Large Neutrino Masses in Cosmology:
DARK SECTOR TO THE RESCUE

Drona Vatsyayan
28th May 2025, PLANCK 2025, Padova

Based on:
JCAP 04 (2025) 054 [Cristina Benso, Thomas Schwetz, **DV**]
arXiv: 2410.23926

CSIC · Universitat de València · IFIC Institut de Física Corpuscular · Generalitat Valenciana · Gen—T

Dark Sectors?

DM vs. Visible Sector

Particle DM

Only **one** DM state makes up the entire dark sector

DM χ $\mathbf{Z}_2$ / $U(1)_{dark}$

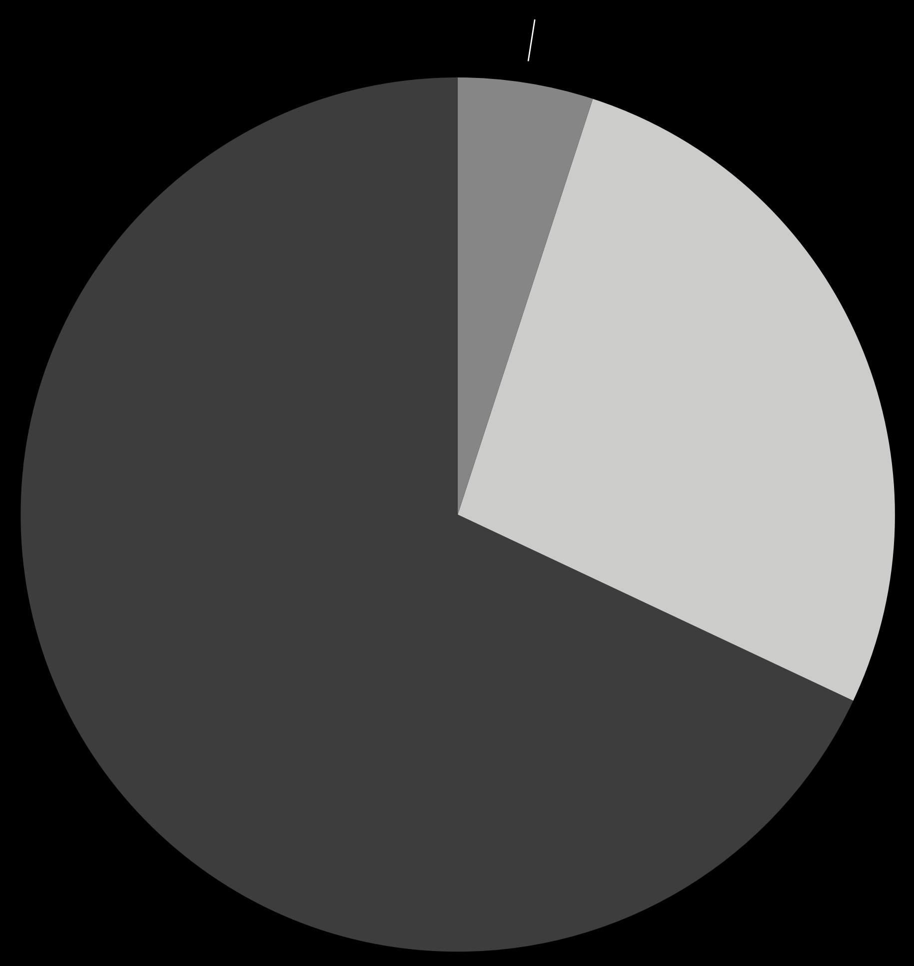
DM states are **symmetric,** abundance set by freeze-out/freeze-in

Visible Sector

Several stable components - stabilising symmetries

Electrons	Electric charge
Protons	Baryon number
Neutrinos	Spin
Photons	Poincare

Abundance set by the **baryon asymmetry** of the universe $\eta_B \sim 10^{-10}$



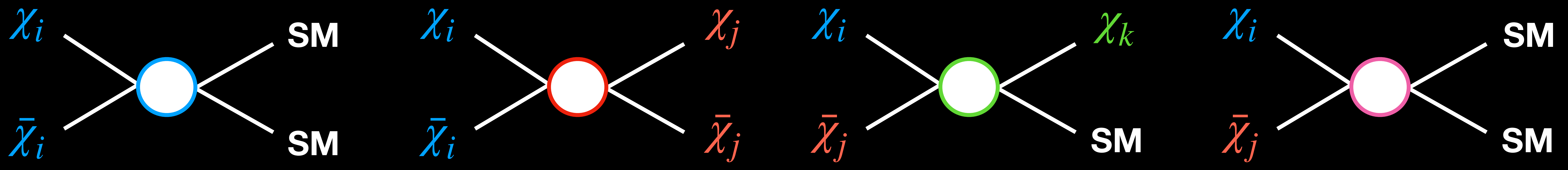
$\rho_{DM} = 5\rho_B$

Why should there be just one particle on the dark side?

Multi-component Dark Sectors

Generic features

Many degrees of freedom → e.g. ***N***-component dark sector, may have the following interactions:



Annihilations | **Conversions** $m_i > m_j$ | **Semi-annihilations** $N \geq 2$ | Co-**annihilations**

Complex system → behaviour characterised by power laws and exponentials

A. Bas, J. Herrero-Garcia, **DV**
JHEP 10 (2022) 075

Relaxation of existing bounds on direct/indirect detection, collider searches, etc.

Neutrino Mass Bounds

Oscillations

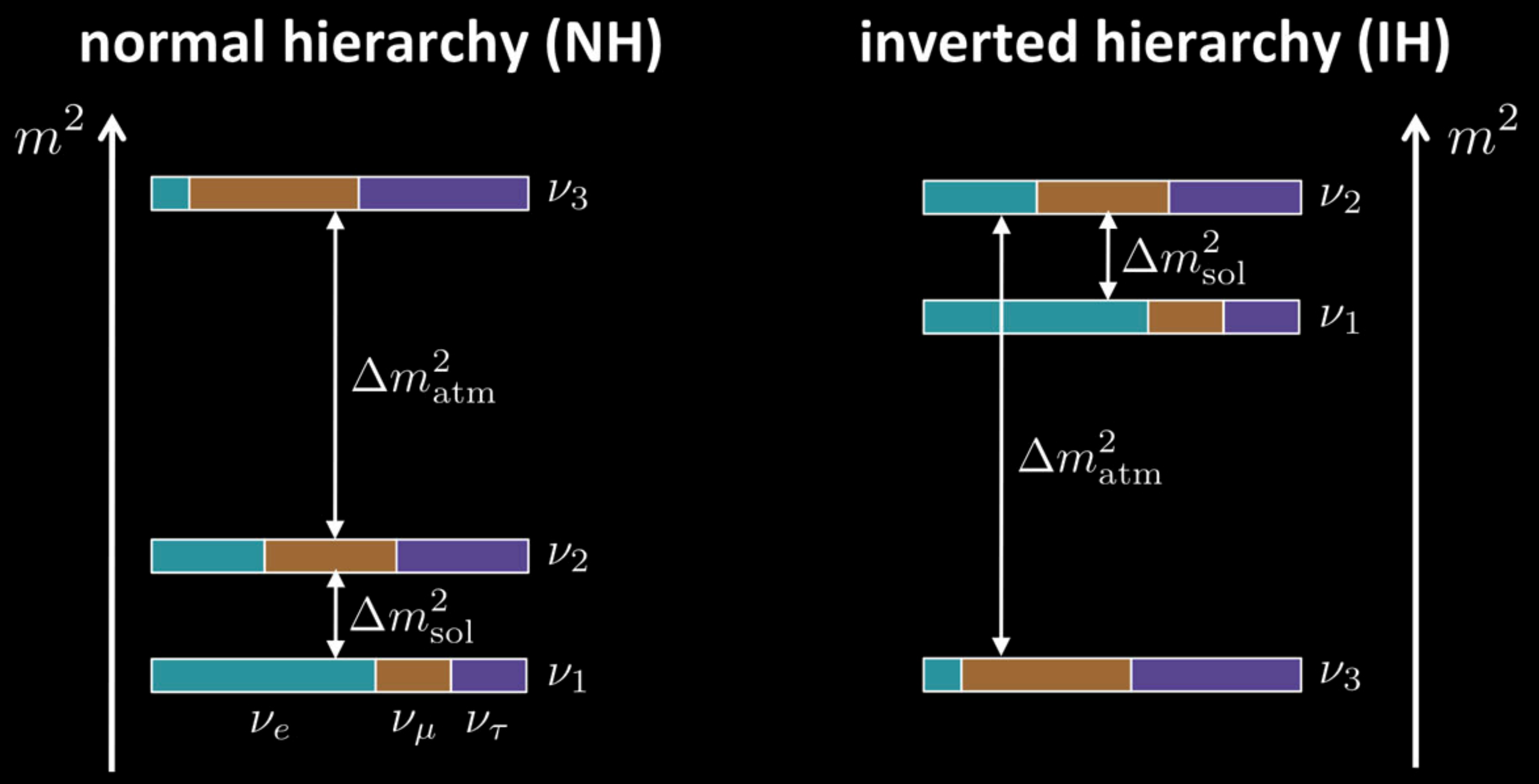
$$\sum_{i=1}^{3} m_i = \begin{cases} m_0 + \sqrt{\Delta m_{21}^2 + m_0^2} + \sqrt{\Delta m_{31}^2 + m_0^2} & \text{(NO)} \\ m_0 + \sqrt{|\Delta m_{32}^2| + m_0^2} + \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2 + m_0^2} & \text{(IO)} \end{cases}$$

$$m_0 \to 0$$

NuFit Collaboration

$$\sum m_\nu > \begin{cases} 0.058 \text{ eV} & \text{Normal ordering} \\ 0.098 \text{ eV} & \text{Inverted ordering} \end{cases}$$

95% CL



Origin of mass unknown! Absolute mass unknown!

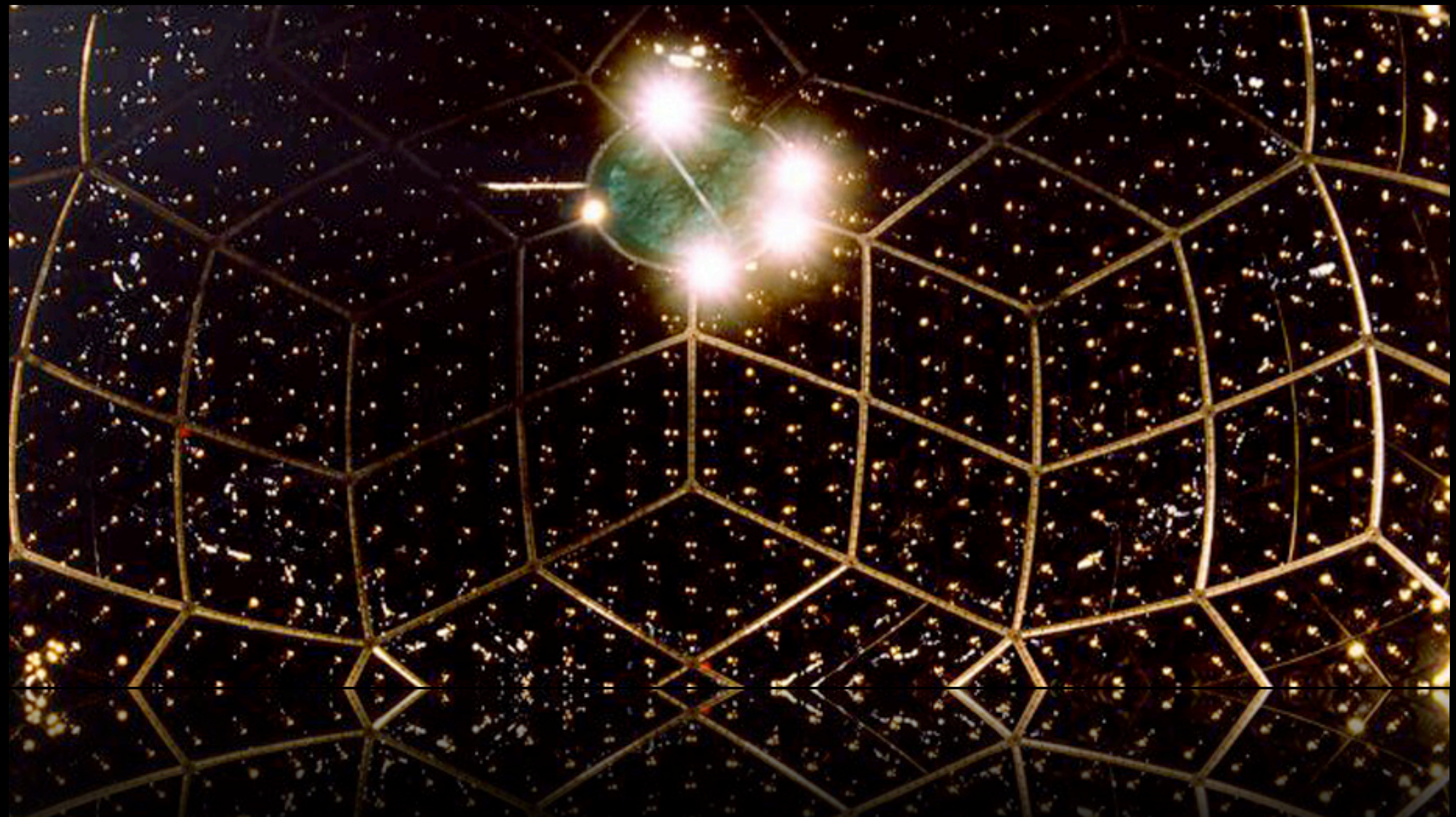
Neutrino Mass Bounds

Terrestrial Experiments

Experiments looking for absolute neutrino mass scale and $0\nu\beta\beta$

KATRIN Experiment

$\sum m_\nu \lesssim 1.35\ \text{eV}$

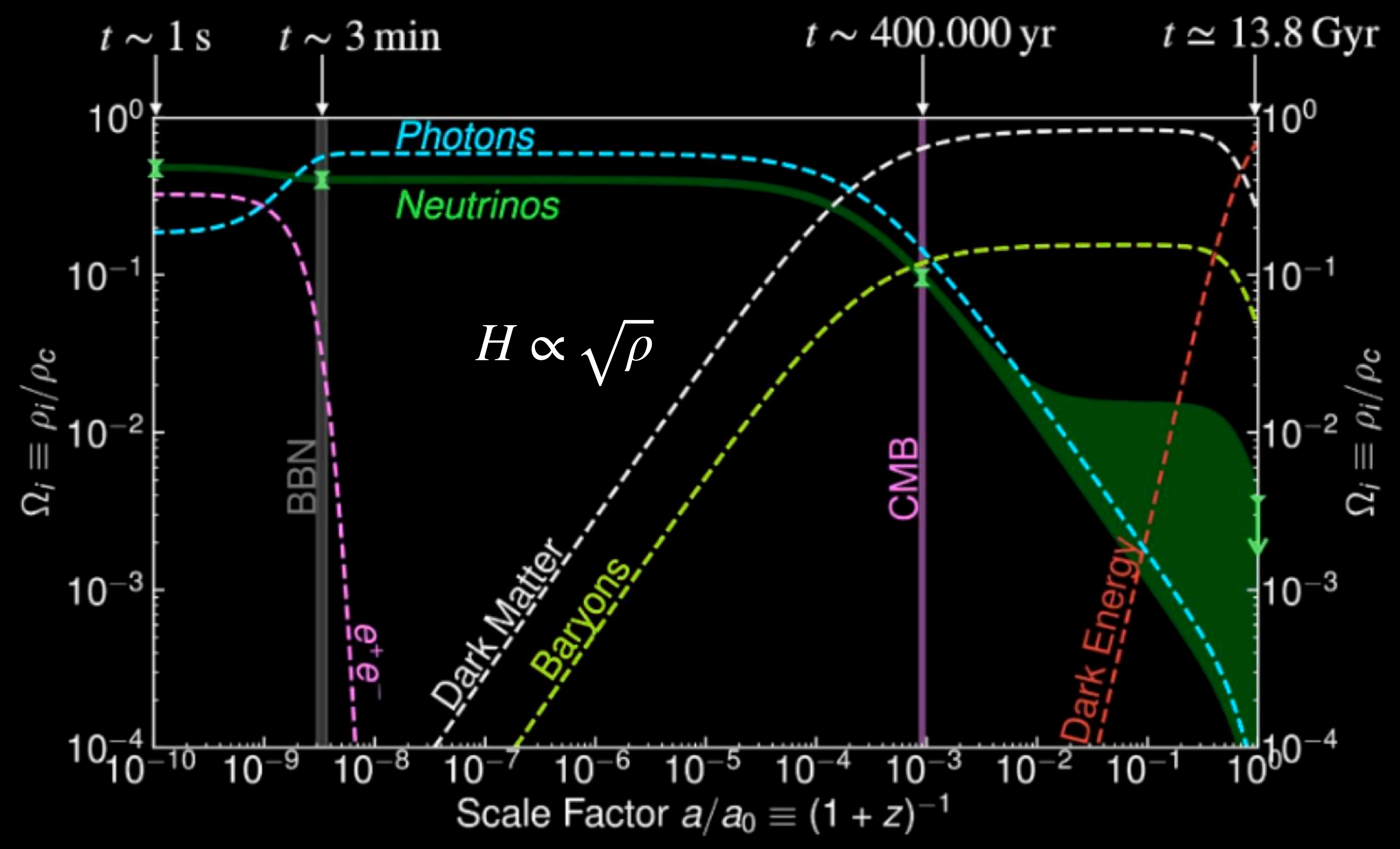


KamLAND-ZEN Experiment

$m_{\text{lightest}} < 0.084 - 0.353\ \text{eV}$

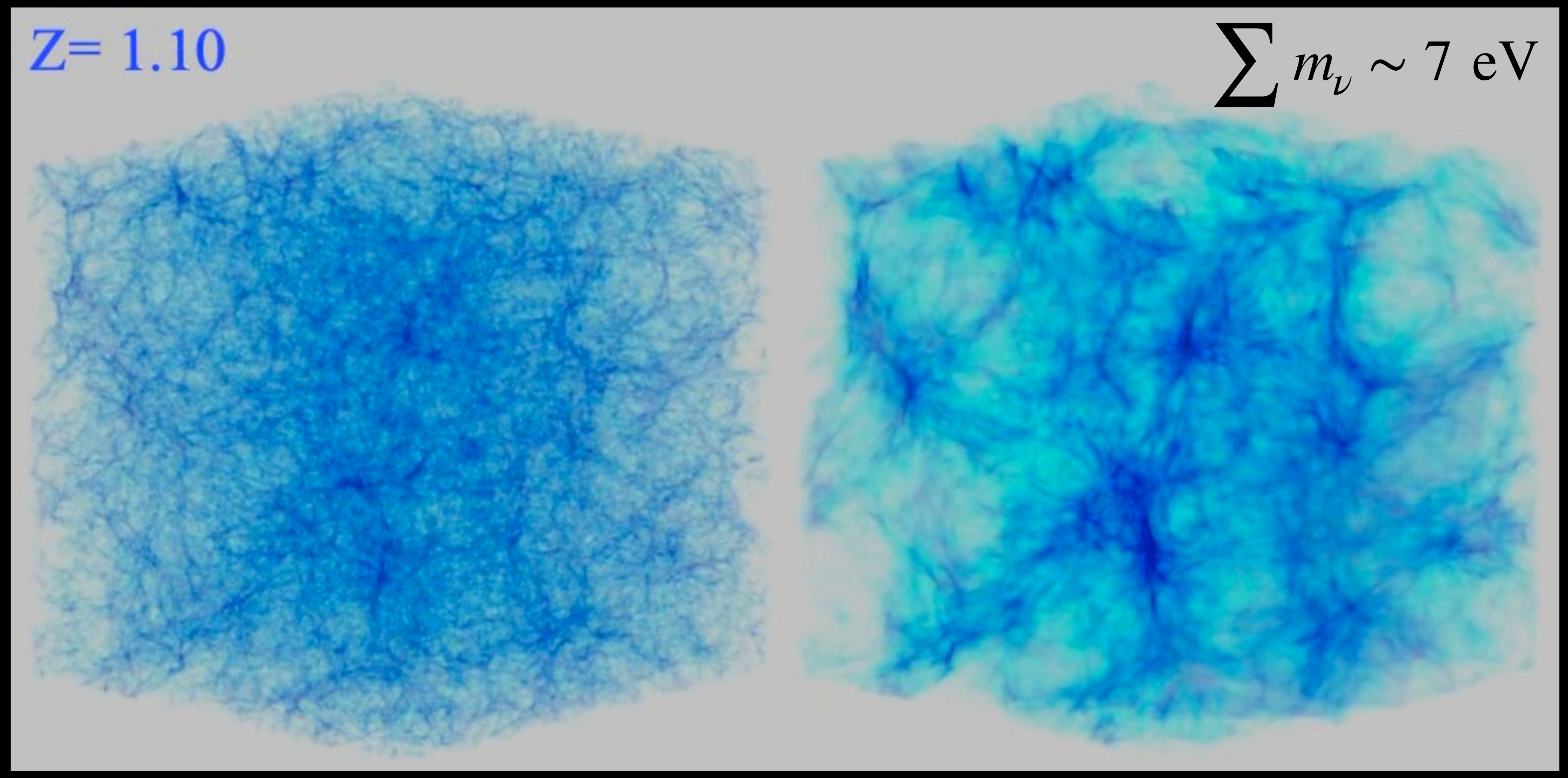
Neutrino Masses
Cosmology

Alter the expansion history of the universe near matter-radiation equality epoch



Credit: Miguel Escudero

Free-streaming affects the growth of structures at late times



Credit: Troels Haugbølle

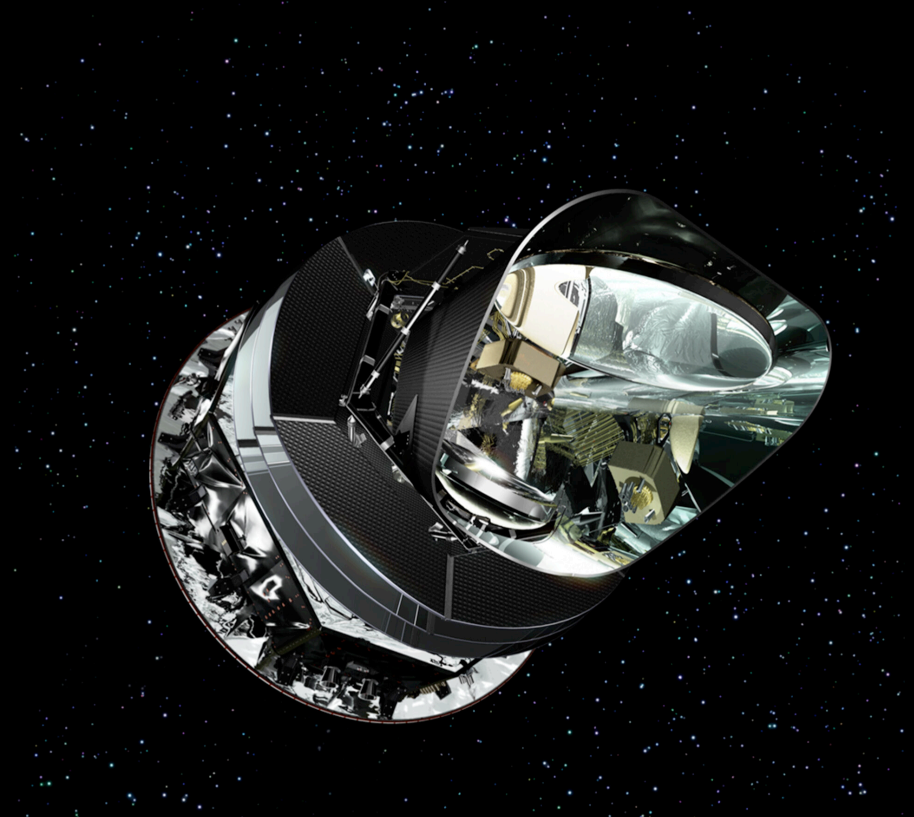
Cosmological observables → CMB + LSS

Neutrino Mass Bounds

Cosmology

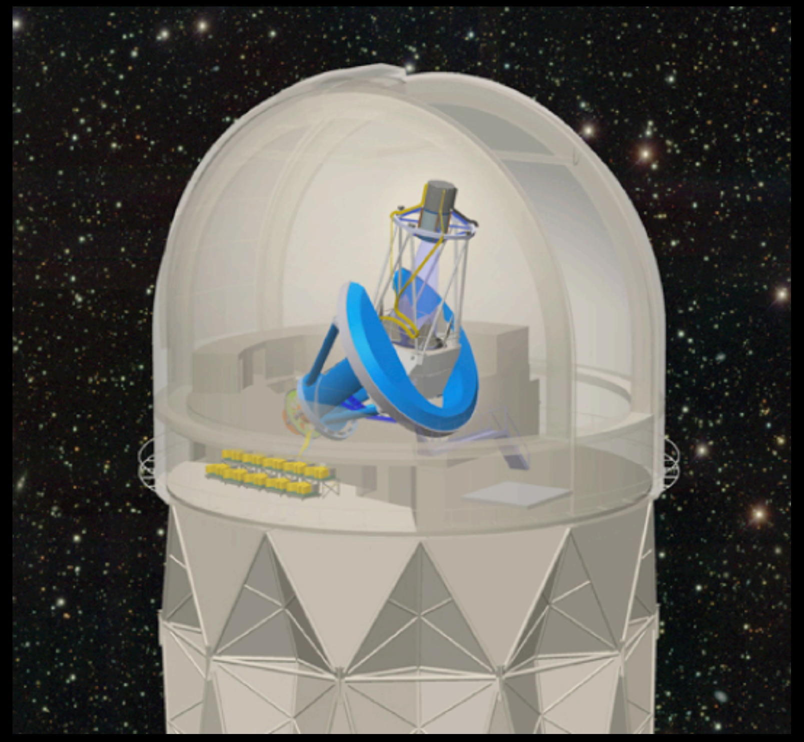
$m_\nu \neq 0 \rightarrow$ Cosmological Implications $\rightarrow$ Suppression of growth of small scale structures; Affect on CMB anisotropies

$\sum m_\nu \equiv \sum_{i=1}^{3} m_i$ can be constrained from cosmological surveys



PLANCK CMB+BAO (2018)

$\sum m_\nu < 0.12$ eV



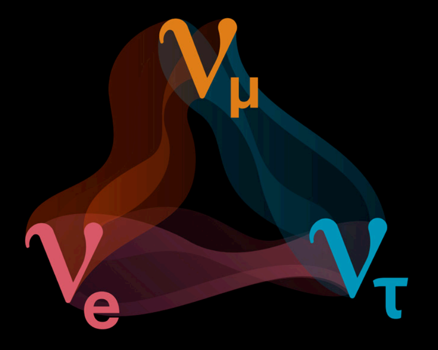
DESI 2025 + CMB

$\sum m_\nu < 0.064$ eV

Neutrino mass bounds from cosmology keep getting stronger

Neutrino Mass Bounds

Oscillations vs. Cosmology



Oscillations

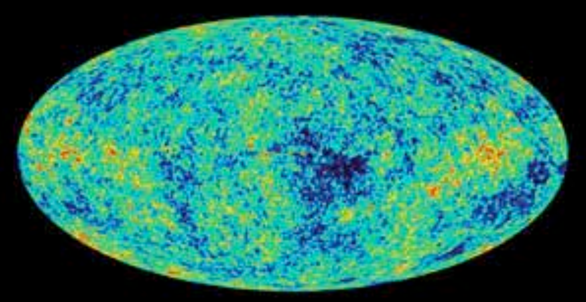
NuFit Collaboration

$$\sum m_\nu > \begin{cases} 0.058 \text{ eV} & \text{Normal ordering} \\ 0.098 \text{ eV} & \text{Inverted ordering} \end{cases}$$

95% CL

$$\sum m_\nu < \begin{cases} 0.12 \text{ eV} & \text{PLANCK CMB+BAO (2018)} \\ 0.064 \text{ eV} & \text{DESI 2025 + CMB} \end{cases}$$

95% CL



Cosmology

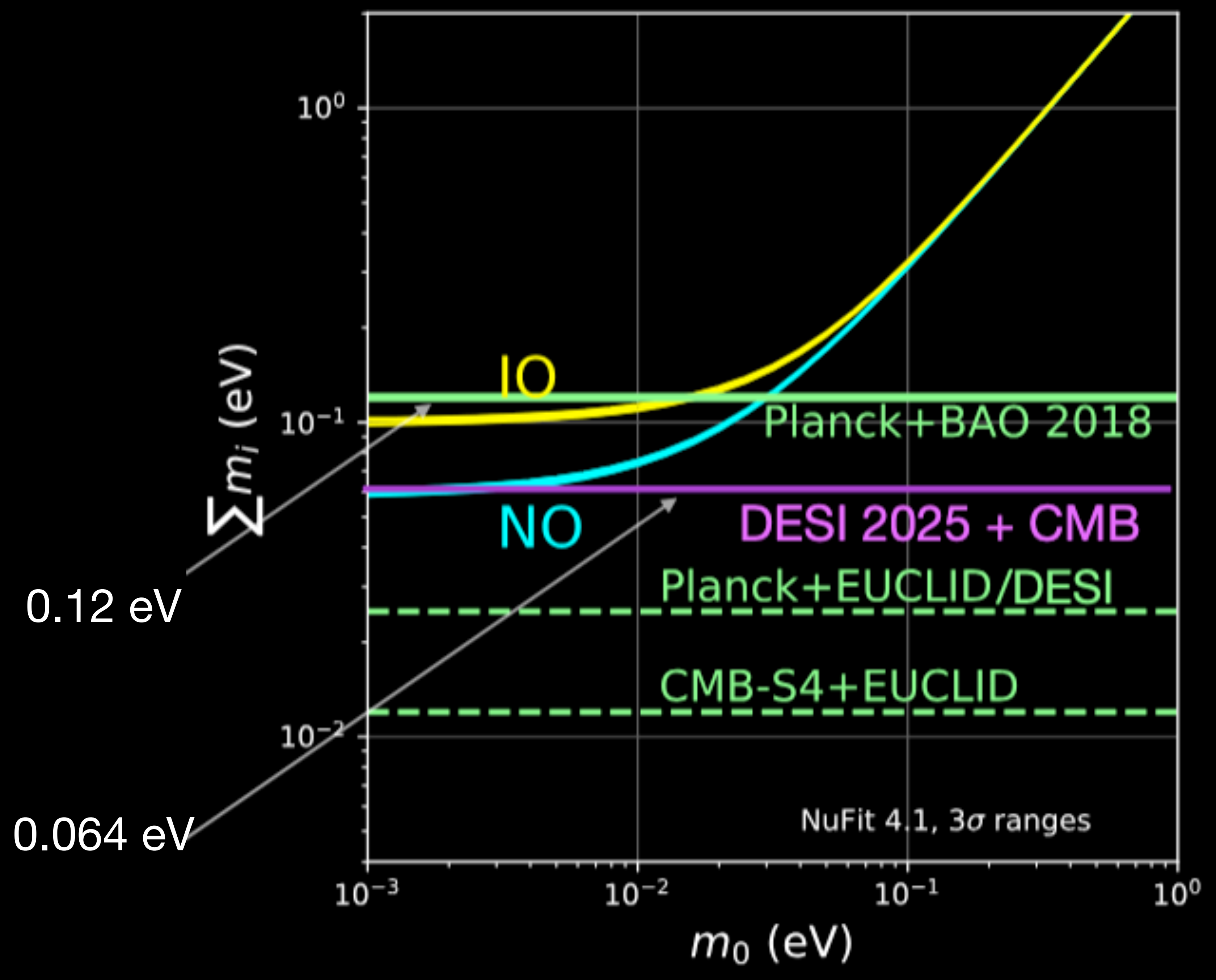
DESI bound is in significant tension with IO, very close to NO

2-zero neutrino mass textures $\rightarrow \sum m_\nu > 0.12$ eV

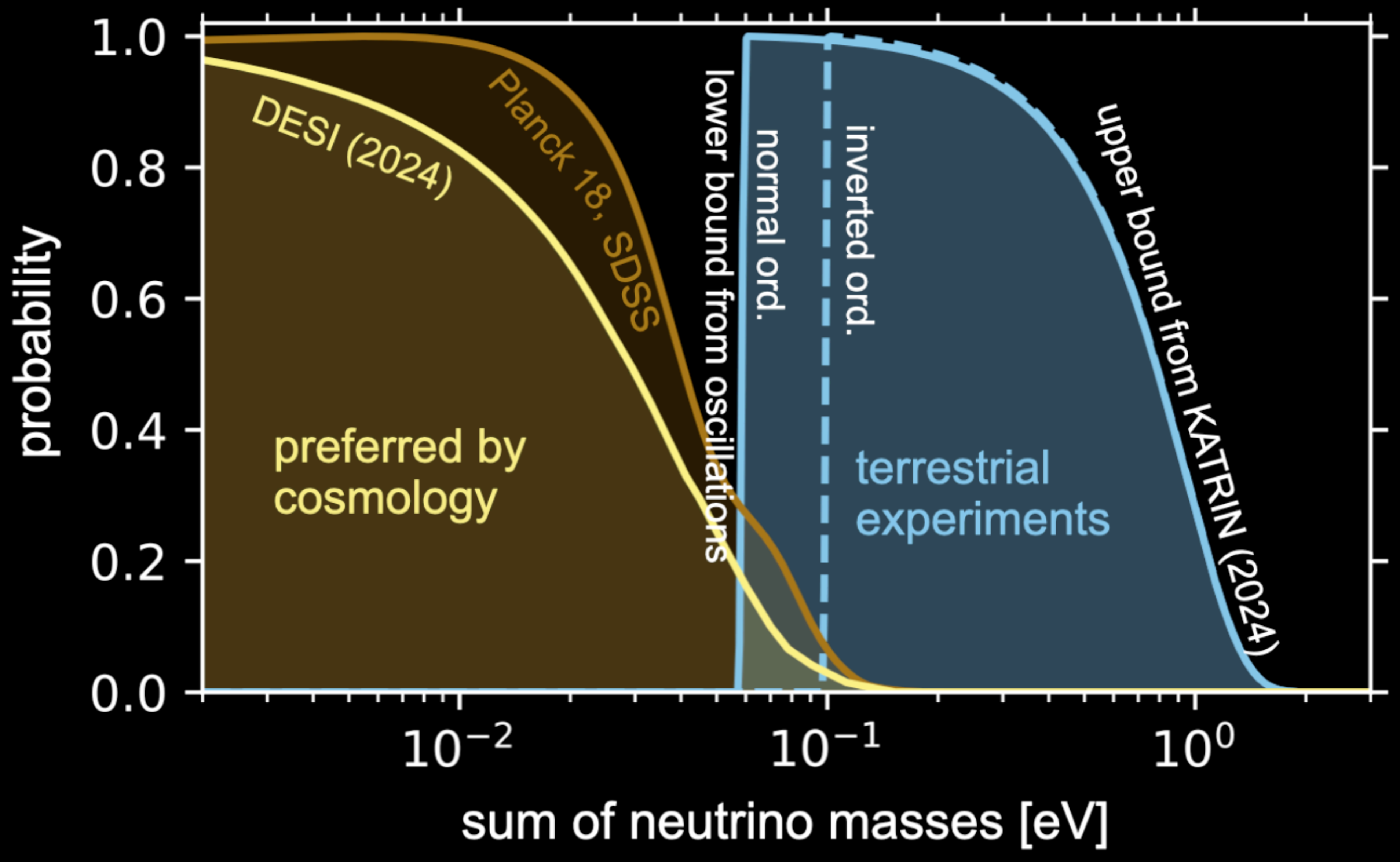
Alcaide et al: 1806.06785;
Lattanzi et al: 2007.01650

Neutrino Mass Bounds

Laboratory vs. Cosmology



Courtesy: Thomas Schwetz (Durham 2025); updated from 2302.14159



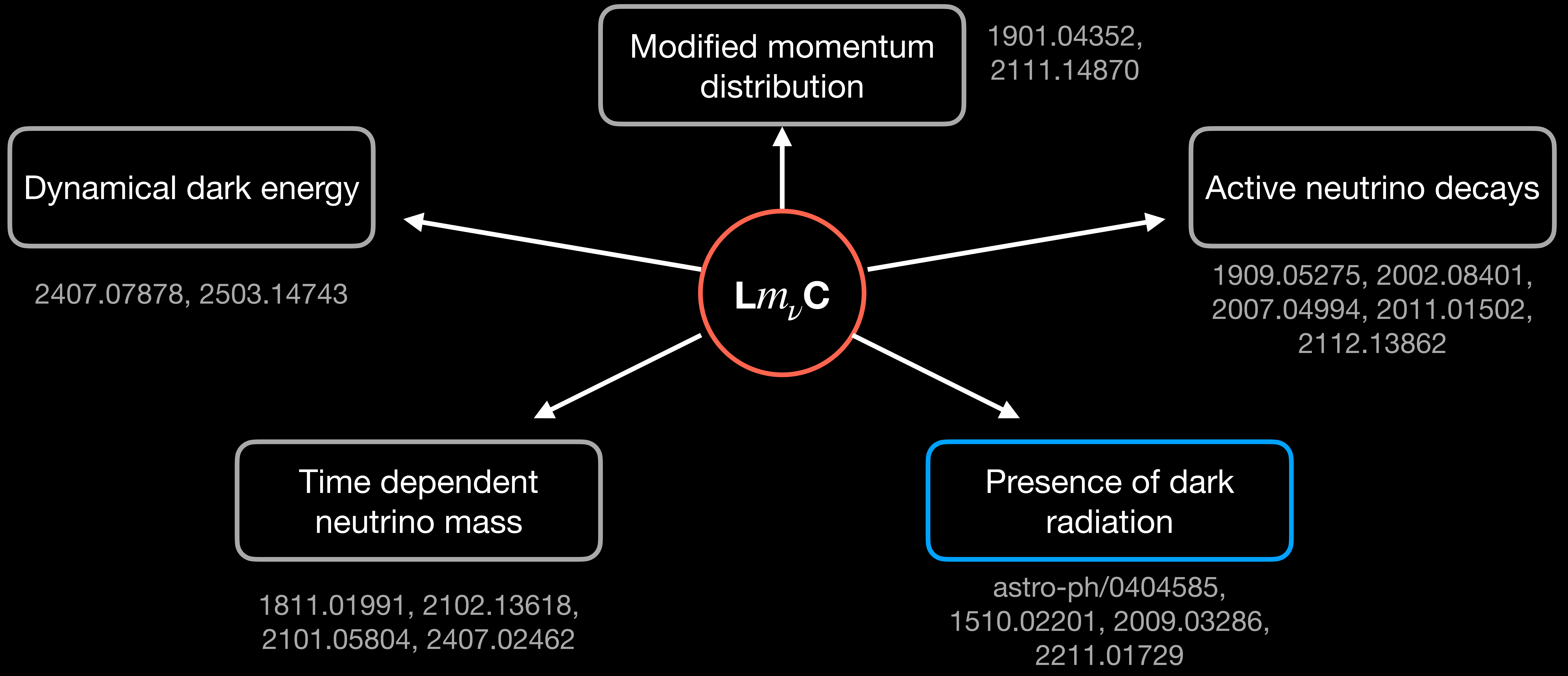
Standard cosmological scenario → We may not observe finite absolute neutrino mass in the laboratory

Can the two be reconciled? Can cosmological bounds be relaxed?

Relaxing the Cosmological ν mass bound

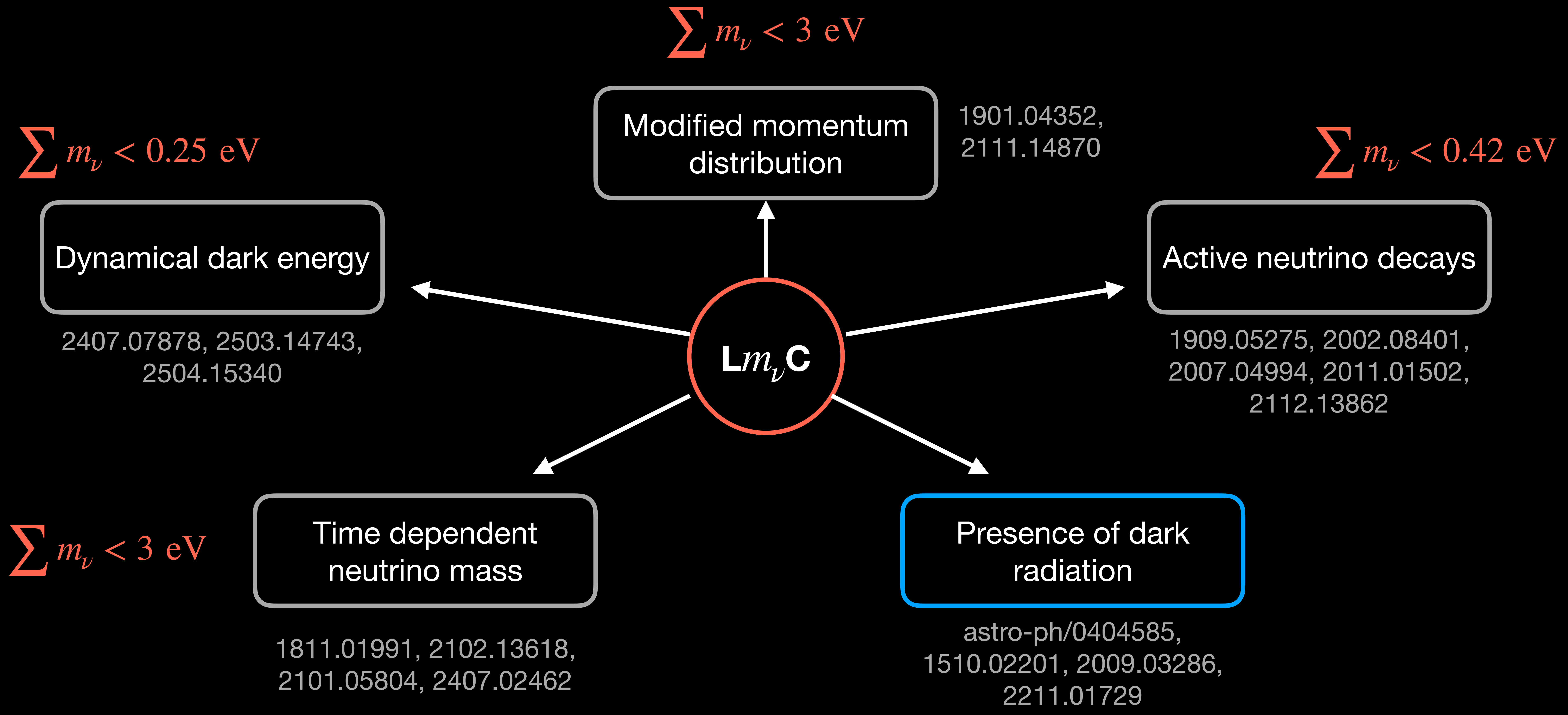
Large ν mass cosmology

Relaxing the cosmological bound requires non-standard scenarios → Large ν mass cosmologies



Relaxing the Cosmological ν mass bound

Large ν mass cosmology



Large m_ν Cosmology
Presence of dark radiation

Cosmological bounds are sensitive to neutrino energy density

$$\Omega_\nu h^2 \equiv \frac{\sum m_\nu n_\nu^0 h^2}{\rho_{\text{critical}}} < 1.3 \times 10^{-3} \ (95\,\%\ \text{CL}) \longrightarrow \sum m_\nu \times \left(\frac{n_\nu^0}{56\ \text{cm}^{-3}}\right) < 0.12\ \text{eV}\ (95\,\%\ \text{CL})$$

PLANCK 2018

Reduce number density of neutrinos → Mass bound can be relaxed

At earlier times for ultra-relativistic νs: Energy density characterised by $N_{\text{eff}} \propto \langle p_\nu \rangle n_\nu$

- 2.99 ± 0.17 PLANCK 2018
- $3.044(1)$ SM prediction

$$N_{\text{eff}} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\text{rad}} - \rho_\gamma}{\rho_\gamma}\right)$$

Compensate decrease in n_ν : Add new light/massless d.o.f → Dark radiation

Large m_ν Cosmology
Presence of dark radiation

BBN ⟷ CMB

Introduce a mechanism to reduce n_ν

Add BSM states: X, χs that thermalise with νs

$100\ \mathrm{keV} \gtrsim T_\gamma \gtrsim 10\ \mathrm{eV}$

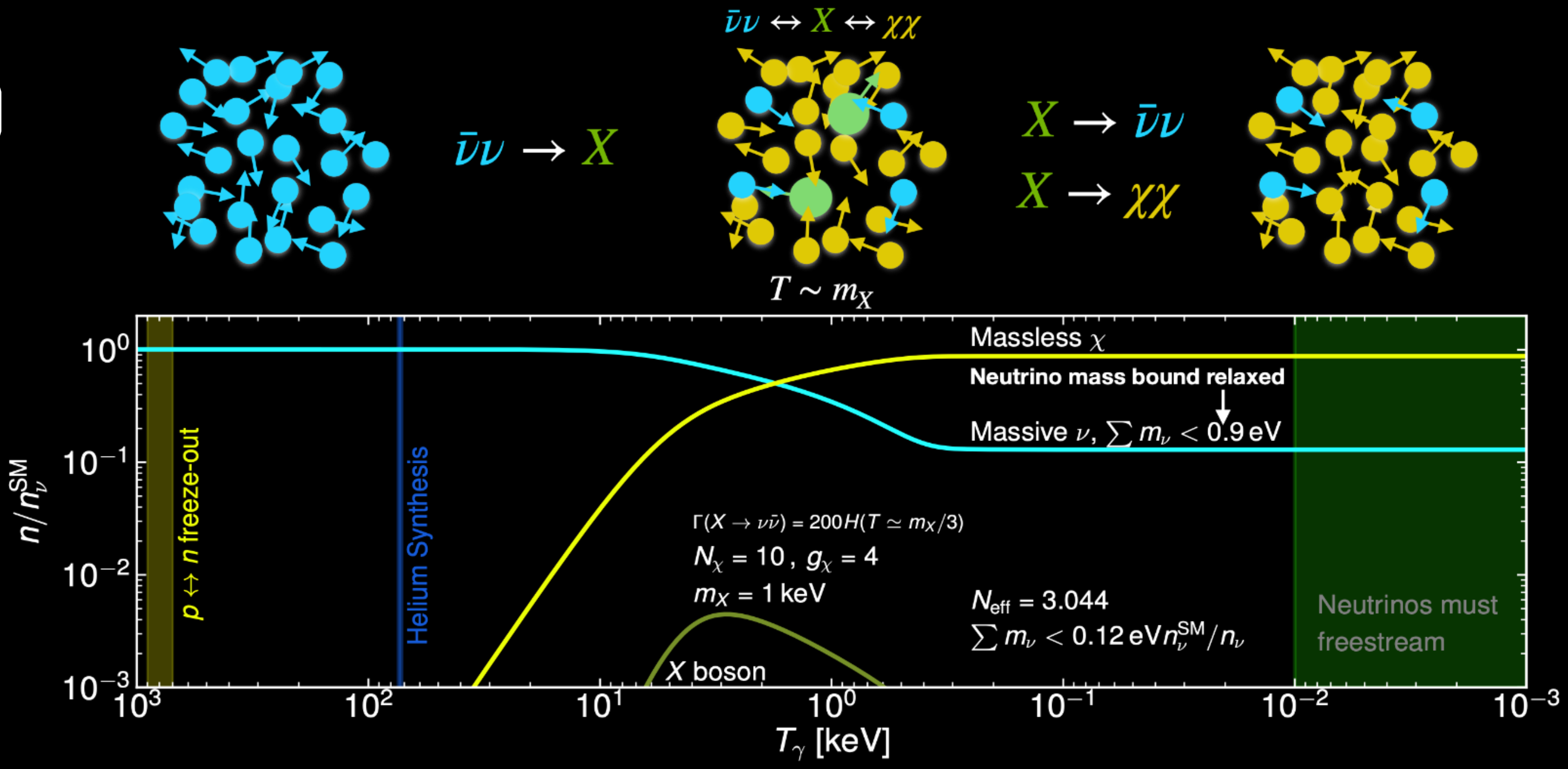
N_{eff} unchanged at earlier times

Post $\nu-$decoupling ($T_\gamma \sim 2\ \mathrm{MeV}$):

Neutrinos cannot be produced anymore:
Production of new states at their expense
→ n_ν reduced

$$\left[\sum m_\nu\right]_{\mathrm{eff}} = \sum m_\nu \frac{n_\nu}{n_\nu^{\mathrm{SM}}}$$

Depends on the dark d.o.f



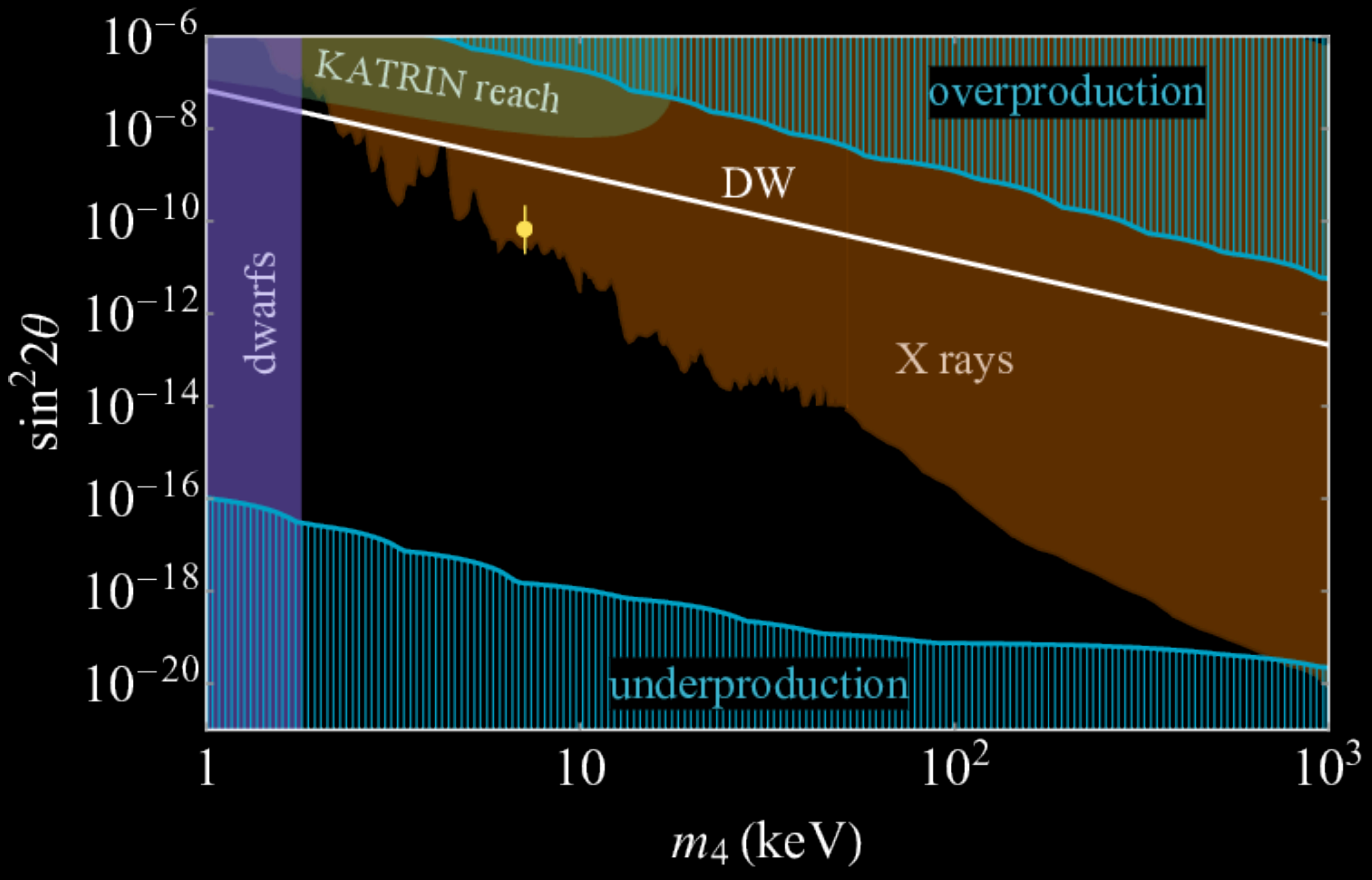
Escudero, Schwetz, Terol-Calvo: 2211.01729

Large m_ν Cosmology

Presence of Dark Matter?

de Gouvêa, Sen, Tangarife, Zhang: 1901.04901

The dark sector can be enlarged to contain a light keV fermionic DM candidate along with the dark radiation → Multi-component DS



Problem keV scale sterile neutrino (ν_s) DM from oscillations: Dodelson Widrow is severely constrained

Solution Non-standard neutrino interactions open up parameter space

Thermal DM below MeV possible if DM comes into thermal equilibrium post ν-decoupling

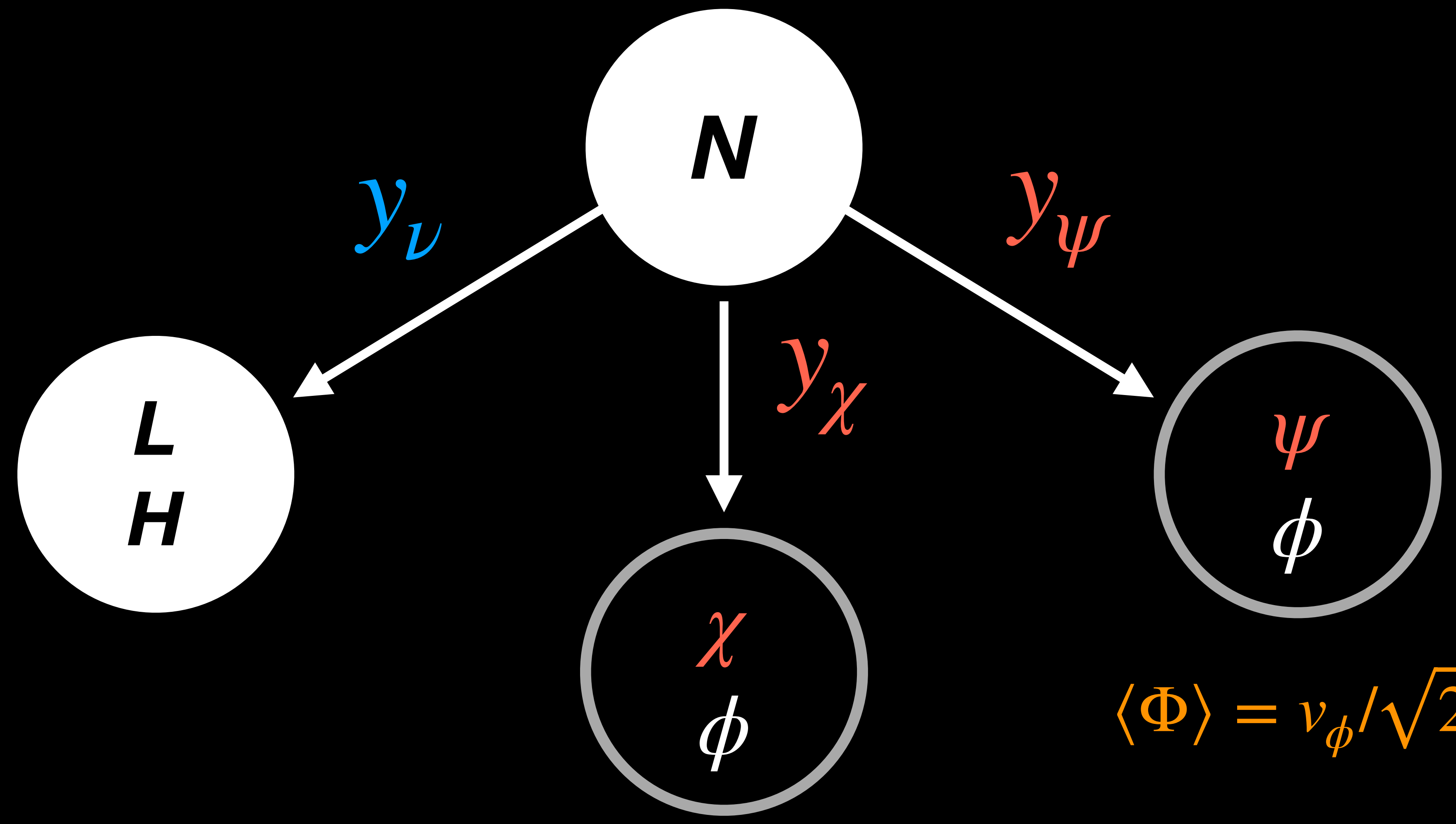
Berlin, Blinov: 1706.07046

The Model

Minimally Extended Type-I Seesaw with U(1)X

Similar to $\nu\Lambda$MDM: Ko and Tang: 1404.0236

	Field	Species	$U(1)_X$
Scalar	Φ	1	$+1$
Fermions	χ	N_χ	-1
	ψ	1	-1
	N	3	0
	N'	1	0



$\langle\Phi\rangle = v_\phi/\sqrt{2}$

+N′

$$-\mathscr{L}_{\rm new} = Y_\nu \bar{N} l_L \tilde{H}^\dagger + Y_\chi \bar{N}\chi_L\Phi + Y_\psi \bar{N}\psi_L\Phi + Y'_\nu \bar{N}' l_L \tilde{H}^\dagger + Y'_\chi \bar{N}'\chi_L\Phi + Y'_\psi \bar{N}'\psi_L\Phi + \frac{1}{2}M\bar{N}N^c + \frac{1}{2}M'\bar{N}'N'^c + {\rm H.c.}$$

$$V(H,\phi) = \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu_\phi^2 |\Phi|^2 + \lambda_\phi |\Phi|^4 + \lambda_{H\phi}|\Phi|^2 H^\dagger H$$

Gauge interaction $\quad \mathscr{L}_{\rm int} = \sum_f Q_f g Z'_\mu \bar{f}\gamma^\mu f \quad g \equiv m_{Z'}/v_\phi$

The Model

Masses & Mixings

Neutral fermion mixing matrix:
$(\chi_L^c, \nu_L^c, \psi_L^c, N', N)$ basis

$$\mathcal{M}_n = \begin{pmatrix} 0 & 0 & 0 & \Lambda' & \Lambda \\ 0 & 0 & 0 & m_D{}' & m_D \\ 0 & 0 & 0 & \kappa' & \kappa \\ \Lambda'^T & m_D^{T\prime} & \kappa'^T & M' & 0 \\ \Lambda^T & m_D^T & \kappa^T & 0 & M \end{pmatrix}$$

$Y_\chi v_\phi/\sqrt{2}$

$Y_\nu v_{\rm EW}/\sqrt{2}$

$Y'_\psi v_\phi/\sqrt{2}$

$M \gg M' \gg m_D \gg \kappa', \Lambda \gg m_D', \Lambda', \kappa$

Massive: $2N_{\rm heavy}$

Massless: $(3 + N_{\rm light} - N_{\rm heavy})$

$m_N \approx M \,, m_{N'} \approx M'$

$m_\chi = 0$ — N_χ massless fermions

$m_\nu \approx m_D M^{-1} m_D^T$

$m_\psi \approx \kappa' M'^{-1} \kappa'^T$

Seesaw induced Majorana masses

$$m_\psi \sim 10 \text{ keV} \left(\frac{\kappa'}{10^4 \text{ keV}}\right)^2 \left(\frac{10 \text{ GeV}}{M'}\right)$$

Small mixing: $\theta_{\nu\chi} = \frac{\Lambda}{m_D}$

$\theta_{\nu\psi} = \frac{m_D{}'}{\kappa'}$: Suppressed mixing

ψ becomes the DM candidate in the model

DM freeze-out in DS

Production & Depletion

$$\nu\nu \leftrightarrow Z' \leftrightarrow \chi\chi$$

DM can be produced by $Z' \leftrightarrow \psi\psi$ $(m_{Z'} > 2m_\psi)$ or $Z'Z' \leftrightarrow \psi\psi$ and $\chi\chi \leftrightarrow \psi\psi$ $(m_\psi > m_{Z'})$

Annihilations $\psi\psi \rightarrow \chi\chi$ and $\psi\psi \rightarrow Z'Z'$ freeze-out at $T_{\rm dark} < m_\psi$

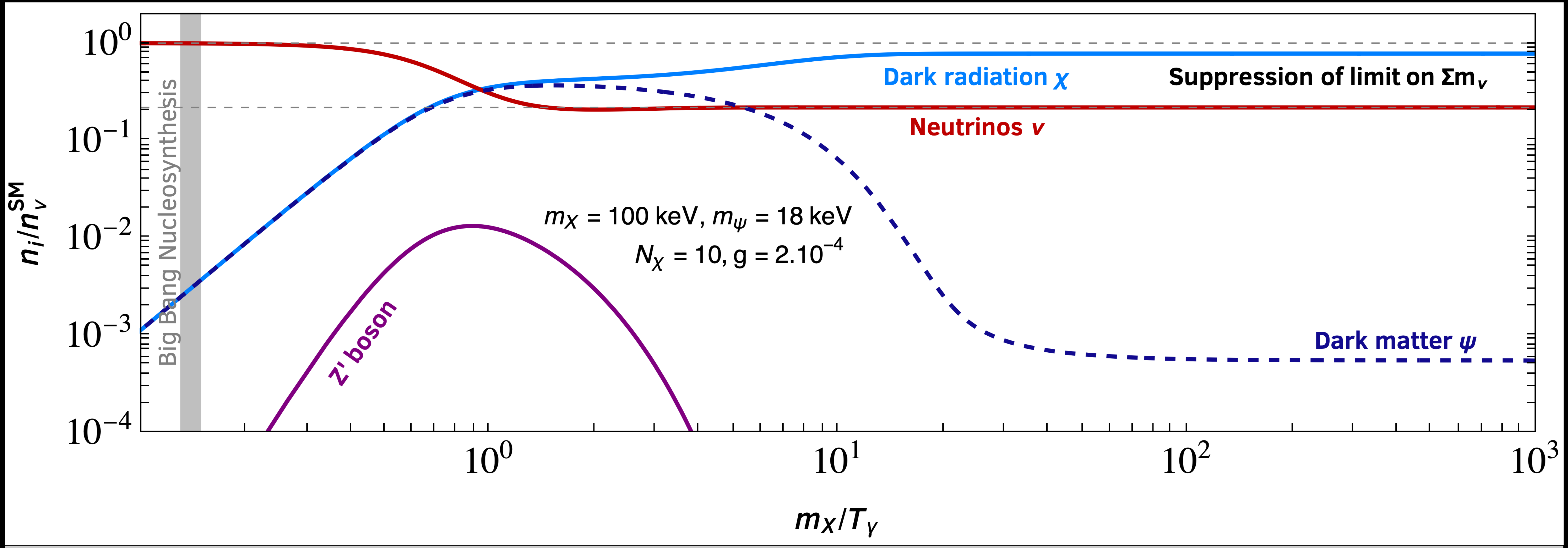
$$\mathscr{L}_{\rm int} = \sum_f Q_f g Z'_\mu \bar{f}\gamma^\mu f$$

$$\lambda_{Z'}^{\nu\nu} \simeq \frac{m_{Z'}}{v_\phi}\theta_{\nu\chi}^2$$

$$\lambda_{Z'}^{\nu\chi} = \frac{m_{Z'}}{v_\phi}\theta_{\nu\chi}$$

$$\lambda_{Z'}^{\psi\psi} = \lambda_{Z'}^{\chi\chi} = \frac{m_{Z'}}{v_\phi}$$

$$\lambda_{Z'}^{\nu\psi} = \frac{m_{Z'}}{v_\phi}\theta_{\nu\psi}$$



DM freeze-out in DS

Relic abundance

ψ comes into thermal equilibrium with the DS and finally freezes out

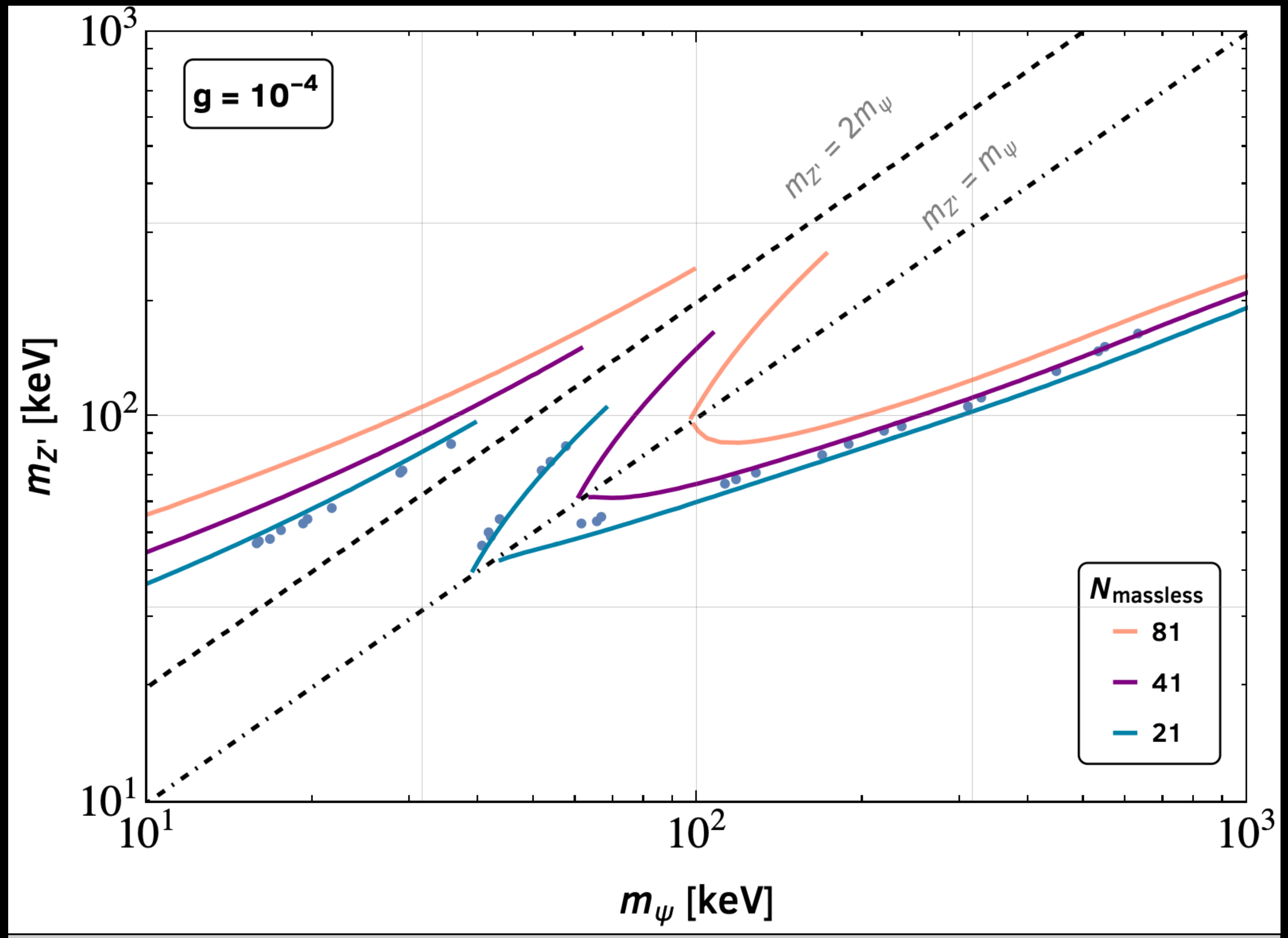
$$\Omega_\psi h^2 \simeq x_f \frac{10^{-10}\,\mathrm{GeV}^{-2}}{\langle \sigma v \rangle_{\rm tot}}$$

Depends on DS temperature

$$(\sigma v)_{\psi\psi\to\chi\chi} \approx \tilde{N} \frac{g^4}{48\pi} \frac{m_\psi^2}{(m_{Z'}^2 - 4m_\psi^2)^2} v^2$$

$$(\sigma v)_{\psi\psi\to Z'Z'} \approx \frac{g^4}{16\pi\, m_\psi^2} \left(1 - \frac{m_{Z'}^2}{m_\psi^2}\right)^{1/2} \left(1 + \frac{m_\psi^4}{m_{Z'}^4} v^2\right)$$

Extreme limits: $m_{Z'} \gg m_\psi$ or vice versa $\to (\sigma v) \propto v_\phi^{-4}$



$$v_\phi \simeq 10^5\,\mathrm{keV} \left(\frac{m_\psi}{15\,\mathrm{keV}}\right)^{1/2} \left(\frac{3.2}{x_f}\right)^{1/2} \times \begin{cases} 2\tilde{N}^{1/4} & (m_{Z'} \gg m_\psi) \\ 2.4 & (m_{Z'} \ll m_\psi) \end{cases}$$

Constraints

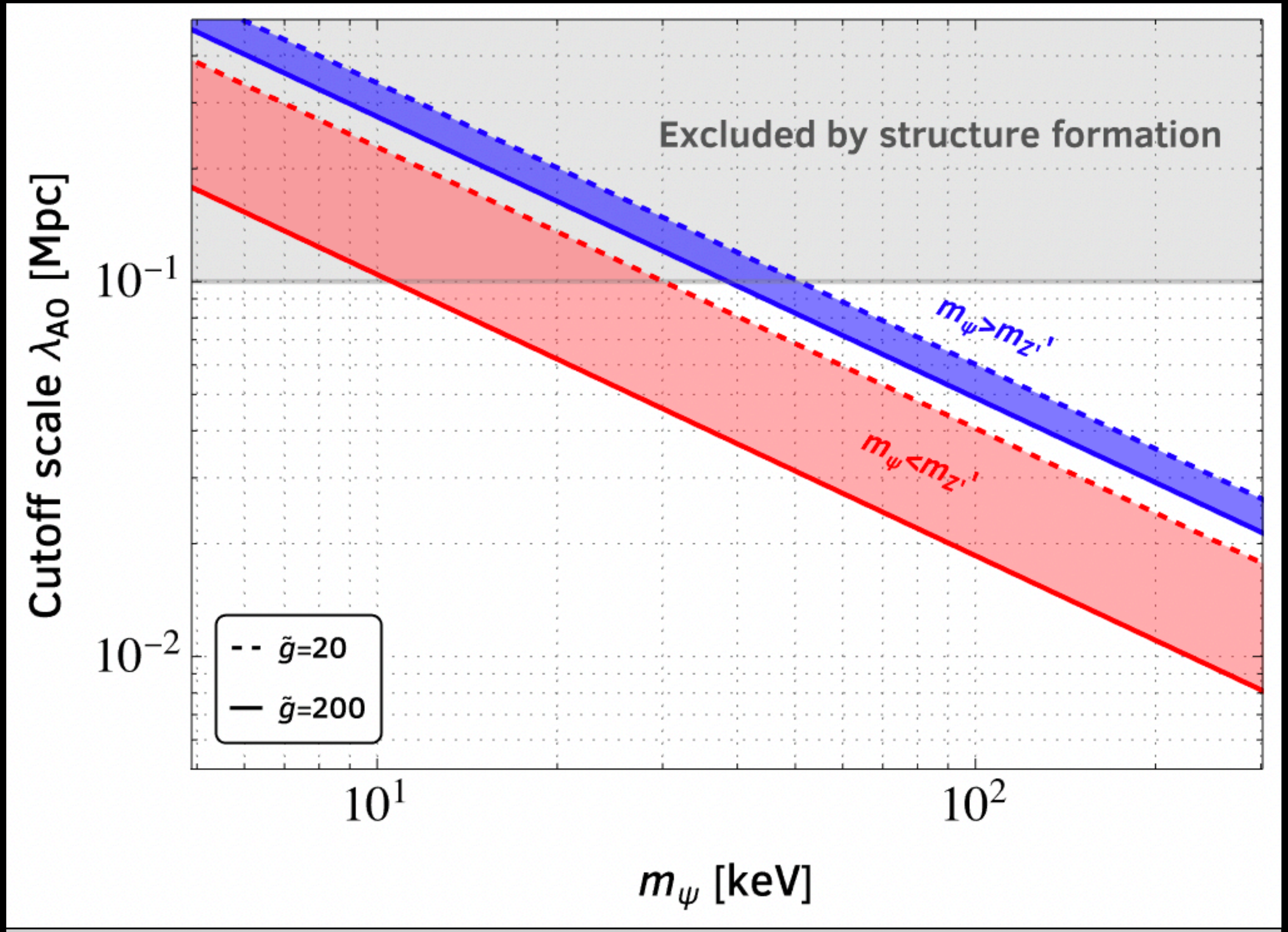
Structure formation

Potentially large free-streaming scale →
Prevent formation of small scale structures

Post freeze-out DM ψ remains in thermal contact with dark radiation χ via elastic processes $\psi\chi \leftrightarrow \psi\chi$

$$M_{\rm hm} = \frac{4\pi}{3}\rho_{\rm DM}\left(\frac{\lambda_{\rm hm}}{2}\right)^3 \approx 1.9\times 10^7\, M_\odot \left(\frac{\lambda_{\rm hm}}{0.1\,{\rm Mpc}}\right)^3$$

Depends on temperature of kinetic decoupling $T_{\rm kd}$



Viable Parameter Space

Putting everything together

Taule, Escudero, Garny: 2207.04062

Thermalisation

νs should thermalise with Z' in $0.7\ \mathrm{MeV} > T_\gamma > 10\ \mathrm{eV}$

BBN constraints

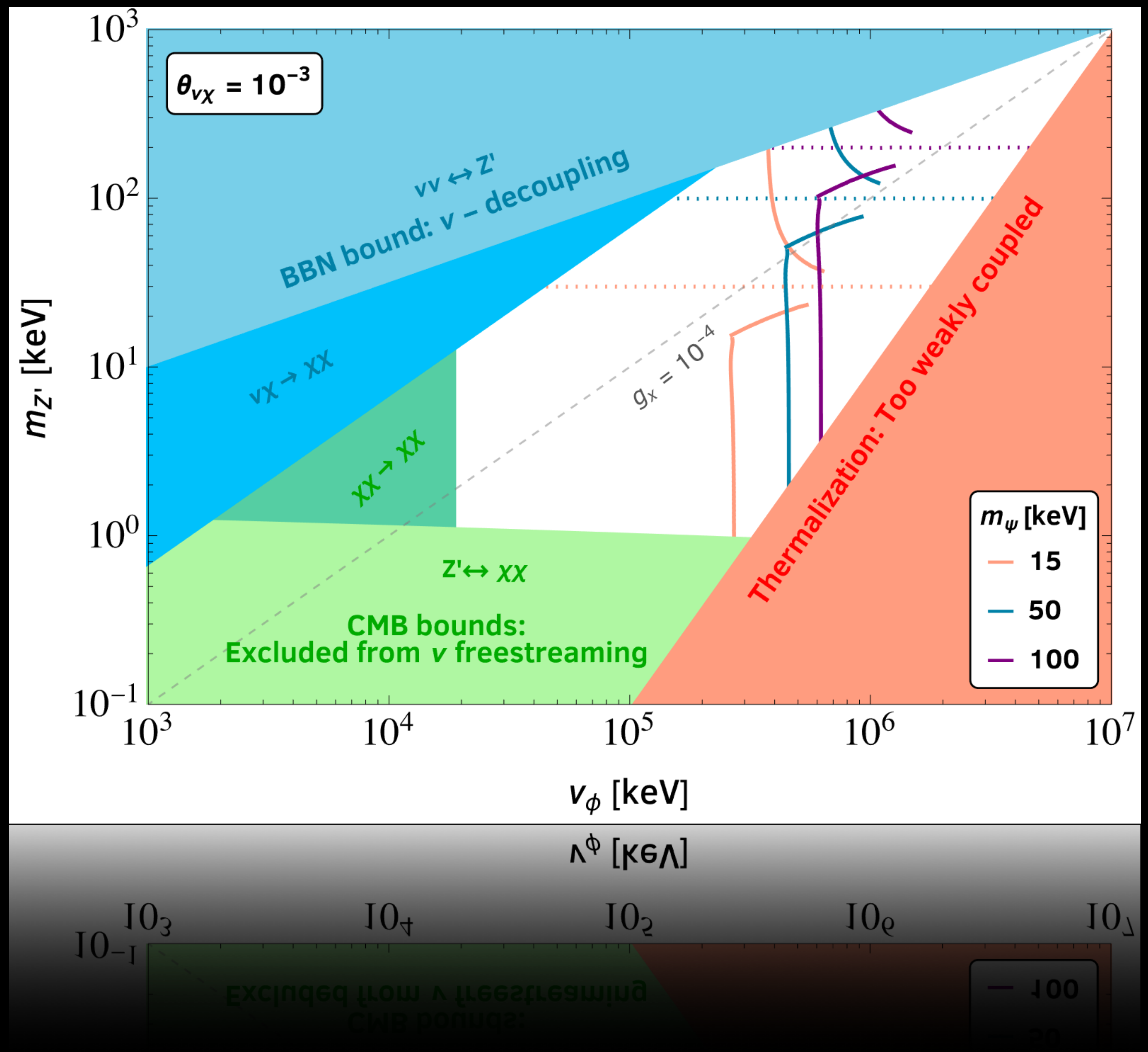
νs should not thermalise with Z'; avoid χ's exponential growth $\nu\chi \leftrightarrow \chi\chi$ at $T_\gamma > 0.7\ \mathrm{MeV}$

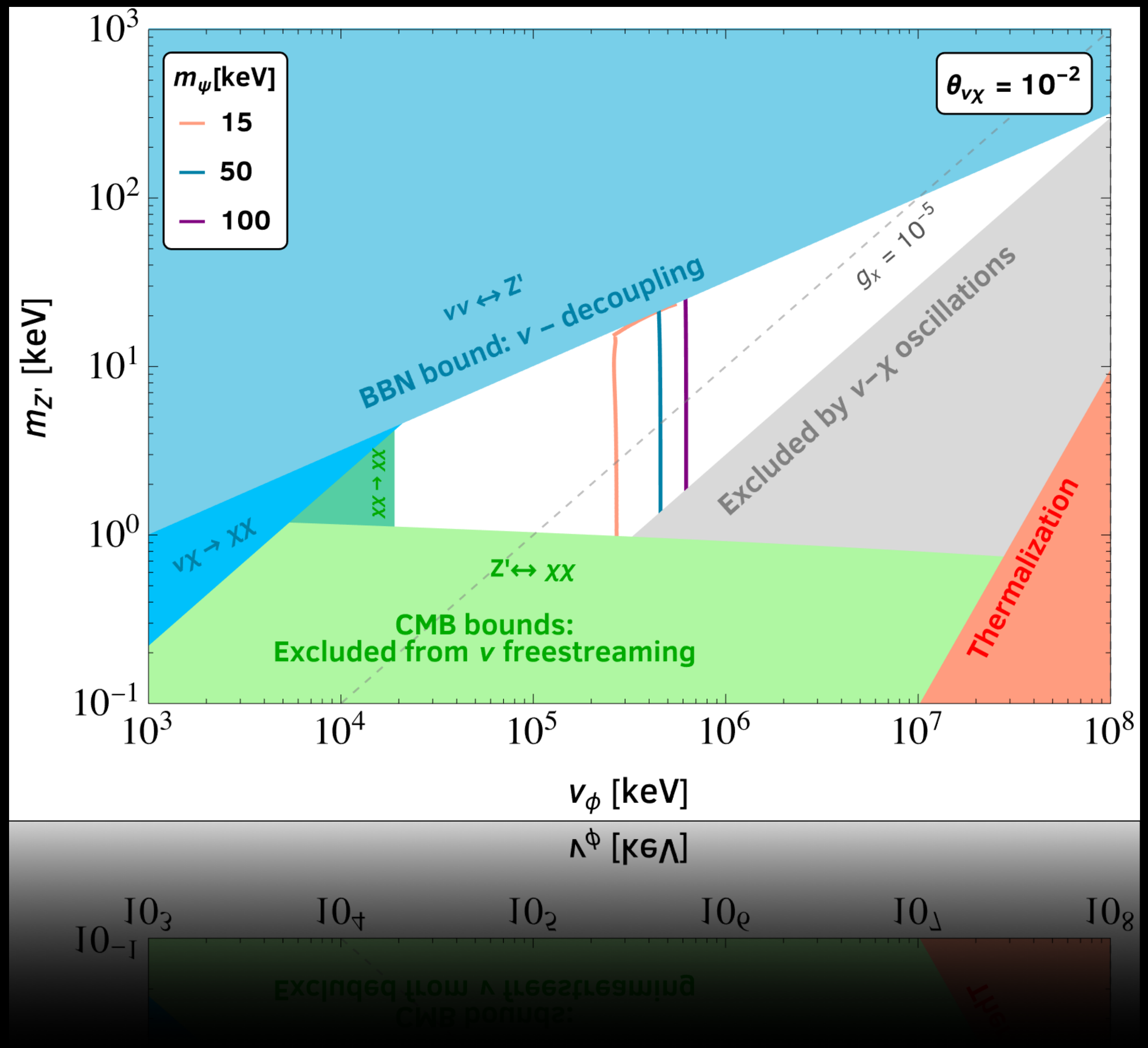
CMB constraints

$\nu\nu \to Z'$ and $Z' \to \chi\chi$ must be inefficient at $z \sim 10^5$; CMB not perturbed by lack of χ free streaming

Active-sterile mixing

Constrain production of χ from oscillations before BBN using $\Delta N_{\rm eff} < 0.3 \ \to 10^{-4} \leq \theta_{\nu\chi} < 10^{-1}$





Neutrino Mass Suppression

$N_{\rm eff}$ and DS Temperature

New degrees of freedom come into equilibrium with neutrinos at $T_\nu^{\rm eq}$ to form a system with $T_{\rm eq}$

$$\rho_\nu(T_\nu^{\rm eq}) = \sum_{f=\nu,\chi,\psi} \rho_f(T_{\rm eq}) + \rho_{Z'}(T_{\rm eq})$$

System evolves adiabatically from $T_{\rm eq}$ to $T_{\rm fin}$ when ψ, Z' become non-relativistic, use $a_{\rm eq}^3 s_{\rm eq}(T_{\rm eq}) = a_{\rm fin}^3 s_{\rm fin}(T_{\rm fin})$

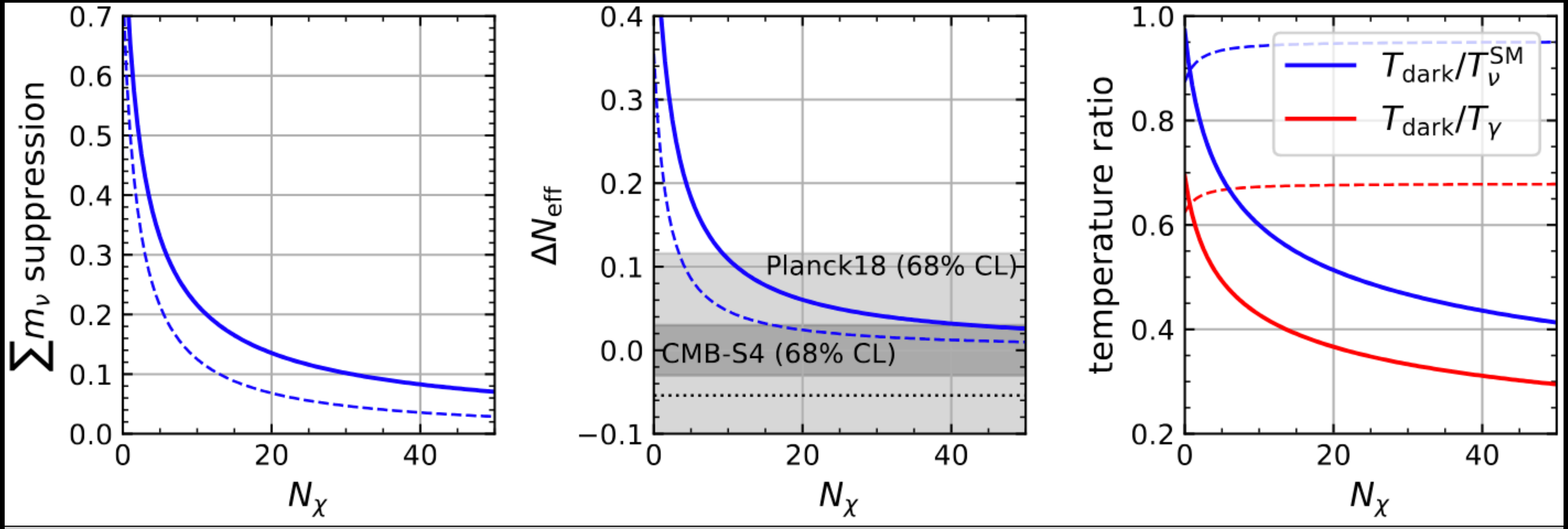
$$\frac{n_\nu}{n_\nu^{\rm SM}} = \left(\frac{T_{\rm dark}}{T_\nu^{\rm SM}}\right)^3 = \frac{g_\nu + \tilde{g} + g_\psi + \frac{8}{7} g_{Z'}}{g_\nu + \tilde{g}} \left(\frac{g_\nu}{g_\nu + \tilde{g} + g_\psi + \frac{8}{7} g_{Z'}}\right)^{3/4}$$

$$N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_{\rm dark}}{\rho_\gamma} = \frac{g_\nu + \tilde{g}}{2}\left(\frac{T_{\rm dark}}{T_\nu^{\rm SM}}\right)^4$$

$$\left[\sum m_\nu\right]_{\rm eff} = \sum m_\nu \frac{n_\nu}{n_\nu^{\rm SM}}$$

Neutrino Mass Suppression

N_{eff} and DS Temperature



M [GeV]	M' [GeV]	m_D [GeV]	κ' [GeV]	Λ [GeV]	v_ϕ [GeV]	m_ψ [keV]	$m_{Z'}$ [keV]	$g = m_{Z'}/v_\phi$	$\theta_{\nu\chi}$	N_χ	n_ν/n_ν^{SM}	ΔN_{eff}
10^{11}	10^2	4.47	0.043	0.004	0.5	18.5	100	2×10^{-4}	10^{-3}	10	0.216	0.109
10^{12}	10^3	14.14	0.23	0.141	0.8	53	77	9.6×10^{-5}	10^{-2}	10	0.216	0.109
10^{13}	10^2	44.7	0.1	0.044	0.6	100	32	5.3×10^{-5}	10^{-3}	20	0.135	0.060

Summary

Comparing cosmology and laboratory bounds on $\sum m_\nu \rightarrow$ Hints of new physics

The cosmological neutrino mass bound can be relaxed with a light dark sector

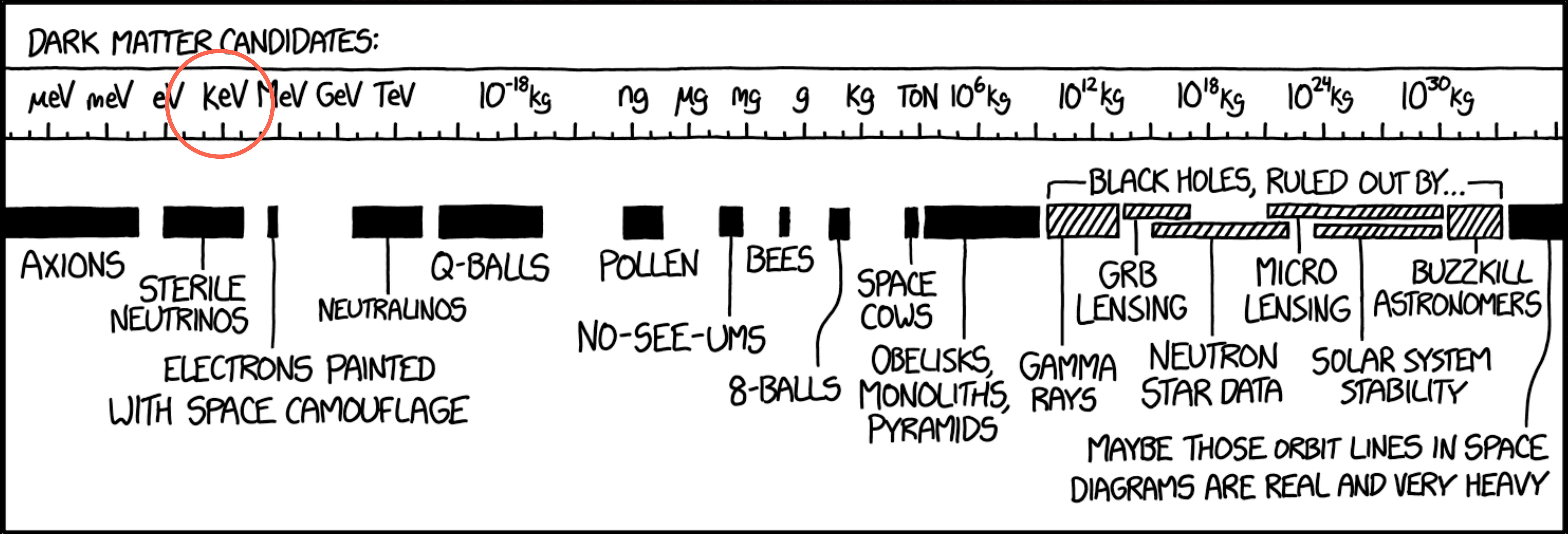
We embed the mechanism in an extended seesaw model $\rightarrow$ ν masses, DM and leptogenesis

Model has a complex particle content though not so numerous free parameters $\rightarrow$ $m_\psi, m_{Z'}, v_\phi, \theta_{\nu\chi}, N_\chi$

Dark sector particles play no role above $T_\gamma \sim 1$ MeV and come into equilibrium after ν-decoupling

DM thermalises with the DS and then freezes out $\rightarrow$ Abundance set by DS gauge interactions, not by mixing with SM neutrinos

Signatures of the model $\rightarrow$ Slightly increased N_{eff} at late times, Suppressed matter power spectrum at small scales (warm DM) xkcd



Thank You

Backup

Neutrinos

In cosmology

$$n_\nu = \frac{g}{(2\pi)^3} \int d^3p \, f_\nu(E, T_\nu)$$

$$\rho_\nu = \frac{g}{(2\pi)^3} \int d^3p \, E \, f_\nu(E, T_\nu)$$

$T \gg \mathrm{MeV}$

Weak interactions and ν's in equilibrium

$$n + \nu_e \leftrightarrow p + e^-$$

$$f_\nu(E, T_\nu) = \frac{1}{1 + \exp[(E - \mu)/T_\nu]}$$

Fermi-Dirac distribution

$T \sim \mathrm{MeV}$

Weak interactions drop out: $\Gamma \sim G_F^2 T^5 < H$

ν's decouple while relativistic at $T \simeq 2$ MeV

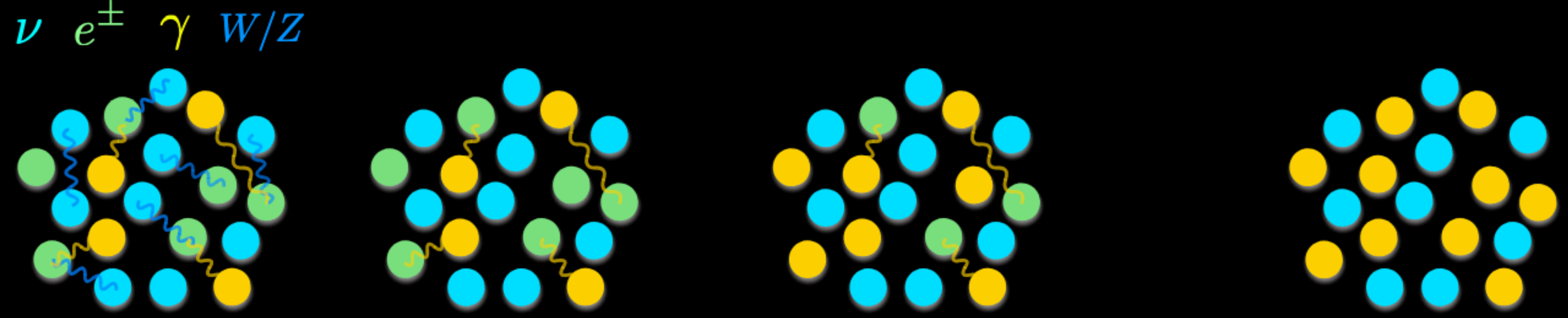
$$f_\nu(E, T_\nu) = \frac{1}{1 + \exp[p/T_\nu]}$$

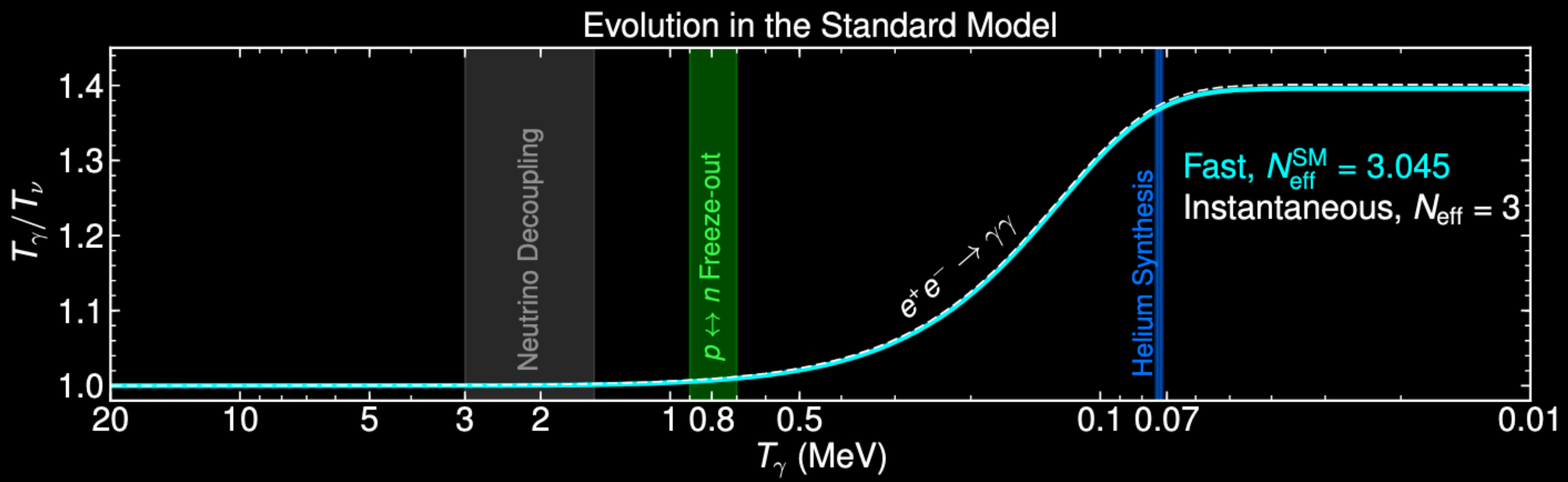
Well-approximated

$$\sum_i \rho_{\nu,i} \equiv N_{\mathrm{eff}} \rho_{\nu,0}$$

Neutrinos

In cosmology





$n_{\nu,0} = 56\ \mathrm{cm}^{-3}$

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

Escudero: 2001.04466

Neutrinos
In cosmology

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{\text{critical}}} \propto \sum \langle E_\nu \rangle n_\nu$$

$T_{\nu,0} \gg m_\nu$

$\langle E_\nu \rangle \simeq \langle \rho_\nu \rangle$, energy density characterised by $N_{\text{eff}} \propto \langle p_\nu \rangle n_\nu$

$$N_{\text{eff}} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\text{rad}} - \rho_\gamma}{\rho_\gamma}\right)$$

$3.044(1)$

SM prediction

2.99 ± 0.17

PLANCK 2018

$T_{\nu,0} \ll m_\nu$

$$\langle \rho_\nu \rangle = m_\nu,\ \rho_\nu = \sum m_\nu n_\nu$$

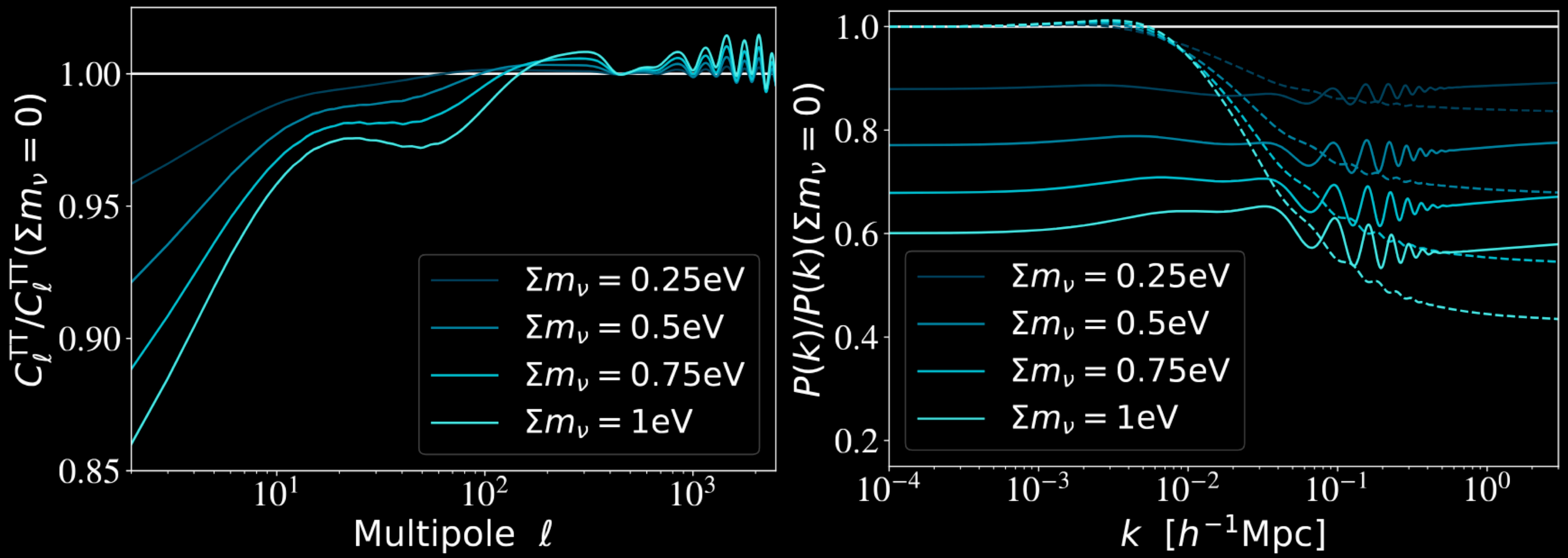
ν's contribute to expansion rate as DM

$$\Omega_\nu h^2 \equiv \frac{\sum m_\nu n_\nu^0 h^2}{\rho_{\text{critical}}} < 1.3 \times 10^{-3}\ (95\ \%\ \text{CL})$$

PLANCK 2018

Neutrino Masses

Cosmology



Credit: PDG 2022; Lesgourges, Verde

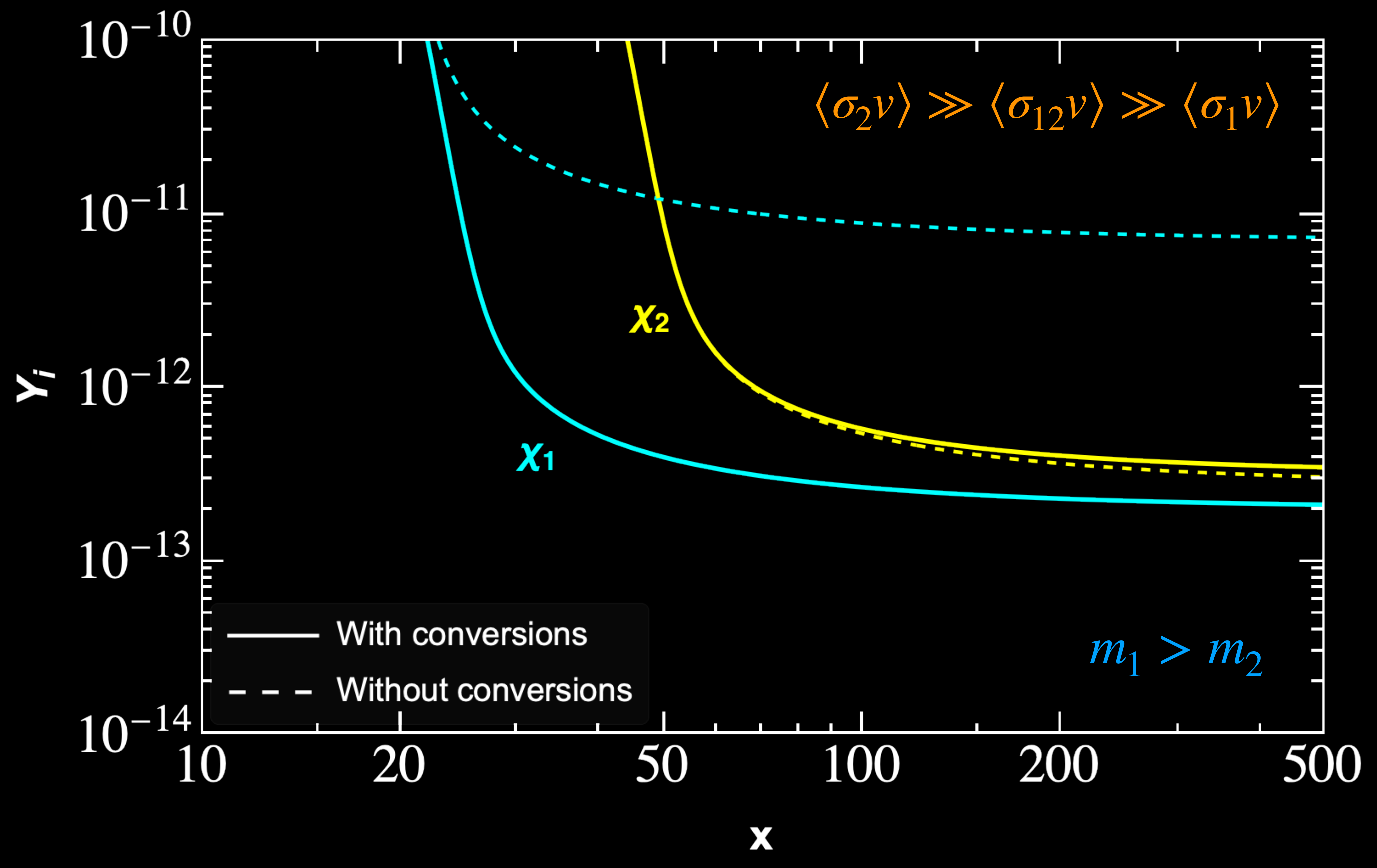
Symmetric Components

2DM

$$\frac{dY_1}{dx} = -\frac{s}{Hx}\left[\langle\sigma_1 v\rangle\,(Y_1^2 - \bar{Y}_1^2) + \langle\sigma_{12}v\rangle(Y_1^2 - \frac{Y_2^2}{\bar{Y}_2^2}\bar{Y}_1^2)\right] \qquad \frac{dY_2}{dx} = -\frac{s}{Hx}\left[\langle\sigma_2 v\rangle\,(Y_2^2 - \bar{Y}_2^2) - \langle\sigma_{12}v\rangle(Y_1^2 - \frac{Y_2^2}{\bar{Y}_2^2}\bar{Y}_1^2)\right]$$

No conversions: DM abundance dominated by the particle with *smallest* $\langle\sigma v\rangle$

$$\Omega h^2 \propto \frac{1}{\langle\sigma_1 v\rangle} + \frac{1}{\langle\sigma_2 v\rangle} \equiv \frac{1}{\langle\sigma v\rangle_{\rm eff}} \simeq \frac{1}{2.2\cdot 10^{-26}\,{\rm cm^3/s}}$$



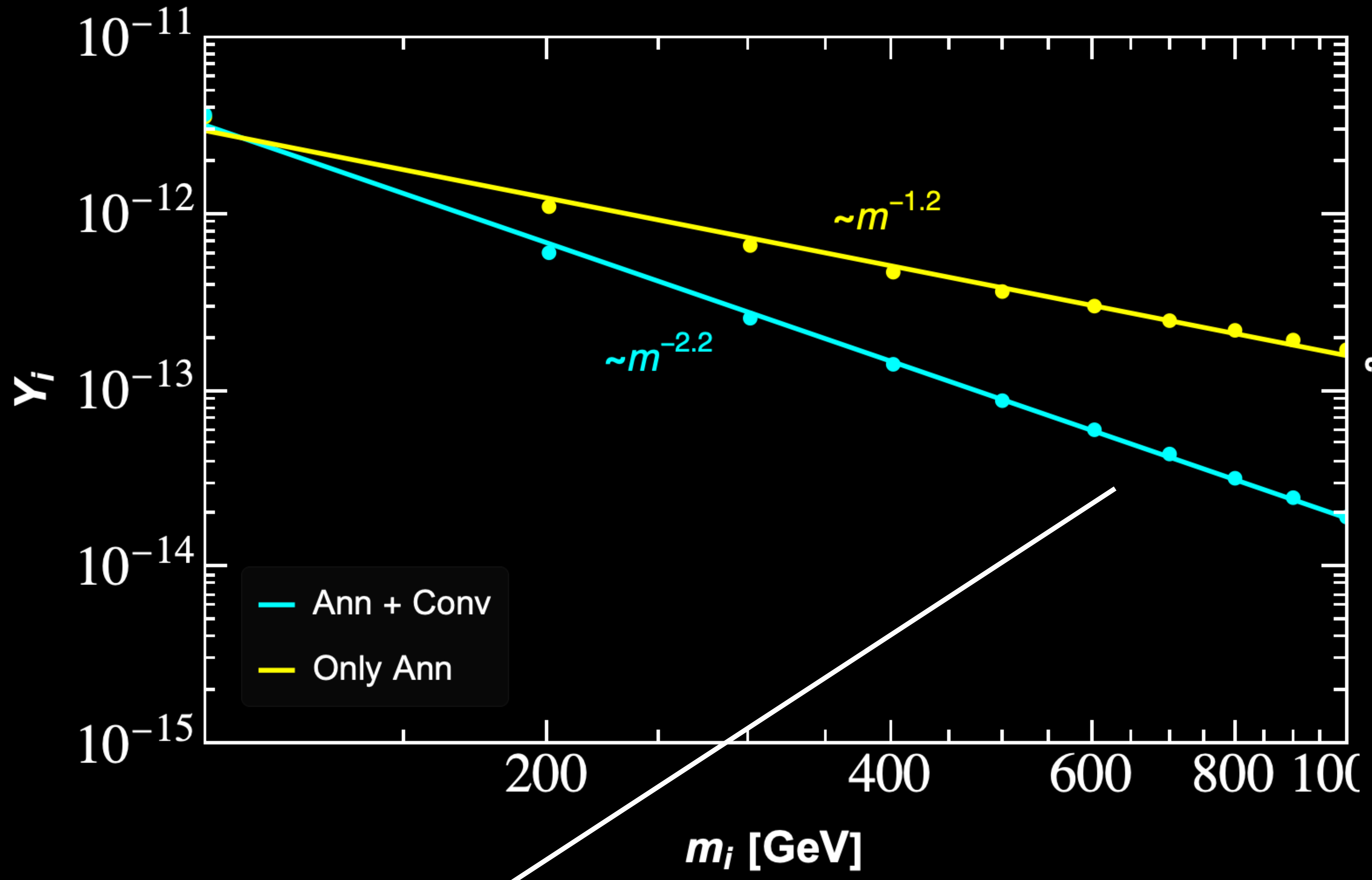
With conversions: heavier components have *reduced* abundance

$$\Omega h^2 \propto \frac{1}{\langle\sigma_1 v\rangle + \langle\sigma_{12} v\rangle} + \frac{1}{\langle\sigma_2 v\rangle} \equiv \frac{1}{\langle\sigma v\rangle_{\rm eff}}$$

Aoki et al: 1207.3318
Bhattacharya et al: 1607.08461

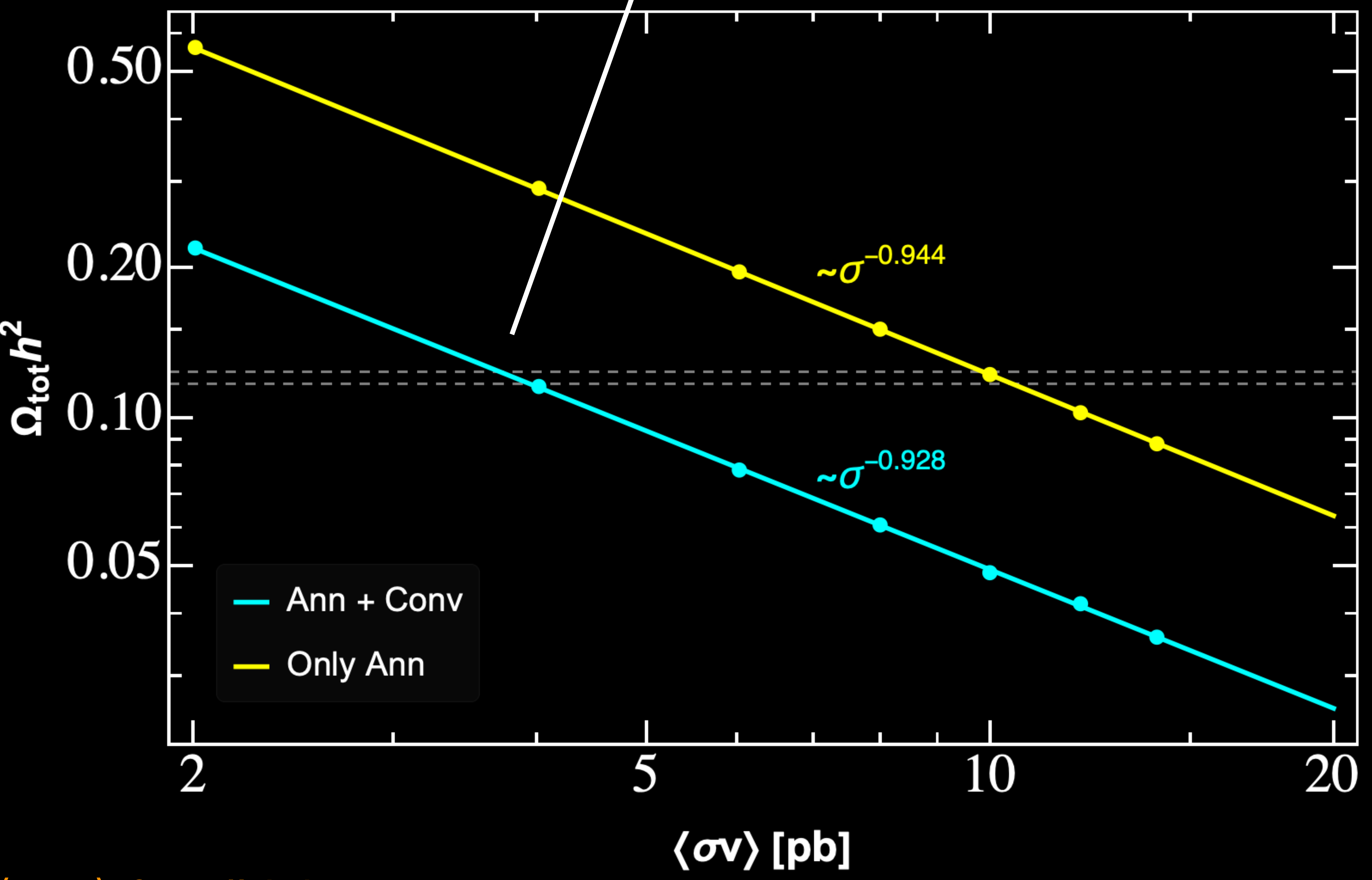
Symmetric Components

10DM



Heavier component yields suppressed due to conversions

Smaller annihilation cross-sections with conversions: relaxes limits!



$\langle\sigma_i v\rangle = \langle\sigma_{ij} v\rangle$ for all i, j

Constraints

Stability & X-ray bounds

DM decay $\rightarrow$ ψ lifetime should be larger than the age of the universe

$m_\psi < m_{Z'}$

$$\psi \rightarrow \nu\chi\chi$$

$$\theta^2_{\nu\psi} < 2 \times 10^{-16} \left(\frac{15\ \mathrm{keV}}{m_\psi}\right)^5 \left(\frac{21}{\tilde{N}}\right) \left(\frac{v_\phi}{2\ \mathrm{GeV}}\right)^4$$

$m_\psi > m_{Z'}$

$$\psi \rightarrow Z'\nu$$

$$\theta^2_{\nu\psi} < 1.2 \times 10^{-30} \left(\frac{m_{Z'}}{10\ \mathrm{keV}}\right)^2 \left(\frac{10^{-4}}{g}\right)^2 \left(\frac{40\ \mathrm{keV}}{m_\psi}\right)^3$$

Sterile νDM mixing with active νs $\rightarrow$ Observable monochromatic X-ray line

$$\psi \rightarrow \nu\gamma$$

$$\Gamma_{\psi\rightarrow\nu\gamma} = \frac{9\,\alpha\,G_F^2}{256 \cdot 4\pi^4} \sin^2(2\theta_{\nu\psi})\, m_\psi^5$$

$$\theta^2_{\nu\psi} \lesssim 7.65 \times 10^{-13} \left(\frac{15\ \mathrm{keV}}{m_\psi}\right)^5$$

$\theta_{\nu\psi}$ should be really suppressed

Model

Masses, Mixing and Parameters

$$m_\chi = 0\,,$$

$$m_\nu = \frac{(m_D\kappa' - m_D{}'\kappa)^2 + (m_D{}'\Lambda - m_D\Lambda')^2 + (\kappa'\Lambda - \kappa\Lambda')^2}{M'(m_D^2 + \kappa^2 + \Lambda^2) + M(m_D{}'^2 + \kappa'^2 + \Lambda'^2)}\,,$$

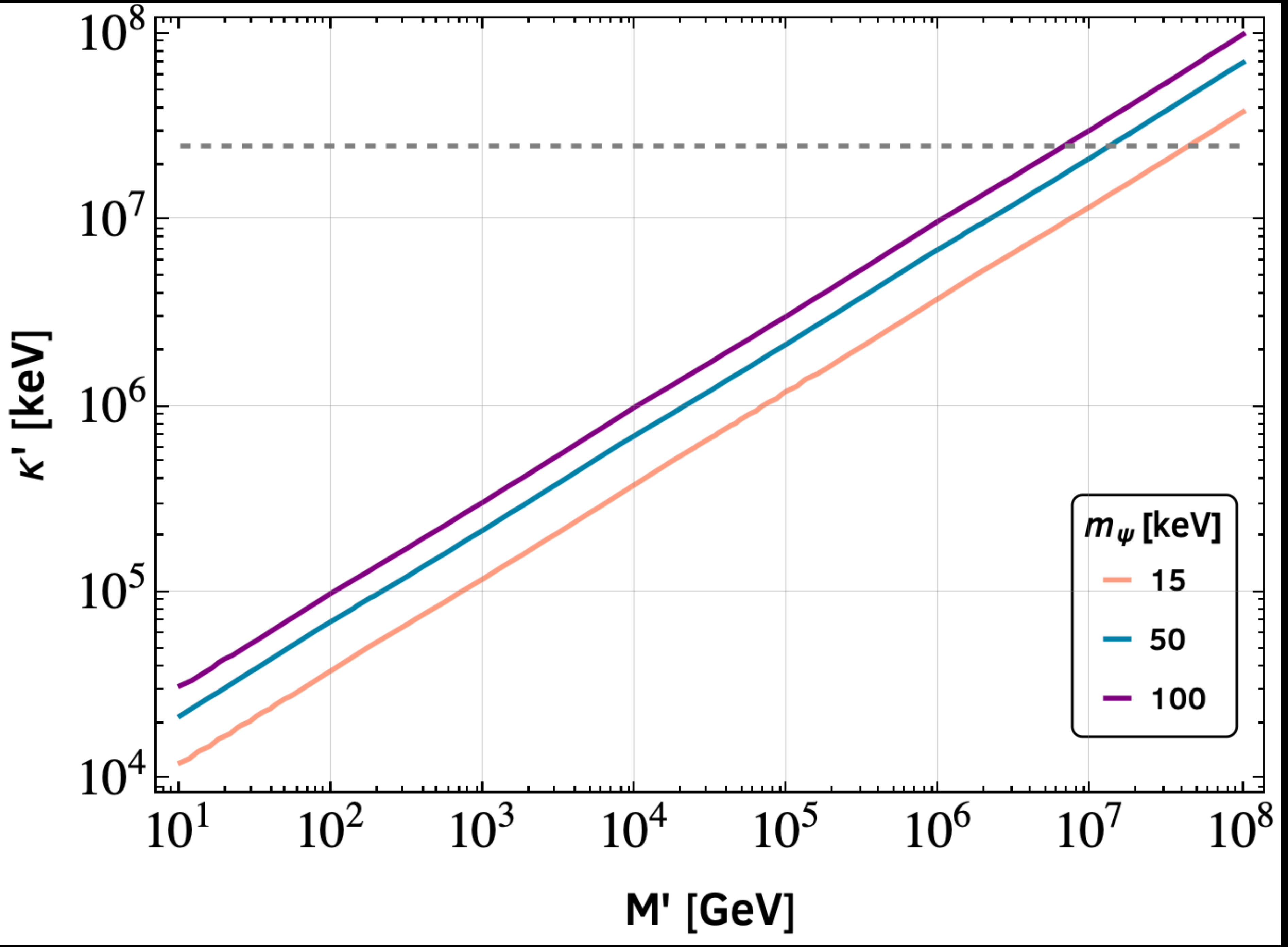
$$m_\psi \approx \frac{m_D^2 + \kappa^2 + \Lambda^2}{M} + \frac{m_D{}'^2 + \kappa'^2 + \Lambda'^2}{M'}\,,$$

$$m_{N'} \approx M'\,,$$

$$m_N \approx M\,.$$

$$\theta_{\nu N} = \frac{m_D}{M}\,, \quad \theta_{\nu\chi} = \frac{\Lambda}{m_D}\,, \quad \theta_{N\chi} = \frac{\Lambda}{M} \sim 0\,,$$

$$\theta_{\nu N'} = \frac{m_D{}'}{M'}\,, \quad \theta_{\nu\psi} = \frac{m_D{}'}{\kappa'}\,, \quad \theta_{N'\psi} = \frac{\kappa'}{M'}\,.$$



DS Freeze-out

Analytical Solution

$$\Omega_\psi h^2 \simeq x_f \frac{10^{-10}\,\mathrm{GeV}^{-2}}{\langle \sigma v \rangle_{\mathrm{tot}}}$$

$x_f = m_\psi / T_{\gamma,f}$

DS Freeze-out temperature

$$x_f \approx \xi \ln[0.04\delta(\delta+2)(g_\psi/g_{\mathrm{eff}}^{1/2})\beta\xi^{3/2}] - \frac{1}{2}\xi \ln\{\xi \ln[0.04\delta(\delta+2)(g_\psi/g_{\mathrm{eff}}^{1/2})\beta\xi^{3/2}]\}$$

$g_{\mathrm{eff}} = g_\gamma + g_{\mathrm{dark}}\xi^4$

$\beta = M_{\mathrm{pl}}\, m_\psi \langle \sigma v \rangle_{\mathrm{tot}}^{x=1}$

$\xi \equiv T_{\mathrm{dark}}/T_\gamma$

DS Freeze-out

Numerical Solution

$$\frac{dY_\nu}{dx} = \frac{\langle\Gamma_\nu\rangle}{Hx}\left(Y_{Z'} - Y_{Z'}^{\rm eq}\frac{Y_\nu^2}{Y_\nu^{\rm eq\,2}}\right) \;,$$

$$\frac{dY_{Z'}}{dx} = \sum_{i=\nu,\chi,\psi} -\frac{\langle\Gamma_i\rangle}{Hx}\left(Y_{Z'} - Y_{Z'}^{\rm eq}\frac{Y_i^2}{Y_i^{\rm eq\,2}}\right) + \frac{s\langle\sigma v\rangle_{\psi\psi\to Z'Z'}}{Hx}\left(Y_\psi^2 - \frac{Y_\psi^{\rm eq\,2}}{Y_{Z'}^{\rm eq\,2}}Y_{Z'}^2\right) \;,$$

$$\frac{dY_\chi}{dx} = \frac{\langle\Gamma_\chi\rangle}{Hx}\left(Y_{Z'} - Y_{Z'}^{\rm eq}\frac{Y_\chi^2}{Y_\chi^{\rm eq\,2}}\right) + \frac{s\langle\sigma v\rangle_{\psi\psi\to\chi\chi}}{Hx}\left(Y_\psi^2 - \frac{Y_\psi^{\rm eq\,2}}{Y_\chi^{\rm eq\,2}}Y_\chi^2\right) \;,$$

$$\frac{dY_\psi}{dx} = \frac{\langle\Gamma_\psi\rangle}{Hx}\left(Y_{Z'} - Y_{Z'}^{\rm eq}\frac{Y_\psi^2}{Y_\psi^{\rm eq\,2}}\right) - \frac{s\langle\sigma v\rangle_{\psi\psi\to\chi\chi}}{Hx}\left(Y_\psi^2 - \frac{Y_\psi^{\rm eq\,2}}{Y_\chi^{\rm eq\,2}}Y_\chi^2\right) - \frac{s\langle\sigma v\rangle_{\psi\psi\to Z'Z'}}{Hx}\left(Y_\psi^2 - \frac{Y_\psi^{\rm eq\,2}}{Y_{Z'}^{\rm eq\,2}}Y_{Z'}^2\right)$$

Constraints
Structure formation

Free-streaming $$\lambda_{\rm FS} \approx \frac{1}{2}\int_{t_{\rm kd}}^{t_{\rm MRE}} dt \frac{v_\psi}{a(t)} \approx \frac{1}{2}\left(\frac{4\pi^3 g_{\rm eff}}{135}\right)^{-1/2} \sqrt{\frac{\xi}{T_{\rm kd} m_\psi}} \frac{M_{\rm pl}}{T_0} \log \frac{T_{\rm kd}}{T_{\rm MRE}}$$

Acoustic oscillations $$\lambda_{\rm AO} = \int_0^{t_{\rm kd}} \frac{dt}{a(t)} = \frac{1}{aH}\bigg|_{\rm kd} \approx \left(\frac{4\pi^3 g_{\rm eff}}{45}\right)^{-1/2} \frac{M_{\rm pl}}{T_{\rm kd} T_0}$$

$$\lambda_{\rm cutoff} = \max(\lambda_{\rm FS}, \lambda_{\rm AO}) < 0.1\,{\rm Mpc}$$