Axion-like particles and CP violation

Based on ArXiv:2311.12158, ArXiv:2312.17310 and work in progress with L. Di Luzio and P. Paradisi

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Axion-Like Particles: Motivations

Axion-Like Particles (ALPs)

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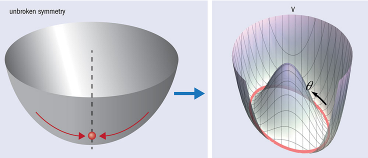

(Softly broken) **shift symmetry**:

- (small) mass terms
- (mainly) derivative interactions

Differently from the QCD axion, ALPs:

- Need not solving the strong CP problem
- Have arbitrary masses and couplings ($f_\phi m_\phi \not\propto f_\pi m_\pi$)

Axion-Like Particles: Motivations

ALPs can address several open problems in particle physics:

- Strong CP problem (**QCD axion**)
- Hierarchy problem (**relaxion**)
- Flavour problem (**axiflavon/flaxion**)
- The observed **dark matter** abundance

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ALPs can be **probed experimentally** via:

- **Higgs and Z boson decay processes** ($h \to Z\phi$, $Z \to \gamma\phi$)
- **Flavour-changing neutral current** processes ($K^{\pm} \to \pi^{\pm}\phi$)
- **Electric Dipole Moments (EDMs)**: **Electric Dipole Moments (EDMs)** are flavour-diagonal, CP-violating observables with (basically) **no SM background**

Probing the CP violating ALP-I

How can we **probe CPV ALPs** at low energies? Start from

$SU(3)_c \times U(1)_{em}$ invariant EFT for a CPV ALP ϕ at the EW scale

$$\mathcal{L}_{\rm ALP}^{\text{dim-5}} \supset e^2 \frac{C_\gamma}{\Lambda} \phi F^{\mu\nu} F_{\mu\nu} + g_s^2 \frac{C_g}{\Lambda} \phi G_a^{\mu\nu} G^a_{\mu\nu} + \frac{v}{\Lambda} y_S^{ij} \phi \bar{f}_i f_j$$
$$e^2 \frac{\tilde{C}_\gamma}{\Lambda} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} + g_s^2 \frac{\tilde{C}_g}{\Lambda} \phi G_a^{\mu\nu} \tilde{G}^a_{\mu\nu} + i \frac{v}{\Lambda} y_P^{ij} \phi \bar{f}_i \gamma_5 f_j + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

[Di Luzio, Gröber, Paradisi,'20]

- CP-even. Non-derivative, shift-symmetry violating
- CP-odd. Derivative, shift-symmetry preserving

Jarlskog invariants: basis-independent measures of CP violation

$$C_a \tilde{C}_b, y_S^{ii} \tilde{C}_a, \; y_P^{ii} C_a, \; y_S^{ii} y_P^{jj}, \; y_S^{ik} y_{SM}^{kk} y_P^{ki}$$

[Di Luzio, Gröber, Paradisi,'20] [Bonnefoy, Grojean, Kley,'22]

Probing the CP violating ALP-II

Three regimes:

- $m_\phi \gtrsim$ few GeV: QCD is perturbative [Di Luzio, Gröber, Paradisi,'20]
- 1GeV $\lesssim m_\phi \lesssim$ few GeV. Dispersive approach?
- $m_\phi \lesssim 1$ GeV: QCD confines and χpt [Di Luzio, GL, Paradisi,'23]

Probing the CP violating ALP-II

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Different approaches are required, but common features are:

- **Renormalization** of $\mathcal{L}_{\text{ALP}}^{\text{dim-5}}$ + **running** of its Wilson coefficients [Chala, Guedes, Ramos, Santiago,'20],[Bakshi, Machado-Rodríguez, Ramos,'23],[Bauer, Neubert, Renner, Schnubel, Tamm,'20],[Bonilla, Brivio, Gavela, Sanz,'20][Bresciani, Brunello, GL, Mastrolia, Paradisi,'24]
- Lagrangian **Matching** on effective low-energy descriptions
- Classification of the **CPV Jarlskog invariants** of the theory
- **Experimental bounds** in terms of the Jarlskog invariants

Light ALPs ($m_\phi \lesssim 1$ GeV) - I

External gauge and scalar **fields** enter as sources in $\mathcal{L}_{\text{QCD}}$:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}^0_{\text{QCD}} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q - \bar{q}(s - i\gamma_5 p)q$$

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These enter $\mathcal{L}_{\chi \mathrm{pt}}$ via

$$\mathcal{L}_{\chi \mathrm{PT}} = \frac{f^2}{4}\mathrm{Tr}\left[D_\mu\Sigma^\dagger D^\mu\Sigma + \Sigma^\dagger\chi + \chi^\dagger\Sigma\right]$$

$$D_\mu\Sigma = \partial_\mu\Sigma + i\Sigma\ell_\mu - ir_\mu\Sigma, \qquad \chi = 2B_0(s + ip)$$

Light ALPs ($m_\phi \lesssim 1$ GeV) - I

External gauge and scalar **fields** enter as sources in $\mathcal{L}_{\rm QCD}$:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q - \bar{q}(s - i\gamma_5 p)q$$

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Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality**

$$\int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}G_\mu\,\exp\left(i\int d^4x\,\mathcal{L}^{\rm ext}_{\rm QCD}\right) = \int \mathcal{D}\Sigma\exp\left(i\int d^4x\,\mathcal{L}^{\rm ext}_{\chi{\rm pt}}\right)\ (*)$$

From quarks to mesons

We want to find the **chiral counterpart** to our Lagrangian

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{C_\gamma}{\Lambda} \phi\, F\, F + e^2 \frac{\tilde{C}'_\gamma}{\Lambda} \phi\, F \tilde{F} + g_s^2 \frac{C_g}{\Lambda} \phi\, G\, G + g_s^2 \frac{\tilde{C}'_g}{\Lambda} \phi\, G \tilde{G} + \frac{\partial_\mu \phi}{\Lambda} \bar{q}\, \gamma^\mu (Y_S + Y_P \gamma_5)\, q + \frac{v}{\Lambda} \phi\, \bar{q}\, y_{q,S}\, q + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality** (*). For instance:

Example

$$\bar{q}_i y_{ij}^S q_j = -y_{ij}^S \frac{\partial \mathcal{L}_{\text{QCD}}}{\partial y_{ij}^S} \longrightarrow -y_{ij}^S \frac{\partial \mathcal{L}_{\chi\text{pt}}}{\partial y_{ij}^S} = -\frac{f_\pi^2}{2} B_0 \text{Tr}\left[y^S (\Sigma + \Sigma^\dagger) \right]$$

Light ALPs ($m_\phi \lesssim 1$ GeV) - II

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{c_\gamma}{\Lambda} \phi F F + e^2 \frac{\tilde{c}_\gamma}{\Lambda} \phi F \tilde{F} + \frac{\partial_\mu \phi}{\Lambda} \bar{q} \gamma^\mu (Y_V + Y_A \gamma_5) q$$
$$- \kappa \frac{\phi}{\Lambda} T^\mu{}_\mu + \frac{v}{\Lambda} \phi \bar{q} \mathcal{Z} q + \bar{q}_L M_q^\phi q_R + \text{h.c.} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Its counterpart is found by using the **duality** in (*)

low-energy CP-violating ALP Lagrangian

$$\mathcal{L}_{\phi\chi} = -\frac{1}{3} \frac{m_\pi^2}{m_\pi^2 - M_\phi^2} \frac{\Delta_{ud}}{f_\pi \Lambda} \Big[-2\partial\phi \left(2\pi^+\pi^-\partial\pi_0 + \pi_0\pi^+\partial\pi^- + \pi_0\pi^-\partial\pi^+\right)$$
$$+ M_\phi^2 \phi \left(\pi_0^3 + 2\pi^+\pi^-\pi_0\right)\Big] + 2\kappa \frac{\phi}{\Lambda} [\partial_\mu \pi^+ \partial^\mu \pi^- + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0]$$
$$- m_\pi^2 \omega \frac{\phi}{\Lambda} [\pi^+\pi^- + \frac{1}{2}\pi_0^2] + C_N^S \frac{\phi}{\Lambda} \bar{N}_v N_v + C_N^A \frac{\partial_\mu \phi}{\Lambda} \bar{N}_v \gamma^\mu \gamma_5 N_v$$
$$+ e^2 \widetilde{C}'_\gamma \frac{\phi}{\Lambda} F \widetilde{F} + e^2 C'_\gamma \frac{\phi}{\Lambda} FF + +i \frac{v}{\Lambda} y_{P,\ell}^{ij} \phi \bar{\ell}_i \gamma_5 \ell_j + \frac{v}{\Lambda} y_{S,\ell}^{ij} \phi \bar{\ell}_i \ell_j$$

CPV Jarlskog invariants ($n_f = 2$)

The **low-energy Jarlskog invariants** are found from $\mathcal{L}_{\phi\chi}$ by multiplying the Wilson coefficients of operators possessing **opposite CP** transformation properties

Example

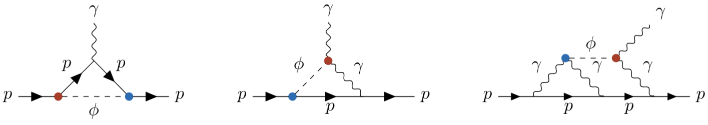
$$\begin{array}{l} c_\gamma FF \xrightarrow{CP} c_\gamma FF \\ \tilde{c}_\gamma F\widetilde{F} \xrightarrow{CP} -\tilde{c}_\gamma F\widetilde{F} \end{array} \longrightarrow \quad c_\gamma \tilde{c}_\gamma \text{ is a Jarlskog invariant!}$$

Matching dictionary between UV and IR Jarlskog invariants!

$$c_\gamma \tilde{c}_\gamma = \left(C_\gamma - \frac{\beta^0_{\mathrm{QED}}}{\beta^0_{\mathrm{QCD}}} C_g \right) \left(\tilde{C}_\gamma - 4N_c \,\mathrm{tr}\,(Q_A Q_q^2)\, \tilde{C}_g \right)$$

Phenomenological applications

- **EDMs** of protons, neutrons, atoms, molecules ...



$$d_p \simeq -\frac{e\,Q_p}{4\pi^2\Lambda^2}\Bigg[C_{\phi pp}\,\tilde{C}_{\phi p} + e^2 m_p c_\gamma\,\tilde{C}_{\phi p}\left(6 + 2\ln\frac{\Lambda_{\rm ren}^2}{m_p^2}\right) + e^2\tilde{c}_\gamma\,C_{\phi pp}\left(2 + \ln\frac{\Lambda_{\rm ren}^2}{m_p^2}\right) + 3\frac{Q_p^2}{\pi^2} m_p e^6 c_\gamma\,\tilde{c}_\gamma \ln^2\frac{\Lambda_{\rm ren}}{m_\phi}\Bigg]$$

$$d_n \sim -\frac{e\,\mathcal{C}_n}{4\pi^2\Lambda^2}\left[-\frac{3}{8}C_{\phi nn}\,\tilde{C}_{\phi n}\frac{\Lambda_{\rm ren}^2}{m_n^2} + e^2(2m_n c_\gamma\,\tilde{C}_{\phi n} + 3\tilde{c}_\gamma\,C_{\phi nn})\ln\frac{\Lambda_{\rm ren}^2}{m_n^2}\right]$$

$$\longrightarrow C_g\,\tilde{C}_g < 4.4\times 10^{-8}$$

Interplay with other precision observables

Interplays with other precision observables are important ...

- **flavour probes**: Kaon decays $K \to \pi\phi\,(\phi \to \text{inv})$

  BR $(K^+ \to \pi^+ + \text{inv})$ and BR $(K_L \to \pi_0 + \text{inv})$ to probe Y_V^{ds}:

  $$|Y_V^{ds}| \lesssim 1.4 \times 10^{-9} \frac{\Lambda}{\text{TeV}} \qquad |\text{Im } Y_V^{ds}| \lesssim 3.6 \times 10^{-9} \frac{\Lambda}{\text{TeV}}$$

  Similarly for $K \to \pi\pi\phi(\phi \to \text{inv}, m_\phi \ll m_\pi)$

  $$|Y_A^{ds}| \lesssim 1.1 \times 10^{-5} \frac{\Lambda}{\text{TeV}} \qquad |\text{Re } Y_A^{ds}| \lesssim 1.7 \times 10^{-6} \frac{\Lambda}{\text{TeV}}$$

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- **magnetic moments**: if $d_e \neq 0$, what about $(g-2)_e$?
- **lepton flavour violation**: what about $\mu \to eee$?

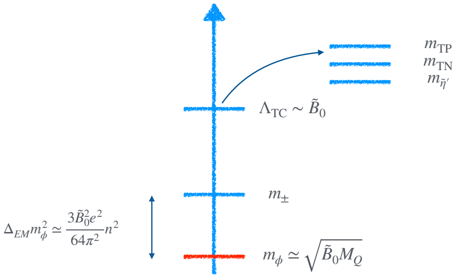
. . . as they provide a handle on individual Wilson coefficients!

CPV ALP from composite dynamics - I

Key idea: new **confining dark sector** with a **sizable theta term**

$$\mathcal{L}_D = -\frac{1}{4}\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a + \bar{\mathcal{Q}} i\gamma^\mu D_\mu \mathcal{Q} - \frac{\theta_{TC}}{32\pi^2}\mathcal{F}^a_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}_a + \bar{\mathcal{Q}}\gamma^\mu(v_\mu + a_\mu\gamma_5)\mathcal{Q} - \mathcal{Q}(s - ip\gamma_5)\mathcal{Q}$$

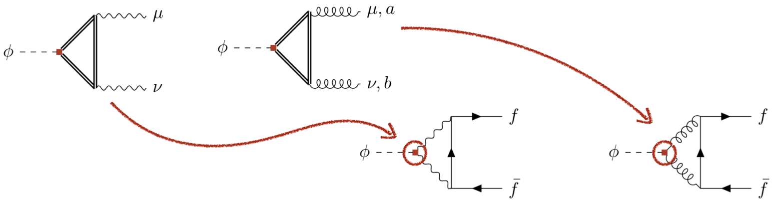
Confinement: dark mesons and baryons.
The **lightest SM gauge singlet** to be identified as the CPV ALP. (Bonus: DM?)
[Abe, Sato, Yamanaka, '24.]



CPV ALP from composite dynamics - II

CPV interactions are generated as for **the SM** π_0:

- $\phi F\tilde{F}$ and $\phi G\tilde{G}$: **anomalous** couplings to gauge bosons
- ϕFF and ϕGG: via **heavy dark baryon loops**
- $\phi\bar{\psi}\psi$ and $\phi\bar{\psi}\gamma_5\psi$: generated radiatively



CPV ALP from relaxion models

CPV interactions from **mixing** between an ALP and the Higgs:

- $\phi F\tilde{F}$, $\phi G\tilde{G}$, $\partial_\mu\phi\,\bar{\psi}\gamma^\mu\psi$ from the shift-symmetry preserving ALP interactions
- ϕFF, ϕGG and $\phi\bar{\psi}\psi$ from the Higgs portal:

$$\mathcal{L}_{\text{ALP-Higgs}} = |H^\dagger H| \left(\mathcal{C}_a \cos\frac{\Phi}{\Lambda} + \mathcal{C}_b \sin\frac{\Phi}{\Lambda} \right) + |H^\dagger H|^2 \left(\mathcal{C}_c \cos\frac{\Phi}{\Lambda} + \mathcal{C}_d \sin\frac{\Phi}{\Lambda} \right)$$

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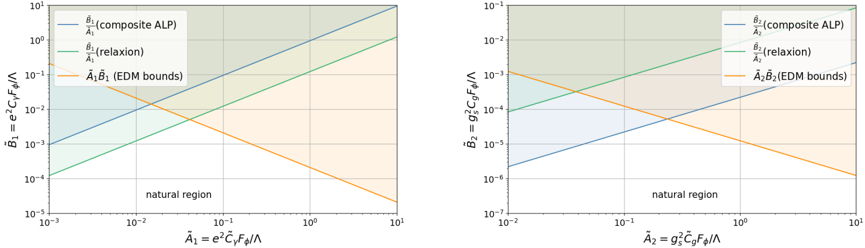

Summary

New physics at the precision frontier: CP-violating ALPs. We have

- Shown that **EDMs** are flavour-diagonal probes of CP violation and offer huge potentialities for discoveries (here ALPs)
- Provided the **matching dictionary** relating the IR couplings in low-energy Lagrangians to the UV couplings at the EW scale
- Classified the **Jarlskog invariants** of the theory
- **Explored the parameter space** for light and heavy ALPs
- Identified the **natural regions** of the parameter space
- `FeynRules` **model** available for both the 2- and the 3-flavors setting in χpt $\rightarrow$ extensive, automatized pheno analyses

Thanks for your attention!

Backup slides

Heavy ALPs ($m_\phi \gtrsim$ few GeV) - I

Running from the EW scale to the ALP mass scale $m_\phi \gtrsim 5$ GeV, then one-loop matching onto [Pospelov, Ritz,'05]

$$\mathcal{L}_{\rm CPV} = \sum_{i,j=u,d,e} C_{ij}(\bar{f}_i f_i)(\bar{f}_j i\gamma_5 f_j) + \alpha_s C_{Ge}\, GG\, \bar{e} i\gamma_5 e + \alpha_s C_{\tilde{G}e}\, G\tilde{G}\, \bar{e}e$$
$$- \frac{i}{2}\sum_{i=u,d,e} d_i\, \bar{f}_i(F\cdot\sigma)\gamma_5 f_i - \frac{i}{2}\sum_{i=u,d} g_s\, d_i^C\, \bar{f}_i(G\cdot\sigma)\gamma_5 f_i + \frac{d_G}{3} f^{abc} G^a \tilde{G}^b G^c$$

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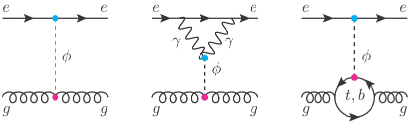

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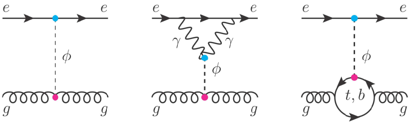

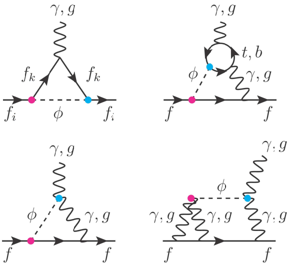

Heavy ALPs ($m_\phi \gtrsim$ few GeV) - II

Bounds are set from:

- **Neutron EDM**: $d_n^{\rm exp} < 1.8 \cdot 10^{-26} e\, cm$,
  $d_n \simeq 0.8 d_u - 0.2 d_d - 0.6 e\, d_u^C - 1.1 e\, d_d^C - 50 \text{ MeV } e\, d_G + 30 \text{ MeV } e\,(C_{ud} - C_{du})$
- **Hg EDM**: $d_{Hg}^{\rm exp} < 6 \cdot 10^{-30} e\, cm$,
  $d_{Hg} \simeq 4 \times 10^{-4} d_n - [2.8 C_S - 2.1 C_P] \times 10^{-22}$
- **ThO electron precession frequency**: $\omega_{ThO}^{\rm exp} < 1.3$ mrad/s ,
  $\omega_{ThO} = 1.2 \text{ mrad/s } \left(\frac{d_e}{10^{-29} \text{e cm}}\right) + 1.8 \text{ mrad/s } \left(\frac{C_S}{10^{-9}}\right)$

with $C_S/v^2 \simeq -17(C_{ue} + C_{de}) + 5 \text{ GeV } C_{Ge},\ C_P/v^2 \simeq 350(C_{eu} + C_{ed}) + 1 \text{ GeV } C_{\tilde{G}e}$.

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For instance ($m_\phi = 5$ GeV, $\Lambda = 1$ TeV):

- $|C_g \tilde{C}_g| < 1.4 \cdot 10^{-6}$ from $d_n, d_{Hg}(d_G)$
- $|y_S^{uu} y_P^{ee}|, |y_S^{dd} y_P^{ee}| < 2.1 \cdot 10^{-13}$ from $\omega_{ThO}(C_S)$

Getting rid of gluons

- Eliminate ϕGG thanks to the **trace anomaly** equation [Leutwyler, Shifman,'89]:

$$T^{\mu}{}_{\mu} = \sum_q m_q \bar{q} q - \frac{\alpha_s}{8\pi}\,\beta^0_{\mathrm{QCD}}\, G_a^{\mu\nu} G^a_{\mu\nu} - \frac{\alpha_{\mathrm{em}}}{8\pi}\,\beta^0_{\mathrm{QED}}\, F^{\mu\nu} F_{\mu\nu}$$

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- Eliminate $\phi G\tilde{G}$ via an **ALP-dependent quark field redefinition**[Georgi, Kaplan, Randall,'86]:

$$q \to q = \exp\left[i\frac{\phi}{\Lambda}\left(Q_V + \lambda_g^* Q_A \gamma_5\right)\right] q'$$

with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal, Tr(Q_A)$= 1/2$, $\lambda_g^* = 32\pi^2 \tilde{C}_g'$).

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- Eliminate ϕGG thanks to the **trace anomaly** equation [Leutwyler, Shifman,'89]:

$$T^{\mu}{}_{\mu} = \sum_q m_q \bar{q} q - \frac{\alpha_s}{8\pi} \beta^0_{\text{QCD}} \, G_a^{\mu\nu} G^a_{\mu\nu} - \frac{\alpha_{\text{em}}}{8\pi} \beta^0_{\text{QED}} \, F^{\mu\nu} F_{\mu\nu}$$

- Eliminate $\phi G \tilde{G}$ via an **ALP-dependent quark field redefinition**[Georgi, Kaplan, Randall,'86]:

$$q \to q = \exp\left[i \frac{\phi}{\Lambda} \left(Q_V + \lambda_g^* Q_A \gamma_5 \right) \right] q'$$

with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal, $\text{Tr}(Q_A) = 1/2$, $\lambda_g^* = 32\pi^2 \tilde{C}_g'$).

- Other **couplings** are **modified** (currents, masses, ...)!

Chiral Lagrangian for the CPV ALP

All of the previous modifications lead to the following

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{c_\gamma}{\Lambda}\, \phi\, FF + e^2 \frac{\tilde{c}_\gamma}{\Lambda}\, \phi\, F\tilde{F} + \frac{\partial_\mu \phi}{\Lambda}\, \bar{q}\gamma^\mu \left(Y_V + Y_A \gamma_5 \right) q$$
$$- \kappa \frac{\phi}{\Lambda}\, T^\mu{}_\mu + \frac{v}{\Lambda}\, \phi\, \bar{q} \mathcal{Z} q + \bar{q}_L M_q^\phi q_R + \text{h.c.} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Its counterpart is found by using the **duality** in (*)

Mesonic Chiral Lagrangian for a CP-violating ALP ϕ at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\chi\text{pt}} = \frac{\partial_\mu \phi}{\Lambda} \left[2\,\text{Tr}(Y_V T_a) j_V^{\mu,a} + 2\,\text{Tr}(Y_A T_a) j_A^{\mu,a} \right] + \frac{f_\pi^2}{2} B_0 \text{Tr} \left[M_\phi \Sigma^\dagger + \Sigma M_\phi{}^\dagger \right]$$
$$+ \kappa \frac{f_\pi^2}{2} \frac{\phi}{\Lambda} \left[\text{Tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + 4 B_0 \text{Tr} \left[M_q (\Sigma + \Sigma^\dagger) \right] \right]$$
$$- \frac{f_\pi^2}{2} \frac{v}{\Lambda} B_0\, \phi\, \text{Tr} \left[\mathcal{Z} (\Sigma + \Sigma^\dagger) \right] + e^2 \frac{c_\gamma}{\Lambda}\, \phi\, FF + e^2 \frac{\tilde{c}_\gamma}{\Lambda}\, \phi\, F\tilde{F} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Matching onto the low-energy Lagrangian ($n_f = 2$)

The $\mathcal{O}(\Lambda^{-2})$ low-energy Lagrangian $\mathcal{L}_{\phi\chi}$ valid for $E < 1-2$ GeV is:

low-energy CP-violating ALP Lagrangian

$$\mathcal{L}_{\phi\chi} = -\frac{1}{3}\frac{m_\pi^2}{m_\pi^2 - M_\phi^2}\frac{\Delta_{ud}}{f_\pi\Lambda}\Big[-2\partial\phi\left(2\pi^+\pi^-\partial\pi_0 + \pi_0\pi^+\partial\pi^- + \pi_0\pi^-\partial\pi^+\right)$$
$$+ M_\phi^2\phi\left(\pi_0^3 + 2\pi^+\pi^-\pi_0\right)\Big] + 2\kappa\frac{\phi}{\Lambda}[\partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0]$$
$$- m_\pi^2\omega\frac{\phi}{\Lambda}[\pi^+\pi^- + \frac{1}{2}\pi_0^2] + C_N^S\frac{\phi}{\Lambda}\bar{N}_v N_v + C_N^A\frac{\partial_\mu\phi}{\Lambda}\bar{N}_v\gamma^\mu\gamma_5 N_v$$
$$+ e^2\widetilde{C}_\gamma'\frac{\phi}{\Lambda}F\widetilde{F} + e^2 C_\gamma'\frac{\phi}{\Lambda}FF + +i\frac{v}{\Lambda}y_{P,\ell}^{ij}\,\phi\bar{\ell}_i\gamma_5\ell_j + \frac{v}{\Lambda}y_{S,\ell}^{ij}\,\phi\bar{\ell}_i\ell_j$$

All the couplings in $\mathcal{L}_{\phi\chi}$ can be expressed in terms of those in $\mathcal{L}_{\mathrm{ALP}}^{\mathrm{dim\text{-}5}}$ or at most of **measurable/computable** quantities.

Example: $Y_A^{ij} = -y_{q,P}^{ij}\frac{v}{m_i+m_j} - 32\pi^2\, Q_A^{ij}\,\tilde{C}_g$

CPV Jarlskog invariants ($n_f = 2$)

The **low-energy Jarlskog invariants** are found from $\mathcal{L}_{\phi\chi}$ by multiplying the Wilson coefficients of operators possessing **opposite CP** transformation properties

Example

$$c_\gamma FF \xrightarrow{CP} c_\gamma FF$$
$$\tilde{c}_\gamma F\widetilde{F} \xrightarrow{CP} -\tilde{c}_\gamma F\widetilde{F}$$
$\longrightarrow$ $c_\gamma \tilde{c}_\gamma$ is a Jarlskog invariant!

	c_γ	$y_{\ell,S}$	κ	$\mathcal{Z}$	$C_{\phi NN}$
$\tilde{c}_\gamma$	$\tilde{c}_\gamma\ c_\gamma$	$\tilde{c}_\gamma\ y_{\ell,S}$	$\tilde{c}_\gamma\ \kappa$	$\tilde{c}_\gamma\ \mathcal{Z}$	$\tilde{c}_\gamma\ C_{\phi NN}$
$y_{\ell,P}$	$y_{\ell,P}\ c_\gamma$	$y_{\ell,P}\ y_{\ell,S}$	$y_{\ell,P}\ \kappa$	$y_{\ell,P}\ \mathcal{Z}$	$y_{\ell,P}\ C_{\phi NN}$
Δ^A_{ud}	$\Delta^A_{ud}\ c_\gamma$	$\Delta^A_{ud}\ y_{\ell,S}$	$\Delta^A_{ud}\ \kappa$	$\Delta^A_{ud}\ \mathcal{Z}$	$\Delta^A_{ud}\ C_{\phi NN}$
$\tilde{C}_{\phi N}$	$\tilde{C}_{\phi N}\ c_\gamma$	$\tilde{C}_{\phi N}\ y_{\ell,S}$	$\tilde{C}_{\phi N}\ \kappa$	$\tilde{C}_{\phi N}\ \mathcal{Z}$	$\tilde{C}_{\phi N}\ C_{\phi NN}$

Table: Jarlskog invariants of the low-energy chiral Lagrangian $\mathcal{L}_{\phi\chi}$

Perturbative vs non-perturbative: matching