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One-loop running in bosonic theories

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CERN

Outline

based on: 2502.14030 in collaboration with
Luigi Bresciani and Nudzeim Selimovic

Outline

Series of papers

Renormalization of general Effective Field Theories

Diagrammatic, also fermion couplings

Fonseca/Olgoso/Santiago: 2501.13185

One-loop Renormalization Group Equations in Generic Effective Field Theories

Diagrammatic, even-dimensional

Misiak/Nalecz: 2501.17134

Anomalous Dimension of a General Effective Gauge Theory

OS-methods/geometric, up to dim-6

JA/Bresciani/Selimovic: 2502.14030

Renormalization Group Equations (RGEs)

RGEs

Scale-dependence of parameters

Log resummation

$\sum_N (\alpha Log(\frac{m^2}{\Lambda^2}))^N$

Renormalization scale dependence

cancelled in matching + running

Idea: General one-loop RGEs

Fields

scalars, vectors

Symmetries

Gauge, Poincaré

Computation

once and for all

Renormalizable Lagrangian

Gauge

2-loop

Luo/Wang/Xiao: hep-ph/0211440
Schienbein/Staub/Steudtner/Svirina: 1809.06797

3-loop

Pickering/Gracey/Jones: hep-ph/0104247
Poole/Thomsen: 1906.04625

4-loop

Bednyakov/Pikelner: 2105.09918

Yukawa

3-loop

Davies/Herren/Thomsen: 2110.05496
Poole/Thomsen: 1901.02749

Scalar theory

6-loop

Bednyakov/Pikelner: 2102.12832
Bednyakov: 2501.14087

On-shell methods

Helicity spinors

$$\not{p} = \begin{pmatrix} 0 & p_\mu(\sigma^\mu)_{\alpha\dot{\alpha}} \\ p_\mu(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix} = \begin{pmatrix} 0 & p_{\alpha\dot{\alpha}} \\ p^{\dot{\alpha}\alpha} & 0 \end{pmatrix}$$

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}, \quad p^{\dot{\alpha}\alpha} = \tilde{\lambda}^{\dot{\alpha}} \lambda^\alpha$$

Lorentz-invariant products

$$\lambda_i{}^\alpha \lambda_{j\alpha} = \langle i\, j \rangle$$
$$\tilde{\lambda}_{i\dot{\alpha}} \tilde{\lambda}_j{}^{\dot{\alpha}} = [i\, j]$$

Mandelstam invariants

$$s_{ij} = 2 p_i \cdot p_j = \langle i\, j \rangle [j\, i]$$

Helicity formalism: RGEs

Form factor

$F_i(\vec{n}; q) = \langle \vec{n} | \mathcal{O}_i(q) | 0 \rangle$

Fundamental relations

Miró/Ingoldby/Riembau: 2005.06983

Analyticity: $F_i^*(\{s_{ij} - i\epsilon\}) = F_i(\{s_{ij} + i\epsilon\})$

Unitarity: $\sum_{\vec{n}} \int \mathrm{dLIPS}_n |\vec{n}\rangle\langle\vec{n}| = \mathbb{1}$

CPT Theorem: $\langle \vec{n}; \mathrm{out} | \mathcal{O}_i(x) | 0 \rangle = \langle 0 | \mathcal{O}_i^\dagger(-x) | \vec{n}; \mathrm{in} \rangle$

RGEs

Caron-Huot/Wilhelm: 1607.06448

$$\left(\delta_{ij}\mu\frac{\partial}{\partial\mu} + \frac{\partial\beta_i}{\partial C_j} - \delta_{ij}\gamma_{i,\mathrm{IR}} + \delta_{ij}\beta_g\frac{\partial}{\partial g}\right) F_i = 0$$

Advantages

On-shell

No need for green’s basis

Gauge invariance

automatic

Zeros in ADM

helicity, length, angular momentum

Outline

Lagrangian

Field content

ϕ, A_μ

Symmetry

gauge, Poincaré

Mass dimension

≤ 6

Renormalizable Lagrangian

$$\mathcal{L}^{(4)} = -\frac{1}{4}F^A_{\mu\nu}F^{A\,\mu\nu} + \frac{\theta g^2}{32\pi^2}F^A_{\mu\nu}\tilde{F}^{B\,\mu\nu} + \frac{1}{2}(D_\mu\phi)_a(D^\mu\phi)^a$$
$$- \Lambda - t_a\phi_a - \frac{m^2_{ab}}{2!}\phi_a\phi_b - \frac{h_{abc}}{3!}\phi_a\phi_b\phi_c - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

Operator classification

Amplitude

$\mathcal{M} \sim \mathcal{K}_i$

$\mathcal{K}_i = \mathcal{K}_i(\{\langle i\,j\rangle, [i\,j]\})$

Three-point function

Benincasa/Cachazo: 0705.4305

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) \sim \begin{cases} \langle 1\,2\rangle^{a_3}\langle 2\,3\rangle^{a_1}\langle 3\,1\rangle^{a_2} & \text{if } h_1 + h_2 + h_3 < 0 \\ [1\,2]^{-a_3}[2\,3]^{-a_1}[3\,1]^{-a_2} & \text{if } h_1 + h_2 + h_3 > 0 \end{cases}$$

$$a_1 = h_1 - h_2 - h_3\,, \qquad a_2 = h_2 - h_3 - h_1\,, \qquad a_3 = h_3 - h_1 - h_2\,,$$

Mass dimension

Durieux/Machado: 1912.08827
Li/Ren/Xiao/Yu/Zheng: 2201.04639

$[\mathcal{K}_i] = [\mathcal{O}_i] - \ell(\mathcal{O}_i) \geq 0$

Example: Dim-5

$[\mathcal{K}_i] = 5 - \ell(\mathcal{O}_i)$:

$$\ell_i = 5 \quad \Longrightarrow \quad \mathcal{K}_i = \begin{cases} 1 \end{cases} \quad \Longrightarrow \quad \mathcal{O}_i = \begin{cases} \phi_1\, \phi_2\, \phi_3\, \phi_4\, \phi_5 \end{cases}$$

$$\ell_i = 4 \quad \Longrightarrow \quad \mathcal{K}_i = \begin{cases} \langle 1\, 2 \rangle \end{cases} \quad \Longrightarrow \quad \mathcal{O}_i = \begin{cases} \psi_{1L}\, \psi_{2L}\, \phi_3\, \phi_4 \end{cases}$$

$$\ell_i = 3 \quad \Longrightarrow \quad \mathcal{K}_i = \begin{cases} \langle 1\, 2 \rangle^2 \\ \langle 1\, 2 \rangle \langle 1\, 3 \rangle \end{cases} \quad \Longrightarrow \quad \mathcal{O}_i = \begin{cases} F_{1L}\, F_{2L}\, \phi_3 \\ F_{1L}\, \psi_{2L}\, \psi_{3L} \end{cases}$$

Dim-5 Lagrangian

$$\mathcal{L}^{(5)} = \left[C_{\phi F^2}\right]_a^{A_\alpha B_\beta} \phi_a F^{A_\alpha}_{\mu\nu} F^{B_\beta\,\mu\nu} + \left[C_{\phi \tilde{F}^2}\right]_a^{A_\alpha B_\beta} \phi_a F^{A_\alpha}_{\mu\nu} \tilde{F}^{B_\beta\,\mu\nu} + \left[C_{\phi^5}\right]_{abcde} \phi_a \phi_b \phi_c \phi_d \phi_e$$

Dim-6 Lagrangian

$$\mathcal{L}^{(6)} = \left[C_{\phi^2 F^2}\right]^{A_\alpha B_\beta}_{ab} \phi_a \phi_b F^{A_\alpha}_{\mu\nu} F^{B_\beta\,\mu\nu} + \left[C_{\phi^2 \tilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} \phi_a \phi_b F^{A_\alpha}_{\mu\nu} \tilde{F}^{B_\beta\,\mu\nu}$$
$$+ \left[C_{\phi^6}\right]_{abcdef} \phi_a \phi_b \phi_c \phi_d \phi_e \phi_f + \left[C_{D^2\phi^4}\right]_{abcd} (D_\mu \phi)_a (D^\mu \phi)_b \phi_c \phi_d$$
$$+ \left[C_{F^3}\right]^{A_\alpha B_\alpha C_\alpha} F^{A_\alpha\,\nu}_{\mu} F^{B_\alpha\,\rho}_{\nu} F^{C_\alpha\,\mu}_{\rho} + \left[C_{\widetilde{F}^3}\right]^{A_\alpha B_\alpha C_\alpha} F^{A_\alpha\,\nu}_{\mu} F^{B_\alpha\,\rho}_{\nu} \widetilde{F}^{C_\alpha\,\mu}_{\rho}$$

Dim-6: Wilson coefficients

Name	Operator	Symmetry
$\mathcal{O}_{\phi^6}$	$\phi_a\phi_b\phi_c\phi_d\phi_e\phi_f$	$[C_{\phi^6}]_{abcdef} = [C_{\phi^6}]_{(abcdef)}$
$\mathcal{O}_{D^2\phi^4}$	$(D_\mu\phi)_a(D^\mu\phi)_b\phi_c\phi_d$	$[C_{D^2\phi^2}]_{abcd} = [C_{D^2\phi^2}]_{(ab)cd} = [C_{D^2\phi^2}]_{ab(cd)}$
$\mathcal{O}_{\phi^2F^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu} F^{B_\beta\,\mu\nu}$	$[C_{\phi^2F^2}]^{A_\alpha B_\beta}_{ab} = [C_{\phi^2F^2}]^{(A_\alpha B_\beta)}_{ab} = [C_{\phi^2F^2}]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{\phi^2\tilde{F}^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu} \tilde{F}^{B_\beta\,\mu\nu}$	$\left[C_{\phi^2\tilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} = \left[C_{\phi^2\tilde{F}^2}\right]^{(A_\alpha B_\beta)}_{ab} = \left[C_{\phi^2\tilde{F}^2}\right]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{F^3}$	$F^{A_\alpha\ \nu}_{\mu} F^{B_\alpha\ \rho}_{\nu} F^{C_\alpha\ \mu}_{\rho}$	$[C_{F^3}]^{A_\alpha B_\alpha C_\alpha} = [C_{F^3}]^{[A_\alpha B_\alpha C_\alpha]}$
$\mathcal{O}_{\tilde{F}^3}$	$F^{A_\alpha\ \nu}_{\mu} F^{B_\alpha\ \rho}_{\nu} \tilde{F}^{C_\alpha\ \mu}_{\rho}$	$[C_{\tilde{F}^3}]^{A_\alpha B_\alpha C_\alpha} = [C_{\tilde{F}^3}]^{[A_\alpha B_\alpha C_\alpha]}$

Outline

RGEs

Order

$\mathcal{O}(\frac{1}{\Lambda^2})$

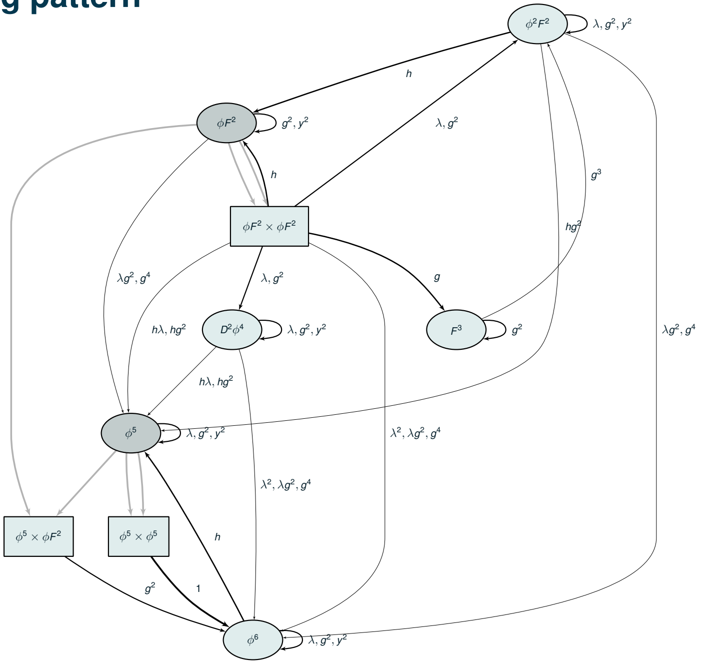
Renormalizable part

dim-5 & dim-6

Dim-5, Dim-6

All contributions

Mixing pattern



Example: $F^3 \rightarrow F^3$

$$\left[\dot{C}_{F^3}\right]^{ABC} = 4g^2\Big(F^{BCDF}\left[C_{F^3}\right]^{ADF} + F^{CADF}\left[C_{F^3}\right]^{BDF} + F^{ABDF}\left[C_{F^3}\right]^{CDF}\Big)$$
$$+ \gamma^{AD}_{c,v}\left[C_{F^3}\right]^{DBC} + \gamma^{BD}_{c,v}\left[C_{F^3}\right]^{ADC} + \gamma^{CD}_{c,v}\left[C_{F^3}\right]^{ABD}$$

$F^{ABCD} = f^{ABE} f^{CDE}$

Outline

SMEFT: $O_G \to O_G$

$$O_G = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$$

With

$[C_{F^3}]^{ABC} = C_G f^{ABC}$

$[\gamma_{c,G}]^{AB} = -g_3^2\, b_{0,3} \delta^{AB}$

One finds

$$\dot{C}_G\, f^{ABC} = 4 g_3^2 \Big(F^{BCDF} f^{ADF} C_G + F^{CADF} f^{BDF} C_G + F^{ABDF} f^{CDF} C_G \Big) + \gamma_{c,v}^{AD} C_G f^{DBC} + \gamma_{c,v}^{BD} \left[C_{F^3}\right]^{ADC} + \gamma_{c,v}^{CD} \left[C_{F^3}\right]^{ABD}$$

$$\Rightarrow \dot{C}_G = (12 N_c - 3 b_{0,3})\, g_3^2\, C_G$$

SMEFT: Higgs sector

Higgs

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix}$$

Generators

$$\theta^A = i \begin{pmatrix} Im(T^A) & Re(T^A) \\ -Re(T^A) & Im(T^A) \end{pmatrix}$$

Higgs Lagrangian

$$\mathcal{L}_{\mathrm{SM}} \supset \frac{1}{2}(D_\mu \phi)_a (D^\mu \phi)_a + \frac{1}{4} m_H^2 \delta_{ab}\, \phi_a \phi_b - \frac{1}{4} \lambda \delta_{(ab}\delta_{cd)}\, \phi_a \phi_b \phi_c \phi_d$$

Reproduced results

SMEFT

Complete bosonic sector

Alonso/Jenkins/Manohar/Trott: 1308.2627, 1312.2014

ALP-SMEFT

$aX, a\widetilde{X} \to H^6, H^4 D^2, X^2 H^2, X^3$

Galda/Neubert/Renner: 2105.01078
Brescian/Brunello/Levati/Mastrolia/Paradisi: 2412.04160

$O(n)$ scalar EFT

λ-dependence

Cao/Herzog/Melia/Nepveu: 2105.12742
Jenkins/Manohar/Naterop/Pagès: 2310.19883

New results: ALP-SMEFT

Lagrangian

$$\mathcal{L}_{\rm ALP}^{(5)} = C_{aG}\, a\, G^A_{\mu\nu} G^{A\,\mu\nu} + C_{a\tilde{G}}\, a\, G^A_{\mu\nu} \tilde{G}^{A\,\mu\nu} + C_{aW}\, a\, W^I_{\mu\nu} W^{I\,\mu\nu}$$
$$+ C_{a\tilde{W}}\, a\, W^I_{\mu\nu} \tilde{W}^{I\,\mu\nu} + C_{aB}\, a\, B_{\mu\nu} B^{\mu\nu} + C_{a\tilde{B}}\, a\, B_{\mu\nu} \tilde{B}^{\mu\nu}$$

mass

$$\dot{m}^2 = 8\, m^4 \left(C_{aB}^2 + C_{a\tilde{B}}^2 + 3\, C_{aW}^2 + 3\, C_{a\tilde{W}}^2 + 8\, C_{aG}^2 + 8\, C_{a\tilde{G}}^2 \right)$$

dim-6: $X^2 H^2$

$$\dot{C}_{H\tilde{B}} = 4\, g_1^2\, C_{aB}\, C_{a\tilde{B}}\,, \quad \dot{C}_{H\tilde{W}} = 4\, g_2^2\, C_{aW}\, C_{a\tilde{W}}\,,$$
$$\dot{C}_{H\tilde{W}B} = 4\, g_1\, g_2 \left(C_{a\tilde{B}} C_{aW} + C_{aB} C_{a\tilde{W}} \right)$$

Outline

Summary

General bosonic EFT

Operators up to dim-6

Complete RGEs

λ, h, g

New results

RGEs for ALP-SMEFT