Growth of Cosmic Strings Beyond Kination

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Why Cosmic Strings?

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In String Theory, **Cosmic String Networks** can form:

- From brane-antibrane annihilation at the end of inflation (see [Cicoli et al:2024] for recent progress)
- From closed strings which grow and percolate into a network [Conlon et al: 2024]

Once formed, networks reach a scaling regime [Martins, Shellard:1996, 2000; Revello, Villa:2024]:

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The network **emits GWs** through production of loops [Allen, Caldwell: 1992; Lewicki et al: 2018; Ghoshal, Revello, Villa: 2025].

Recently proposed to fit the PTA signal [Ellis et al.:2023; Avgoustidis et al.:2025]

In scaling regime: can give information about the cosmological history of the Early Universe!

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Goal: study the mechanism underlying formation of a string network through **comoving growth** of elementary strings in Type IIB String Theory.

Setup: population of closed circular loops nucleated after inflation can grow to cosmological size and form a network if their tension decreases sufficiently fast [Conlon et al.:2024; Revello, Villa:2024]

Well-established case: fundamental strings with tension controlled by kinating volume of the internal dimensions.

We analyse the problem using Dynamical System Techniques, and find this mechanism works in more general situations!

Overview

- **Part I: Strings with Time-Dependent Tension**
- **Part II: Strings with Field-Dependent Tension and Dynamical System Approach**
- **Part III: Cosmic Strings in Type IIB String Theory**

Part I:
Strings with Time-Dependent Tension

Equations of Motion

Nambu-Goto action for a string with spacetime-dependent tension:

$$S_{\rm NG} = -\int {\rm d}^2\sigma\, \mu(x)\, \sqrt{-\gamma}$$

with $x^\mu = x^\mu(\sigma)$ and $\gamma_{ab} = \dfrac{\partial x^\mu}{\partial \sigma^a}\dfrac{\partial x^\nu}{\partial \sigma^b}\, g_{\mu\nu}$ in FLRW spacetime ${\rm ds}^2 = {\rm dt}^2 - {\rm a(t)}^2 {\rm d}\mathbf{x}^2$

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Varying the action [Emond et al. :2021]:

$$\frac{1}{\mu\sqrt{-\gamma}} \partial_a(\mu\sqrt{-\gamma}\gamma^{ab} x^\mu_{,b}) + \Gamma^\mu_{\nu\rho} \gamma^{ab} x^\nu_{,a} x^\rho_{,b} - \frac{\partial^\mu \mu}{\mu} = 0$$

Equations of Motion of the Circular Loop

Gauge-fixing:

$$\sigma^0 = x^0 = t \qquad \text{and} \qquad \frac{\partial \mathbf{x}}{\partial \sigma^0} \cdot \frac{\partial \mathbf{x}}{\partial \sigma^1} = 0$$

Circular-loop ansatz:

$$\mathbf{x}(t) = R(t)\,(\cos\theta, \sin\theta, 0)$$

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Circular-loop ansatz:

$$\mathbf{x}(t) = R(t)\,(\cos\theta, \sin\theta, 0)$$

Equations of motion $(\mu(x) \equiv \mu(t))$:

$$\frac{\dot{\epsilon}}{\epsilon} = H - a^2 \dot{R}^2 \left(2H + \frac{\dot{\mu}}{\mu}\right)$$

$$\ddot{R} + H\dot{R} + \frac{R}{\epsilon^2} + \left(2H + \frac{\dot{\mu}}{\mu}\right)(1 - a^2 \dot{R}^2)\dot{R} = 0$$

Where:

$$\epsilon^2 \equiv \frac{a^2 R^2}{1 - a^2 \dot{R}^2} \quad \Rightarrow \quad \boxed{\epsilon(t) = a(t) R_{\max}(t)}$$

Physiscal Radius of the String

Growth Condition

Growth in comoving coordinates if:

$$\frac{\dot{\epsilon}}{\epsilon} > H$$

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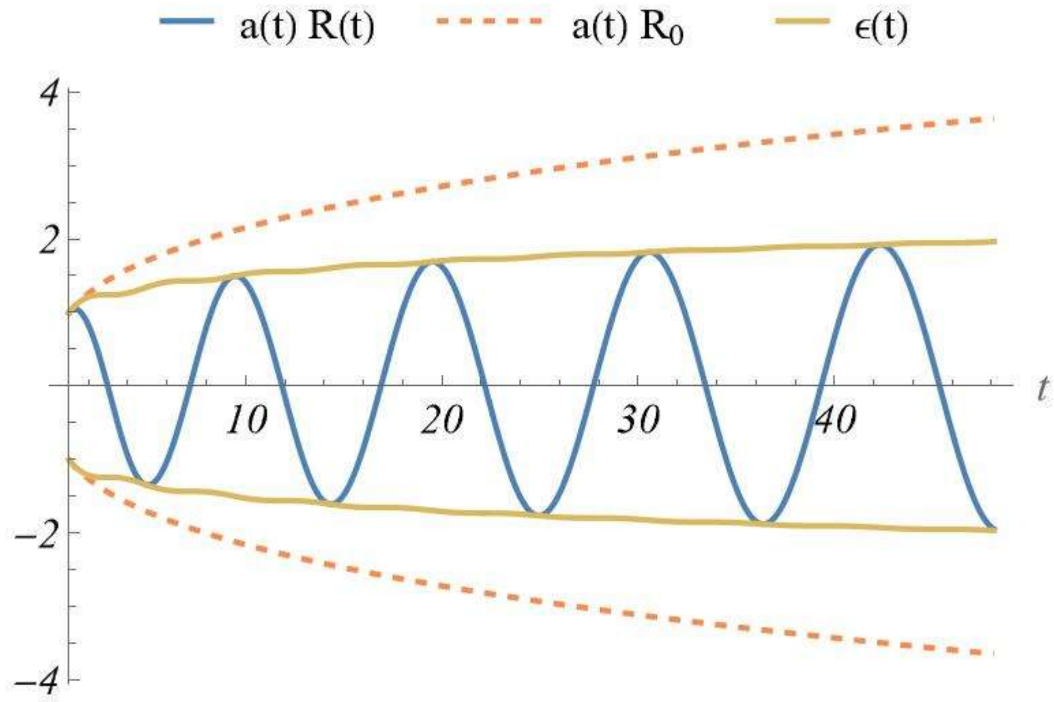
In expanding Universe, growth assured by the **growth condition** [Conlon et al.: 2024]:

$$\boxed{2H + \frac{\dot{\mu}}{\mu} < 0}$$

If string tension decreases sufficiently fast with the scale factor, the physical radius of the string grows faster than Hubble radius and loops can percolate!

$$\mu \sim a^{-q},\, q > 2 \implies$$ **Net Growth!**

$$\mu \sim a^{-q}$$



$$q < 2$$

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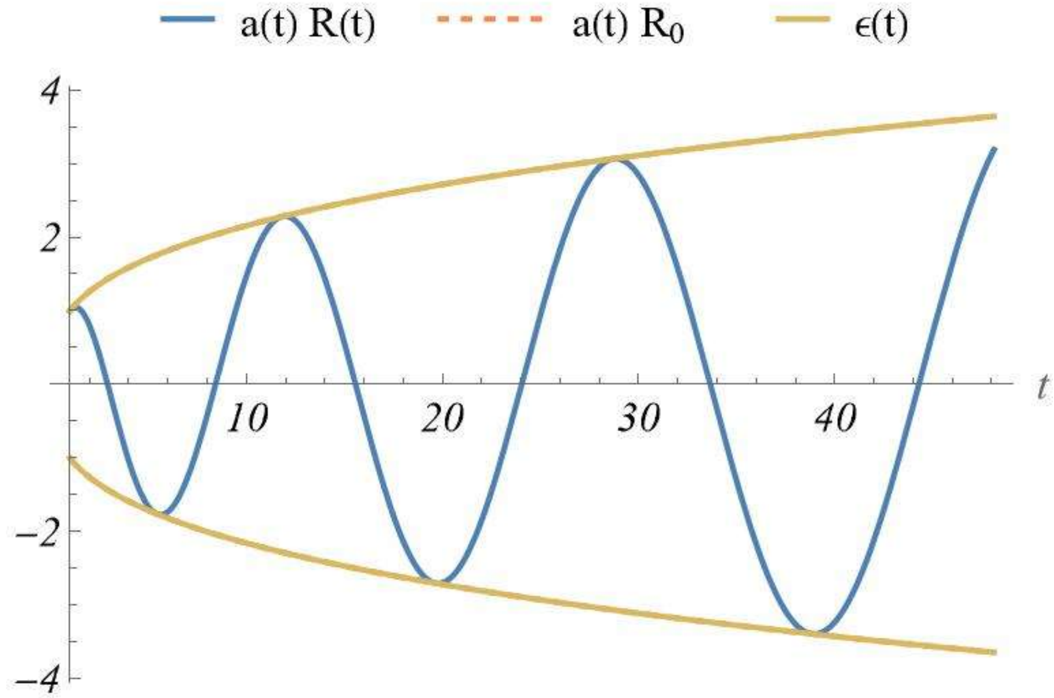

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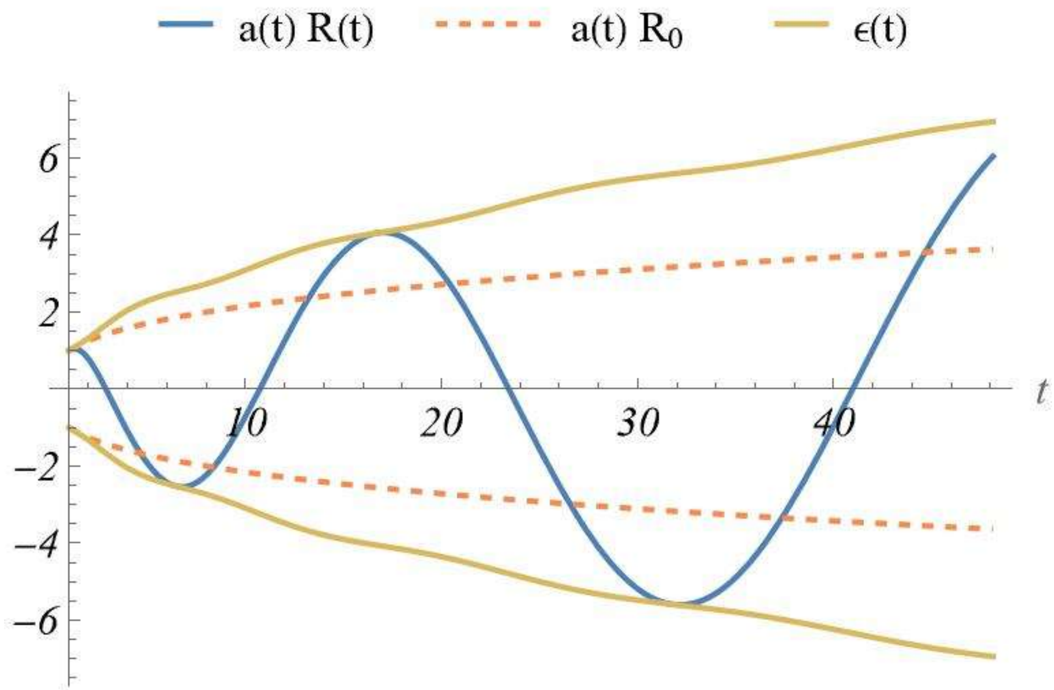

$$q > 2$$

Part II:
Strings with Field-Dependent Tension and Dynamical System Approach

Field-Dependent Tension

Suppose time-variation of tension set by time-dependence of a scalar field ϕ

Consider a population of string loops with exponential tension:

$$\mu = \mu_0 \, e^{-\xi \frac{\phi}{M_p}}$$

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In number of efoldings $N = \ln a$ the growth condition becomes $(M_p = 1)$:

$$2 - \xi\phi' < 0$$

Depends on ξ and dynamics of ϕ!

We can study it using Dynamical System (DS) Techniques!

The Dynamical System Setup

Flat FLRW Universe filled with background perfect fluid with eos:

$$p_{\rm f} = \omega \rho_{\rm f}$$

Initial population of string loops with negligible energy density (however, see [Conlon et al.:2025]) :

$$\rho_s \ll \rho_\phi, \rho_{\rm f}$$

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$$X = \frac{\phi'}{\sqrt{6}} \qquad Y = \frac{1}{H}\sqrt{\frac{V(\phi)}{3}}$$

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Friedmann equation:

$$X^2 + Y^2 + \frac{\rho_{\rm f}}{3H^2} = 1 \Rightarrow X^2 + Y^2 \leq 1$$

The Dynamical System Setup

Consider an exponential potential:

$$V = V_0\, e^{-\lambda\phi}$$

Klein-Gordon and Continuity eq. in phase space [Copeland, Liddle, Wands:1998]:

$$X' = -3X + \lambda\sqrt{\frac{3}{2}}Y^2 + \frac{3}{2}X\left[2X^2 + (\omega+1)(1 - X^2 - Y^2)\right]$$

$$Y' = -\lambda\sqrt{\frac{3}{2}}\,XY + \frac{3}{2}Y\left[2X^2 + (\omega+1)(1 - X^2 - Y^2)\right].$$

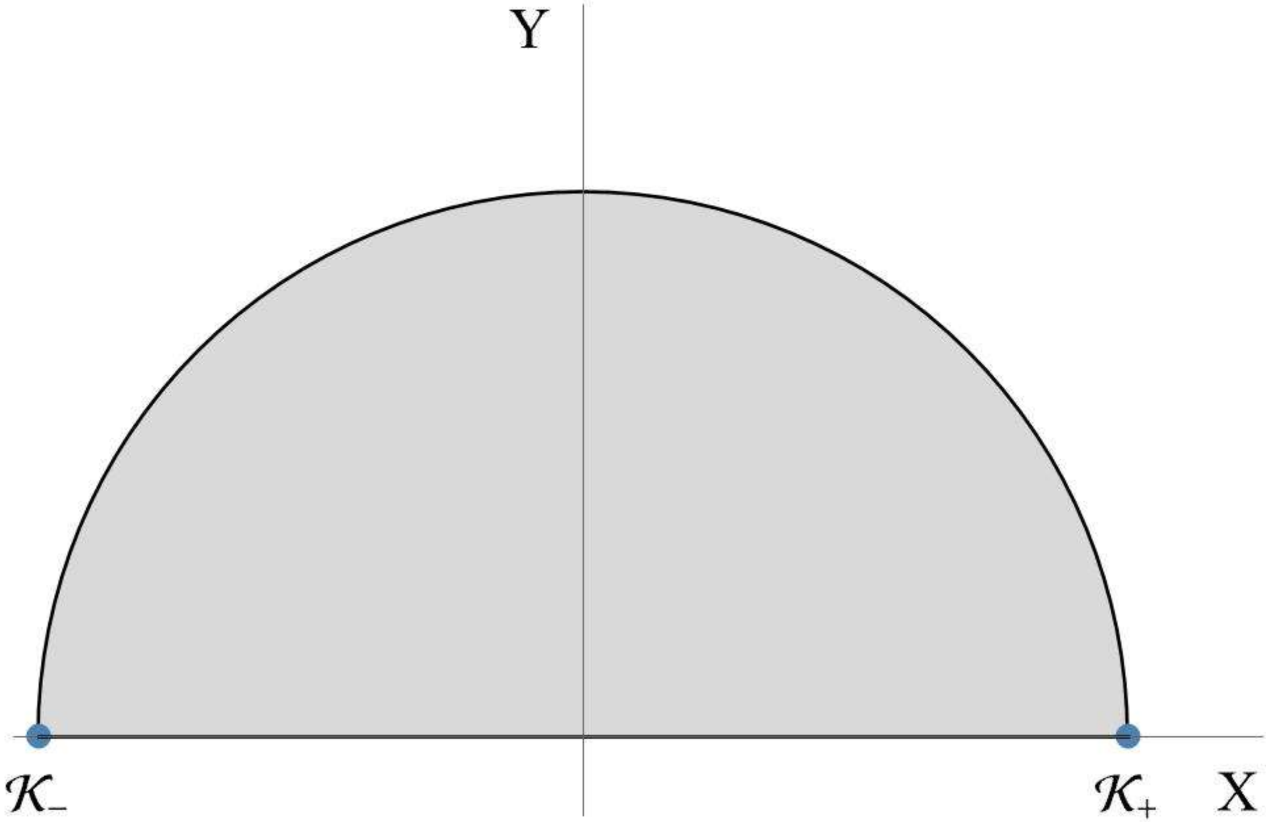
Fixed points fully classified!

Phase Space and Fixed Points

$$X = \frac{\phi'}{\sqrt{6}}$$

$$Y = \frac{1}{H}\sqrt{\frac{V(\phi)}{3}}$$

FP	X	Y
$\mathcal{K}_+$	1	0
$\mathcal{K}_-$	-1	0
$\mathcal{F}$	0	0
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}}\frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$



Kinating fixed points

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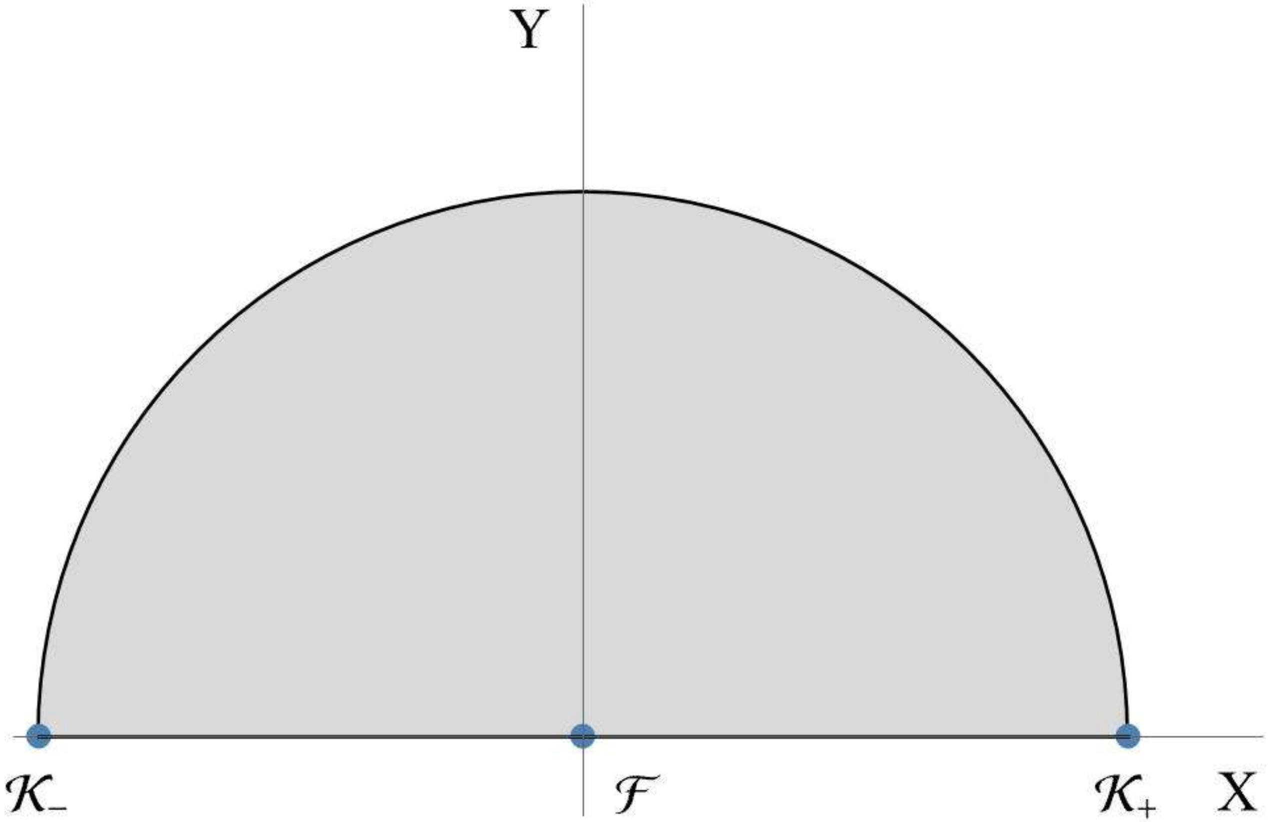

Fluid domination fixed point

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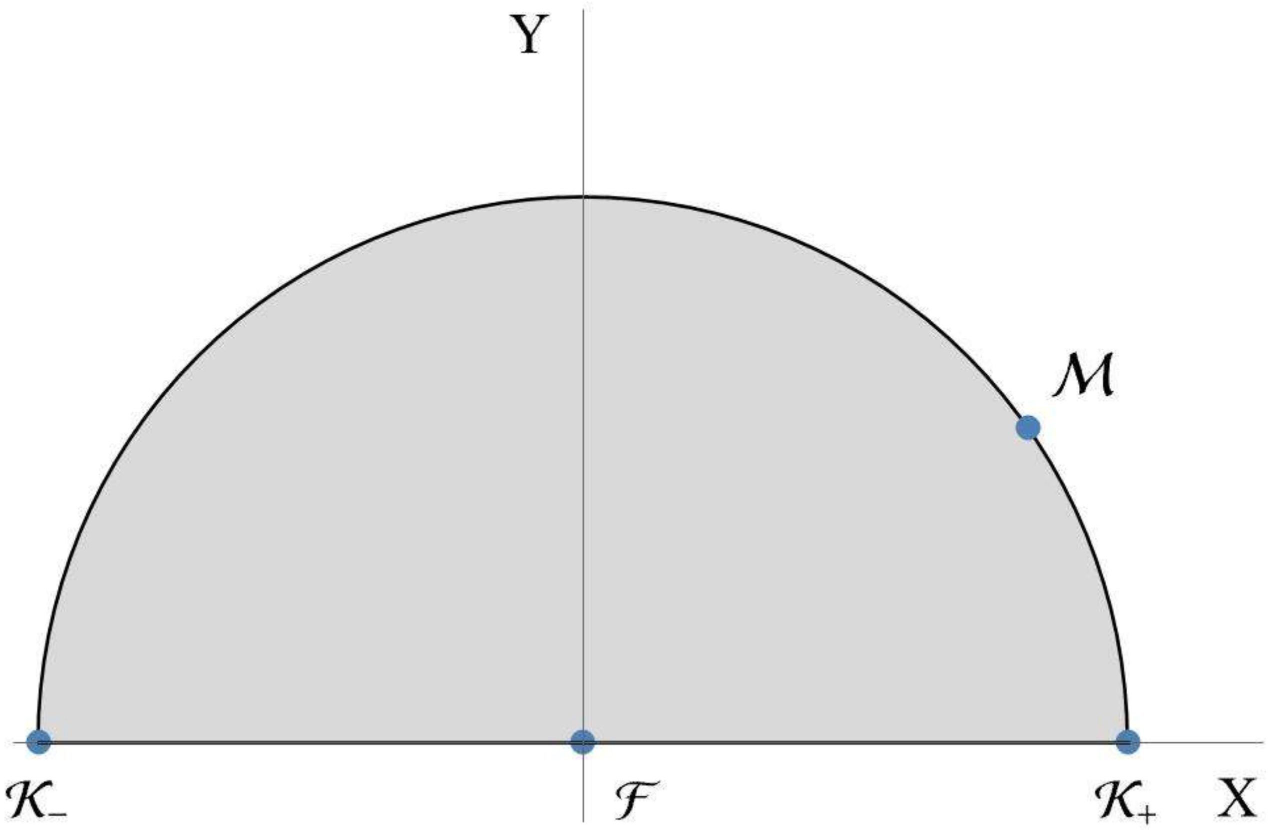

Field domination fixed point

Phase Space and Fixed Points

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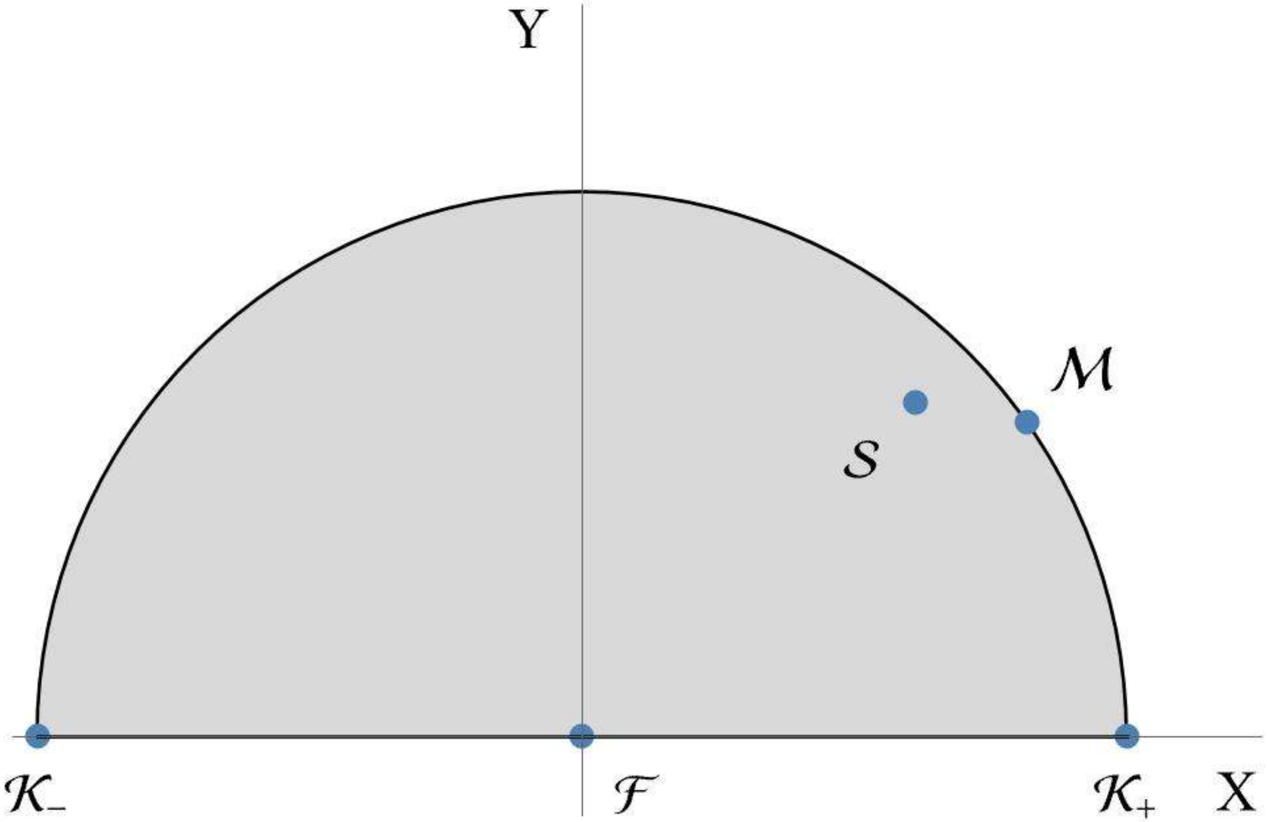

Scaling fixed point

Growth Condition and Phase Space Constraints

In phase space, the growth condition identifies a **Growth Region (GR):**

$$X > \sqrt{\frac{2}{3}\frac{1}{\xi}}$$

If $\xi > 0$

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Can we push the fixed points of the dynamical system within the GR?

Fixed Points of Growth

FP	X	Y	Existence	Existence and Growth when $\lvert\xi\rvert > \sqrt{2/3}$
$\mathcal{K}_+$	1	0	$\forall\lambda$ and $\forall\omega$	$\forall\lambda$ and $\forall\omega$
$\mathcal{K}_-$	-1	0	$\forall\lambda$ and $\forall\omega$	$\forall\lambda$ and $\forall\omega$
$\mathcal{F}$	0	0	$\forall\lambda$ and $\forall\omega$	never
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	$\lvert\lambda\rvert < \sqrt{6}$	$\frac{2}{\lvert\xi\rvert} < \lvert\lambda\rvert < \sqrt{6}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}}\frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$	$\lambda^2 > 3(\omega+1)$	$\sqrt{3(\omega+1)} < \lvert\lambda\rvert < \frac{3}{2}(\omega+1)\lvert\xi\rvert$ with $\sqrt{\omega+1} > \frac{2}{\sqrt{3}\lvert\xi\rvert}$

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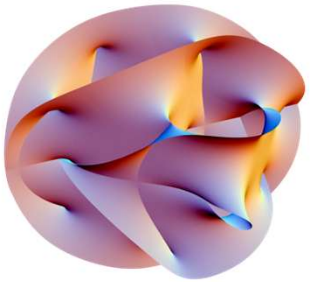
Depending on the values of λ, ω and ξ, $\mathcal{K}_\pm$, $\mathcal{M}$ and $\mathcal{S}$ can be in the GR!

Part III:
Cosmic Strings in Type IIB String Theory

Type IIB Compactifications and Moduli

D = 10 TYPE IIB
STRING THEORY

⇓



Calabi-Yau

⇓

D=4 $\mathcal{N}$=1
SUGRA EFT

Type IIB Compactifications and Moduli

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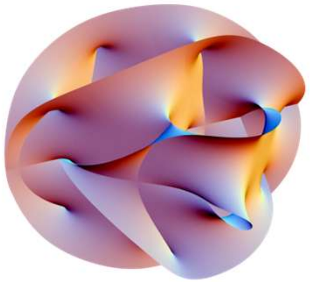

Calabi-Yau

Moduli
Scalar Fields
in the EFT

D=4 $\mathcal{N}$=1
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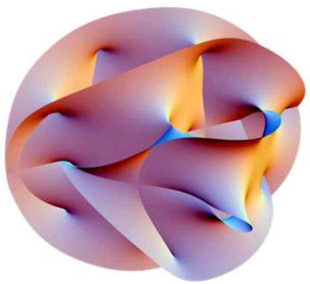

Calabi-Yau

D=4 $\mathcal{N}$=1 SUGRA EFT

Moduli
Scalar Fields in the EFT

Complex Structure Moduli: Shape of Extra Dimension

Kähler Moduli: Size of cycles on Extra Dimensions

Type IIB Compactifications and Moduli

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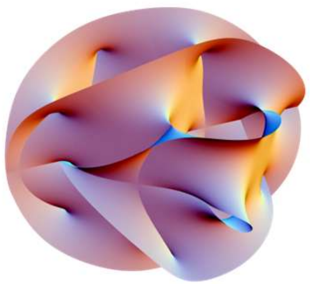

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Scalar Fields in the EFT

Complex Structure Moduli:
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Kähler Moduli:
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No free parameters in Stringy EFT:
μ function of moduli!

Fundamental Strings

Fundamental strings (F-strings) in 4d are produced by string-scale energy effects.

Tension (in Einstein frame):

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Depends on two moduli: $\mathcal{V}$ and $g_s = e^{\varphi}$

In terms of the dilaton:

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No GR in the phase space of φ

Volume Mode: Growth Region

Tension dependence on $\mathcal{V}$

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In the simplest case:

$$\mathcal{V} = \tau^{3/2}$$

Canonically normalised field:

$$\frac{\chi}{M_p} = \sqrt{\frac{3}{2}} \ln \tau = \sqrt{\frac{2}{3}} \ln \mathcal{V}$$

Tension dependence on χ

$$\mu \sim M_p^2 e^{-\sqrt{\frac{3}{2}} \frac{\chi}{M_p}} \Longrightarrow \boxed{\xi = \sqrt{\frac{3}{2}} > \sqrt{\frac{2}{3}}}$$

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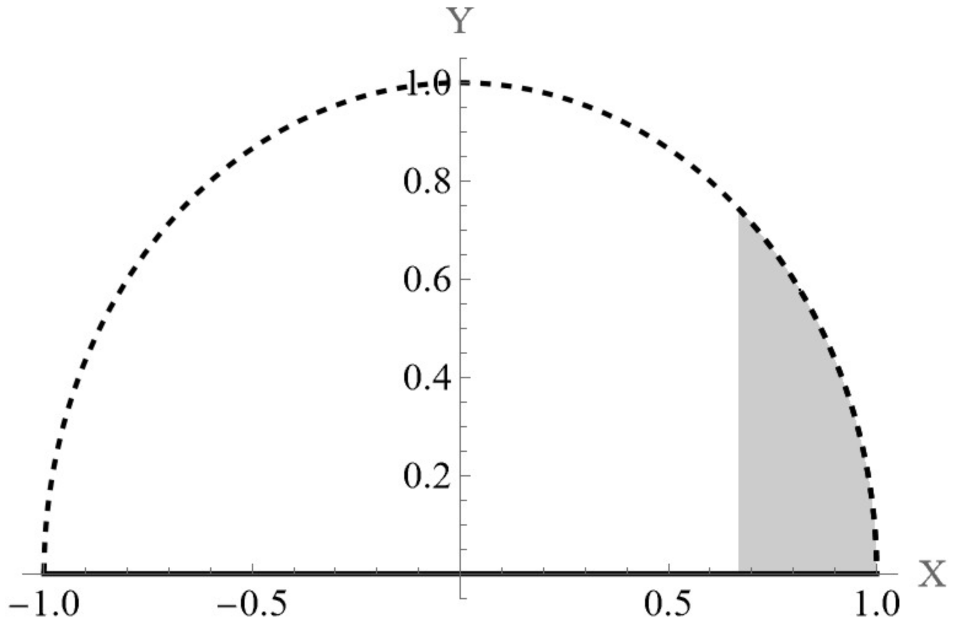
Growth region exists!

$$X > \frac{2}{3}$$



Volume Mode: Kination

$$\mu \sim M_p^2 e^{-\sqrt{\frac{3}{2}}\frac{\chi}{M_p}}$$

Typical potential for volume $(M_p = 1)$:

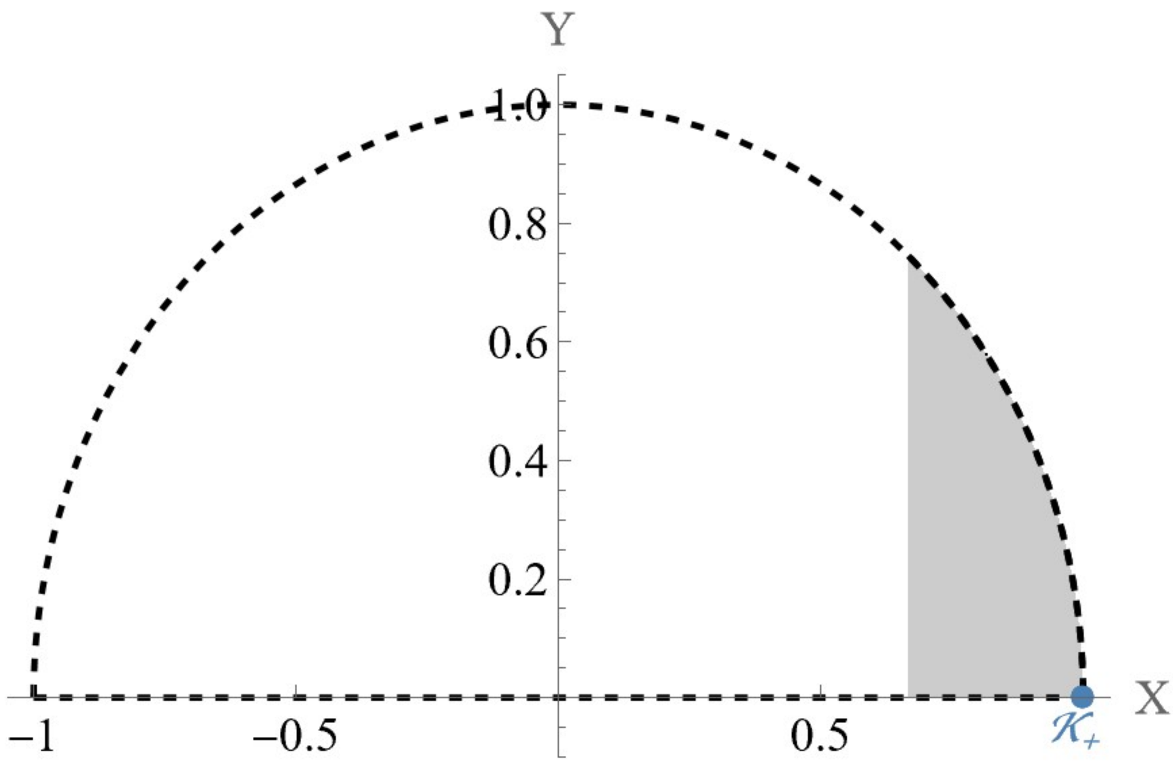
$$V \sim \frac{\alpha}{\mathcal{V}^p} \Rightarrow V = V_0 e^{-\lambda\chi}$$

We can use our DS techniques!

$\mathcal{K}_+ = (1, 0)$ fixed point in the growth region [Conlon et al.: 2024]

But it is unstable!

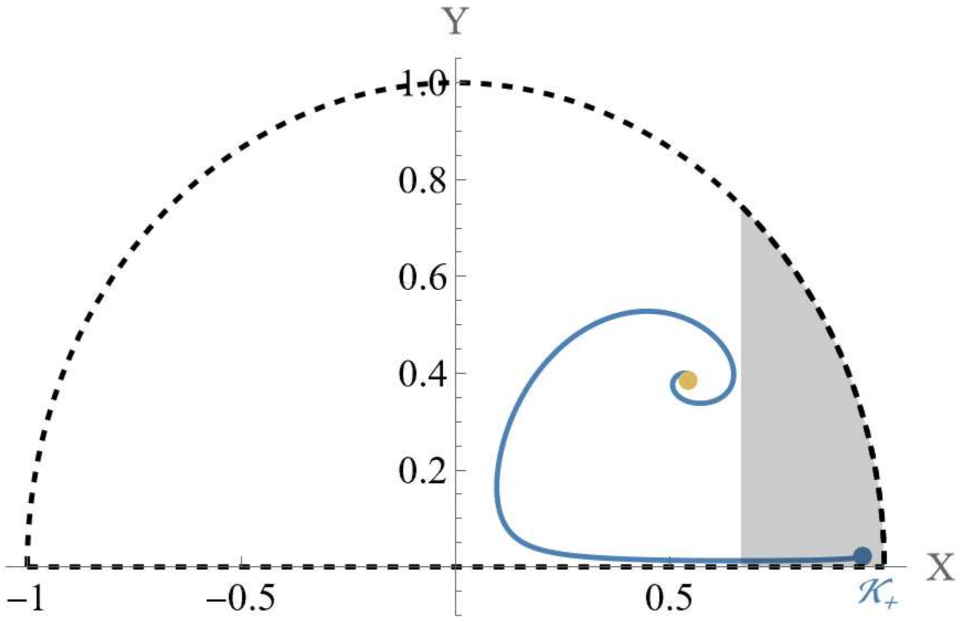
$$\rho_{\text{kin}} \sim a^{-6}$$



Volume Mode: Almost Kination

$$V = V_0 e^{-\lambda\chi}$$

Starting close to $\mathcal{K}_+$, the system spends some time in the **GR**, but then exits.



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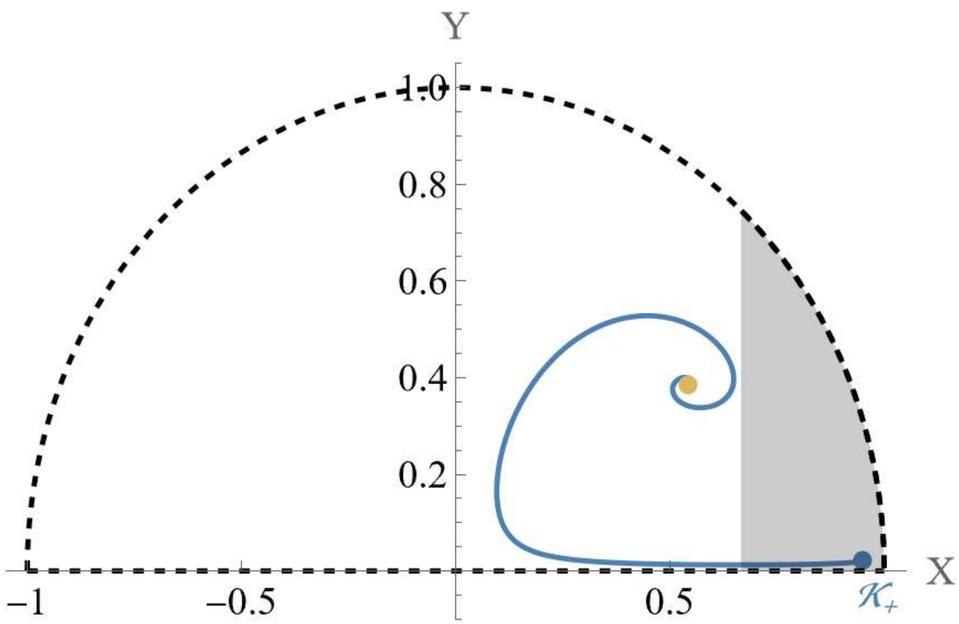

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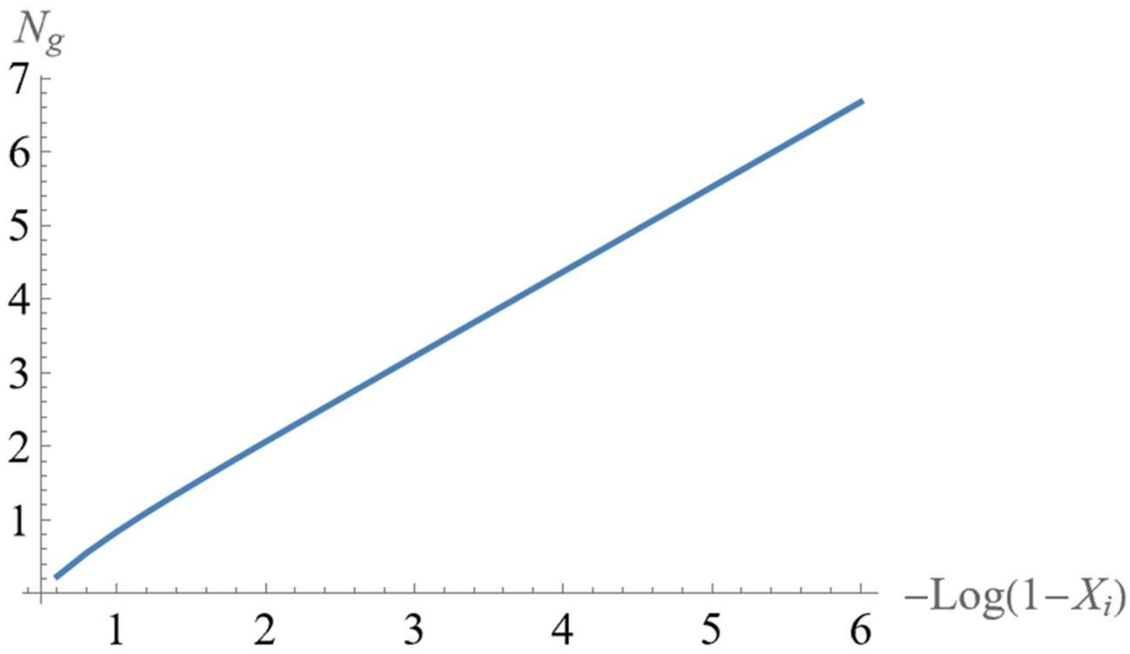


Volume Mode: Field Domination Fixed Point

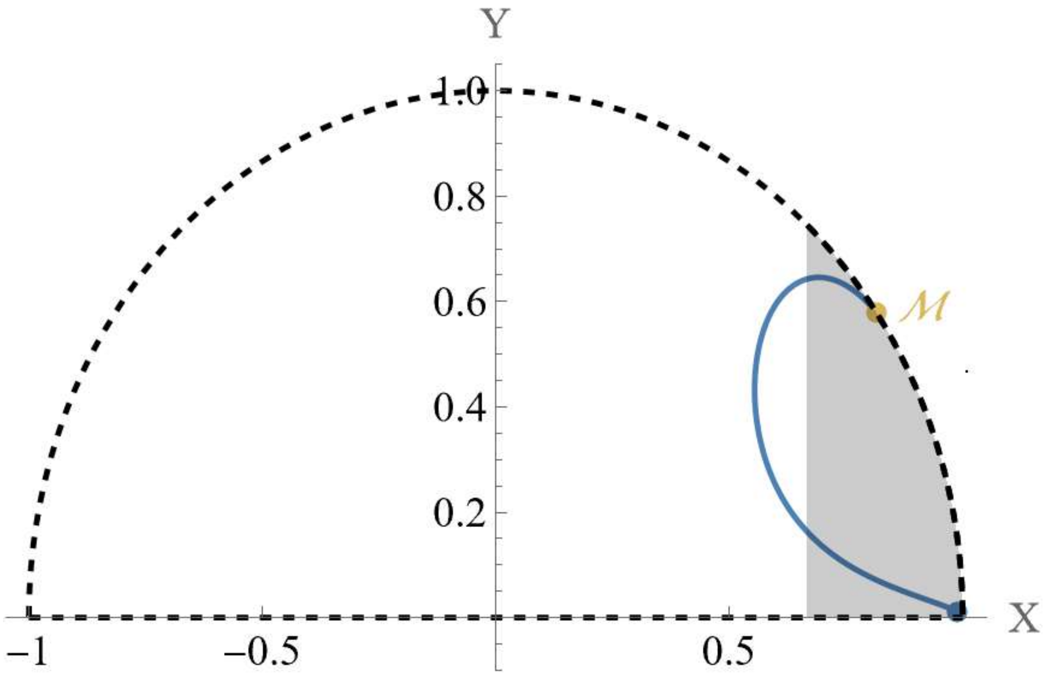
FP	X	Y	Existence	Existence and Growth when $\lvert\xi\rvert > \sqrt{2/3}$
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	$\lvert\lambda\rvert < \sqrt{6}$	$\frac{2}{\lvert\xi\rvert} < \lvert\lambda\rvert < \sqrt{6}$

$$\xi = \sqrt{\frac{3}{2}}$$

$\mathcal{M}$ in GR for:

$$2\sqrt{\frac{2}{3}} < \lambda < \sqrt{6} \implies \frac{4}{3} < p < 2$$

For such values, $\mathcal{M}$ is **unstable**



$\lambda = 2$

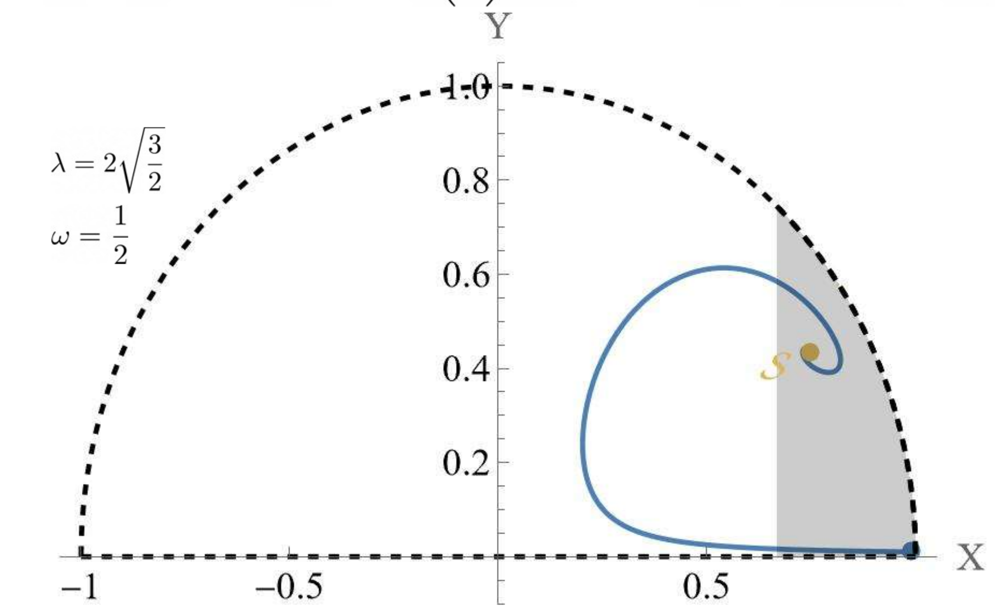
Volume Mode: Scaling Fixed Point

FP	X	Y	Existence	Existence and Growth when $\|\xi\| > \sqrt{2/3}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}}\frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$	$\lambda^2 > 3(\omega+1)$	$\sqrt{3(\omega+1)} < \|\lambda\| < \frac{3}{2}(\omega+1)\|\xi\|$ with $\sqrt{\omega+1} > \frac{2}{\sqrt{3}\|\xi\|}$

$$\xi = \sqrt{\frac{3}{2}}$$

$\mathcal{S}$ is in GR for:

$$\sqrt{3(\omega+1)} < \lambda < (\omega+1)\left(\frac{3}{2}\right)^{3/2} \implies \sqrt{2(\omega+1)} < p < \frac{3}{2}(\omega+1)$$



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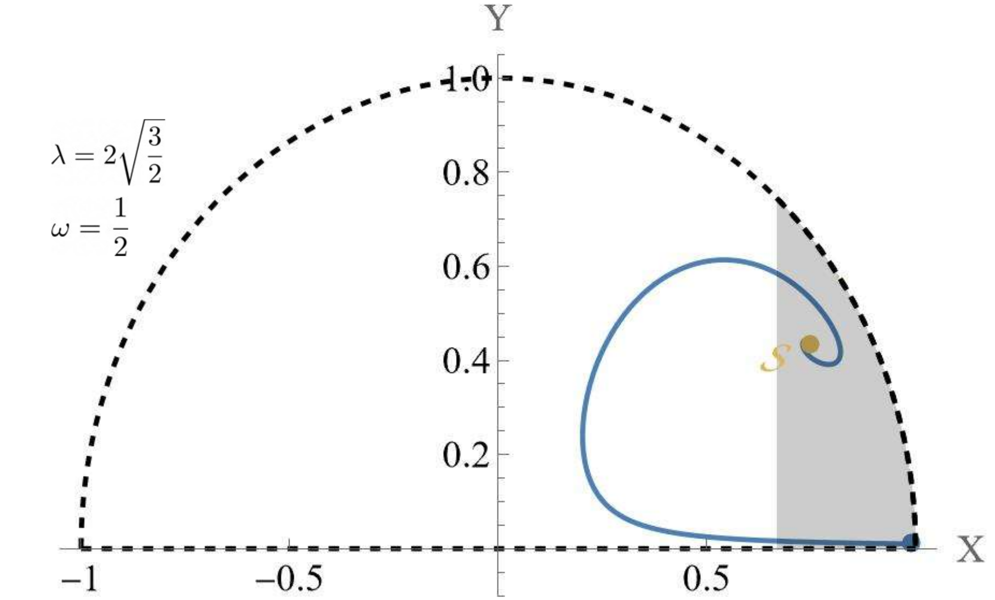
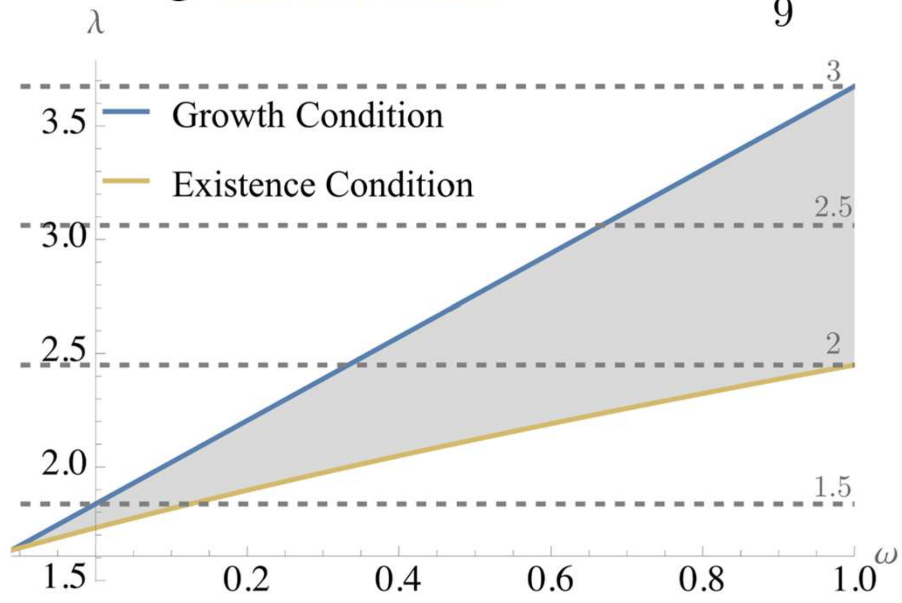


$\mathcal{S}$ is stable in GR for all $\omega > -\frac{1}{9}$



Effective Strings in Type IIB

Other source of strings in the EFT: p-brane wrapping a (p-1)-cycle Σ_{p-1}

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- A D3-brane wraps a 2-cycle of volume: $t\,\ell_s^{-2} \implies \mu \sim M_s^2\, t$
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The simlest case: $\mathcal{V} \propto t^3 \propto \tau^{3/2}$ allows for no GR [Conlon et al:2024]:

- For D3's: $$\boxed{\xi = \sqrt{\frac{2}{3}}}$$
- For 5-branes: $$\boxed{\xi < \sqrt{\frac{2}{3}}}$$

Fibred Calabi-Yau's

Next-to-simplest case: CY with a fibred structure!

$\mathbb{P}^1$ base fibred by a $\mathbb{T}^4$ or a $K3$ surface.

$$\mathcal{V} = \frac{1}{2} k_{122}\, t_1 t_2^2 = \kappa\, \tau_b \sqrt{\tau_f}$$

Relation between 2- and 4-cycles:

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t_i} \quad \Longrightarrow \quad t_2 = \sqrt{\frac{2\tau_f}{k_{122}}} \qquad t_1 = \frac{1}{\sqrt{2}k_{122}} \frac{\tau_b}{\sqrt{\tau_f}}$$

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Idea: keep volume fixed and vary the orthogonal direction

$$u = \frac{1}{\kappa} \frac{\tau_f}{\tau_b} = e^{\sqrt{3}\phi/M_p}$$

Canonically Normalised Cycles and Tension

Tension linear in cycle volume: just check canonical normalisation at fixed volume! $(M_p = 1)$

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NS5 wrapping 4-cycles:

$$\tau_b = \left(\frac{\mathcal{V}}{\sqrt{u}}\right)^{2/3} = \mathcal{V}^{2/3}\, e^{-\frac{1}{\sqrt{3}}\phi}$$

$$\tau_f = (u\mathcal{V})^{2/3} = \mathcal{V}^{2/3}\, e^{\frac{2}{\sqrt{3}}\phi}$$

D3 wrapping 2-cycles:

$$t_1 \sim \mathcal{V}^{1/3}\, e^{-\frac{2}{\sqrt{3}}\phi}$$

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$$|\xi| > \sqrt{\frac{2}{3}}$$

Same $|\xi|$ mirror GRs!

Growth Region and Kination

$$\mu \sim M_s^2 \, e^{\pm \frac{2}{\sqrt{3}} \phi / M_p}$$

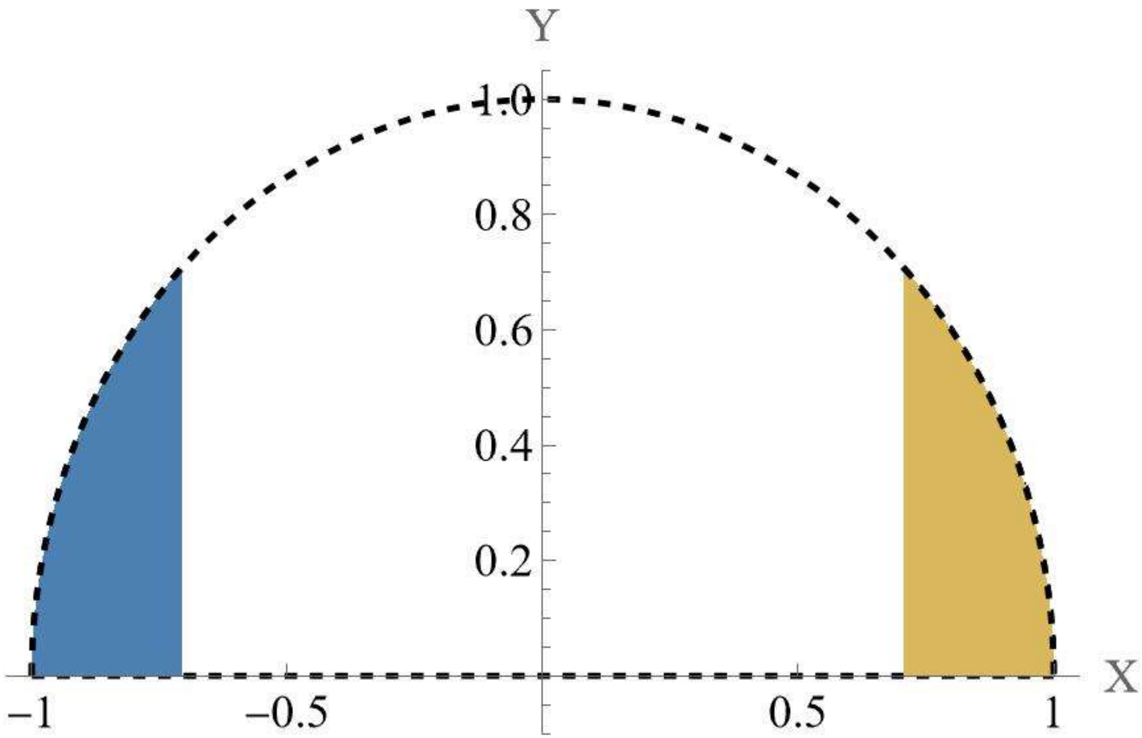
Growth Regions:

$$X < -\frac{1}{\sqrt{2}}$$

(fibre 4-cycle)

$$X > \frac{1}{\sqrt{2}}$$

(base 2-cycle)



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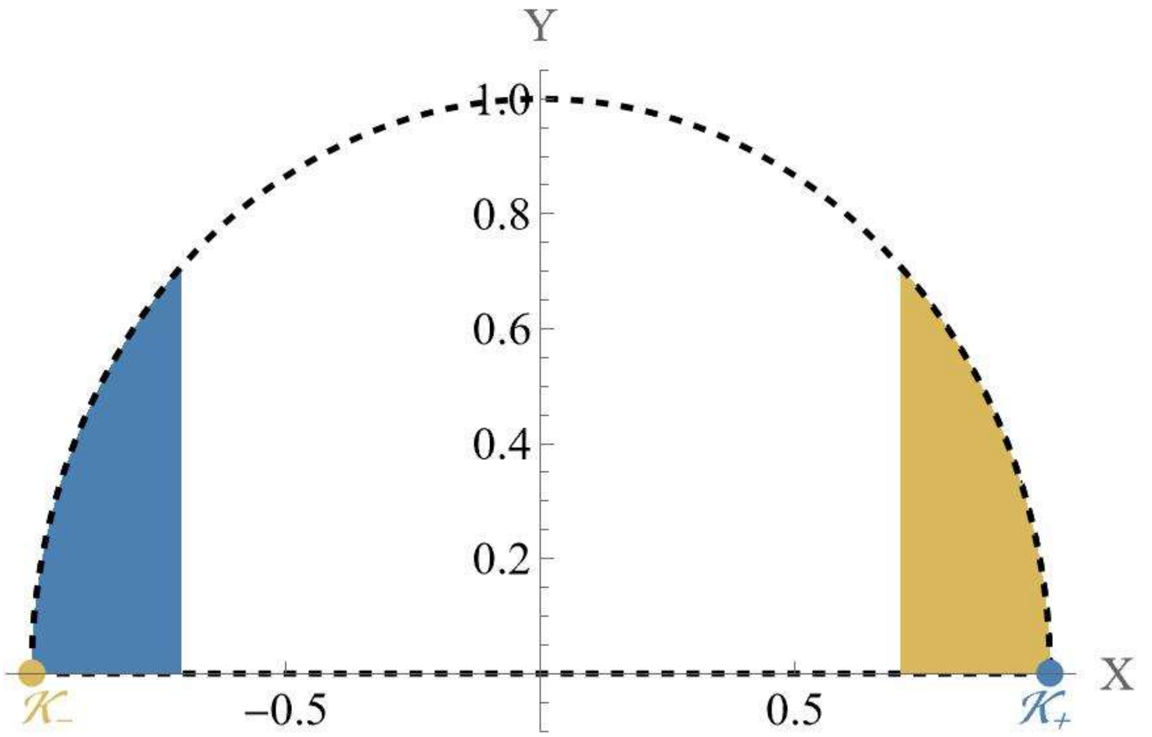
(fibre 4-cycle) (base 2-cycle)

Perturbative potential:

$$V \sim \frac{\beta}{\tau_f^q} \sim V_0\, e^{-\lambda\phi}$$

Kinating fixed points $\mathcal{K}_\pm$ within one GR.



Field Domination Fixed Points

FP	X	Y	Existence	Existence and Growth when $\lvert\xi\rvert > \sqrt{2/3}$
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	$\lvert\lambda\rvert < \sqrt{6}$	$\frac{2}{\lvert\xi\rvert} < \lvert\lambda\rvert < \sqrt{6}$

$$|\xi| = \frac{2}{\sqrt{3}}$$

$\mathcal{M}$ in GR for:

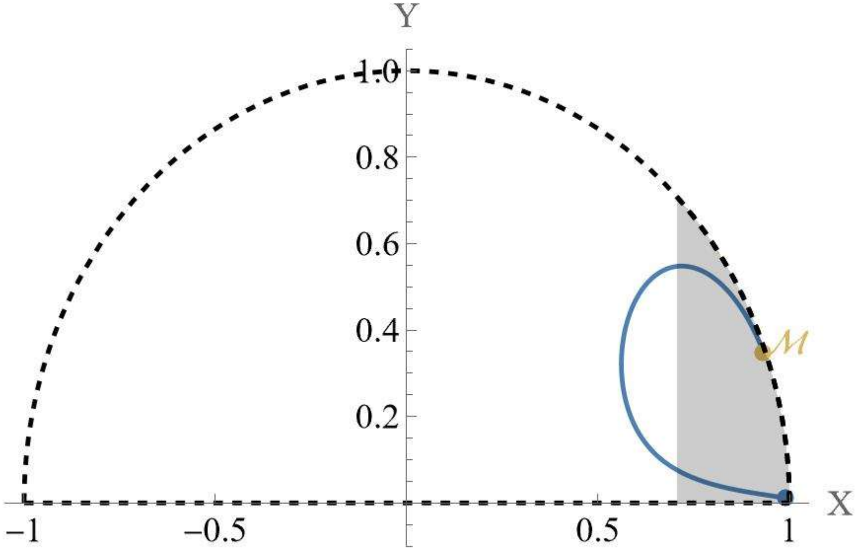
$$\sqrt{3} < \lambda < \sqrt{6} \implies \frac{3}{2} < q < \frac{3}{\sqrt{2}}$$

(base 2-cycle)

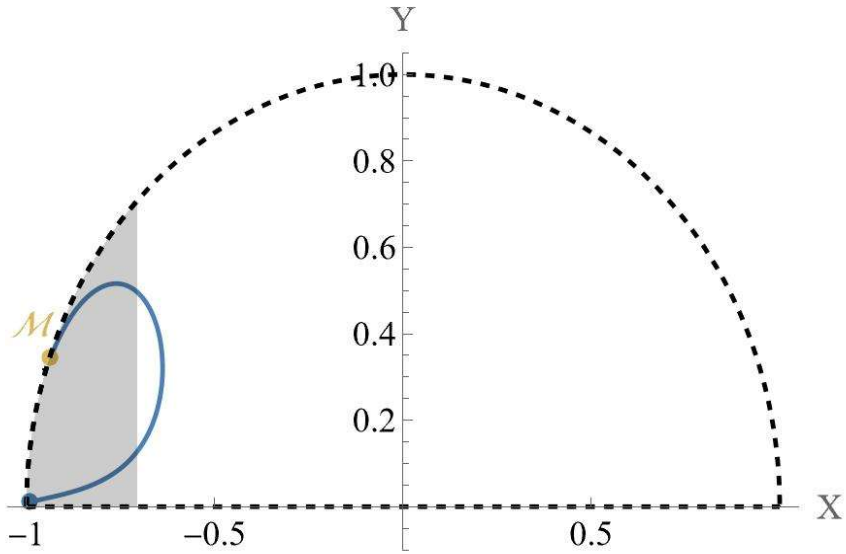
$$-\sqrt{6} < \lambda < -\sqrt{3} \implies -\frac{3}{\sqrt{2}} < q < -\frac{3}{2}$$

(fibre 4-cycle)

Unstable! If $\mathcal{S}$ exists, the system will evolve toward it



$$|\lambda| = \frac{4}{\sqrt{3}}$$



Scaling Fixed Points

FP	X	Y	Existence	Existence and Growth when $\lvert\xi\rvert > \sqrt{2/3}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}}\frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$	$\lambda^2 > 3(\omega+1)$	$\sqrt{3(\omega+1)} < \lvert\lambda\rvert < \frac{3}{2}(\omega+1)\lvert\xi\rvert$ with $\sqrt{\omega+1} > \frac{2}{\sqrt{3}\lvert\xi\rvert}$

$$|\xi| = \frac{2}{\sqrt{3}}$$

$\mathcal{S}$ is in GR for:

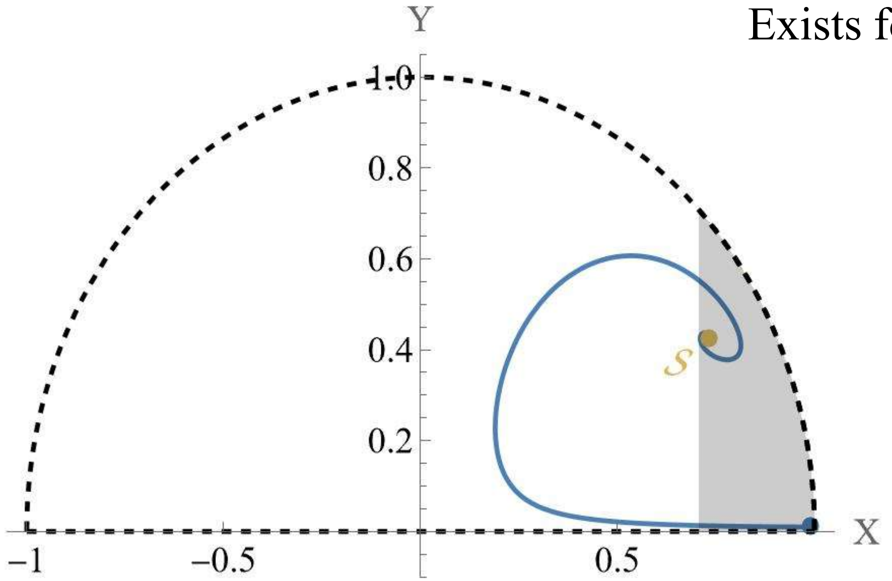
$$\sqrt{3(\omega+1)} < \lambda < \sqrt{3}(\omega+1) \implies \frac{1}{2}\sqrt{\omega+1} < q < \frac{1}{2}(\omega+1)$$

(base 2-cycle)

$$-\sqrt{3}(\omega+1) < \lambda < -\sqrt{3(\omega+1)} \implies -\frac{1}{2}(\omega+1) < q < -\frac{1}{2}\sqrt{\omega+1}$$

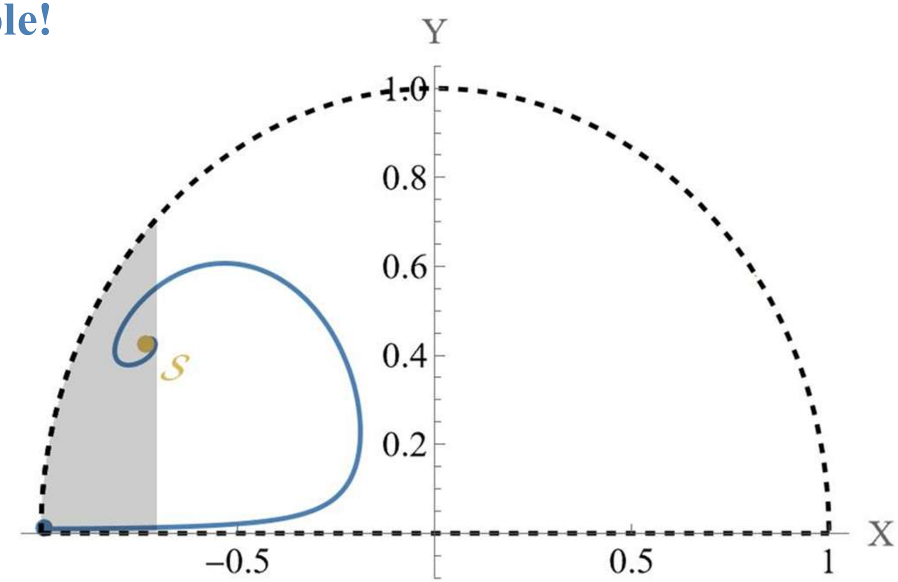
(fibre 4-cycle)

Exists for all $\omega > 0$ **Stable!**

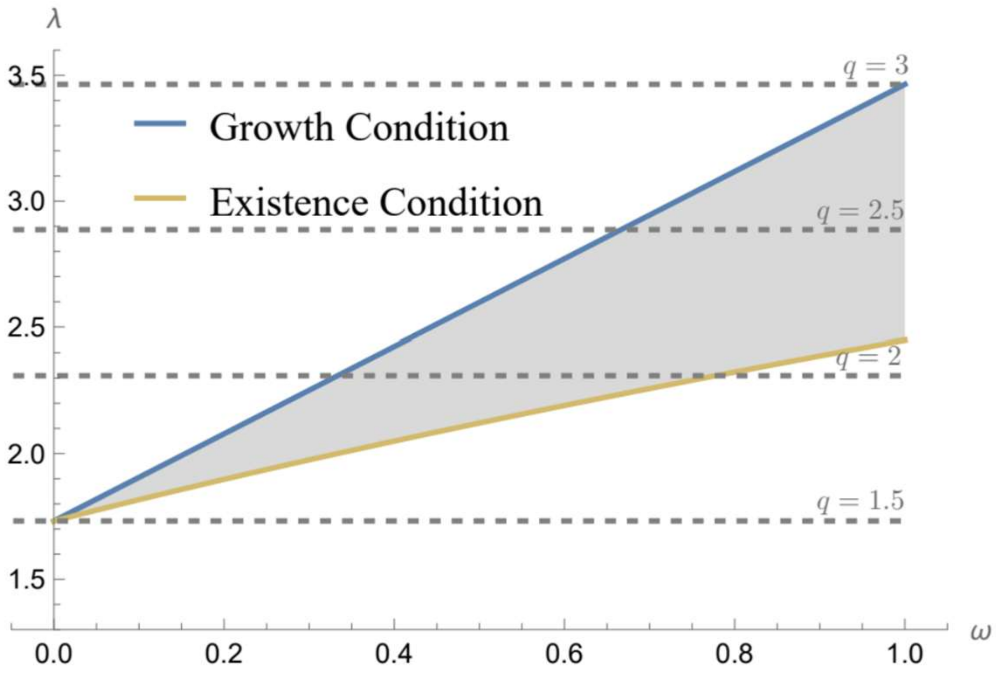


$|\lambda| = 2$

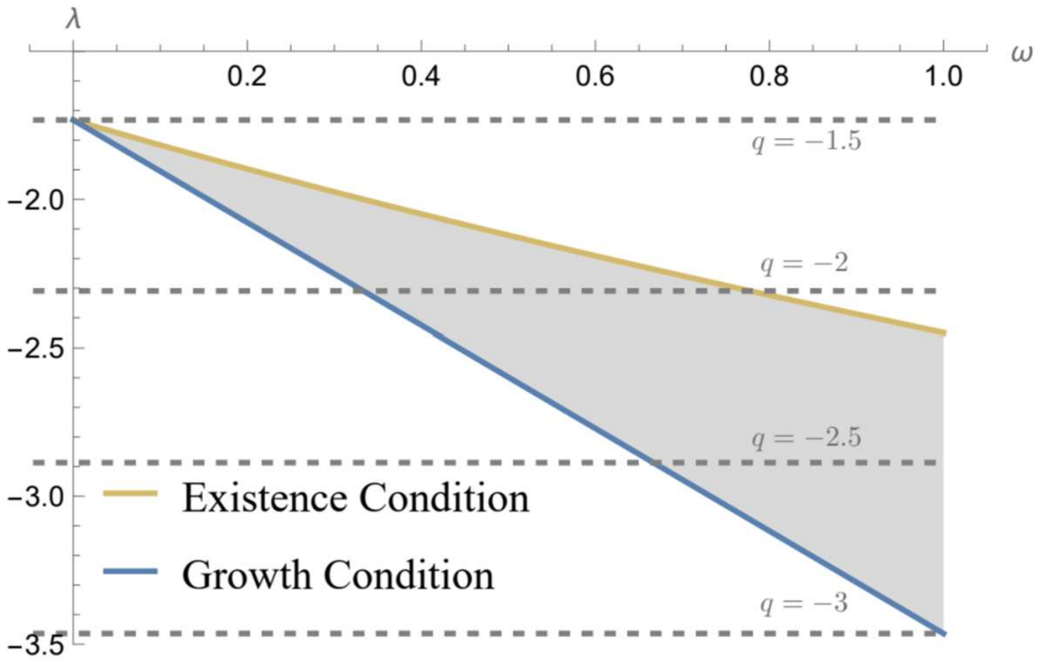
$\omega = 1/3$



Scaling Fixed Points in GR



(base 2-cycle)



(fibre 4-cycle)

Summary

- Cosmic Strings from Superstring theory are phenomenologically interesting!
- Their tension depends on a modulus and can be studied as a Dynamical System
- Fundamental Strings can Grow even if the volume is not kinating, but scaling
- Effective strings from Fibred CYs can also grow in comoving size when the cycle evolves

Outlook

- What happens when the string fluid is no longer negligible? [Conlon et al: 2025]
- Can Effective String give rise to observable signals in GW or particle emission?
- Backreaction of the network on the evolution of the modulus?
- How do we produce the initial population of strings? How do we estimate their number? How long should they grow?

Thank You For Your Attention!

Backup

Cosmic Strings

In **QFT,** solitonic field configurations in the presence of **U(1) SSB** [Kibble: 1976]:

- **Global Strings:** from global U(1) SSB, also called axion strings [Vilenkin, Everett: 1982; Lazarides, Shaqfi: 1982].
- **Local Strings:** from gauge U(1) SSB [Nielsen, Olsen: 1973]

In **String Theory**, 1-dimensional objects in the EFT:

- **Fundamental (F-) Strings:** fundamental strings of string theory in 4D
- **D1-Strings:** D-branes with 1 extended spacelike dimension
- **EFT (or Axion) Strings:** Dp-branes wrapped around (p-1)-cycles in the internal dimensions

+ Local Strings realized from SSB in gauge sectors

Constraints and Observational Features

Tension of a Cosmic String Network strongly constrained by CMB and PTA.

Constraint from CMB [Charnok et al.:2016]:

$$G\mu \lesssim 10^{-7} - 10^{-8}$$

Constraint from PTA [Ellis et al.:2023; Marfatia et al. :2023; Avgoustidis et al.:2025]:

$$G\mu \lesssim 10^{-11} - 10^{-12}$$

Best fit possible for cosmic superstrings in Large Volume Scenario! [Ghoshal, Revello, Villa:2025]

(Super)String Theory Crash Course

Superstring Theory is one the most promising candidates for Quantum Gravity.

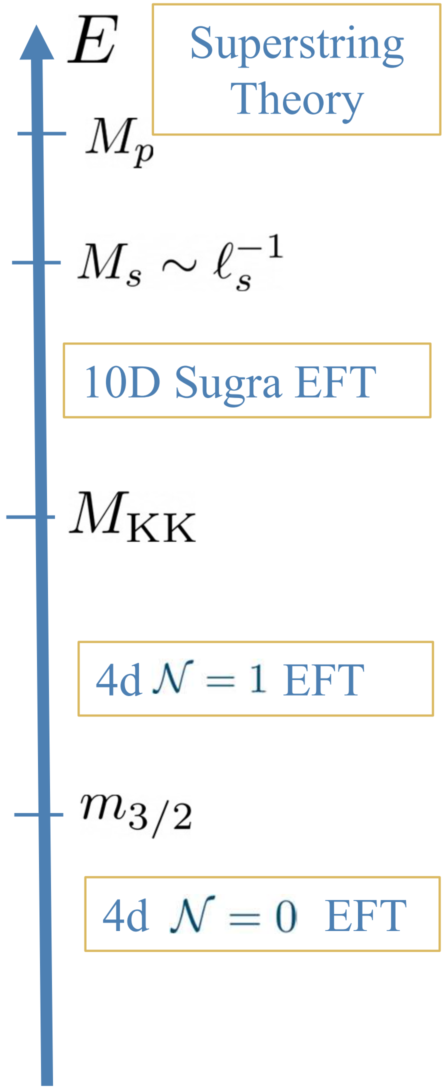
The fundamental objects are one-dimensional strings with length: $\ell_s = 2\pi\sqrt{\alpha'}$

Quantization of the Superstring automatically gives a massless spin-2 field: graviton!

Multiple Types of Superstring Theory, related by dualities

Requires 10 (or 11) dimensions for consistency (Weyl anomaly cancellation)

Needs compactification to make contact with phenomenology!



Tension of Effective Strings

Tension From dim. reduction can get a NG action out of DBI-like part of actions:

$$S_p \simeq -T_p \int_{\mathcal{W}_{p+1}} \sqrt{-g_{(p+1)}}\,(1+\dots)\mathrm{d}^{p+1}\zeta \quad \xrightarrow{\mathcal{W}_{p+1} = \Sigma_{p-1} \times \mathcal{W}_2} \quad S_p \supset -T_p \int_{\mathcal{W}_2} \sqrt{-g_{(2)}}\,\mathrm{Vol}(\Sigma_{p-1})\mathrm{d}^2\sigma$$

$$\text{If } \mathrm{Vol}(\Sigma_{p-1}) = t_{p-1}\,\ell_s^{p-1}$$

$$\mu \sim T_p\, M_s^{-p+1}\, t_{p-1} \quad \xrightarrow{T_p \sim M_s^{p+1}} \quad \boxed{\mu \sim M_s^2\, t_{p-1}}$$