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Revisited axion contribution to dark radiation using momentum-dependent evolution

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Based on **JHEP 02 (2025) 108** [2410.18186]

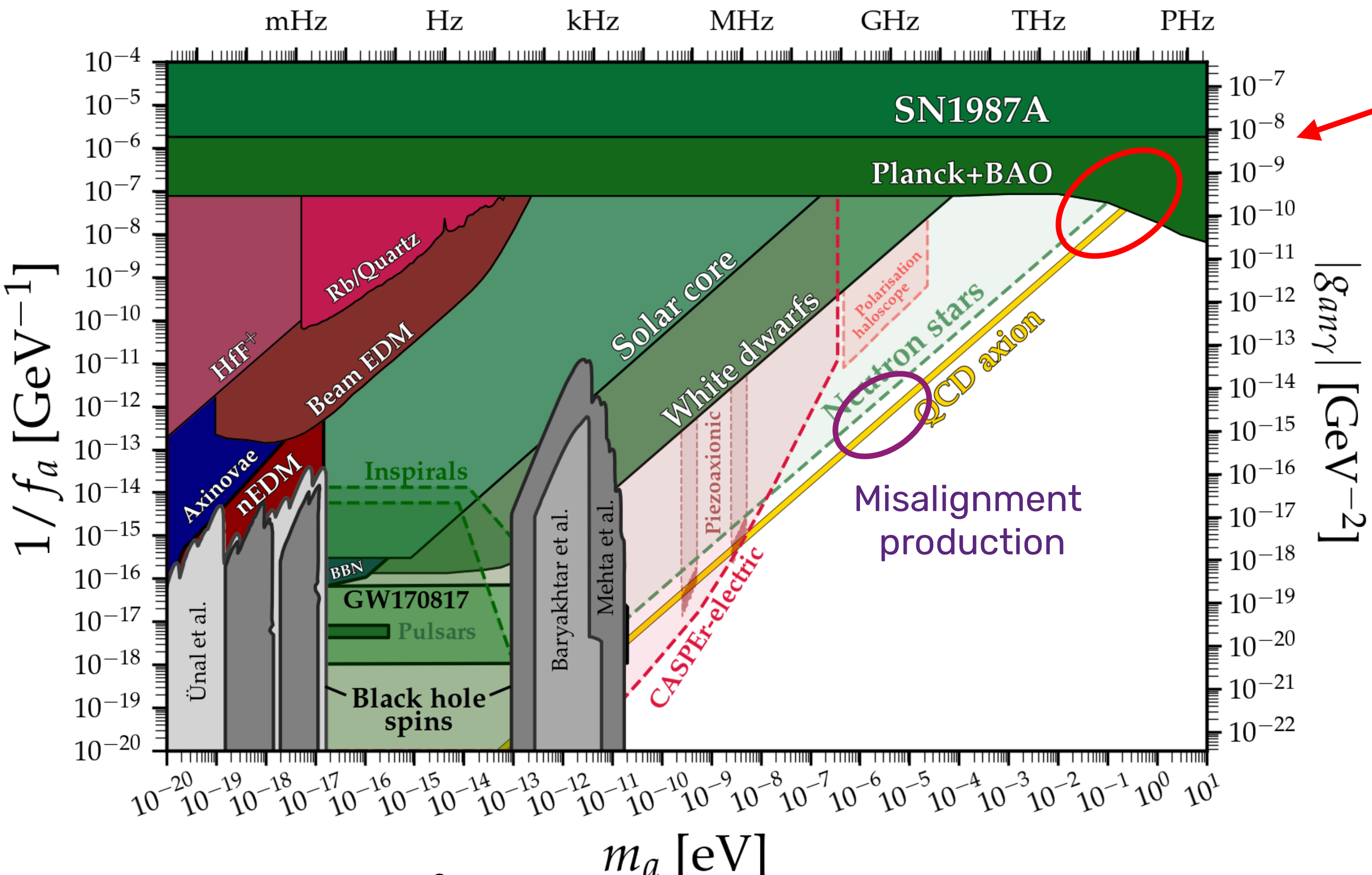
with **M. Badziak** (Warsaw U.)

28/05/2025

PLANCK 2025

UNIVERSITY OF WARSAW

QCD axion and thermal production



$$m_a = \frac{m_\pi f_\pi}{2 f_a}$$

Efficient production from the interactions in the early Universe plasma (thermal)

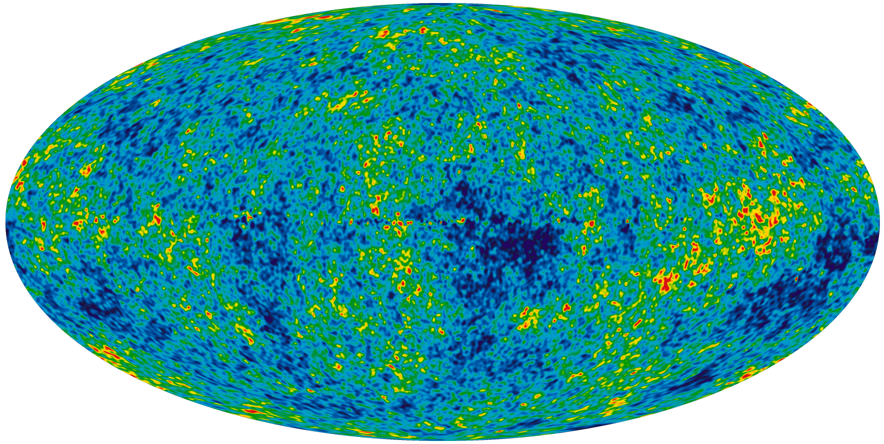
$$\Gamma \propto \frac{1}{f_a^2}$$

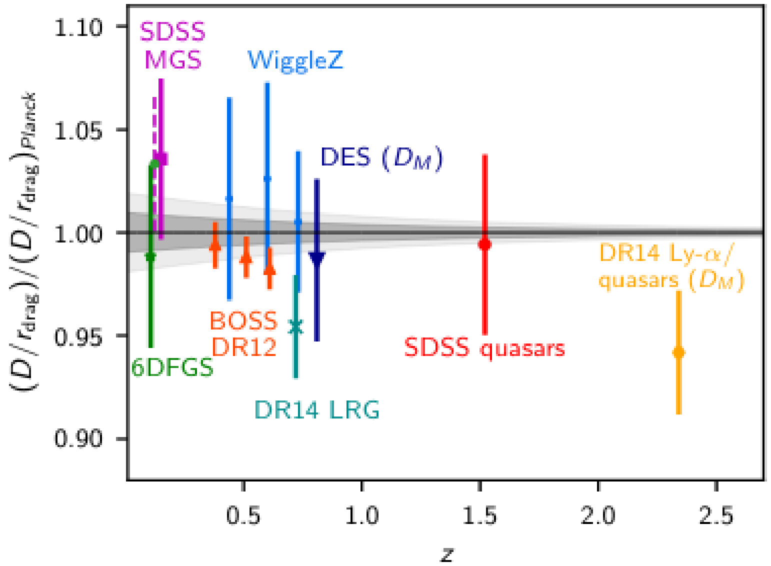
Such axions can't be DM because they are too hot

But their abundance can put constraints on the models

CMB and baryon acoustic oscillations

Additional radiation energy density affects the CMB spectrum (especially low-l mode polarization) and baryon acoustic oscillation (BAO) geometry

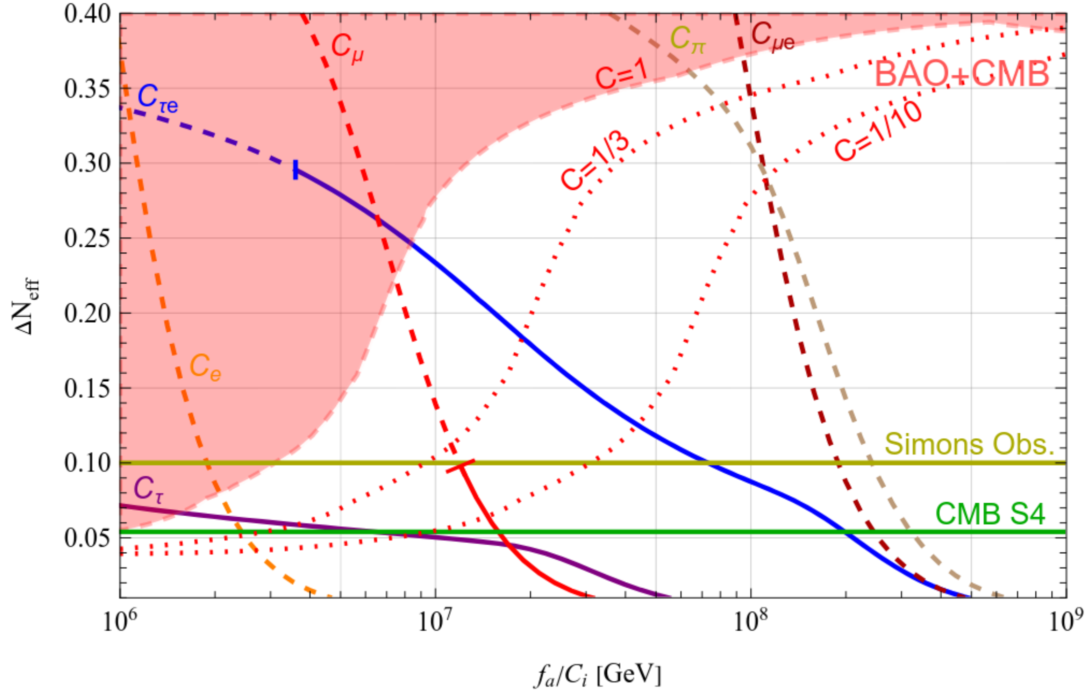




Can be effectively represented as $\Delta N_{\rm eff} \propto \rho_a$

Planck constraint $\Delta N_{\rm eff} \leq 0.3 \text{ (95\% C.L.)}$ PLANCK, 1807.06209

Constraints on axion interactions with leptons

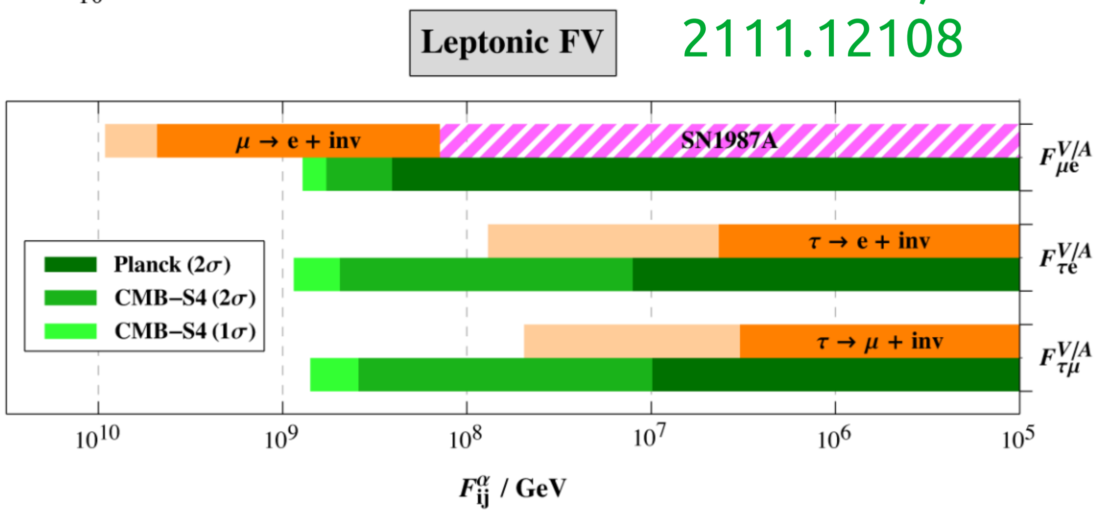


Badziak+, 2403.05621

The abundance of axions is calculated using the Boltzmann equation for the axion number density

$$\Delta N_{\rm eff} \propto n_a^{4/3}$$

D'Eramo+, 2111.12108



Sets constraints on axion-lepton interactions (can be compared with other searches)

Standard approach

Solving the Boltzmann equation for the density of axions in the expanding Universe

$$\frac{dY}{dx} = \frac{(1+\tilde{g})}{sHx}\left[\Gamma_+ - \Gamma_-(Y)\right] \quad \begin{cases} \Gamma_+ = n_{\rm eq}^2 \langle \sigma v \rangle & \text{annihilations} \\ \Gamma_+ = n_{\rm eq}/\tau & \text{decays} \end{cases}$$

$$x = \frac{m}{T} \qquad Y = \frac{n}{s} \qquad \tilde{g} = \frac{1}{3}\frac{d\ln h_s}{d\ln x}$$

However, this approach is based on the assumptions:

1. Axions are in kinetic equilibrium with the SM
2. Equilibrium distributions have the Maxwell-Boltzmann shape

Axion interactions

At tree-level axions have interaction vortices with the SM particles with just one branch

$$\mathcal{L}_{\rm int}^{(a)} = \frac{1}{2f}\,\partial_\mu a\, J^\mu_a + \frac{a}{f}\sum_X \frac{\alpha_X}{8\pi} C_{XX} X_{\mu\nu}\tilde{X}^{\mu\nu}$$

Lagrangian has to satisfy the shift symmetry $a \to a + 2\pi$

Thus, the rates of all the reactions involving two axions are supressed by $(1/f_a)^2$ factor (incl. elastic scatterings)

Full Boltzmann equation

To take everything into account we need to solve the Boltzmann equation for the distribution function

$$\tilde{H}(x\partial_x - \tilde{g}q\partial_q) f_a(x,q) = C[f_a]$$

If all the particles in reactions have equilibrium-shaped distributions we reproduce the standard Boltzmann equation for the density by integrating over $g_i \int \frac{d^3p_i}{(2\pi)^3}$

$$\frac{g_i}{s} \int \frac{d^3p_i}{(2\pi)^3} f_i = Y \qquad \frac{dY}{dx} = \frac{(1+\tilde{g})}{sHx} \sum_i \gamma_i \left[1 - \frac{Y}{Y_{\mathrm{eq}}}\right]$$

Integral of the collision term C[f]

Contribution to dark radiation (ΔN_{eff})

1) General formula

$$\Delta N_{\text{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_a}{\rho_\gamma} \qquad \rho_a = \frac{g_a}{2\pi^2}\int dE_a\, E_a^3\, f_a(E_a)$$

2) Simplified formula (assuming axions in thermal equilibrium and instantaneous decoupling)

$$\Delta N_{\text{eff}} = \frac{4}{7}\left(\frac{11}{4}\cdot\frac{2\pi^4 h_s(x)}{45\zeta(3)}Y_a\right)^{4/3} \approx 74.85\, Y_a^{4/3}$$

Contribution to dark radiation (ΔN_{eff})

The simple formula misses the fact that even if axions have a thermal shape of the distribution, their abundance is not equilibrium

$$f_a = \frac{A}{\exp(E/T) - 1} \qquad A \equiv n_a / n_a^{\rm eq}$$

(normalization factor)

$$n_a = A \cdot g_a \frac{\zeta(3)}{\pi^2} T^3 \,, \qquad \rho_a = A \cdot g_a \frac{\pi^2}{30} T^4$$

$$\rho_a = \frac{g_a}{A^{1/3}} \cdot \frac{\pi^2}{30} \left(\frac{\pi^2 n_a}{\zeta(3) g_a} \right)^{4/3}$$

Simple formula underestimates the ΔN_{eff} by a factor $A^{1/3}$

Lepton-flavour violating decays

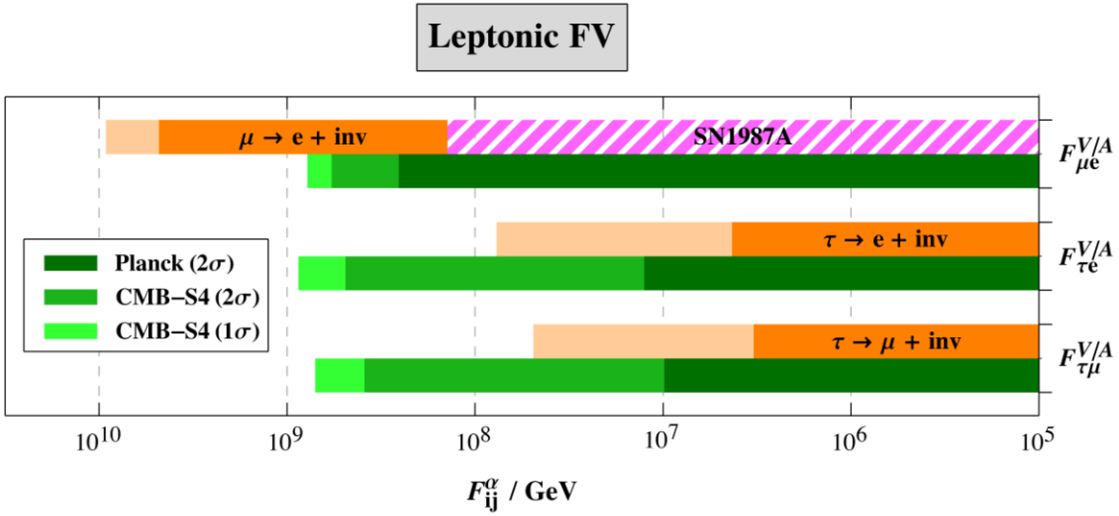
Axions can be produced via LFV decays

$$\mathcal{L}_{\text{eff}} = \frac{\partial_\mu a}{2 f_a} \overline{f}_i \gamma^\mu \left(C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5 \right) f_j - \frac{m_a^2}{2} a^2$$

Causing $l_i^\pm \to l_j^\pm + a$

D'Eramo+, 2111.12108

We concentrate on tau decays as muon decays are severely constrained by laboratory measurements and SN1987A



Collision term for decay (j → i+a)

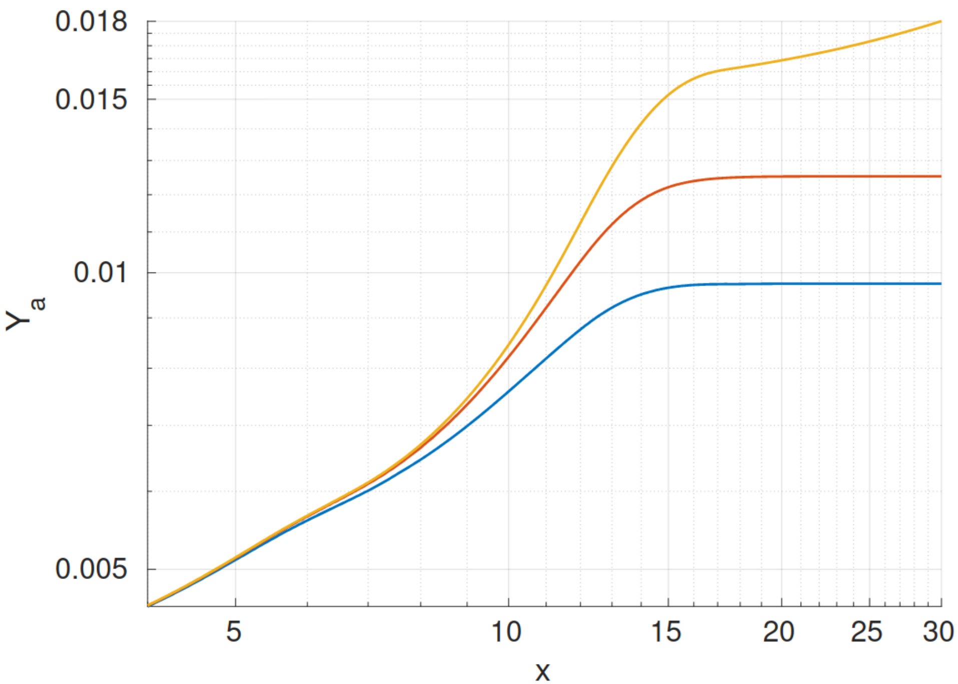
$$C[f_a] = \frac{1}{2g_a E_a}\Big[\int \frac{d^3p_j}{(2\pi)^3 2E_j} \int \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(\mathcal{P}_j - \mathcal{P}_i - \mathcal{P}_a)\, |M|^2_{j\to ia}\, f_j (1\pm f_a)(1\pm f_i)$$
$$- \frac{d^3p_j}{(2\pi)^3 2E_j} \int \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(\mathcal{P}_j - \mathcal{P}_i - \mathcal{P}_a)\, |M|^2_{j\leftarrow ia}\, f_i f_a (1\pm f_j)\Big]$$

Without any assumptions, this expression simplifies to an analytical formula (due to the simplicity of the amplitude squared and the kinematics of the 2-body decay)

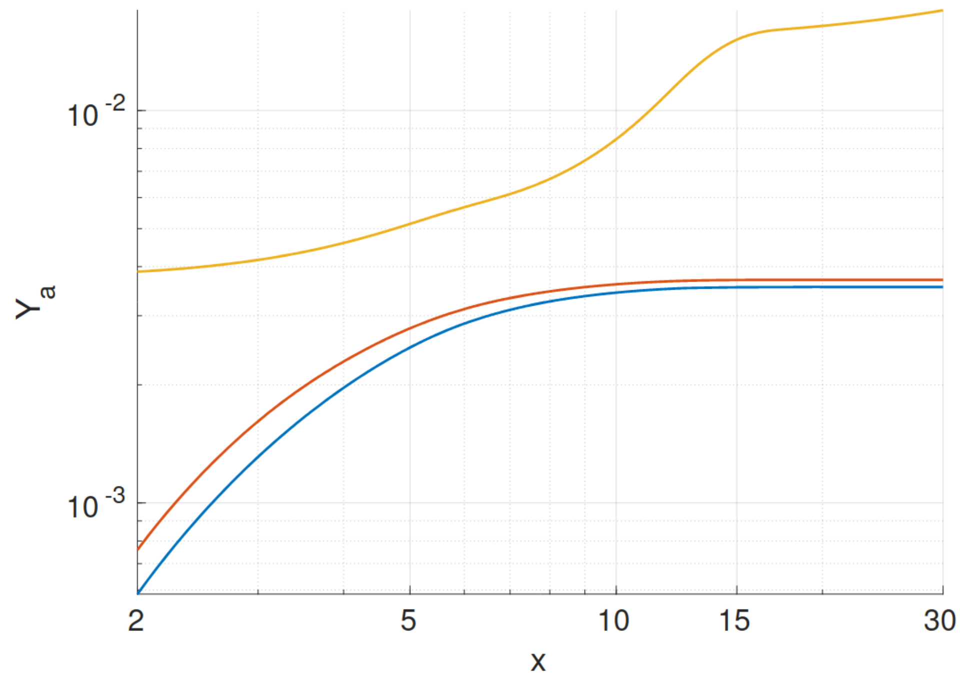
$$C[f_a] = \frac{x|\mathcal{M}|^2}{8\pi q^2} \log\left[\frac{1+\exp(-\epsilon_1(x,q))}{1+\exp(-\epsilon_2(x,q))}\right](f_a - f_a^{\mathrm{eq}})$$

$$x = m_\tau/T \qquad q = p/T$$

Results for the abundance Y



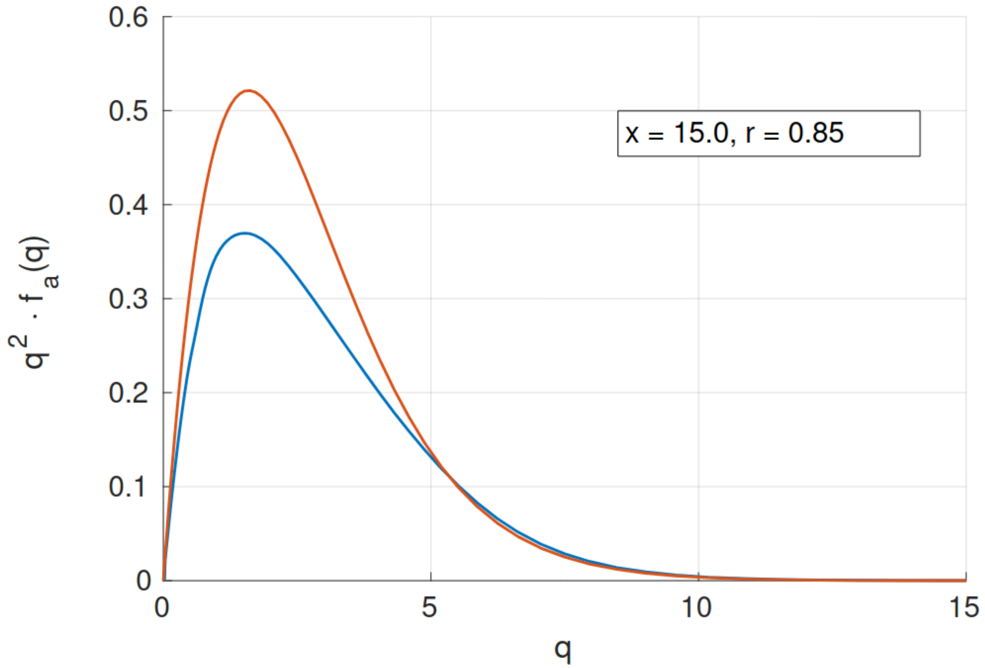
$$f/C_{\tau\mu} = 10^7\,\mathrm{GeV}$$



$$f/C_{\tau\mu} = 2 \cdot 10^8\,\mathrm{GeV}$$

nBE solution
fBE solution
Equilibrium abundance

Axion distribution functions



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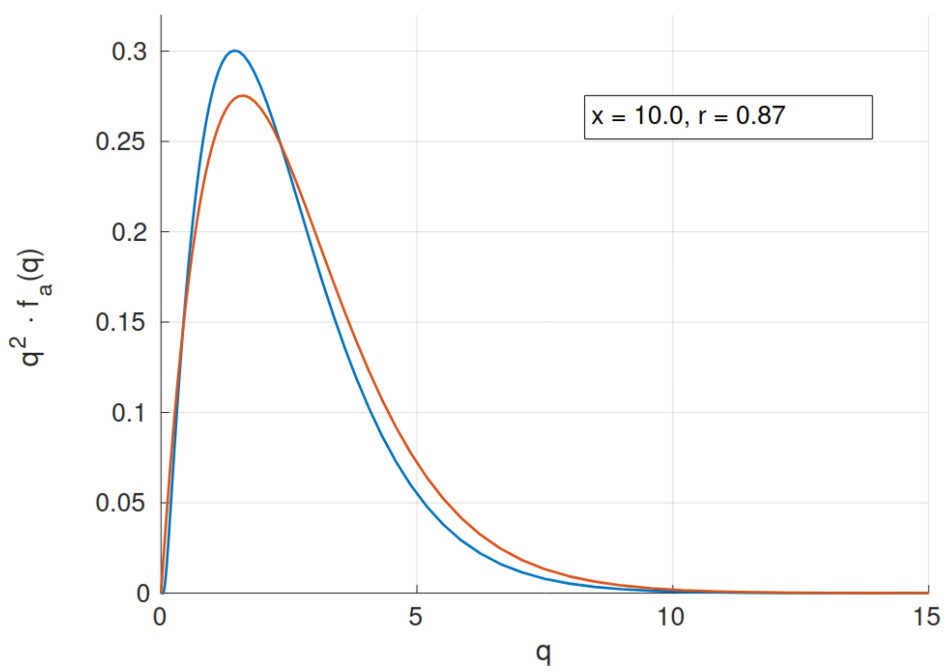

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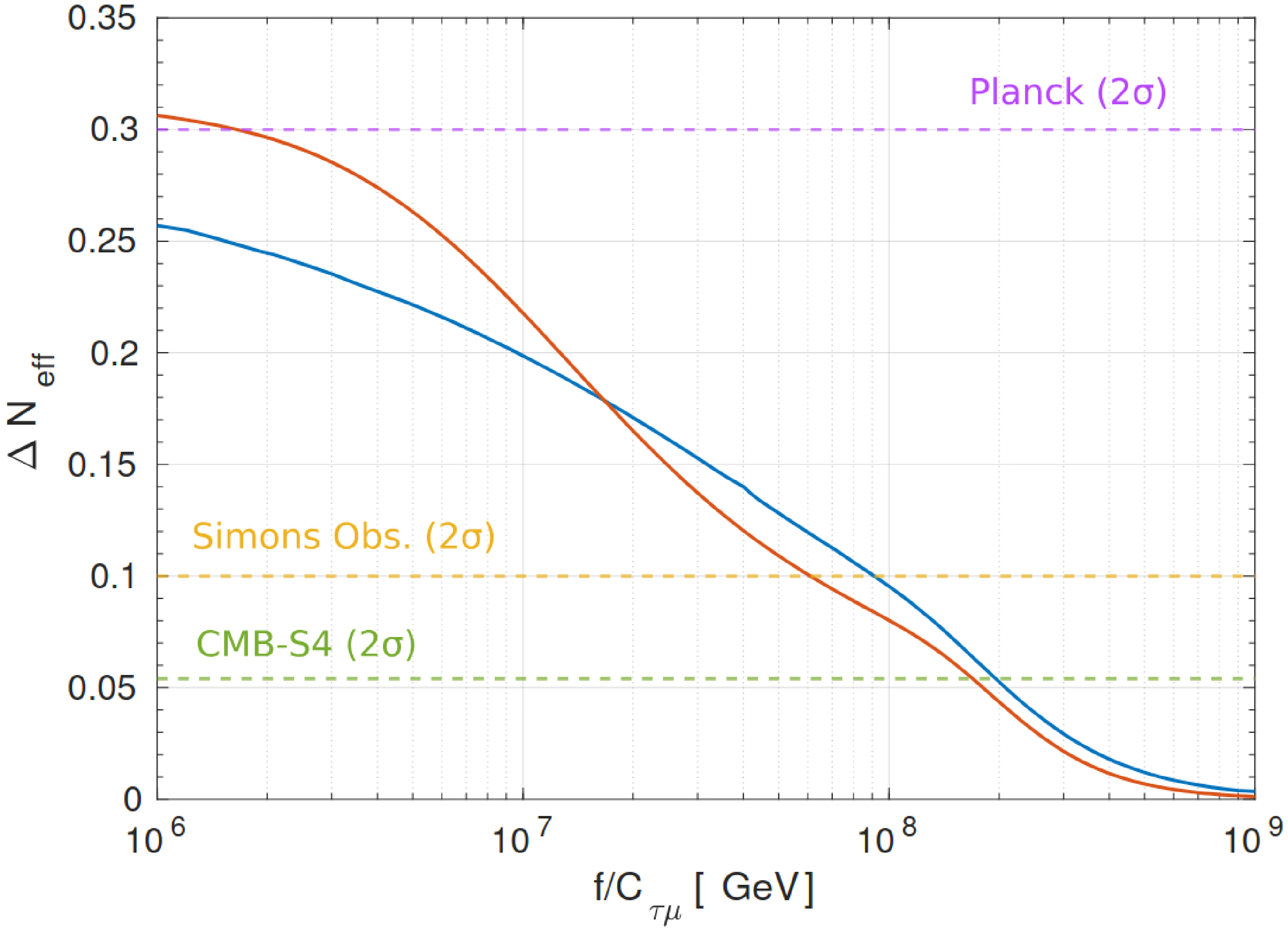
Equilibrium shape (Bose-Einstein)
Realistic shape

Difference in the ΔN_{eff}



Diagonal interactions with muons

$$\mu^+\mu^- \to \gamma a$$

Annihilation of leptons into axion

$$\mu^\pm\gamma \to \mu^\pm a$$

Primakoff scattering

These processes in the early Universe can be *a probe of the axion coupling* to muons.

- Electron coupling is tightly constrained by XENONnT and white dwarf luminosity function XENON, 2207.11330 Bertolami+, 1406.7712
- Muon (and tau) couplings are less constrained (by SN1987A)

Caputo+, 2109.03244

$$\frac{f_a}{|C_\mu|} \gtrsim 1.2 \times 10^7\,\mathrm{GeV}$$

Collision term for i + j → k + a

$$C[f_a] = \frac{1}{2g_a E_a}\Big[\int d\Pi_k d\Pi_i d\Pi_j\,(2\pi)^4\delta^{(4)}(\mathcal{P}_i+\mathcal{P}_j-\mathcal{P}_a-\mathcal{P}_k)\,|M|^2_{ij\to ak}\,f_i f_j(1+f_a)(1\pm f_k)$$
$$-\int d\Pi_k d\Pi_i d\Pi_j\,(2\pi)^4\delta^{(4)}(\mathcal{P}_i+\mathcal{P}_j-\mathcal{P}_a-\mathcal{P}_k)\,|M|^2_{ij\leftarrow ak}\,f_a f_k(1-f_j)(1-f_i)\Big]$$

Simplifies to

$$C[f_a] = \frac{1}{2g_a E_a}\left(1-\frac{f_a}{f_a^{\rm eq}}\right)\gamma_{\rm ann}$$

$$\gamma_{\rm ann} = \int d\Pi_k d\Pi_i d\Pi_j\,(2\pi)^4\delta^{(4)}(\mathcal{P}_i+\mathcal{P}_j-\mathcal{P}_a-\mathcal{P}_k)\,|M|^2_{ij\to ak}\,f_i^{\rm eq} f_j^{\rm eq}(1\pm f_k^{\rm eq})$$

Collision term for i + j → k + a

General expression for the differential rate

$$\gamma_{ij\to ak} = \frac{1}{p_a}\int dE_k\,\frac{(1\pm f_k(E_k))}{16\,(2\pi)^4}\int\frac{ds}{p_k^*\sqrt{s}}\int dt\,|\mathcal{M}|^2\int d\cos\phi\,\frac{f_i^*\cdot f_j^*}{\sqrt{1-\cos\phi^2}}$$

computed using **CollCalc** (our own designed code)

Maxwell-Boltzmann approximation

$$\gamma_{ij\to ak} = \frac{\exp(-q)}{(2\pi)^2}\int dE_k\;E_k\,f_k(E_k)\int ds\;\sigma_{ak\to ij}(s)\,v_{\mathrm{Mol}}$$

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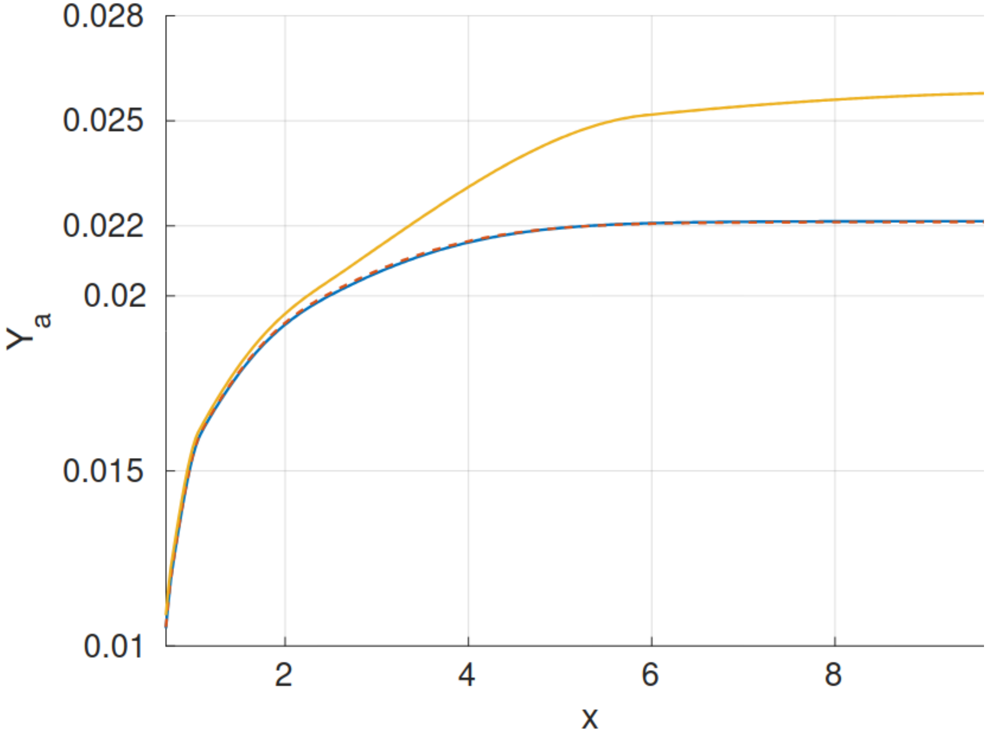

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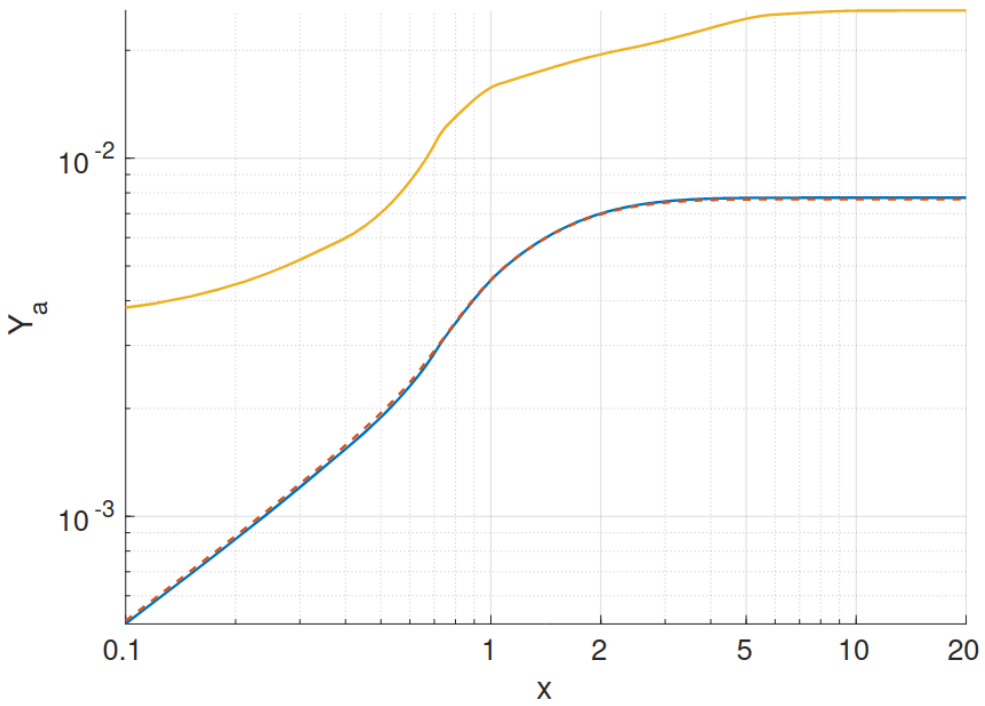

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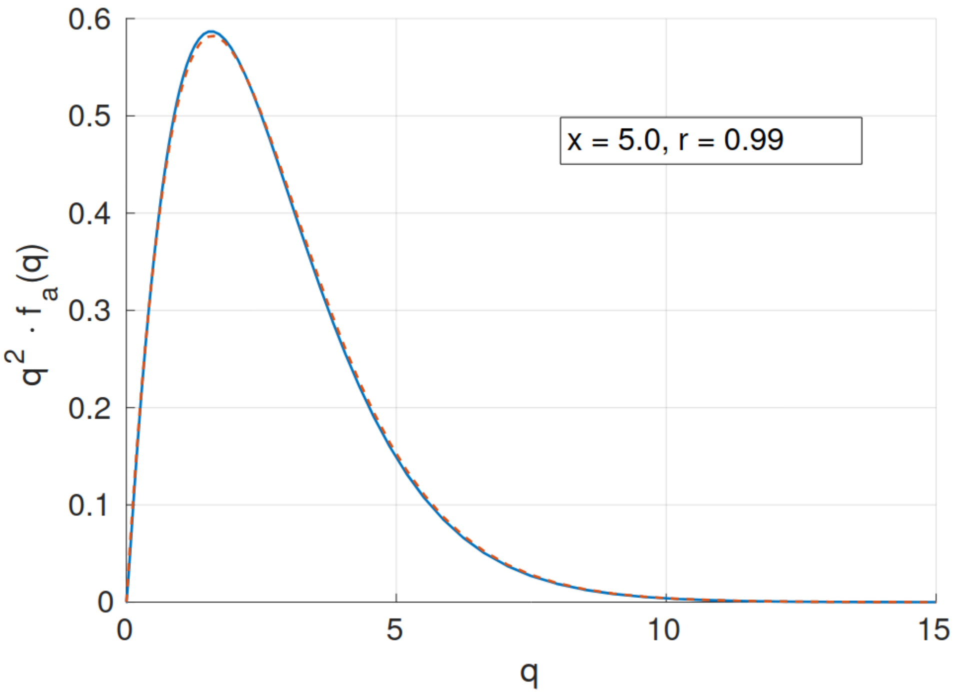

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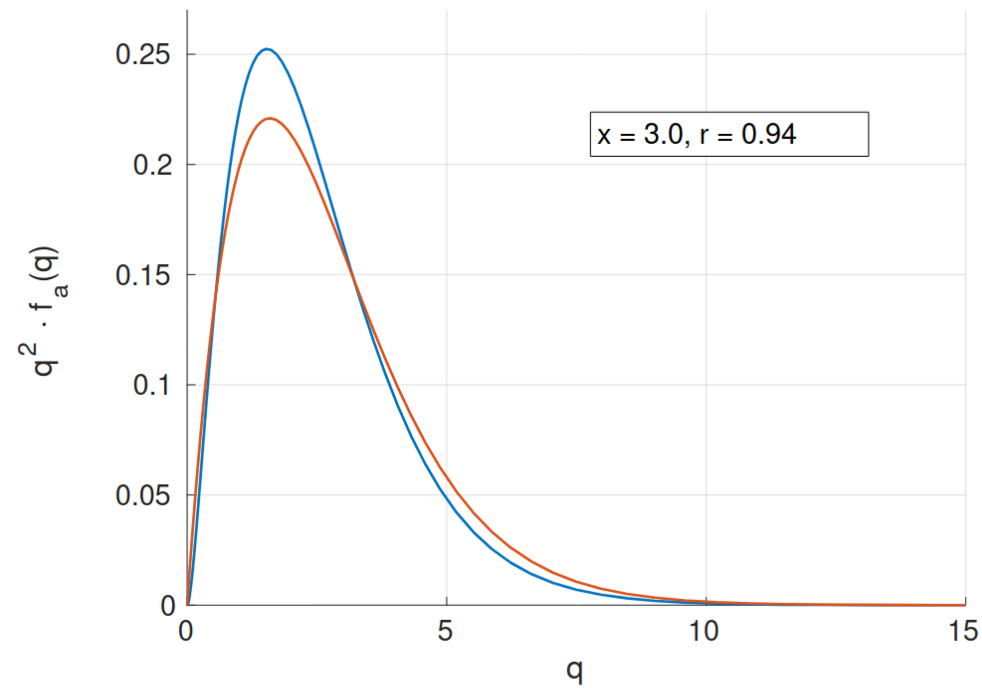

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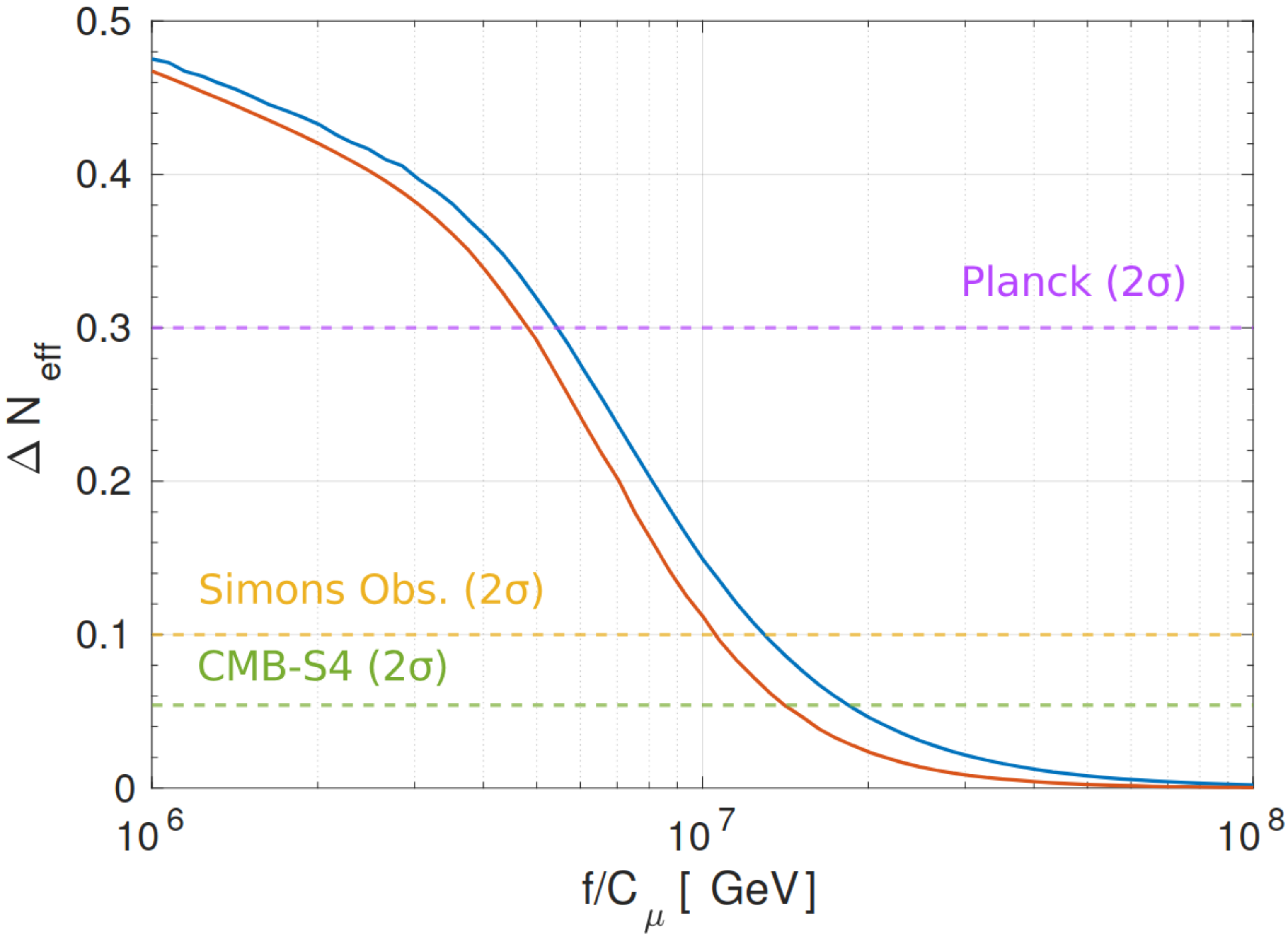
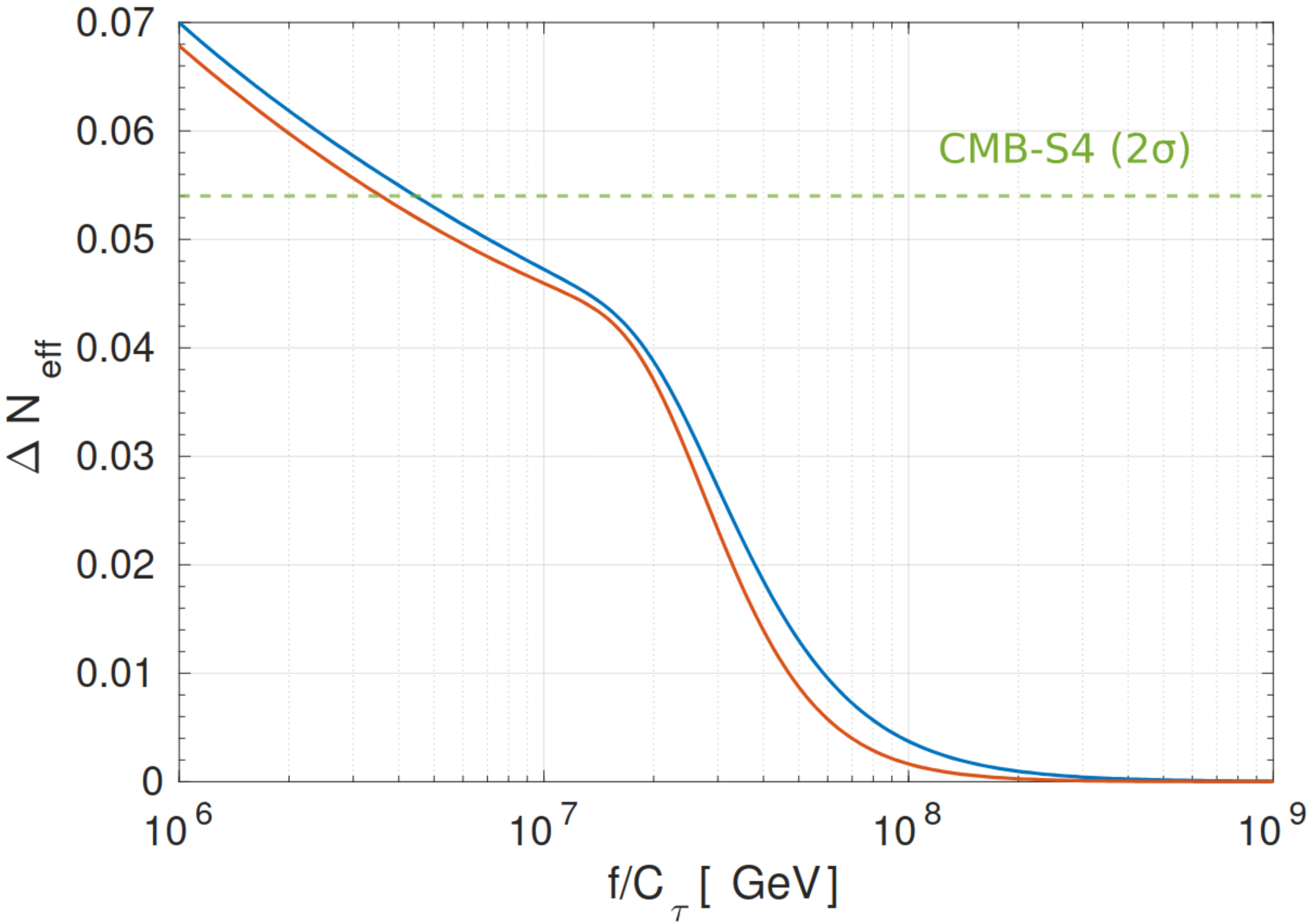
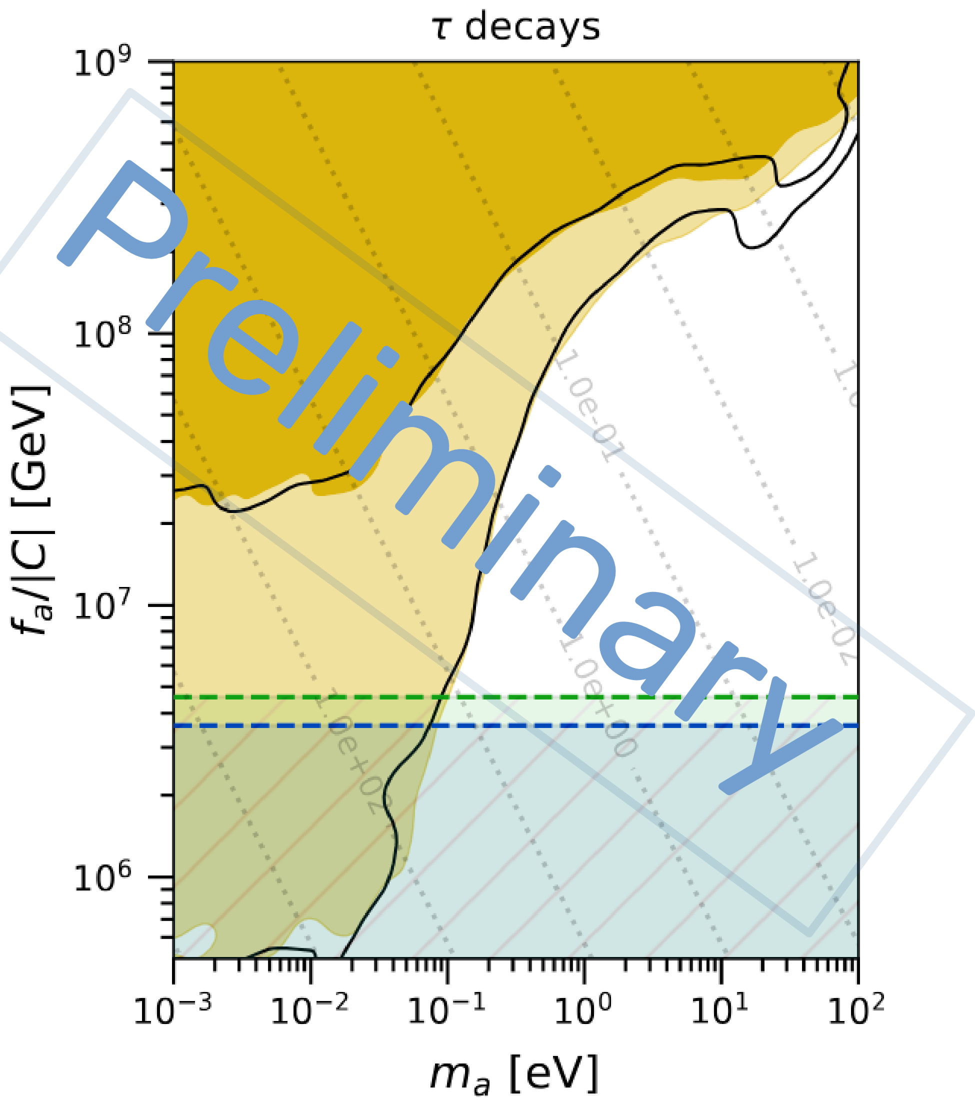


Difference in the ΔN_{eff} (tau scatterings)



Future prospects



Full cosmological scan of axion-lepton models using CLASS/Monte-Python

- Includes the impact of axion mass around recombination
- More precise constraints

Constraints on $f_a/|C|$ vs. m_a for axions produced via tau decays

*In progress with **M. Badziak, A. Gomulka** and **K. Szafranski***

Similar studies

- A. Notari, F. Rompineve, and G. Villadoro, "*Improved Hot Dark Matter Bound on the QCD Axion*", Phys. Rev. Lett. 131 (2023), no. 1 011004 [2211.03799]

  The impact of axion-pion scatterings on ΔN_{eff} using momentum-dependent calculation

- K. Bouzoud and J. Ghiglieri, *"Thermal axion production at hard and soft momenta"*, JHEP 01 (2025) 163 [2404.06113]

  Axion-gluon scatterings above QCD transition

- F. D'Eramo and A. Lenoci, "*Back to the phase space: thermal axion dark radiation via couplings to standard model fermions*", Phys.Rev.D 110 (2024) 11, 116028 [2410.21253]

  Axion-lepton and axion-quark scatterings (LF conserving)

Numerical packages

We are actively developing numerical packages to solve the full phase-space Boltzmann equation (fBE) in the early Universe

- **PyBolt** (with A. Gomulka and M. Lukawski)

  Solves fBE and nBE for a given model and interaction processes (in Python)

  https://github.com/Maxim-Laletin/PyBolt

- **CollCalc** (with K. Szafranski)

  Calculates collision integrals in full generality for annihilation and co-annihilation processes (in C++)

  https://github.com/Maxim-Laletin/CollCalc

Conclusion

- Axion lepton-flavour conserving and violating interactions can be **probed** and constrained by cosmological observations, competing againts the laboratory and astrophysical tests

- We revisit the axion production using a more general momentum-dependent approach to **recalculate the constraints** on the models with axion interactions

- The approach we consider is important for the studies of thermal axions and other BSM particles