ALP production from Abelian Gauge interactions: Beyond the HTL.

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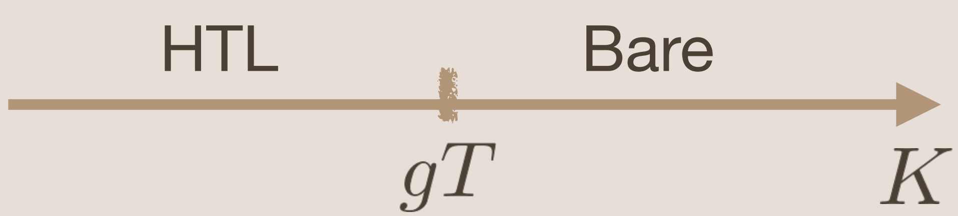
Rates in the early universe

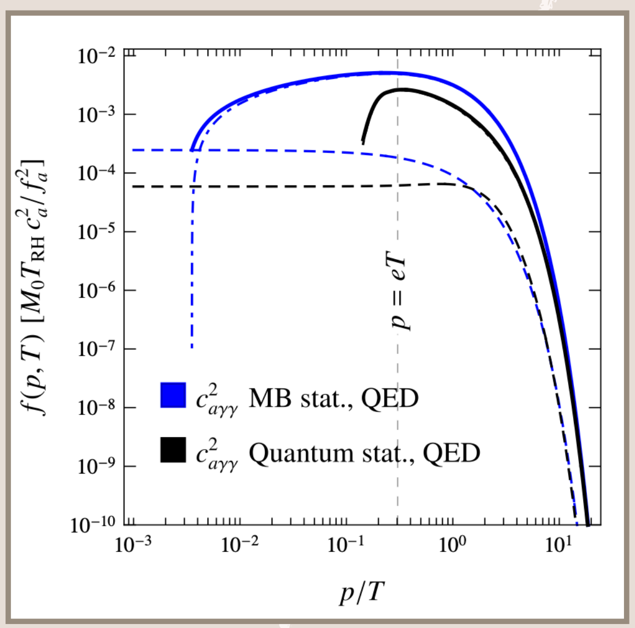
- DM , Baryon asymmetry, thermal GW…
- ❗Massless particles: IR divergences

+

p~gT : perturbative approach not reliable.



✘ **Unphysical rates for p<gT (soft ALP)**

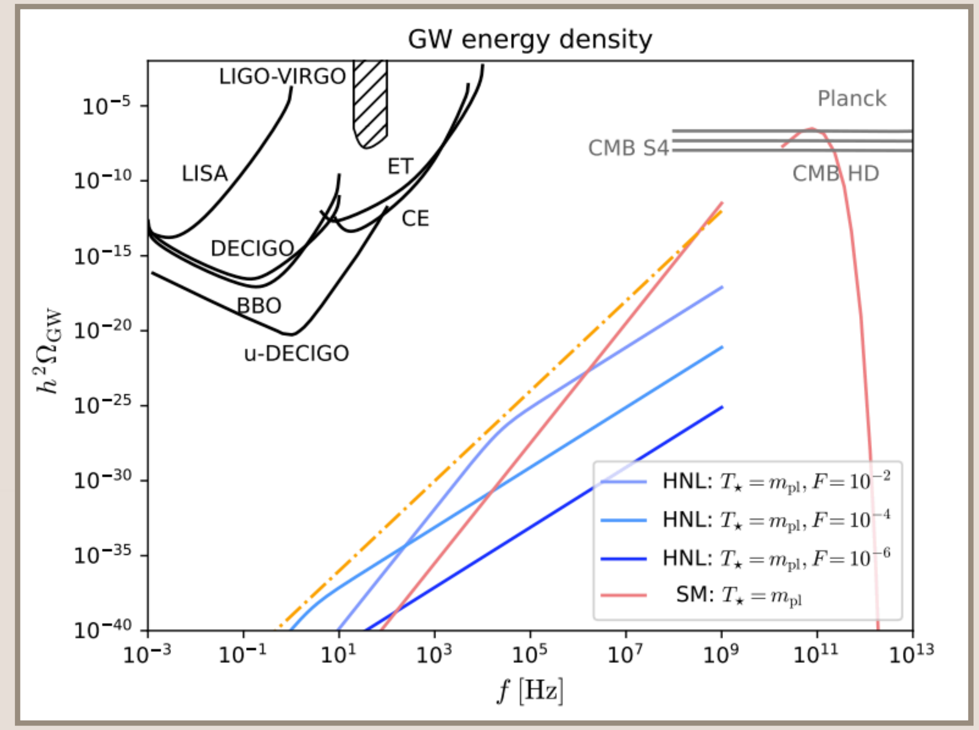


📖 S.Baumholzer, V.Brdarand E. Morgante

See also

📖 E.Braaten and M.H. Thoma

- Thermal GW spectrum



📖 M.Drewes, Y.Georis, J. Klarick and P.Klose

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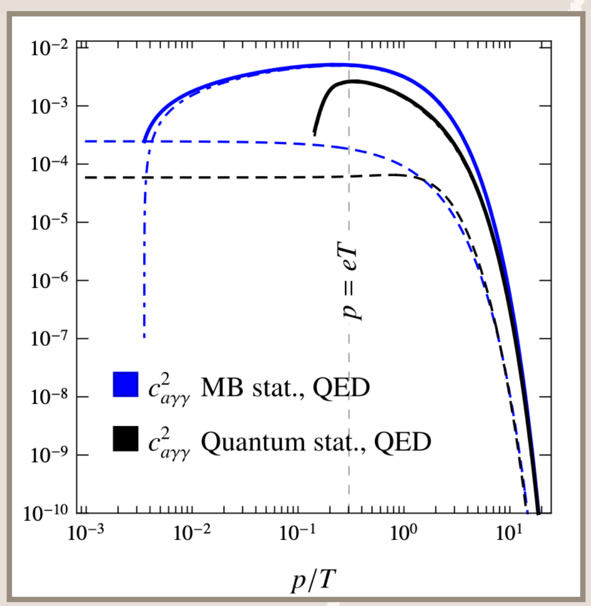

Today:

Full form of 1PI-resumed TFT propagator.

📖 arXiv:**2502.01729**: Becker, Harz, Morgante, CPI, Schwaller.

Different approach and non-abelian

📖 K.Bouzoud and J.Ghiglieri

Model: "photophilic" axion

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{2} m_a^2 a^2 - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{c_{aBB}}{4 f_a} a B_{\mu\nu} \tilde{B}^{\mu\nu}$$

- $B_{\mu\nu}$: field strength of the abelian gauge field.

→ "photon": $\mathrm{U}(1)_Y$ → DM relic abundance

Applied as well to SM photon or any other U(1) dark field

We need:

$$\left[\frac{\partial}{\partial t} - H p \frac{\partial}{\partial p}\right] f(p,t) = \mathcal{C}(p)$$

$$\mathcal{C}(p) = \frac{\Pi^<(P)}{2p_0} = \frac{\epsilon(p_0)}{p_0(1 - e^{p_0/T})} \operatorname{Im}\Pi(P)$$

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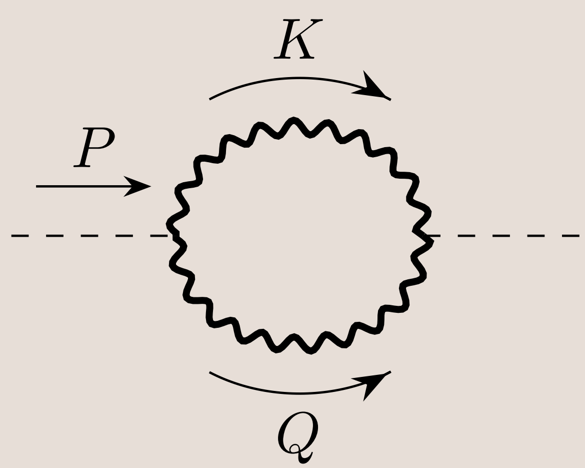
Integrate over $z = m_a/T$

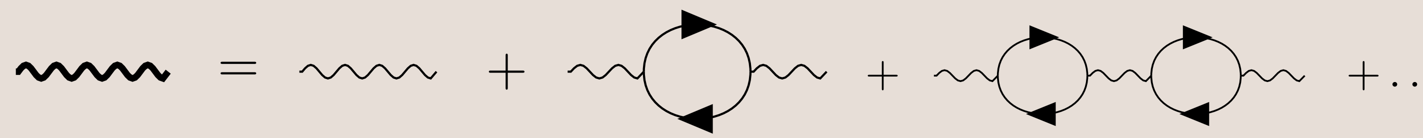
We need:

$$\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right] f(p,t) = \mathcal{C}(p) \quad \rightarrow \quad \frac{\partial f(p/T, z)}{\partial z} = \frac{1}{2p_0 z H}\Pi^<(P,z)$$

$$\mathcal{C}(p) = \frac{\Pi^<(P)}{2p_0} = \frac{\epsilon(p_0)}{p_0(1-e^{p_0/T})}\,\mathrm{Im}\,\Pi(P)$$

Thermal axion production.

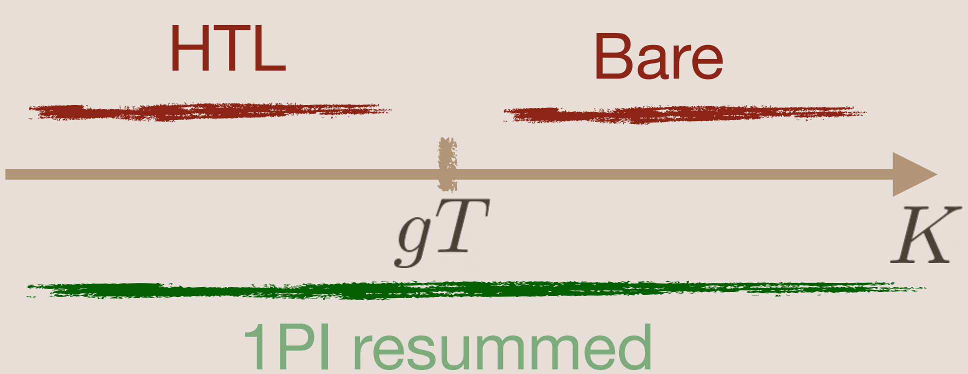




$$^*D^<_{\mu\nu}(K) = f_B(k_0)\left[\mathcal{P}^t_{\mu\nu}\rho_t(K) + \mathcal{P}^l_{\mu\nu}\frac{k^2}{K^2}\rho_l(K) + \xi\frac{k_\mu k_\nu}{K^2}\right]$$

Spectral densities

♦ Our implementation:



! We resumed **both** gauge bosons

♦ The gauge dependent parallel polarization cancels

♦ Our spectral densities are gauge independent

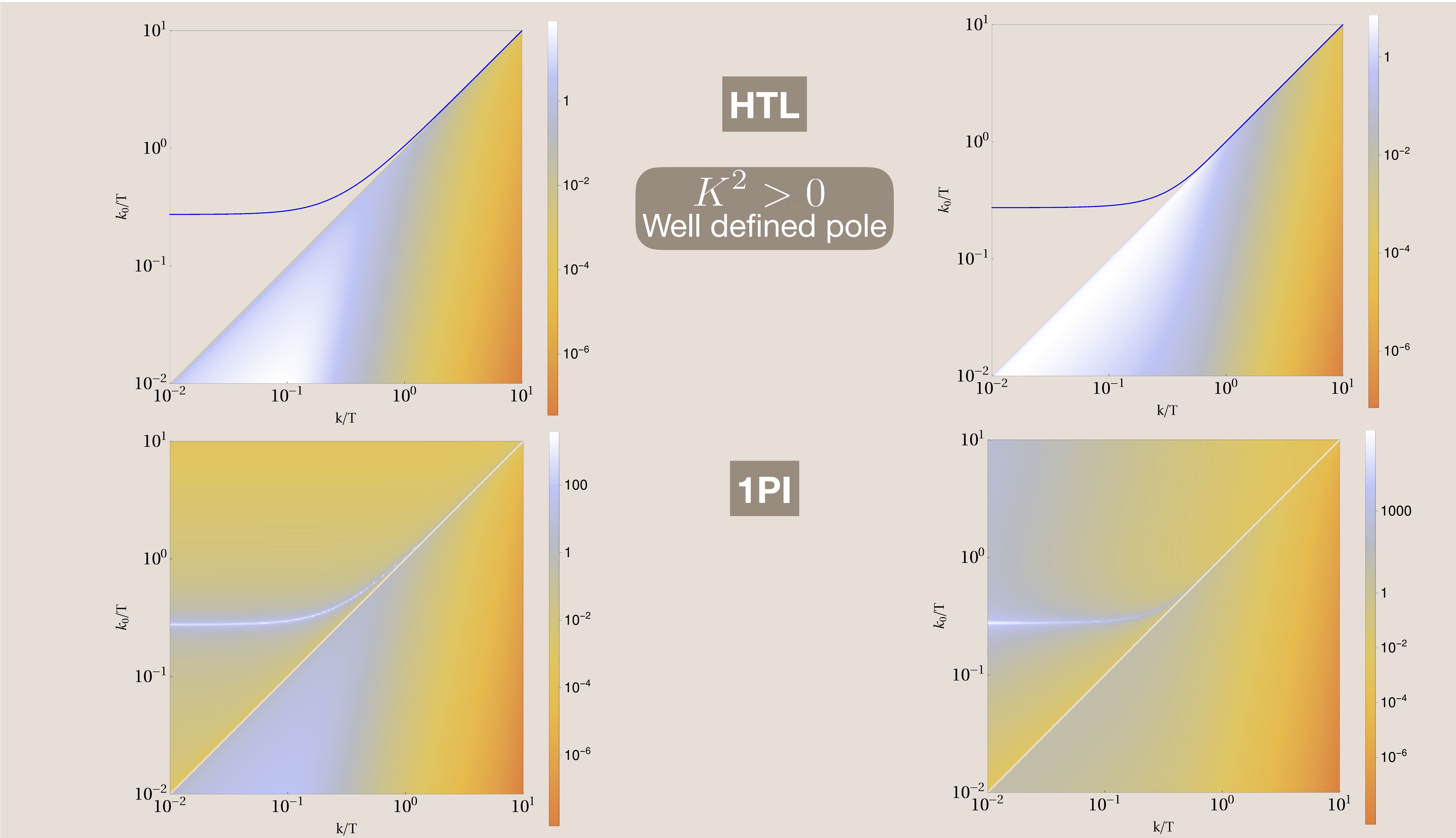
→ **gauge independent**

Different approach and non-abelian

📖 K.Bouzoud and J.Ghiglieri

$$\rho_t = -2\frac{\mathrm{Im}\,\pi_0 + \mathrm{Im}\,\pi_t}{(K^2 - \mathrm{Re}\,\pi_0 - \mathrm{Re}\,\pi_t)^2 + (\mathrm{Im}\,\pi_0 + \mathrm{Im}\,\pi_t)^2}$$

$$\rho_l = -2\frac{K^2}{k^2}\frac{\mathrm{Im}\,\pi_0 + \mathrm{Im}\,\pi_l}{(K^2 - \mathrm{Re}\,\pi_0 - \mathrm{Re}\,\pi_l)^2 + (\mathrm{Im}\,\pi_0 + \mathrm{Im}\,\pi_l)^2}$$



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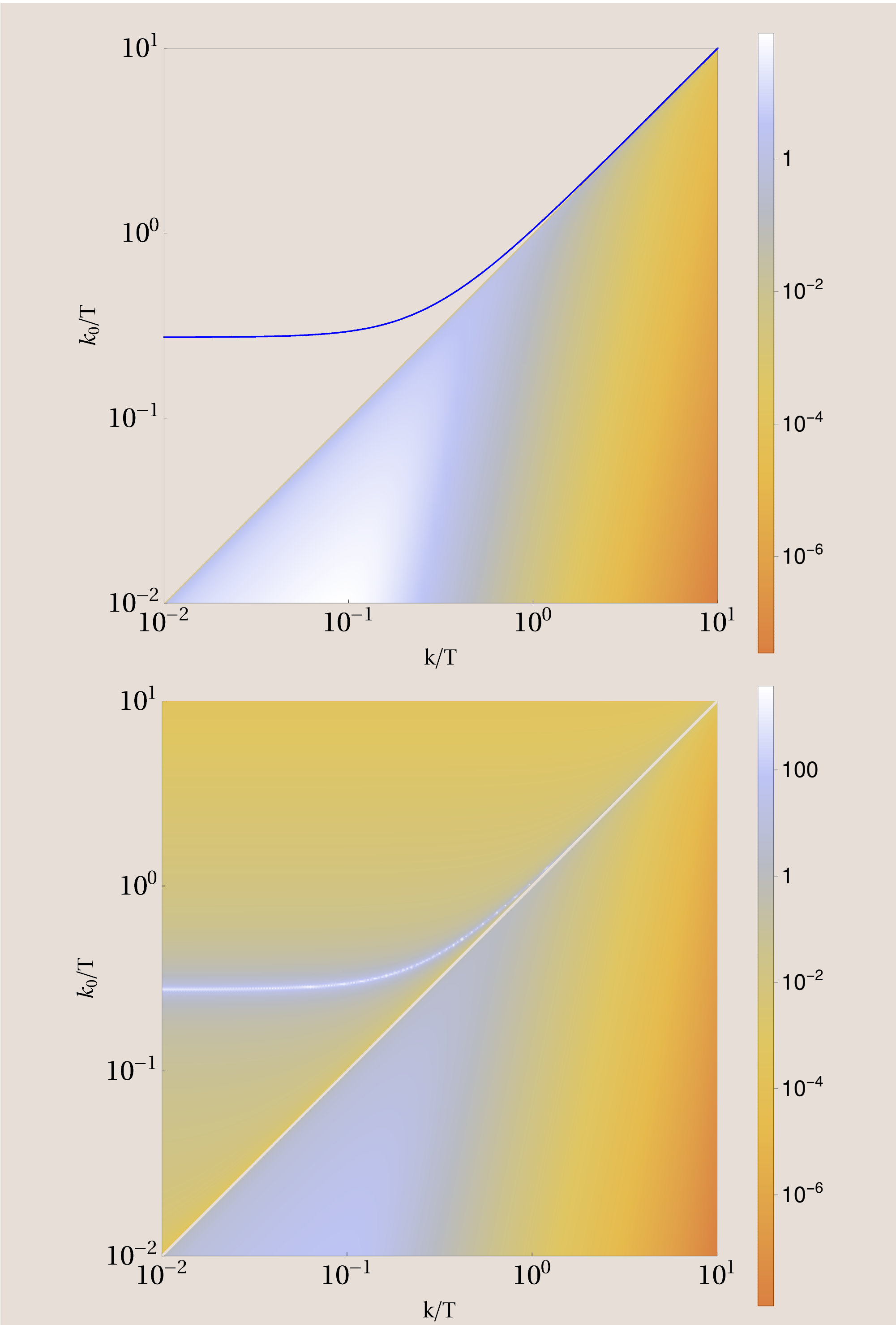
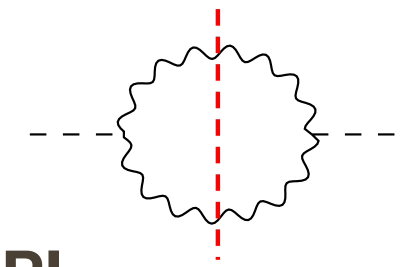


1 loop, included

$\Pi^{<}_{TT}(p)$:

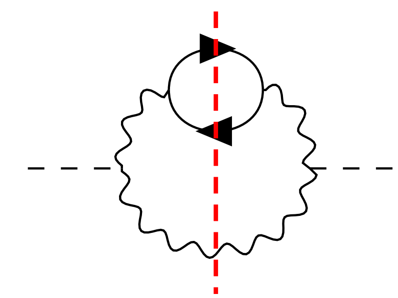


(Inverse) Decays for **HTL**

includes 2⟷3 scatterings for **1PI**

2 loops, included

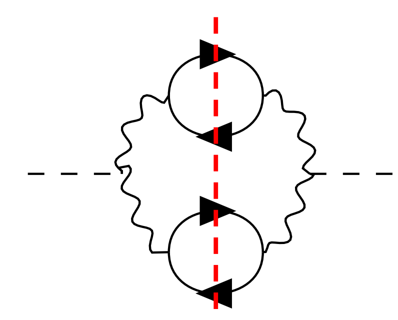
$\Pi^{<}_{TS}(p)$:



2⟷2 scatterings

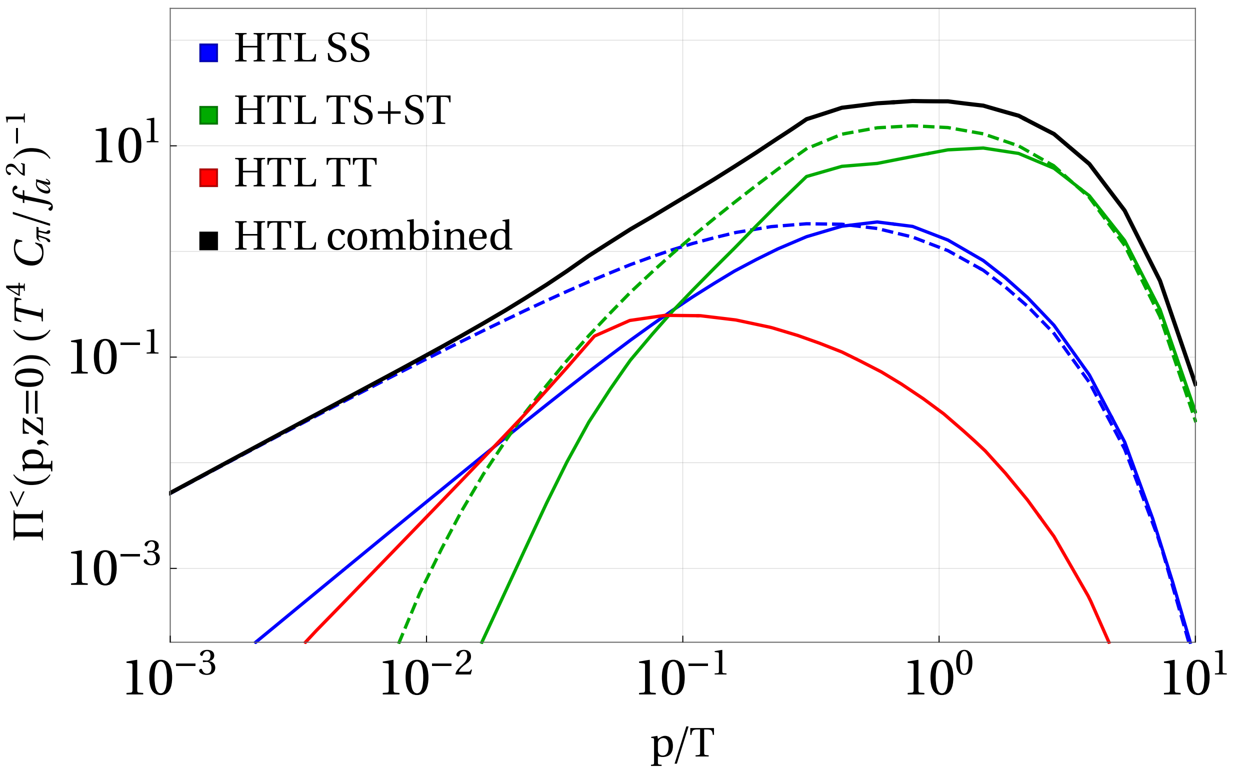
3 loops, included

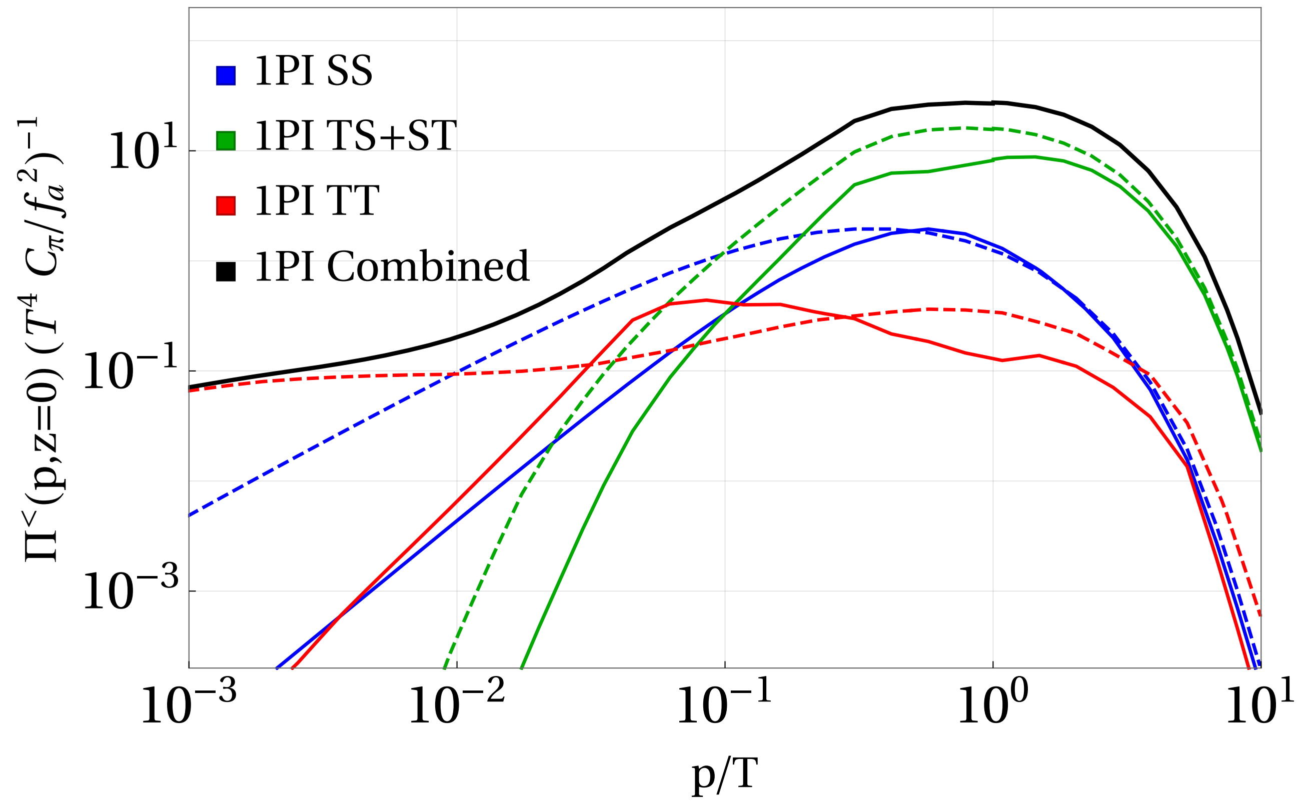
$\Pi^{<}_{SS}(p)$:



2⟷3 scatterings

Results: Production rate.





- (blue) $\Pi^<_{SS}(p)$: 2⟷3 scatterings
- (green) $\Pi^<_{TS}(p)$: 2⟷2 scatterings
- (red) $\Pi^<_{TT}(p)$: (Inverse) Decays

  includes 2⟷3 scatterings for **1PI**

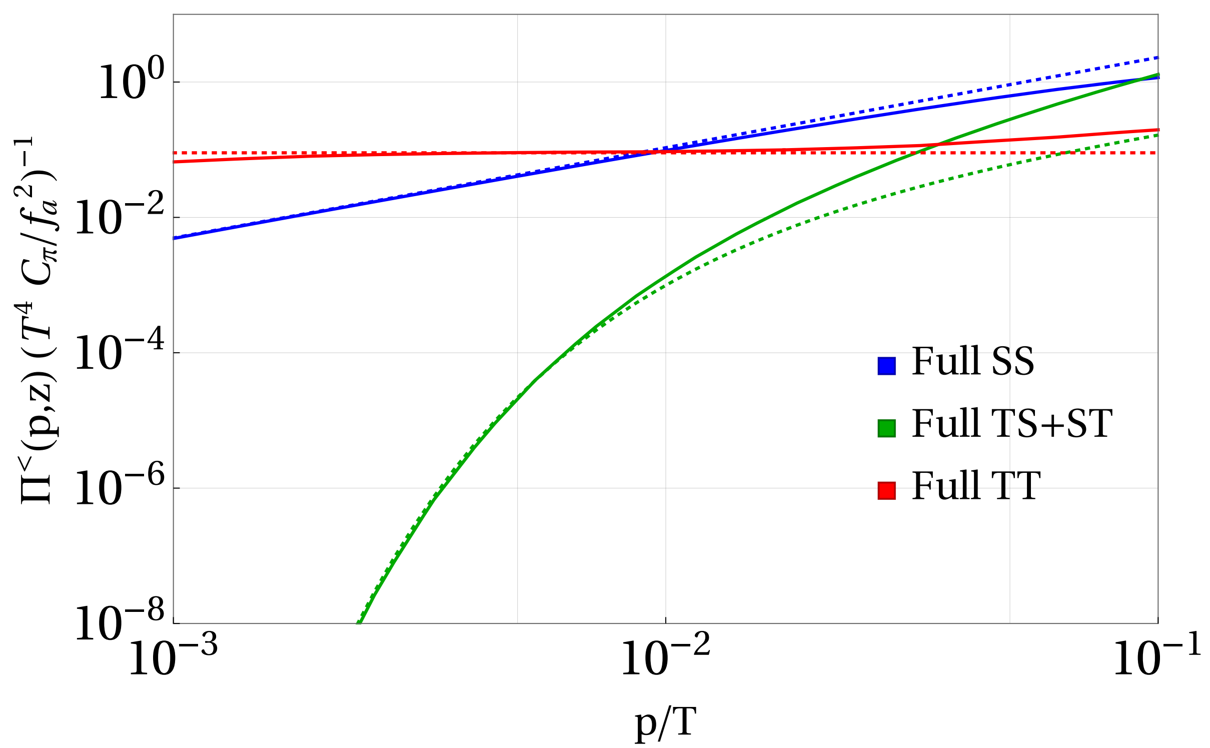
- Production dominated by high temperatures around T_{rh}

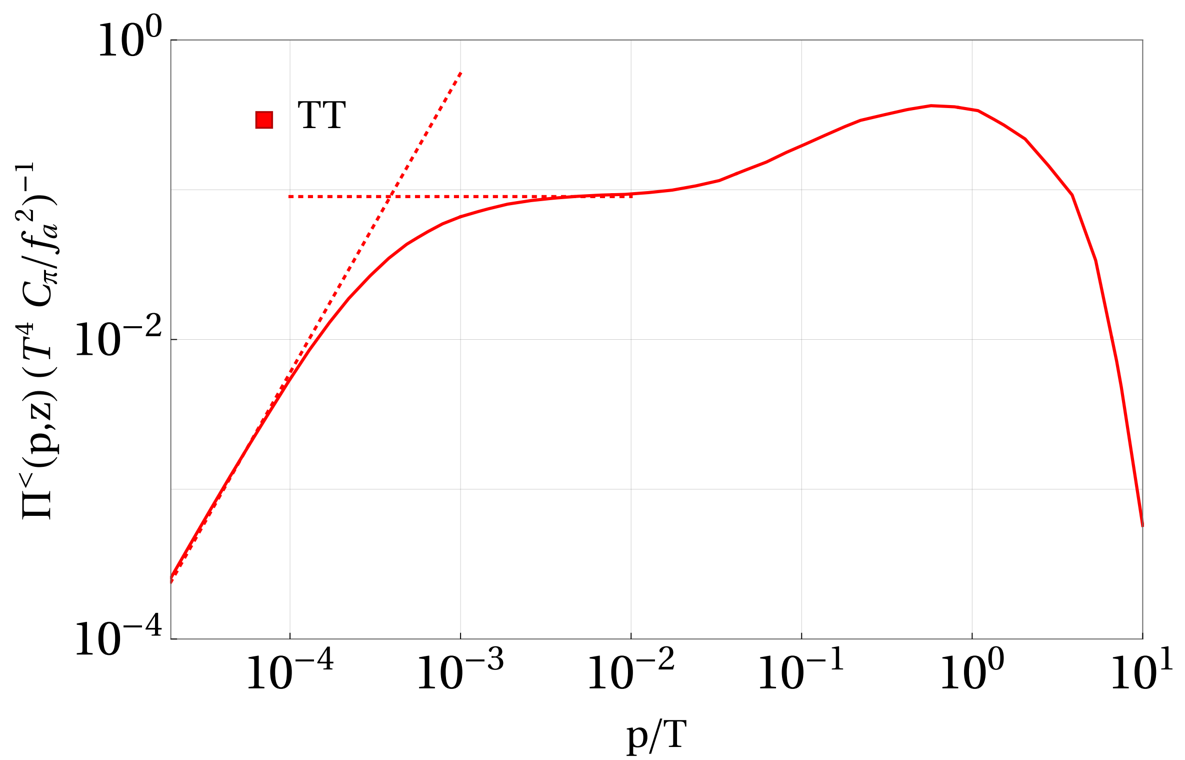
  → $z = m_a/T = 0$

——— Longitudinal-Transversal

- - - - - Transversal-Transversal

Results: Production rate.





Small p scaling

$\Pi^<_{SS}(p) \propto p^{\frac{4}{3}}$ 2⟷3

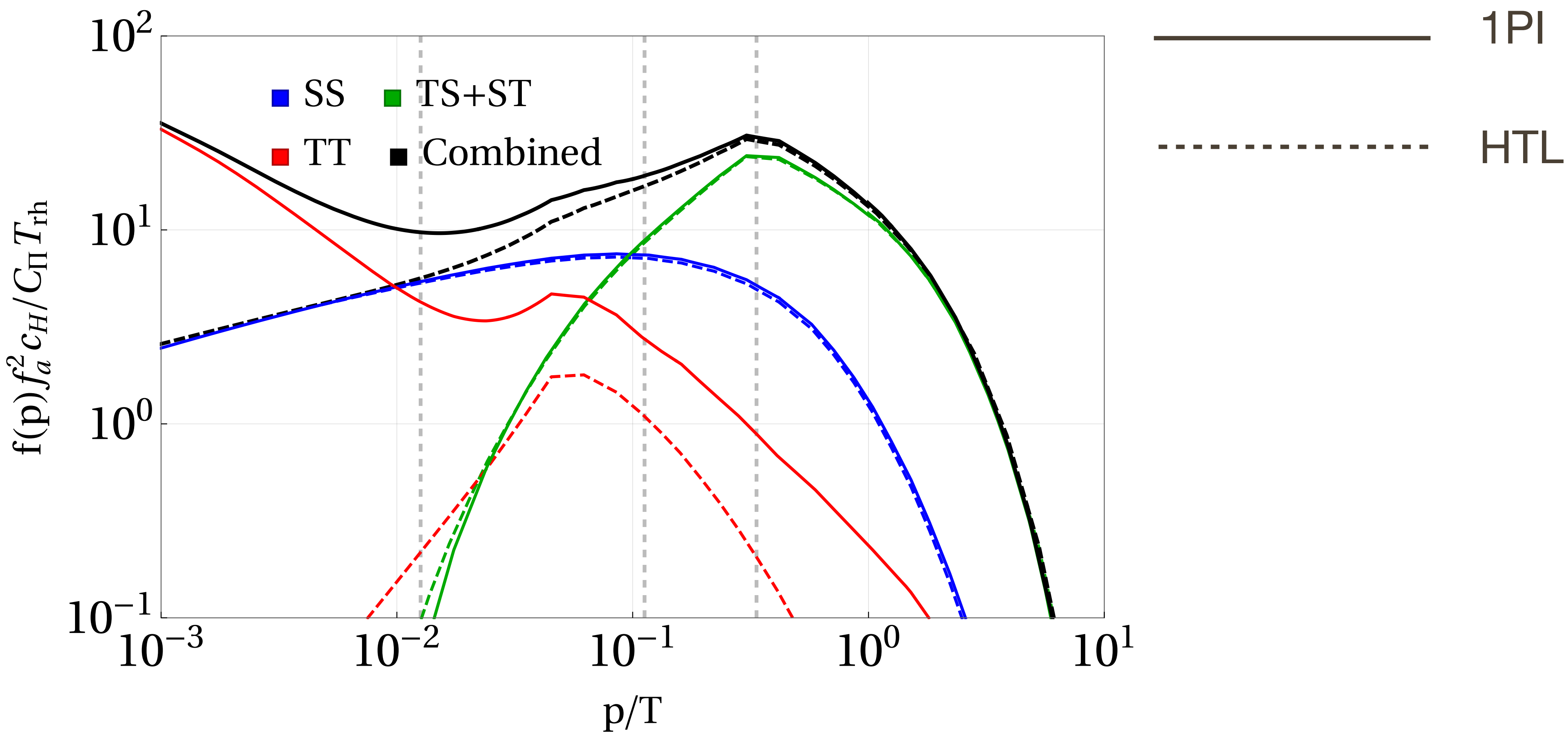
$\Pi^<_{TS}(p) \propto p \exp\left(-\frac{m_V^2}{4p}\right)$ 2⟷2

$\Pi^<_{TT}(p) \propto \begin{cases} p^0 & p \gtrsim p_c \\ p^2 & p \lesssim p_c \end{cases}$ 2⟷3

$p_c \approx 5 \times 10^{-4}$

- At soft momentum $(p < gT)$ 2⟷2 scatterings suffer a kinematic suppression.
- 1PI captures time-like gauge boson effects

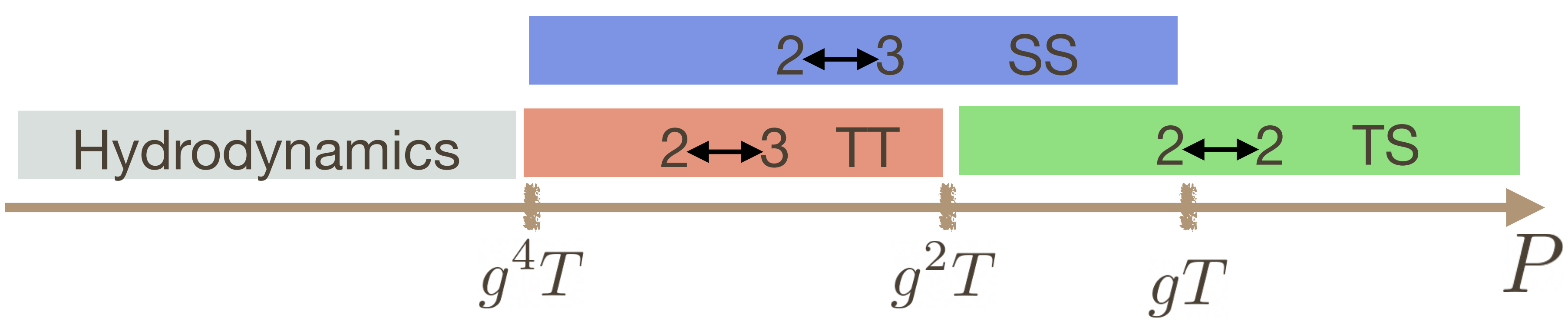
Results: Distribution function.



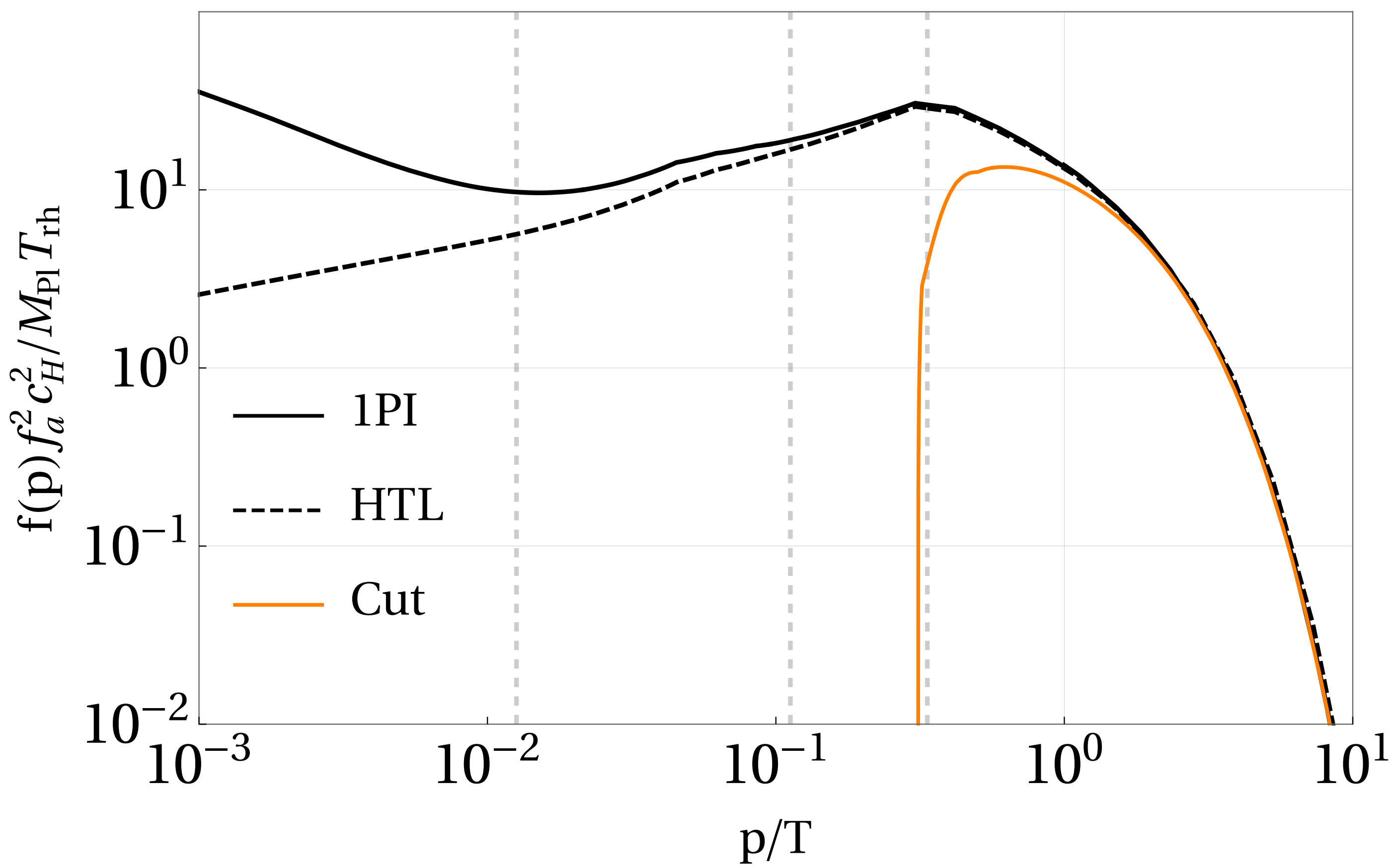
Scales

$P \sim T$ “Hard”

$P \sim gT$ “Soft”



Results: Comparision.



$$\langle p \rangle_{1PI} = 3.06$$
$$\langle p \rangle_{HTL} = 3.17$$
$$\langle p \rangle_{Cut} = 3.14$$
$$\langle p \rangle_{BE} = 2.70$$

- Mild impact on Lyman-α constraints.

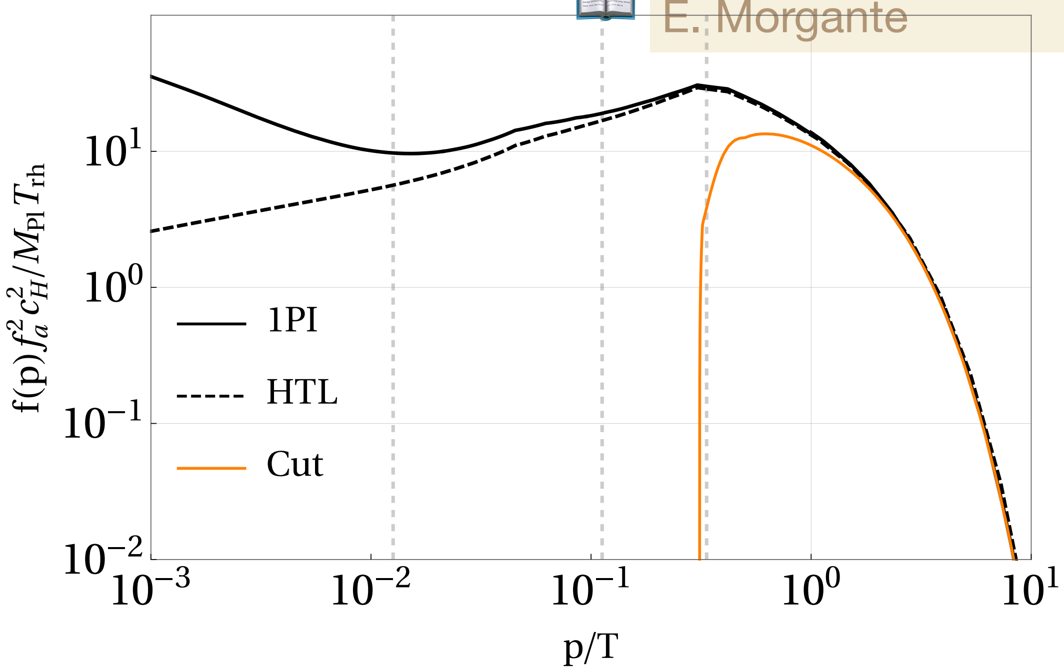
$$\lambda_{\rm fs} \propto \frac{\langle p \rangle / T}{m_a}$$

$$\langle p \rangle = \frac{\int dp\, p^3 f_a(p)}{\int dp\, p^2 f_a(p)}$$

📖 S.Baumholzer, V.Brdar, and E. Morgante

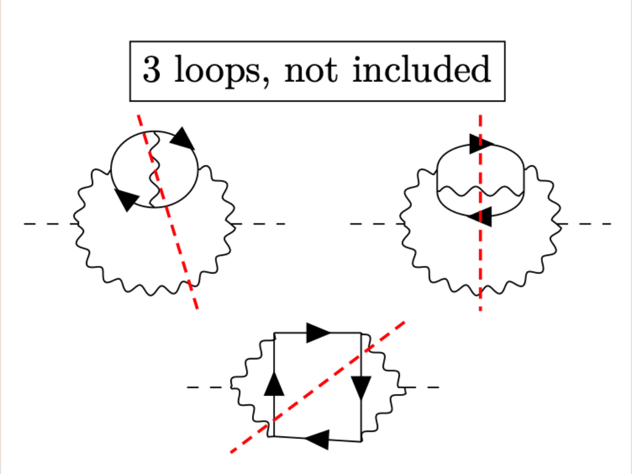
However…..

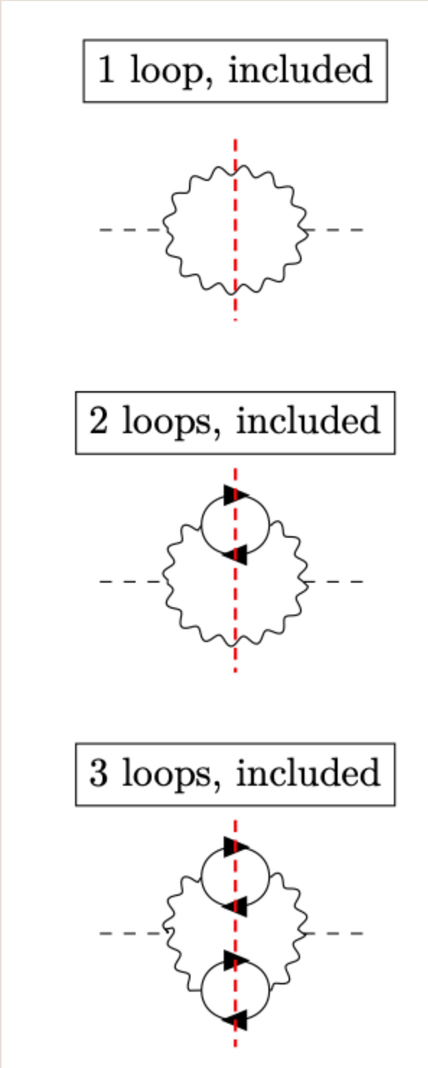
S.Baumholzer, V.Brdar, and E. Morgante



However we still don't have the LO calculation….

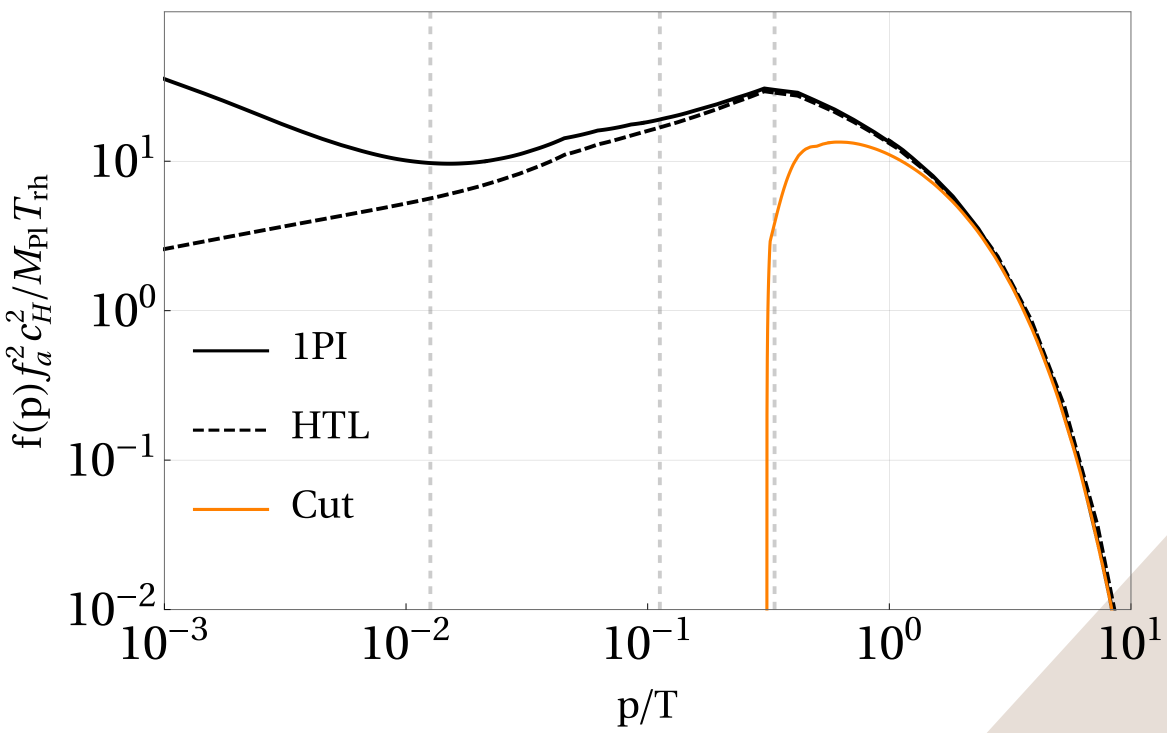
- We include soft ALP radiation from virtual photons
- We do not include soft ALP radiation from real photons yet → Goes beyond 1-loop 1PI resummation





Conclusions.

- A gap exist for $g^4T \leq P \leq gT$
- Our HTL and 1PI calculations resolve negative rates → Below gT
- 1PI captures contributions from 2 time-like gauge bosons.
- $2 \leftrightarrow 3$ scatterings dominate for soft ALPs but we need to keep working for full accuracy at LO for $P < g^2T$
- Mild impact on Lyman-α constraints.



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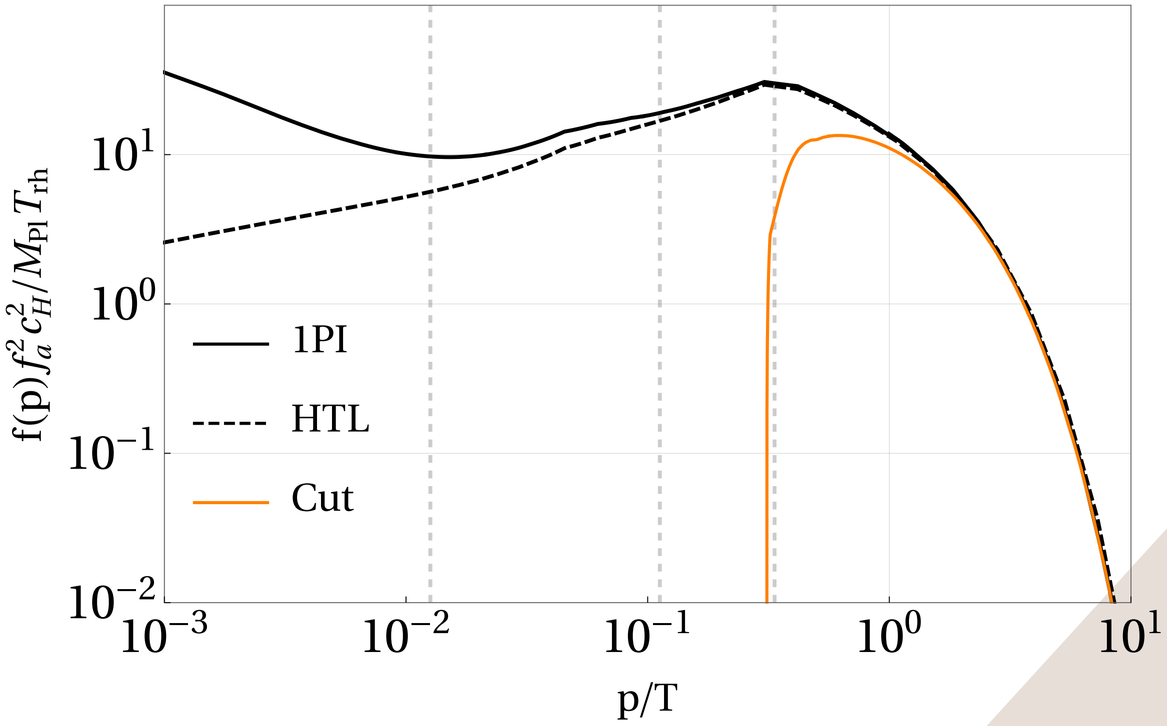
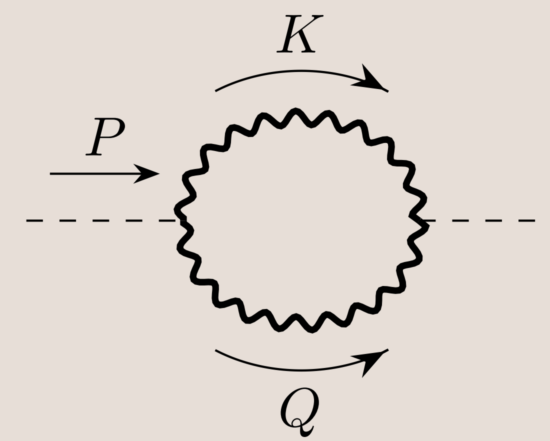


Thank you!

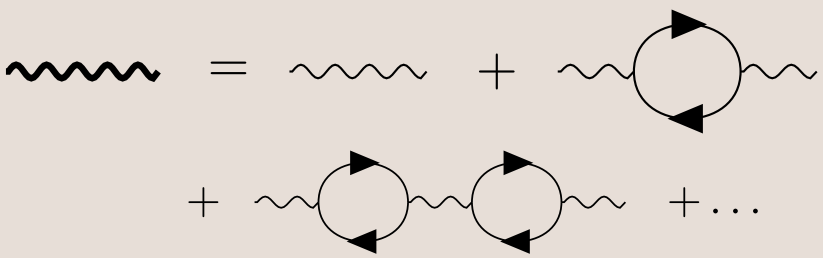
Thermal axion production.

$$\Pi^<(P) = \frac{C_\Pi}{f_a^2 p} \int \frac{d^4K}{2\pi^4} \epsilon^{\mu\nu\alpha\beta} \epsilon^{\mu'\nu'\alpha'\beta'} K_\alpha Q_\beta K_{\alpha'} Q_{\beta'} {}^*D^<_{\mu\mu'}(K) {}^*D^<_{\nu\nu'}(Q)$$



- ${}^*D^<_{\mu\nu}(K) = f_B(k_0) \left[\mathcal{P}^t_{\mu\nu} \rho_t(K) + \mathcal{P}^l_{\mu\nu} \frac{k^2}{K^2} \rho_l(K) + \xi \frac{k_\mu k_\nu}{K^2} \right]$
- $C_\Pi = \frac{c_{aBB} d_i \alpha_i^2}{8(2\pi)^5} = \frac{c_{aBB} \alpha_Y^2}{8(2\pi)^5}$
- $f_B(k_0) = \frac{1}{e^{k_0/T} - 1}$

$$\Pi^<(P) = \frac{C_\Pi}{f_a^2 p} \int_{-\infty}^{\infty} dk^0 \int_0^\infty dk \int_{|p-k|}^{|p+k|} dq\, kq f_B(k^0) f_B(p^0 - k^0) \left[\mathcal{I}_{lt} + \mathcal{I}_{tt} \right]$$



- $\mathcal{I}_{lt} = \left(\rho_t(K)\rho_l(Q) + \rho_l(K)\rho_t(Q) \right) \left[(k+q)^2 - p^2 \right] \left[p^2 - (k-q)^2 \right]$
- $\mathcal{I}_{tt} = \rho_t(K)\rho_t(Q) \left[\left(\frac{k_0}{k} \right)^2 + \left(\frac{q_0}{q} \right)^2 \left((k^2 - p^2 + q^2)^2 + 4k^2 q^2 \right) + 8k^0 q^0 (k^2 + q^2 - p^2) \right]$