
Implications for phenomenology





Bound from SN 1987A

Have observed a core-collapse (type II) SN in 1987 in the Large Magellanic Cloud



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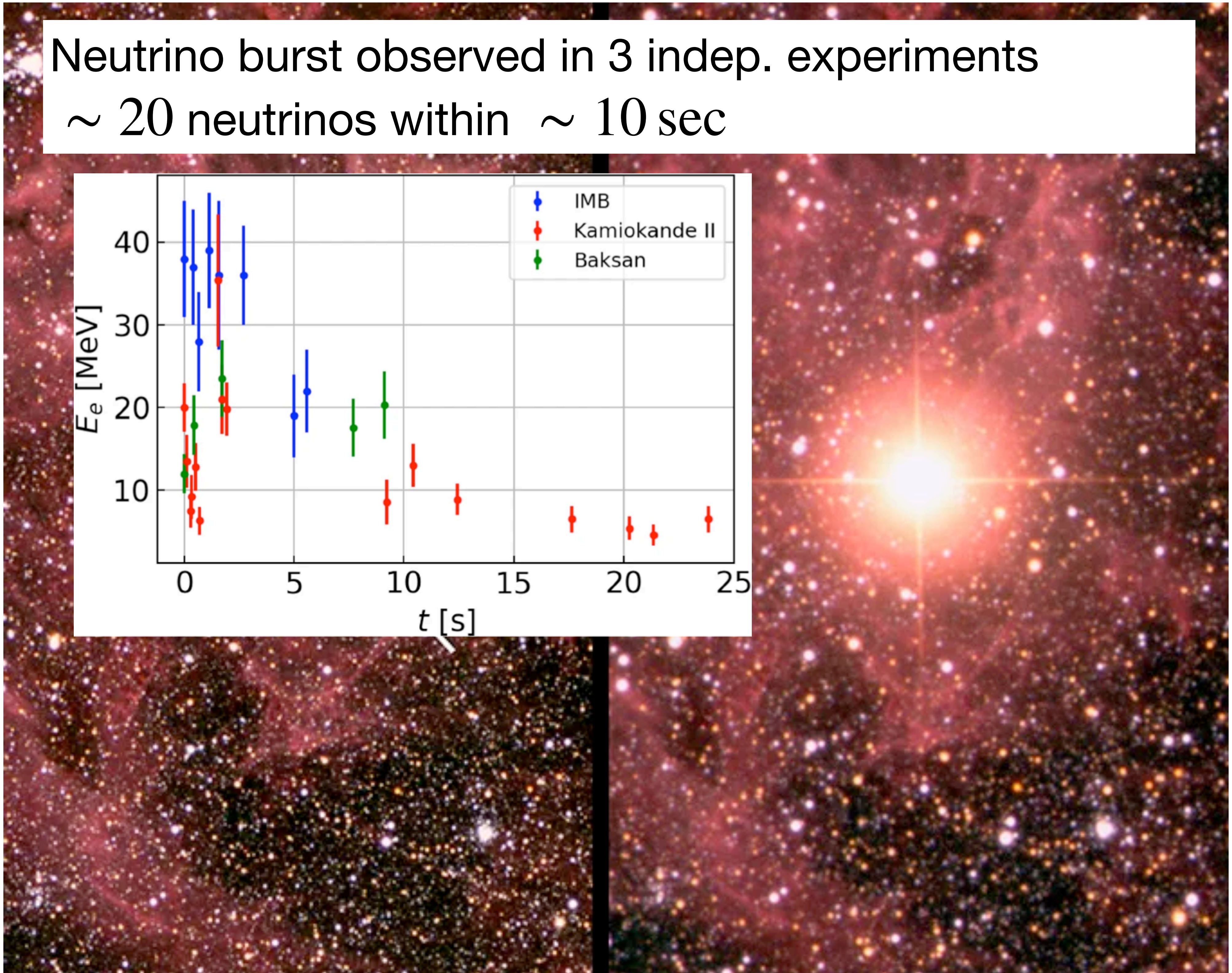

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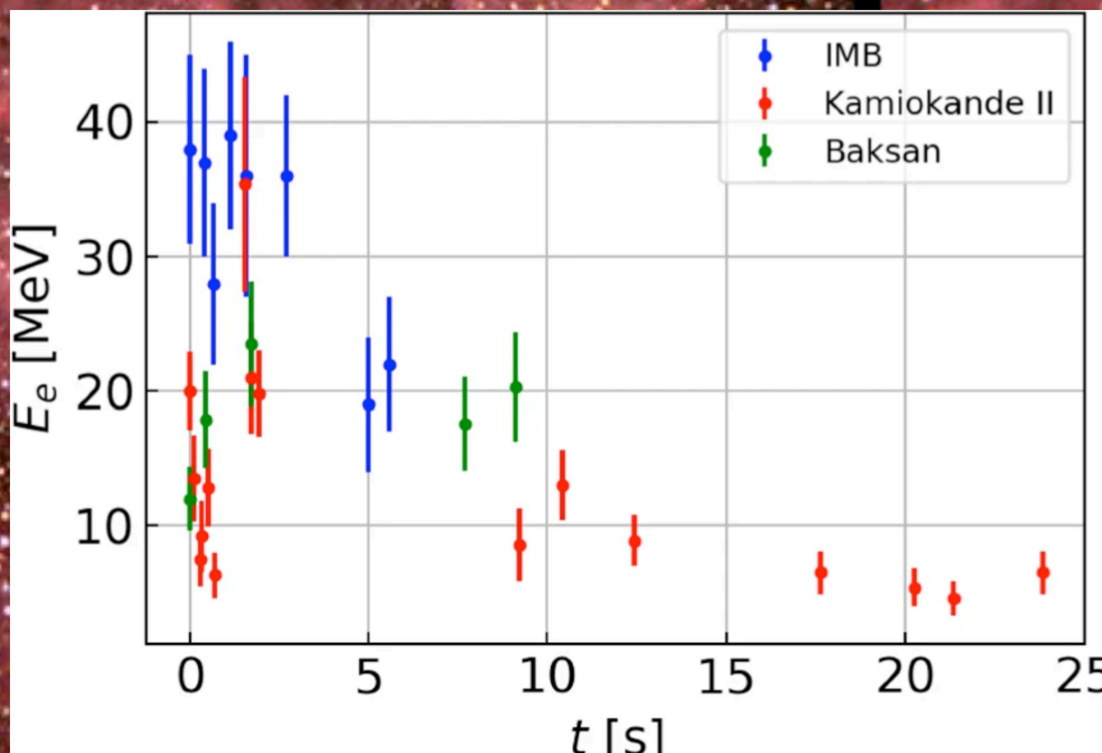

This gives a constraint on the luminosity of possible new particles $L_a \lesssim 3 \times 10^{52}\,\mathrm{erg\,s^{-1}}$

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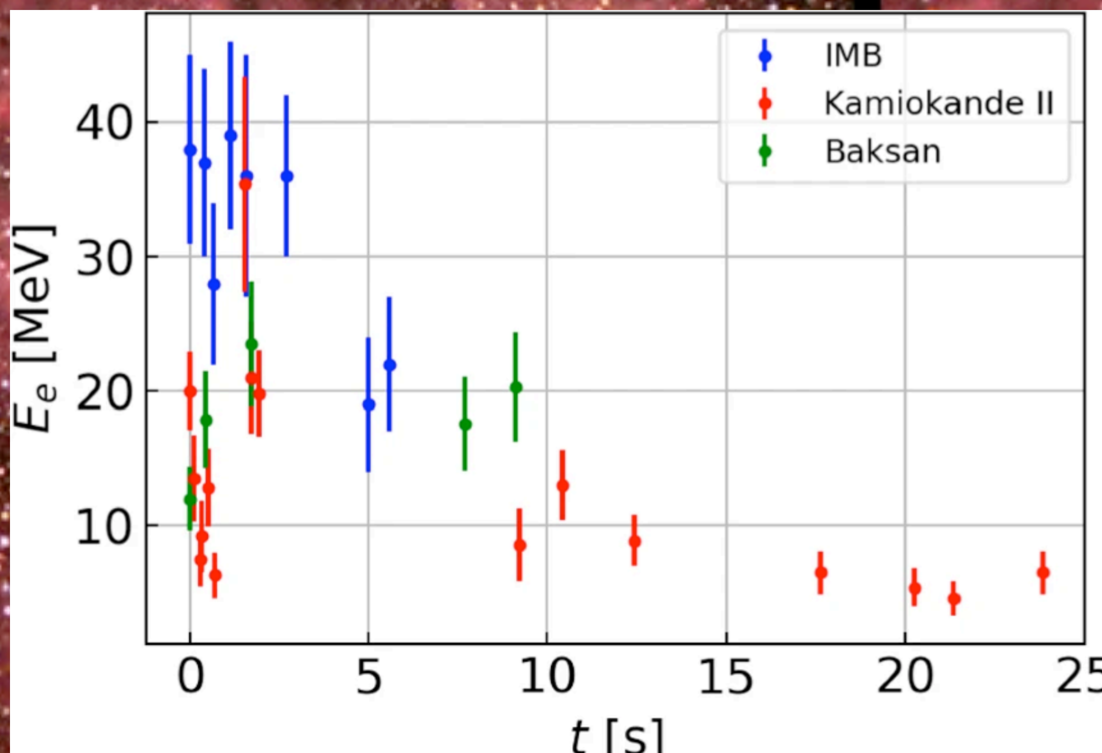

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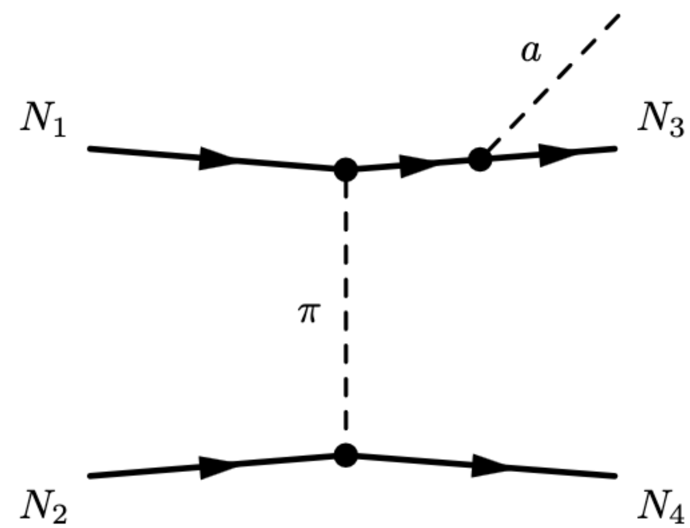
For a QCD axion this constrains m_a and f_a

Raffelt and Seckel ‘88
Chang et al ‘18
Carenza et al ‘19

Implications for supernova bound

Calculation of axion emissivity of supernova

typically



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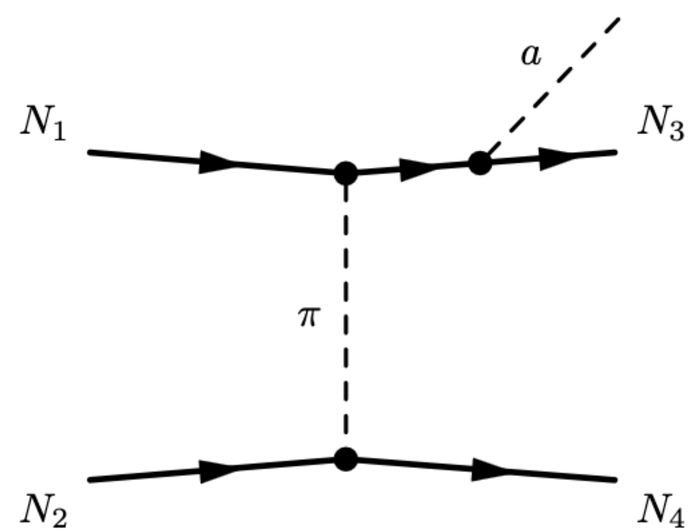

What has been done? Included corrections **phenomenologically**

Implications for supernova bound

Raffelt and Seckel '88
Chang et al '18
Carenza et al '19

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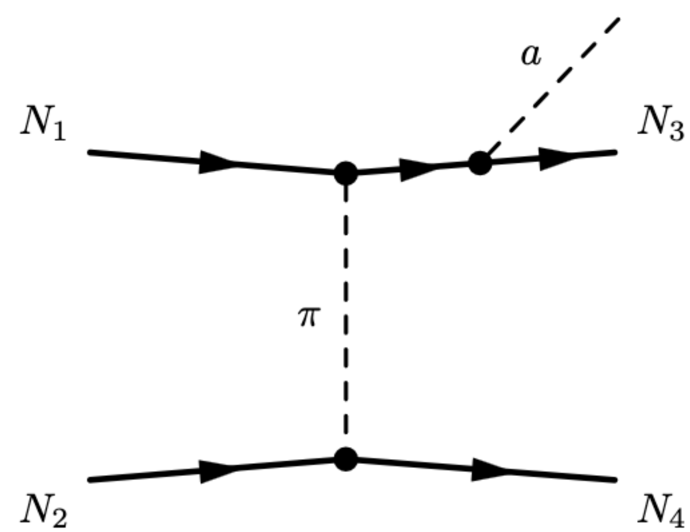
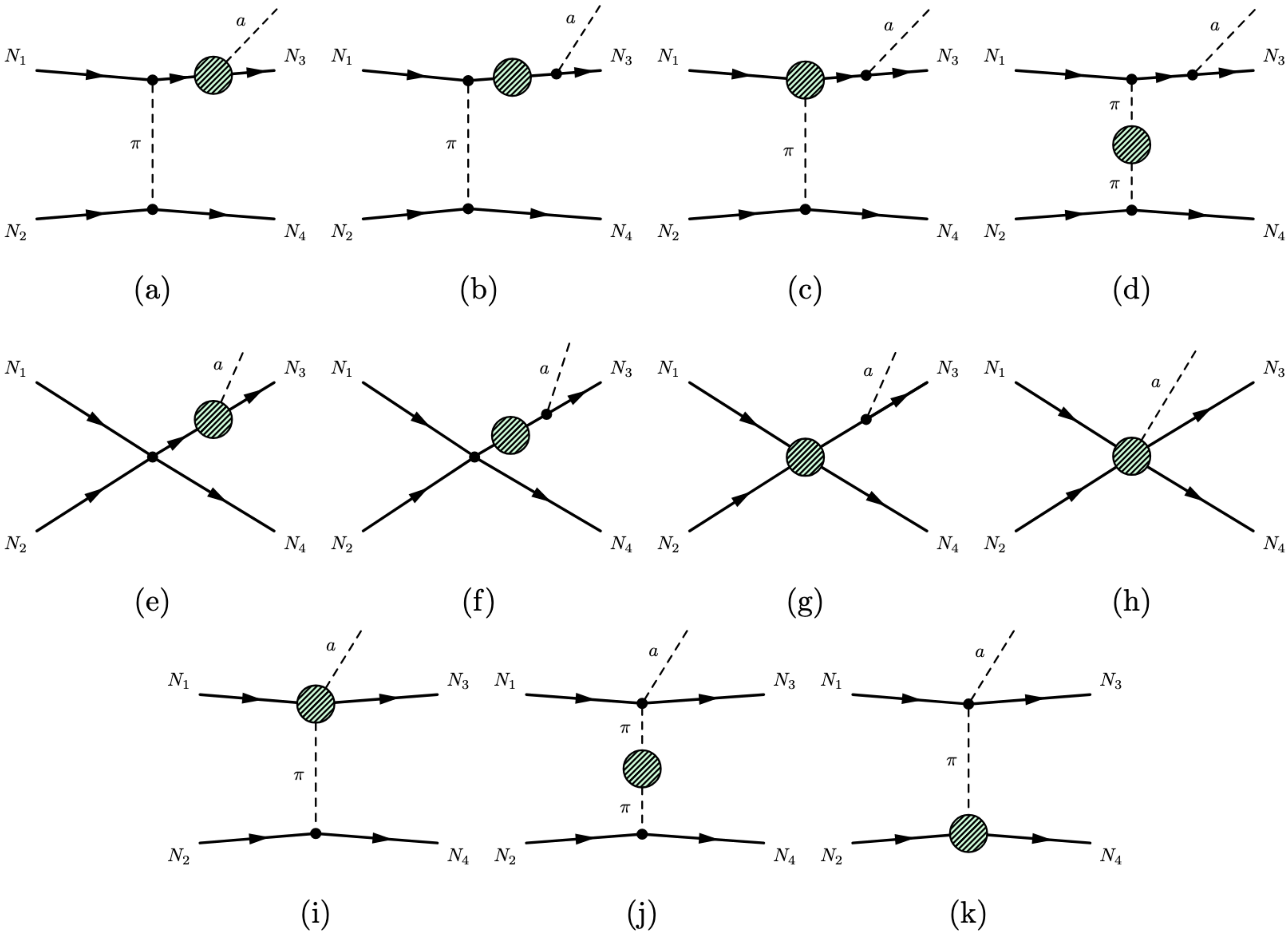


What has been done? Included corrections **phenomenologically**

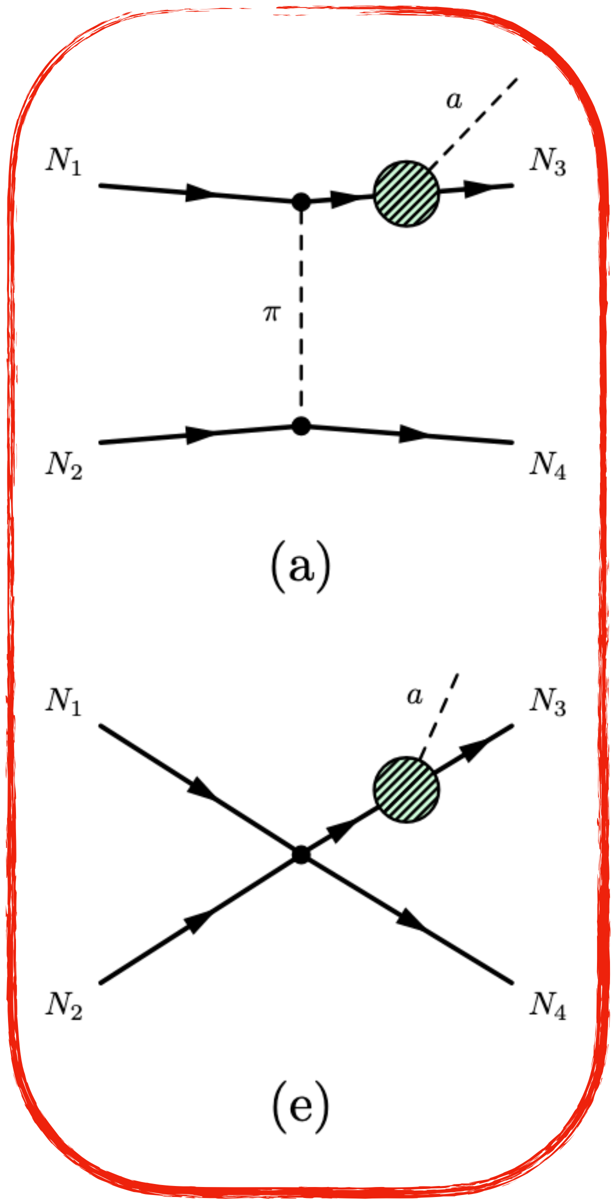
- Multiply rate by fudge factors: $\Gamma_a = \Gamma_a^{\text{tree}} \gamma_f \gamma_p \gamma_h$ Chang, Essig, McDermott ('18)

Supernova bound revisited

Relevant diagrams up to NLO



Supernova bound revisited



Outlined for the first time

Springmann, MS, Stelzl, Weiler ('24)

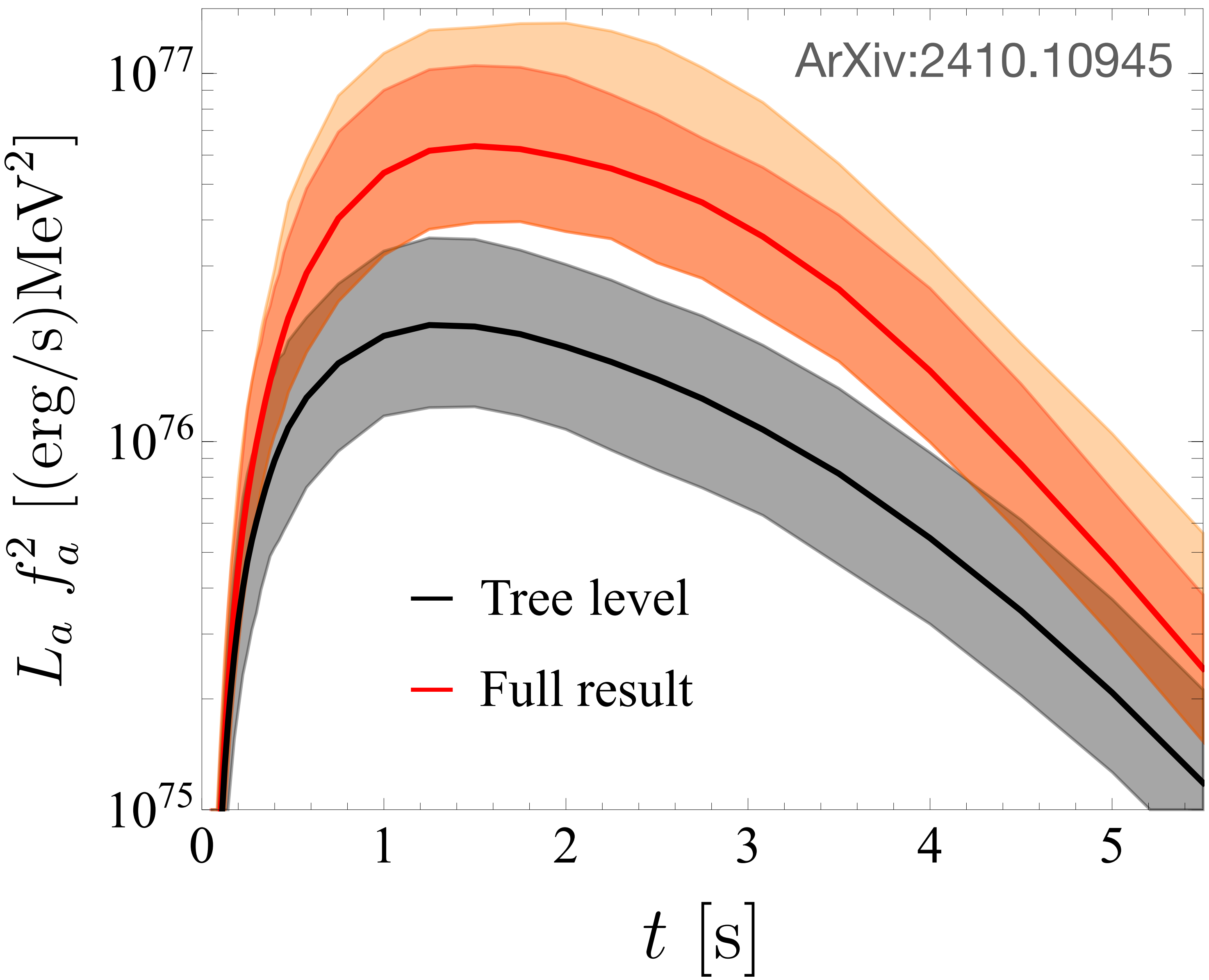
Modified couplings

Focus on these for now

Fully systematic evaluation should take into account all diagrams up to given order

Implications for supernova bound

example: KSVZ axion



$$L_a = \int dr 4\pi r^2 \dot{\epsilon}_a(r)$$

Supernova profile from https://wwwmpa.mpa-garching.mpg.de/ccsnarchive/

SN bound on axion mass O(few) stronger

Adds theory uncertainty!

Implications for supernova bound

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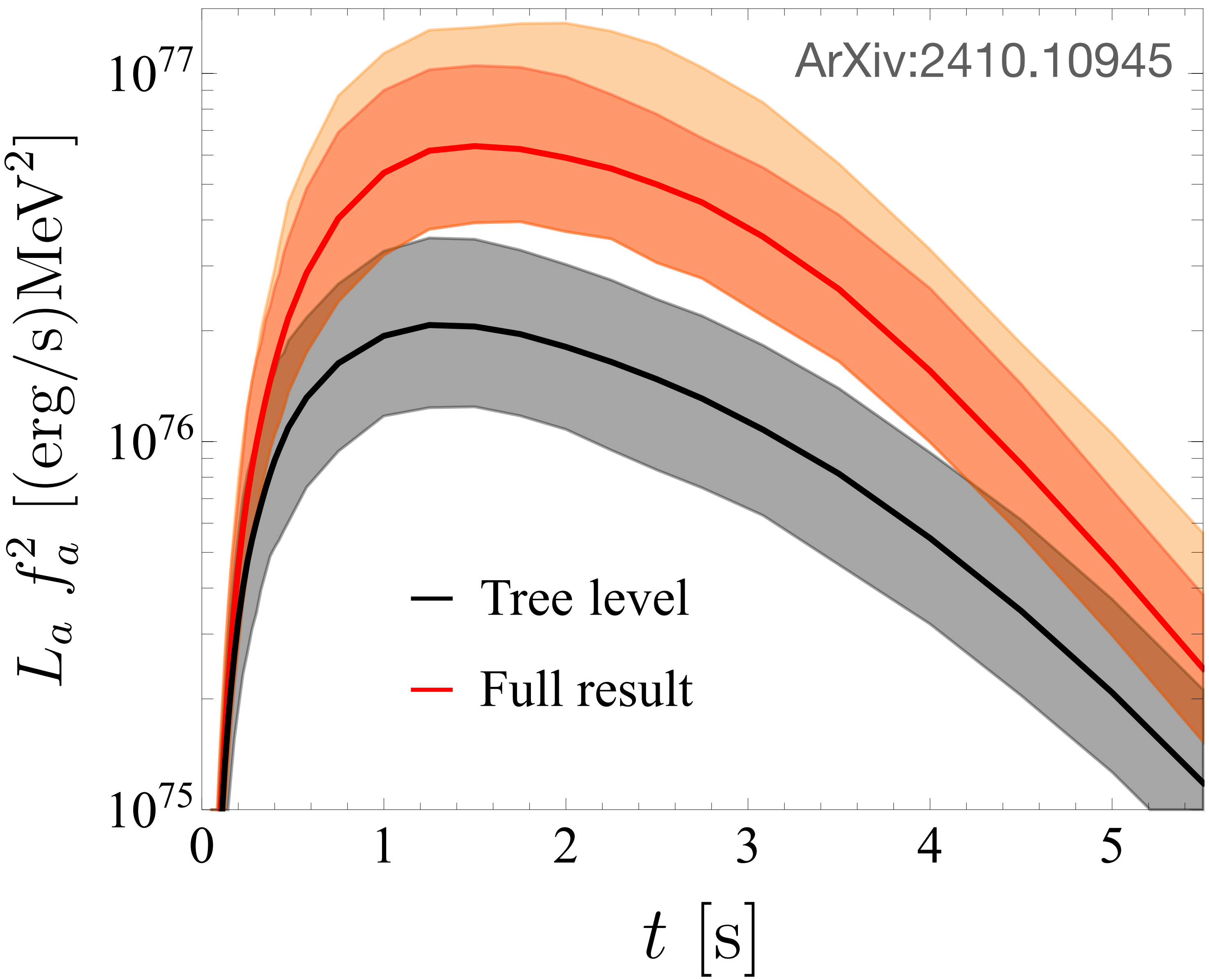

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Adds theory uncertainty!

Tree level: $f_a \gtrsim 6.1^{+1.7}_{-1.4} \times 10^8\,\mathrm{GeV}, \quad m_a \lesssim 9.8^{+3.0}_{-2.2}\,\mathrm{meV}.$

Vertex corrections: $f_a \gtrsim 1.0^{+0.5}_{-0.2} \times 10^9\,\mathrm{GeV}, \quad m_a \lesssim 5.9^{+1.8}_{-2.0}\,\mathrm{meV}.$

Model dependence - QCD axion-nucleon coupling

Requiring that the axion solve the strong CP problem, i.e.

$$\mathcal{L}_{\rm QCD} \supset \frac{a}{f_a}\frac{g^2}{32\pi^2}G^{\mu\nu}\tilde{G}_{\mu\nu}$$

Independent of the Axion model, what is the strongest constraint on f_a from SN observation?

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The $aG\tilde{G}$ term in the Lagranian induces a nEDM

$$\mathcal{L}_a^{\rm EDM} = -\frac{i}{2}\frac{C_{aN\gamma}}{m_N}\frac{a}{f_a}\bar{N}\gamma_5\sigma_{\mu\nu}NF^{\mu\nu}$$

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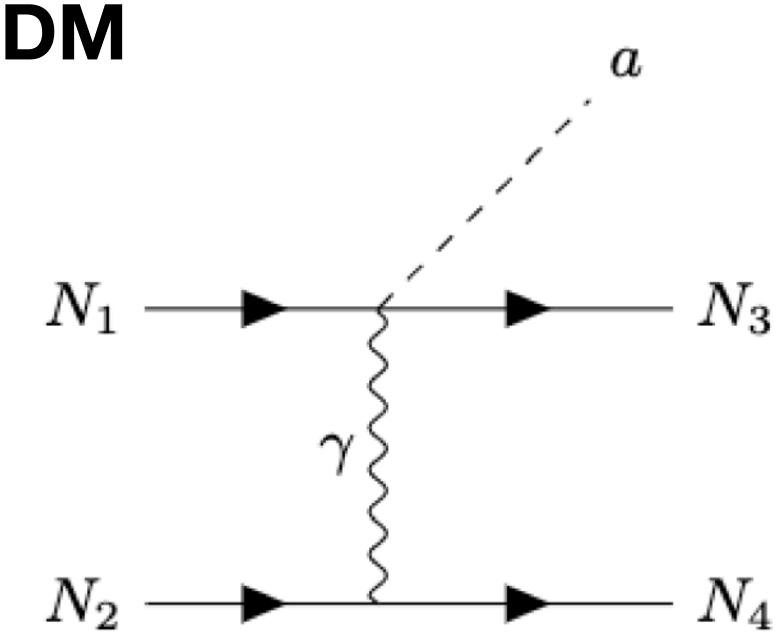

Bound on axion from EDM operator in SN:

Lucente, Mastrototaro, Carenza, DiLuzio, Giannotti, Mirizzi

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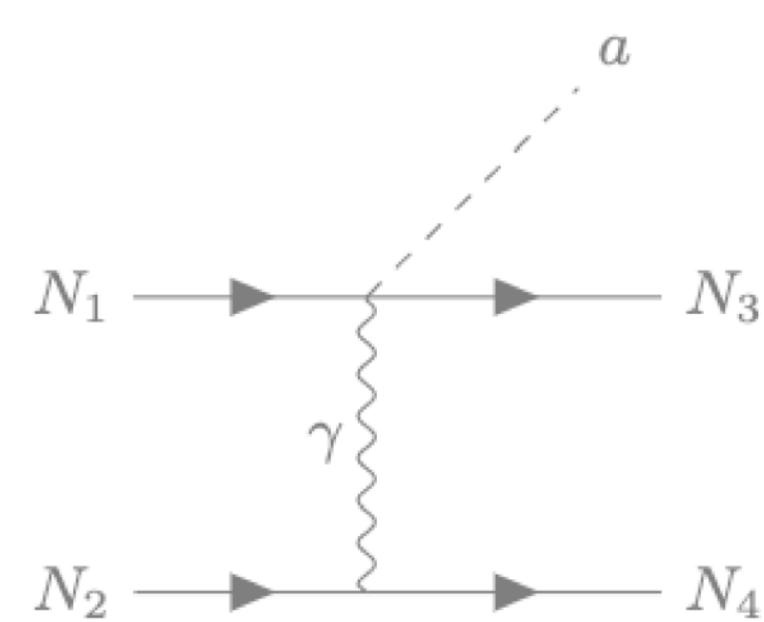
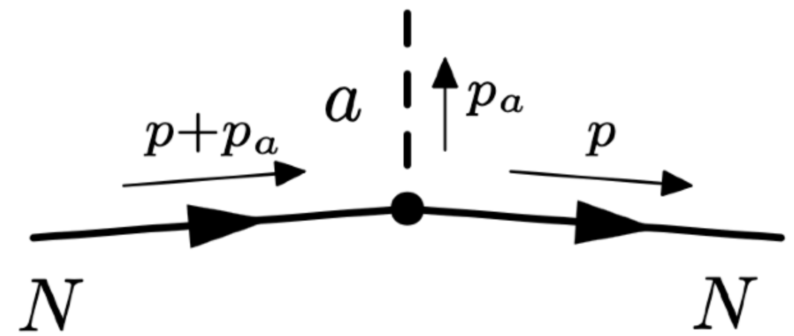
We found that this is not the dominant process



Model dependence - QCD axion-nucleon coupling

LO:

$$c_p = g_0 c_{u+d} + g_A c_{u-d}$$
$$c_n = g_0 c_{u+d} - g_A c_{u-d}$$



$$c_{u\pm d} = (c_u \pm c_d)/2$$

$$c_q \equiv c_q^0 - [Q_a]_q$$

$$Q_a = \frac{\mathrm{Diag}[1, z]}{1+z}, \quad z \equiv \frac{m_u}{m_d}$$

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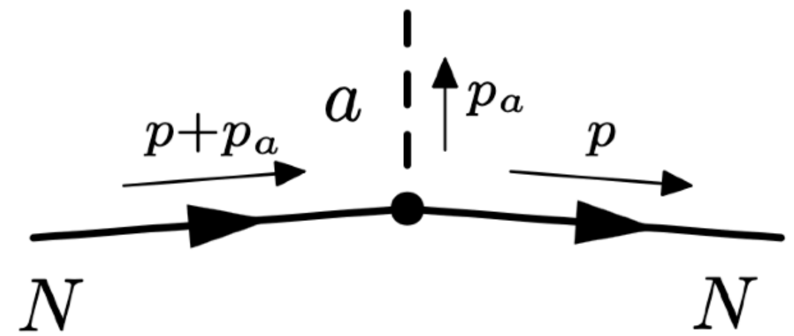

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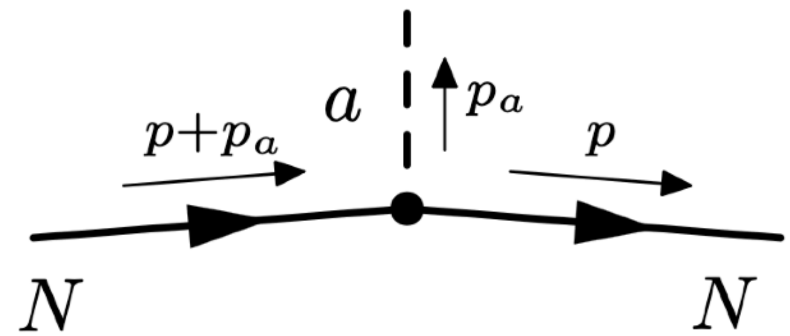

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Model-dependent constants can be tuned to 0, leading to $c_n \simeq c_p \simeq 0$.

DiLuzio, Mescia, Nardi, Panci, Ziegler ('17)

Badziak, Harigaya ('23)

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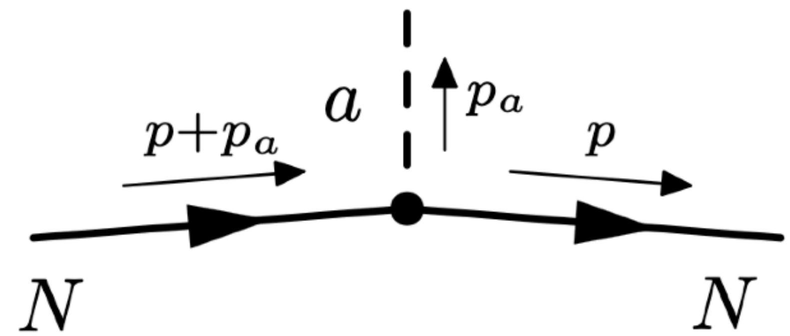

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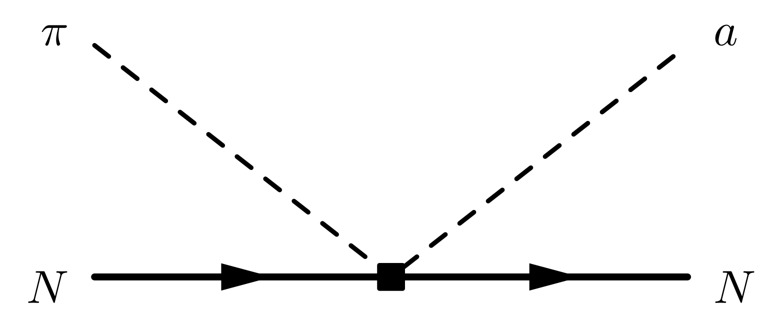
What is left at NLO?

Model dependence - QCD axion-nucleon coupling

Isospin breaking term survives independent of the type of axion model

$$\mathcal{L}_{\pi N}^{(2)} \supset \hat{c}_5 \bar{N} \tilde{\chi}_+ N$$

with $\widetilde{\chi}_+ \supset -m_\pi^2 \frac{4z}{(1+z)^2} \left(\frac{\pi^a a}{f_\pi f_a} \right) \tau^a$



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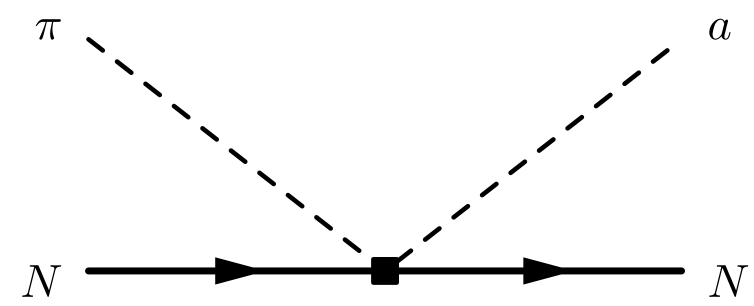
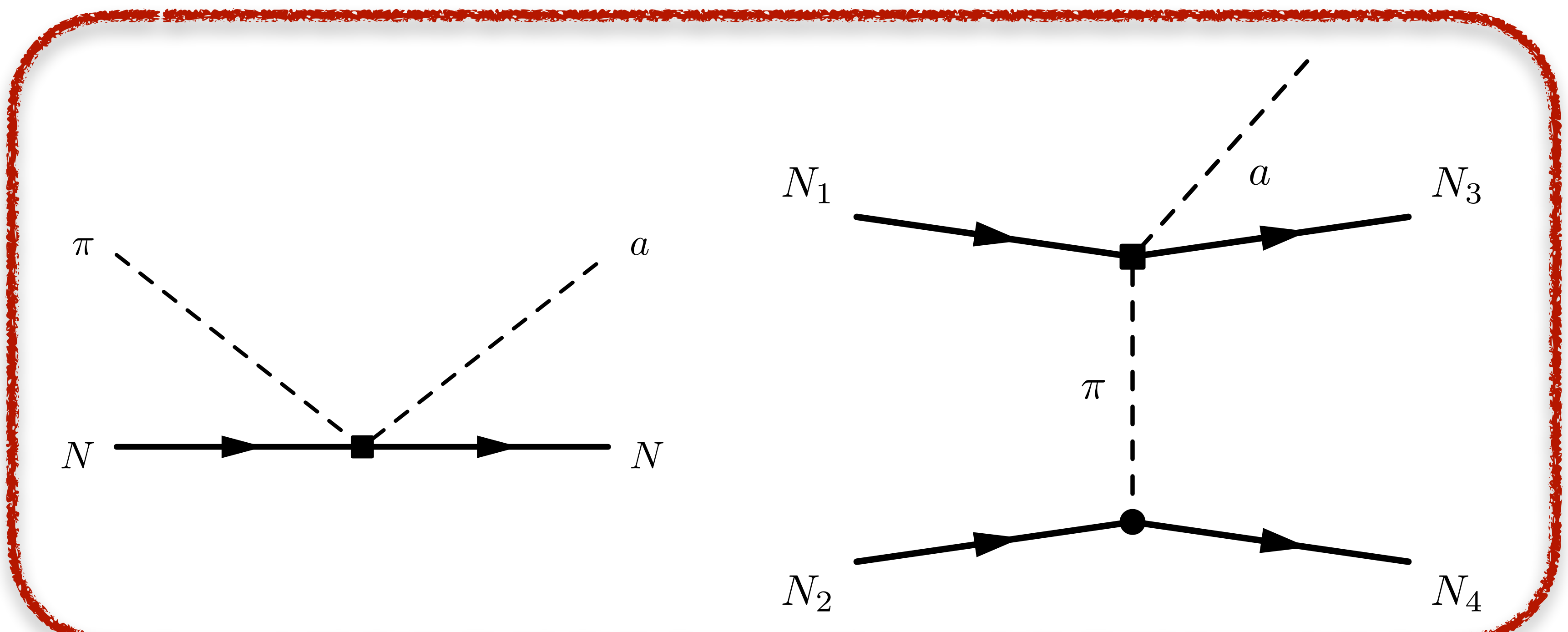




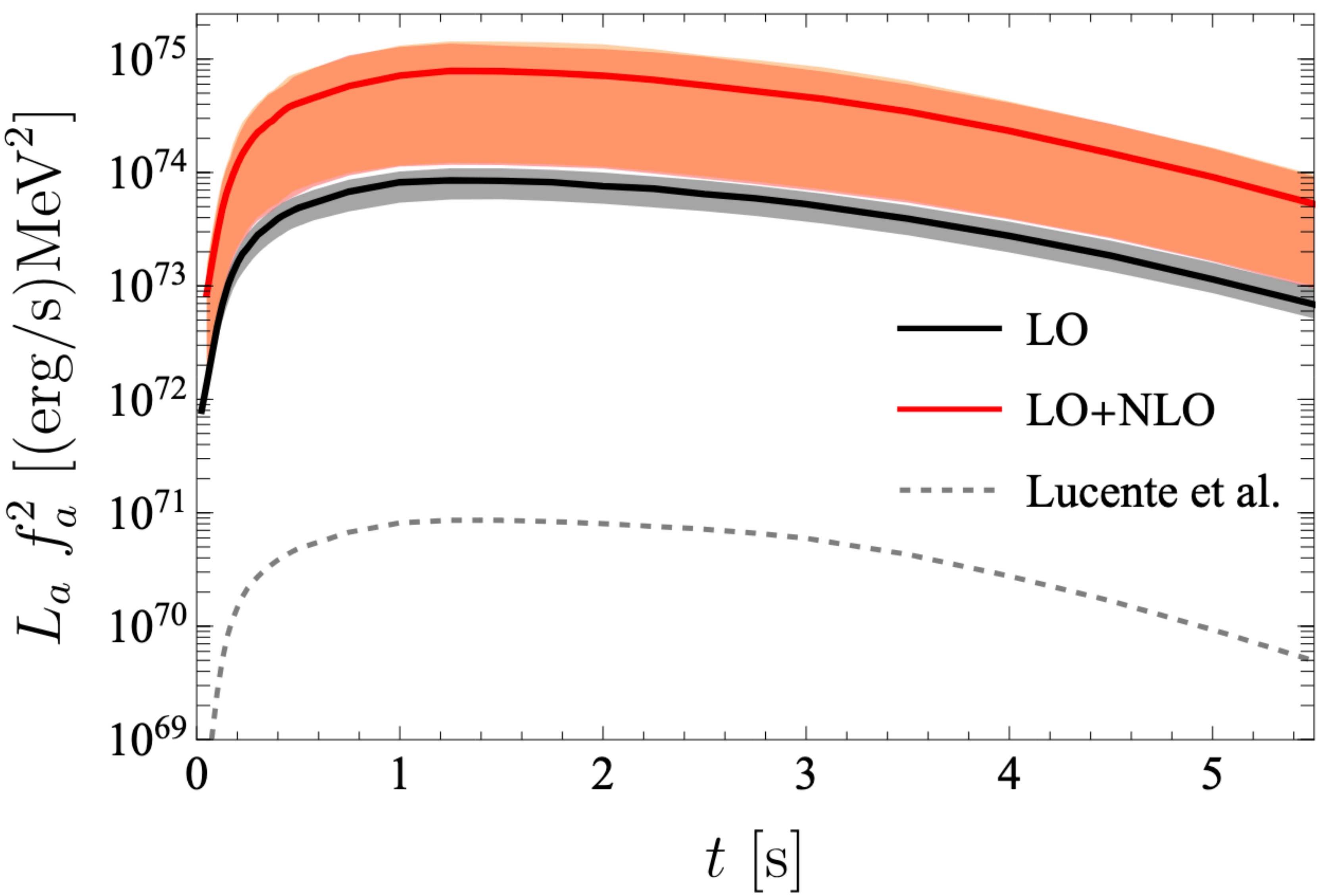
Dominant production channels

These diagrams dominate for the model independent SN bound

Astrophobic axions

- Loose the loop-suppression compared to EDM operator

$$L_a^{\text{tree},\hat{c}_5} \simeq (4\pi)^4 L_a^{\text{EDM}} \simeq 10^4 L_a^{\text{EDM}}$$



ArXiv:2410.19902

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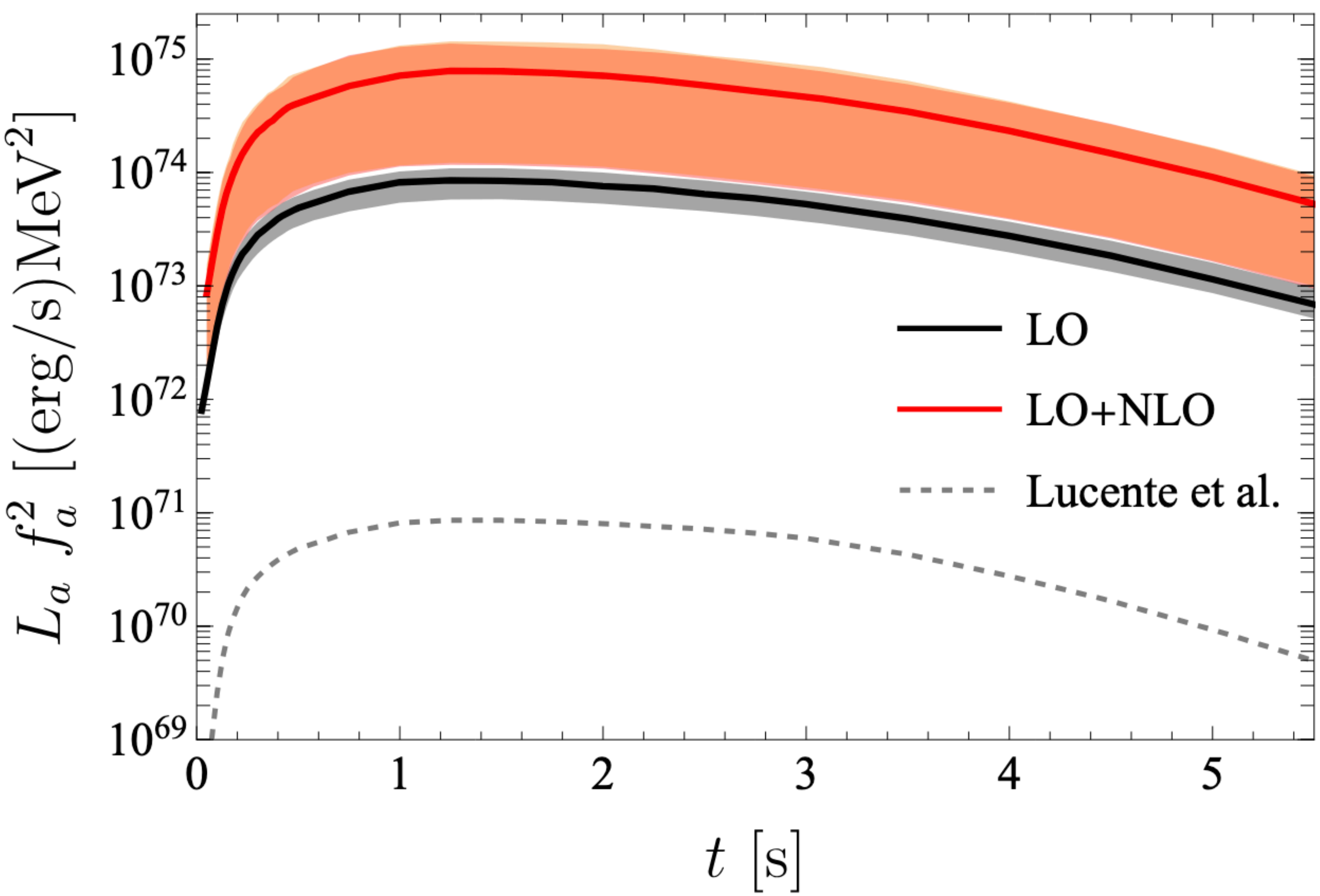

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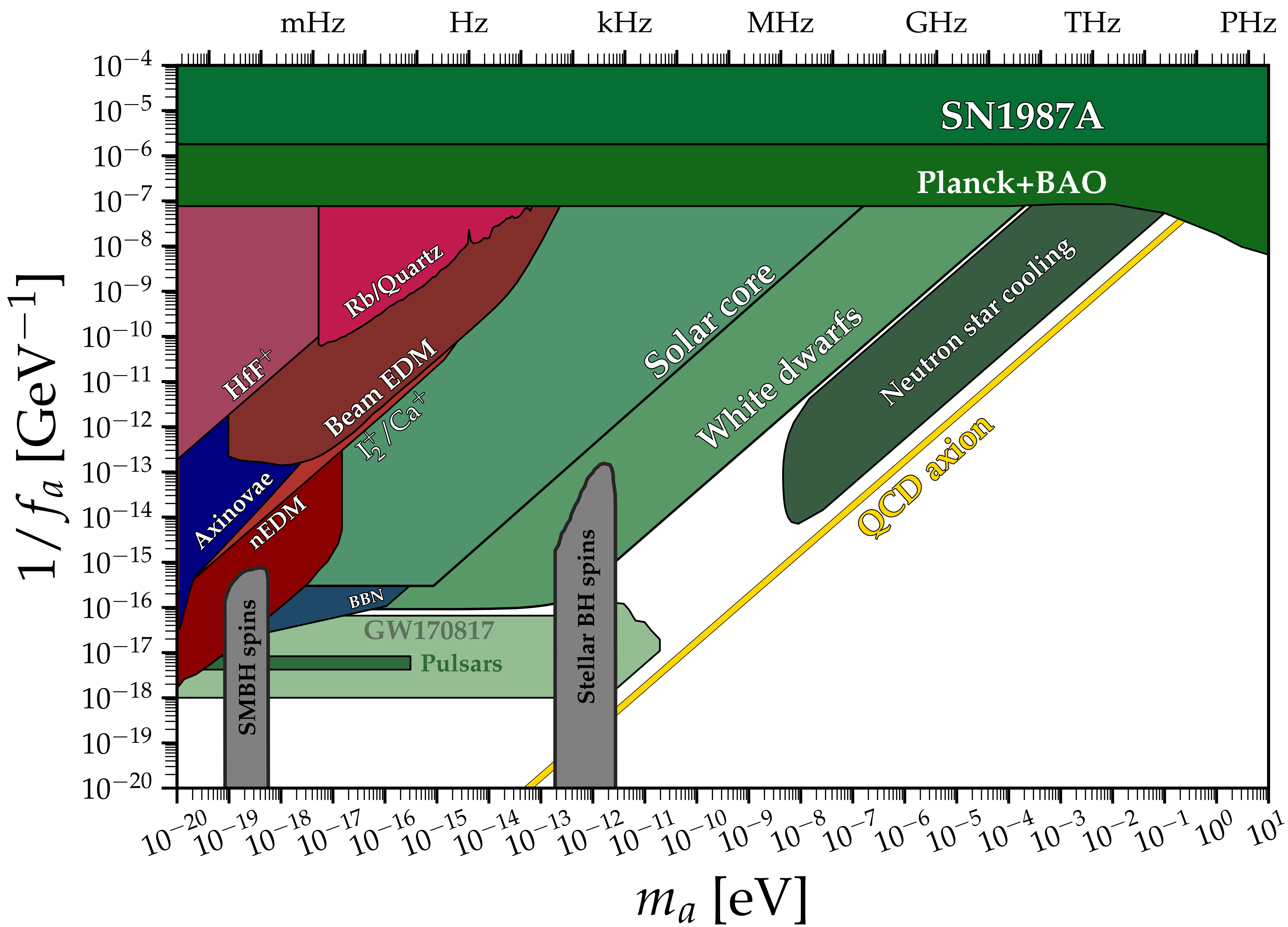
Strong universal bound on QCD axions: $f_a > 1.1^{+0.4}_{-0.6} \times 10^8\,\text{GeV}, \quad (68\%\ \text{C.L.})$

Astrophobic axions



Exclusion Plot from https://github.com/cajohare

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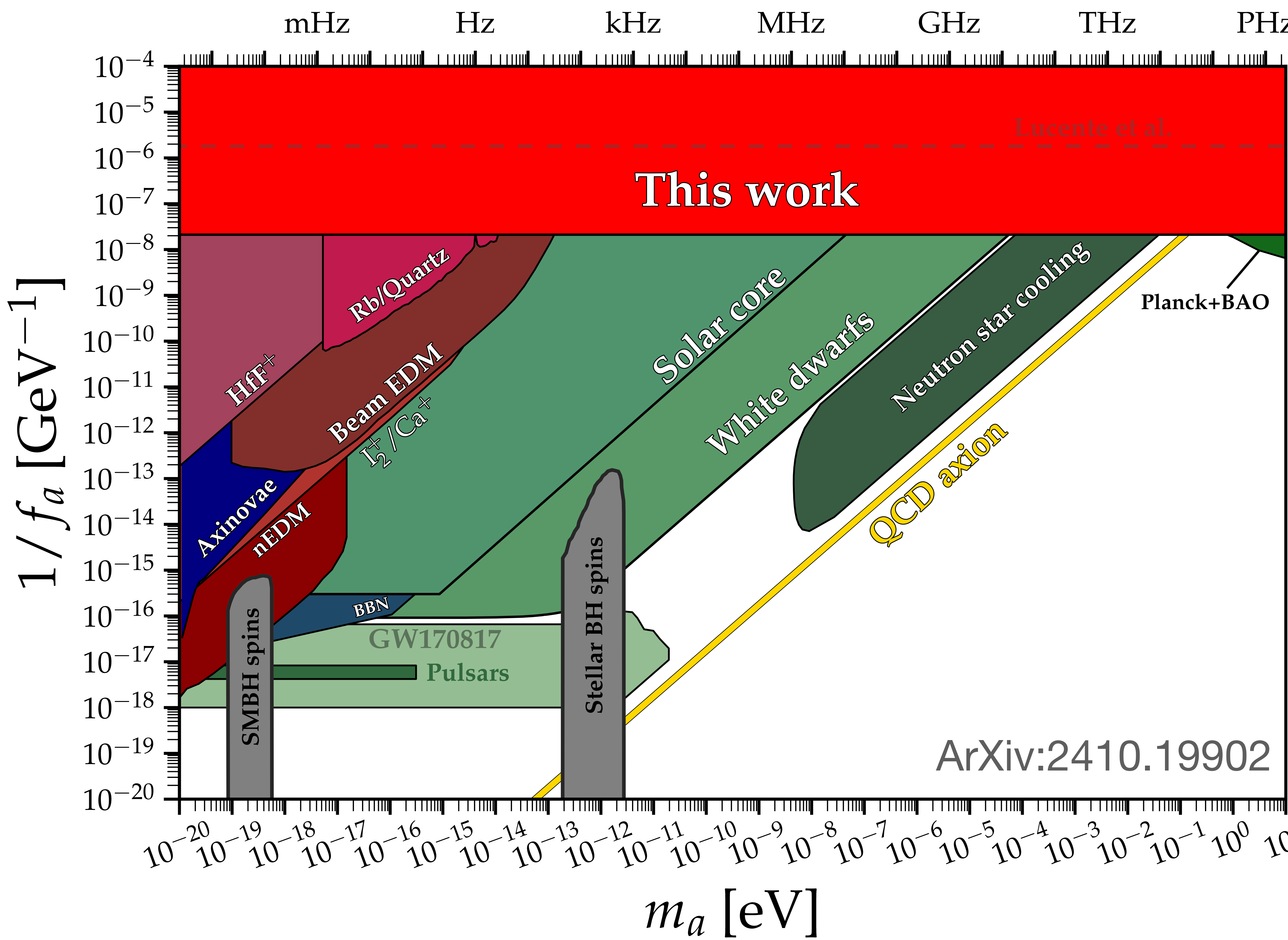

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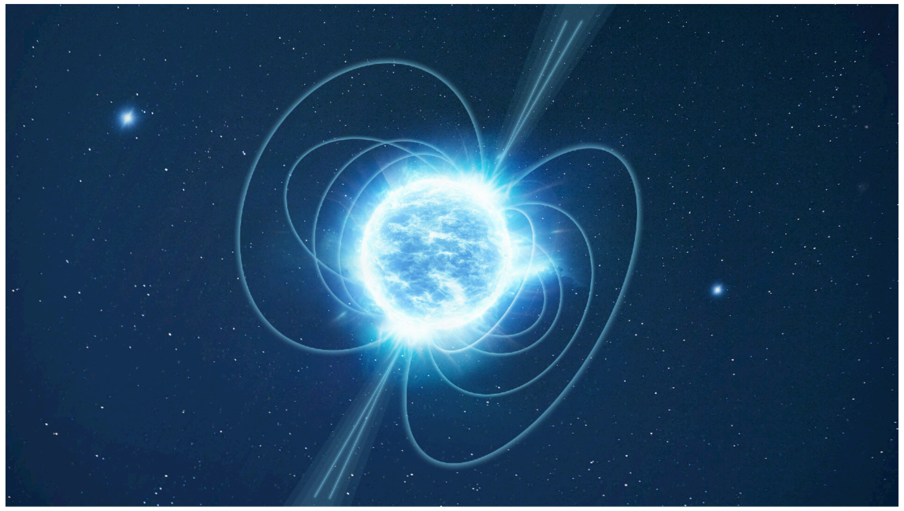
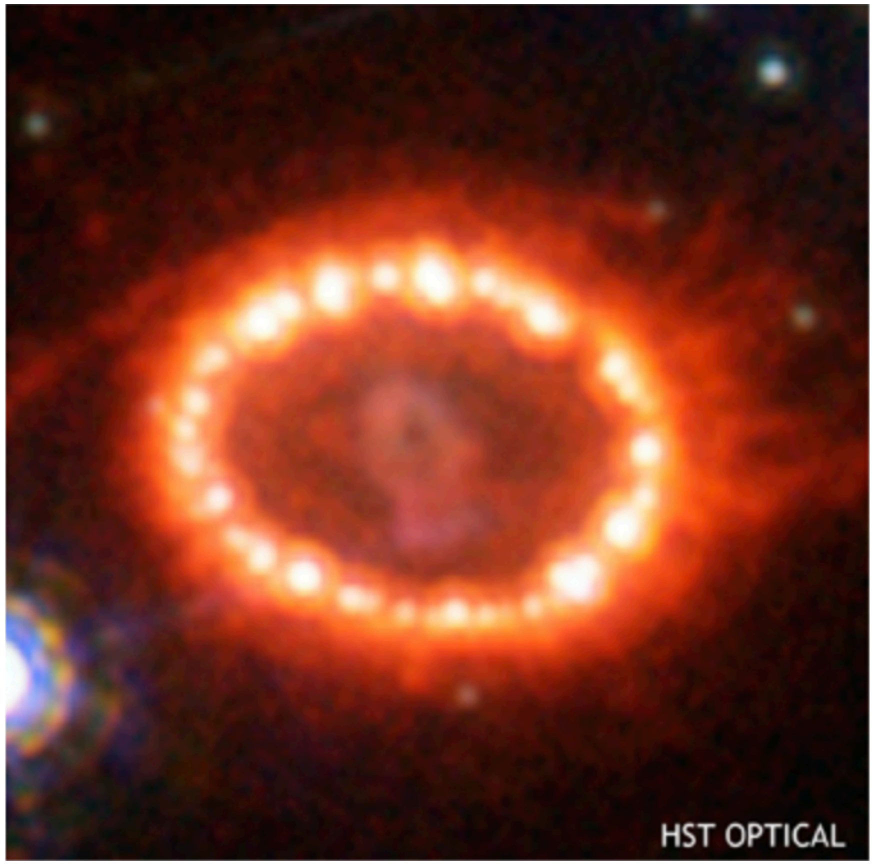
Conclusions



- QCD axion couplings are density dependent!
- Systematic calculation of axion couplings within ChPT
- Significant changes of supernova bound
- Large uncertainty at high densities



Backup slides

Model dependence - QCD axion-nucleon coupling

LO: $$\hat{\mathcal{L}}^{(1)}_{\pi N} = \bar{N}\left(iv \cdot D + g_A S \cdot u + g_0 S \cdot \hat{u}\right) N$$

$$\hat{u}_\mu = c_{u+d}\left(\frac{\partial_\mu a}{f_a}\right)\mathbf{1} + \dots$$

$$u_\mu = -\left(\frac{\partial_\mu \pi^a}{f_\pi}\right)\tau^a + c_{u-d}\left(\frac{\partial_\mu a}{f_a}\right)\tau_3$$

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NLO:

$$\hat{\mathcal{L}}^{(2)}_{\pi N} = \bar{N}\left[-\frac{1}{2m_N}\left(D^2 - (v\cdot D)^2 + ig_A\{S\cdot D, v\cdot u\} + ig_0\{S\cdot D, v\cdot\hat{u}\}\right)\right.$$
$$+ \hat{c}_1\langle\chi_+\rangle + \frac{\hat{c}_2}{2}(v\cdot u)^2 + \hat{c}_3(u\cdot u) + \frac{\hat{c}_4}{2} i\epsilon^{\mu\nu\rho\sigma}[u_\mu, u_\nu]v_\rho S_\sigma$$
$$\left. + \hat{c}_5\widetilde{\chi}_+ + \frac{\hat{c}_8}{4}(v\cdot u)(v\cdot\hat{u}) + \hat{c}_9(u\cdot\hat{u})\right]N,$$

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Which interactions does this give rise to?

Comparison with current literature

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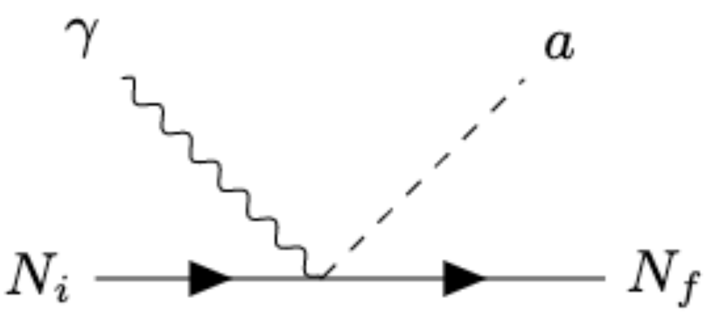

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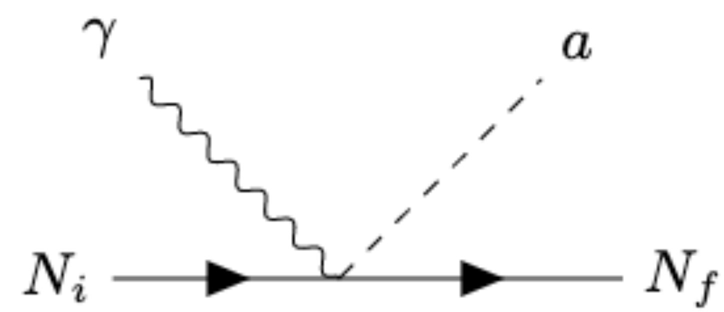
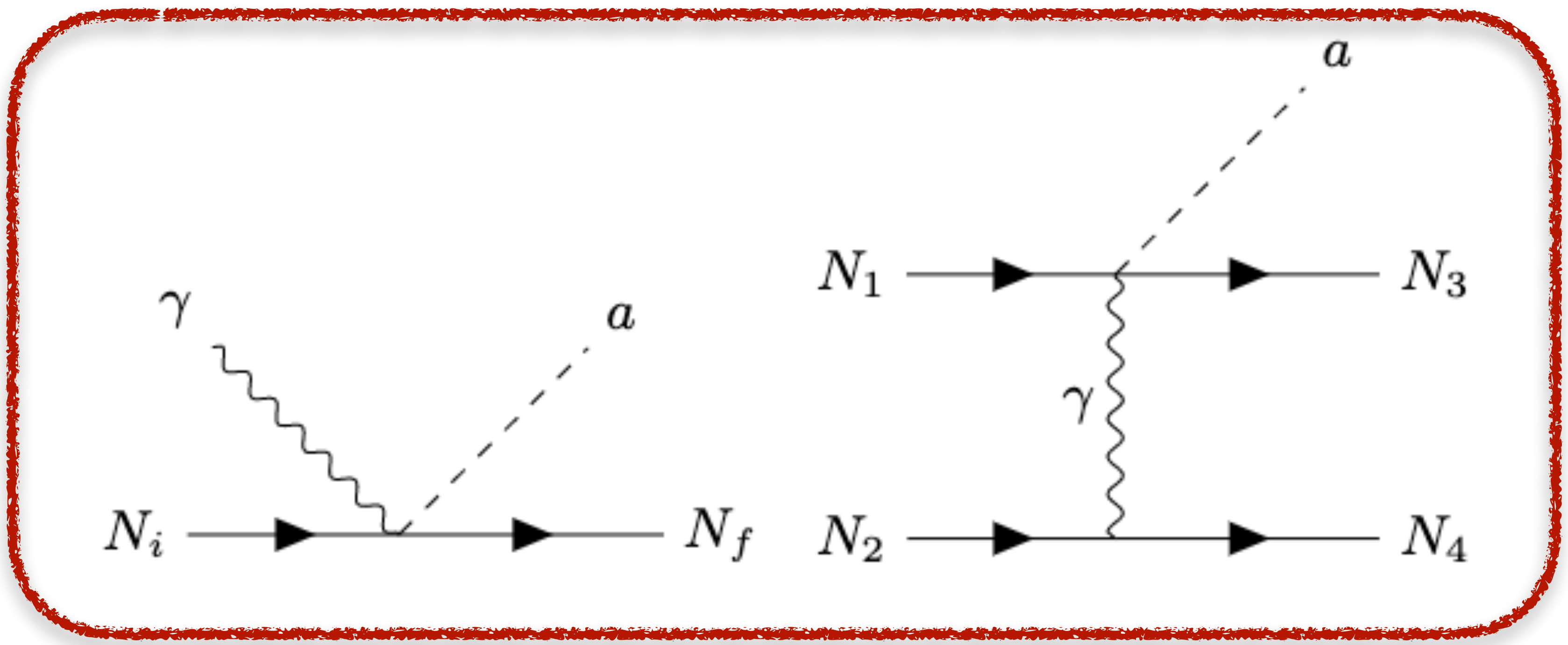


see Lucente et al. ‘22



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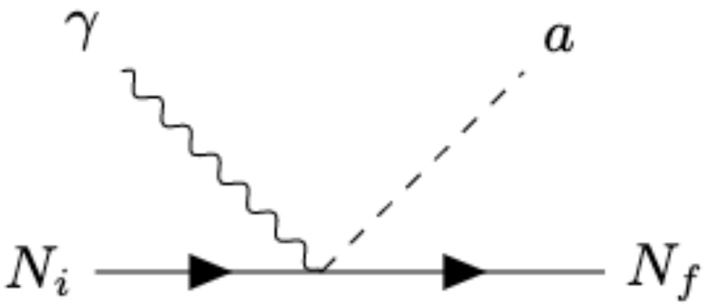

<u>This term is subleading and induced at 1-loop by another operator</u>

Crewther, Vecchia, Veneziano, Witten ('79)

Schwartz, QFT

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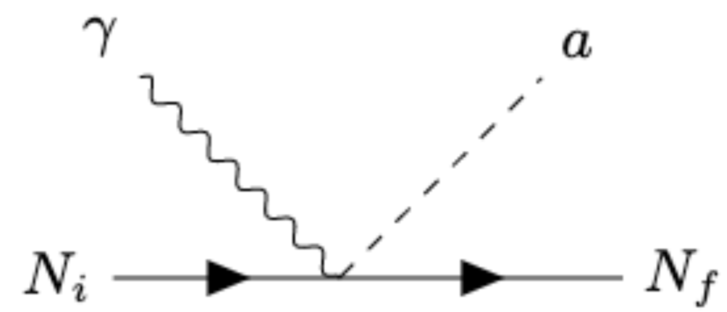

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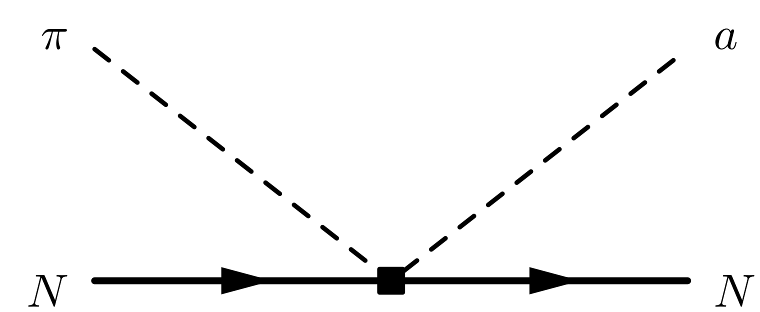

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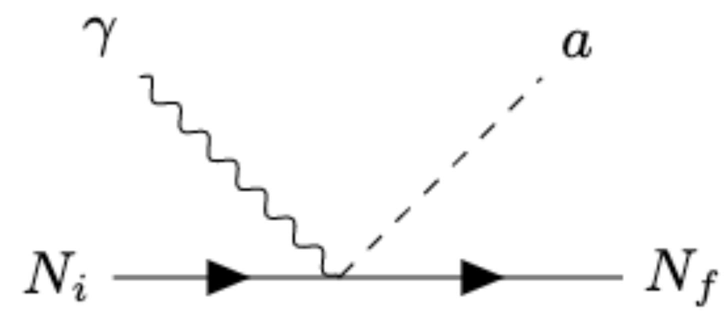


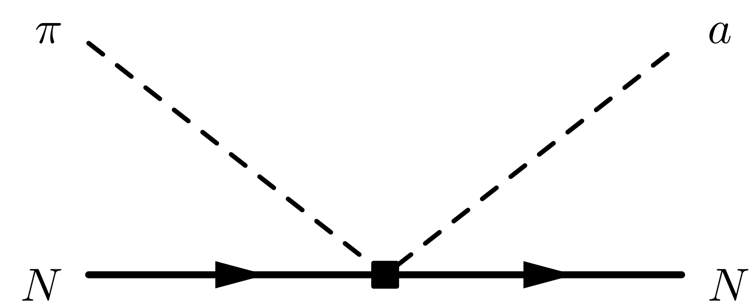
<u>This term is subleading and induced at 1-loop by another operator</u>

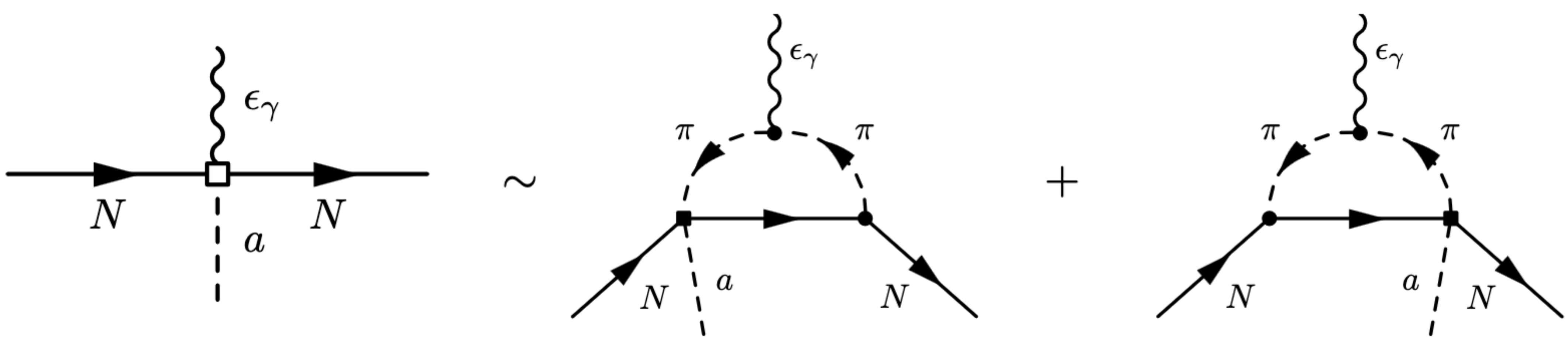
Crewther, Vecchia, Veneziano, Witten ('79)

Schwartz, QFT

$$\mathcal{L}_{\pi N}^{(2)} \supset \hat{c}_5 \bar{N} \tilde{\chi}_+ N$$

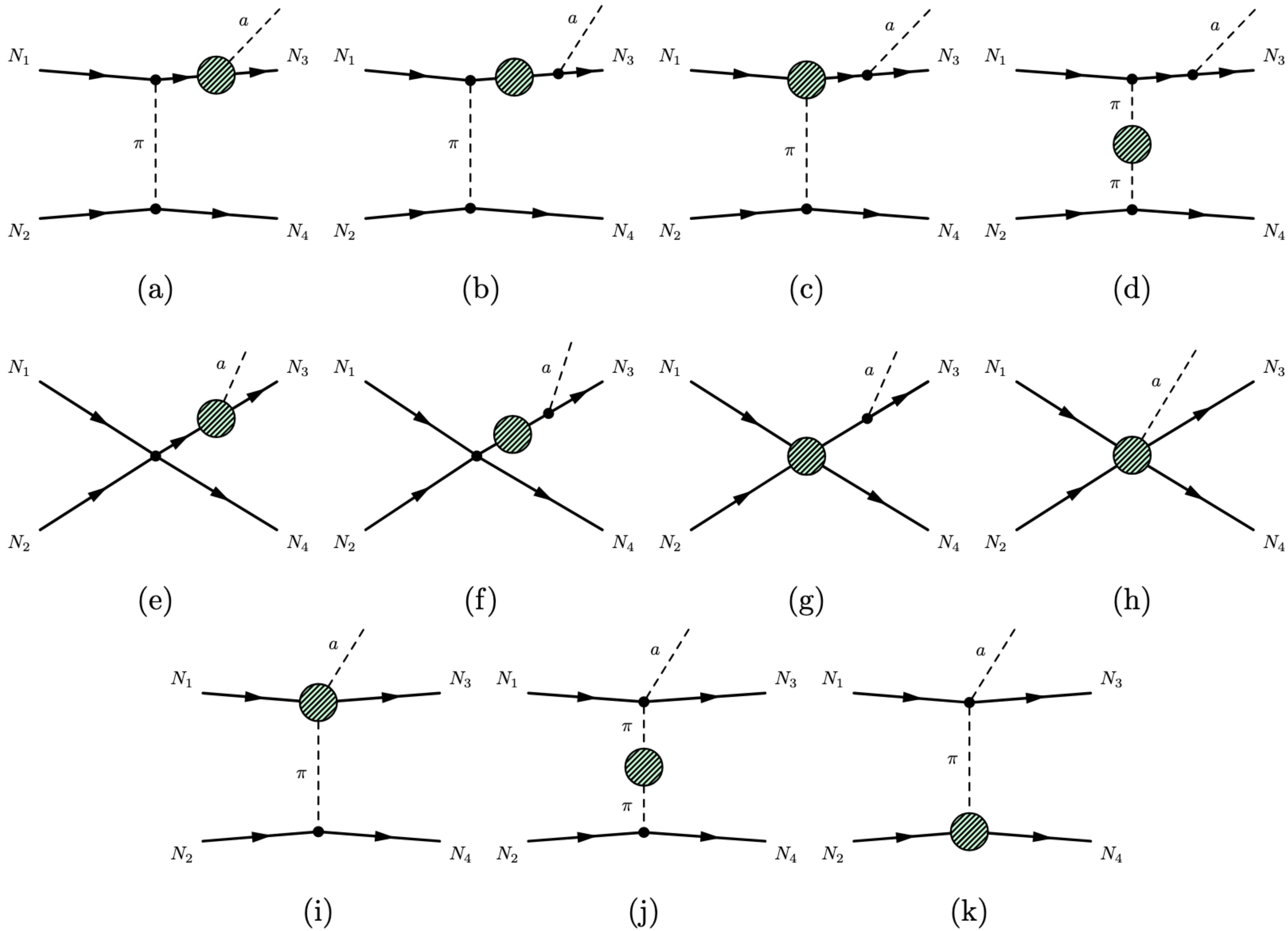
with $\tilde{\chi}_+ \supset -m_\pi^2 \frac{4z}{(1+z)^2} \left(\frac{\pi^a a}{f_\pi f_a} \right) \tau^a$





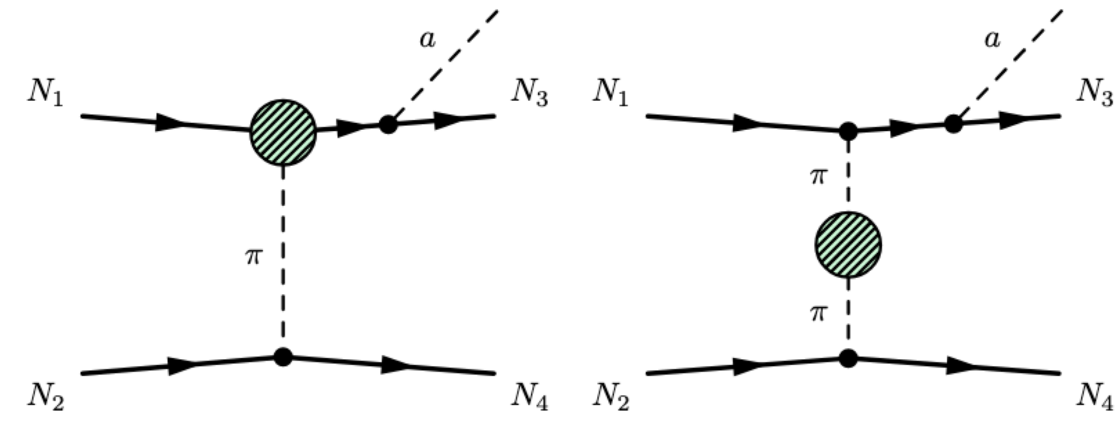
Supernova bound revisited

Relevant diagrams up to NLO



Supernova bound revisited

Neglected



(c) (d)

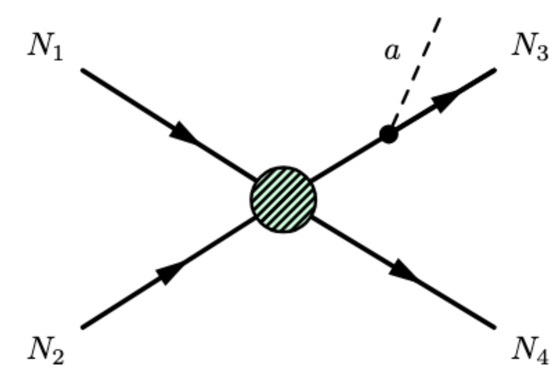
Modification of nuclear interaction:

- Fudge factor γ_p

Chang, Essig, McDermott ('18)

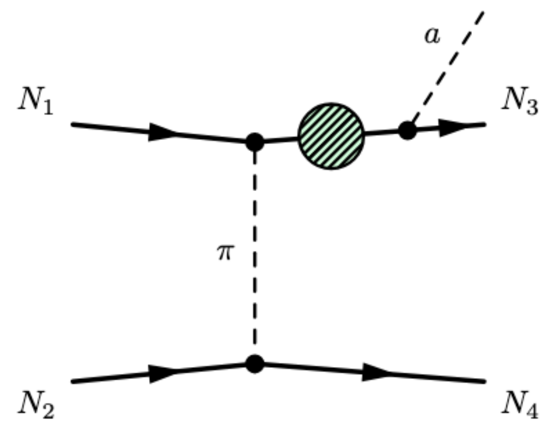
- Phenomenologically modelled

Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

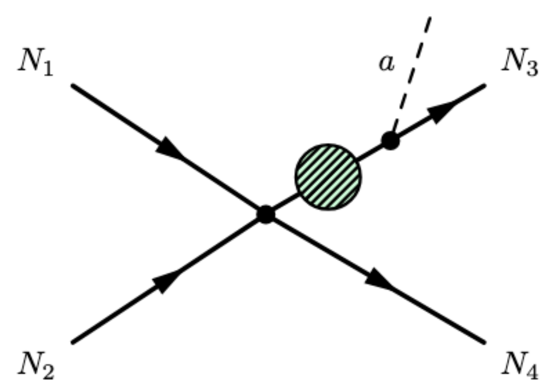


(g)

Supernova bound revisited



(b)



(f)

Modelled as nucleon re-scatterings

- Fudge factor γ_h

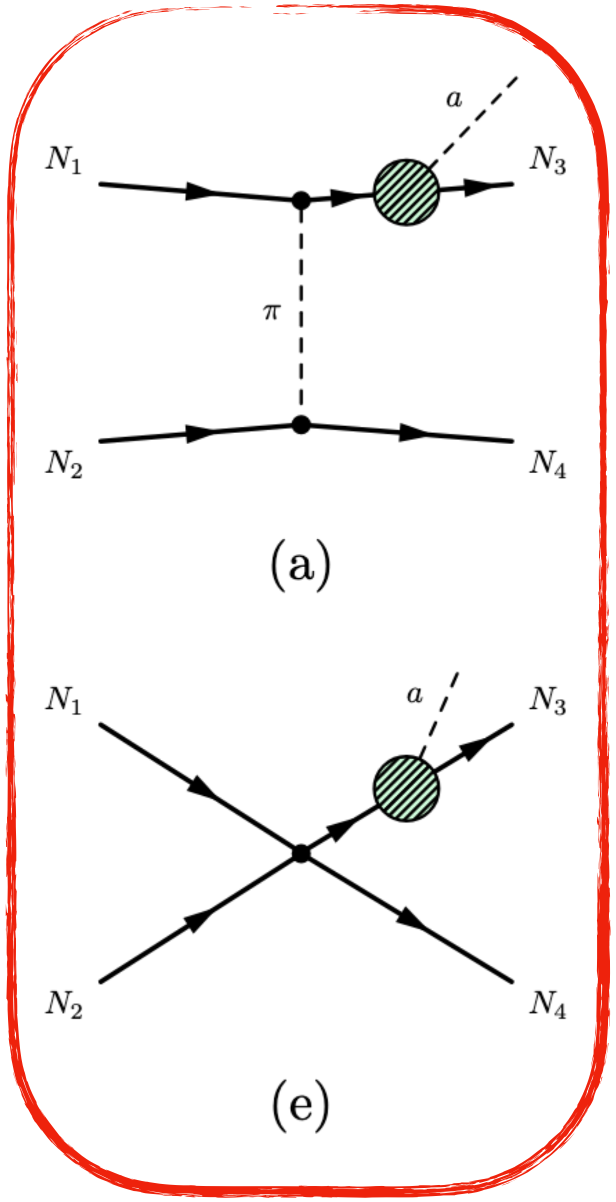
  Raffelt, Seckel ('88)

  Chang, Essig, McDermott ('18)

- Phenomenologically

  Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

Supernova bound revisited



Outlined for the first time

Springmann, MS, Stelzl, Weiler ('24)

Modified couplings

Focus on these for now

Fully systematic evaluation should take into account all diagrams up to given order