Universität Münster

Probing Axion Inflation via Gravitational-Wave Production

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Cosmological gravitational waves

Consider transverse-traceless perturbations on top of a flat FLRW spacetime:

$$ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\mu = a^2(\eta)\left[\mathrm{d}\eta^2 - (\delta_{ij} + h_{ij}^{TT}(\mathbf{x},\eta))\mathrm{d}x^i \mathrm{d}x^j\right]$$

Einstein's equations: the TT-metric perturbations(=gravitational waves) obey[1]:

$$h_{ij}''(\mathbf{x},\eta) + 2\mathcal{H}h_{ij}'(\mathbf{x},\eta) - \Delta h_{ij}(\mathbf{x},\eta) = 2\frac{a^2}{M_\mathrm{P}}\Sigma_{kl}^{TT}(\mathbf{x},\eta)$$

Σ_{ij}^{TT} is the **anisotropic** part of the energy-momentum tensor

$$T^i{}_j = -p\delta^i{}_j - \Sigma^i{}_j, \qquad \delta^j{}_i\Sigma^i{}_j = 0,$$

describing the matter present in the system.

[1] I suppress the subscript "TT" from now on

Anisotropic stress induced by Abelian gauge fields

Consider Abelian gauge-fields sourced by some arbitrary previous mechanism, i.e.,

$$\mathcal{L} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

without specifying the interactions sourcing the gauge fields.

It is straight forward to derive that

$$\Sigma_{ij}^{EM} = -(E_i E_j + B_i B_j), \qquad F_{0i} = a^2 E^i, \qquad F_{ij} = -a^2 \epsilon_{ijk} B^k$$

Thus, with no other sources present,

$$h_{ij}''(\mathbf{x}, \eta) + 2\mathcal{H} h_{ij}'(\mathbf{x}, \eta) - \Delta h_{ij}(\mathbf{x}, \eta) = -2 \frac{a^2}{M_{\mathrm{P}}} (E_i E_j + B_i B_j)^{TT}$$

The gauge-field-induced tensor power spectrum

$$h''_\lambda(\mathbf{k},\eta) + 2\mathcal{H}h'_\lambda(\mathbf{k},\eta) + k^2 h_\lambda(\mathbf{k},\eta) = -2\frac{a^2}{M_\mathrm{P}}\Pi^{ij}_\lambda(\mathbf{k})(E_iE_j + B_iB_j)^{TT}$$

- tensor power spectrum splits into two contributions:
  - vacuum: $\mathcal{P}^{vac}_{T,\lambda}(k,\eta) = \frac{k^2}{\pi^2}|h^{vac}_\lambda(\mathbf{k},\eta)|^2$
  - induced:

$$\mathcal{P}^{ind}_{T,\lambda}(k,\eta) = \frac{k^3}{2\pi^2 M_\mathrm{P}^4}\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\sum_{\alpha,\beta=\pm1}\left|1+\lambda\alpha\frac{\mathbf{k}\cdot\mathbf{p}}{kp}\right|^2\left|1+\lambda\beta\frac{(\mathbf{k}-\mathbf{p})\cdot\mathbf{k}}{|\mathbf{k}-\mathbf{p}|p}\right|^2$$
$$\times\left|\int_{-\infty}^{\eta}\mathrm{d}\tau\frac{G_k(\eta,\tau)}{a^2(\tau)}\left[A'_\alpha(p,\tau)A'_\beta(|\mathbf{k}-\mathbf{p}|,\tau) + \alpha\beta p|\mathbf{k}-\mathbf{p}|A_\alpha(p,\tau)A_\beta(|\mathbf{k}-\mathbf{p}|,\tau)\right]\right|^2$$

- Model dependence only in gauge-field mode function $A_\alpha(k,\eta)$.

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Relation to GW spectrum at $f = \mathcal{H}_{in}/(2\pi a_{today})$:

$$\Omega_{GW}(f) = \frac{\pi^2}{3H_0^2}f^2|T_{GW}(f)|^2\mathcal{P}_T(k,\eta_{k,in}) = \Omega^{vac}_{GW}(f) + \Omega^{ind.}_{GW}(f)$$

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The typical axion inflation model

Pseudoscalar ϕ (inflaton) coupled to gauge fields in expanding background:

$$\mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\beta\phi}{4M_{\mathrm{P}}}F_{\mu\nu}\tilde{F}^{\mu\nu}\Big]$$

Classical equations of motion + Friedmann eq. (neglecting $\nabla\phi$-terms):

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{\beta}{M_{\mathrm{P}}}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$$

$$\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\operatorname{rot}\boldsymbol{B} + \frac{\beta}{M_{\mathrm{P}}}\dot{\phi}\boldsymbol{B} = 0, \qquad \operatorname{div}\boldsymbol{E} = 0,$$

$$\dot{\boldsymbol{B}} + 2H\boldsymbol{B} + \frac{1}{a}\operatorname{rot}\boldsymbol{E} = 0, \qquad \operatorname{div}\boldsymbol{B} = 0$$

$$H^2 = \frac{1}{3M_{\mathrm{P}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle\right)$$

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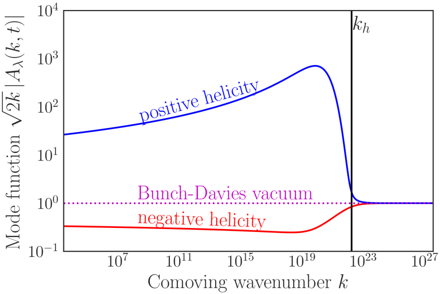
Helical gauge-field production

$$\ddot{A}^k_\pm + H\dot{A}^k_\pm + \left[\left(\frac{k}{a}\right)^2 \mp \left(\frac{k}{a}\right)\frac{\beta}{M_\mathrm{P}}\dot{\phi}\right]A^k_\pm = 0$$

- helicity dependence!
- relevant scale: $k_h/a = \beta/M_\mathrm{P}|\dot{\phi}|$
- excitation over vacuum

  $\rightarrow$ **tachyonic instability!**



One helicity $+/-$ **exponentially** amplified! [Anber & Sorbo, PRD 81 (2010)]

Phenomenological Consequences

Amplification of only **one circular polarisation**!
$\Longrightarrow$ generation of **helical gauge fields**

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Phenomenological consequences:

- production of **cosmological magnetic fields** [Anber & Sorbo, JCAP 10 (2006)]
- Production of **circular-polarised gravitational waves**
  [Barnaby et al., PRL 106 (2011), Garcia-Bellido et al., JCAP 01 (2024)]
- **non-Gaussianities** in scalar-power spectrum [Barnaby et al., PRL 106 (2011)]
- decay of gauge fields $\stackrel{?}{\rightarrow}$ **baryon asymmetry of the universe**
  [Giovanni et al., PRD 57 (1998)]
- Axion coupling leads to **effective reheating** of the universe
  [Adshead et al., JCAP 12 (2015)]

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Focus on gravitational wave production

A typical GW spectrum from axion inflation

The GW spectrum will take two contributions:

- vacuum contribution: $\Omega_{GW}^{vac} \propto H^2$
- induced contribution:
  - for weak BR: $\Omega_{GW}^{ind} \propto H^4 \frac{e^{4\pi\xi}}{\xi^6}$, $\xi = \frac{\beta}{M_P}\frac{\dot{\phi}}{2H} \sim \beta$
  - Ω_{GW}^{ind} oscillatory, non-linear

decreasing inflation scale (H)

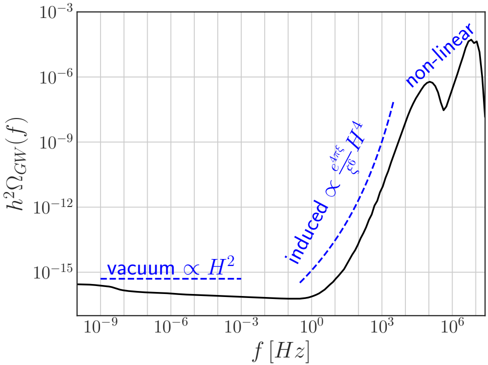
- decrease amplitude
- later rise in Ω_{GW}^{ind}

increasing coupling (β)

- earlier rise in Ω_{GW}^{ind}

non-linearities:

- prolong inflation
  $\rightarrow$ shift spectrum to left



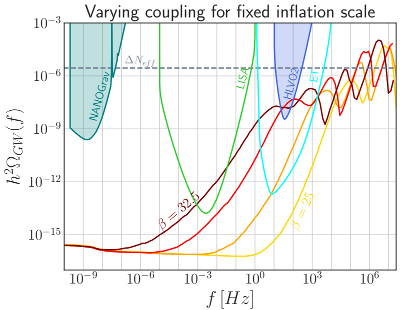
Detectability of gravitational waves

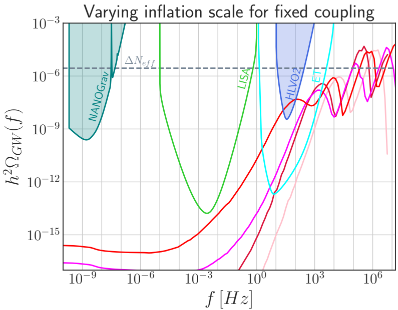
changing **coupling strength**:

- induced contribution observable
- ΔN_{eff} violation

changing **inflation scale**:

- decrease only linear rise
- non-linearities insensitve





Maximal amplitude near insensitive to parameters $\rightarrow$ **conflict with ΔN_{eff}!**

Introducing charged fields: Schwinger pair creation

So far "sterile" gauge-field: no coupling to charged fields

- realistic gauge-fields **couple to fermions**
- particularly: $\mathrm{U}(1)_\mathrm{Y} \to$ coupling to **SM fermions**!

Strong EM fields from axion inflation $\to$ **Schwinger pair production**

- continuous Fermion production during axion inflation
- energy-transfer from gauge fields to fermions
  $\to$ **damping of gauge-field production**

avoid overproduction of gravitational waves[2]!

[2] Fermions may also contribute anisotropic stress: *open question...*

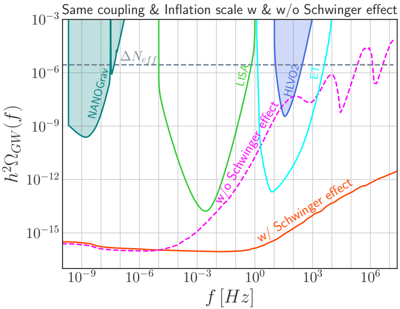
Gravitational waves: including Schwinger pair creation

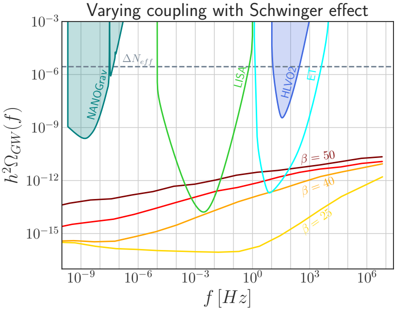
$\mathrm{U}(1)_\mathrm{Y}$ vs. "sterile" $\mathrm{U}(1)$

- reduce $\Omega_{GW}^{ind} \rightarrow$ no ΔN_{eff} violation

Increasing the coupling strength

- observable signals!





Detectable signal across frequencies & no conflict with ΔN_{eff}!

Conclusion and Outlook

Gauge-fields in the early universe may **source gravitational waves**

- tensor power spectrum $\rightarrow$ analytic and model independent
- tensor power spectrum directly linked to Ω_{GW}

Axion inflation: strong gauge-field production during inflation

- helical gauge-field production $\rightarrow$ helical GW production.
- GW production: too strong or not observable?

Fermion production during axion inflation via Schwinger pair production:

- Fermions couple to gauge fields $\rightarrow$ dampened gauge-field production
- natural model if considering $U_{\mathrm{Y}}(1)$ gauge fields
- reduced GW production: avoid ΔN_{eff} bounds but still observable

3n+1 ODEs to obtain dynamics of axion inflation!

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\frac{\beta}{M_\mathrm{P}}\mathcal{G}^{(0)}$$

$$\dot{\mathcal{E}}^{(n)} + (n+4)H\,\mathcal{E}^{(n)} - \frac{2\beta}{M_\mathrm{P}}\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = S_{\mathcal{E}}^{(n)}$$

$$\dot{\mathcal{G}}^{(n)} + (n+4)H\,\mathcal{G}^{(n)} - \frac{\beta}{M_\mathrm{P}}\dot{\phi}\,\mathcal{B}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} = S_{\mathcal{G}}^{(n)}$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = S_{\mathcal{B}}^{(n)}$$

$$H^2 = \frac{1}{3M_\mathrm{P}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\left(\mathcal{E}^{(0)} + \mathcal{B}^{(0)}\right)\right)$$

Homogeneous evolution of scalar field,
electric- & magnetic field, and Universe expansion

Schwinger effect in expanding background

Charged particles $\rightarrow$ induced current: [Domcke et al. JHEP 02 (2020)]

$$|\boldsymbol{J}| = \frac{(g|Q|)^2}{6\pi^2 H} |\boldsymbol{E}||\boldsymbol{B}| \coth\left(\frac{\pi|\boldsymbol{B}|}{|\boldsymbol{E}|}\right) e^{-\frac{\pi m^2}{g|Q||\boldsymbol{E}|}}$$

- induced current enters Ampère's law:

$$\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\,\mathrm{rot}\,\boldsymbol{B} + \frac{\beta}{M_{\mathrm{P}}}\,\dot{\phi}\,\boldsymbol{B} + \boxed{\boldsymbol{J}} = 0$$

- damping of gauge-field production!

Effective treatment of the induced current

How to treat $\boldsymbol{J}$ in a quantum system?

- Effective approach: Ohm's law: $|\boldsymbol{J}| \propto |\boldsymbol{E}||\boldsymbol{B}| \rightarrow \hat{\boldsymbol{J}} = \sigma_E \hat{\boldsymbol{E}} \,/\, \sigma_B \hat{\boldsymbol{B}}$
- significant choice: electric or magnetic picture:

$$\ddot{A}^k_\pm + \left(H + \sigma_E\right)\dot{A}^k_\pm + \left[\left(\frac{k}{a}\right)^2 \mp \frac{k}{a}\left(\frac{\beta}{M_\mathrm{P}}\dot{\phi} + \sigma_B\right)\right]A^k_\pm = 0$$

- Assumption: $\boldsymbol{E}||\boldsymbol{B} \rightarrow$ true at late times, never exactly
- boost to collinear frame, compute $|\boldsymbol{J}|$, boost back to comoving frame!
- Release the assumption of $\boldsymbol{E}||\boldsymbol{B} \rightarrow \boldsymbol{J} \propto |\boldsymbol{E}||\boldsymbol{B}|$ no longer true!

mixed picture: $\boldsymbol{J} = \sigma_E \boldsymbol{E} + \sigma_B \boldsymbol{B}$

Identifying a new scale in the problem

Effective description: **global, k-indep. damping** $\sigma_{E/B}$

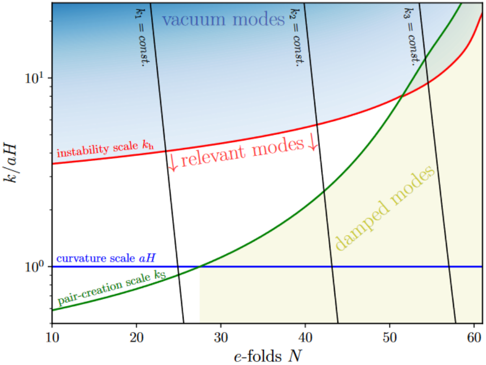
- gauge-modes of any wavelength are equally damped (even vacuum. . .)

More realistic: fermions produced at **typical separation** $\delta x_S \sim (|eQE|)^{-1/2}$

- gauge-modes with wavelengths $\lambda \leq \delta x_S$ should not be damped

Three scales:

- Hubble: $k_H = aH$
- Instability: $k_h \approx a\frac{\beta|\dot{\phi}|}{M_P}$
- Schwinger pair-creation scale: $k_S = a\sqrt{|eQE|}$



Scale dependent Schwinger damping & GEF

$$\dot{\mathcal{E}}^{(n)} + (4+n)H\mathcal{E}^{(n)} + 2\mathcal{G}^{(n+1)} - 2I_{,\phi}\dot{\phi}\mathcal{G}^{(n)} + 2\sigma_E\bar{\mathcal{E}}^{(n)} - 2\sigma_B\bar{\mathcal{G}}^{(n)} = S_{\mathcal{E}}^{(n)}\,,$$
$$\dot{\mathcal{G}}^{(n)} + (4+n)H\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - I_{,\phi}\dot{\phi}\mathcal{B}^{(n)} + \sigma_E\bar{\mathcal{G}}^{(n)} - \sigma_B\bar{\mathcal{B}}^{(n)} = S_{\mathcal{G}}^{(n)}\,,$$
$$\dot{\mathcal{B}}^{(n)} + (4+n)H\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = S_{\mathcal{B}}^{(n)}\,,$$

$$\bar{\mathcal{E}}^{(n)} = \int_0^{\min(k_\mathrm{S},k_\mathrm{h})} \frac{\mathrm{d}k}{k}\frac{k^{n+3}}{2\pi^2 a^{n+2}}\sum_\lambda \lambda^n|\dot{A}_\lambda(t,k)|^2\theta(k_\mathrm{S}-aH)$$

$$\bar{\mathcal{G}}^{(n)} = \int_0^{\min(k_\mathrm{S},k_\mathrm{h})} \frac{\mathrm{d}k}{k}\frac{k^{n+4}}{2\pi^2 a^{n+3}}\sum_\lambda \lambda^{n+1}\mathrm{Re}[\dot{A}_\lambda(t,k)A_\lambda^*(t,k)]\theta(k_\mathrm{S}-aH)$$

$$\bar{\mathcal{B}}^{(n)} = \int_0^{\min(k_\mathrm{S},k_\mathrm{h})} \frac{\mathrm{d}k}{k}\frac{k^{n+5}}{2\pi^2 a^{n+4}}\sum_\lambda \lambda^n|A_\lambda(t,k)|^2\theta(k_\mathrm{S}-aH)$$

The tensor power spectrum: from emission to today

- wavenumber red-shifts to give frequency today: $f = k/(2\pi a_{today})$
- After sources turns off: GW modes evolve according to

$$h''_\lambda(\mathbf{k},\eta) + 2\mathcal{H}h'_\lambda(\mathbf{k},\eta) + k^2 h_\lambda(\mathbf{k},\eta) = 0$$

- overdamping for modes with $k \ll \mathcal{H} \to \mathcal{P}_T(k \ll \mathcal{H}(\eta),\eta)$ **conserved**
- during RH/RD/MD: $\mathcal{H}$ shrinks $\to \mathcal{P}_T$ evolves after $k \sim \mathcal{H}(\eta_{k,in})$.
- Transfer function $T_{GW}(f)$: evolve $\mathcal{P}_T(k,\eta_{k,in})$ until today.

Relation to GW spectrum at $f = \mathcal{H}_{in}/(2\pi a_{today})$:

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