Functional Methods Beyond One-Loop Order

Based on [2311.13630] and [2412.12270]

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Universität Basel

Swiss National Science Foundation

Outline

- **Motivation**
  - Higher-loop computations in EFTs
- **Functional formalism at one- and two-loop order**
  - Generating functional, effective action, topologies
- **Applications**
  - Scalar toy model
  - Euler-Heisenberg Lagrangian

Motivation

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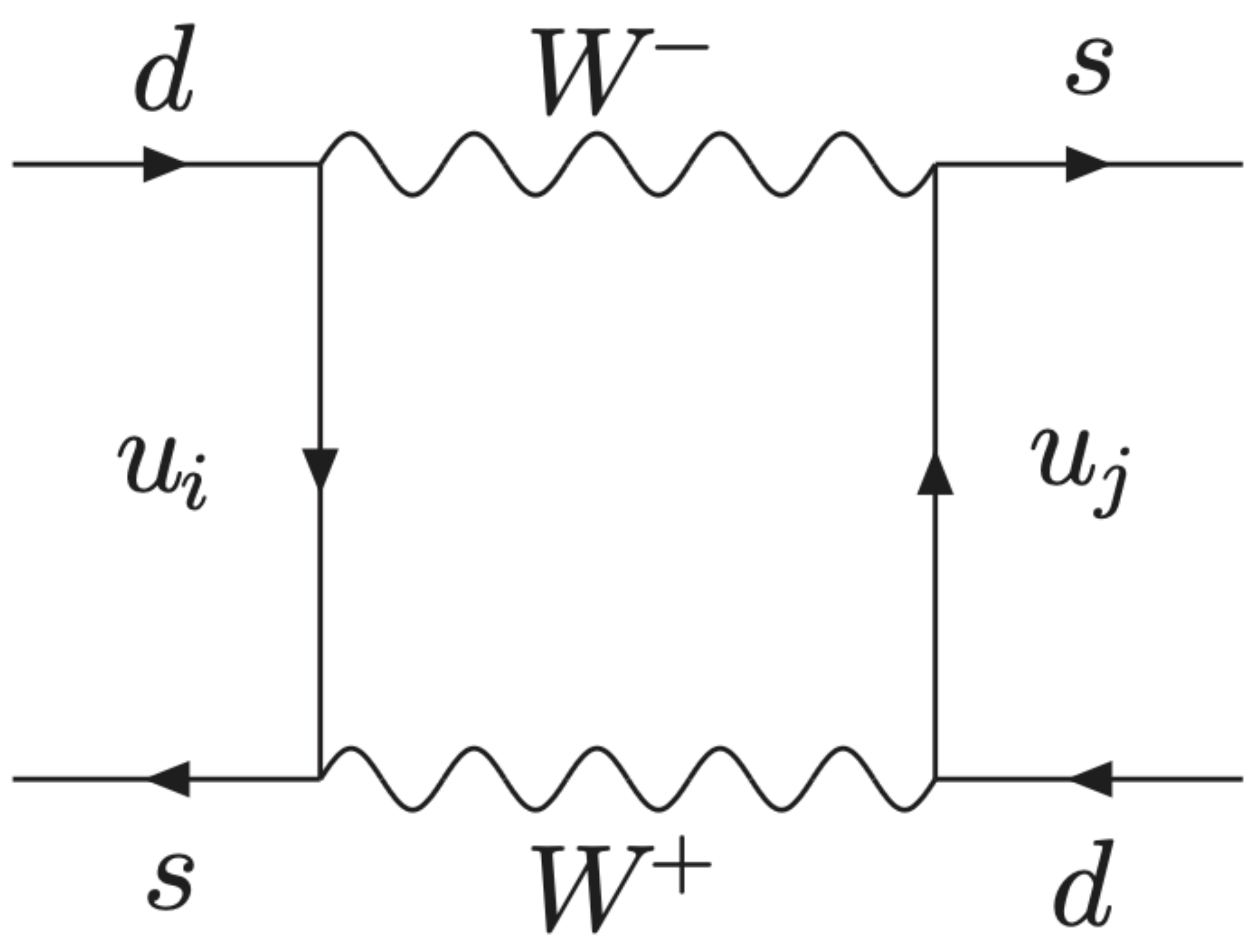

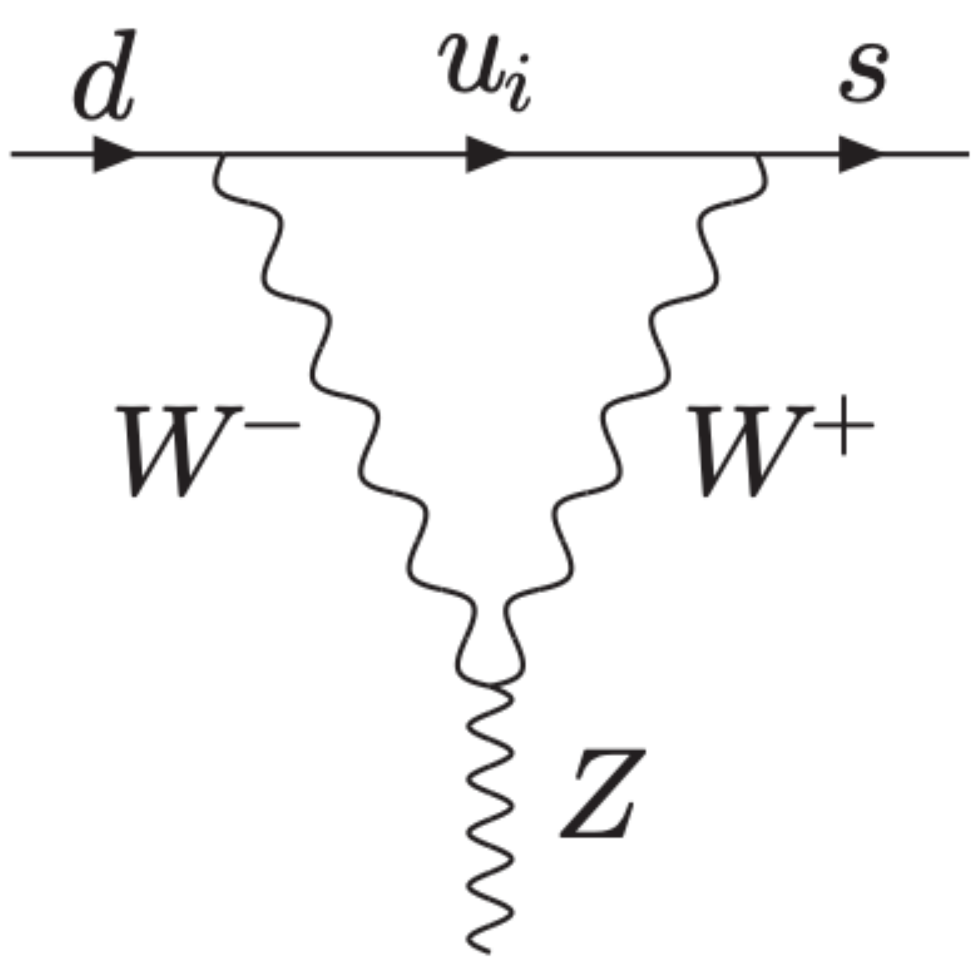

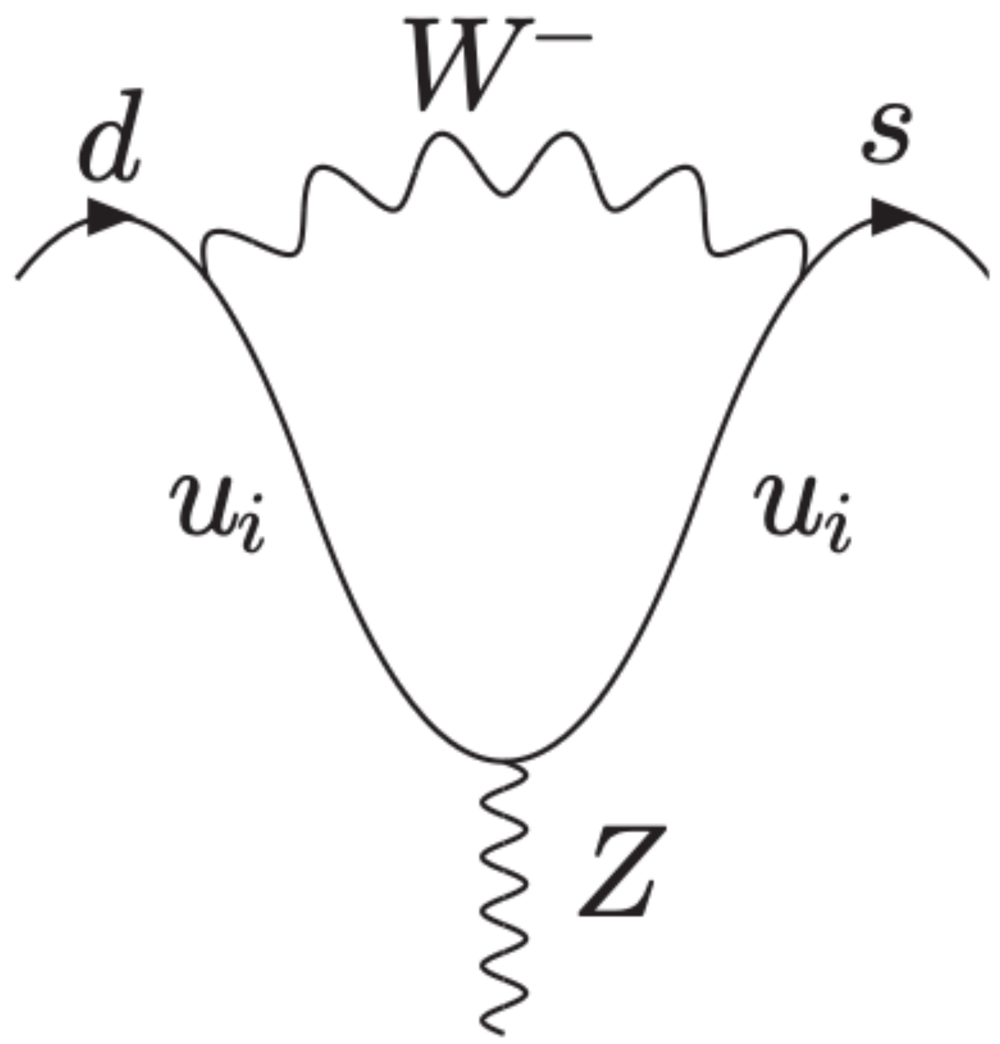

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  - Precision demands
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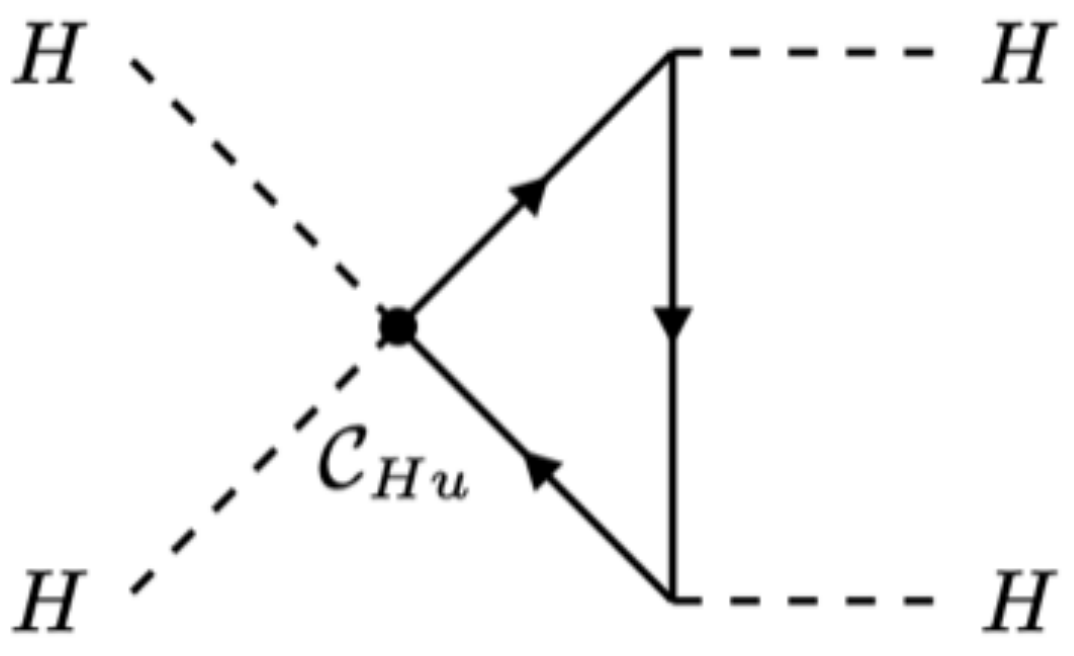
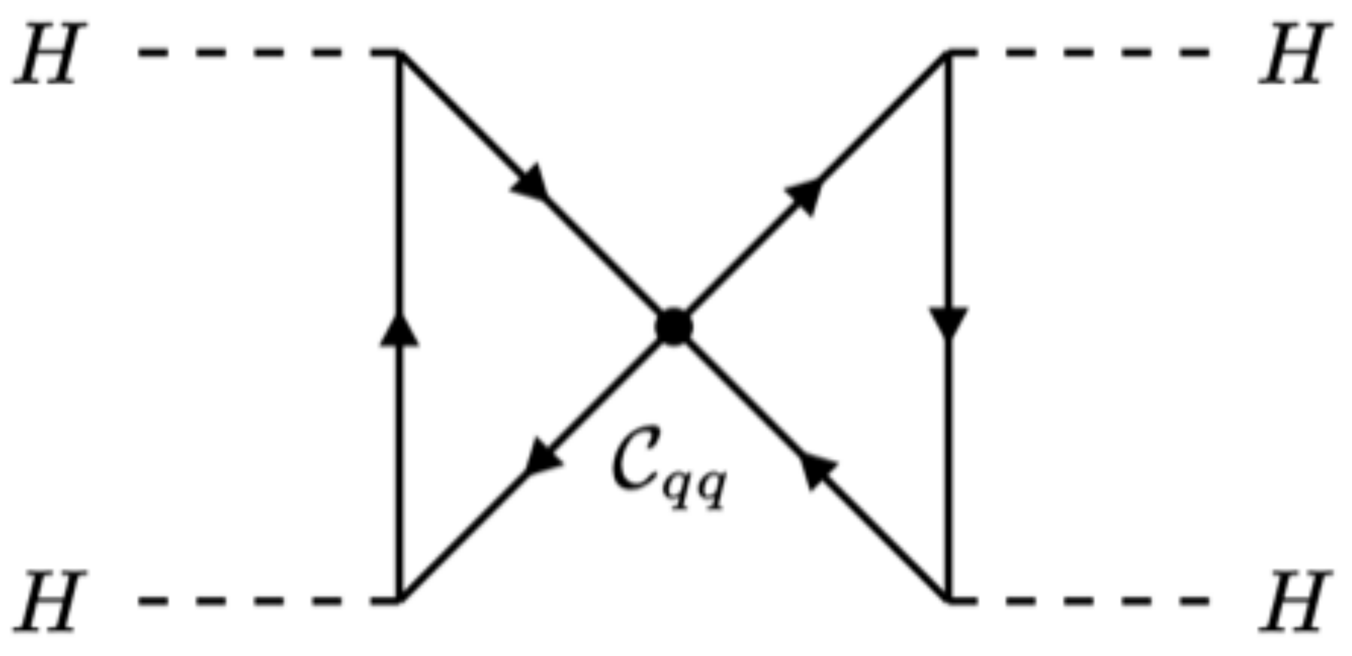
  - Operator mixing in EFT

$$16\pi^2\mu\frac{d\mathcal{C}_i}{d\mu} = \gamma_{ij}\mathcal{C}_j$$





[2302.11584]

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  - UV completions - neutrino mass models, ALP models

Diagrammatic vs functional approach

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- **Diagrammatic approach**
  - Based on equating the amplitudes
  - Valid at any loop order
  - Possible both on- and off-shell
  - Redundancies in EFT basis construction
  - ✶ Symmetry factors

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- **Functional approach**
  - Based on equating effective action

$$\Gamma_{\mathrm{UV}}(\Phi_H, \phi_L) = \Gamma_{\mathrm{EFT}}(\phi_L)$$

  - EFT basis automatically obtained
  - ✶ Symmetry factors automatically taken care of
  - ✶ Automation

Theoretical framework

Functional formalism

- **Generating vacuum functional**

$$e^{i\hbar^{-1}W[\mathcal{J}]} = \int [\mathcal{D}\eta] \exp\left[\frac{i}{\hbar}\left(S[\eta] + \mathcal{J}_I \eta_I\right)\right], \qquad S = \sum_{\ell=0}^{\infty} \hbar^{\ell} S^{(\ell)}$$

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Field configurations

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$$S[\eta+\bar{\eta}] = S[\bar{\eta}] + \eta_I \frac{\delta S}{\delta \eta_I}[\bar{\eta}] + \frac{1}{2}\eta_I\eta_J \frac{\delta^2 S}{\delta\eta_I \delta\eta_J}[\bar{\eta}] + \frac{1}{3!}\eta_I\eta_J\eta_K \frac{\delta^3 S}{\delta\eta_I\delta\eta_J\delta\eta_K}[\bar{\eta}] + \frac{1}{4!}\eta_I\eta_J\eta_K\eta_L \frac{\delta^4 S}{\delta\eta_I\delta\eta_J\delta\eta_K\delta\eta_L}[\bar{\eta}]$$

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- **Quantum effective action**
  - Generating functional for 1PI diagrams
  - Legendre transformation $\quad \Gamma[\hat{\eta}] = W[\mathcal{J}] - \mathcal{J}_I\hat{\eta}_I, \qquad \hat{\eta}_I \equiv \dfrac{\delta W}{\delta \mathcal{J}_I}$

Quantum effective action

- **Up to $\mathcal{O}(\hbar^2)$ quantum effective action is expressed as**

Fuentes-Martín, Moreno-Sánchez, AP, Thomsen [2412.12270]

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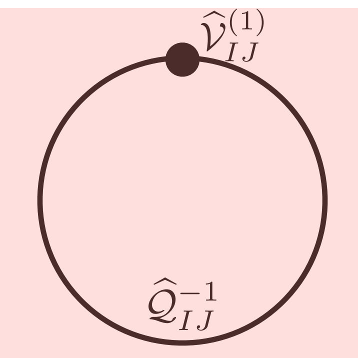
Counterterm topology



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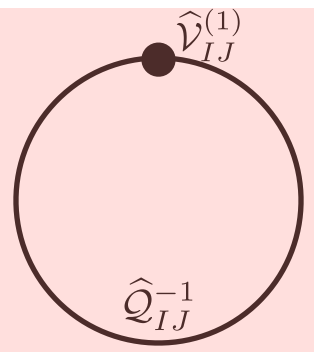

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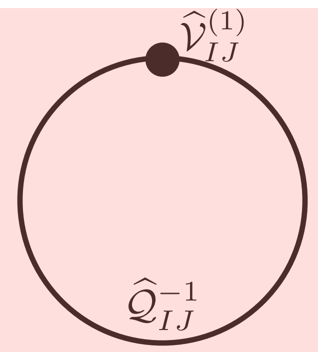

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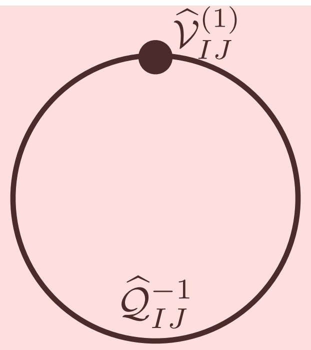

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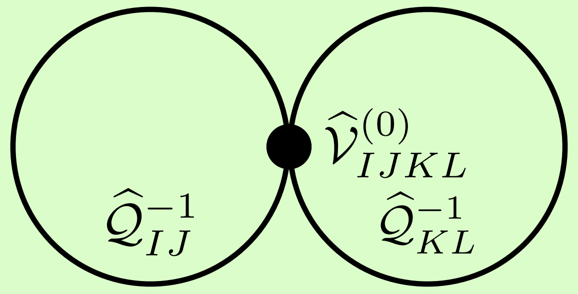
Figure-8 topology



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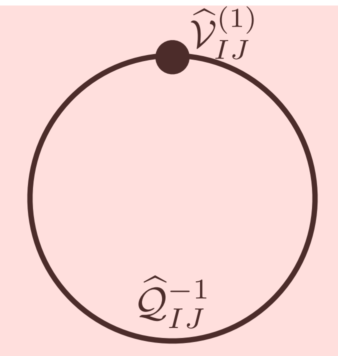
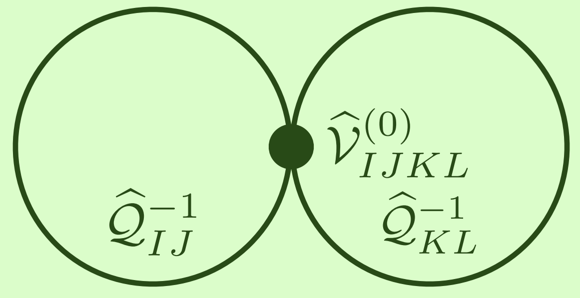
Counterterm topology



$$G_{\mathrm{ct.}} = \mathcal{Q}^{-1}_{IJ}\mathcal{V}^{(1)}_{JI} = \int_{x,y}\delta_{ba}(y,x)Q^{-1}_{ac}(x,\,P_x)V^{(1)}_{cd}(x,\,P_x)\delta_{db}(x,y)$$

Figure-8 topology



$$G_{\mathrm{f8.}} = \sum_{m,n,r}(-1)^n \int_x \int_{k\ell} V^{(\underline{m},\underline{n},\underline{r})}_{abcd}(x)\left[Q^{-1}_{ae}(x,\,P_x+k)(P_x+k)^{\underline{m}}U_{eb}(x,y)\right]_{y=x} \times \left[(P_x+\ell)^{\overleftarrow{\underline{n}}}Q^{-1}_{cf}(x,P_x+\ell)(P_x+\ell)^{\underline{r}}U_{fd}(x,z)\right]_{z=x}$$

Quantum effective action

- **Extraction of the relevant two-loop topologies**

Fuentes-Martín, Moreno-Sánchez, AP, Thomsen [2412.12270]

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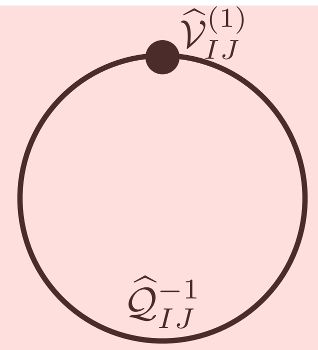

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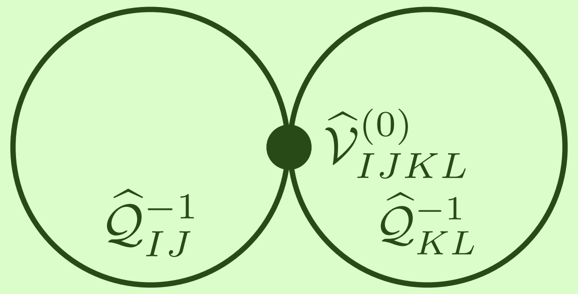

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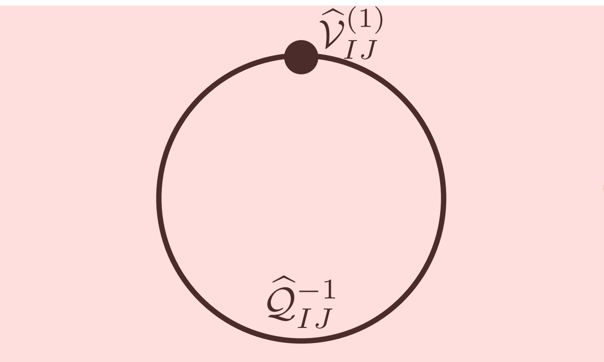

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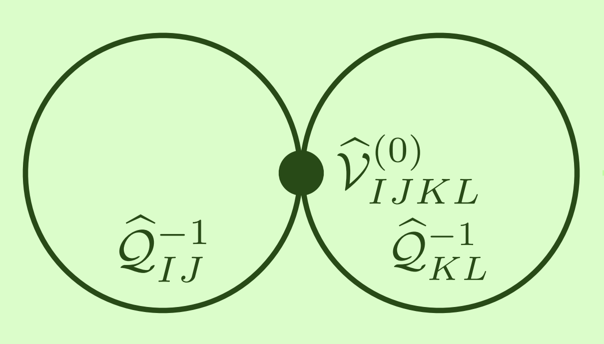

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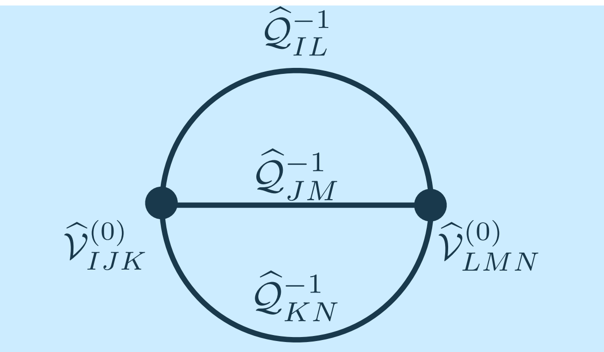
Sunset topology



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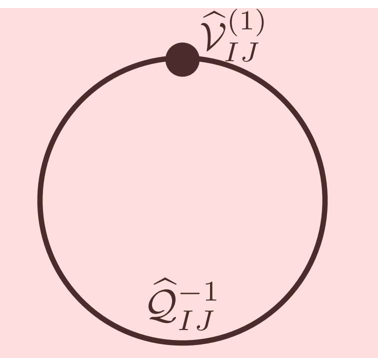

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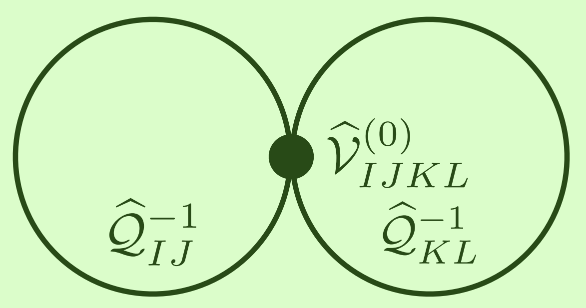

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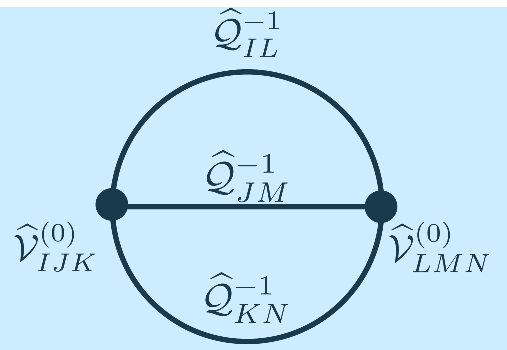

$$G_{\mathrm{ss.}} = \sum_{m,n,m',n'}(-1)^{m+n}\int_x\int_{k\ell}V^{(\underline{m},\underline{n})}_{abc}(x)Q^{-1}_{aa'}(y,\,P_y-k-\ell)V^{(\underline{m}',\underline{n}')}_{a'b'c'}(y)$$
$$\times\left[(P_x+k)^{\underleftarrow{m}}Q^{-1}_{be}(x,\,P_x+k)(P_x+k)^{\underline{m}'}U_{eb'}(x,y)\right]$$
$$\times\left[(P_x+\ell)^{\underleftarrow{n}}Q^{-1}_{cf}(x,\,P_x+\ell)(P_x+\ell)^{\underline{n}'}U_{fc'}(x,y)\right]\Big|_{y=x}$$

Applications

Applications: Scalar toy model

- **Model involving one heavy (Φ) and one light scalar (ϕ)**

$$\mathcal{L}_{\rm UV} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} M^2 \Phi^2 - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_\Phi}{4!}\Phi^4 - \frac{\lambda_{\Phi\phi}}{4}\Phi^2\phi^2 - \frac{\kappa}{3!}\Phi^3 + \mathcal{L}_{\rm UV}^{\rm ct}$$

Fuentes-Martín, AP, Thomsen [2311.13630]

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Fuentes-Martín, AP, Thomsen [2311.13630]

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$$\begin{aligned}
\delta_\phi^{(1)} &= \delta_\Phi^{(1)} = 0\,, & \delta_\kappa^{(1)} &= \frac{3\lambda_\Phi\kappa}{2\epsilon}\,,\\
\delta_{m_\phi}^{(1)} &= \frac{\lambda_\phi m_\phi^2 + \lambda_{\Phi\phi}M^2}{2\epsilon}\,, & \delta_{\lambda_\Phi}^{(1)} &= \frac{3\lambda_\Phi^2 + 3\lambda_{\Phi\phi}^2}{2\epsilon}\,,\\
\delta_M^{(1)} &= \frac{\lambda_\Phi M^2 + \lambda_{\Phi\phi}m_\phi^2 + \kappa^2}{2\epsilon}\,, & \delta_{\lambda_\phi}^{(1)} &= \frac{3\lambda_\phi^2 + 3\lambda_{\Phi\phi}^2}{2\epsilon}\,,\\
\delta_{\lambda_{\phi\Phi}}^{(1)} &= \frac{\lambda_\phi\lambda_{\Phi\phi} + \lambda_\Phi\lambda_{\Phi\phi} + 4\lambda_{\Phi\phi}^2}{2\epsilon}\,, & \delta_{\kappa'}^{(1)} &= \frac{\lambda_{\Phi\phi}\kappa}{2\epsilon}\,,\\
\delta_{\kappa_T}^{(1)} &= \frac{M^2\kappa}{2}\left(\frac{1}{\epsilon} - \overline{\ln}\,M^2 + 1\right)
\end{aligned}$$

Fuentes-Martín, AP, Thomsen [2311.13630]

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$$\begin{aligned} V^{(1)}_{\Phi\Phi} &= -\delta^{(1)}_M - \frac{\delta^{(1)}_{\lambda_{\Phi\phi}}}{2}\phi^2\,, \\ V^{(1)}_{\Phi\phi} &= -\delta^{(1)}_{\kappa_{\Phi\phi}}\phi\,, \\ V^{(1)}_{\phi\phi} &= -\delta^{(1)}_{m_\phi} - \frac{\delta^{(1)}_{\lambda_\phi}}{2}\phi^2 \end{aligned}$$

$$\begin{aligned} V_{\Phi\Phi\Phi\Phi} &= -\lambda_\Phi\,, \\ V_{\Phi\Phi\phi\phi} &= -\lambda_{\Phi\phi}\,, \\ V_{\phi\phi\phi\phi} &= -\lambda_\phi\,, \\ V_{\Phi\Phi\Phi\phi} &= V_{\Phi\phi\phi\phi} = 0 \end{aligned}$$

$$\begin{aligned} V_{\Phi\Phi\Phi} &= -\kappa\,, \\ V_{\Phi\Phi\phi} &= -\lambda_{\Phi\phi}\phi\,, \\ V_{\phi\phi\phi} &= -\lambda_\phi\phi\,, \\ V_{\Phi\phi\phi} &= 0 \end{aligned}$$

Fuentes-Martín, AP, Thomsen [2311.13630]

Applications: Scalar toy model

Fuentes-Martín, AP, Thomsen [2311.13630]

- **Matching at two-loop level**

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{c_6}{6!}\phi^6 + \mathcal{L}_{\mathrm{EFT}}^{\mathrm{ct}}$$

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$$\begin{aligned} m^2 = m_\phi^2 - \frac{\lambda_{\Phi\phi}}{32\pi^2}\left(1 - \overline{\ln}\, M^2\right) M^2 + \frac{1}{(16\pi^2)^2}\bigg[&\left(5 - 4\,\overline{\ln}\, M^2 + \overline{\ln}^{\,2} M^2\right)\frac{\lambda_{\Phi\phi}^2}{2} M^2 - \left(\overline{\ln}\, M^2 - \overline{\ln}^{\,2} M^2\right)\frac{\lambda_\Phi \lambda_{\Phi\phi}}{4} M^2 \\ &+ \left(1 - 2\sqrt{3}\,\mathrm{Cl}_2 - 2\,\overline{\ln}\, M^2 + \overline{\ln}^{\,2} M^2\right)\frac{\lambda_{\Phi\phi}}{8}\kappa^2 - \left(\frac{11}{4} + \overline{\ln}\, M^2 + \overline{\ln}^{\,2} M^2\right)\frac{\lambda_{\Phi\phi}^2}{4} m_\phi^2 + \left(5 + 6\,\overline{\ln}\, M^2\right)\frac{\lambda_{\Phi\phi}^2 m_\phi^4}{36 M^2}\bigg] \end{aligned}$$

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Fuentes-Martín, AP, Thomsen [2311.13630]

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- **EFT RGEs at two-loop order**

Applications: Scalar toy model

Fuentes-Martín, AP, Thomsen [2311.13630]

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$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{c_6}{6!}\phi^6 + \mathcal{L}_{\mathrm{EFT}}^{\mathrm{ct}}$$

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- **EFT RGEs at two-loop order**

$$\begin{aligned} \delta^{(2)}_{m^2,1} &= -\frac{\lambda^2 m^2}{4}, & \delta^{(2)}_{m^2,2} &= \frac{4m^2\lambda^2 + m^4 c_6}{8}, \\ \delta^{(2)}_{\lambda,1} &= -\frac{9\lambda^3 + 5m^2\lambda c_6}{6}, & \delta^{(2)}_{\lambda,2} &= \frac{9\lambda^3 + 11m^2\lambda c_6}{4}, \\ \delta^{(2)}_{c_6,1} &= -\frac{215\lambda^2 c_6}{12}, & \delta^{(2)}_{c_6,2} &= \frac{135\lambda^2 c_6}{4}, \\ \delta^{(2)}_{\phi,1} &= -\frac{\lambda^2}{24}, & \delta^{(2)}_{\phi,2} &= 0 \end{aligned}$$

$$\begin{aligned} \gamma_\phi &= \frac{1}{2}\frac{\mathrm{d}\ln(1+\delta_\phi)}{\mathrm{d}\ln\mu} = -\frac{\partial \delta^{(2)}_{\phi,1}}{\partial\lambda} = \frac{1}{(16\pi^2)^2}\frac{\lambda^2}{12}, \\ \beta_{m^2} &= \frac{m^2\lambda}{16\pi^2} - \frac{5}{6}\frac{m^2\lambda^2}{(16\pi^2)^2}, \\ \beta_\lambda &= \frac{3\lambda^2 + m^2 c_6}{16\pi^2} - \frac{1}{3}\frac{17\lambda^3 + 10\lambda m^2 c_6}{(16\pi^2)^2}, \\ \beta_{c_6} &= \frac{15\lambda c_6}{16\pi^2} - \frac{427}{6}\frac{\lambda^2 c_6}{(16\pi^2)^2} \end{aligned}$$

$\rightarrow$ **RGBeta** [2101.08265]

Applications: Euler-Heisenberg Lagrangian

Fuentes-Martín, Moreno-Sánchez, AP, Thomsen [2412.12270]

- **Matching QED onto theory with self-interacting photons**

$$\mathcal{L}_{\rm QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \mathcal{L}_{\rm ct}.$$

Applications: Euler-Heisenberg Lagrangian

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$$V^{(1)}_{A_\mu A_\nu} = -g^{\mu\nu}\delta^{(1)}_F P_x^2\,, \qquad V^{(1)}_{\psi\psi^c} = V^{(1)}_{\psi^c\psi} = C\left(\delta^{(1)}_\psi \not{P}_x - \delta^{(1)}_m\right), \qquad V_{A_\mu\psi\psi^c} = -V_{A_\mu\psi^c\psi} = e\,C\gamma^\mu$$

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- **Full two-loop matching result**

$$\begin{aligned}\mathcal{L}_{\text{EFT}} = &-\frac{1}{4}\left[1 + \frac{e^2}{16\pi^2}\frac{4}{3}\log\frac{\mu^2}{m^2} + \frac{e^4}{(16\pi^2)^2}\left(\frac{13}{3} - 4\log\frac{\mu^2}{m^2}\right)\right]F^{\mu\nu}F_{\mu\nu}\\ &+\left[\frac{1}{16\pi^2}\frac{7}{90}\frac{e^4}{m^4} + \frac{1}{(16\pi^2)^2}\frac{e^6}{m^4}\left(\frac{247}{810} - \frac{14}{15}\log\frac{\mu^2}{m^2}\right)\right](F^{\mu\nu}F_{\nu\sigma})(F^{\rho\sigma}F_{\mu\rho})\\ &-\left[\frac{1}{16\pi^2}\frac{1}{36}\frac{e^4}{m^4} + \frac{1}{(16\pi^2)^2}\frac{e^6}{m^4}\left(\frac{49}{324} - \frac{1}{3}\log\frac{\mu^2}{m^2}\right)\right](F^{\mu\nu}F_{\mu\nu})^2\,.\end{aligned}$$

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Summary

- **Necessity of performing computations beyond one-loop order**
- **Extension of the well-established functional formalism beyond one-loop order**
- **Extraction of relevant topologies in the effective action**
- **Analysis of two simple examples**
- **Avenues for future work**
  - Implementation of γ_5
  - More applications: SMEFT, LEFT

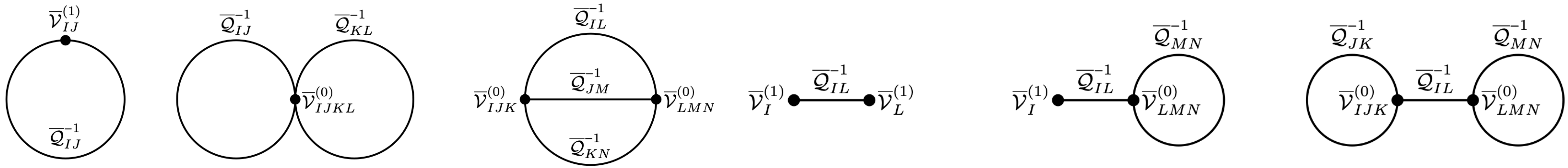
Backup slides

Generating functional

- **Expanded version of generating functional**

$$
\begin{aligned}
W[\mathcal{J}] = {} & \overline{S}^{(0)} + \mathcal{J}_I \overline{\eta}_I + \hbar\,\overline{S}^{(1)} + \frac{i\hbar}{2}\mathrm{STr}\log\overline{\mathcal{Q}} + \hbar^2\overline{S}^{(2)} + \frac{i\hbar^2}{2}\zeta_{II}\overline{\mathcal{Q}}^{-1}_{IJ}\overline{\mathcal{V}}^{(1)}_{JI} \\
& + \frac{\hbar^2}{12}\zeta_{IN}\zeta_{IM}\zeta_{JN}\,\overline{\mathcal{Q}}^{-1}_{LI}\overline{\mathcal{Q}}^{-1}_{MJ}\overline{\mathcal{Q}}^{-1}_{NK}\overline{\mathcal{V}}^{(0)}_{IJK}\overline{\mathcal{V}}^{(0)}_{LMN} - \frac{\hbar^2}{8}\overline{\mathcal{Q}}^{-1}_{IJ}\overline{\mathcal{Q}}^{-1}_{KL}\overline{\mathcal{V}}^{(0)}_{IJKL} \\
& - \frac{\hbar^2}{2}\zeta_{II}\left(\overline{\mathcal{V}}^{(1)}_{I} + \frac{i}{2}\overline{\mathcal{Q}}^{-1}_{JK}\overline{\mathcal{V}}^{(0)}_{IJK}\right)\overline{\mathcal{Q}}^{-1}_{IL}\left(\overline{\mathcal{V}}^{(1)}_{L} + \frac{i}{2}\overline{\mathcal{Q}}^{-1}_{MN}\overline{\mathcal{V}}^{(0)}_{LMN}\right) + \mathcal{O}(\hbar^3)
\end{aligned}
$$

- **Topologies**



Gauge fixing

- **Background field gauge**

$$\Gamma[\underline{\eta},\, \hat{\eta}] = -i \log \int [\mathcal{D}\eta][\mathcal{D}\boldsymbol{\omega}]\, \exp\left[i \left(S[\hat{\eta}+\eta] + S^G_{\text{fix}}[\eta,\, \boldsymbol{\omega},\, \hat{\eta}] + \int_x \mathcal{J}_I(\eta_I - \underline{\eta}_I) \right) \right]$$

- **Faddeev-Popov procedure**

$$S^G_{\text{fix}}[\eta, \boldsymbol{\omega}, \hat{\eta}] = -\frac{1}{2g^2\xi}\left(\widehat{D}^\mu A^A_\mu\right)^2 - \overline{\omega}_A \widehat{D}^\mu \left(\widehat{D}_\mu \omega^A + f^A{}_{BC} A^B_\mu \omega^C\right)$$

- **Action**

$$S[\hat{\eta}+\eta] + S^G_{\text{fix}}[\eta, \boldsymbol{\omega}, \hat{\eta}] = \widehat{S} + \sum_{\ell=0}^{\infty} \left(\Omega_I \widehat{\mathcal{V}}^{(\ell)}_I + \frac{1}{2} \Omega_I \widehat{\mathcal{V}}^{(\ell)}_{IJ} \Omega_J + \sum_{n=3}^{\infty} \frac{1}{n!} \Omega_{I_1} \ldots \Omega_{I_n} \widehat{\mathcal{V}}^{(\ell)}_{I_1 \ldots I_n} \right)$$

Gauge connection and integrals

- **Parallel transport**

$$f_a(x) = [U_\gamma(x,\, y)]_{ab}\, f_b(y)\,, \qquad U_\gamma(x,\, y) = \mathscr{P}\exp\left[i\int_\gamma \mathrm{d}z^\mu \widehat{A}_\mu(z)\right]$$

- **Covariant evaluation of integrals**

$$\mathcal{A}_{IJ}\mathcal{B}_{JK} = \int_y A_{ad}(x, P_x)\delta_{db}(x, y)B_{bf}(y, P_y)\delta_{fc}(y, z) = A_{ab}(x, P_x)B_{bd}(x, P_x)\delta_{dc}(x, z)$$

- **Covariant evaluation of the log term**

$$(\log \mathcal{A})_{IJ} = -\sum_{n=1}^{\infty}\frac{1}{n}(\mathcal{I}-\mathcal{A})^n_{IJ} = -\sum_{n=1}^{\infty}\frac{1}{n}\big(\mathbb{1} - A(x, P_x)\big)^n_{ac}\delta_{cb}(x, y) = \big(\log A(x, P_x)\big)_{ac}\delta_{cb}(x, y)$$