# Gravitational wave from current-carrying domain walls

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#### Introduction

#### Domain wall

• remnant of breaking of discrete sym.



fig. from https://www.ctc.cam.ac.uk/outreach/origins/cosmic\_structures\_two.php

• easily dominates energy density of universe,  $\rho_{\rm DW} \propto a(t)^{-2}$ 

 $\rightarrow$  inconsistent with cosmological observations

 $\rightarrow$  need to decay!

#### Domain wall w/ bias

• one popular way: to introduce bias energy



• less energy domain is favored  $\rightarrow$  bias  $\Delta V$  gives pressure to DW

### GW from DW

• bias  $\Delta V$  drives "+v domain" to shrink @ $t \sim \sigma / \Delta V \equiv t_{ann}$ 



fig. from slide by Saikawa

• This motion radiates GW!

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(quadrupole formula)
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$$\frac{dE_{\rm GW}}{dt} \sim G(\ddot{Q}_{ij})^2 \sim G\sigma_{\rm DW}^2 R^2 \qquad Q_{ij} \equiv$$

$$Q_{ij} \equiv \int d^3x \,\rho(x) \left( \hat{x}_i \hat{x}_j - \frac{1}{3} \delta_{ij} \right)$$

DW size  $R \sim t$ 

#### GW from DW and future observatories



We might observe GW from DW in the near future!!

But this is just a standard story! ↓ When considering "current-carrying DW", the story might change very much!

#### GW from current-carrying DW

#### **DW+fermion**

[Jackiw-Rebbi '76] [Kaplan '92]

• Dirac fermion coupled with DW

$$\mathcal{L} = \bar{\psi} \left[ i \gamma^{\mu} \partial_{\mu} - g \phi_{\rm DW}(x) \right] \psi$$



#### DW+fermion

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$$\mathcal{L} = \bar{\psi} \left[ i \gamma^{\mu} \partial_{\mu} - g \phi_{\rm DW}(x) \right] \psi$$

• Solving only *x*-dependence:

$$\therefore \begin{cases} \psi_0(x) = C \,\varphi \exp\left(-\int_0^x dx' g \phi_{\rm DW}(x')\right) \\ i \gamma^1 \varphi = -\varphi \\ \text{localized fermion on DV} \end{cases}$$



### **DW+fermion**

+v



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- In y, z directions (along DW), it behaves as massless particles
- → DW carries current if  $\psi$  has any charge (DW = superconducting membrane)



-v

### **Current-carrying DW is ubiquitous**

#### **Topological insulator**



Fig from Tokura, Yasuda & Tsukazaki, Nat Rev Phys **1**, 126–143 (2019)

e.g., [Thouless+ '82]

#### hep-th, Lattice QCD



Fig from slide by Y. Kuramashi https://aics.riken.jp/aicssite/wp-content/uploads/ 2013/07/110523\_kuramashi.pdf

[Jackiw-Rebbi '76] [D.B. Kaplan '92] [Shamir '93]

#### Even in hep-ph:

#### 2HDM with $\mathbb{Z}_2$ sym, SO(10) GUT etc...

[Battye-Brawn-Pilaftsis, '11] [Eto-Kurachi-Nitta, '18] [Battye-Pilaftsis-Viatic '20, '20] [Law-Pilaftsis, '21]

[Sassi-Moortgat-Pick, '23] [Fu+, '24]

#### [Kibble-Lazarides-Shafi, '82] [Lazarides-Shafi, '85]

#### But not well studied for cosmological application!

#### Domain wall w/ bias (again)

• one popular way: to introduce bias energy



• less energy domain is favored  $\rightarrow$  bias  $\Delta V$  gives pressure to DW

### What changes?

• bias  $\Delta V$  drives "+v domain" to shrink



...But DW captures and stores particles!

- free particle in plasma
- captured particle

### What changes?

• bias  $\Delta V$  drives "+v domain" to shrink



...But DW captures and stores particles!

- free particle in plasma
- captured particle
- trapped # of particles:  $Q \sim \frac{4\pi}{3} n_{\psi} \left( R_0^3 R(t)^3 \right)$  w/ trap rate~1

Hubble size  $R_0$  DW size R(t) # density  $n_w$ 

• DW becomes a sphere as it shrinks









### Simulation of spheron formation

- Solve EOM of  $\phi$  coupled w/ fermion in 1+1D system

$$\partial_{\mu}\partial^{\mu}\phi + \gamma\partial_{t}\phi + \frac{\partial V}{\partial\phi} + g\langle\bar{\psi}\psi\rangle_{Q} = 0 \qquad \gamma: \text{ friction parameter}$$
(Hubble friction or thermal)



#### Quantum decay

• Spheron is not absolutely stable due to quantum effects

Ex.) decay of trapped particles  $\rightarrow$  spheron shrinks



- obtains quadrupole due to non-zero angular momentum
  - $\rightarrow$  radiate GW until relaxed w/ smaller R
  - $\rightarrow$  again loses particles $\rightarrow$  ...  $\rightarrow$  continue to disappear

### GW from spheron

• Typical frequency: 
$$f_{\rm emit} \sim 1/R_{\rm sph}$$

• GW quadrupole formula: 
$$\frac{dE_{\rm GW}}{dt} \sim G(\ddot{Q}_{ij})^2 \sim GM_{\rm DW}^2 R_{\rm sph}^{-2}$$

$$\Omega_{\rm GW,max} \sim \frac{8\pi G}{3H_0^2} \left(\frac{GM_{\rm DW}^2}{R_{\rm sph}^2}\right) \tau_{\rm rad} \frac{1}{t_{\rm ann}^3} \left(\frac{a(t_{\rm ann})}{a(t_{\rm dec})}\right)^3 \left(\frac{a(t_{\rm dec})}{a(t_0)}\right)^4 \left(\frac{g_{*0}}{g_{*}(t_{\rm dec})}\right)^{\frac{1}{3}}$$
  
time scale of radiation number density of spheron @decay  
 $\tau_{\rm rad} = \min\left[t_{\rm dec}, \frac{M_{\rm DW}}{\dot{E}_{\rm GW}}\right]$ 

#### **GW** spectrum



#### Parameter space with SNR > 10



#### **PTA and LISA**

- DW network interpretation of PTA signal
- → peak from spheron can appear in LISA range!





- Conventional collapsing DW radiate GW
- Taking into account particles trapped on DW, a metastable object "spheron" may survive until decaying
- Decaying spheron radiates GW, which gives secondary peak in GW spectrum! → GW mountains



#### Backup

### Friction on DW

• suppose the trapped particles  $\psi$  interacts w/ plasma



- DW is in over-damp regime if  $F_f\gtrsim\sigma_{\rm DW}/t_{\rm ann}$  , leading to sphere



→ The formation efficiency would be  $\mathcal{O}(1)$ 

#### Quantum decay

- Spheron is not absolutely stable due to quantum effects
  - (i): fission



non-perturbative→suppressed

- It loses charge Q
  - → shrink by **radiating GW!**
  - $\rightarrow$  stabilized w/ smaller R
  - → again loses Q → .....

(ii): decay of charge carrier



perturbative decay



#### Total GW spectrum

 We have a superposition of both GW from conventional DW and from spheron



#### **SNR** plot



→ spheron improves SNR significantly

We have two conditions to have enough particles on DW (condition 1)

• Naively net total charge is  $0 \rightarrow$  pair-annihilation on DW

 $\rightarrow$  need charge asymmetry on DW!

• Assuming max. asymmetry, trapped net charge:  $Q \sim N$ 

(condition 2)

- $\left[i\gamma^{\mu}\partial_{\mu} g\phi_{\rm DW}(x)\right]\psi = 0$
- To be captured, (momentum of  $\psi$  in bulk)  $\lesssim m_{\psi}$

 $\rightarrow \psi$  should be non-relativistic

something like asymmetric DM?

#### **DW** solution

• real scalar  $\phi$ 

$$\mathcal{L} = \frac{1}{2} \left( \partial_{\mu} \phi \right)^2 - \lambda \left( \phi^2 - v^2 \right)^2$$



• static solution of EOM:

$$\partial_{\mu}\partial^{\mu}\phi + 4\lambda \left(\phi^2 - v^2\right)\phi = 0$$
  
 $\therefore \phi_{\rm DW}(x) = v \tanh\left[\sqrt{2\lambda}vx\right]$ 

 $\phi_{\mathrm{DW}}(x)$ 

DW at x = 0

#### GW from DW (simulation)

[Hiramatsu-Kawasaki-Ssikawa, 1002.1555]







## Model example: SO(10) GUT

- SO(10) GUT [Kibble-Lazarides-Shafi, '82] [Lazarides-Shafi, '85]  $SO(10) \xrightarrow{54} SU(4)_C \times SU(2)_L \times SU(2)_R \times C \xrightarrow{126} G_{SM} \xrightarrow{10} SU(3)_c \times U(1)_{EM}$ gives cosmic strings (diluted by inflation) gives DW
- DW connects two different vacua associated with  ${\cal C}$
- C acts as  $\phi_{10} \rightarrow -\phi_{10}$

→SM/GUT fermions feels flipped mass when crossing DW

#### **DW fermion**

[Kaplan '92]

• Dirac eq is solved as 
$$\begin{bmatrix} i\gamma^{\mu}\partial_{\mu} - g\phi \end{bmatrix} \Psi = 0$$
$$\Psi_{0} = C \exp\left(-\int_{0}^{x} dx' g\phi_{DW}(x')\right) \begin{pmatrix} \chi(y, z, t) \\ -i\sigma_{1}\chi(y, z, t) \end{pmatrix}$$
$$\therefore \begin{pmatrix} \partial_{0} - \partial_{z} & i\partial_{y} \\ -i\partial_{y} & \partial_{0} + \partial_{z} \end{pmatrix} \chi(y, z, t) = 0$$
$$\chi(y, z, t): \text{ two-comp spinor}$$

• After Fourier trsf., this has non-trivial solution only when

$$p_0^2 = p_y^2 + p_z^2$$

 $\rightarrow$  propagate as massless dof in *y*, *z* direction

### Fermion on DW sphere

[Aoki-Fukaya '21]

•  $S^2$  DW embedded in 3+1D  $\rightarrow$  almost same argument



$$\gamma^{\mu}\partial_{\mu} \rightarrow \gamma^{a} \left( \partial_{a} + \frac{1}{4} \sum_{bc} \omega_{bc,a} \gamma^{b} \gamma^{c} \right) + \text{(normal directions)}$$
$$\gamma^{a} = \gamma^{\mu} e^{a}_{\mu}$$
$$e^{a}_{\mu} = \frac{\partial y^{a}}{\partial x^{\mu}}$$