Froggatt-Nielsen ALP

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What is the lowest FN scale compatible with present experimental constraints?

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Answer:

The current energy frontier (TeV)

Outline

1. Introduction: flavor puzzle and Froggatt-Nielsen models

2. Z_N Froggatt-Nielsen

3. The minimal model: Z_4
   - Setup
   - Phenomenology

4. Conclusion

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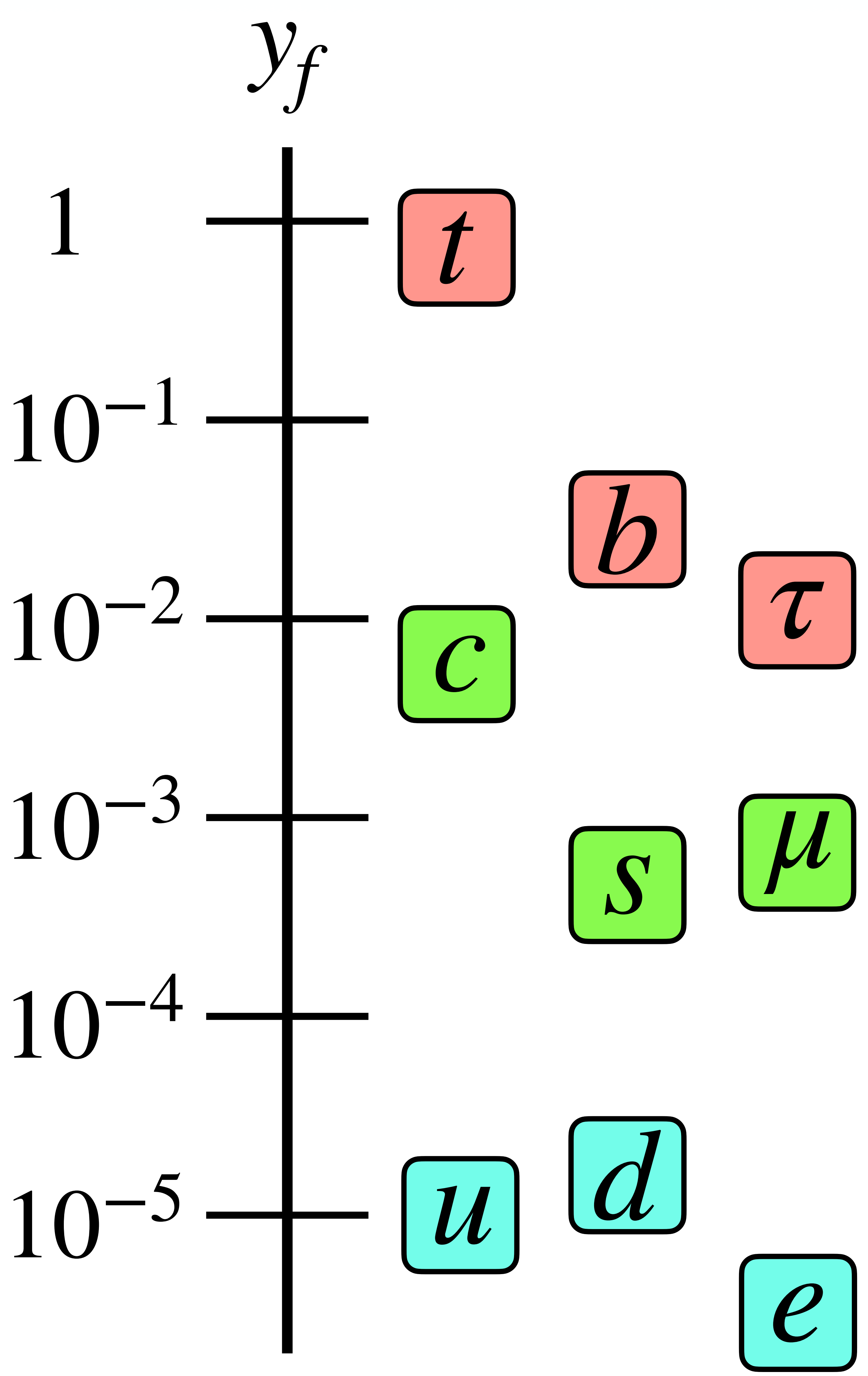
1. The flavor puzzle and Froggatt-Nielsen models



Standard Model

$SU(3) \times SU(2) \times U(1)$ gauge theory, 3 generations

<u>Gauge sector:</u> $U(3)^5$ symmetric

<u>Higgs sector (Yukawa)</u>: $U(3)^5$ broken (partially)

Why is the breaking so hierarchical?

The flavor puzzle

Froggatt-Nielsen

$$\mathscr{L}_{\rm SM} \supset \sum_{F,f} y^f_{ij}\, \bar{F}_{L,i} H f_{R,j} + \{\tilde{H}\} + \text{h.c.}$$

FN idea: $y^f_{ij} \longrightarrow x^f_{ij}\, \epsilon^{n^f_{ij}}$ $\qquad x^f_{ij} \sim \mathcal{O}(1)$

$$\boxed{U(1)_{\rm FN}}$$

Froggatt, Nielsen (1978)

$$[F_i] = Q^F_i \qquad [f_j] = Q^f_j \qquad [\epsilon] = 1$$

$U(1)_{\rm FN}$-breaking spurion

Selection rules $\leftrightarrow n^f_{ij} \leftrightarrow$ flavor pattern

Froggatt-Nielsen

Dynamical realization: $\epsilon \longrightarrow \frac{\langle \Phi \rangle}{M}$ $[\Phi] = 1$

$$\left(y^f_{ij} \longrightarrow x^f_{ij}\, \epsilon^{n^f_{ij}} \right)$$

$$\Phi = \frac{v_\Phi + \rho}{\sqrt{2}} e^{i \frac{a}{v_\Phi}} \qquad \left(\langle \Phi \rangle = \frac{v_\Phi}{\sqrt{2}} \right)$$

$$\Big\downarrow$$

$U(1)_{\rm FN}$ SSB $\leftrightarrow$ **ALP** a

Froggatt-Nielsen

- If $U(1)_{\mathrm{FN}}$ has anomaly with QCD $\longrightarrow$ Strong CP for free!

flaxion/axiflavon

Calibbi, Goertz, Redigolo, Ziegler, Zupan (2016)

Ema, Hamaguchi, Moroi, Nakayama (2016)

Froggatt-Nielsen

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- Bound: $\mathscr{L} \sim \epsilon^{n_{sd}} e^{ia/v_\Phi} \bar{s}_L H d_R +$ h.c.

E787+E949 (2007) 95% CL

$$\text{Br}(K^+ \to \pi^+ a) < 9.5 \times 10^{-11}$$

$$\boxed{\Rightarrow v_\Phi \gtrsim 10^{12}\ \textbf{GeV} \times \epsilon^{n_{sd}}}$$

Camalich, Pospelov, Vuong, Ziegler, Zupan (2020)

Can we lower this?

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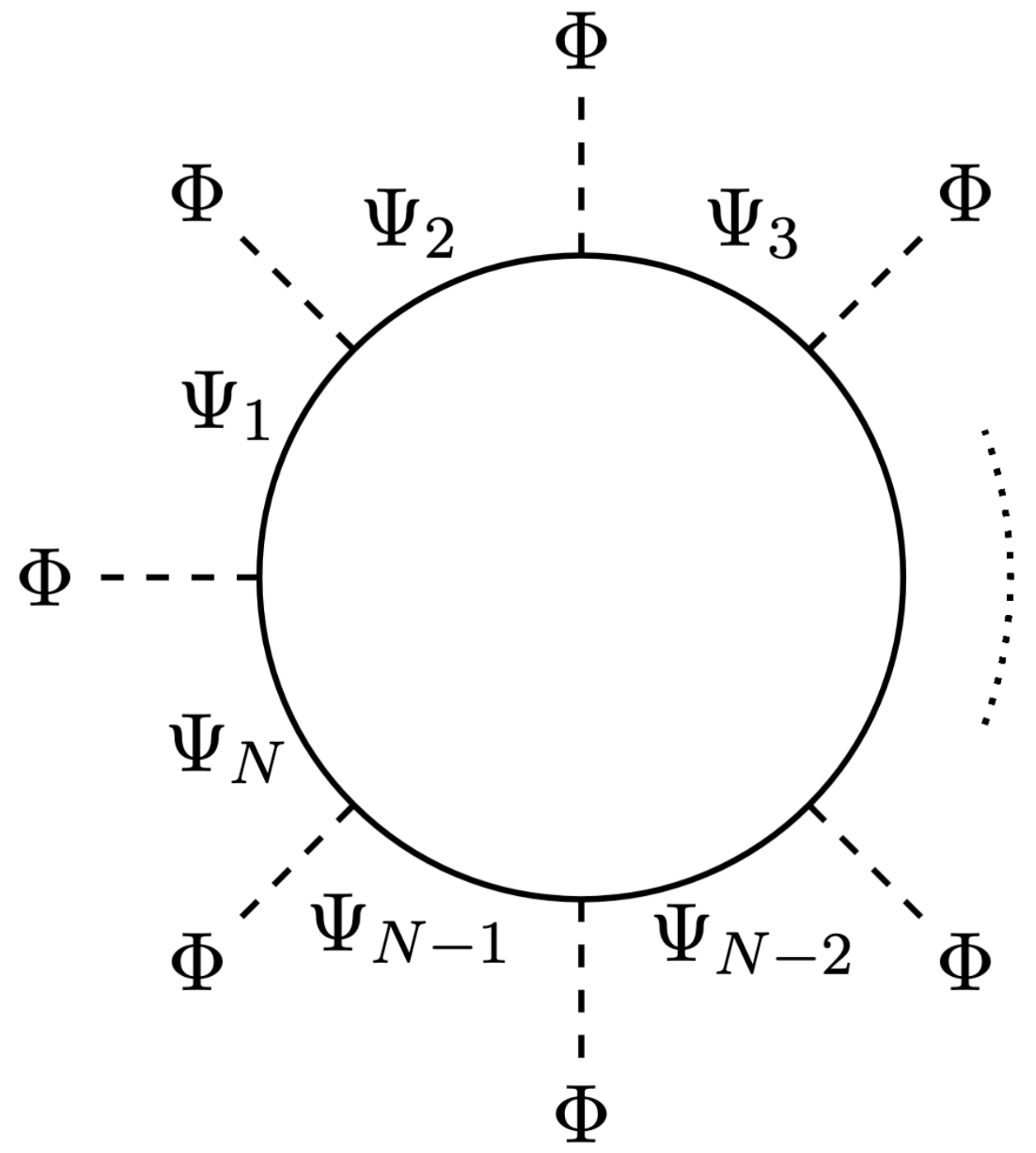
Discrete Froggatt-Nielsen models

Simplest setup: $U(1)_{\mathrm{FN}} \to Z_N$ symmetry

$$\mathscr{L} \supset -\frac{1}{4}\frac{\lambda_N'}{M_\Phi^{N-4}}\left[\Phi^N + (\Phi^*)^N\right]$$

Discrete Froggatt-Nielsen models

Simplest setup: $U(1)_{\rm FN} \to Z_N$ symmetry



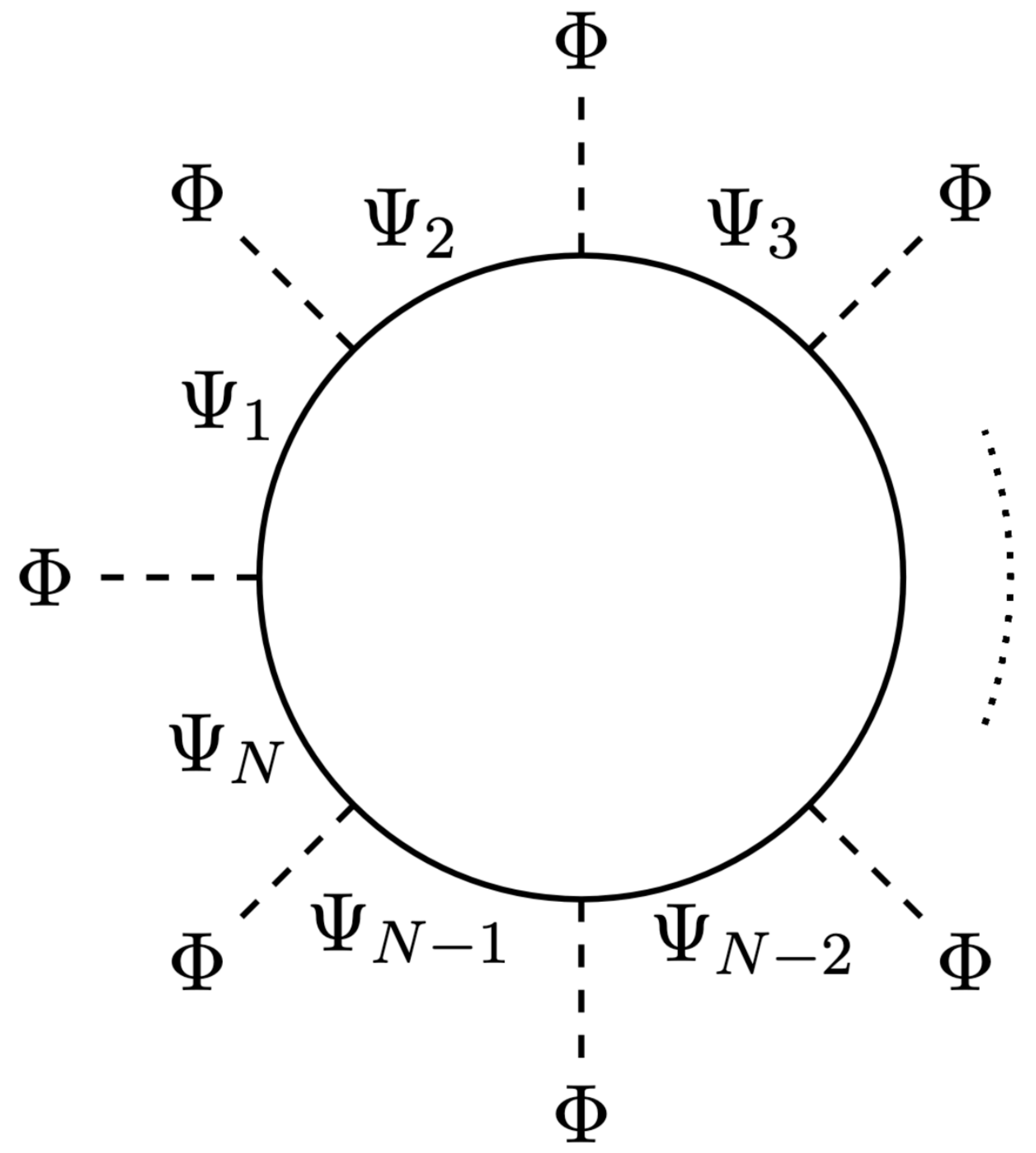
$$\mathscr{L} \supset -\frac{1}{4}\frac{\lambda'_N}{M_\Phi^{N-4}}\left[\Phi^N + (\Phi^*)^N\right]$$

Concrete models:

$$M_\Phi \sim M$$

Discrete Froggatt-Nielsen models

Simplest setup: $U(1)_{\rm FN} \to Z_N$ symmetry



$$\mathscr{L} \supset -\frac{1}{4}\frac{\lambda'_N}{M_\Phi^{N-4}}\left[\Phi^N + (\Phi^*)^N\right]$$

$$m_a^2 \sim \lambda'_N v_\Phi^2 \left(\frac{v_\Phi}{M_\Phi}\right)^{N-4} \sim \lambda'_N v_\Phi^2 \epsilon^{N-4} \qquad \left(\lambda'_N \sim \frac{\prod_k y_k}{16\pi^2}\right)$$

Concrete models:

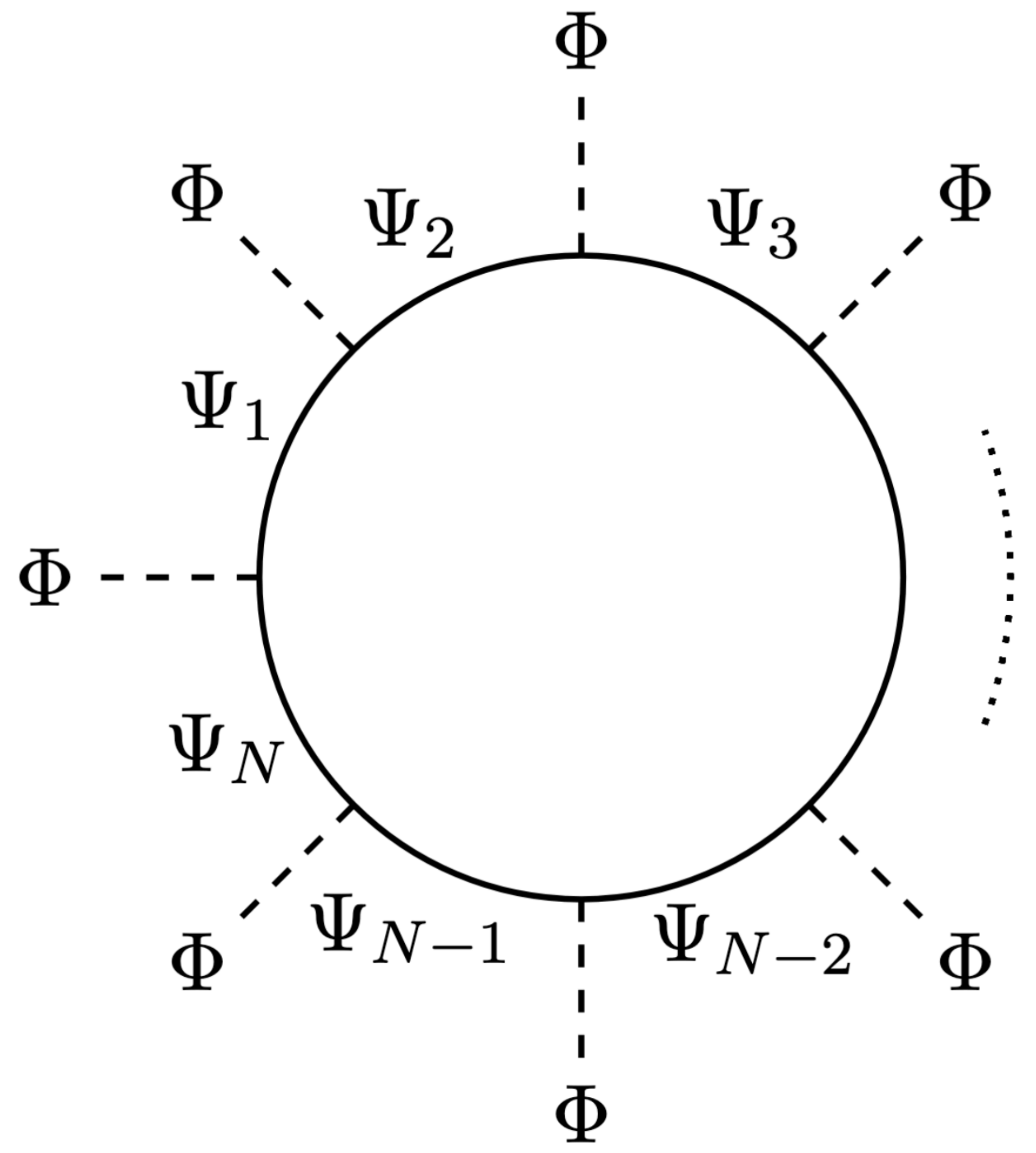
$M_\Phi \sim M$

$K \to \pi a$ not kinematically allowed!

(Strong CP solved externally)

Discrete Froggatt-Nielsen models

Simplest setup: $U(1)_{\rm FN} \to Z_N$ symmetry



Important: $\Phi^{n^f_{ij}} \sim (\Phi^*)^{N-n^f_{ij}} \qquad (Z_N)$

max $[n^f_{ij}] \leq N/2$

required to not spoil hierarchies

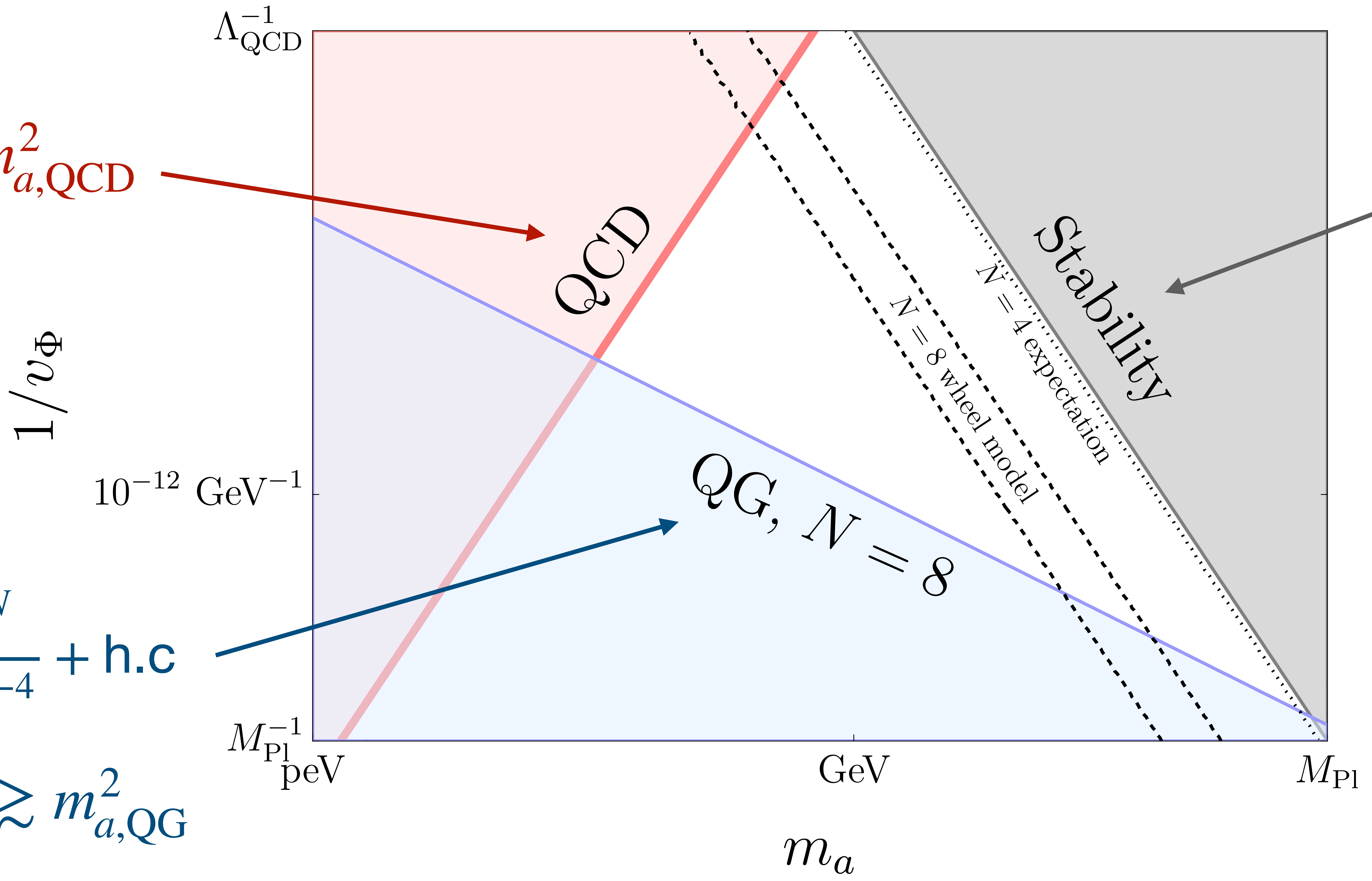
(Exceptions e.g. SUSY)

Concrete models:

$M_\Phi \sim M$

Theory constraints (no fine-tunings)

$m_a^2 \gtrsim m_{a,\mathrm{QCD}}^2$



$m_a^2 \lesssim \lambda v_\Phi^2$

$V_\Phi \supset \dfrac{\Phi^N}{M_P^{N-4}} + \text{h.c}$

$m_a^2 \gtrsim m_{a,\mathrm{QG}}^2$

$\lambda = 0.5$

$\lambda_4' = \lambda/4$

$\lambda_8' = \dfrac{\prod_k y_k}{16\pi^2}$

$(\, y_k \sim U(0,1)\,,$

10th – 90th percentiles)

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Minimal model: Z_4 "$q+\bar{e}$"

($\max(n_{ij}) \leq N/2$)

Z_4 charge: $q_1, \bar{e}_1 \sim 2 \;|\; q_2, \bar{e}_2 \sim 1 \;|\; q_3, \bar{e}_3 \sim 0$

$$Y_d = \begin{pmatrix} 0.55\epsilon^2 & 2.5\epsilon^2 & (0.73-1.8i)\epsilon^2 \\ 0 & 0.049\epsilon & 0.10\epsilon \\ 0 & 0 & 0.011 \end{pmatrix} \qquad Y_u = \begin{pmatrix} 0.25\epsilon^2 & z_{u_2}\epsilon^2 & z_{u_3}\epsilon^2 \\ 0 & 0.57\epsilon & y_{u_3}\epsilon \\ 0 & 0 & 0.71 \end{pmatrix} \qquad Y_e = \begin{pmatrix} 0.15\epsilon^2 & 0 & 0 \\ z_{e_2}\epsilon^2 & 0.14\epsilon & 0 \\ z_{e_3}\epsilon^2 & y_{e_3}\epsilon & 0.01 \end{pmatrix}$$

$$\begin{cases} y_{u,d,e} \sim \epsilon^2 & \text{(1st gen)} \\ y_{c,s,\mu} \sim \epsilon & \text{(2nd gen)} \\ y_{t,b,\tau} \sim 1 & \text{(3rd gen)} \end{cases}$$

$$\boxed{\epsilon \simeq 4.4 \times 10^{-3}}$$

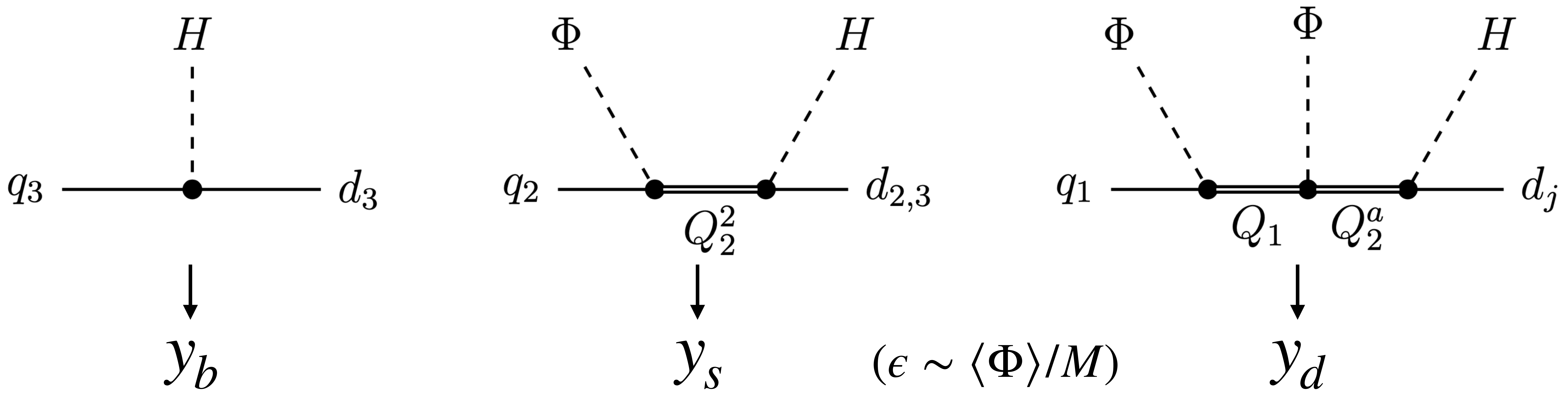
- "Acceptable" fit ($O(10^{-2})$ tuning)
- Pattern analogous to $U(2)_{q+e}$ models

Antusch, Greljo, Stefanek, Thomsen (2023)

Minimal model: Z_4 "$q + \bar{e}$"

<u>**UV completion:**</u> <u>VLQ</u> Q_2^a, Q_1 + <u>VLL</u> E_2^a, E_1

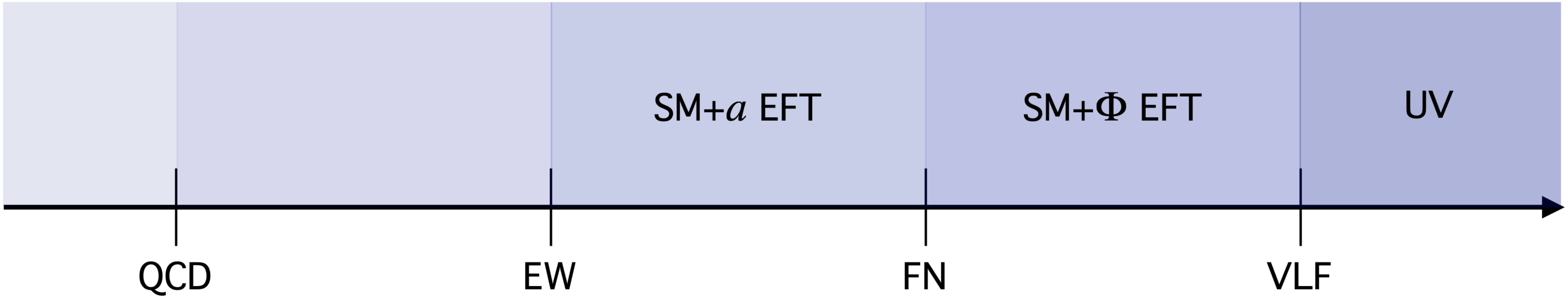
Z_4 charge: $(0\ ,\ 1)$ $(0\ ,\ 1)$



$(\epsilon \sim \langle\Phi\rangle/M)$

up quarks, leptons analogous

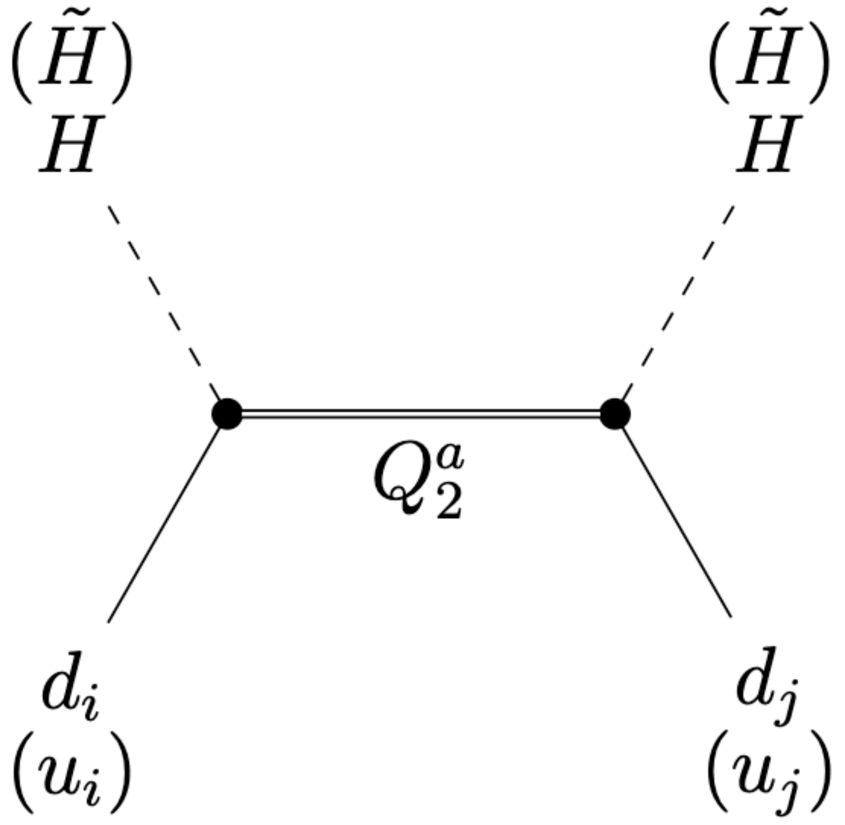
Phenomenogy: tower of EFTs

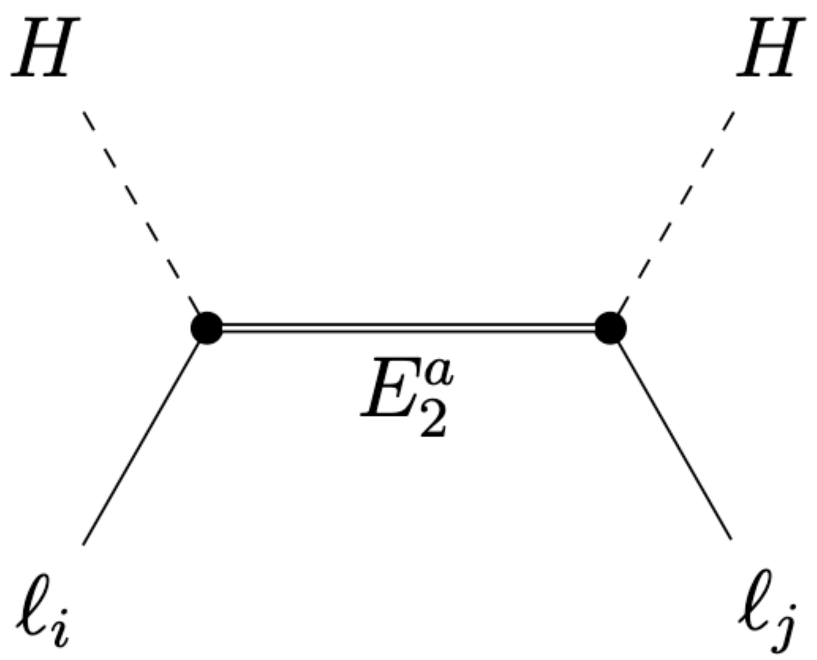


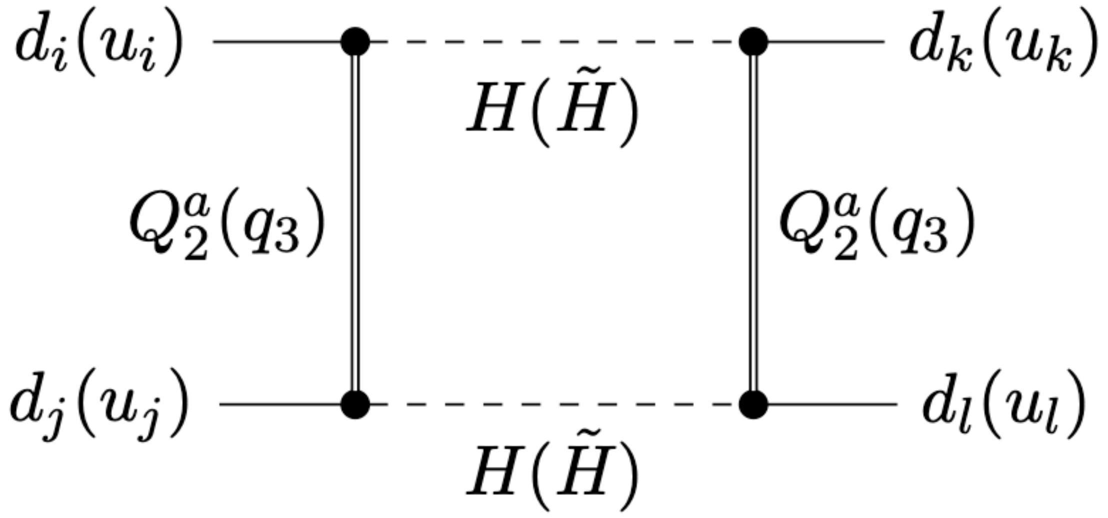
<u>Interplay between UV (VLF) and ALP effects</u>

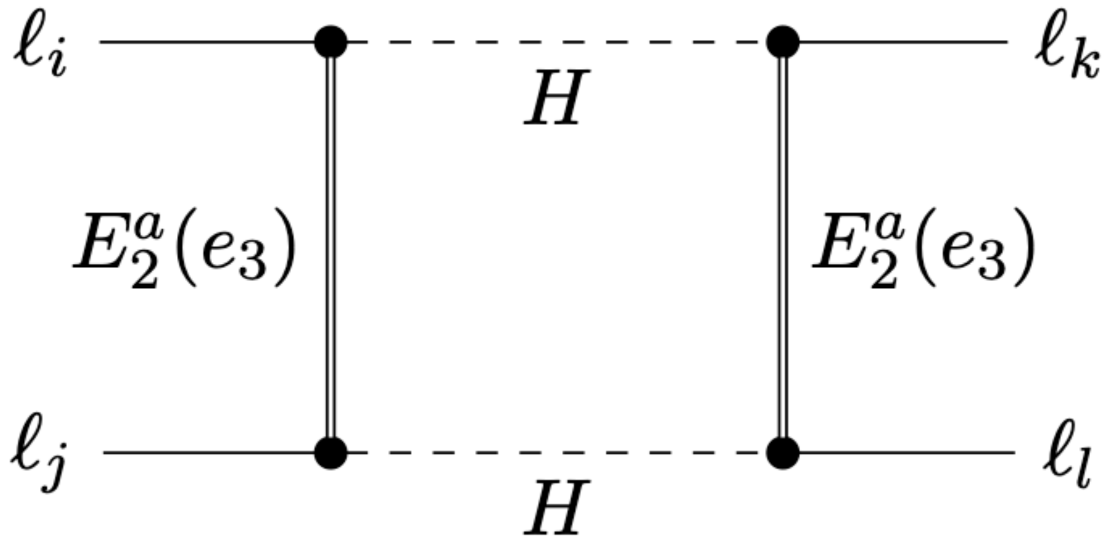
Phenomenogy: tower of EFTs

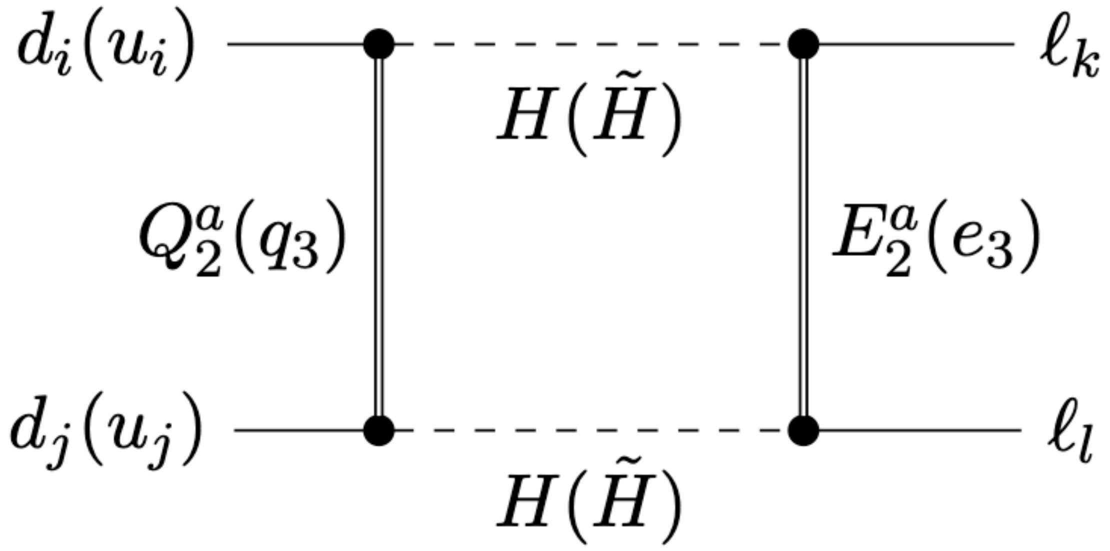
- **VLF**





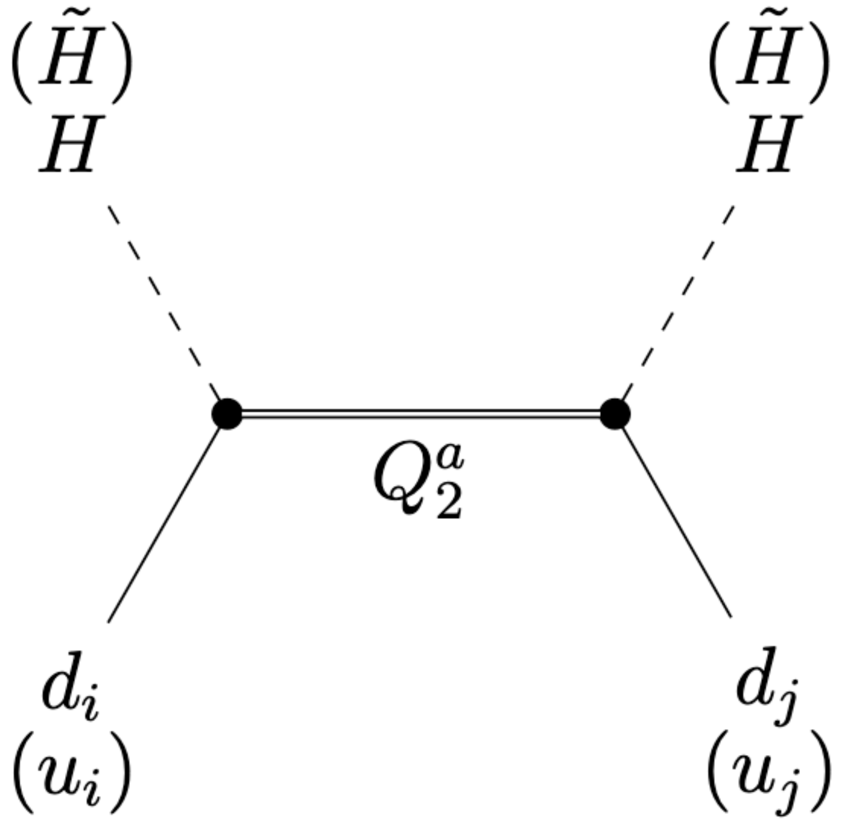


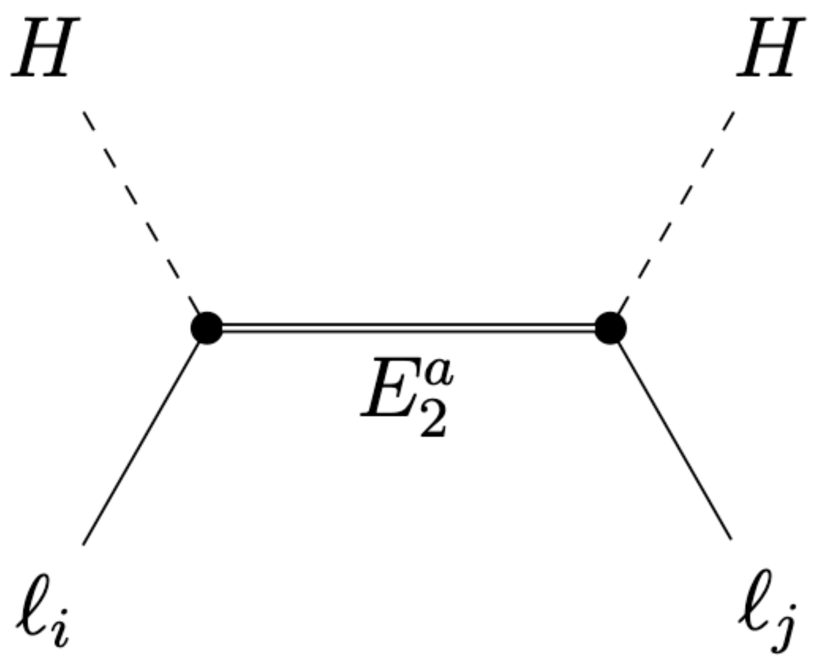


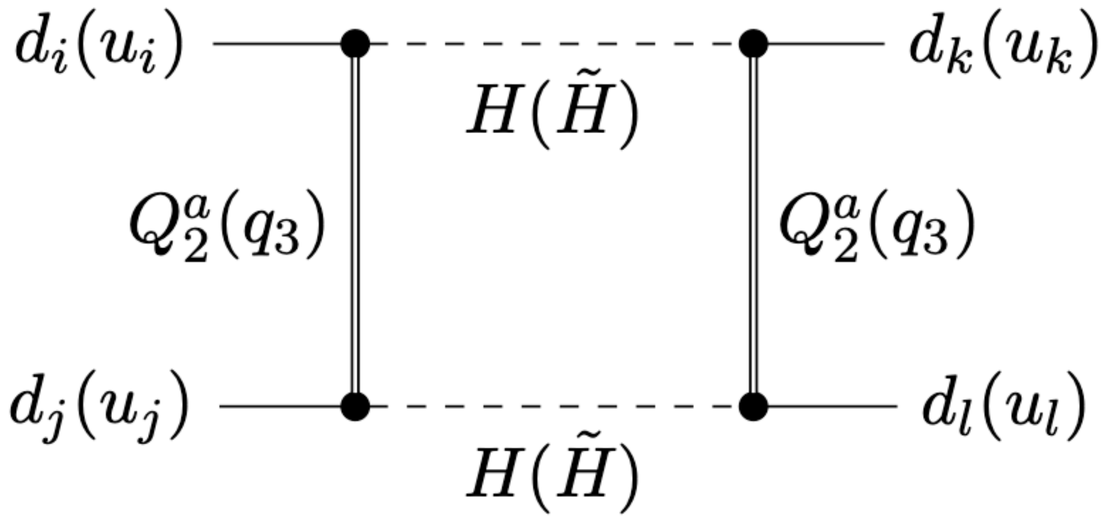


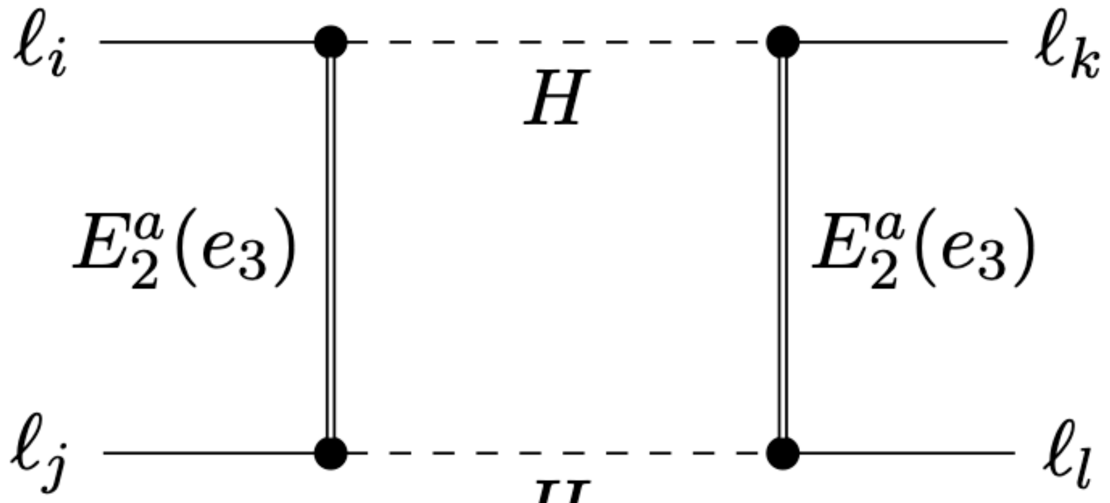
Phenomenogy: tower of EFTs

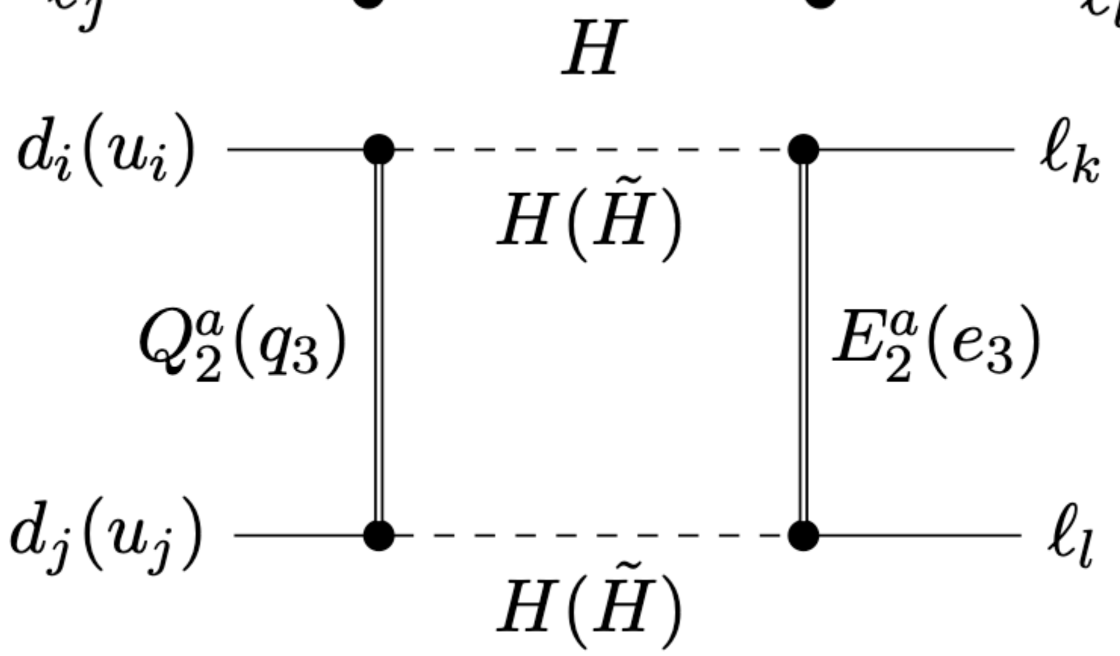
- **VLF**









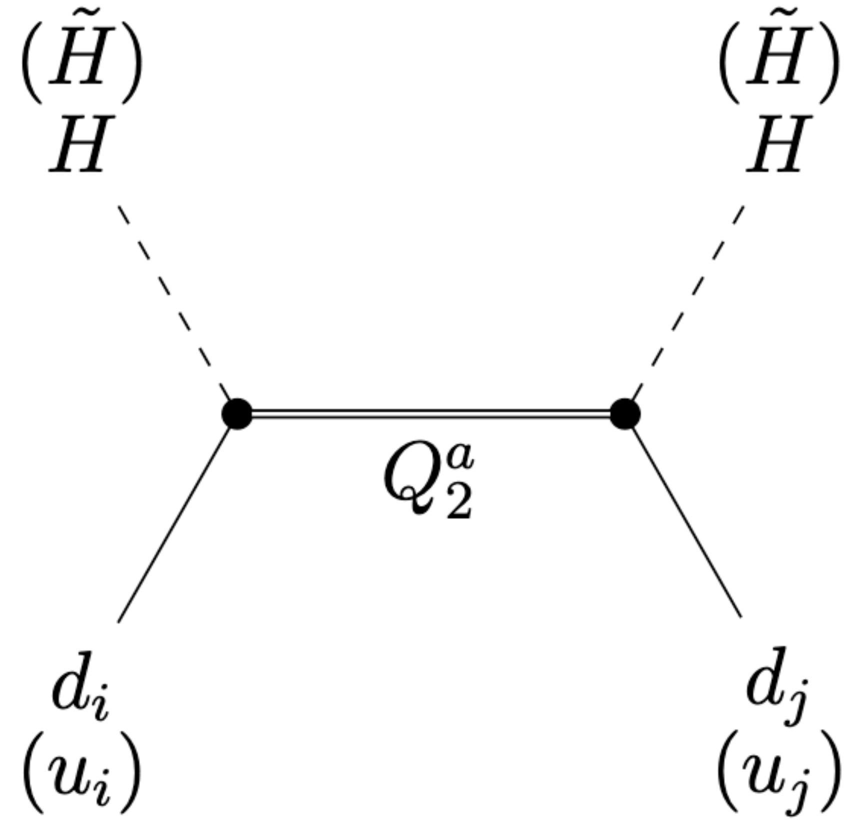


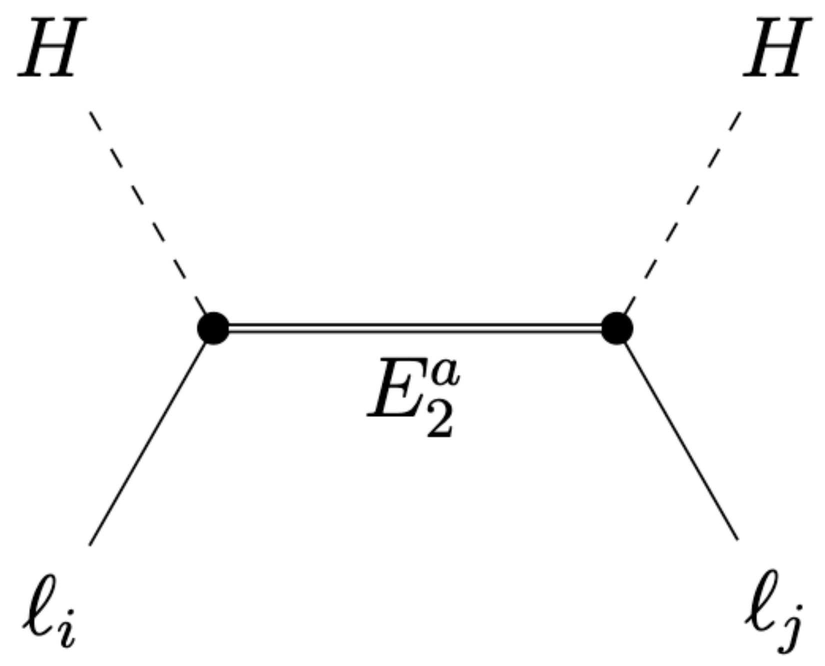
<u>Observables</u>:

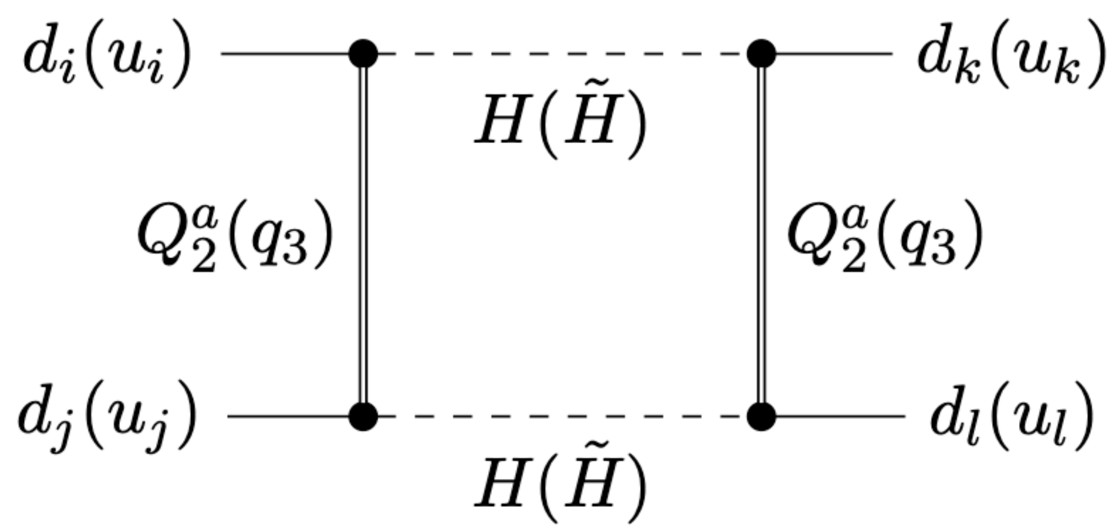
- Rare meson decays
- Rare lepton decays
- Meson mixings
- …

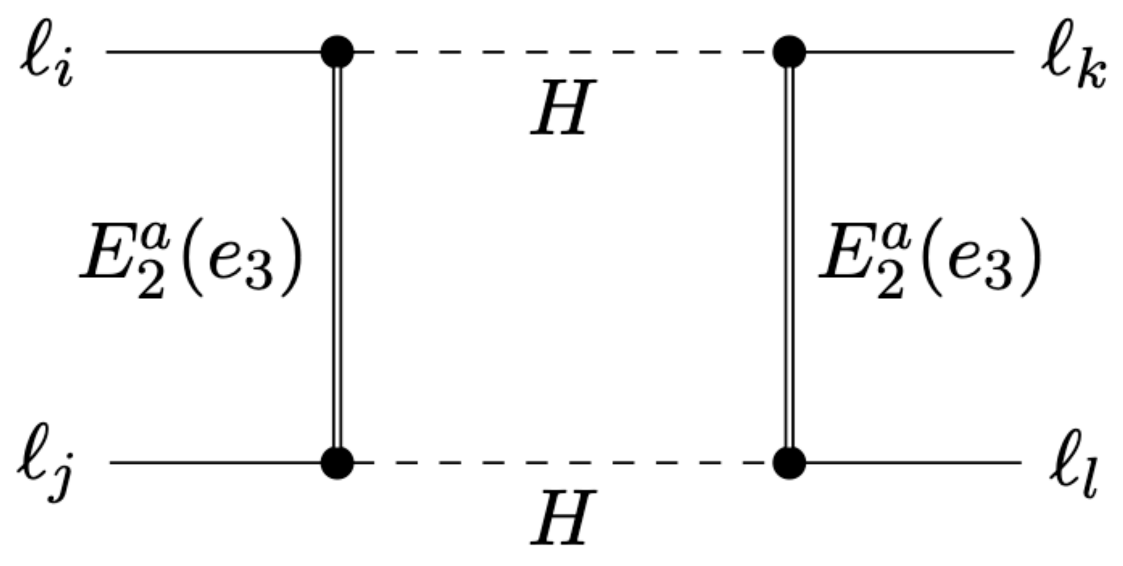
Phenomenogy: tower of EFTs

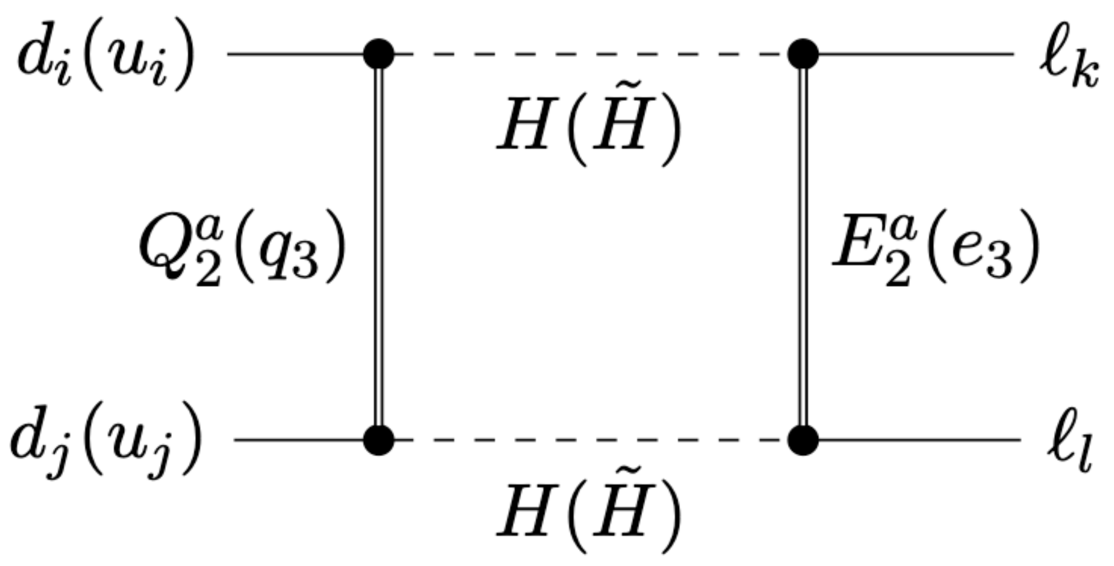
- **VLF**











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- …

$\boxed{v_\Phi \sim \epsilon M_{Q,E}}$ $\Rightarrow$ $\boxed{v_\Phi \gtrsim 3\ \textbf{TeV}}$

Leading: ϵ_K

Phenomenogy: tower of EFTs

- VLF
- **ALP**

$$\mathscr{L} \supset \sum_{f=u,d,e} \frac{1}{v_\Phi} c^f_{ij} \bar{f}_{L,i}(\rho + ia) f_{R,j} + \text{h.c.} + \text{h.o.}$$

See also:

Björkeroth, Chun, King (2018)

Bauer, Schnell, Plehn (2016)

Bauer, Neubert, Renner, Schnubel, Tamm (2017,2020,2022)

….

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….

- **c^f_{ij} fixed by FN mechanism**
- **Free parameters:** m_a, v_Φ

Here $m_a \sim v_\Phi$ natural expectation ($\lambda_4 \sim 1$)

$$c^d \sim \frac{1}{v_\Phi} \begin{pmatrix} m_d & m_s \frac{m_d}{m_s} & m_b \frac{m_d}{m_b} \\ m_d \frac{m_d}{m_s} & m_s & m_b \frac{m_s}{m_b} \\ m_d \frac{m_d}{m_d} & m_s \frac{m_s}{m_b} & m_b \frac{m_s^2}{m_b^2} \end{pmatrix}$$

($c^u, (c^e)^t$ analogous)

Phenomenogy: tower of EFTs

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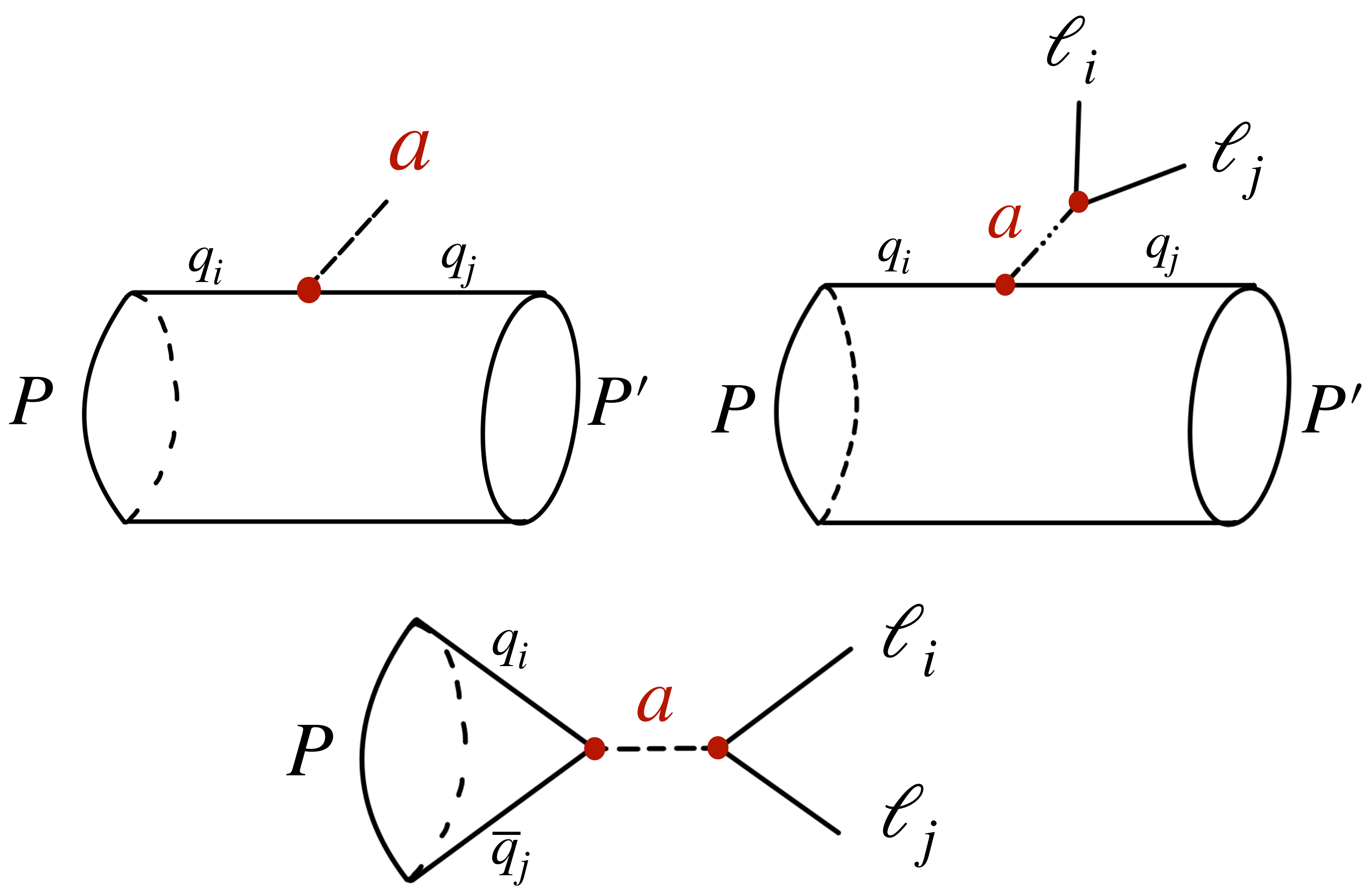
$$\mathscr{L} \supset \sum_{f=u,d,e} \frac{1}{v_\Phi} c^f_{ij}\, \bar{f}_{L,i}(\rho + ia) f_{R,j} + \text{h.c.} + \text{h.o.}$$



(Leading)

$K \to \pi a, B \to K a$

Displaced $B \to K^{(*)}\mu\mu$

$K_L \to \mu\mu, K_L \to \mu e$

Rare meson decays

Phenomenogy: tower of EFTs

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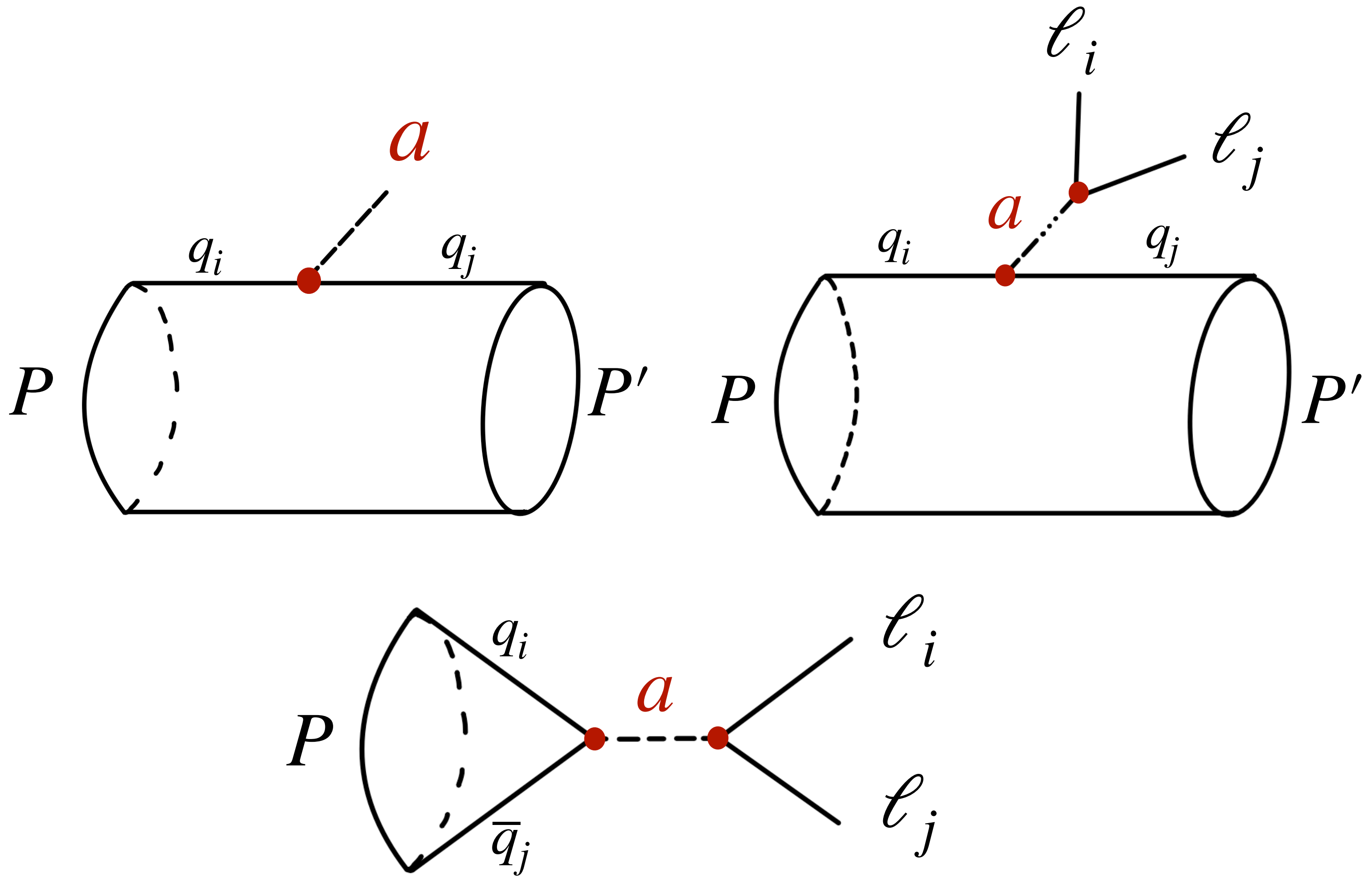

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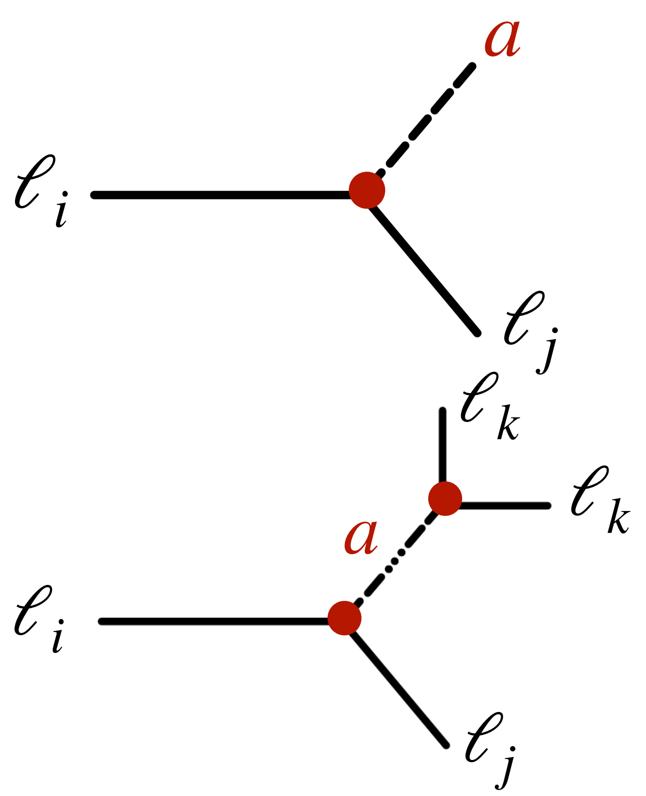

(Leading)

$\tau \to \mu a$

$\tau \to 3\mu$ (displaced?)

Rare lepton decays

Phenomenogy: tower of EFTs

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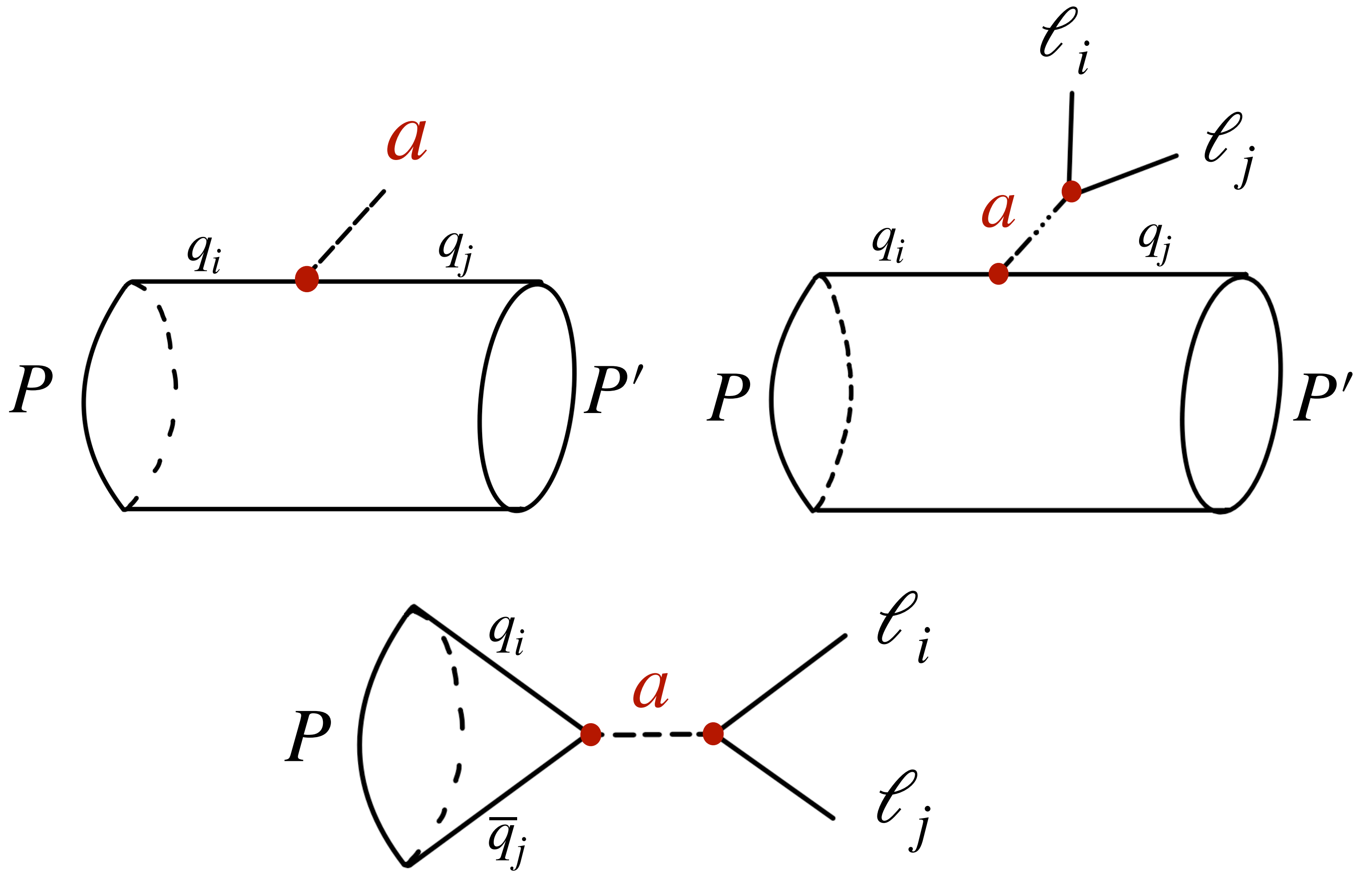
$$\mathscr{L} \supset \sum_{f=u,d,e} \frac{1}{v_\Phi} c^f_{ij}\, \bar{f}_{L,i}(\rho + ia) f_{R,j} + \text{h.c.} + \text{h.o.}$$



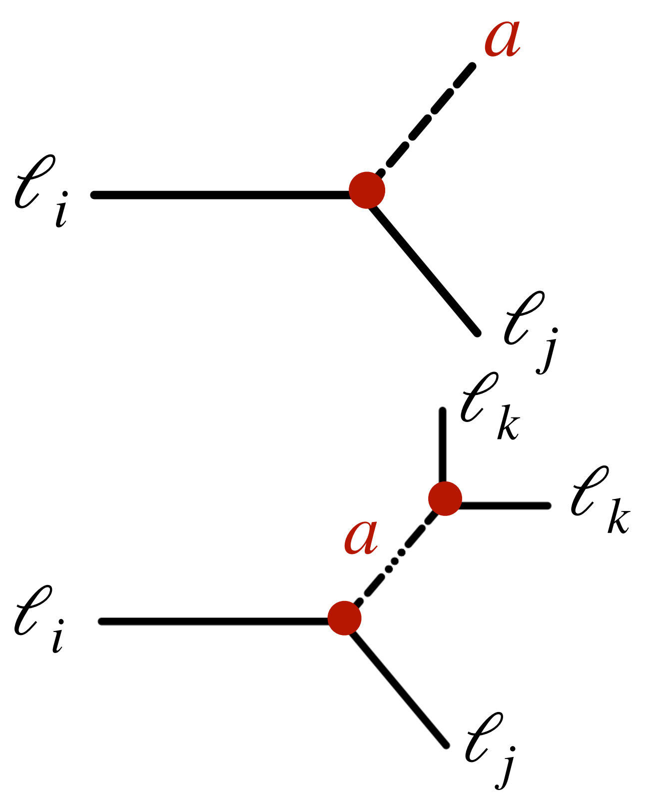
(Leading)

$K \to \pi a, B \to Ka$

<u>Displaced</u> $B \to K^{(*)}\mu\mu$

$K_L \to \mu\mu, K_L \to \mu e$

Rare meson decays



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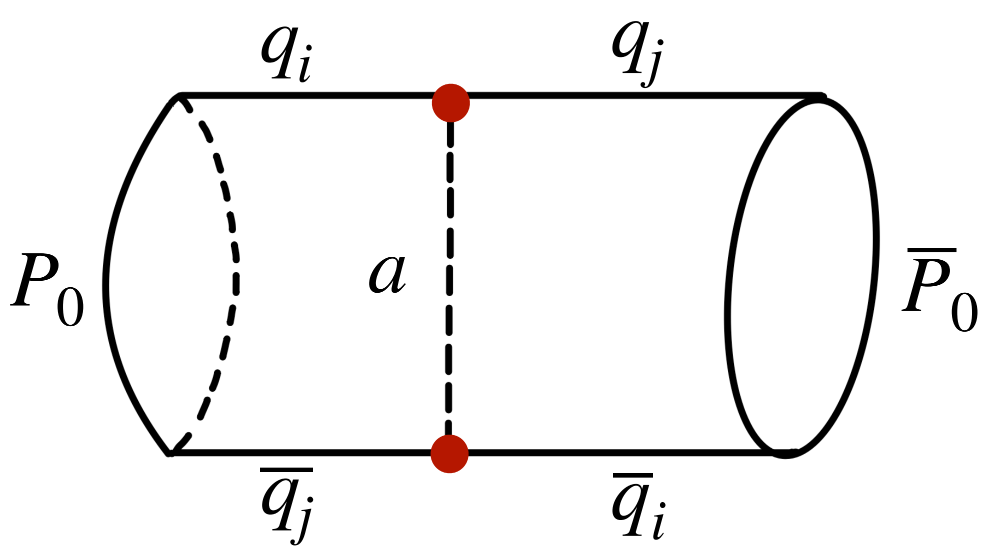


(Leading)

$K - \bar{K}\ \ (\epsilon_K)$

$\Delta F = 2$

Phenomenogy: tower of EFTs

- VLF
- **ALP**

$$\mathscr{L} \supset \sum_{f=u,d,e} \frac{1}{v_\Phi} c^f_{ij} \bar{f}_{L,i}(\rho + ia) f_{R,j} + \text{h.c.} + \text{h.o.}$$

+ $\ell_i \to \ell_j \gamma$ ⟶ $\tau \to \mu\gamma, \mu \to e\gamma, \ldots$

+ $\mu \to e$ **conversion**

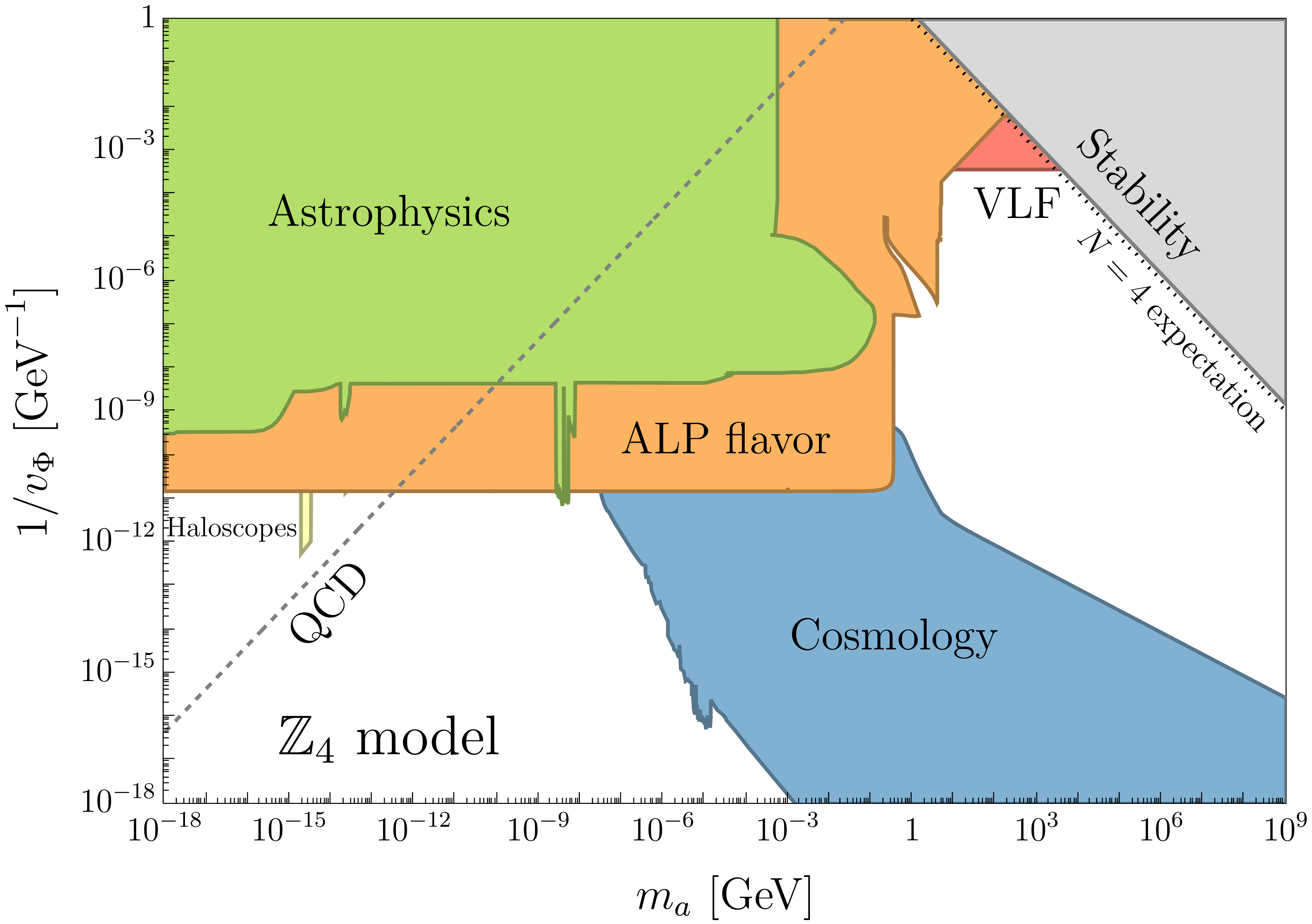
+ …

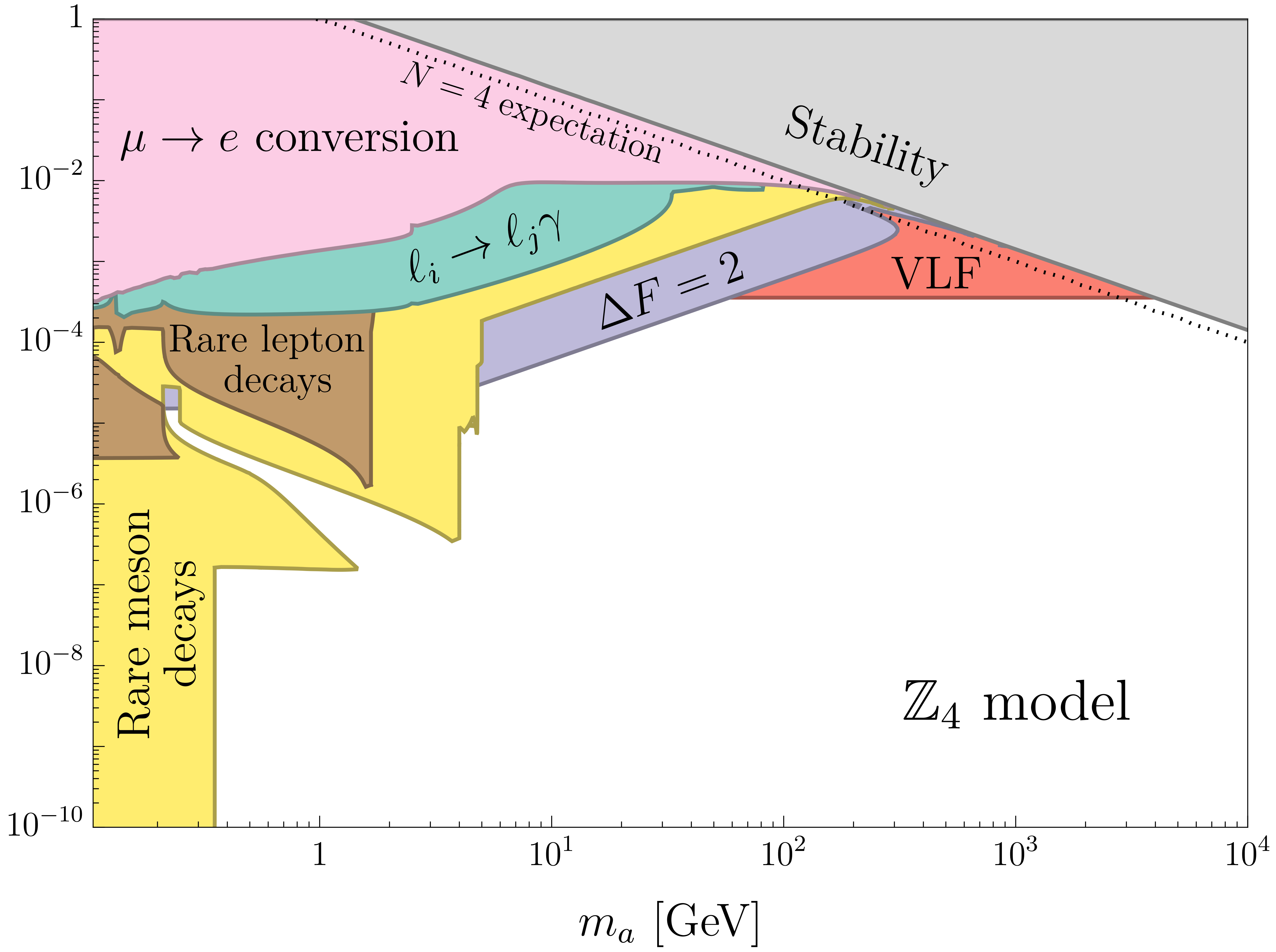
+ Cosmology ⟶ BBN ($\Gamma_a \gtrsim H(T_{\text{BBN}})$, X-rays, …

+ Astrophysics ⟶ SN1987, …

+ Haloscopes + …

3. The minimal model: Z_4





"Natural" Z_4 probed by UV completion, $v_\Phi \sim$ few TeV allowed!

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4. Conclusion

- FN models: popular, high-scale flavor models
- *Discrete* FN models allow massive ALP, lower FN scale
- Z_4: minimal, predictive framework with simple UV completion
- Phenomenology: interplay between VLF and ALP crucial
- *FN models can live at few TeV scale!*

Thank you for your attention!

Backup

Z_N potential

$$V(\Phi) = -m^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 - \frac{1}{4}\frac{\lambda'_N}{M_\Phi^{N-4}}\left[\Phi^N + (\Phi^*)^N\right] + \dots$$

$$\mathscr{L}_{\rm UV} \supset \sum_k y_k \Phi \bar{\Psi}_{L,k+1}\psi_{R,k} \longrightarrow \quad \frac{\lambda'_N}{M_\Phi^{N-4}} \sim \frac{1}{M_\psi^{N-4}}\frac{\prod_k y_k}{16\pi^2}$$

$$\frac{m_a}{v_\Phi} \sim \frac{N}{8\pi}\epsilon^{N/2-2}\sqrt{\prod_k y_k}$$

- $\epsilon^{N/2} \sim (y_e/y_t) \longrightarrow$ large N disfavored $\left(\epsilon \sim (y_e/y_t)^{2/N}\right)$

Z_8 model

$$q_{1,2,3} \sim (2,1,0) \qquad \bar{u}_{1,2,3} \sim (2,1,0) \qquad \bar{d}_{1,2,3}(2,2,2)$$

$$\ell_{1,2,3} \sim (4,4,4) \qquad \bar{e}_{1,2,3} \sim (0,1,2)$$

$$\begin{cases} \hat{y}^{d,e}_{ii} \sim (\epsilon^4, \epsilon^3, \epsilon^2) \\ \hat{y}^{u}_{ii} \sim (\epsilon^4, \epsilon^2, 1) \end{cases}$$

$$\epsilon \simeq 6.6 \times 10^{-2}$$

$$Y_d \simeq \begin{pmatrix} 0.55\epsilon^4 & 2.5\epsilon^4 & (0.73-1.84i)\epsilon^4 \\ 0 & 0.74\epsilon^3 & 1.51\epsilon^3 \\ 0 & 0 & 2.4\epsilon^2 \end{pmatrix}, \quad Y_u \simeq \begin{pmatrix} 0.25\epsilon^4 & z_{u_2}\epsilon^3 & z_{u_3}\epsilon^2 \\ y_{u_1}\epsilon^3 & 0.57\epsilon^2 & y_{u_3}\epsilon \\ x_{u_1}\epsilon^2 & x_{u_2}\epsilon & 0.71 \end{pmatrix}$$

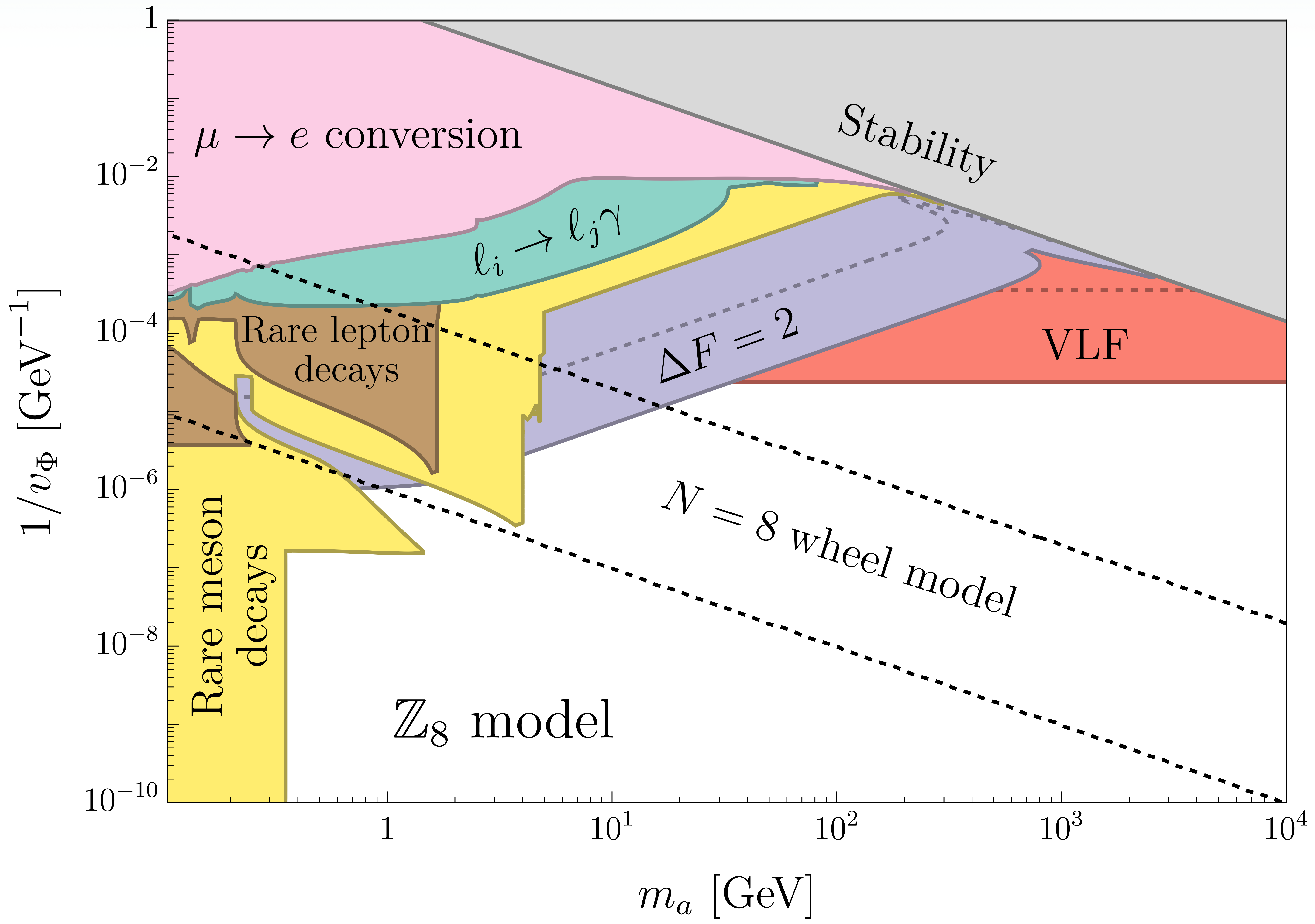
$$Y_e \simeq \begin{pmatrix} 0.15\epsilon^4 & 0 & 0 \\ z_{e_2}\epsilon^4 & 2.1\epsilon^3 & 0 \\ z_{e_3}\epsilon^4 & y_{e_3}\epsilon^3 & 2.3\epsilon^2 \end{pmatrix}.$$

Z_8 model

UV completion:

$$[L_N] = 0, \quad [L^a_{N-1}] = -1, \quad [L^i_{N-2}] = -2, \quad [L^i_{N-3}] = -3$$

$$\begin{aligned} [Q^a_2] &= 0, \quad [Q_1] = 1, \\ [U^a_2] &= 0, \quad [U_1] = -1, \\ [D^i_2] &= 0, \quad [D^i_1] = -1. \end{aligned}$$



$\mu \to e$ conversion
Stability
$\ell_i \to \ell_j\gamma$
Rare lepton decays
$\Delta F = 2$
VLF
$N = 8$ wheel model
Rare meson decays
$\mathbb{Z}_8$ model
$1/v_\Phi$ [GeV^{-1}]
m_a [GeV]
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1
10^1
10^2
10^3
10^4